Camparison of Three Break Point Tests: Chow Test, Andrew Test and Hansen Test

by

Lingfeng Li, Shaowen-Lai, Yutao Chen

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Chapter 1

Introduction

When we want to use a model to describe the relations among of a set of data during a given period, for example, linear regression model, one thing we may concern is if the structure of model remains unchanged. If there is a significant break in model structure, fitting the model with data of the whole period is not a good idea. Thus, the test of the existence of structural break become important. Many people have come up with different kinds of break tests. We will discuss three different tests in this paper: Chow (1960) test, Andrew (1993) test and Hansen (1992) test. Chow test is the earliest break test, and the idea is running OLS regression on the data before and after a given time and using the SSR of the restricted model and unrestricted model to construct F-statistic. However, the Chow test require a given break time. If we do not know the exact break time and choose a wrong break time to do this test, we may have low probability to reject the null hypothesis. Andrew test is a kind of extension to Chow test. It takes the maximum of Chow test's F-statistic among a period of time and uses this maximum as a statistic. This method does not require specifying the break time. What's more, we may not only concern about the stability of parameters, but also the variance of disturbance term. Hansen test can handle this well. We can do the test on each parameter or the variance of error term individually. We also can do a joint test of all the parameters and the variance. In the following sections, we will conduct tests with simulated data to see if these three tests can identify the existence of structural break correctly.

Chapter 2

Description of Different Tests

2.1 Chow Test

We consider the OLS model:

$$Y_t = X_t\beta_t + \epsilon_t \quad (t = 1, 2, \dots, T) \tag{2.1}$$

Where $X_t = \begin{pmatrix} X_{1,t} & X_{2,t} & \dots & X_{m,t} \end{pmatrix}$, $\beta_t = \begin{pmatrix} \beta_1 & \beta_2 & \dots & \beta_m \end{pmatrix}^T$ and $\epsilon_t \sim N(0, \sigma^2)$. Chow test also assume that the data is stationary, i.e (X_t, ϵ_t) are i.i.d distributed.

We want to test if the parameters β_t are constants during time period $[1, T]$, so the Null hypothesis is:

$$H_0 : \beta_t = \beta_0 \ \textit{are constants}$$

Ro construct a statistic, first we should partition the Data into 2 parts $T_1 = 1, 2, \dots, \tau$ and $T_2 = \tau + 1, \dots, T$, where τ is a given date. Then, we can run regression on these 2 time period and compute the sum of square residual SSR_{T_1} and SSR_{T_2}. So the SSR under unrestricted model is $SSR_{un} = SSR_{T_1} + SSR_{T_2}$. We also need to run OLS on the entire time period T to compute the SSR under restricted model SSR_{res}. The F statistic is constructed by:

$$F = \frac{(SSR_{res} - SSR_{un})/m}{SSR_{un}/(T - 2m)}$$

Under restricted model, the DOF is m and under unrestricted model, the DOF is $2m$. The F follow a $F(m, T - 2m)$ distribution when H_0 holds.

2.2 Andrew Test

Andrew test can be viewed as an extension to Chow test. For Chow test, the break time should be given. But when the break time is unknown, we can do Andrew test. The idea is picking the maximum F-statistic during a given time interval:

$$A_T = \underset{\tau_1 \leq \tau \leq \tau_2}{SUP} F(\tau)$$

Where $F(\tau)$ is the F-statistic computed in Chow test with given possible break time τ. The critical value could be found in Andrew(1993).

2.3 Hansen Test

We also consider OLS model (2.1) here, but the $\epsilon_t \sim N(0, \sigma_t^2)$ where σ_t may not be a constant. The first order condition for OLS is:

$$\sum_{t=1}^{T} x_{i,t}\hat{e}_t = 0 \quad (i = 1, 2, \ldots, m)$$
$$\sum_{t=1}^{T} (\hat{e}_t^2 - \hat{\sigma}^2) = 0$$

Let $f_{i,j} = x_{i,j}\hat{e}_j$ $(i = 1, 2, \ldots, m)$ and $f_{m+1,j} = \hat{e}_t^2 - \hat{\sigma}^2$, then the first order condition can be written as $\sum_{t=1}^{T} f_{i,t} = 0$ $(i = 1, 2, \ldots, m+1)$. Then we can do a CUSUM type of test. Let $S_{i,t} = \sum_{j=1}^{t} f_{i,j}$, then the statistics are:

$$L_i = \frac{1}{TV_i}\sum_{t=1}^{T} S_{i,t}^2 \quad (i = 1, 2, \ldots, m+1)$$
$$V_i = \sum_{t=1}^{T} f_{i,t}^2$$
$$L = \frac{1}{T}\sum_{t=1}^{T} \mathbf{S_t}'\mathbf{V^{-1}}\mathbf{S_t}$$
$$\mathbf{V} = \sum_{t=1}^{T} \mathbf{f_t}(\mathbf{f_t})'$$
$$\mathbf{S_t} = (S_{1,t}, S_{2,t}, \ldots, S_{m+1,t})'$$
$$\mathbf{f_t} = (f_{1,t}, f(2,t), \ldots, f_{m+1,t})'$$

L_i is the statistic for testing whether β_i or σ_t $(i = m+1)$ is constant, and L is the statistic for testing if all the parameters are constant jointly. The critical value of L_i

TABLE 1: ASYMPTOTIC CRITICAL VALUES FOR L_C[3]

Degrees of Freedom (k_1)	Significance Level 1%	2.5%	5%	7.5%	10%	20%
1	.748	.593	.470	.398	.353	.243
2	1.07	.898	.749	.670	.610	.469
3	1.35	1.16	1.01	.913	.846	.679
4	1.60	1.39	1.24	1.14	1.07	.883
5	1.88	1.63	1.47	1.36	1.28	1.08
6	2.12	1.89	1.68	1.58	1.49	1.28
7	2.35	2.10	1.90	1.78	1.69	1.46
8	2.59	2.33	2.11	1.99	1.89	1.66
9	2.82	2.55	2.32	2.19	2.10	1.85
10	3.05	2.76	2.54	2.40	2.29	2.03
11	3.27	2.99	2.75	2.60	2.49	2.22
12	3.51	3.18	2.96	2.81	2.69	2.41
13	3.69	3.39	3.15	3.00	2.89	2.59
14	3.90	3.60	3.34	3.19	3.08	2.77
15	4.07	3.81	3.54	3.38	3.26	2.95
16	4.30	4.01	3.75	3.58	3.46	3.14
17	4.51	4.21	3.95	3.77	3.64	3.32
18	4.73	4.40	4.14	3.96	3.83	3.50
19	4.92	4.60	4.33	4.16	4.03	3.69
20	5.13	4.79	4.52	4.36	4.22	3.86

FIGURE 2.1: Table of Critical Values(Hansen(1990), Table 1)

and L could be found at figure 2.1. For L, the degree of freedom is $m+1$ and for each L_i, the degree of freedom is 1.

Chapter 3

Testing the Methods With Simulated Data and Market Data

3.1 Chow Test

3.1.1 Test with Linear Regression Model

The test data is generated from a linear regression model:

$$Y_t = X_t\beta_1 + \epsilon_t \qquad t = 1, 2, \ldots, \frac{3T}{10}$$
$$Y_t = X_t\beta_2 + \epsilon_t \qquad t = \frac{3T}{10} + 1, \ldots, T$$

Where $X_t = (1, x_{1,t}, x_{2,t})$ is i.i.d and $x_{i,t} \sim unif[0,1]$, $\epsilon_t \sim N(0,1)$ and is also i,i,d, $\beta_1 = (0.2, 1, 0.5)$, $\beta_2 = (0.4, 0.5, 0.35)$ and $T = 1000$.

We simulated 500 times and compute the probability of rejecting the H_0 with 5% significance level when choosing a different estimated break point. The result is in table 3.1. From the results of simulation, we can find that when the estimated break point g0 closer to $0.3T$, the probability of rejecting H_0 go higher, which is reasonable because the structure

Table 3.1: Results of Chow Test

The estimated break point	The probability of reject
0.7	0.236
0.5	0.566
0.4	0.69
0.35	0.844
0.3	0.924

Table 3.2: Results of Chow Test

The estimated break point	The probability of reject
0.7	0.354
0.5	0.688
0.4	0.830
0.35	0.888
0.3	0.92

3.1.2 Test with ARIMA Model

Data is generated by model:

$$
\begin{aligned}
&Y_t = a_{11} + a_{12}Y_{t-1} + b_1\epsilon_{t-1} + \epsilon_t \qquad t = 1, 2, \ldots, \frac{3T}{10}\\
&Y_t = a_{21} + a_{22}Y_{t-1} + b_2\epsilon_{t-1} + \epsilon_t \qquad t = \frac{3T}{10} + 1, \ldots, T\\
&Y_0 = 1, \epsilon_0 = 0
\end{aligned}
$$

Where $\epsilon \sim N(0,1)$, $a11 = 0.2$, $a12 = 0.8$, $b_1 = 0.5$, $a21 = 0.22$, $a22 = 0.68$, $b_2 = 0.45$ and $T = 1000$.

We also simulated 500 times and compute the probability of rejecting the H_0 with 5% significance level when choosing a different estimated break point. The result is in table 3.2. We can see the probability of rejecting the H_0 is lower when estimated break point is larger.

3.2 Andrew Test

3.2.1 Generate Data with Fixed Break Time

We generated the data from the same linear regression model and ARIMA model with the Chow Test. The break time is fixed at $t = 0.3T$.

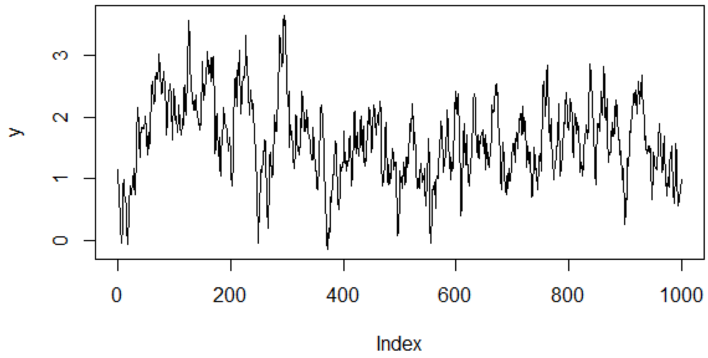

FIGURE 3.1: Figure of a ARIMA Model Sample Path

In the linear regression model, Andrew test can figure out the where the break point is. The estimated location of the break point is close to the true value. The Monte Carlo test shows that we have 83.2 percentage to reject the Null hypothesis in 500 sample paths.

The figure 3.1 shows a sample path of the Y we generated with ARIMA model. As we can see, the fluctuation of the path is larger from $t = 0$ to $t = 0.3$ and the fluctuation becomes relatively smaller form 0.3 to 1.

We also conduct a Monte Carlo test. The result shows that we have 92.4 percentage to reject the Null hypothesis in the 500 sample path. Like a supremum of a series of Chow test, Andrew test is as accurate as Chow test with a good estimated break point. With the same parameters, Chow test have 92 percentage to reject the Null hypothesis while Andrew test have 92.4 percentage. The result is as what we expect.

3.2.2 Simulate Break With Poisson Process

Since Andrew test does not require specifying the break time, it should be able to apply on stochastic break time model. Therefore, we use a Poisson process to simulate the break time to see if the Andrew test can identify the break point correctly. It is natural to use a Poisson process to simulate break points because in real world, the structural break may happen randomly.

The data is generated from model:

$$Y_t = X_t(\beta \cdot \Pi_{i=1}^{N(t)} \alpha) + \epsilon_t$$

Where $N(t)$ is counting process, $\beta = (0.2, 0.9, 0.5)$ and $\alpha = (1.1, 0.95, 0.95)$. Each time the Poisson process jump, the coefficients of the model are multiplied by a constant vector α (The multiplication here means dot product).

The result shows that we have 89.6 percentage to reject the Null hypothesis in the 500 sample path.

Test the data set under ARIMA model with Poisson process:

$$Y_t = a_1 \Pi_{i=1}^{N(t)} \alpha_1 + a_2 \Pi_{i=1}^{N(t)} \alpha_2 Y_{t-1} + b_1 \Pi_{i=1}^{N(t)} \beta_1 \epsilon_{t-1} + \epsilon_t$$

Where $N(t)$ is counting process, $(a_1, a_2, b_1) = (0.2, 0.9, 0.5)$ and $(\alpha_1, \alpha_2, \beta_1) = (1.1, 0.95, 0.95)$. Each time the Poisson process jump, the coefficients of the model are multiplied by a constant vector α (The multiplication here means dot product).

The result shows that we have 0.976 percentage to reject the Null hypothesis in the 500 sample path.

3.3 Hansen Test

3.3.1 Simulate the break point of variance

Hansen test can detect the break point of the variance of the data set. We want to test this ability. Set all the parameters of the model as a constant, then we generate the error term with a standard normal distribution before the break point and change it into a normal distribution with higher variation after the break point. Then we test this model with the Hansen test:

The test data is generated from a linear regression model:

$$\begin{aligned} Y_t &= X_t \beta + \epsilon_t^1 \quad t = 1, 2, \ldots, \frac{3T}{10} \\ Y_t &= X_t \beta + \epsilon_t^2 \quad t = \frac{3T}{10} + 1, \ldots, T \end{aligned}$$

Where $X_t = (1, x_{1,t}, x_{2,t})$ is i.i.d and $x_{i,t} \sim unif[0, 1]$, $\epsilon_t^1 \sim N(0, 1)$, $\epsilon_t^1 \sim N(0, 2)$ and is also i,i,d, $\beta = (0.2, 1, 0.5)$, and $T = 1000$.

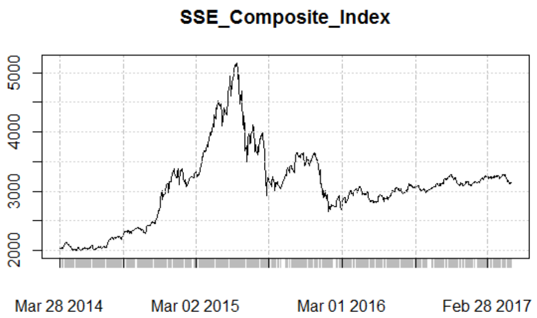

FIGURE 3.2: Figure of the SSE Composite Index

TABLE 3.3: Results of Chow Test

The estimated break point	P Value
0.407	$1.21E-02$
0.42	0.1067629
0.45	0.2136999
0.35	0.1234726

Monte Carlo result show thats we reject the Null hypothesis in 88.8% of 500 sample paths. This result illustrate that Hansen test is very sensitive to the change of the variance in the model.

3.4 Test with market data

The data we applied here is the Shanghai Stock Exchange Composite Index from 2014/03/28 to 2017/04/28. The figure of the data is shown in the Figure3.2. First, we used the ADF test to test the stationary of the Data. The original data is not stationary, so we use the first difference of logarithm of orginal data.

We need to choose an estimated break point in the Chow test. After observing the orginal data, we estimate that there is a break point around the 2015/06.

The result of the Chow test with different estimated break point is in table 3.3. The p value of Andrew test is less than 0.005. The p value of Hansen test is 0.0422.

These results are close to what we expected because around the date 2015/06 there was a crisis in Chinese stock market. The parameters of the ARIMA model changed a lot because the environment of the stock market totally changed. The stock price increased dramatically in the first half year in 2015 and then it undergone a huge drop in the later 2015.

Appendix A

An Appendix

Code of Chow test

```
n <- 1000
a1=0.2;
a2=0.95;
b1=0.5;
p=0;
parameters<-t(c(a1,a2,1,b1))
chow_result=0
for(k in 1:500)
{
    parameters<-t(c(a1,a2,1,b1))
    z=rnorm(n, mean = 0, sd = 0.2)
    y<-c(parameters%*%c(1,1,z[1],0))
    for(i in 2:n)
    {
        y[i]=parameters%*%c(1,y[i-1],z[i],z[i-1])
    }
    parameters<-t(c(a1*1.1,a2*0.97,1,b1*0.95))
    bp=300
    for(i in bp:n)
    {
        y[i]=parameters%*%c(1,y[i-1],z[i],z[i-1])
    }
    #"Nyblom-Hansen""Chow"
    r<-sctest(y[2:1000]~1+y[1:999]+z[1:999], type ="Chow", h 0.15,alt.boundary = FALSE,
    + functional = c("max", "range","maxL2", "meanL2"), from = 0.15,to = NULL, point=0.7,
    + asymptotic = FALSE)
    if (r$p.value<0.05) chow_result=chow_result+1
    p[k]=r$p.value
}
chow_result/500
```

Code of Andrew test

```
n <- 1000
a1=0.2;
a2=0.95;
b1=0.5;
p=0;
parameters<-t(c(a1,a2,1,b1))
chow_result=0
for(k in 1:500)
{
    parameters<-t(c(a1,a2,1,b1))
    z=rnorm(n, mean = 0, sd = 0.2)
    y<-c(parameters%*%c(1,1,z[1],0))
    for(i in 2:n)
    {
        y[i]=parameters%*%c(1,y[i-1],z[i],z[i-1])
    }
    parameters<-t(c(a1*1.1,a2*0.97,1,b1*0.95))
    bp=300
    for(i in bp:n)
    {
        y[i]=parameters%*%c(1,y[i-1],z[i],z[i-1])
    }
}

scus.seat <- gefp(y[2:1000]~1+y[1:999]+z[1:999])
sctest(scus.seat, functional = supLM(from=0.2))
plot(scus.seat, functional = supLM(from=0.2))
plot(y,type='l')
```

Code of Hansen test

```
n <- 1000
a1=0.2;
a2=1;
b1=0.5;
parameters<-t(c(a1,a2,1,b1))
x=matrix(runif(n*3),n,3)
y<-0
result=0
for(k in 1:500)
{
  parameters<-c(a1,a2,b1,1)
  z=rnorm(n)
  xx<-cbind(x[1:1000,1],x[1:1000,2],x[1:1000,3],z[1:1000])
  y[1:1000]<-c(xx%*%parameters)
  parameters<-c(a1,a2,b1,sqrt(2))
  bp=300
  xx<-cbind(x[bp:1000,1],x[bp:1000,2],x[bp:1000,3],z[bp:1000])
  y[bp:1000]<-c(xx%*%parameters)
  r<-sctest(y~x[1:1000,1]+x[1:1000,2]+x[1:1000,3], type ="Nyblom-Hansen",
  h = 0.15,alt.boundary = FALSE, functional = c("max", "range","maxL2", "meanL2"),
  from = 0.15,to = NULL, asymptotic = FALSE)
  if (r$p.value<0.10) result=result+1
}
result/500
```

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