

## Article

# Scaling-Up Quantum Heat Engines Efficiently via Shortcuts to Adiabaticity

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**Abstract:** The finite-time operation of a quantum heat engine that uses a single particle as a working medium generally increases the output power at the expense of inducing friction that lowers the cycle efficiency. We propose to scale up a quantum heat engine utilizing a many-particle working medium in combination with the use of shortcuts to adiabaticity to boost the nonadiabatic performance by eliminating quantum friction and reducing the cycle time. To this end, we first analyze the finite-time thermodynamics of a quantum Otto cycle implemented with a quantum fluid confined in a time-dependent harmonic trap. We show that nonadiabatic effects can be controlled and tailored to match the adiabatic performance using a variety of shortcuts to adiabaticity. As a result, the nonadiabatic dynamics of the scaled-up many-particle quantum heat engine exhibits no friction, and the cycle can be run at maximum efficiency with a tunable output power. We demonstrate our results with a working medium consisting of particles with inverse-square pairwise interactions that includes non-interacting and hard-core bosons as limiting cases.

**Keywords:** quantum thermodynamics; shortcut to adiabaticity; interacting Bose gas

## 1. Introduction

Quantum thermodynamics resembles a fruitful crucible of research fields where the foundations of physics, information science and statistical mechanics merge [1,2]. It is further spurred by the development of quantum technologies that have facilitated the realization and control of thermal machines and related devices exhibiting quantum dynamics. A prominent example is that of quantum heat engines (QHE) that transform thermal energy into mechanical work.

In both classical and quantum domains, the performance of heat engines is characterized by the efficiency and power of the cycle. Studies to date have been limited to the optimization of thermodynamic cycles operating with a single-particle working medium. After the pioneering works [3,4], the quantum Otto cycle has been analyzed in detail [5–12]. It is of relevance to current experimental efforts aimed at realizing a QHE with a single trapped ion [13–15]. Nonetheless, it is worth pointing out that a universal behavior emerges among different types of cycles in the limit of small action per cycle [16]. These works show that when a single-particle QHE is operated in a finite amount of time, nonadiabatic excitations act as quantum friction, reducing the efficiency of the engine. As a result, the maximum efficiency is achieved for long cycle times, in the adiabatic limit, when the output power of the QHE vanishes. This state of affairs is already present in classical heat engines and gave rise to the field of finite-time thermodynamics.

Scaling up a QHE arises as a natural strategy to compensate the tradeoff between efficiency and power. Yet, the analysis of QHE with a many-particle working medium remains essentially

Assuming the initial state to be at equilibrium so that  $\langle \{z_i, p_i\}(0) \rangle = 0$ , it follows that:

$$\langle \hat{H}_{\text{LCD}}(t) \rangle = \frac{Q_{\text{LCD}}^*(t)}{b_{\text{ad}}^2} \langle \hat{H}(0) \rangle. \quad (50)$$

Under LCD, the nonadiabatic factor along the process is given by:

$$Q_{\text{LCD}}^*(t) = 1 + \frac{\dot{\omega}^2}{\omega^4} = 1 + \frac{1}{4} \left( \frac{\ddot{\omega}}{\omega^3} - \frac{\dot{\omega}^2}{\omega^4} \right) \quad (51)$$

and reduces explicitly to unity at the beginning and end of the LCD protocol, provided  $\dot{\omega}(0) = \dot{\omega}(\tau) = 0$ . This is illustrated in Figure 2 for the driving protocol  $\omega(t)$  varying from  $\omega_1$ – $\omega_2$  and satisfying these boundary conditions together with  $\ddot{\omega}(0) = \ddot{\omega}(\tau) = 0$ . The explicit expression for  $\omega(t)$  resembles that for the scaling factor in Equation (38) and reads:

$$\omega(t) = \omega_1 + \alpha_3 t^3 + \alpha_4 t^4 + \alpha_5 t^5, \quad (52)$$

where  $\alpha_3 = 10(\omega_2 - \omega_1)/\tau^3$ ,  $\alpha_4 = -15(\omega_2 - \omega_1)/\tau^4$  and  $\alpha_5 = 6(\omega_2 - \omega_1)/\tau^5$ . The nonadiabatic nature of the STA designed by LCD becomes apparent in Figure 2a, where transient excitations are generated during the protocol and canceled out upon its completion. At variance with accidental STA, LCD protocols can be engineered for arbitrary small values of  $\tau$  provided that the modulation of the trap frequency  $\Omega(t)$  can be implemented; see Figure 2b.

#### 4. Discussion

Along the compression and expansion strokes of a quantum Otto cycle, the nonadiabatic dynamics of a many-particle working medium can be efficiently characterized by the nonadiabatic factor  $Q^*(t)$  whenever the dynamics is scale-invariant, as is the case for the family of models described by Hamiltonian (1). The adiabatic performance is matched whenever  $Q^*(\tau) = 1$  at the end of the compression and expansion strokes governed by unitary dynamics. We have seen that the condition  $Q^*(\tau) = 1$  can be fulfilled without resorting to adiabatic dynamics. In particular, a variety of STA can be exploited as an alternative. This possibility has prompted us to introduce two alternative schemes to boost the performance of a many-particle QHE. In both approaches, the fixed resources are given by the temperature of the hot and cold reservoirs, together with the particle number and the inter-particle interactions. One can opt for the optimization of the output power of the QHE run in finite-time  $\tau$  as a function of the frequency ratio of  $\omega_1/\omega_2$ . Under the resulting condition for maximum finite-time output power, varying the value of  $\tau$  for a given specific functional form of  $\omega(t)$  can lead to accidental STA protocols satisfying  $Q^*(\tau) = 1$ , for a set of discrete values  $\tau = \tau_n$  ( $n = 1, 2, 3, \dots$ ), as discussed in Section 3.1. As an alternative, one can directly opt for maximizing the output power of the QHE assuming zero friction ( $Q^*(\tau) = 1$ ) and then use STA in many-particle systems to consistently match the adiabatic performance and reach the maximum efficiency of the cycle. Such STA protocols can be engineered by a variety of techniques, including reverse engineering of the scale-invariant dynamics (Section 3.2.1), counterdiabatic driving (Section 3.2.2) and local counterdiabatic driving (Section 3.2.3). Both approaches lead to the operation of the many-particle QHE at maximum efficiency and optimal output power.

#### 5. Conclusions

Finite-time thermodynamics aims at optimizing the nonadiabatic performance of thermal machines, required for any realistic application. Conditions for optimal performance generally depend on the specific characteristics of the working medium, such as the number of particles and the interaction strength. At the quantum level, the required optimization involves tailoring thermal