Lecture 2

3.1-1

Let f(n) and g(n) be asymptotically nonnegative functions. Using the basic definition of Θ-notation, prove that $max(f(n), g(n)) = \Theta(f(n) + g(n))$**.**

trivially : 46

3.1-4

Is $2^{n+1} = O(2^n)$**)? Is** $2^{2n} = O(2^n)$**?**

$2^{n+1} = 2 * 2^n = O(2^n)$

$2^{2n} = 4^n$ IS NOT $O(2^n)$

2 By Getting rid of the asymtotically insignificant parts of the expressions, give a simplified asymtotic tight bounds (theta notation) for the following running times. Here $k >= 1, e > 0, c > 1$ **are constants**

- **1.** $0.001n^2 + 70000n$

  $\Theta(n2)$
- **2.** $2^n + n^{10000}$ $\theta(2^n)$
- **3.** $n^k + c^n$ $\theta(c^n)$ Because any exponential function dominates any polynomial function.
- **4.** $lg^k n + n^e$ $\theta(n^e)$
- **5.** $2^n + 2^{n/2}$ $\theta(2^n)$
- **6.** $nlg * c + c^{lg*n}$ $\theta(c^{lg*n}) = \theta(n^{lg*c})$ because $n^{lg*c} = c^{lg*n}$

Exercise 2.1 Consider the following algorithm:

```
DoSomething(n:int):int
A:int[1..n]
for i ← 1 to n
    do A[i] ← i
i ← n
while i > 1 do
    x ← A[1]
    for j ← 1 to n − 1
        do A[j] ← A[j + 1]
    A[n] ← x
    i ← ⌈i/2⌉
return A[1]
```

What is DoSomething(8)? What is the (asymptotic) running time of DoSomething?

Lines number	Algorithm	cost	times
1	$A : int[1..n]$	c1 = 0	
2	$for\ i \leftarrow 1\ to\ n$	c2	n
3	$do\ A[i] \leftarrow i$	c3	n-1
4	$i \leftarrow n$	c4 = 0	
5	$while\ i > 1\ do$	c5	$\sum_{j=2}^{n} t_j$
6	$x \leftarrow A[1]$	c6	$\sum_{j=2}^{n} t_j - 1$
7	$for\ j \leftarrow 1\ to\ n-1$	c7	n + noget af det ovenstående?
8	$do\ A[j] \leftarrow A[j+1]$	c8	$n-1$
9	$A[n] \leftarrow x$	c9	$\sum_{j=2}^{n} t_j - 1$
10	$i \leftarrow [i/2]$	c10	$\sum_{j=2}^{n} t_j - 1$
11	$return\ A[1]$	c11 = 0	

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Svar = $O(n * lg * n)$

Exercise 2.2 - Consider the following algorithm:

$\textsc{FillTable}(A{:}int[1..n,1..n])$

```
for i ← 1 to n do
    for j ← i downto 1 do
        A[j, i − j + 1] ← i
        A[n − j + 1, n − i + j] ← i
```

Assume that A is with size 8x8, how does A look like after FillTable(A)? What is the (asymtotic) running time of FillTable?

Runtime = $O(n^2)$