

Problem 1:

1. F
2. T
3. F

Problem 2:

1. $[-50, 50]$
2. $[0, 1]$

Problem 3:

1. F

Problem 4:

1. D

Problem 5:

1. It is the same for 2 and 3 classes

Problem 6:

1. Model 1 is the better fit

Problem 7 (20 points)

Problem Description

A projectile is launched with input x- and y-velocity components. A dataset is provided, which contains launch velocity components as input and whether a target was hit (0/1) as an output. This data has a nonlinear decision boundary.

You will use gradient descent to train a logistic regression model on the dataset to predict whether any given launch velocity will hit the target.

Fill out the notebook as instructed, making the requested plots and printing necessary values.

You are welcome to use any of the code provided in the previous problems.

Summary of deliverables:

Functions (described in later section)

- `sigmoid(h)`
- `map_features(data)`
- `loss(data,y,w)`
- `grad_loss(data,y,w)`
- `grad_desc(data, y, w0, iterations, stepsize)`

Results:

- Print final `w` after training on the training data
- Plot of loss throughout training
- Print model percent classification accuracy on the training data
- Print model percent classification accuracy on the testing data
- Plot that shows the training data as data points, along with a decision boundary

Imports and Utility Functions:

```
In [ ]: import numpy as np
import matplotlib.pyplot as plt

def plot_data(data, c, title="", xlabel="$x_1$", ylabel="$x_2$", classes=["", ""], alpha=1):
    N = len(c)
    colors = ['royalblue', 'crimson']
    symbols = ['o', 's']

    plt.figure(figsize=(5,5), dpi=120)

    for i in range(2):
        x = data[:,0][c==i]
        y = data[:,1][c==i]

        plt.scatter(x,y,color=colors[i],marker=symbols[i],edgecolor="black",linewidths=0)

    plt.legend(loc="upper right")
```

```

plt.xlabel(xlabel)
plt.ylabel(ylabel)
ax = plt.gca()
plt.xlim([-0.05,1.05])
plt.ylim([-0.05,1.05])
plt.title(title)

def plot_contour(w):
    res = 500
    vals = np.linspace(-0.05,1.05,res)
    x,y = np.meshgrid(vals,vals)
    XY = np.concatenate((x.reshape(-1,1),y.reshape(-1,1)),axis=1)
    prob = sigmoid(map_features(XY) @ w.reshape(-1,1))
    pred = np.round(prob.reshape(res, res))
    plt.contour(x, y, pred)

```

Load Data

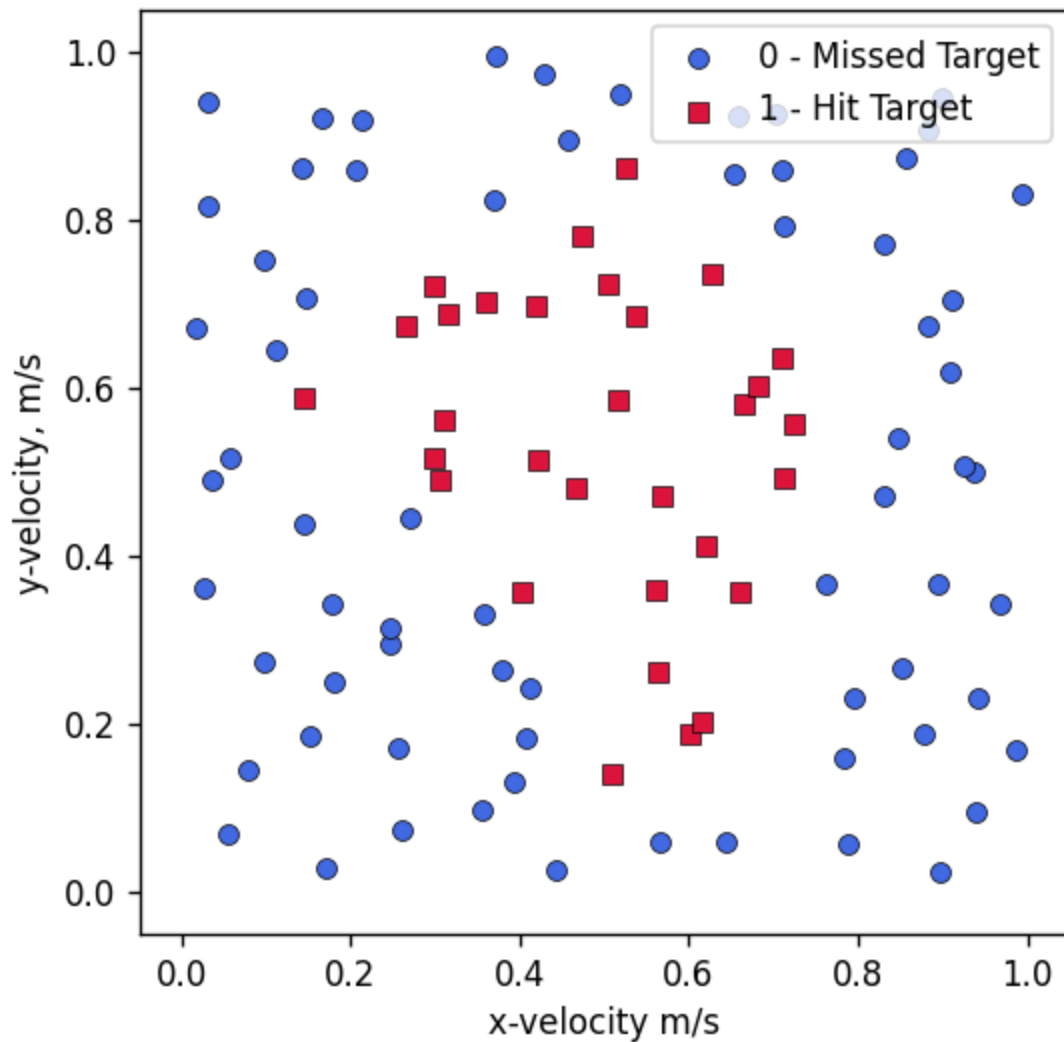
This cell loads the dataset into the following variables:

- `train_data` : Nx2 array of input features, used for training
- `train_gt` : Array of ground-truth classes for each point in `train_data`
- `test_data` : Nx2 array of input features, used for testing
- `test_gt` : Array of ground-truth classes for each point in `test_data`

```

In [ ]: train = np.load("data/w3-hw1-data-train.npy")
test = np.load("data/w3-hw1-data-test.npy")
train_data, train_gt = train[:, :2], train[:, 2]
test_data, test_gt = test[:, :2], test[:, 2]
format = dict(xlabel="x-velocity m/s", ylabel="y-velocity, m/s", classes=["0 - Missed Ta
plot_data(train_data, train_gt, **format)

```



Helper Functions

Here, implement the following functions:

sigmoid(h) :

- Input: `h` , single value or array of values
- Returns: The sigmoid of `h` (or each value in `h`)

map_features(data) :

- Input: `data` , Nx2 array with rows (x_i, y_i)
- Returns: Nx45 array, each row with $(1, x_i, y_i, x_i^2, x_i y_i, y_i^2, x_i^3, x_i^2 y_i, \dots)$ with all terms through 8th-order

loss(data, y, w) :

- Input: `data` , Nx2 array of un-transformed input features
- Input: `y` , Ground truth class for each input
- Input: `w` , Array with 45 weights
- Returns: Loss: $L(x, y, w) = \sum_{i=1}^n -y^{(i)} \cdot \ln(g(w'x^{(i)})) - (1 - y^{(i)}) \cdot \ln(1 - g(w'x^{(i)}))$

grad_loss(data, y, w) :

- Input: `data` , Nx2 array of un-transformed input features
- Input: `y` , Ground truth class for each input
- Input: `w` , Array with 45 weights
- Returns: Gradient of loss with respect to weights: $\frac{\partial L}{\partial w_j} = \sum_{i=1}^n (g(w'x^{(i)}) - y^{(i)})x_j^{(i)}$

```
In [ ]: def sigmoid(h):
        return (np.divide(1, (np.add(np.exp(-h), 1))))

def map_features(data):
    X = np.ones_like(data[:,0]).reshape(-1,1)
    for i in range(1,9):
        for j in range(1,i):
            X = np.concatenate([np.multiply(np.power(data[:,0], i-j+1), np.power(data[:,
            X = np.concatenate([np.power(data[:,0], i).reshape(-1,1), X], axis=1)
            X = np.concatenate([np.power(data[:,1], i).reshape(-1,1), X], axis=1)
    return X

def loss(data, y, w):
    loss = 0
    X = map_features(data)
    wt_x = np.sum((w * X), axis=1)
    J1 = -np.log(sigmoid(wt_x)) * y
    J2 = -np.log(1-sigmoid(wt_x)) * (1-y)
    L = np.sum(J1 + J2)
    return L

def grad_loss(data, y, w):
    X = map_features(data)
    wt_x = np.sum((w * X), axis=1)
    return np.sum((sigmoid(wt_x) - y).reshape(-1,1) * X, axis=0)
```

Gradient Descent

Now, write a gradient descent function with the following specifications:

`grad_desc(data, y, w0, iterations, stepsize)` :

- Input: `data` , Nx2 array of un-transformed input features
- Input: `y` , array of size N with ground-truth class for each input
- Input: `w0` , array of weights to use as an initial guess (size)
- Input: `iterations` , number of iterations of gradient descent to perform
- Input: `stepsize` , size of each gradient descent step
- Return: Final `w` array after last iteration
- Return: Array containing loss values at each iteration

```
In [ ]: def grad_desc(data, y, w0, iterations, stepsize):
        loss_data = [loss(data, y, w0)]
        for i in range(iterations):
            w0 = w0 - stepsize*grad_loss(data, y, w0)
            loss_data.append(loss(data, y, w0))
        return w0, np.array(loss_data)
```

Training

Run your gradient descent function and plot the loss as it converges. You may have to tune the step size and iteration count.

Also print the final vector `w`.

```
In [ ]: n = 10000
step = 0.001
w0 = np.ones(45)
w, loss_data = grad_desc(train_data, train_gt, w0, n, step)
```

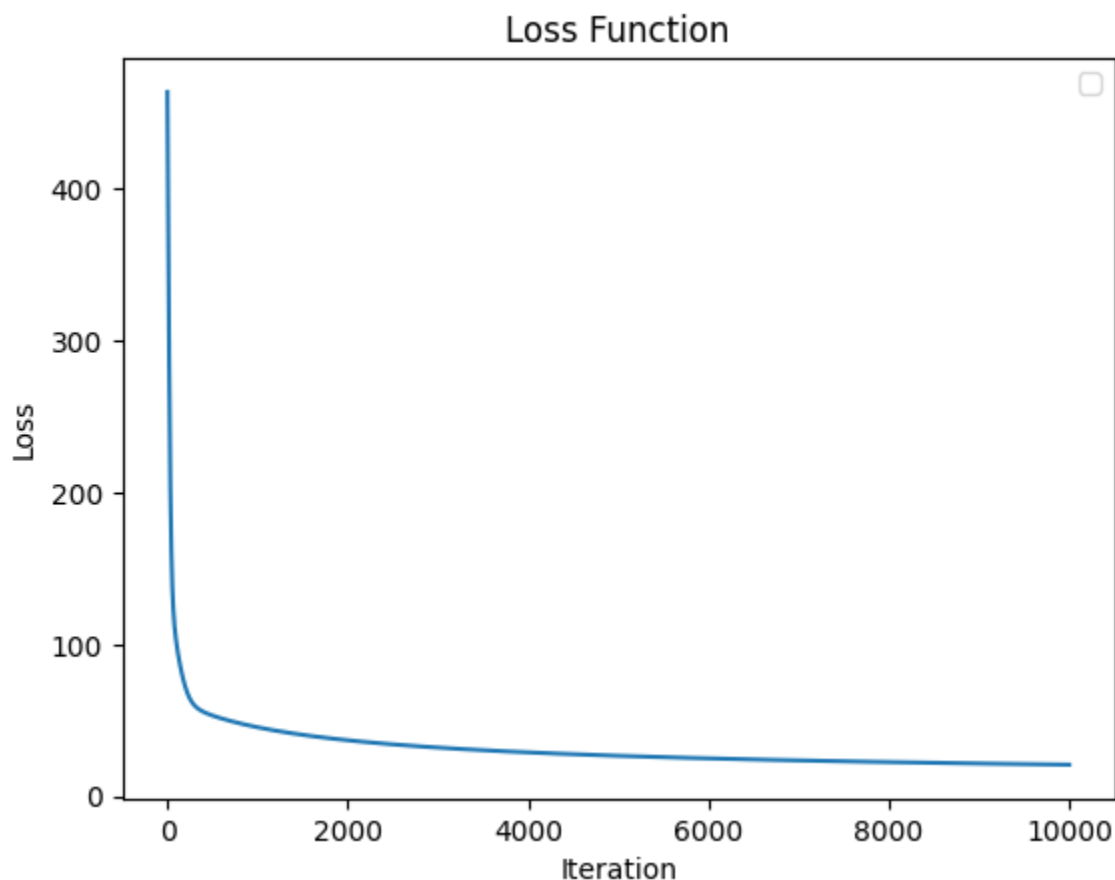
```
In [ ]: print(w)

fig = plt.figure()
plt.plot(np.linspace(0,n,n+1), loss_data)
plt.legend(loc="upper right")
plt.xlabel("Iteration")
plt.ylabel("Loss")
ax = plt.gca()
plt.title("Loss Function")
```

No artists with labels found to put in legend. Note that artists whose label start with an underscore are ignored when legend() is called with no argument.

```
[-3.26554133 -3.5337449 -0.51234658 -0.3572217 -0.28210304 -0.25189916
 -0.26960482 -0.35976476 -0.58358581 -3.40617231 -3.91440032 -0.61523439
 -0.48559149 -0.4364916 -0.44216112 -0.5250694 -0.75410276 -3.36403926
 -4.15806968 -0.68355583 -0.59989799 -0.59919526 -0.67814224 -0.91082355
 -2.98944277 -4.0869536 -0.67893617 -0.67494082 -0.7658958 -1.00785672
 -2.0432341 -3.37561706 -0.53314337 -0.66655089 -0.94465884 -0.17912172
 -1.45528233 -0.12482279 -0.50419784  2.94119513  2.50481334  0.7597856
  6.47904007  8.25949274 -5.71198449]
```

```
Out[ ]: Text(0.5, 1.0, 'Loss Function')
```



Accuracy

Compute the accuracy of the model, as a percent, for both the training data and testing data

```
In [ ]: train_preds = np.round(sigmoid(np.sum((w * map_features(train_data)), axis=1))).astype(int)
test_preds = np.round(sigmoid(np.sum((w * map_features(test_data)), axis=1))).astype(int)
train_accuracy = np.sum(train_preds == train_gt) / len(train_gt) * 100
test_accuracy = np.sum(test_preds == test_gt) / len(test_gt) * 100
print("    Train Accuracy: ", train_accuracy, r"%")
print("    Test Accuracy: ", test_accuracy, r"%")
```

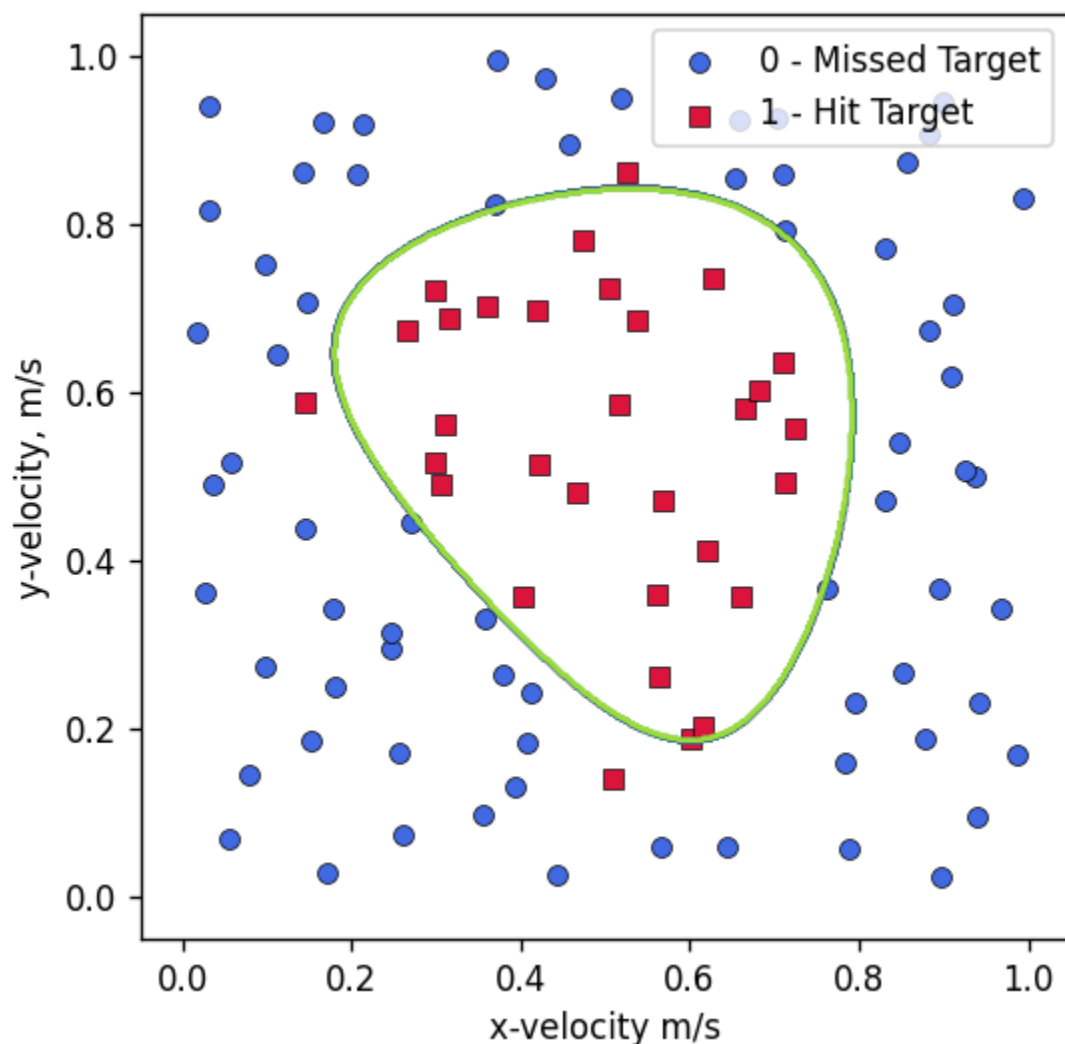
Train Accuracy: 97.0 %

Test Accuracy: 96.0 %

Visualize Results

Use the provided plotting utilities to plot the decision boundary with the data.

```
In [ ]: # You may have to modify this code, i.e. if you named 'w' differently)
plot_data(train_data, train_gt, **format)
plot_contour(w)
```



M3-HW2

September 23, 2023

1 Problem 8 (20 Points)

1.1 Problem Description

Several molecular dynamics simulations have been carried out for a material, and the phase (solid/liquid/vapor) at different temperature/pressure combinations has been recorded.

You will use gradient descent to train a One-vs-Rest logistic regression model on data with 3 classes. Fill out the notebook as instructed, making the requested plots and printing necessary values.

You are welcome to use any of the code provided in the previous problems.

Summary of deliverables:

- 3 binomial classification w vectors, corresponding to each class
- Function `classify(xy)` that evaluates all 3 models at a given array of points, returning the class prediction as the model with the highest probability
- Print model percent classification accuracy on the training data
- Print model percent classification accuracy on the testing data
- Plot that shows the training data as data points, along with the class of a grid of points in the background, as in the lecture activity.

1.1.1 Imports and Utility Functions:

```
[ ]: import numpy as np
import matplotlib.pyplot as plt
from matplotlib.colors import ListedColormap

def plot_data(x, y, c, title="Phase of simulated material"):
    xlim = [0, 52.5]
    ylim = [0, 1.05]
    markers = [dict(marker="o", color="royalblue"), dict(marker="s",
    ↪ color="crimson"), dict(marker="^", color="limegreen")]
    labels = ["Solid", "Liquid", "Vapor"]

    plt.figure(dpi=150)

    for i in range(1+max(c)):
        plt.scatter(x[c==i], y[c==i], s=60, **(markers[i]), edgecolor="black",
    ↪ linewidths=0.4, label=labels[i])
```



```

plt.title(title)
plt.legend(loc="upper right")
plt.xlim(xlim)
plt.ylim(ylim)
plt.xlabel("Temperature, K")
plt.ylabel("Pressure, atm")
plt.box(True)

def plot_colors(classify, res=40):
    xlim = [0,52.5]
    ylim = [0,1.05]
    xvals = np.linspace(*xlim,res)
    yvals = np.linspace(*ylim,res)
    x,y = np.meshgrid(xvals,yvals)
    XY = np.concatenate((x.reshape(-1,1),y.reshape(-1,1)),axis=1)

    color = classify(XY).reshape(res,res)

    cmap = ListedColormap(["lightblue","lightcoral","palegreen"])
    plt.pcolor(x, y, color, shading="nearest", zorder=-1,
    ↪cmap=cmap,vmin=0,vmax=2)
    return

```

1.2 Load Data

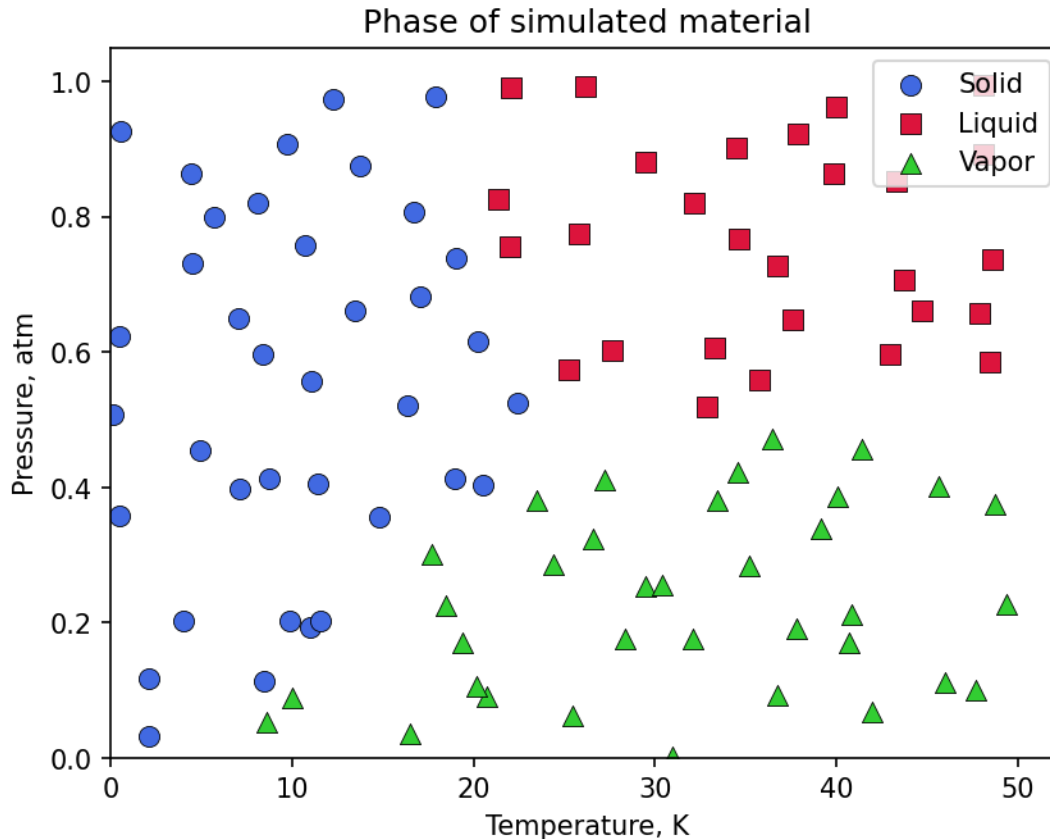
This cell loads the dataset into the following variables: - **train_data**: Nx2 array of input features, used for training - **train_gt**: Array of ground-truth classes for each point in **train_data** - **test_data**: Nx2 array of input features, used for testing - **test_gt**: Array of ground-truth classes for each point in **test_data**

In the class arrays, 0 = solid, 1 = liquid, 2 = vapor.

```

[ ]: train = np.load("data/w3-hw2-data-train.npy")
test = np.load("data/w3-hw2-data-test.npy")
train_data, train_gt = train[:, :2], train[:, 2].astype(int)
test_data, test_gt = test[:, :2], test[:, 2].astype(int)
plot_data(train_data[:, 0], train_data[:, 1], train_gt)

```



1.3 Gradient Descent

Here, write all of the necessary code to perform gradient descent and train 3 logistic regression models for a 1-vs-rest scenario. Use linear decision boundaries (features should only be 1, temperature, pressure)

Feel free to reuse code from the first problem or lecture activities.

We have provided the following function to help with the one-vs-all method:

`convert_to_binary_dataset(classes, A):` - Input: data, Nx2 array of temperature-pressure data - Input: classes, array (size N) of class values for each point in data - Input: A, the class (0, 1, or 2 here) to use as '1' in the binary dataset - Returns: `classes_binary`, copy of classes where class A is 1, and all other classes are 0.

```
[ ]: def convert_to_binary_dataset(classes, A):
    classes_binary = (classes == A).astype(int)
    return classes_binary
```

```
[ ]: def sigmoid(h):
    return (np.divide(1, (np.add(np.exp(-h), 1))))
```

```

def transform(data, w):
    xs = data[:,0]
    ys = data[:,1]
    ones = np.ones_like(xs)
    h = w[0]*ones + w[1]*xs + w[2]*ys
    return h

def loss(data, y, w):
    wt_x = transform(data,w)
    J1 = -np.log(sigmoid(wt_x)) * y
    J2 = -np.log(1-sigmoid(wt_x))*(1-y)
    L = np.sum(J1 + J2)
    return L

def gradloss(data, y, w):
    wt_x = transform(data,w)
    return np.array([np.sum((sigmoid(wt_x) - y) * np.ones(len(data[:,0]))), np.
↪sum((sigmoid(wt_x) - y) * data[:,0]), np.sum((sigmoid(wt_x) - y) * data[:,
↪1]))])

def grad_desc(data, y, w0=np.array([0,0,0]), iterations=100, stepsize=0.1):
    for i in range(iterations):
        w0 = w0 - np.multiply(stepsize,gradloss(data,y,w0))
    return w0

```

2 Training

Train your 3 models and print the w vector corresponding to each class

```

[ ]: # def get_ovr_prob_function(xy, c, A, iterations=100, stepsize=0.1):
#     c_new = (c == A).astype(int)

#     w = grad_desc(xy,c_new, iterations=100, stepsize=0.1)
#     print("Class ", A, " w: ", w, r"%")

#     def prob(xy):
#         pred = np.round(sigmoid(transform(xy, w))).astype(int)
#         return pred.flatten()

#     return prob
from sklearn.linear_model import LogisticRegression

def get_ovr_prob_function(xy, c, A):
    c_new = (c == A).astype(int)

    model = LogisticRegression(max_iter=10000, tol=1e-6, C=10)
    model.fit(xy,c_new)

```

```

def prob(x):
    pred = model.predict_proba(x)[:,-1]
    return pred.flatten()

return prob

def generate_ovr_prob_functions(x, c):
    A = np.unique(c)
    classifier_list = []
    for A_i in A:
        classifier_list.append(get_ovr_prob_function(x, c, A_i))
    return classifier_list

probs = generate_ovr_prob_functions(train_data, train_gt)

```

2.1 Classification function

Write a function `classify(x)` that will evaluate each model and select the appropriate class.

```

[ ]: def classify(x):
    results = []
    for prob in probs:
        results.append(prob(x))

    results = np.array(results)
    maximum = np.max(results, axis=0)
    return np.array([np.where(results[:,i] == maximum[i])[0] for i in
↪range(len(maximum))]).flatten()

```

2.2 Accuracy

Compute and print the accuracy on the training and testing sets as a percent

```

[ ]: train_preds = classify(train_data)
test_preds = classify(test_data)
train_accuracy = np.sum(train_preds == train_gt) / len(train_gt) * 100
test_accuracy = np.sum(test_preds == test_gt) / len(test_gt) * 100
print("    Train Accuracy: ", train_accuracy, r"%")
print("    Test Accuracy: ", test_accuracy, r"%")

```

```

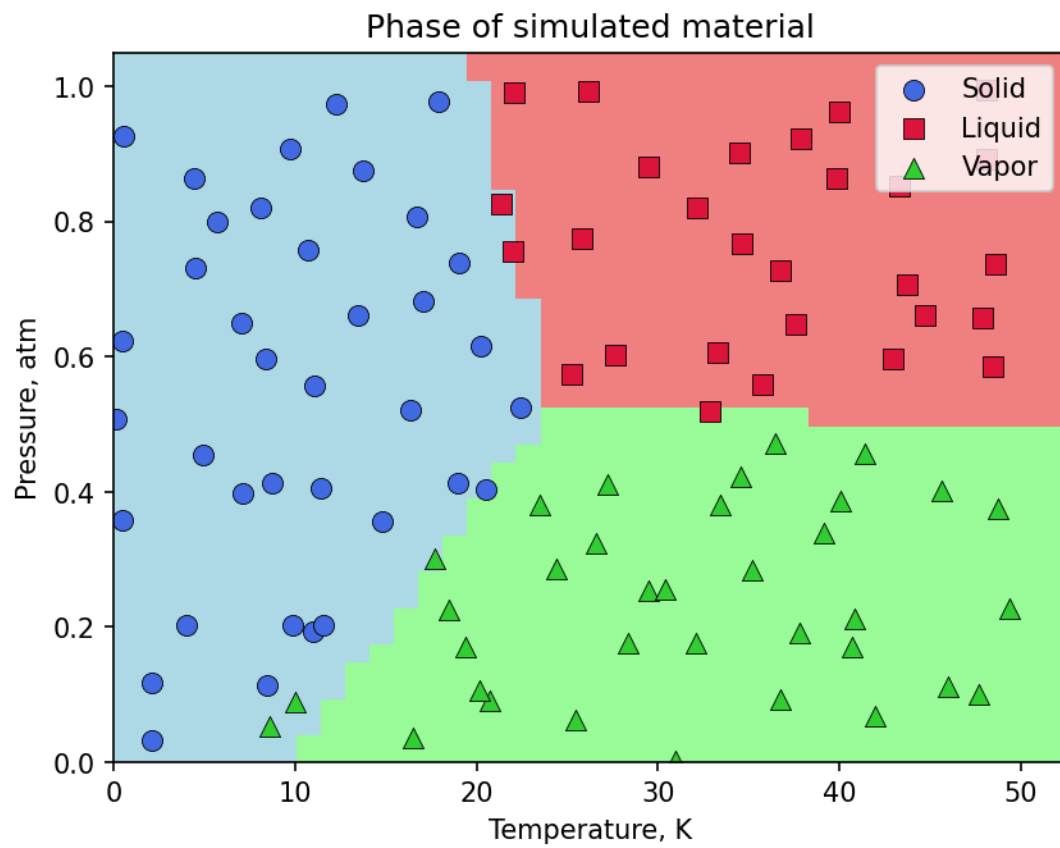
Train Accuracy:  94.0 %
Test Accuracy:  92.0 %

```

2.3 Plot results

Run this cell to visualize the data along with the results of `classify()`

```
[ ]: plot_data(train_data[:,0], train_data[:,1], train_gt)
      plot_colors(classify)
```



M3-HW3

September 23, 2023

1 Problem 9 (20 Points)

1.1 Problem description

So far, we have worked with ~2 dimensional problems with 2-3 classes. Most often in ML, there are many more explanatory variables and classes than this. In this problem, you'll be training logistic regression models on a database of grayscale images of hand-drawn digits, using SciKit-Learn. Now there are 400 (20x20) input features and 10 classes (digits 0-9).

As usual, you can use any code from previous problems.

1.2 Summary of deliverables

- OvR model accuracy on training data
- OvR model accuracy on testing data
- Multinomial model accuracy on training data
- Multinomial model accuracy on testing data

1.2.1 Imports and Utility Functions:

```
[ ]: import numpy as np
import matplotlib.pyplot as plt
from sklearn.linear_model import LogisticRegression

def visualize(xdata, index, title=""):
    image = xdata[index,:].reshape(20,20).T
    plt.figure()
    plt.imshow(image, cmap = "binary")
    plt.axis("off")
    plt.title(title)
    plt.show()
```

1.3 Load data

The following cell loads in training and testing data into the following variables: - **x_train**: 4000x400 array of input features, used for training - **y_train**: Array of ground-truth classes for each point in **x_train** - **x_test**: 1000x400 array of input features, used for testing - **y_test**: Array of ground-truth classes for each point in **x_test**

You can visualize a digit with the `visualize(x_data, index)` function.

```
[ ]: x_train = np.load("data/w3-hw3-train_x.npy")
      y_train = np.load("data/w3-hw3-train_y.npy")
      x_test = np.load("data/w3-hw3-test_x.npy")
      y_test = np.load("data/w3-hw3-test_y.npy")

      visualize(x_train, 1234)
```



1.4 Logistic Regression Models

Use sklearn's `LogisticRegression` to fit a multinomial logistic regression model on the training data. You may need to increase the `max_iter` argument for the model to converge.

Train 2 models: one using the One-vs-Rest method, and another that minimizes multinomial loss. You can do these by setting the `multi_class` argument to "ovr" and "multinomial", respectively.

More information: https://scikit-learn.org/stable/modules/generated/sklearn.linear_model.LogisticRegression.html

```
[ ]: model_1 = LogisticRegression(max_iter=10000, multi_class="ovr")
      model_2 = LogisticRegression(max_iter=10000, multi_class="multinomial")

      model_1.fit(x_train, y_train)
      model_2.fit(x_train, y_train)
```

```
[ ]: LogisticRegression(max_iter=10000, multi_class='multinomial')
```

1.5 Accuracy

Compute and print the accuracy of each model on the training and testing sets as a percent.

```
[ ]: train_preds_1 = model_1.predict(x_train)
      train_preds_2 = model_2.predict(x_train)
      test_preds_1 = model_1.predict(x_test)
      test_preds_2 = model_2.predict(x_test)

      train_accuracy_1 = np.sum(train_preds_1 == y_train) / len(y_train) * 100
      test_accuracy_1 = np.sum(test_preds_1 == y_test) / len(y_test) * 100
      train_accuracy_2 = np.sum(train_preds_2 == y_train) / len(y_train) * 100
      test_accuracy_2 = np.sum(test_preds_2 == y_test) / len(y_test) * 100
      print("    Train Accuracy (Model 1): ", train_accuracy_1, r"%")
      print("    Test Accuracy (Model 1): ", test_accuracy_1, r"%")
      print("    Train Accuracy (Model 2): ", train_accuracy_2, r"%")
      print("    Test Accuracy (Model 2): ", test_accuracy_2, r"%")
```

```
Train Accuracy (Model 1):  94.72500000000001 %
Test Accuracy (Model 1):  90.7 %
Train Accuracy (Model 2):  96.475 %
Test Accuracy (Model 2):  91.4 %
```


M3-L1-P1

September 23, 2023

1 Problem 1 (5 points)

```
[ ]: import numpy as np
import matplotlib.pyplot as plt
```

1.1 Sigmoid function

Define a function, `sigmoid(h)`, which computes and returns the sigmoid $g(h)$ given an input h . Recall the mathematical formulation of sigmoid:

$$g(h) = \frac{1}{1 + e^{-h}}$$

```
[ ]: def sigmoid(h):
    return (np.divide(1, (np.add(np.exp(-h), 1))))
```

1.2 Transformation function

In logistic regression, we transform the input before applying the sigmoid function. This transformation can take many forms, but here let's define a function `transform_quadratic(x,w)` that takes in an input x , and a weight vector \mathbf{w} , and returns the sum $w_0 + w_1 x + w_2 x^2$.

```
[ ]: def transform_quadratic(x, w):
    sum = 0
    for i in range(len(w)):
        sum += w[i]*x**i
    return sum
```

1.3 Example

Now, we will use both `sigmoid()` and `transform_quadratic()` in a logistic regression context.

Suppose a logistic regression model states that:

$$P(y = 1 \mid x) = g(\mathbf{w}'x),$$

for $g(h)$ the sigmoid function and $\mathbf{w} = [4, -3, 2]$.

Use the functions you wrote to compute $P(y = 1 \mid x = 1.2)$ and $P(y = 1 \mid x = 7)$. Print these probabilities.

```
[ ]: w = [4,-3,2]
      for x in [1.2, 7.]:
          P = sigmoid(transform_quadratic(x,w))
          print(f"x = {x:3} --> P(y=1) = {P}")
```

x = 1.2 --> P(y=1) = 0.9637362836253517

x = 7.0 --> P(y=1) = 1.0

Problem 2 (5 points)

```
In [ ]: import numpy as np
import matplotlib.pyplot as plt

def plot_data(data, c, title="", xlabel="$x_1$", ylabel="$x_2$", classes=["", ""], alpha=1):
    N = len(c)
    colors = ['royalblue', 'crimson']
    symbols = ['o', 's']

    plt.figure(figsize=(5,5),dpi=120)

    for i in range(2):
        x = data[:,0][c==i]
        y = data[:,1][c==i]

        plt.scatter(x,y,color=colors[i],marker=symbols[i],edgecolor="black",linewidths=0)

    plt.legend(loc="upper right")
    plt.xlabel(xlabel)
    plt.ylabel(ylabel)
    ax = plt.gca()
    ax.set_xticklabels([])
    ax.set_yticklabels([])
    plt.xlim([-0.05,1.05])
    plt.ylim([-0.05,1.05])
    plt.title(title)

def plot_contour(predict, mapXY = None):
    res = 500
    vals = np.linspace(-0.05,1.05,res)
    x,y = np.meshgrid(vals,vals)
    XY = np.concatenate((x.reshape(-1,1),y.reshape(-1,1)),axis=1)
    if mapXY is not None:
        XY = mapXY(XY)
    contour = predict(XY).reshape(res, res)
    plt.contour(x, y, contour)
```

Generate Dataset

(Don't edit this code.)

```
In [ ]: def get_line_dataset():
    np.random.seed(4)
    x = np.random.rand(90)
    y = np.random.rand(90)

    h = 1/.9*x + 1/0.9*y - 1

    d = 0.1
    x1, y1 = x[h<=-d], y[h<=-d]
    x2, y2 = x[np.abs(h)<d], y[np.abs(h)<d]
    x3, y3 = x[h>d], y[h>d]

    c1 = np.ones_like(x1)
```

```

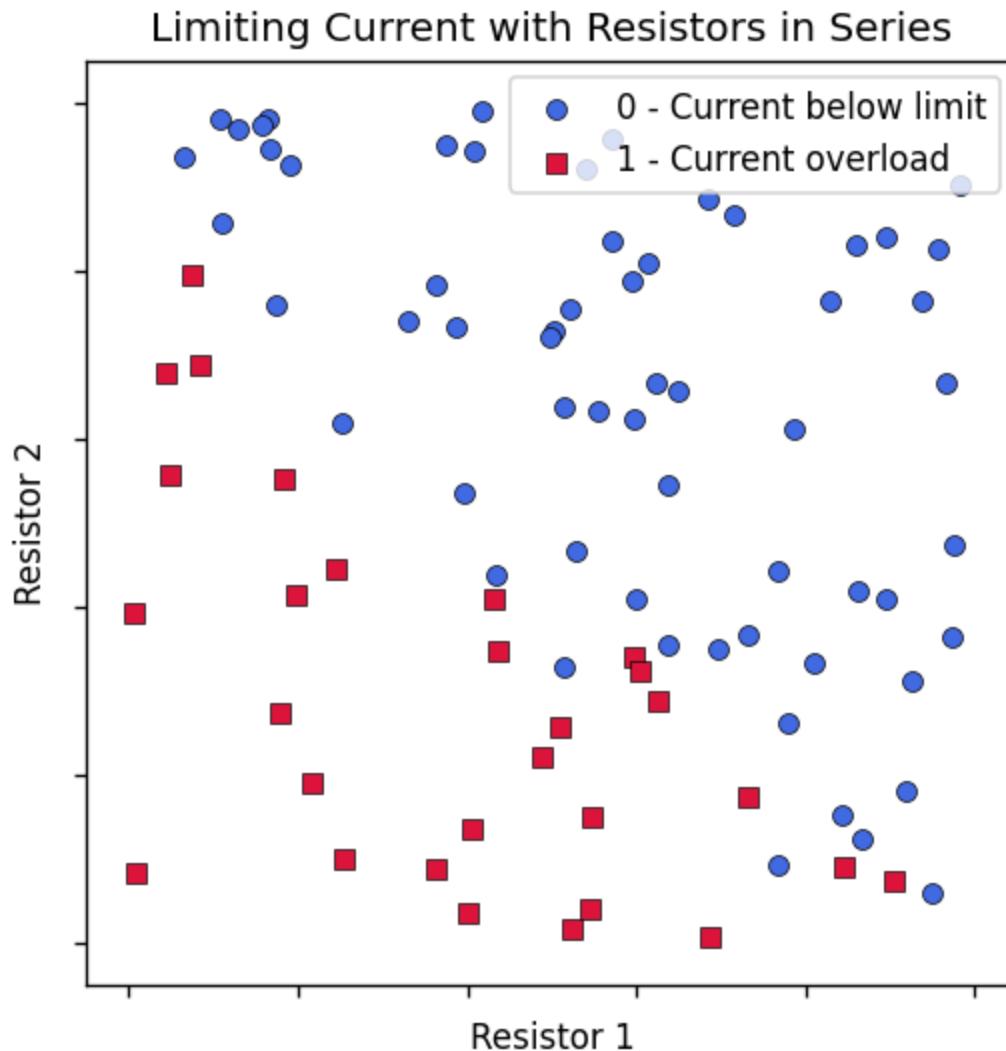
c2 = (np.random.rand(len(x2)) > 0.5).astype(int)
c3 = np.zeros_like(x3)
xs = np.concatenate([x1,x2,x3],0)
ys = np.concatenate([y1,y2,y3],0)
c = np.concatenate([c1,c2,c3],0)
return np.vstack([xs,ys]).T,c

```

```

In [ ]: data, classes = get_line_dataset()
format = dict(title="Limiting Current with Resistors in Series", xlabel="Resistor 1", y1
plot_data(data, classes, **format)

```



Define helper functions

First, fill in code to complete the following functions. You may use code you wrote in the previous question.

- `sigmoid(h)` to compute the sigmoid of an input `h`
- (Given) `transform(data, w)` to add a column of ones to `data` and then multiply by the 3-element vector `w`
- (Given) `loss(data, y, w)` to compute the logistic regression loss function:

$$L(x, y, w) = \sum_{i=1}^n -y^{(i)} \cdot \ln(g(w'x^{(i)})) - (1 - y^{(i)}) \cdot \ln(1 - g(w'x^{(i)}))$$

- `gradloss(data, y, w)` to compute the gradient of the loss function with respect to `w`:

$$\frac{\partial L}{\partial w_j} = \sum_{i=1}^n (g(w'x^{(i)} - y^{(i)}) x_j^{(i)})$$

```
In [ ]: def sigmoid(h):
    return np.divide(1, (np.add(np.exp(-h), 1)))

def transform(data, w):
    xs = data[:,0]
    ys = data[:,1]
    ones = np.ones_like(xs)
    h = w[0]*ones + w[1]*xs + w[2]*ys
    return h

def loss(data, y, w):
    wt_x = transform(data,w)
    J1 = -np.log(sigmoid(wt_x)) * y
    J2 = -np.log(1-sigmoid(wt_x))*(1-y)
    L = np.sum(J1 + J2)
    return L

def gradloss(data, y, w):
    wt_x = transform(data,w)
    return np.array([np.sum((sigmoid(wt_x) - y) * np.ones(len(data[:,0]))), np.sum((sigmoid(wt_x) - y) * data[:,0]), np.sum((sigmoid(wt_x) - y) * data[:,1])])
```

Gradient Descent

Now you'll write a gradient descent loop. Given a number of iterations and a step size, continually update `w` to minimize the loss function. Use the `gradloss` function you wrote to compute a gradient, then move `w` by `stepsize` in the direction opposite the gradient. Return the optimized `w`.

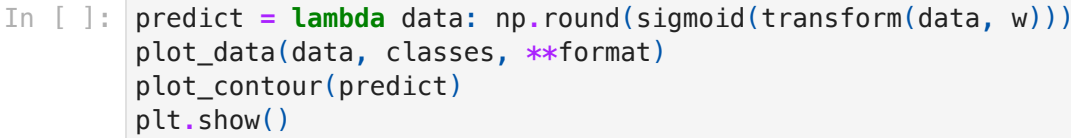
```
In [ ]: def grad_desc(data, y, w0=np.array([0,0,0]), iterations=100, stepsize=0.1):
    for i in range(iterations):
        w0 = w0 - np.multiply(stepsize, gradloss(data, y, w0))
    return w0
```

Test your classifier

Run these cells to find the optimal `w`, compute the accuracy on the training data, and plot a decision boundary.

```
In [ ]: w = grad_desc(data, classes)
preds = np.round(sigmoid(transform(data, w))).astype(int)
accuracy = np.sum(preds == classes) / len(classes) * 100
print("      w = ", w)
print("True Classes: ", classes.astype(int))
print(" Predictions: ", preds)
print("    Accuracy: ", accuracy, r"%")
```

```
In [ ]: predict = lambda data: np.round(sigmoid(transform(data, w)))
plot_data(data, classes, **format)
plot_contour(predict)
plt.show()
```



M3-L1-P3

September 23, 2023

1 Problem 3 (5 points)

```
[ ]: import numpy as np
import matplotlib.pyplot as plt

def plot_data(data, c, title="",
    ↪xlabel="$x_1$", ylabel="$x_2$", classes=["", ""], alpha=1):
    N = len(c)
    colors = ['royalblue', 'crimson']
    symbols = ['o', 's']

    plt.figure(figsize=(5,5), dpi=120)

    for i in range(2):
        x = data[:,0][c==i]
        y = data[:,1][c==i]

        plt.
    ↪scatter(x,y,color=colors[i],marker=symbols[i],edgecolor="black",linewidths=0.
    ↪4,label=classes[i],alpha=alpha)

    plt.legend(loc="upper right")
    plt.xlabel(xlabel)
    plt.ylabel(ylabel)
    ax = plt.gca()
    ax.set_xticklabels([])
    ax.set_yticklabels([])
    plt.xlim([-0.05,1.05])
    plt.ylim([-0.05,1.05])
    plt.title(title)

def plot_contour(predict, mapXY = None):
    res = 500
    vals = np.linspace(-0.05,1.05,res)
    x,y = np.meshgrid(vals,vals)
    XY = np.concatenate((x.reshape(-1,1),y.reshape(-1,1)),axis=1)
```

```

if mapXY is not None:
    XY = mapXY(XY)
contour = predict(XY).reshape(res, res)
plt.contour(x, y, contour)

```

1.1 Generate Dataset

(Don't edit this code.)

```

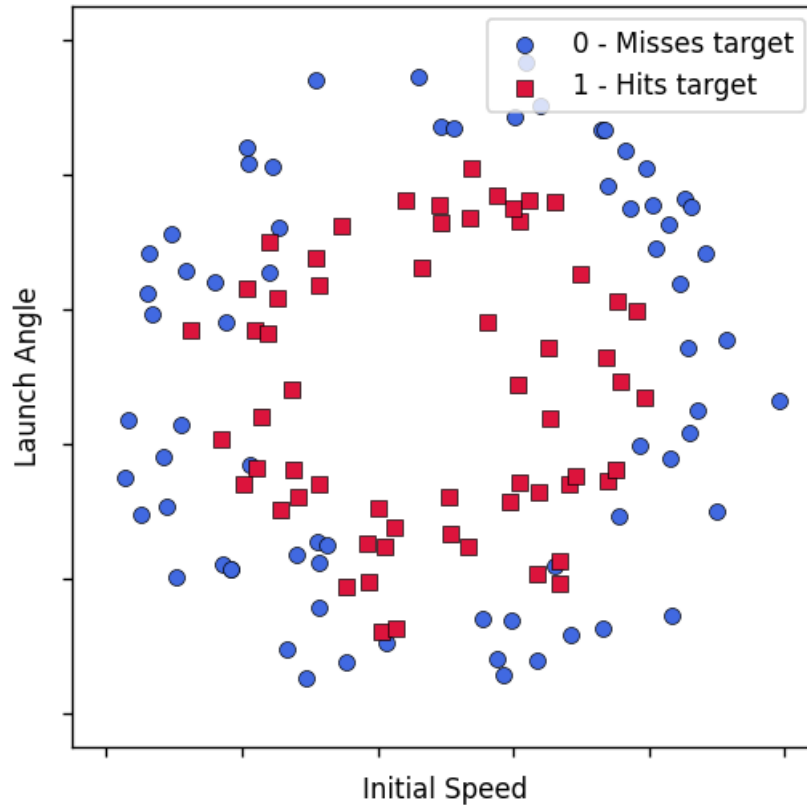
[ ]: def sample_ring(N,x,y,ro,ri):
    theta = np.random.rand(N)*2*np.pi
    r = np.random.rand(N)
    r = np.sqrt(r*(ro**2-ri**2)+ri**2)
    xs = x + r * np.cos(theta)
    ys = y + r * np.sin(theta)
    return xs, ys

def get_ring_dataset():
    np.random.seed(0)
    c0 = sample_ring(70,0.5,0.5,0.5,0.3)
    c1 = sample_ring(60,0.45,0.47,0.36,0.15)
    xs = np.concatenate([c0[0],c1[0]],0)
    ys = np.concatenate([c0[1],c1[1]],0)
    c = np.concatenate([np.zeros(70),np.ones(60)],0)
    return np.vstack([xs,ys]).T, c

[ ]: data, classes = get_ring_dataset()
format = dict(xlabel="Initial Speed",ylabel="Launch Angle", classes=["0 - Misses target", "1 - Hits target"])

plot_data(data, classes, **format)

```

1.2 Feature Expansion

Define a function to expand 2 features into more features For the features x_1 and x_2 , expand into:

- 1 - $x_1 - x_2 - x_1^2 - x_2^2 - \sin(x_1) - \cos(x_1) - \sin(x_2) - \cos(x_2) - \sin^2(x_1) - \cos^2(x_1) - \sin^2(x_2) - \cos^2(x_2)$
- $\exp(x_1) - \exp(x_2)$

```
[ ]: def feature_expand(x):
    x1 = x[:,0].reshape(-1, 1)
    x2 = x[:,1].reshape(-1, 1)

    # YOUR CODE GOES HERE:
    columns = [np.ones_like(x1), x1, x2, x1**2, x2**2,
               np.sin(x1), np.cos(x1), np.sin(x2), np.cos(x2),
               np.sin(x1)**2, np.cos(x1)**2, np.sin(x2)**2, np.cos(x2)**2,
               np.exp(x1), np.exp(x2)] # Add all expanded features to this list

    X = np.concatenate(columns, axis=1)
    return X

features = feature_expand(data)
print("Dataset size:", np.shape(data))
```

```
print("Expanded dataset size:", np.shape(features))
```

Dataset size: (130, 2)

Expanded dataset size: (130, 15)

1.3 Logistic Regression

Use SciKit-Learn's Logistic Regression model to learn the decision boundary for this data, using regularization. (The `C` argument controls regularization strength.)

Train this model on your expanded feature set.

Details about how to use this are here: https://scikit-learn.org/stable/modules/generated/sklearn.linear_model.LogisticRegression.html

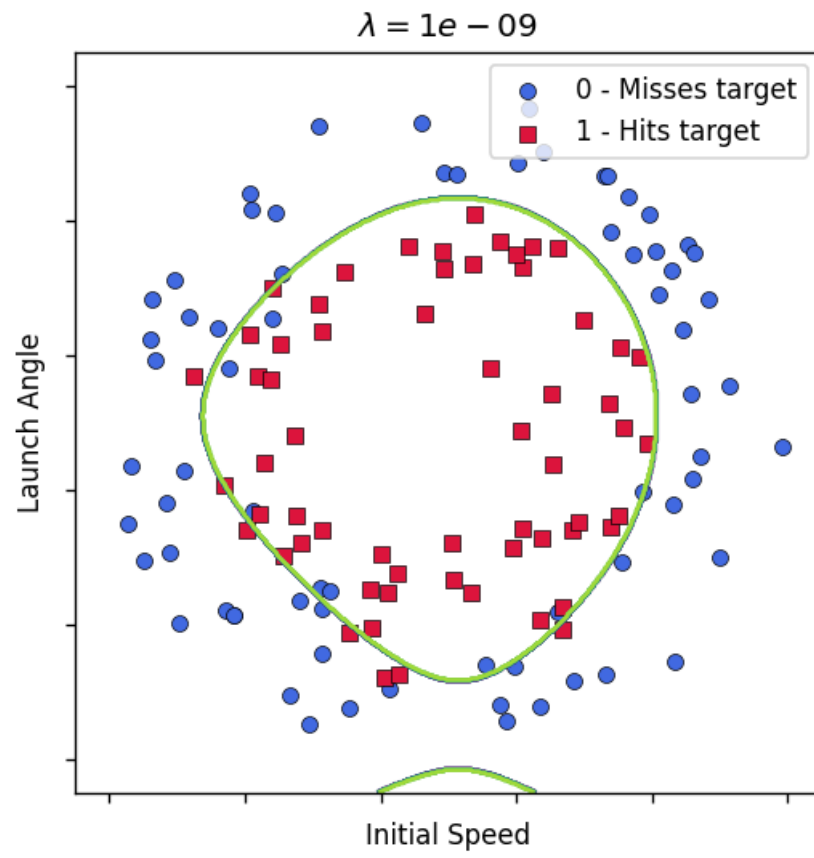
Notes: - λ is related to sklearn's regularization strength C by: $\lambda = 1/C$ - You may want to increase the maximum number of iterations

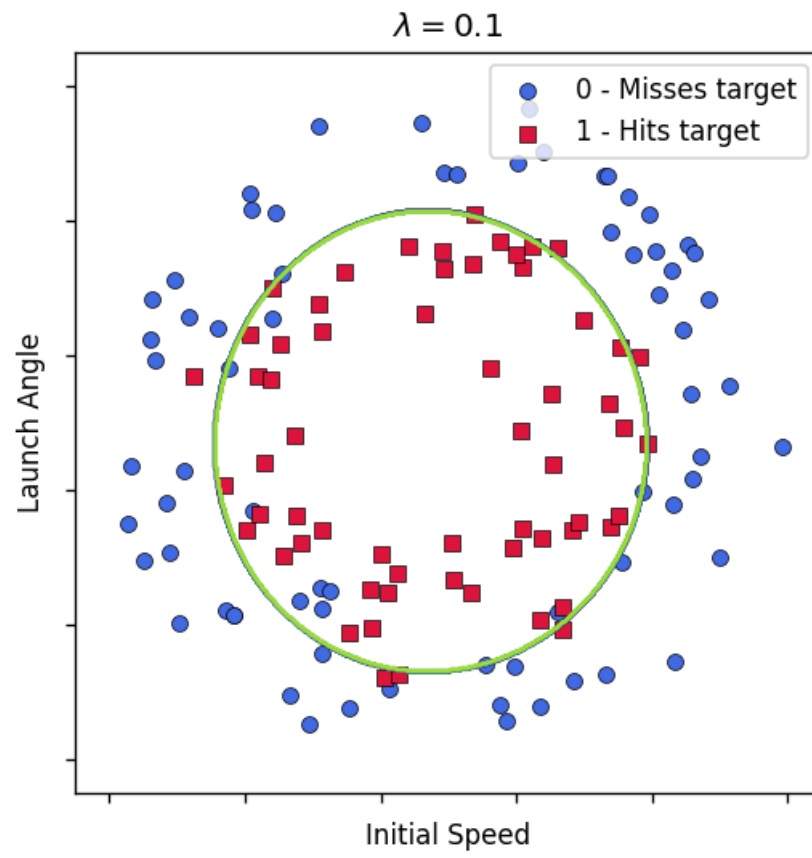
```
[ ]: from sklearn.linear_model import LogisticRegression

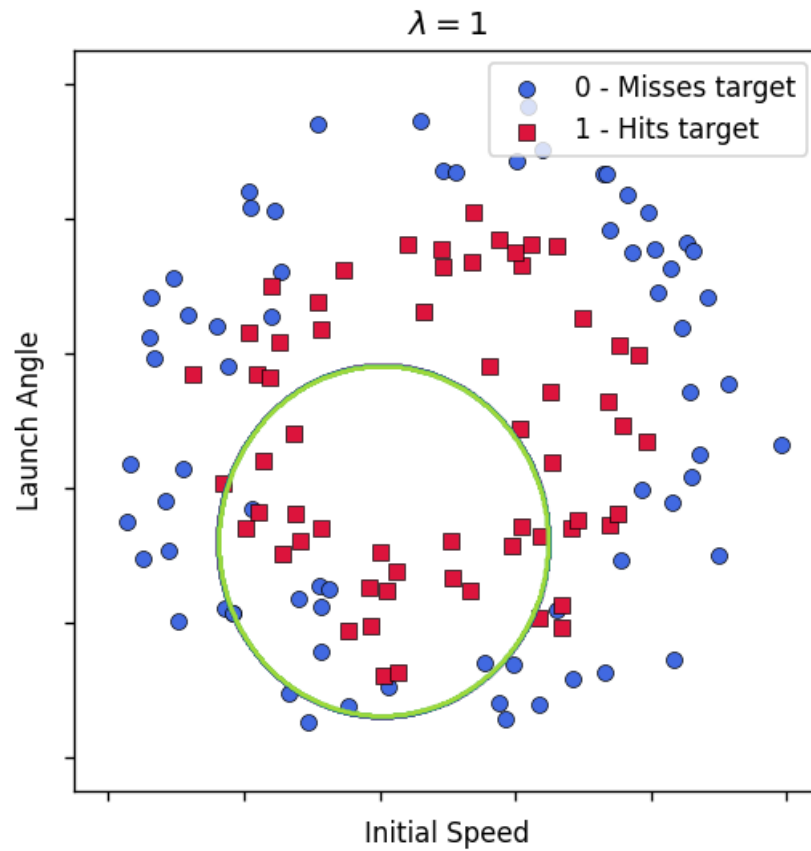
def get_logistic_regressor(features, classes, L = 1):
    # YOUR CODE GOES HERE
    # - Instantiate model with regularization
    # - Fit model to expanded data
    model = LogisticRegression(C=1/L, max_iter=10000)
    model.fit(features, classes)

    return model

[ ]: for L in [1e-9, 1e-1, 1]:
    model = get_logistic_regressor(features, classes, L)
    plot_data(data, classes, **format, title=f"$\lambda={L}$")
    plot_contour(model.predict, feature_expand)
    plt.show()
```







As λ increases, note what happens to the decision boundary. Why does this occur?

Lambda smooths out the fitted boundary. This results in not over fitting but if taken too far, as with $\lambda = 1$, the boundary doesn't fit the data well at all.

M3-L2-P1

September 23, 2023

1 Problem 4 (5 points)

```
[ ]: import numpy as np
import matplotlib.pyplot as plt
from matplotlib.colors import ListedColormap
from scipy.stats import mode
from sklearn.linear_model import LogisticRegression
```

2 One-vs-One Multinomial Classification

2.1 Load Dataset

(Don't edit this)

- (x,y) values are stored in rows of xy
- class values are in c

```
[ ]: x = np.array([7.4881350392732475,16.351893663724194,22.427633760716436,29.
↪04883182996897,35.03654799338904,44.45894113066656,6.375872112626925,18.
↪117730007820796,26.036627605010292,27.434415188257777,38.71725038082664,43.
↪28894919752904,7.680445610939323,18.45596638292661,17.110360581978867,24.
↪47129299701541,31.002183974403255,46.32619845547938,9.781567509498505,17.
↪90012148246819,26.186183422327638,31.59158564216724,35.41479362252932,45.
↪805291762864556,3.182744258689332,15.599210213275237,17.833532874090462,33.
↪04668917049584,36.018483217500716,42.146619399905234,4.64555612104627,16.
↪942336894342166,20.961503322165484,29.284339488686488,30.98789800436355,44.
↪17635497075877,])
y = np.array([0.11120957227224215,0.1116933996874757,0.14437480785146242,0.
↪11818202991034835,0.0859507900573786,0.09370319537993416,0.
↪2797631195927265,0.216022547162927,0.27667667154456677,0.27706378696181594,0.
↪2310382561073841,0.22289262976548535,0.40154283509241845,0.
↪4063710770942623,0.427019677041788,0.41386015134623205,0.46883738380592266,0.
↪38020448107480287,0.5508876756094834,0.5461309517884996,0.5953108325465398,0.
↪5553291602539782,0.5766310772856306,0.5544425592001603,0.705896958364552,0.
↪7010375141164304,0.7556329589465274,0.7038182951348614,0.7096582361680054,0.
↪7268725170660963,0.9320993229847936,0.8597101275793062,0.9337944907498804,0.
↪8596098407893963,0.9476459465013396,0.8968651201647702,])
xy = np.vstack([x,y]).T
```

```
c = np.  
↪array([0,2,2,2,2,2,0,2,2,2,2,2,0,0,2,0,1,2,0,0,1,1,1,2,0,1,0,1,1,1,0,0,1,1,1,1,])
```

2.2 Binomial classification function

You are given a function that performs binomial classification by using sklearn's LogisticRegression tool: `classify = get_binomial_classifier(xy, c, A, B)`

To use it, input: - `xy`, an array in which each row contains (x,y) coordinates of data points - `c`, an array that specifies the class each point in `xy` belongs to - `A`, the class of the first group (0, 1, or 2 in this problem) - `B`, the class of the second group (0, 1, or 2 in this problem), but different from `A`

The function outputs a classifier function (`classify()` in this case), used to classify any new `xy` into group `A` or `B`, such as by using `classify(xy)`.

```
[ ]: def get_binomial_classifier(xy, c, A, B):  
    assert A != B  
    xyA, xyB = xy[c==A], xy[c==B]  
    cA, cB = c[c==A], c[c==B]  
    model = LogisticRegression()  
    xy_new = np.concatenate([xyA, xyB], 0)  
    c_new = np.concatenate([cA, cB], 0)  
    model.fit(xy_new, c_new)  
  
    def classify(xy):  
        pred = model.predict(xy)  
        return pred  
  
    return classify
```

2.3 Coding a 1v1 classifier

Now you will create a one-vs-one classifier to do multinomial classification. This will generate binomial classifiers for each pair of classes in the dataset. Then to predict the class of a new point, classify it using each of the binomial classifiers, and select the majority winner as the class prediction.

Complete the two functions we have started: - `generate_all_classifiers(xy, c)` which returns a list of binary classifier functions for all possible pairs of classes (among 0, 1, and 2 in this problem) - `classify_majority(classifiers, xy)` which loops through a list of classifiers and gets their predictions for each point in `xy`. Then using a majority voting scheme at each point, return the overall class predictions for each point.

```
[ ]: def generate_all_classifiers(xy, c):  
    # YOUR CODE GOES HERE  
    # Use get_binomial_classifier() to get binomial classifiers for each pair  
    ↪of classes,  
    # and return a list of these classifiers  
    fun1 = get_binomial_classifier(xy, c, 0, 1)
```

```

    fun2 = get_binomial_classifier(xy, c, 0, 2)
    fun3 = get_binomial_classifier(xy, c, 1, 2)
    return [fun1, fun2, fun3]

def classify_majority(classifiers, xy):
    results = np.zeros([len(xy), len(classifiers)])
    for i in range(len(classifiers)):
        classifier = classifiers[i]
        results_i = classifier(xy)
        for j in range(len(results_i)):
            results[j, results_i[j]] += 1

    check = np.repeat(np.array([[0, 1, 2]]), results.shape[0], axis=0)
    final_results = []
    for i in range(results.shape[0]):
        final_results.append(np.where(results[i,:] == np.max(results[i,:],
↪))) [0][0])
    return np.array(final_results).flatten()

```

2.3.1 Trying out our multinomial classifier:

```

[ ]: classifiers = generate_all_classifiers(xy, c)
    preds = classify_majority(classifiers, xy)
    accuracy = np.sum(preds == c) / len(c) * 100
    print("True Classes:", c)
    print(" Predictions:", preds)
    print("    Accuracy:", accuracy, r"%")

```

```

True Classes: [0 2 2 2 2 2 0 2 2 2 2 2 0 0 2 0 1 2 0 0 1 1 1 2 0 1 0 1 1 1 0 0 1
1 1 1]
Predictions: [0 0 2 2 2 2 0 0 2 2 2 2 0 0 0 2 2 2 0 0 1 1 1 1 0 0 0 1 1 1 0 0 1
1 1 1]
Accuracy: 80.55555555555556 %

```

2.3.2 Plotting a Decision Boundary

Here, we have made some plotting functions – run these cells to visualize the decision boundaries.

```

[ ]: def plot_data(x, y, c, title="Phase of simulated material", newfig=True):
    xlim = [0, 52.5]
    ylim = [0, 1.05]
    markers = [dict(marker="o", color="royalblue"), dict(marker="s",
↪color="crimson"), dict(marker="^", color="limegreen")]
    labels = ["Solid", "Liquid", "Vapor"]

    if newfig:
        plt.figure(dpi=150)

```



```

    for i in range(1+max(c)):
        plt.scatter(x[c==i], y[c==i], s=60, **(markers[i]), edgecolor="black",
↪linewidths=0.4,label=labels[i])

    plt.title(title)
    plt.legend(loc="upper right")
    plt.xlim(xlim)
    plt.ylim(ylim)
    plt.xlabel("Temperature, K")
    plt.ylabel("Pressure, atm")
    plt.box(True)

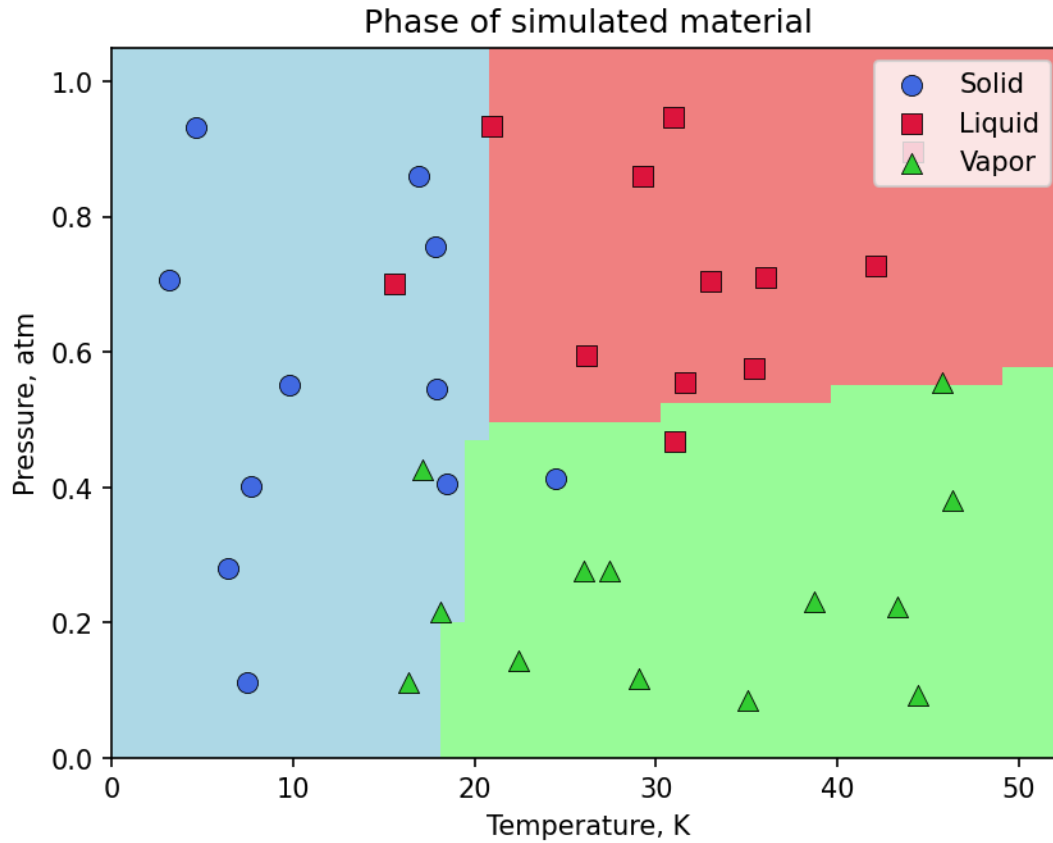
def plot_colors(classifiers, res=40):
    xlim = [0,52.5]
    ylim = [0,1.05]
    xvals = np.linspace(*xlim,res)
    yvals = np.linspace(*ylim,res)
    x,y = np.meshgrid(xvals,yvals)
    XY = np.concatenate((x.reshape(-1,1),y.reshape(-1,1)),axis=1)
    if type(classifiers) == list:
        color = classify_majority(classifiers,XY).reshape(res,res)
    else:
        color = classifiers(XY).reshape(res,res)
    cmap = ListedColormap(["lightblue","lightcoral","palegreen"])
    plt.pcolor(x, y, color, shading="nearest", zorder=-1,
↪cmap=cmap,vmin=0,vmax=2)
    return

```

```

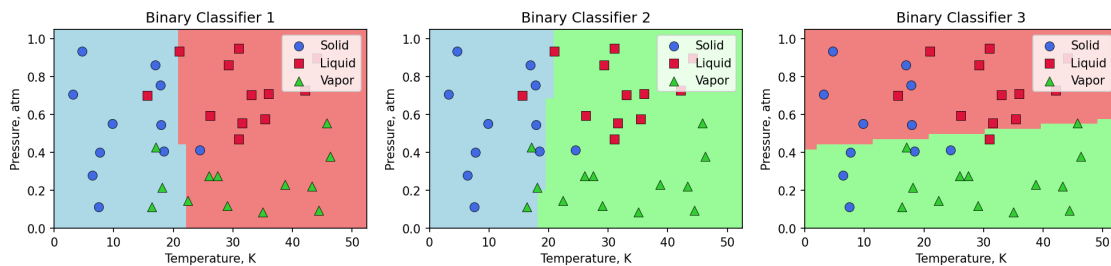
[ ]: plot_data(x,y,c)
     plot_colors(classifiers)
     plt.show()

```



We can also look at the results of each binary classifier:

```
[ ]: plt.figure(figsize=(16,3),dpi=150)
for i in range(3):
    plt.subplot(1,3,i+1)
    plot_data(x, y, c, title=f"Binary Classifier {i+1}", newfig=False)
    plot_colors(classifiers[i])
plt.show()
```



M3-L2-P2

September 23, 2023

1 Problem 5 (5 points)

```
[ ]: import numpy as np
import matplotlib.pyplot as plt
from matplotlib.colors import ListedColormap
from sklearn.linear_model import LogisticRegression
```

2 One-vs-All (One-vs-Rest) Multinomial Classification

2.1 Load Dataset

(Don't edit this)

- (x,y) values are stored in rows of xy
- class values are in c

```
[ ]: x = np.array([7.4881350392732475,16.351893663724194,22.427633760716436,29.
↪04883182996897,35.03654799338904,44.45894113066656,6.375872112626925,18.
↪117730007820796,26.036627605010292,27.434415188257777,38.71725038082664,43.
↪28894919752904,7.680445610939323,18.45596638292661,17.110360581978867,24.
↪47129299701541,31.002183974403255,46.32619845547938,9.781567509498505,17.
↪90012148246819,26.186183422327638,31.59158564216724,35.41479362252932,45.
↪805291762864556,3.182744258689332,15.599210213275237,17.833532874090462,33.
↪04668917049584,36.018483217500716,42.146619399905234,4.64555612104627,16.
↪942336894342166,20.961503322165484,29.284339488686488,30.98789800436355,44.
↪17635497075877,])
y = np.array([0.11120957227224215,0.1116933996874757,0.14437480785146242,0.
↪11818202991034835,0.0859507900573786,0.09370319537993416,0.
↪2797631195927265,0.216022547162927,0.27667667154456677,0.27706378696181594,0.
↪2310382561073841,0.22289262976548535,0.40154283509241845,0.
↪4063710770942623,0.427019677041788,0.41386015134623205,0.46883738380592266,0.
↪38020448107480287,0.5508876756094834,0.5461309517884996,0.5953108325465398,0.
↪5553291602539782,0.5766310772856306,0.5544425592001603,0.705896958364552,0.
↪7010375141164304,0.7556329589465274,0.7038182951348614,0.7096582361680054,0.
↪7268725170660963,0.9320993229847936,0.8597101275793062,0.9337944907498804,0.
↪8596098407893963,0.9476459465013396,0.8968651201647702,])
xy = np.vstack([x,y]).T
```

```
c = np.  
↪array([0,2,2,2,2,2,0,2,2,2,2,2,0,0,2,0,1,2,0,0,1,1,1,2,0,1,0,1,1,1,0,0,1,1,1,1,])
```

2.2 Binomial classification function

You are given a function that performs binomial classification by using sklearn's LogisticRegression tool: `prob = get_ovr_prob_function(xy, c, A)`

To use it, input: - `xy`, an array in which each row contains (x,y) coordinates of data points - `c`, an array that specifies the class each point in `xy` belongs to - `A`, the class of the group (0, 1, or 2 in this problem) – classifies into `A` or “rest”

The function outputs a probability function (`prob()` in this case), used to determine the probability that each `xy` is class `A` or [not `A`], such as by using `prob(xy)`.

```
[ ]: def get_ovr_prob_function(xy, c, A):  
      c_new = (c == A).astype(int)  
  
      model = LogisticRegression()  
      model.fit(xy,c_new)  
  
      def prob(xy):  
          pred = model.predict_proba(xy)[: ,1]  
          return pred.flatten()  
  
      return prob
```

2.3 Coding an OvR classifier

Now you will create a one-vs-rest classifier to do multinomial classification. Binomial predictions will be made for each class vs. the rest of the classes. The class whose binomial prediction gives the highest probability is the selected class.

Complete the two functions we have started: - `generate_ovr_prob_functions(xy, c)` which returns a list of binary classifier probability functions for all possible classes (0, 1, and 2 in this problem) - `classify_ovr(probs, xy)` which loops through a list of ovr classifier probabilities and gets the probability of belonging to each class, for each point in `xy`. Then taking the highest probability for each, return the overall class predictions for each point.

```
[ ]: def generate_ovr_prob_functions(xy, c):  
      A = np.unique(c)  
      classifier_list = []  
      for A_i in A:  
          classifier_list.append(get_ovr_prob_function(xy, c, A_i))  
      return classifier_list  
  
      def classify_ovr(probs, xy):  
          results = []  
          for prob in probs:
```

```

        results.append(prob(xy))

    results = np.array(results)
    maximum = np.max(results, axis=0)
    return np.array([np.where(results[:,i] == maximum[i])[0] for i in
↪range(len(maximum))]).flatten()

```

2.3.1 Trying out our multinomial classifier:

```

[ ]: probs = generate_ovr_prob_functions(xy, c)
    preds = classify_ovr(probs, xy)
    accuracy = np.sum(preds == c) / len(c) * 100
    print("True Classes:", c)
    print(" Predictions:", preds)
    print("    Accuracy:", accuracy, r"%")

```

```

True Classes: [0 2 2 2 2 2 0 2 2 2 2 2 0 0 2 0 1 2 0 0 1 1 1 2 0 1 0 1 1 1 0 0 1
1 1 1]
Predictions: [0 0 2 2 2 2 0 0 2 2 2 2 0 0 0 2 2 2 0 0 1 1 1 1 0 0 0 1 1 1 0 0 1
1 1 1]
Accuracy: 80.55555555555556 %

```

2.3.2 Plotting multinomial classifier results

Here, we have made some plotting functions – run these cells to visualize the decision boundaries.

```

[ ]: def plot_data(x, y, c, title="Phase of simulated material", newfig=True):
    xlim = [0,52.5]
    ylim = [0,1.05]
    markers = [dict(marker="o", color="royalblue"), dict(marker="s",
↪color="crimson"), dict(marker="^", color="limegreen")]
    labels = ["Solid", "Liquid", "Vapor"]

    if newfig:
        plt.figure(dpi=150)

    for i in range(1+max(c)):
        plt.scatter(x[c==i], y[c==i], s=60, **(markers[i]), edgecolor="black",
↪linewidths=0.4,label=labels[i])

    plt.title(title)
    plt.legend(loc="upper right")
    plt.xlim(xlim)
    plt.ylim(ylim)
    plt.xlabel("Temperature, K")
    plt.ylabel("Pressure, atm")
    plt.box(True)

```

```

def plot_ovr_colors(probs, res=40):
    xlim = [0,52.5]
    ylim = [0,1.05]
    xvals = np.linspace(*xlim,res)
    yvals = np.linspace(*ylim,res)
    x,y = np.meshgrid(xvals,yvals)
    XY = np.concatenate((x.reshape(-1,1),y.reshape(-1,1)),axis=1)

    color = classify_ovr(probs,XY).reshape(res,res)

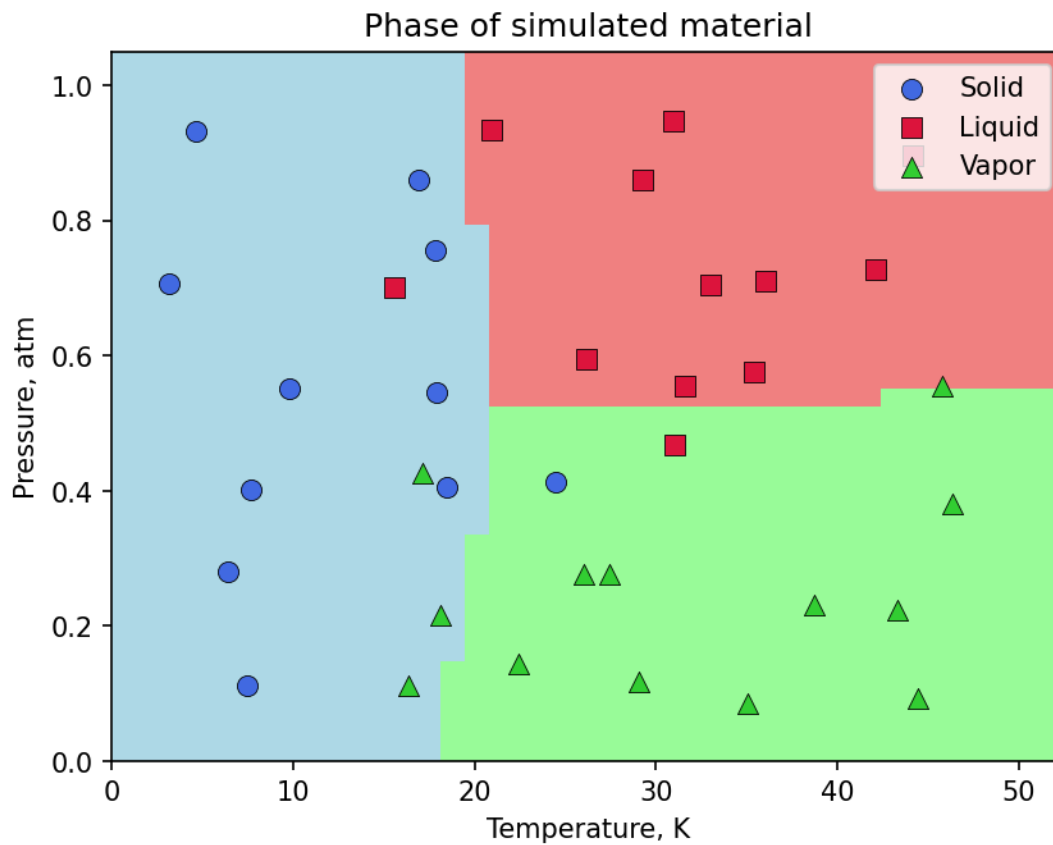
    cmap = ListedColormap(["lightblue","lightcoral","palegreen"])
    plt.pcolor(x, y, color, shading="nearest", zorder=-1,
    cmap=cmap,vmin=0,vmax=2)
    return

```

```

[ ]: plot_data(x,y,c)
     plot_ovr_colors(probs)
     plt.show()

```



M3-L2-P3

September 23, 2023

1 Problem 6 (5 points)

```
[ ]: import numpy as np
import matplotlib.pyplot as plt
from matplotlib.colors import ListedColormap
from sklearn.linear_model import LogisticRegression
```

2 Multinomial Classification in SciKit-Learn

2.1 Load Dataset

(Don't edit this)

- (x,y) values are stored in rows of xy
- class values are in c

```
[ ]: x = np.array([7.4881350392732475,16.351893663724194,22.427633760716436,29.
↪04883182996897,35.03654799338904,44.45894113066656,6.375872112626925,18.
↪117730007820796,26.036627605010292,27.434415188257777,38.71725038082664,43.
↪28894919752904,7.680445610939323,18.45596638292661,17.110360581978867,24.
↪47129299701541,31.002183974403255,46.32619845547938,9.781567509498505,17.
↪90012148246819,26.186183422327638,31.59158564216724,35.41479362252932,45.
↪805291762864556,3.182744258689332,15.599210213275237,17.833532874090462,33.
↪04668917049584,36.018483217500716,42.146619399905234,4.64555612104627,16.
↪942336894342166,20.961503322165484,29.284339488686488,30.98789800436355,44.
↪17635497075877,])
y = np.array([0.11120957227224215,0.1116933996874757,0.14437480785146242,0.
↪11818202991034835,0.0859507900573786,0.09370319537993416,0.
↪2797631195927265,0.216022547162927,0.27667667154456677,0.27706378696181594,0.
↪2310382561073841,0.22289262976548535,0.40154283509241845,0.
↪4063710770942623,0.427019677041788,0.41386015134623205,0.46883738380592266,0.
↪38020448107480287,0.5508876756094834,0.5461309517884996,0.5953108325465398,0.
↪5553291602539782,0.5766310772856306,0.5544425592001603,0.705896958364552,0.
↪7010375141164304,0.7556329589465274,0.7038182951348614,0.7096582361680054,0.
↪7268725170660963,0.9320993229847936,0.8597101275793062,0.9337944907498804,0.
↪8596098407893963,0.9476459465013396,0.8968651201647702,])
xy = np.vstack([x,y]).T
```

```
c = np.
↪array([0,2,2,2,2,2,0,2,2,2,2,2,0,0,2,0,1,2,0,0,1,1,1,2,0,1,0,1,1,1,0,0,1,1,1,1,])
```

2.2 Logistic Regression

SciKit-Learn's Logistic Regression model will perform multinomial classification automatically.

Create an sklearn `LogisticRegression()` class and train this model on the dataset

Details about how to use this are here: https://scikit-learn.org/stable/modules/generated/sklearn.linear_model.LogisticRegression.html

```
[ ]: from sklearn.linear_model import LogisticRegression
```

```
def get_logistic_regressor(features, classes):
    # YOUR CODE GOES HERE
    # - Instantiate model with regularization
    # - Fit model to data
    model = LogisticRegression()
    model.fit(features, classes)

    return model
```

```
[ ]: model = get_logistic_regressor(xy, c)
preds = model.predict(xy)
accuracy = np.sum(preds == c) / len(c) * 100
print("True Classes:", c)
print(" Predictions:", preds)
print("    Accuracy:", accuracy, r"%")
```

```
True Classes: [0 2 2 2 2 2 0 2 2 2 2 2 0 0 2 0 1 2 0 0 1 1 1 2 0 1 0 1 1 1 0 0 1
1 1 1]
Predictions: [0 0 2 2 2 2 0 0 2 2 2 2 0 0 0 2 2 2 0 0 1 1 1 1 0 0 0 1 1 1 0 0 1
1 1 1]
Accuracy: 80.55555555555556 %
```

2.3 Plotting Multinomial Classifier Results

Here, we have made some plotting functions – run these cells to visualize the decision boundaries.

```
[ ]: def plot_data(x, y, c, title="Phase of simulated material", newfig=True):
    xlim = [0,52.5]
    ylim = [0,1.05]
    markers = [dict(marker="o", color="royalblue"), dict(marker="s",
↪color="crimson"), dict(marker="^", color="limegreen")]
    labels = ["Solid", "Liquid", "Vapor"]

    if newfig:
        plt.figure(dpi=150)
```



```

    for i in range(1+max(c)):
        plt.scatter(x[c==i], y[c==i], s=60, **(markers[i]), edgecolor="black",
↳ linewidths=0.4, label=labels[i])

plt.title(title)
plt.legend(loc="upper right")
plt.xlim(xlim)
plt.ylim(ylim)
plt.xlabel("Temperature, K")
plt.ylabel("Pressure, atm")
plt.box(True)

def plot_sklearn_colors(model, res=40):
    xlim = [0,52.5]
    ylim = [0,1.05]
    xvals = np.linspace(*xlim,res)
    yvals = np.linspace(*ylim,res)
    x,y = np.meshgrid(xvals,yvals)
    XY = np.concatenate((x.reshape(-1,1),y.reshape(-1,1)),axis=1)

    color = model.predict(XY).reshape(res,res)

    cmap = ListedColormap(["lightblue","lightcoral","palegreen"])
    plt.pcolor(x, y, color, shading="nearest", zorder=-1,
↳ cmap=cmap,vmin=0,vmax=2)
    return

```

```

[ ]: plot_data(x,y,c)
     plot_sklearn_colors(model)
     plt.show()

```

