HW1 Programming Problem 5 (30 points)

Problem Description

Here, you will perform *weighted* KNN regression.

After you write your own code for weighted KNN regression, you will also try out sklearn's built-in KNN regressor.

Fill out the notebook as instructed, making the requested plots and printing necessary values.

Summary of deliverables:

Functions:

- `weighted_knn(w1, w2, k)`

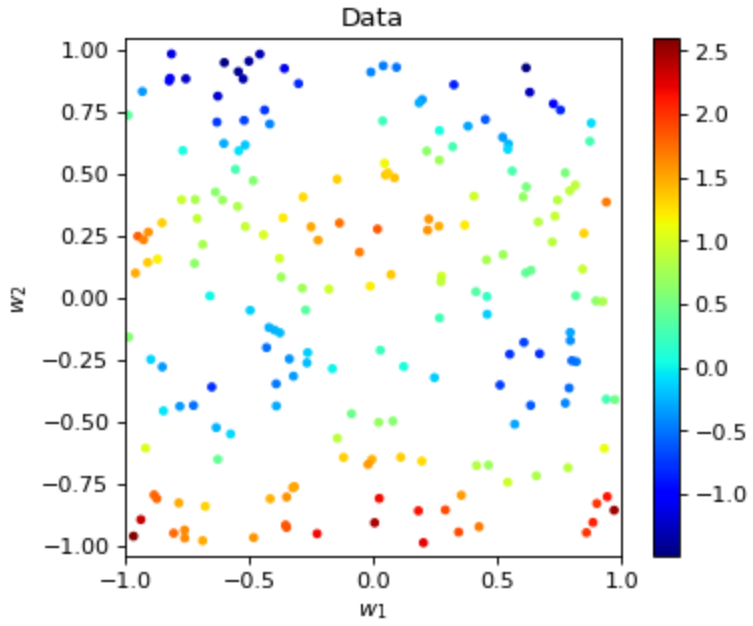
Plots:

- 3 plots of by-hand KNN results
- 3 plots of sklearn.

In [ ]:
```
import numpy as np
import matplotlib.pyplot as plt
from sklearn.neighbors import KNeighborsRegressor

# Data generation -- don't change
np.random.seed(42)
N = 200
w1_data = np.random.uniform(-1,1,N)
w2_data = np.random.uniform(-1,1,N)
L_data = np.cos(4*w1_data) + np.sin(5*w2_data) + 2*w1_data**2 - w2_data/2
# (end of data generation)

plt.figure(figsize=(5,4.2),dpi=80)
plt.scatter(w1_data,w2_data,s=10,c=L_data,cmap="jet")
plt.colorbar()
plt.axis("equal")
plt.xlabel("$w_1$")
plt.ylabel("$w_2$")
plt.xlim(-1,1)
plt.ylim(-1,1)
plt.title("Data")
plt.show()
```



Weighted KNN function

Here, define a function, `weighted_knn(w1, w2, k)`, which takes in a point at [`w1`, `w2`] and a `k` value, and returns the weighted KNN prediction.

- As in the lecture activity, data is in the variables `w1_data`, `w2_data`, and `L_data`.
- You can create as many helper functions as you want
- The key difference between unweighted and weighted KNN is summarized below:

Unweighted KNN

1. Find the k data points closest to the target point w
2. Get the output values at each of these points
3. Average these values together: this is the prediction at w

Weighted KNN

1. Find the k data points closest to the target point w
2. Compute the proximity of each of these points as $\text{prox}_i = 1/(\text{distance}(w, w_i) + 1e-9)$
3. For each w_i, multiply prox_i by the output value at w_i, and divide by the sum of all k proximities
4. Add all k of these results together: this is the prediction at w

In [ ]:
```
def distance(w1, w2):
    w1_diff = w1_data - w1
    w2_diff = w2_data - w2
    return np.sqrt(np.square(w1_diff) + np.square(w2_diff))

def weighted_knn(w1, w2, k):
    d = distance(w1, w2)
    indecies = np.argpartition(d, k)[:k]
    prox = np.divide(1, d[indecies] + 1e-9)
    return np.sum(np.divide(L_data[indecies].T*prox, np.sum(prox)))
```

Plotting

Now create 3 plots showing KNN regressor predictions for k values [1, 5, 25].

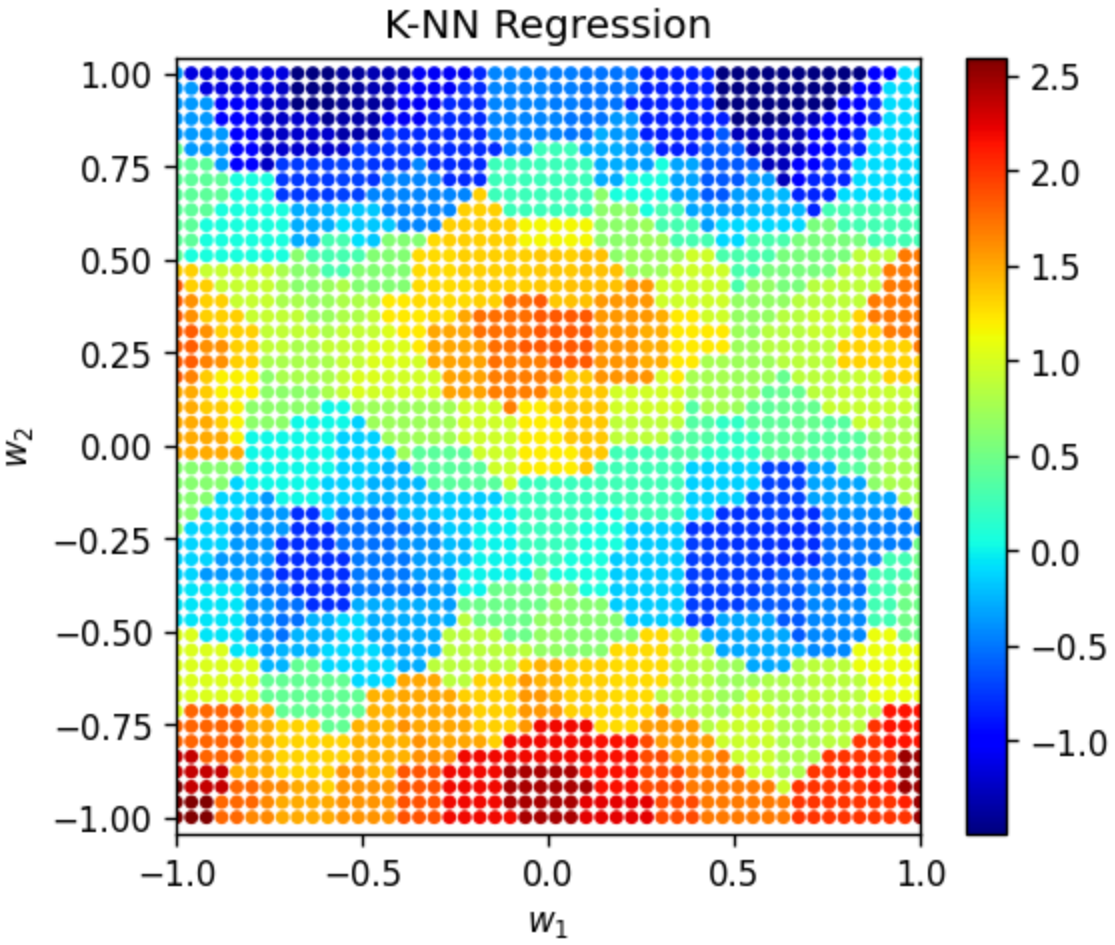
You should plot a 50x50 grid of points on a grid for `w1` and `w2` values between -1 and 1. Consult the lecture activity for how to do this.

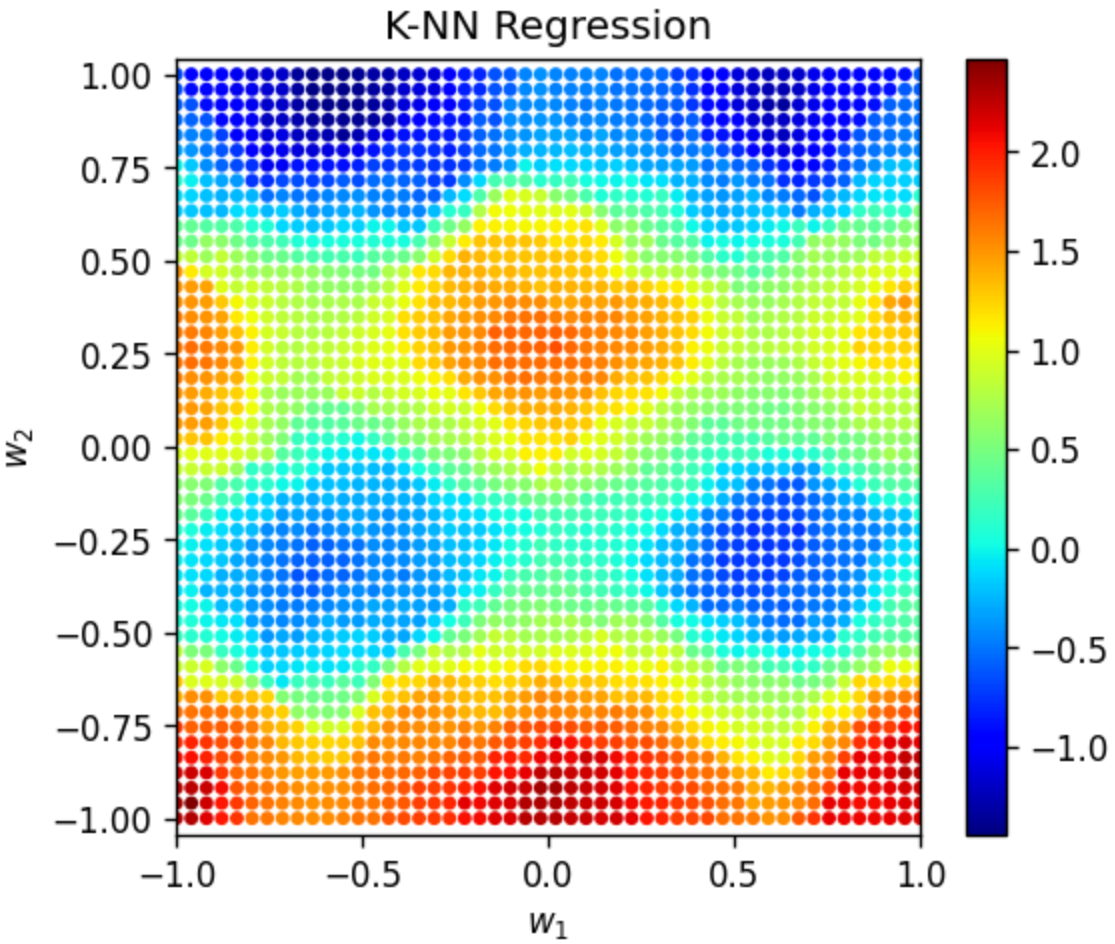
We recommend creating a function, e.g. `plot(k)`, so that you need to rewrite less code.

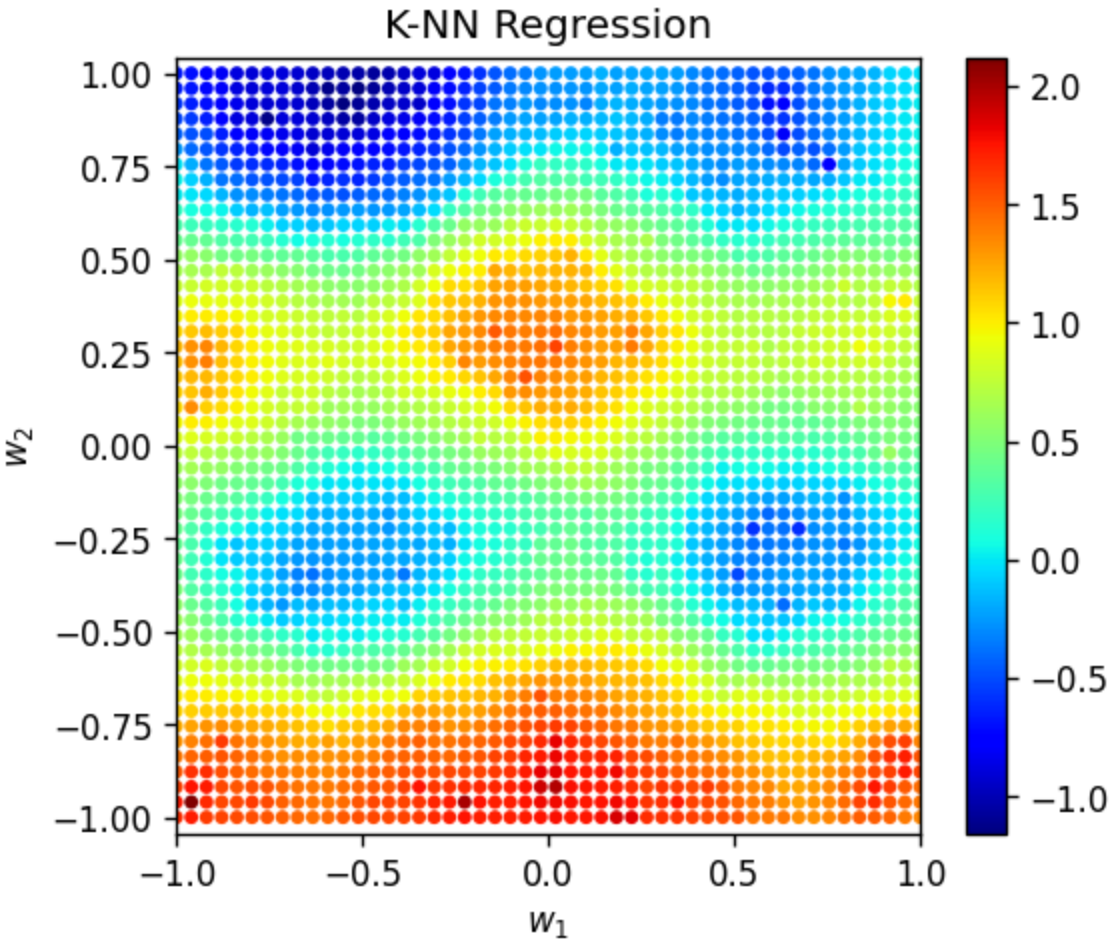
In [ ]:
```
w1_vals = np.linspace(-1,1,50)
w2_vals = np.linspace(-1,1,50)
w1s, w2s = np.meshgrid(w1_vals, w2_vals)
w1_grid, w2_grid = w1s.flatten(), w2s.flatten()

def plot(k:list):
    for i in range(len(k)):
        L_grid = np.zeros_like(w1_grid)
        for j in range(len(L_grid)):
            L_grid[j] = weighted_knn(w1_grid[j], w2_grid[j],k[i])
        plt.figure(figsize=(5,4.2),dpi=120)
        plt.scatter(w1_grid,w2_grid,s=10,c=L_grid,cmap="jet")
        plt.colorbar()
        plt.axis("equal")
        plt.xlabel("$w_1$")
        plt.ylabel("$w_2$")
        plt.xlim(-1,1)
        plt.ylim(-1,1)
        plt.title("K-NN Regression")
    plt.show()

plot([1, 5, 25])
```


K-NN Regression
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
w_2
−1.0
−0.5
0.0
0.5
1.0
w_1
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0


K-NN Regression
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
w_2
−1.0
−0.5
0.0
0.5
1.0
w_1
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0



Using SciKit-Learn

We can also use sklearn's `KNeighborsRegressor()`, which is a very efficient implementation of KNN regression.

The code to do this has been done for one case below. First, make note of how this is done.

In [ ]:
```
model = KNeighborsRegressor(n_neighbors = 1, weights="distance")
X = np.vstack([w1_data,w2_data]).T
model.fit(X, L_data)

# Get a prediction at a point (0, 0):
print(model.predict(np.array([[0,0]])))
```

```
[1.19743607]
```

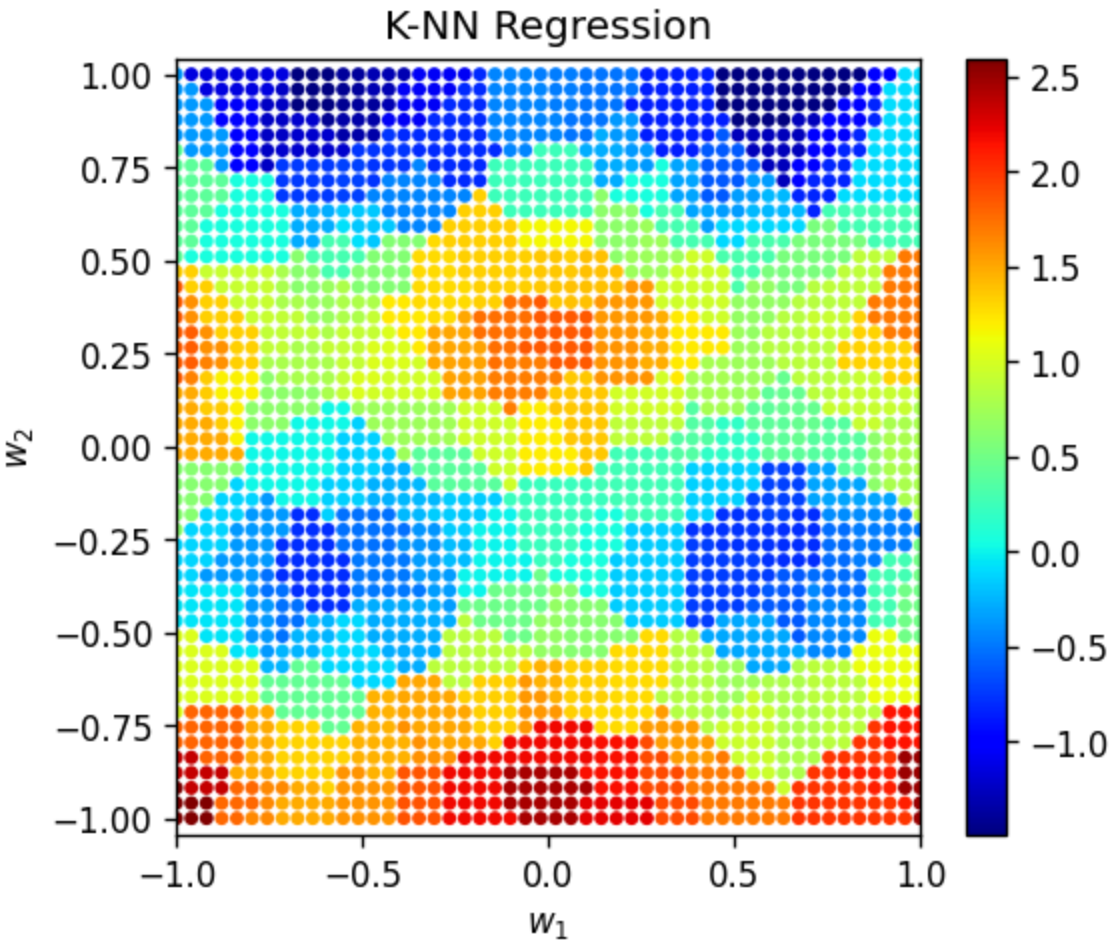
Now create 3 plots for the same values of k as before, using this KNN implementation instead. You can make sure these are visually the same as your from-scratch KNN regressor.

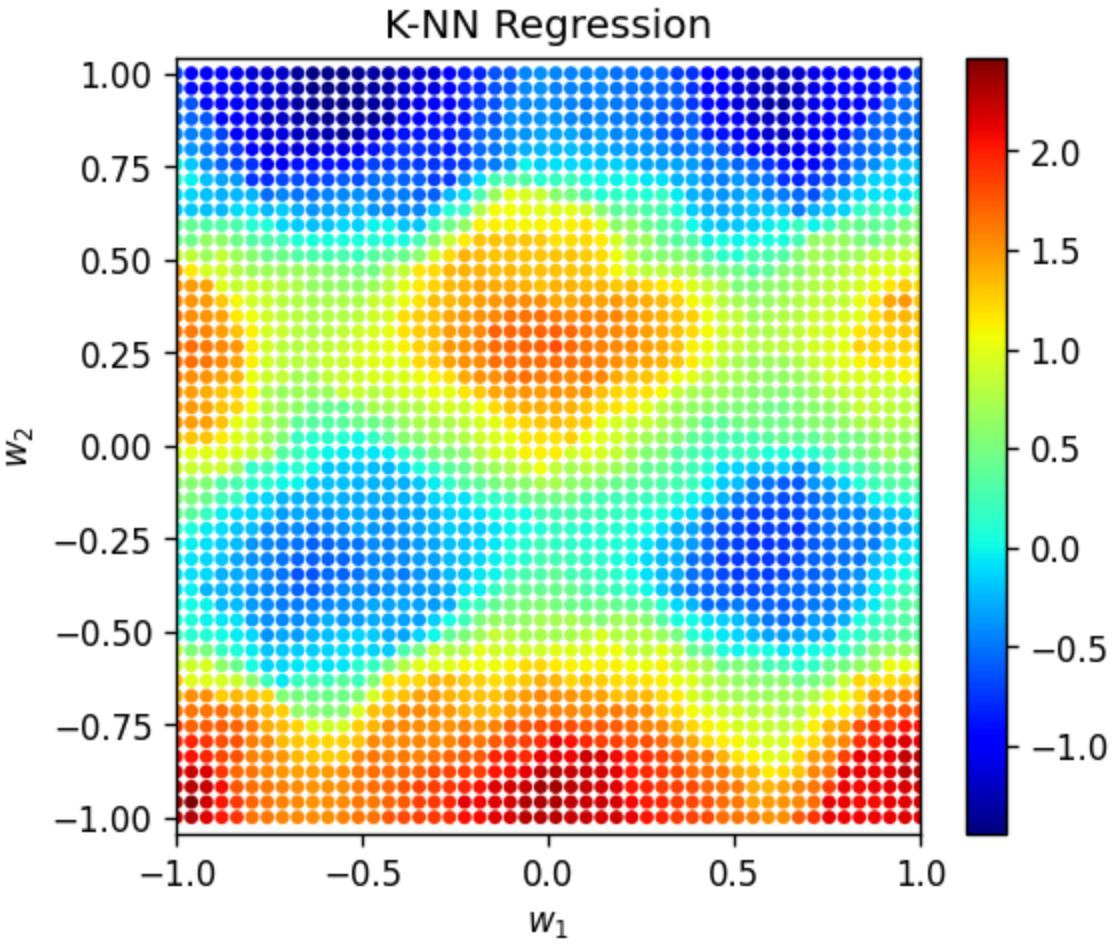
In [ ]:
```
def plot_kNeighborsRegressor(k:list):
    for i in range(len(k)):
        model = KNeighborsRegressor(n_neighbors = k[i], weights="distance")
        X = np.vstack([w1_data,w2_data]).T
        model.fit(X, L_data)

        L_grid = np.zeros_like(w1_grid)
        for j in range(len(L_grid)):
            L_grid[j] = model.predict(np.array([[w1_grid[j], w2_grid[j]]]))
        plt.figure(figsize=(5,4.2),dpi=120)
```

```
        plt.scatter(w1_grid,w2_grid,s=10,c=L_grid,cmap="jet")
        plt.colorbar()
        plt.axis("equal")
        plt.xlabel("$w_1$")
        plt.ylabel("$w_2$")
        plt.xlim(-1,1)
        plt.ylim(-1,1)
        plt.title("K-NN Regression")
    plt.show()

plot_kNeighborsRegressor([1, 5, 25])
```

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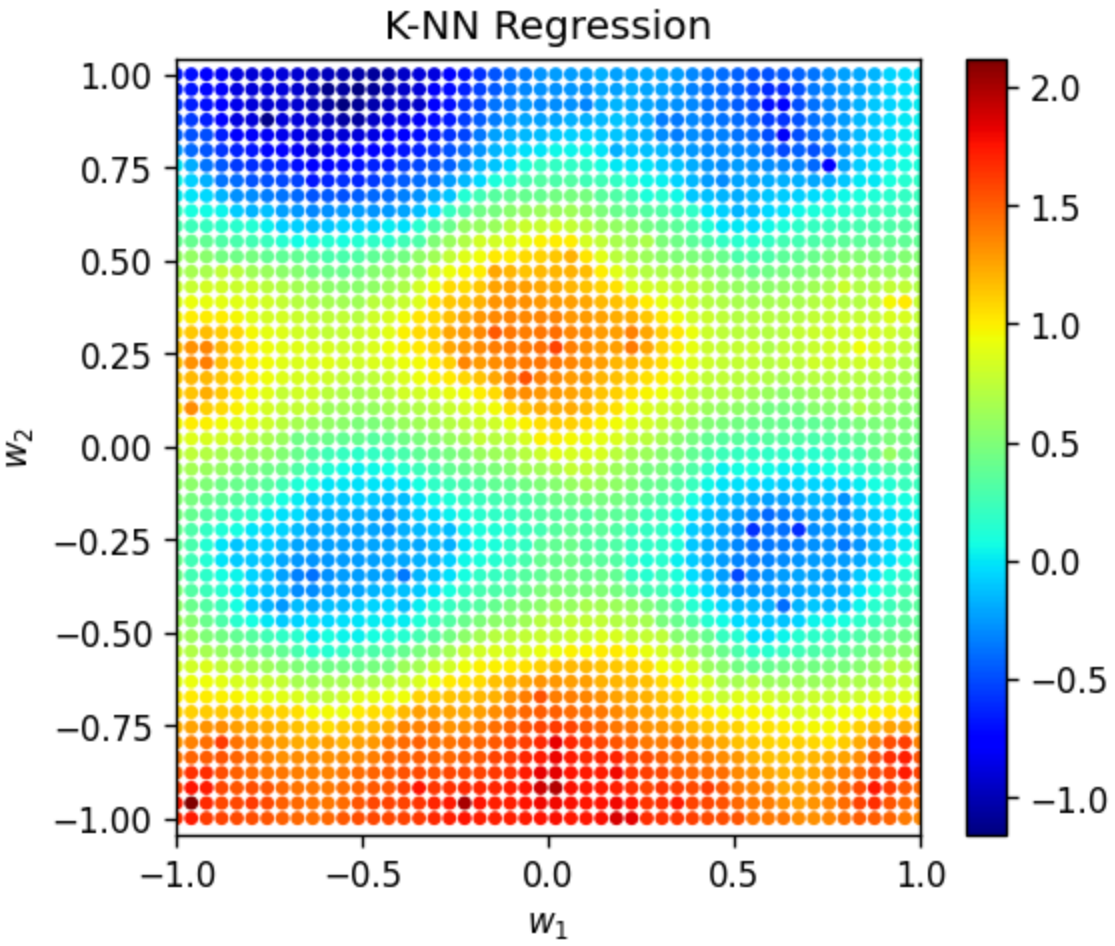
0.00
−0.25
−0.50
−0.75
−1.00
w_2
−1.0
−0.5
0.0
0.5
1.0
w_1
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0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w_2
−1.0
−0.5
0.0
0.5
1.0
w_1
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0