M5-L1-P3

October 3, 2023

1 Problem 3 (6 Points)

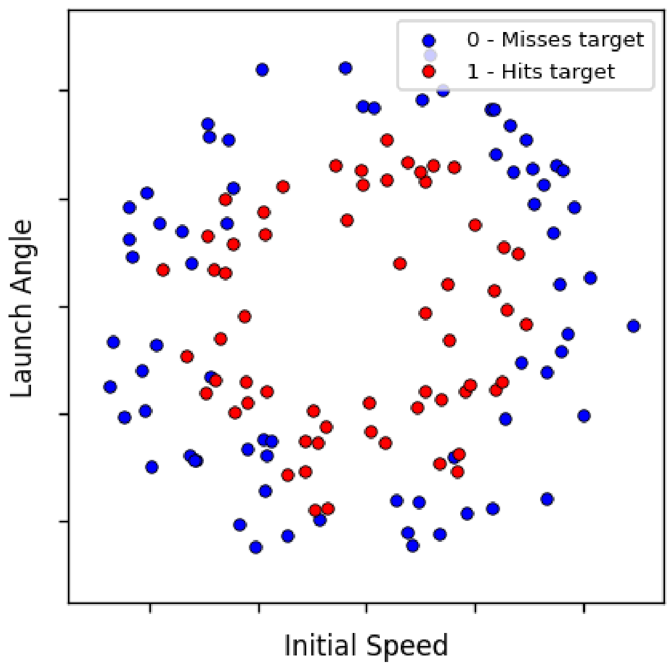
Let's revisit the initial speed vs. launch angle data from the logistic regression module. This time, you will train a decision tree classifier to predict whether a projectile launched with a given speed and angle will hit a target.

Run this cell to load the data and decision tree tools:

[ ]:
```
import numpy as np
import matplotlib.pyplot as plt
from sklearn.tree import DecisionTreeClassifier, plot_tree
from matplotlib.colors import ListedColormap

y = np.array([0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1,
  1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
  1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
  1, 1, 1, 1, 1, 1, 1, 1])
```

```python
x1 = np.array([0.02693745, 0.41186575, 0.10363585, 0.08489663, 0.09512868, 0.
↪31121109, 0.16015486, 0.75698706, 0.86103276, 0.25450354, 0.59727713, 0.
↪11117203, 0.2118569 , 0.90002177, 0.88339852, 0.81076366, 0.9134383 , 0.
↪66078219, 0.57511227, 0.83446708, 0.87207792, 0.63484916, 0.17641653, 0.
↪58623713, 0.77185587, 0.27969298, 0.76628621, 0.78704918, 0.03260164, 0.
↪24102818, 0.45931531, 0.5553572 , 0.0615199 , 0.05104643, 0.85777048, 0.
↪18454679, 0.17247071, 0.18382613, 0.83261753, 0.29546316, 0.24476501, 0.
↪06188762, 0.35479775, 0.84468926, 0.26562408, 0.31266695, 0.61840113, 0.
↪79493902, 0.3079022 , 0.20639025, 0.08952284, 0.11775381, 0.99160872, 0.
↪85210361, 0.60150808, 0.72871228, 0.32553542, 0.49231061, 0.06757372, 0.
↪51293352, 0.73524444, 0.80625762, 0.31447886, 0.73980573, 0.64020137, 0.
↪20844947, 0.68399447, 0.8614671 , 0.73138609, 0.8282699 , 0.6382059 , 0.
↪2402172 , 0.2191855 , 0.60897248, 0.50482995, 0.40076302, 0.69944178, 0.
↪68322982, 0.38699737, 0.7942779 , 0.66176057, 0.59454139, 0.60979337, 0.
↪28162158, 0.561978  , 0.6360264 , 0.53396978, 0.22126403, 0.20591415, 0.
↪75288355, 0.35277133, 0.12387452, 0.41024511, 0.66943243, 0.6534378 , 0.
↪6677045 , 0.75920895, 0.31393471, 0.40585142, 0.60007637, 0.22901595, 0.
↪65065447, 0.53630916, 0.6078229 , 0.50733494, 0.49252727, 0.30893962, 0.
↪69164516, 0.38543013, 0.73631178, 0.6231992 , 0.31464876, 0.20309569, 0.
↪46454817, 0.73854501, 0.25778844, 0.16899741, 0.276636  , 0.42571213, 0.
↪34623966, 0.25249608, 0.53763073, 0.57613609, 0.75106557, 0.42734051, 0.
↪27302061, 0.49041099, 0.44201602, 0.78100287, 0.23748921])-0.5
x2 = np.array([0.3501823 , 0.10349458, 0.20137442, 0.37973165, 0.71062143, 0.
↪25377085, 0.64055034, 0.29218012, 0.41610854, 0.72074402, 0.13748866, 0.
↪42862148, 0.36870966, 0.29806405, 0.68347154, 0.68944199, 0.55280589, 0.
↪21861136, 0.07986956, 0.14388321, 0.44971031, 0.07738745, 0.57988363, 0.
↪05595551, 0.74979864, 0.23396347, 0.83605613, 0.39598089, 0.43543082, 0.
↪65389891, 0.94361628, 0.13925514, 0.62396066, 0.29410959, 0.54243565, 0.
↪21246836, 0.22169931, 0.21435268, 0.37728635, 0.05211104, 0.8104757 , 0.
↪6829834 , 0.07475538, 0.63703731, 0.09345901, 0.15598365, 0.96578717, 0.
↪80986228, 0.94065416, 0.83852381, 0.30622388, 0.65524094, 0.4640243 , 0.
↪76279551, 0.8840741 , 0.86703352, 0.2497341 , 0.87174298, 0.59292618, 0.
↪86911399, 0.8654347 , 0.75457663, 0.2220472 , 0.7832285 , 0.90191786, 0.
↪81549632, 0.11524284, 0.75269284, 0.12477074, 0.72641957, 0.32692003, 0.
↪70036832, 0.56839658, 0.34169059, 0.3212157 , 0.304839  , 0.65177393, 0.
↪34079171, 0.1943221 , 0.46750584, 0.75934886, 0.31240097, 0.73073311, 0.
↪32049905, 0.58032973, 0.20709977, 0.24701365, 0.36393944, 0.63103063, 0.
↪61059462, 0.18643247, 0.56799519, 0.24591095, 0.22541827, 0.4384616 , 0.
↪19224338, 0.49279951, 0.63452085, 0.12069456, 0.74973512, 0.44061972, 0.
↪54129865, 0.73561255, 0.48845014, 0.26644964, 0.7272455 , 0.67658067, 0.
↪3527117 , 0.25076322, 0.52805314, 0.76158356, 0.34050983, 0.3398095 , 0.
↪6608739 , 0.34343993, 0.30274956, 0.40601433, 0.36011736, 0.27654899, 0.
↪72299134, 0.61689563, 0.8099134 , 0.76758364, 0.36026671, 0.12536261, 0.
↪48062248, 0.75285467, 0.76160529, 0.59633481, 0.56288792])-0.5
X = np.vstack([x1, x2]).T

def plot_data(X,y):
```

```
    colors=["blue","red"]
    labels = ["0 - Misses target", "1 - Hits target"]
    for i in range(2):
        plt.
 ↪scatter(X[y==i,0],X[y==i,1],s=20,c=colors[i],edgecolors="black",linewidths=.
 ↪5,label=labels[i])
        plt.xlabel("Initial Speed")
        plt.ylabel("Launch Angle")
        plt.legend(loc="upper right",prop={'size':8})
        ax = plt.gca()
        ax.set_xticklabels([])
        ax.set_yticklabels([])
        plt.xlim([-0.55,.55])
        plt.ylim([-0.55,.55])

plt.figure(figsize=(4,4),dpi=120)
plot_data(X,y)
plt.show()
```

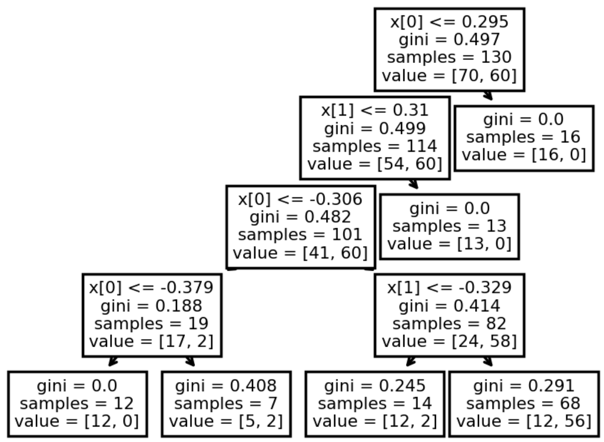


1.1 Training a decision tree classifier.

Below, a decision tree of max depth 4 is trained, and the tree is visualized with `plot_tree()`.

[ ]:
```
dt = DecisionTreeClassifier(max_depth=4)
dt.fit(X,y)
```

```
plt.figure(figsize=(4,3),dpi=250)
plot_tree(dt)
plt.show()
```



1.2 Accuracy on training data

Compute the accuracy on the training data with the provided function `get_dt_accuracy(dt, X, y)`. Print the result.

[ ]:
```
def get_dt_accuracy(dt, X, y):
    pred = dt.predict(X)
    return 100*np.sum(pred == y)/len(y)

print(f"Decision tree accuracy: {get_dt_accuracy(dt,X,y)}%")
```

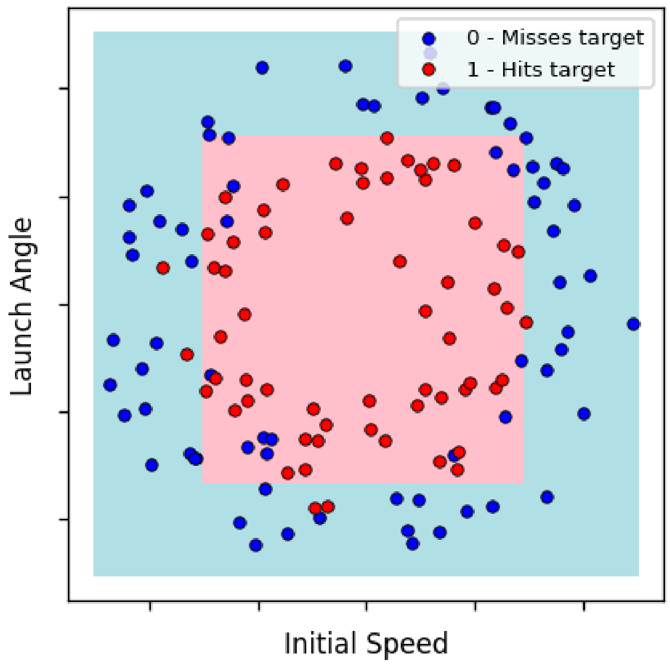
```
Decision tree accuracy: 87.6923076923077%
```

1.3 Visualizing tree predictions

By evaluating the model on a meshgrid of results, we can look at how our model performs on the input space:

[ ]:
```
vals = np.linspace(-.5,.5,100)
x1grid, x2grid = np.meshgrid(vals, vals)
X_test = np.vstack([x1grid.flatten(), x2grid.flatten()]).T

pred = dt.predict(X_test)

plt.figure(figsize=(4,4),dpi=120)
bgcolors = ListedColormap(["powderblue","pink"])
```

```
plt.pcolormesh(x1grid, x2grid, pred.reshape(x1grid.shape),␣
  ↪shading="nearest",cmap=bgcolors)
plot_data(X,y)
plt.show()
```



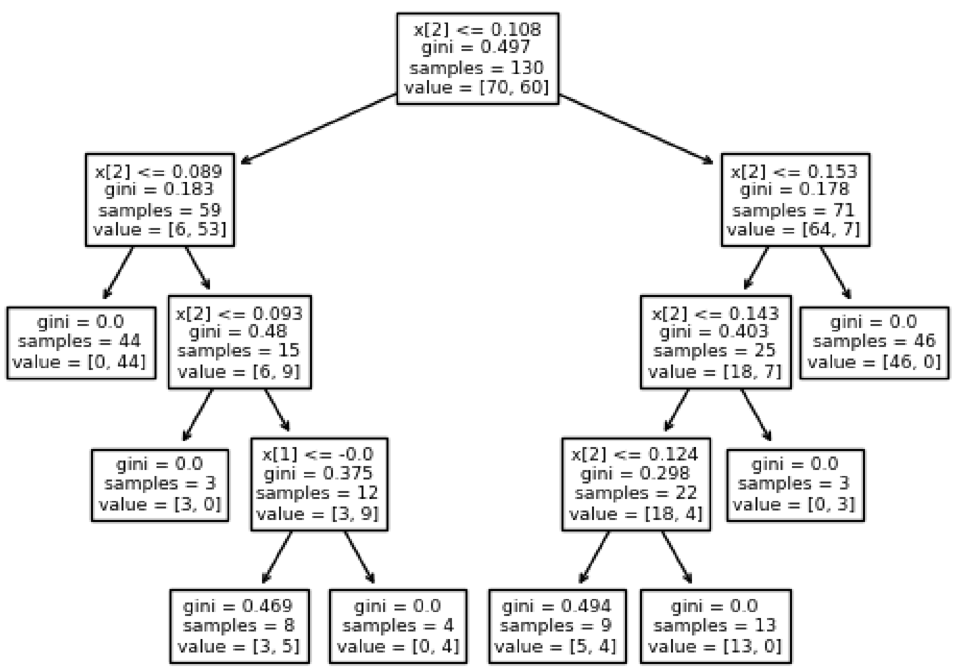
1.4 Expanded feature set

Now, we will add a third feature that (for this problem) happens to be very useful. That feature is $x_1^2 + x_2^2$. A new training input `X_ex` is generated below containing this additional feature.

Train a new decision tree, max depth 4, on this data. Then visualize the tree with `plot_tree()`.

[ ]:
```
def feature_expand(X):
    x1 = X[:,0].reshape(-1, 1)
    x2 = X[:,1].reshape(-1, 1)
    columns = [x1, x2, x1*x1 + x2*x2]
    return np.concatenate(columns, axis=1)

X_ex = feature_expand(X)

model2 = DecisionTreeClassifier(max_depth=4)
model2.fit(X_ex,y)

plot_tree(model2)
```

[ ]: [Text(0.5, 0.9, 'x[2] <= 0.108\ngini = 0.497\nsamples = 130\nvalue = [70, 60]'),
 Text(0.16666666666666666, 0.7, 'x[2] <= 0.089\ngini = 0.183\nsamples =
59\nvalue = [6, 53]'),
 Text(0.08333333333333333, 0.5, 'gini = 0.0\nsamples = 44\nvalue = [0, 44]'),
 Text(0.25, 0.5, 'x[2] <= 0.093\ngini = 0.48\nsamples = 15\nvalue = [6, 9]'),
 Text(0.16666666666666666, 0.3, 'gini = 0.0\nsamples = 3\nvalue = [3, 0]'),
 Text(0.3333333333333333, 0.3, 'x[1] <= -0.0\ngini = 0.375\nsamples = 12\nvalue
= [3, 9]'),
 Text(0.25, 0.1, 'gini = 0.469\nsamples = 8\nvalue = [3, 5]'),
 Text(0.4166666666666667, 0.1, 'gini = 0.0\nsamples = 4\nvalue = [0, 4]'),
 Text(0.8333333333333334, 0.7, 'x[2] <= 0.153\ngini = 0.178\nsamples = 71\nvalue
= [64, 7]'),
 Text(0.75, 0.5, 'x[2] <= 0.143\ngini = 0.403\nsamples = 25\nvalue = [18, 7]'),
 Text(0.6666666666666666, 0.3, 'x[2] <= 0.124\ngini = 0.298\nsamples = 22\nvalue
= [18, 4]'),
 Text(0.5833333333333334, 0.1, 'gini = 0.494\nsamples = 9\nvalue = [5, 4]'),
 Text(0.75, 0.1, 'gini = 0.0\nsamples = 13\nvalue = [13, 0]'),
 Text(0.8333333333333334, 0.3, 'gini = 0.0\nsamples = 3\nvalue = [0, 3]'),
 Text(0.9166666666666666, 0.5, 'gini = 0.0\nsamples = 46\nvalue = [46, 0]')]



1.5 Accuracy on training data: expanded features

Compute the accuracy of this new model its training data. It should have increased. Note that the useful features to expand will vary significantly from problem to problem.

```
[ ]: print(f"Decision tree 2 accuracy: {get_dt_accuracy(model2,X_ex,y)}%")
```

```
Decision tree 2 accuracy: 94.61538461538461%
```

1.6 Visualizing expanded feature results

Use your model to make a prediction called `pred` on the data `X_test_ex`, an expanded meshgrid of points, as indicated. This code will plot the class decisions. Note the difference between this and the previous model, which only had speed and angle as features.

```
[ ]: X_test_ex = feature_expand(X_test)

pred = model2.predict(X_test_ex)

plt.figure(figsize=(4,4),dpi=120)
bgcolors = ListedColormap(["powderblue","pink"])
plt.pcolormesh(x1grid, x2grid, pred.reshape(x1grid.shape),␣
 ↪shading="nearest",cmap=bgcolors)
plot_data(X,y)
plt.show()
```