M3-L2-P1

September 23, 2023

1 Problem 4 (5 points)

[ ]:
```
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.colors import ListedColormap
from scipy.stats import mode
from sklearn.linear_model import LogisticRegression
```

2 One-vs-One Multinomial Classification

2.1 Load Dataset

(Don't edit this)

- (x,y) values are stored in rows of `xy`
- class values are in `c`

[ ]:
```
x = np.array([7.4881350392732475,16.351893663724194,22.427633760716436,29.
 ↪04883182996897,35.03654799338904,44.45894113066656,6.375872112626925,18.
 ↪117730007820796,26.036627605010292,27.434415188257777,38.71725038082664,43.
 ↪28894919752904,7.680445610939323,18.45596638292661,17.110360581978867,24.
 ↪47129299701541,31.002183974403255,46.32619845547938,9.781567509498505,17.
 ↪90012148246819,26.186183422327638,31.59158564216724,35.41479362252932,45.
 ↪805291762864556,3.182744258689332,15.599210213275237,17.833532874090462,33.
 ↪04668917049584,36.018483217500716,42.146619399905234,4.64555612104627,16.
 ↪942336894342166,20.961503322165484,29.284339488686488,30.98789800436355,44.
 ↪17635497075877,])
y = np.array([0.11120957227224215,0.1116933996874757,0.14437480785146242,0.
 ↪11818202991034835,0.0859507900573786,0.09370319537993416,0.
 ↪2797631195927265,0.216022547162927,0.27667667154456677,0.27706378696181594,0.
 ↪2310382561073841,0.22289262976548535,0.40154283509241845,0.
 ↪4063710770942623,0.427019677041788,0.41386015134623205,0.46883738380592266,0.
 ↪38020448107480287,0.5508876756094834,0.5461309517884996,0.5953108325465398,0.
 ↪5553291602539782,0.5766310772856306,0.5544425592001603,0.705896958364552,0.
 ↪7010375141164304,0.7556329589465274,0.7038182951348614,0.7096582361680054,0.
 ↪7268725170660963,0.9320993229847936,0.8597101275793062,0.9337944907498804,0.
 ↪8596098407893963,0.9476459465013396,0.8968651201647702,])
xy = np.vstack([x,y]).T
```

```
c = np.
 ↪array([0,2,2,2,2,2,0,2,2,2,2,2,0,0,2,0,1,2,0,0,1,1,1,2,0,1,0,1,1,1,0,0,1,1,1,1,])
```

2.2 Binomial classification function

You are given a function that performs binomial classification by using sklearn's `LogisticRegression` tool: `classify = get_binomial_classifier(xy, c, A, B)`

To use it, input: - `xy`, an array in which each row contains (x,y) coordinates of data points - `c`, an array that specifies the class each point in `xy` belongs to - `A`, the class of the first group (0, 1, or 2 in this problem) - `B`, the class of the second group (0, 1, or 2 in this problem), but different from `A`

The function outputs a classifier function (`classify()` in this case), used to classify any new `xy` into group A or B, such as by using `classify(xy)`.

[ ]:
```
def get_binomial_classifier(xy, c, A, B):
    assert A != B
    xyA, xyB = xy[c==A], xy[c==B]
    cA, cB = c[c==A], c[c==B]
    model = LogisticRegression()
    xy_new = np.concatenate([xyA, xyB], 0)
    c_new = np.concatenate([cA, cB], 0)
    model.fit(xy_new,c_new)

    def classify(xy):
        pred = model.predict(xy)
        return pred

    return classify
```

2.3 Coding a 1v1 classifier

Now you will create a one-vs-one classifier to do multinomial classification. This will generate binomial classifiers for each pair of classes in the dataset. Then to predict the class of a new point, classify it using each of the binomial classifiers, and select the majority winner as the class prediction.

Complete the two functions we have started: - `generate_all_classifiers(xy, c)` which returns a list of binary classifier functions for all possible pairs of classes (among 0, 1, and 2 in this problem) - `classify_majority(classifiers, xy)` which loops through a list of classifiers and gets their predictions for each point in `xy`. Then using a majority voting scheme at each point, return the overall class predictions for each point.

[ ]:
```
def generate_all_classifiers(xy, c):
    # YOUR CODE GOES HERE
    # Use get_binomial_classifier() to get binomial classifiers for each pair␣
 ↪of classes,
    # and return a list of these classifiers
    fun1 = get_binomial_classifier(xy, c, 0, 1)
```

```
    fun2 = get_binomial_classifier(xy, c, 0, 2)
    fun3 = get_binomial_classifier(xy, c, 1, 2)
    return [fun1, fun2, fun3]

def classify_majority(classifiers, xy):
    results = np.zeros([len(xy), len(classifiers)])
    for i in range(len(classifiers)):
        classifier = classifiers[i]
        results_i = classifier(xy)
        for j in range(len(results_i)):
            results[j,results_i[j]] += 1

    check = np.repeat(np.array([[0, 1, 2]]), results.shape[0], axis=0)
    final_results = []
    for i in range(results.shape[0]):
        final_results.append(np.where(results[i,:] == np.max(results[i,:
 ↪]))[0][0])
    return np.array(final_results).flatten()
```

2.3.1 Trying out our multinomial classifier:

[ ]:
```
classifiers = generate_all_classifiers(xy, c)
preds = classify_majority(classifiers, xy)
accuracy = np.sum(preds == c) / len(c) * 100
print("True Classes:", c)
print(" Predictions:", preds)
print("    Accuracy:", accuracy, r"%")
```

```
True Classes: [0 2 2 2 2 2 0 2 2 2 2 2 0 0 2 0 1 2 0 0 1 1 1 2 0 1 0 1 1 1 0 0 1
1 1 1]
 Predictions: [0 0 2 2 2 2 0 0 2 2 2 2 0 0 0 2 2 2 0 0 1 1 1 1 0 0 0 1 1 1 0 0 1
1 1 1]
    Accuracy: 80.55555555555556 %
```

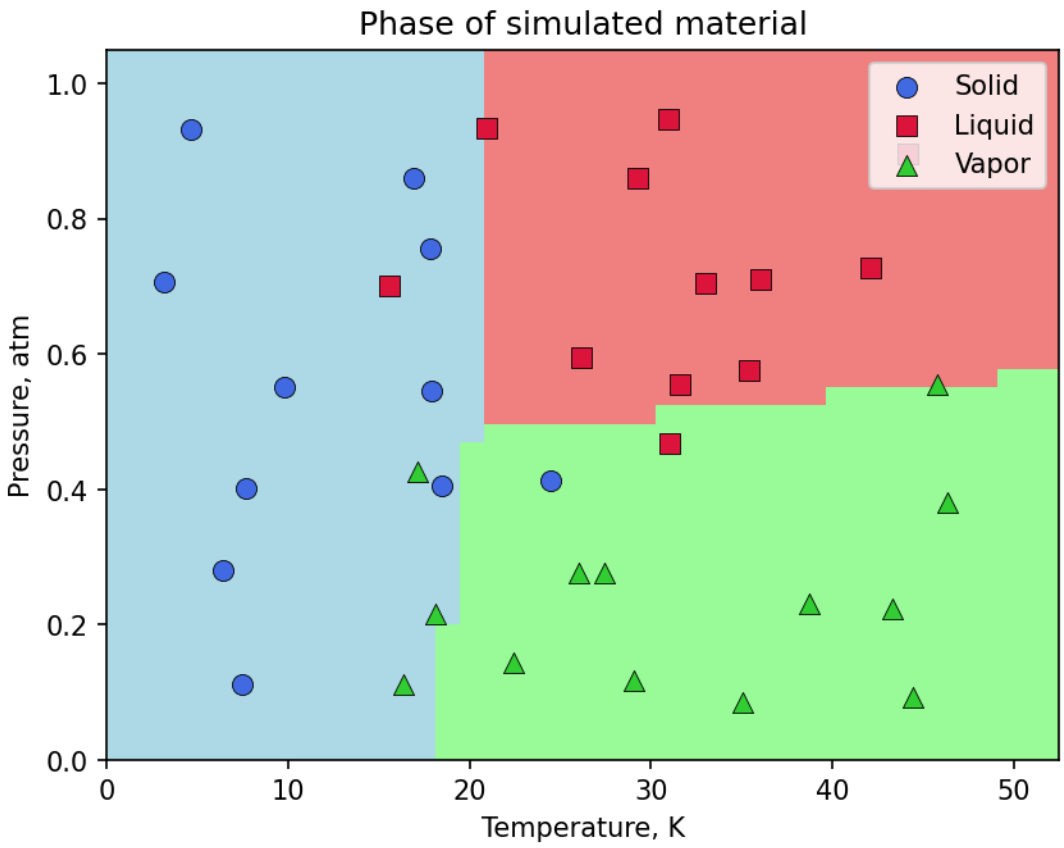
2.3.2 Plotting a Decision Boundary

Here, we have made some plotting functions – run these cells to visualize the decision boundaries.

[ ]:
```
def plot_data(x, y, c, title="Phase of simulated material", newfig=True):
    xlim = [0,52.5]
    ylim = [0,1.05]
    markers = [dict(marker="o", color="royalblue"), dict(marker="s",␣
 ↪color="crimson"), dict(marker="^", color="limegreen")]
    labels = ["Solid", "Liquid", "Vapor"]

    if newfig:
        plt.figure(dpi=150)
```

```
    for i in range(1+max(c)):
        plt.scatter(x[c==i], y[c==i], s=60, **(markers[i]), edgecolor="black",␣
 ↪linewidths=0.4,label=labels[i])

    plt.title(title)
    plt.legend(loc="upper right")
    plt.xlim(xlim)
    plt.ylim(ylim)
    plt.xlabel("Temperature, K")
    plt.ylabel("Pressure, atm")
    plt.box(True)

def plot_colors(classifiers, res=40):
    xlim = [0,52.5]
    ylim = [0,1.05]
    xvals = np.linspace(*xlim,res)
    yvals = np.linspace(*ylim,res)
    x,y = np.meshgrid(xvals,yvals)
    XY = np.concatenate((x.reshape(-1,1),y.reshape(-1,1)),axis=1)
    if type(classifiers) == list:
        color = classify_majority(classifiers,XY).reshape(res,res)
    else:
        color = classifiers(XY).reshape(res,res)
    cmap = ListedColormap(["lightblue","lightcoral","palegreen"])
    plt.pcolor(x, y, color, shading="nearest", zorder=-1,␣
 ↪cmap=cmap,vmin=0,vmax=2)
    return
```

[ ]:
```
plot_data(x,y,c)
plot_colors(classifiers)
plt.show()
```



We can also look at the results of each binary classifier:

[ ]:
```
plt.figure(figsize=(16,3),dpi=150)
for i in range(3):
    plt.subplot(1,3,i+1)
    plot_data(x, y, c, title=f"Binary Classifier {i+1}", newfig=False)
    plot_colors(classifiers[i])
plt.show()
```