M5-HW2

October 3, 2023

1 Problem 7 (30 Points)

1.1 Problem Description

In this problem, you are given a dataset with two input features and one output. You will use a regression tree to make predictions for this data, evaluating each model on both training and testing data. Then, you will repeat this for multiple random forests.

Fill out the notebook as instructed, making the requested plots and printing necessary values.

You are welcome to use any of the code provided in the lecture activities.

Summary of deliverables:

- RMSE function
- Create 4 decision tree prediction surface plots
- Create 4 random forest prediction surface plots
- Print RMSE for train and test data for 4 decision tree models
- Print RMSE for train and test data for 4 random forest models
- Answer the 3 questions posed throughout

Imports and Utility Functions:

[ ]:
```
import numpy as np
import matplotlib.pyplot as plt
from sklearn.tree import DecisionTreeRegressor
from sklearn.ensemble import RandomForestRegressor
from mpl_toolkits.mplot3d import Axes3D
from matplotlib import cm

def make_plot(X,y,model, title=""):
    res = 100
    xrange = np.linspace(min(X[:,0]),max(X[:,0]),res)
    yrange = np.linspace(min(X[:,1]),max(X[:,1]),res)
    x1,x2 = np.meshgrid(xrange,yrange)
    xmesh = np.vstack([x1.flatten(),x2.flatten()]).T
    z = model.predict(xmesh).reshape(res,res)

    fig = plt.figure(figsize=(12,10))
    plt.subplots_adjust(left=0.3,right=0.9,bottom=.3,top=.9)
```

```
    ax = fig.add_subplot(111, projection='3d')
    ax.plot_surface(x1,x2,z,cmap=cm.coolwarm,linewidth=0,alpha=0.9)
    ax.scatter(X[:,0],X[:,1],y,'o',c='black')
    ax.set_xlabel('$x_1$')
    ax.set_ylabel('$x_2$')
    ax.set_zlabel('y')
    plt.title(title)
    plt.show()
```

1.2 Load the data

Use the `np.load()` function to load "w5-hw2-train.npy" (training data) and "w5-hw2-test.npy" (testing data). The first two columns of each are the input features. The last column is the output. You should end up with 4 variables, input and output for each of the datasets.

[ ]:
```
train_data = np.load("data/w5-hw2-train.npy")
test_data = np.load("data/w5-hw2-test.npy")

X_train = train_data[:,0:2]
y_train = train_data[:,-1]
X_test = test_data[:,0:2]
y_test = test_data[:,-1]

print(f"X_train dims: {X_train.shape}")
print(f"y_train dims: {y_train.shape}")

print(f"X_test dims: {X_test.shape}")
print(f"y_test dims: {y_test.shape}")
```

```
X_train dims: (1000, 2)
y_train dims: (1000,)
X_test dims: (500, 2)
y_test dims: (500,)
```

1.3 RMSE function

Complete a root-mean-squared-error function, `RMSE(y, pred)`, which takes in two arrays, and computes the RMSE between them:

[ ]:
```
def RMSE(y, pred):
    return np.sum((pred-y)**2/len(y))
```

1.4 Regression trees

Train 4 regression trees in sklearn, with max depth values [2,5,10,25]. Train your models on the training data.

Plot the predictions as a surface plot along with test points – you can use the provided function: `make_plot(X, y, model, title)`.

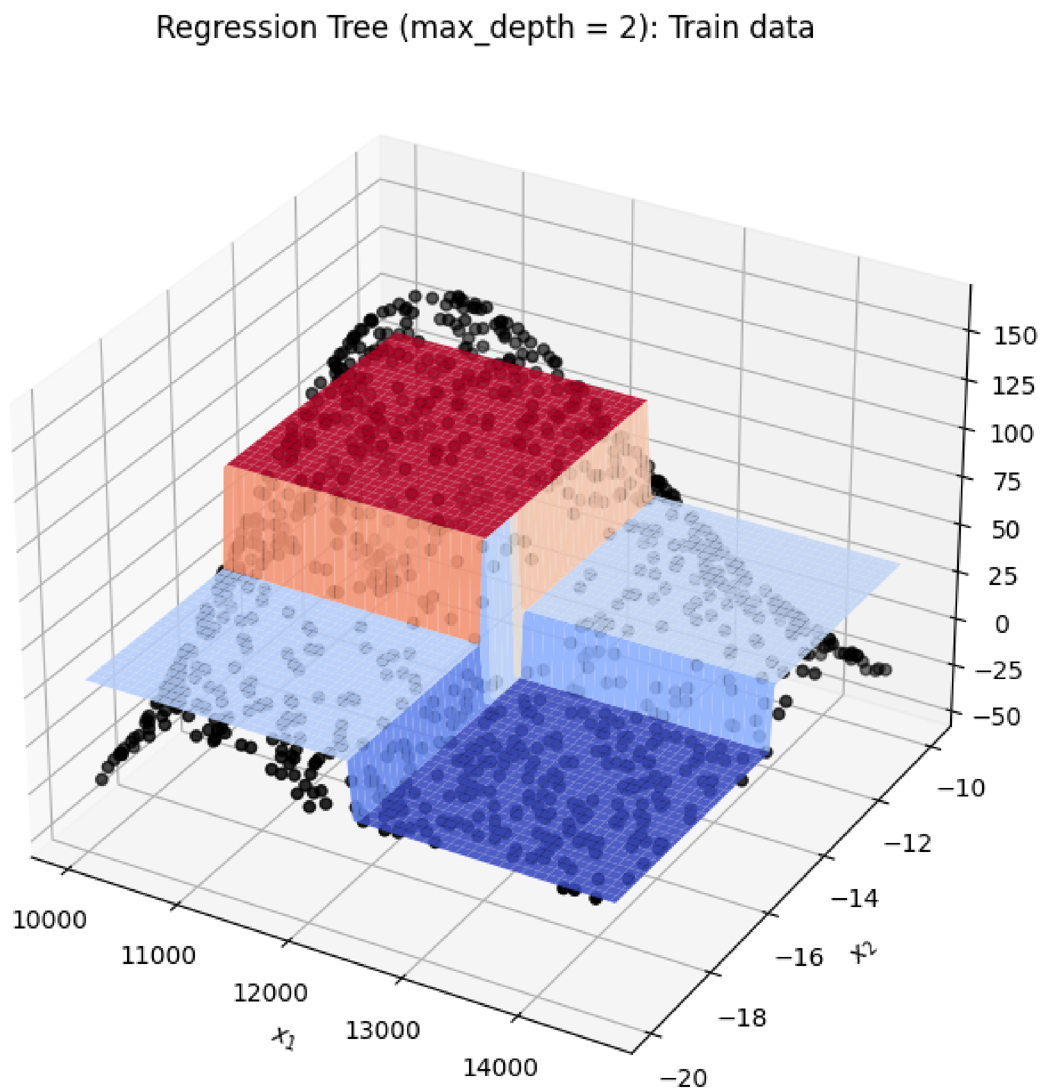
For each model, compute the train and test RMSE by calling your RMSE function. Print these results.

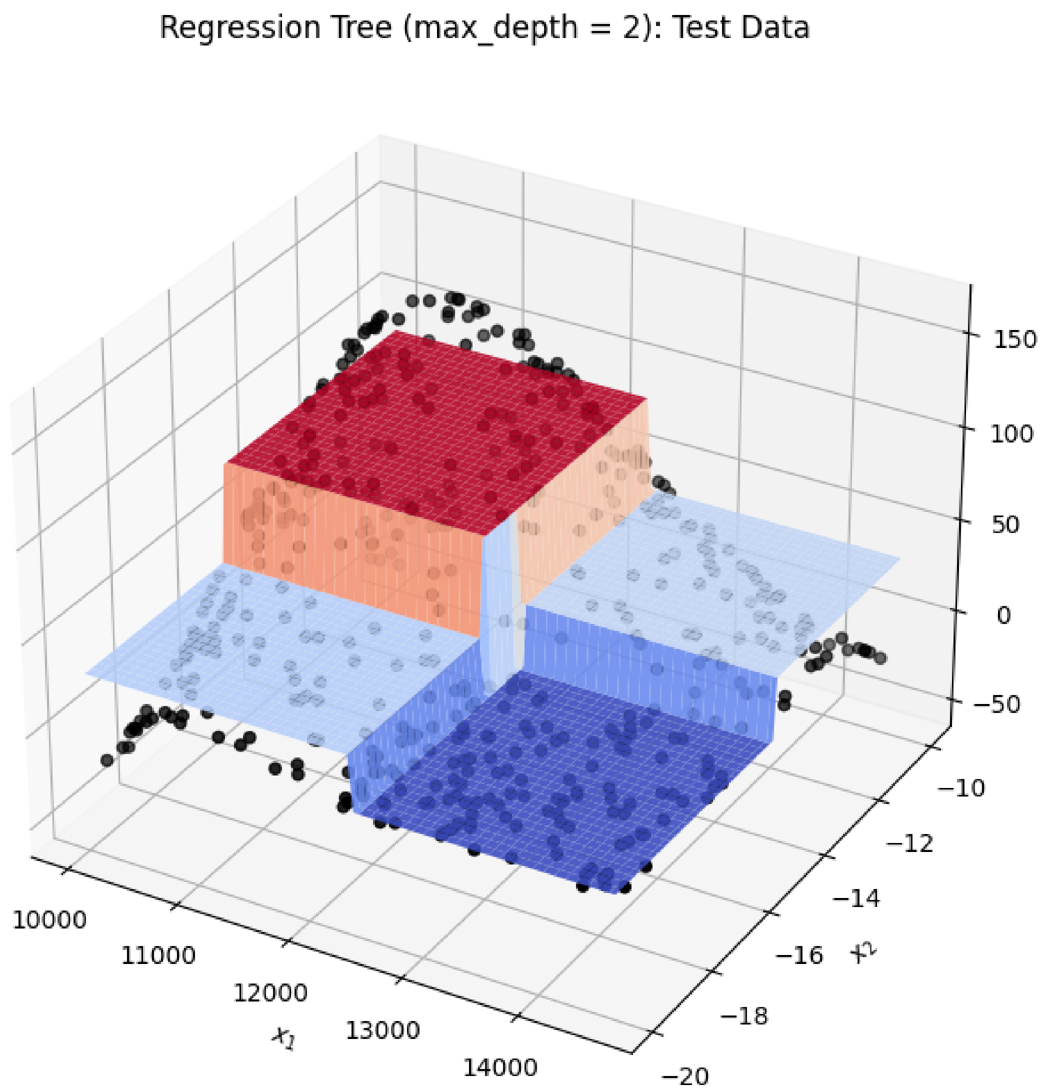
```
[ ]: for max_depth in [2,5,10,25]:
         rt = DecisionTreeRegressor(max_depth=max_depth)
         rt.fit(X_train, y_train)

         print(f"Training RMSE: {RMSE(y_train, rt.predict(X_train))}")
         print(f"Test RMSE: {RMSE(y_test, rt.predict(X_test))}")

         make_plot(X_train, y_train, rt, f"Regression Tree (max_depth = {max_depth}):
      ↪ Train data")
         make_plot(X_test, y_test, rt, f"Regression Tree (max_depth = {max_depth}):␣
      ↪Test Data")
```

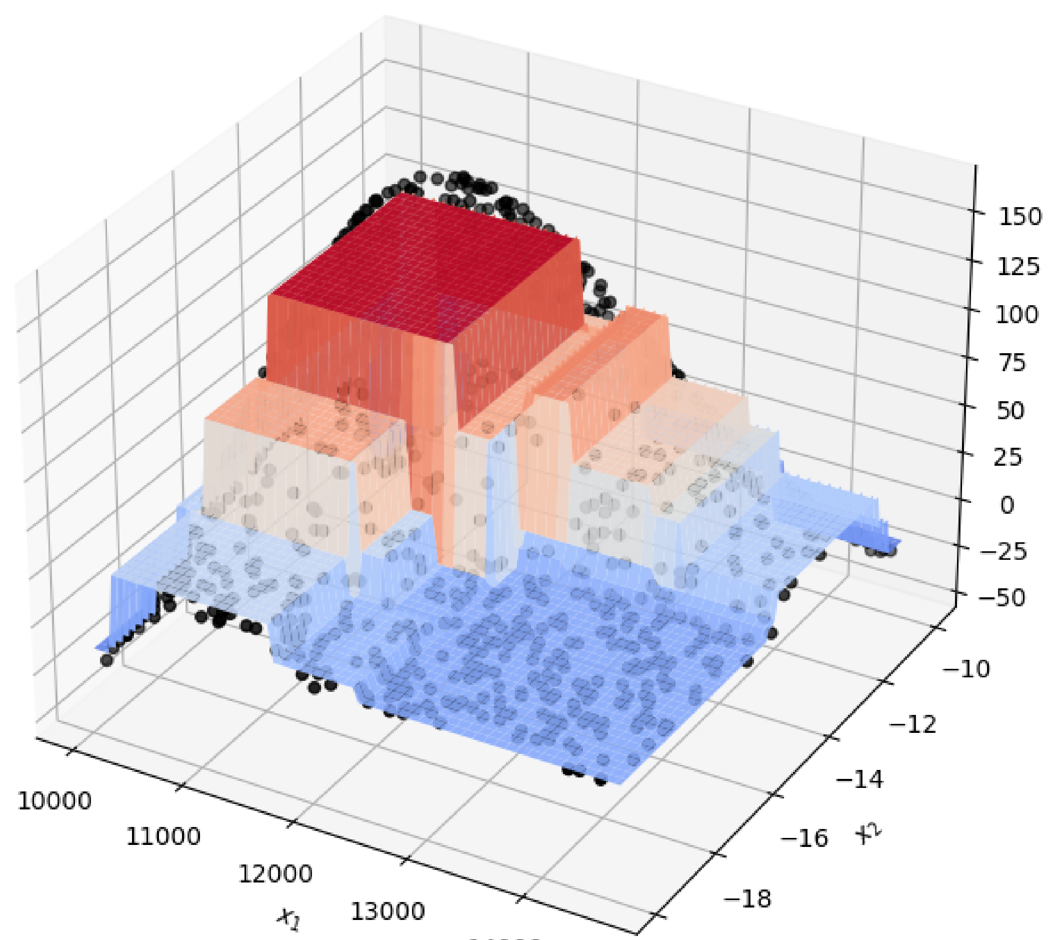
```
Training RMSE: 1258.2521346863323
Test RMSE: 1409.9175176708613
```

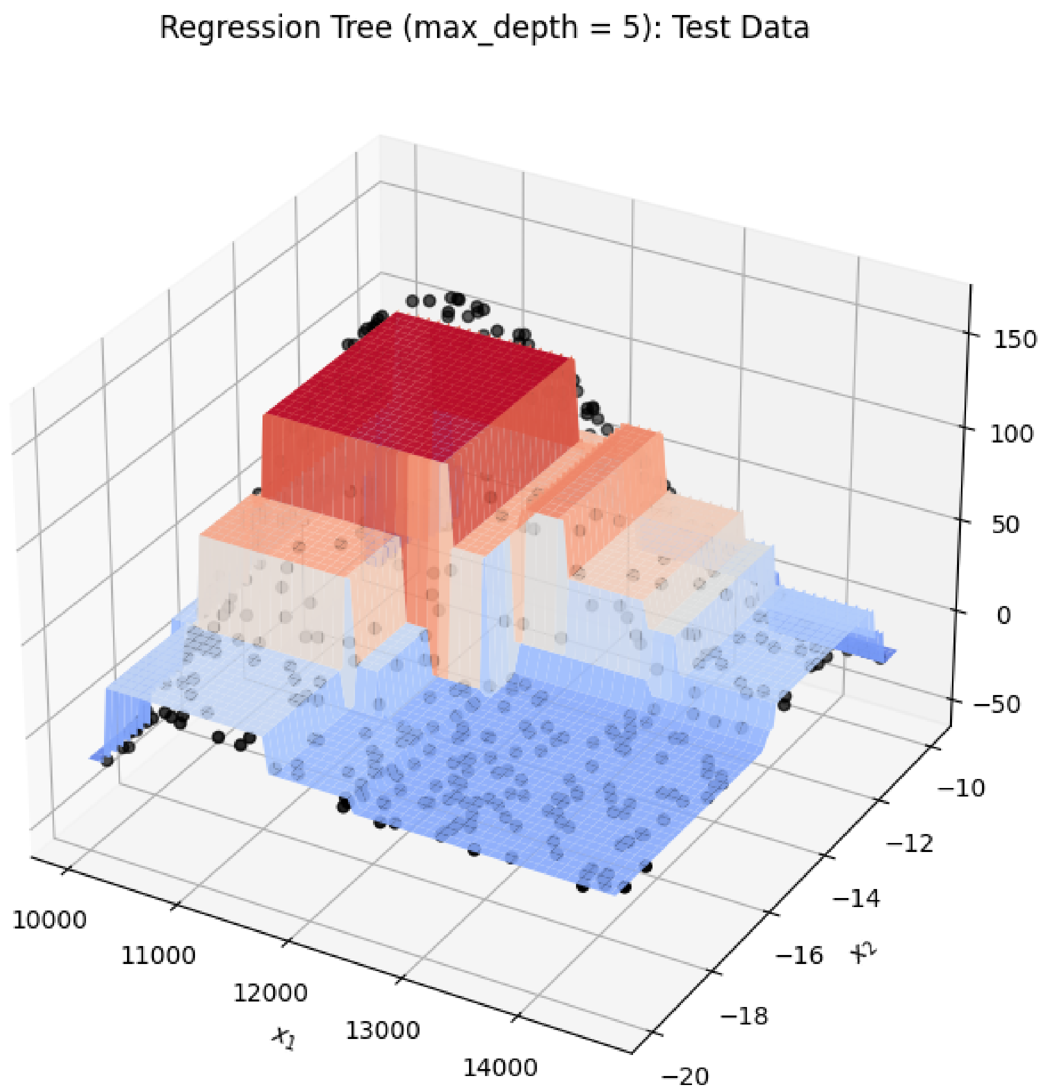

Regression Tree (max_depth = 2): Train data
150
125
100
75
50
25
0
−25
−50
−10
−12
−14
−16
−18
−20
x_2
10000
11000
12000
13000
14000
x_1



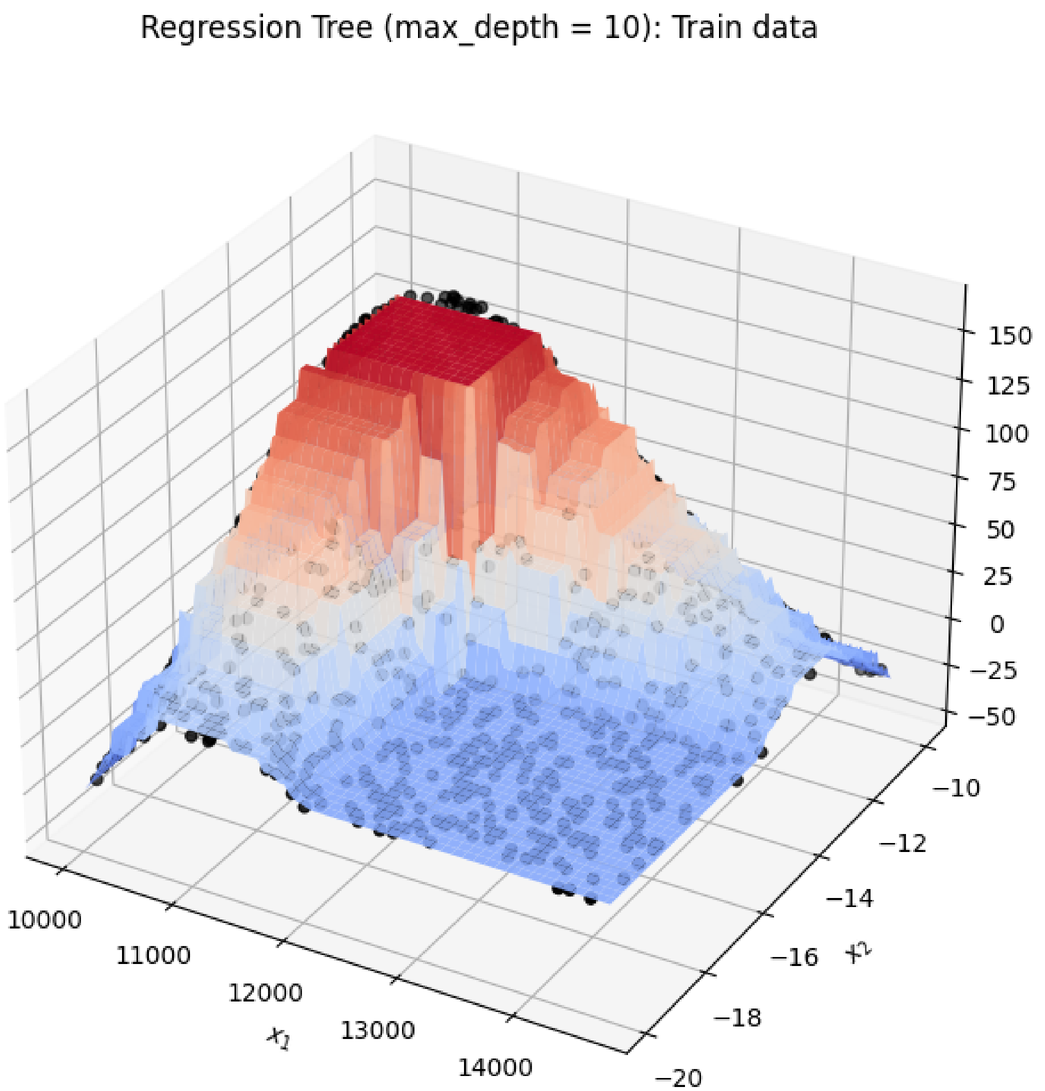

```
Training RMSE: 321.58076944302604
Test RMSE: 362.1164449338508
```

Regression Tree (max_depth = 5): Train data

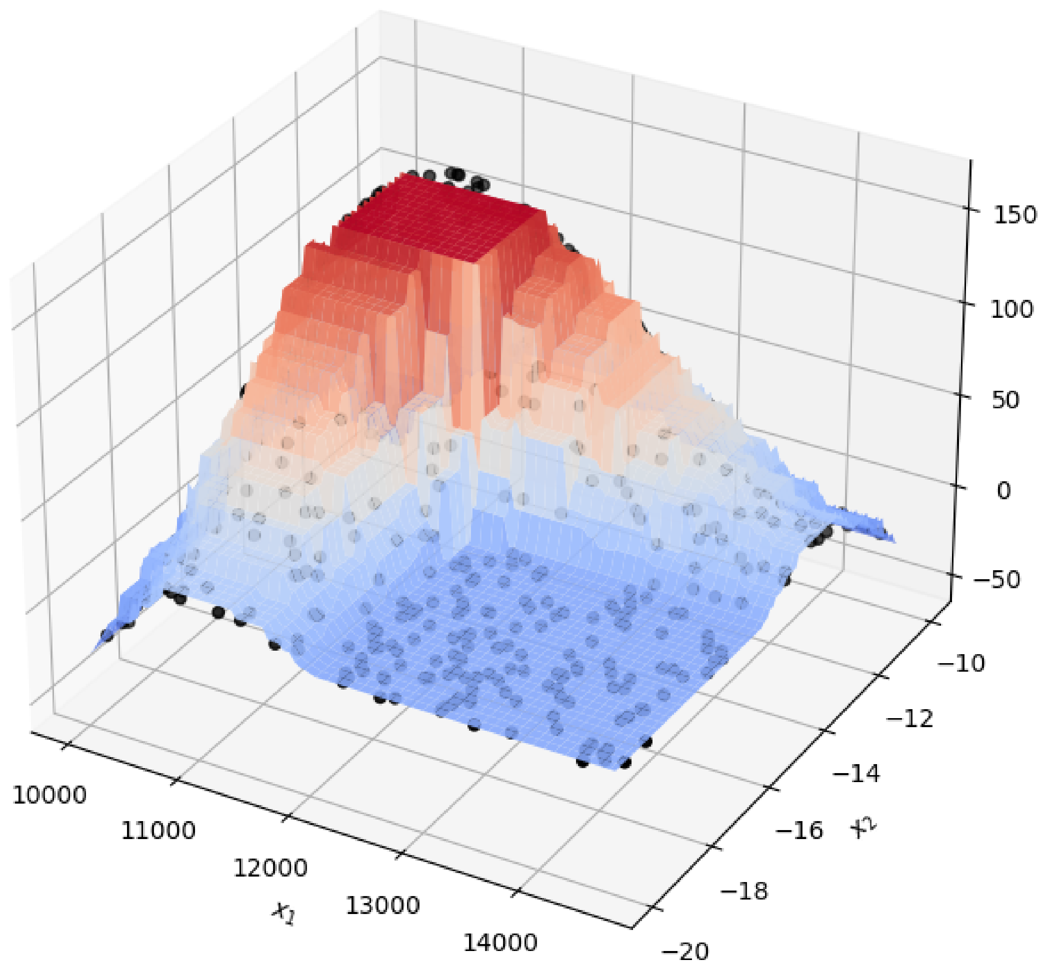




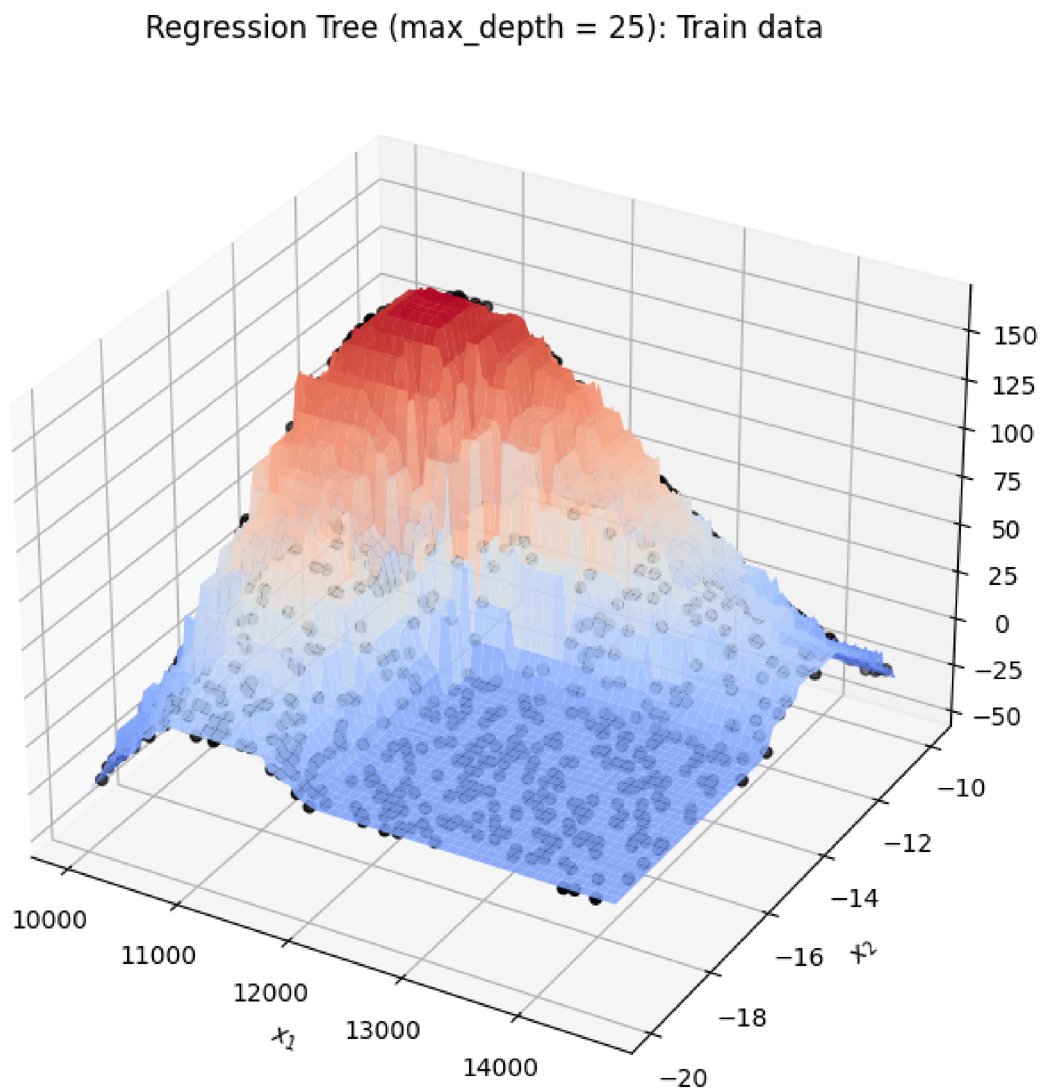

```
Training RMSE: 19.511080867453217
Test RMSE: 60.27832648762853
```

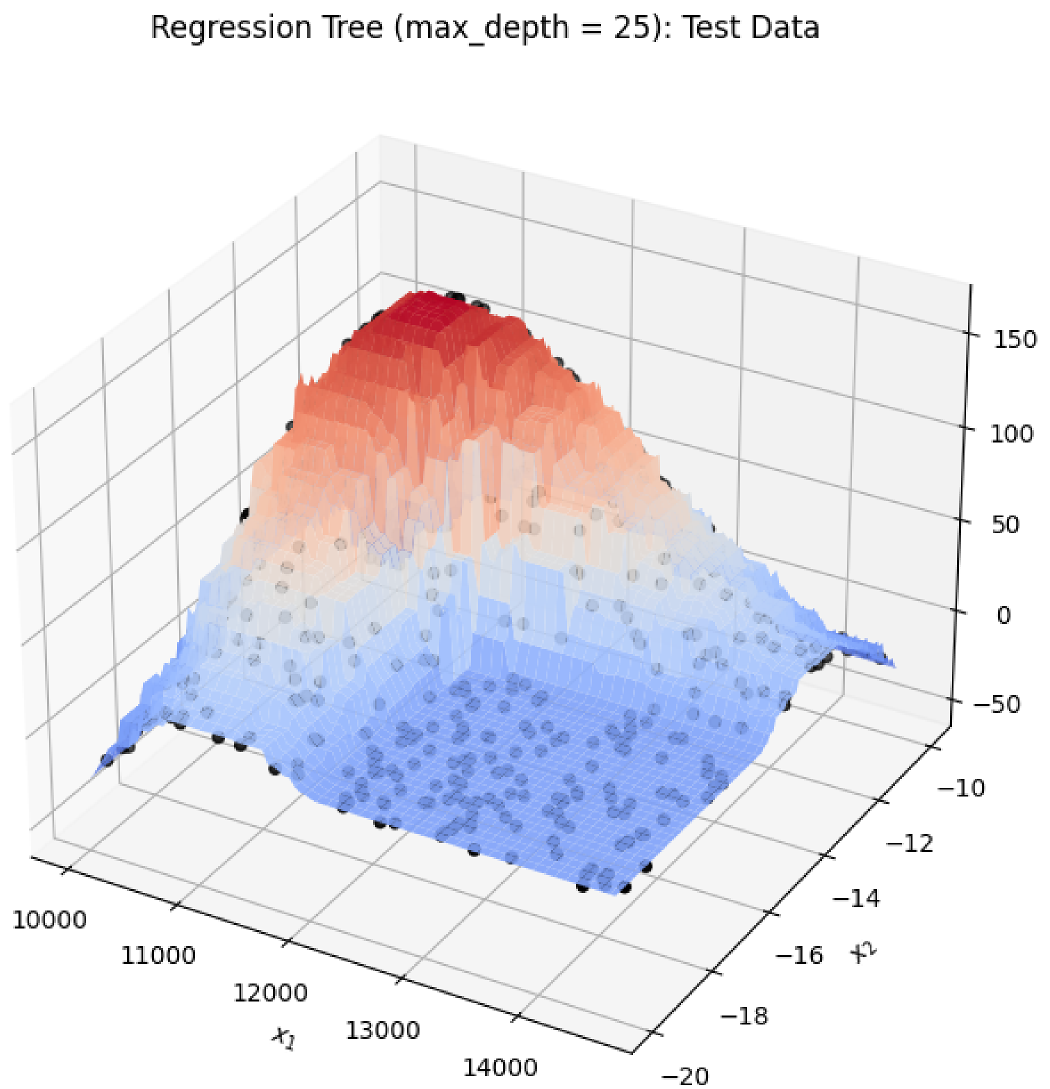

Regression Tree (max_depth = 10): Train data
10000
11000
12000
13000
14000
x_1
−10
−12
−14
−16
−18
−20
x_2
150
125
100
75
50
25
0
−25
−50

Regression Tree (max_depth = 10): Test Data




```
Training RMSE: 0.0
Test RMSE: 31.337753188593794
```


Regression Tree (max_depth = 25): Train data
150
125
100
75
50
25
0
−25
−50
−10
−12
−14
−16
−18
−20
x_2
10000
11000
12000
13000
14000
x_1



1.4.1 Question

- Which of your regression trees performed the best on testing data?

The max_depth 25 decision tree performed the best because it had the smoothest fit to the data with the least RMSE.

1.5 Regression trees

Train 4 random forests in sklearn. For all of them, use the max depth values from your best-performing regression tree. The number of estimators should vary, with values [5, 10, 25, 100].

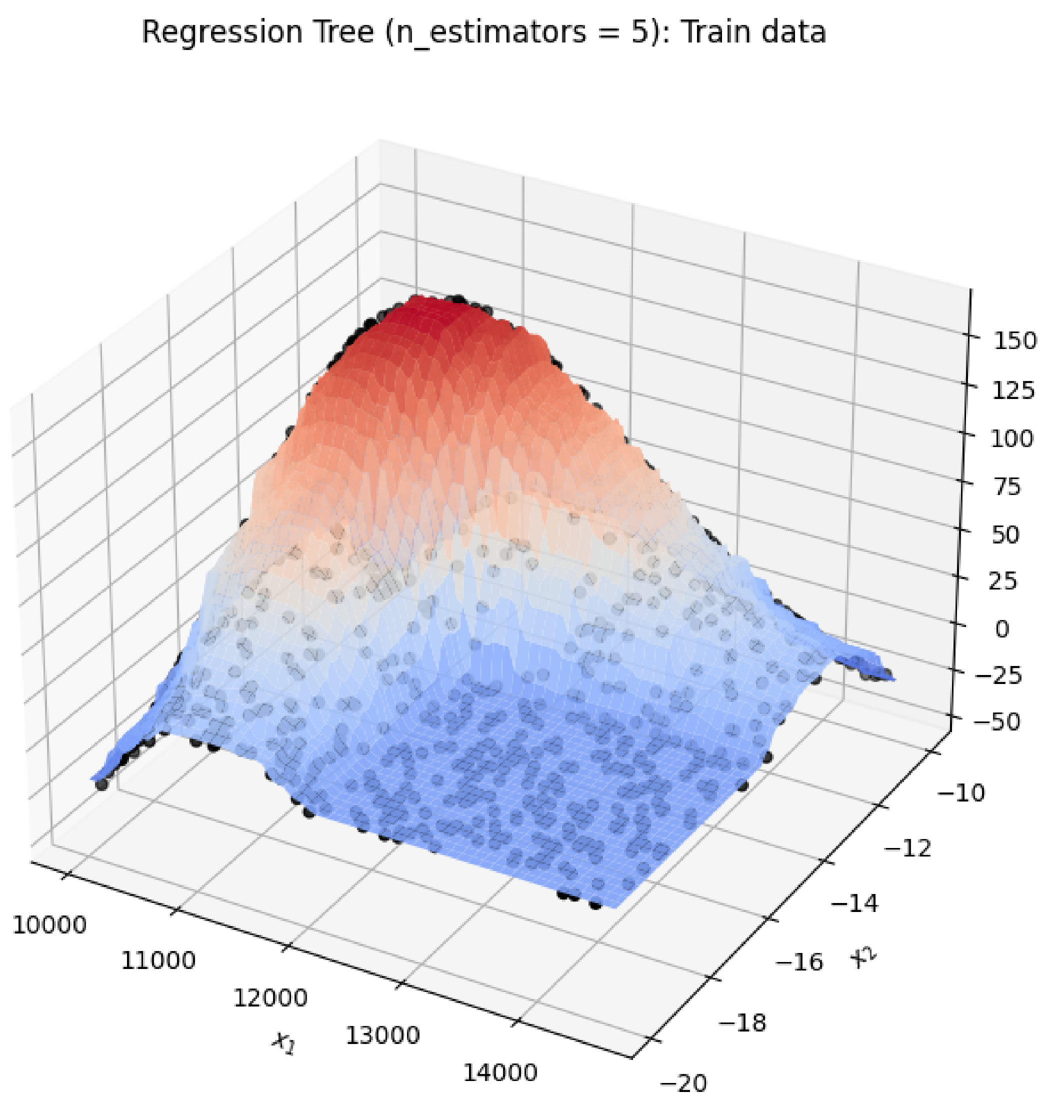
Plot the predictions as a surface plot along with test points. Once again, for each model, compute the train and test RMSE by calling your RMSE function. Print these results.

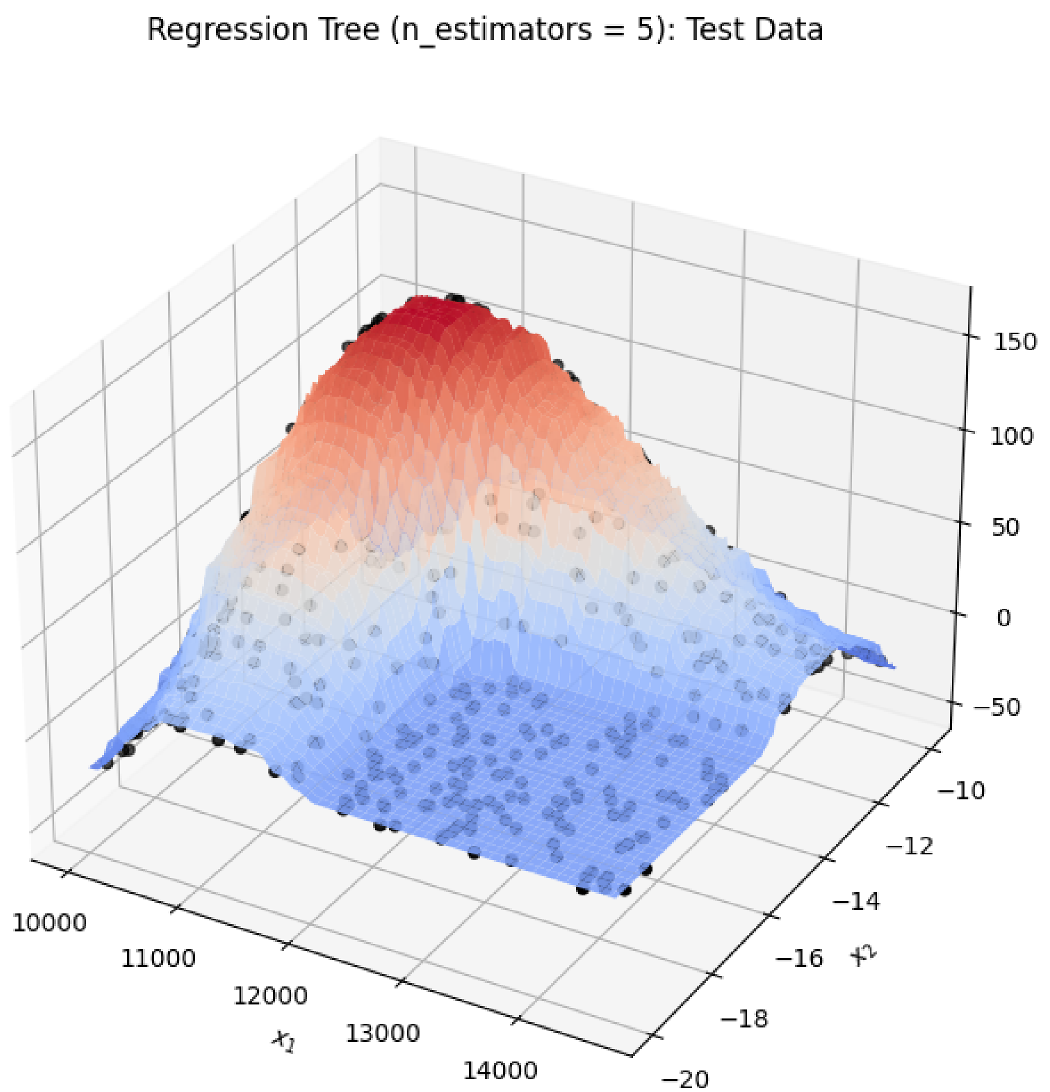
[ ]:
```
for n_estimators in [5, 10, 25, 100]:
    rt = RandomForestRegressor(max_depth=25, n_estimators=n_estimators)
    rt.fit(X_train, y_train)

    print(f"Training RMSE: {RMSE(y_train, rt.predict(X_train))}")
    print(f"Test RMSE: {RMSE(y_test, rt.predict(X_test))}")

    make_plot(X_train, y_train, rt, f"Regression Tree (n_estimators =␣
 ↪{n_estimators}): Train data")
    make_plot(X_test, y_test, rt, f"Regression Tree (n_estimators =␣
 ↪{n_estimators}): Test Data")
```

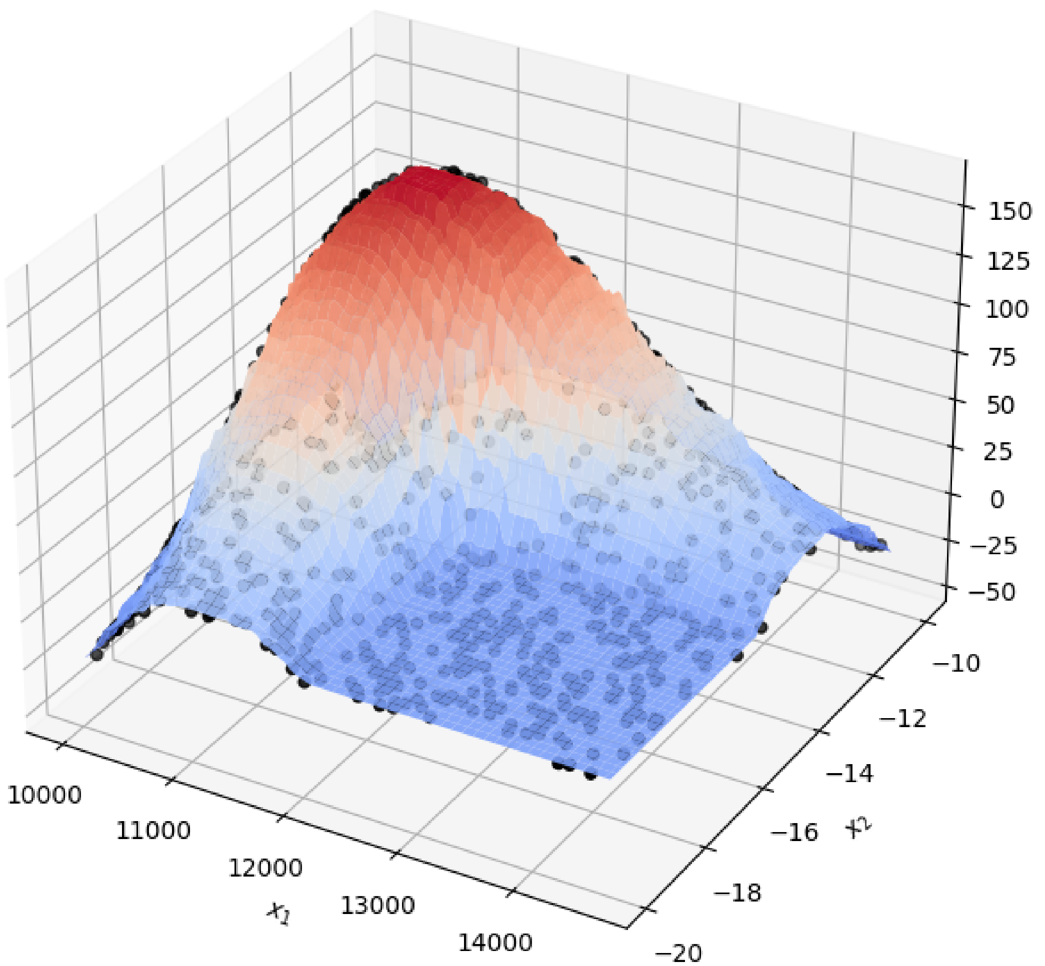
```
Training RMSE: 5.354118817818067
Test RMSE: 19.513572189506608
```

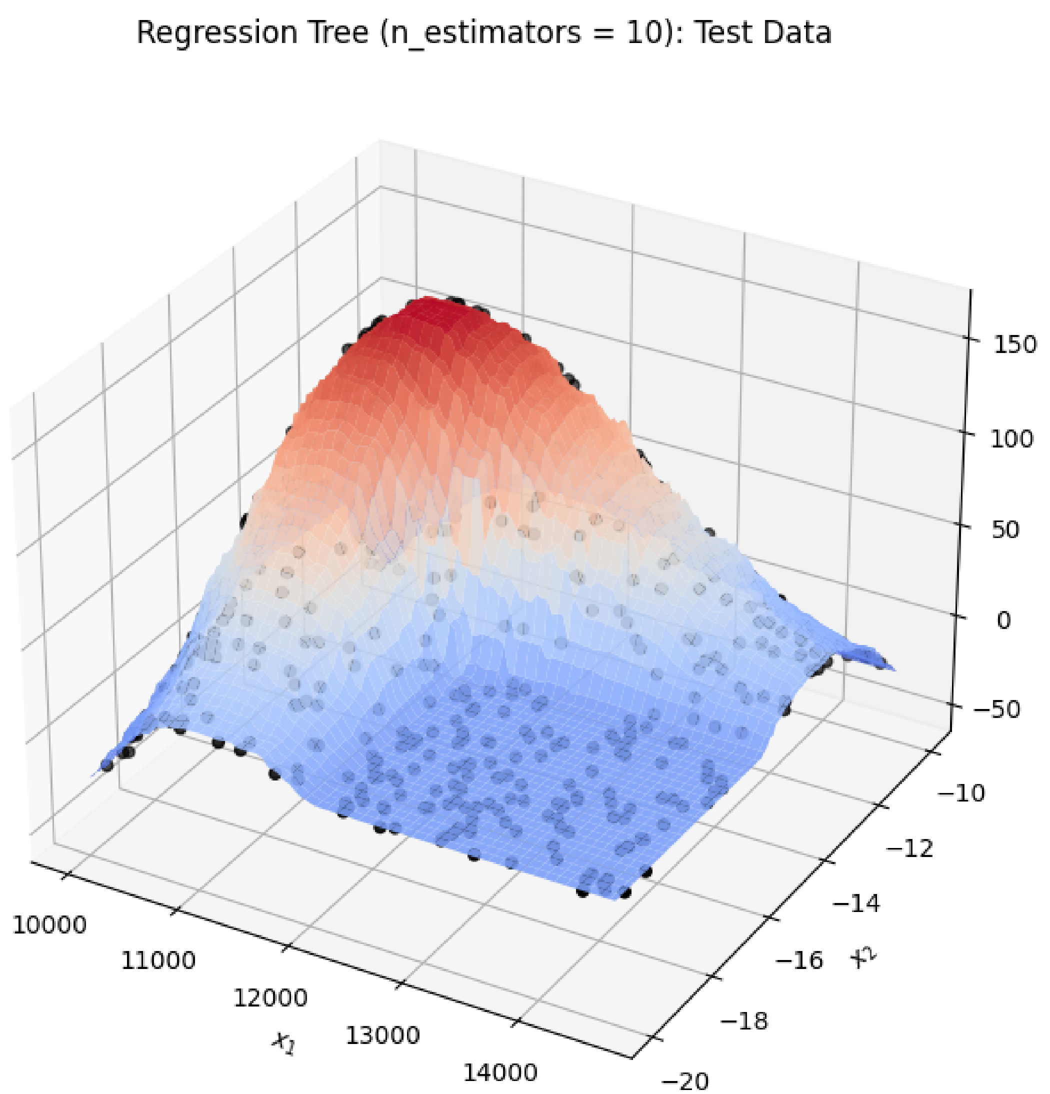





```
Training RMSE: 3.9516607826369325
Test RMSE: 14.113734218124138
```

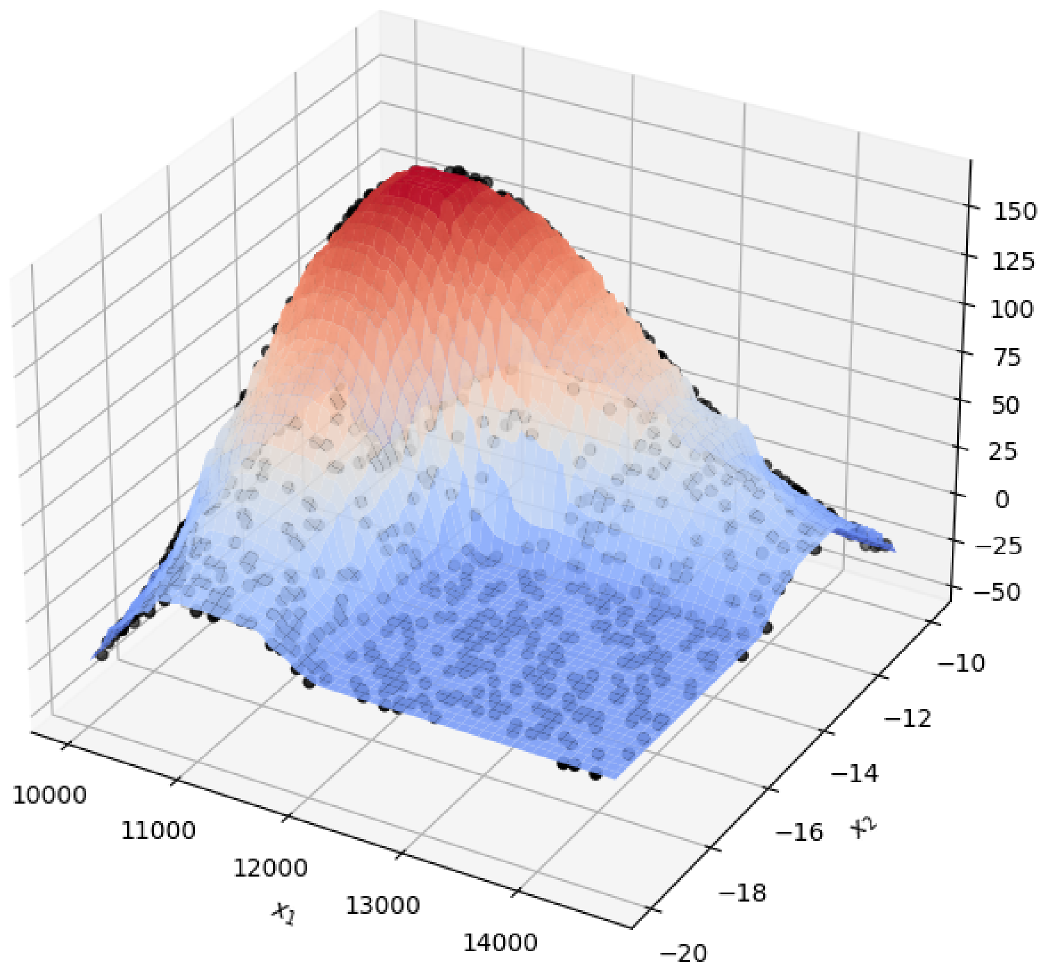
Regression Tree (n_estimators = 10): Train data



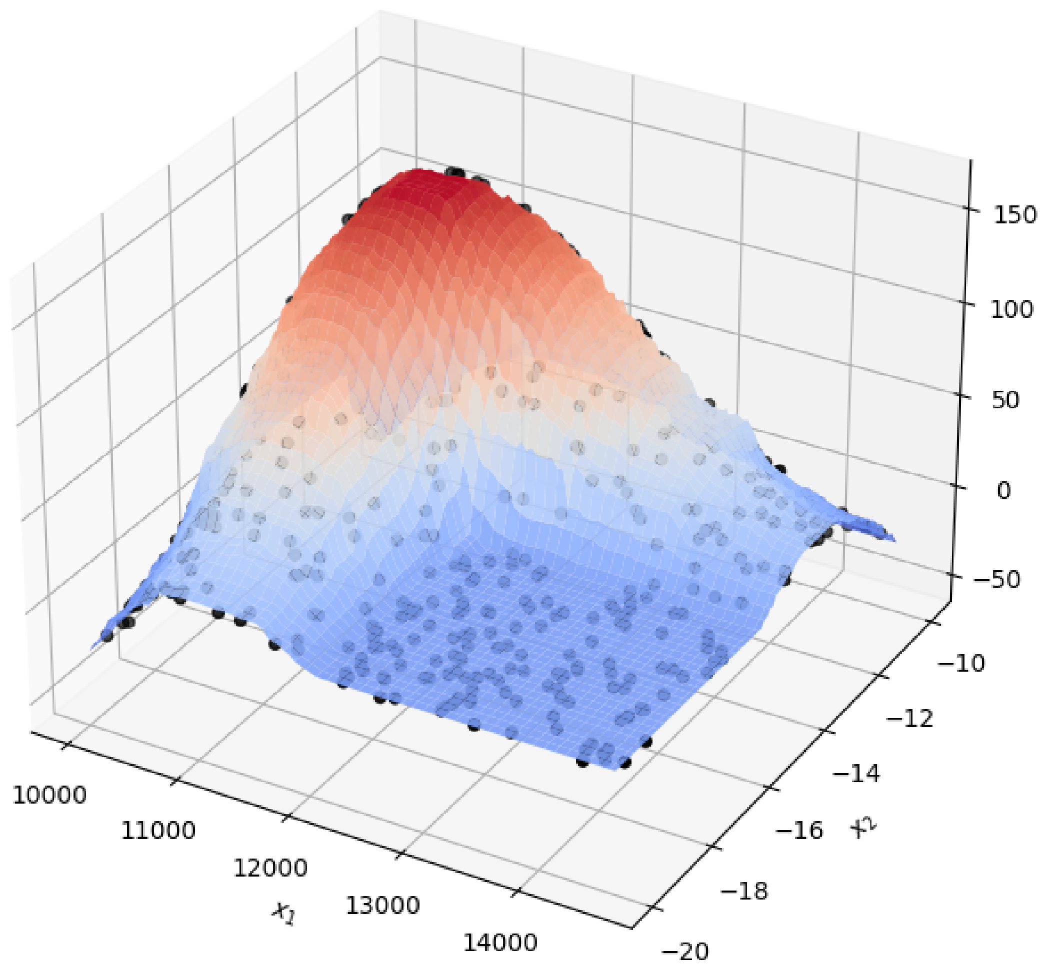



```
Training RMSE: 2.6950201787501014
Test RMSE: 10.918871512775619
```

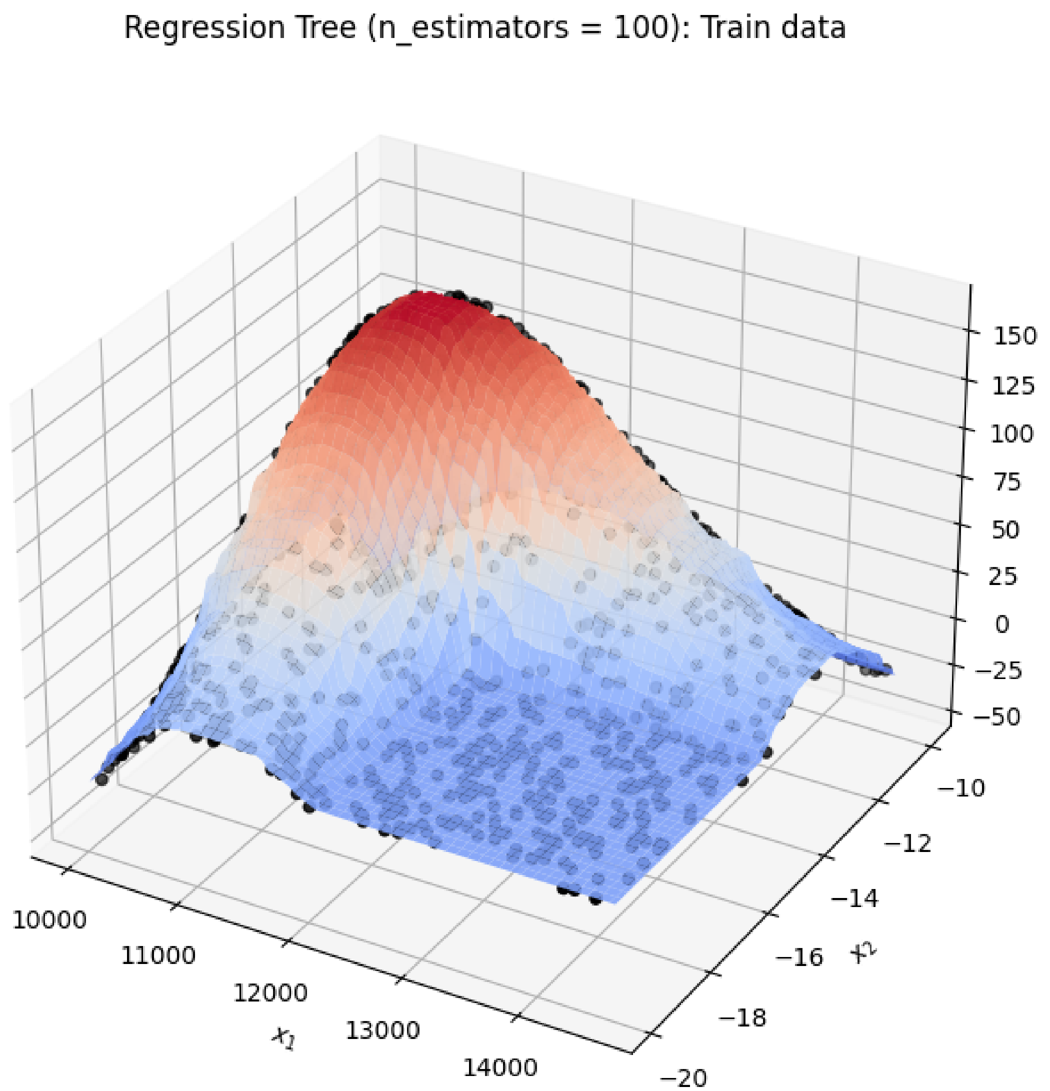
Regression Tree (n_estimators = 25): Train data



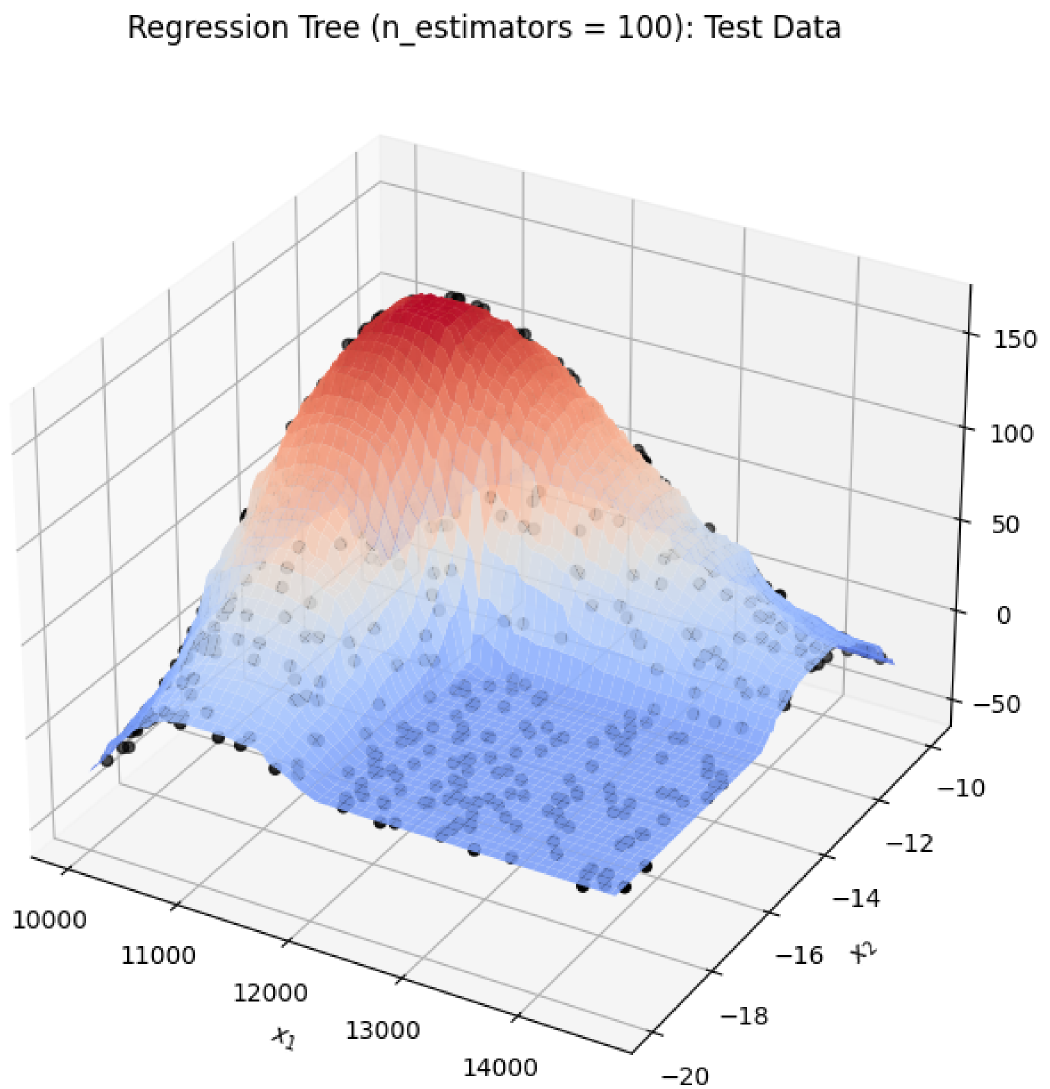
Regression Tree (n_estimators = 25): Test Data




```
Training RMSE: 1.9168457195102795
Test RMSE: 9.493781501499932
```



Regression Tree (n_estimators = 100): Train data
150
125
100
75
50
25
0
−25
−50
−10
−12
−14
−16
−18
−20
x_2
10000
11000
12000
13000
14000
x_1



1.5.1 Questions

- Which of your random forests performed the best on testing data?

  The 100 estimator random forest model performed the best on the testing data with a RMSE of 1.91. This also had the smoothest fit through the training data of the 4 models.

- How does the random forest prediction surface differ qualitatively from that of the decision tree?

  The random forest fit is much smoother than the decision tree fit with less sharp edges. This is to be expected since it is averaging many fits as opposed to a single fit in the case of the decision tree regressor.