M4-L1-P1

October 1, 2023

1 Problem 1 (5 points)

In this problem, you will perform support vector classification on a linearly separable dataset. You will do so without using an SVM package

That is, you will be solving the large margin linear classifier optimization problem:

$$\min_{w,b} \qquad \frac{1}{2}||w||^2$$

$$\text{subject to:} \quad y_i(w^T x_i + b) \geq 1$$

As described in lecture, you will convert the problem into a form compatible with the quadratic programming solver in the `cvxopt` package in Python:

$$\min \quad \frac{1}{2}x^T P x + q^T x$$

$$\text{subject to:} \quad Gx \preceq h; Ax = b$$

Your job in this notebook is to define `P`, `q`, `G`, and `h` from above.

Please install the `cvxopt` package. (You can do that in the notebook directly with `!pip install cvxopt`) Then run the next cell to make the necessary imports.

[ ]:
```
# Import modules
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.colors import ListedColormap

from cvxopt import matrix, solvers
solvers.options['show_progress'] = False

def plot_boundary(x, y, w1, w2, b, e=0.1):
    x1min, x1max = min(x[:,0]), max(x[:,0])
    x2min, x2max = min(x[:,1]), max(x[:,1])

    xb = np.linspace(x1min,x1max)
```

```
    y_0 = 1/w2*(-b-w1*xb)
    y_1 = 1/w2*(1-b-w1*xb)
    y_m1 = 1/w2*(-1-b-w1*xb)

    cmap = ListedColormap(["purple","orange"])

    plt.scatter(x[:,0],x[:,1],c=y,cmap=cmap)
    plt.plot(xb,y_0,'-',c='blue')
    plt.plot(xb,y_1,'--',c='green')
    plt.plot(xb,y_m1,'--',c='green')
    plt.xlabel('$x_1$')
    plt.ylabel('$x_2$')
    plt.axis((x1min-e,x1max+e,x2min-e,x2max+e))
```

1.1 Load the data

[ ]:
```
x1 = np.array([0.0478, 1.4237, 0.2514, 0.2549, 0.3378, 0.5349, 0.7319, 0.7768,
       0.6593, 0.9807, 0.877 , 0.8321, 0.6524, 1.4231, 1.2814, 1.3021,
       1.1915, 1.0913, 1.4438, 0.0959, 0.0752, 0.1789, 0.2549, 0.324 ,
       0.4934, 0.5971, 0.6005, 0.718 , 0.5452, 0.2272, 0.7802, 0.9565,
       0.1028, 0.0579, 0.1927, 0.3862])

x2 = np.array([ 0.9555, -0.396 ,  0.8968,  0.7987,  0.7251,  0.5453,  0.5371,
        0.7088,  0.8028,  0.766 ,  0.439 ,  0.1733,  0.3082,  0.9213,
        0.6515,  0.3777,  0.1896, -0.1374,  0.112 ,  0.3368,  0.1569,
        0.2101,  0.3368,  0.2509,  0.1651,  0.0343, -0.1169, -0.2355,
       -0.3009, -0.3091, -0.3418, -0.3377, -0.3091, -0.0188,  0.0547, -0.3091])

y = np.array([ 1,  1,  1,  1,  1,  1,  1,  1,  1,  1,  1,  1,  1,  1,  1,  1, ␣
 ↪1,
        1,  1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1,␣
 ↪-1])

X = np.vstack([x1,x2]).T
```

1.2 Quadratic Programming

Create the P, q, G, and h matrices as described in the lecture: - `P` (3x3): Identity matrix, but with 0 instead of 1 for the bias (third) row/column - `q` (3x1): Vector of zeros - `G` (Nx3): Negative y multiplied element-wise by [`x1`, `x2`, `1`] - `h` (Nx1): Vector of -1

Make sure the sizes of your matrices match the above. Use numpy arrays. These will be converted into `cvxopt` matrices later.

[ ]:
```
# YOUR CODE GOES HERE

P = np.eye(3)
P[-1,-1] = 0
```

```python
q = np.zeros([3,1])
G = np.array([np.array(-y*x1), np.array(-y*x2), -y*np.ones(len(x1))]).T
h = -1*np.ones(len(x1)).reshape(-1,1)

print("P: ",P.shape)
print("q: ",q.shape)
print("G: ",G.shape)
print("h: ",h.shape)
```

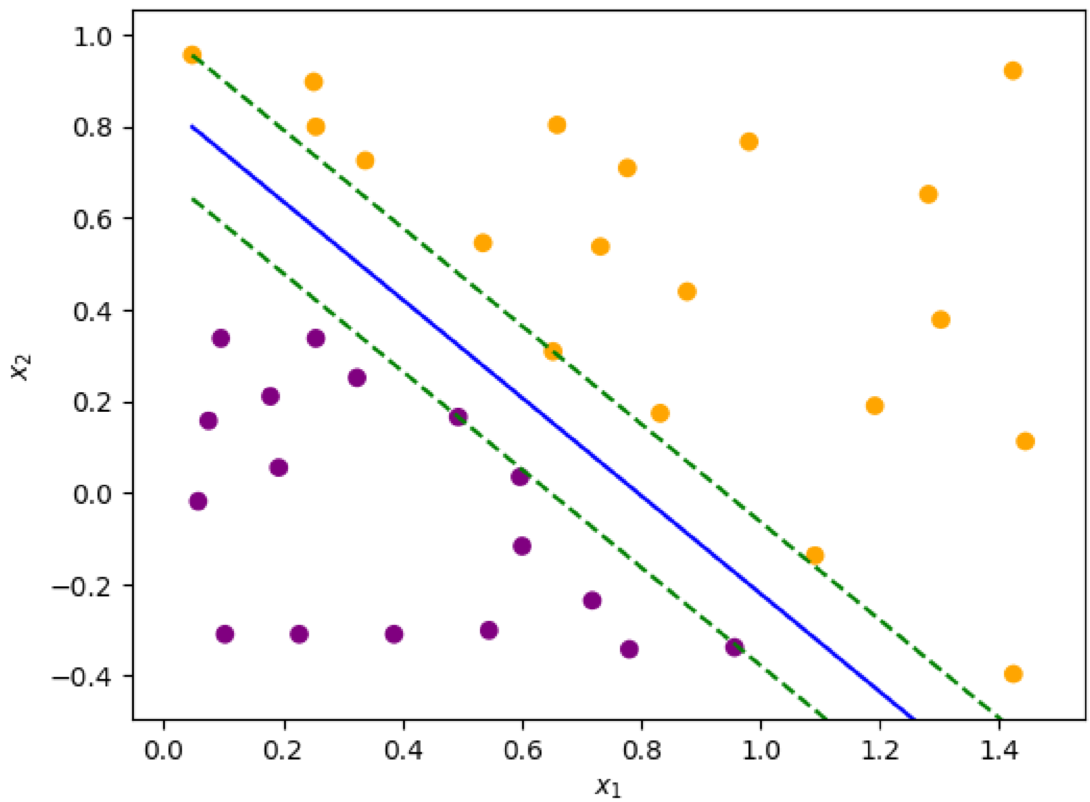
```
P:  (3, 3)
q:  (3, 1)
G:  (36, 3)
h:  (36, 1)
```

1.3 Using cvxopt for QP

Now we convert these arrays into `cvxopt` matrices and solve the quadratic programming problem. Then we get the weights `w1`, `w2`, and `b` and plot the decision boundary.

[ ]:
```python
z = solvers.qp(matrix(P),matrix(q),matrix(G),matrix(h))
w1 = z['x'][0]
w2 = z['x'][1]
b = z['x'][2]

plot_boundary(X, y, w1, w2, b)
```



1.4 Using the SVM

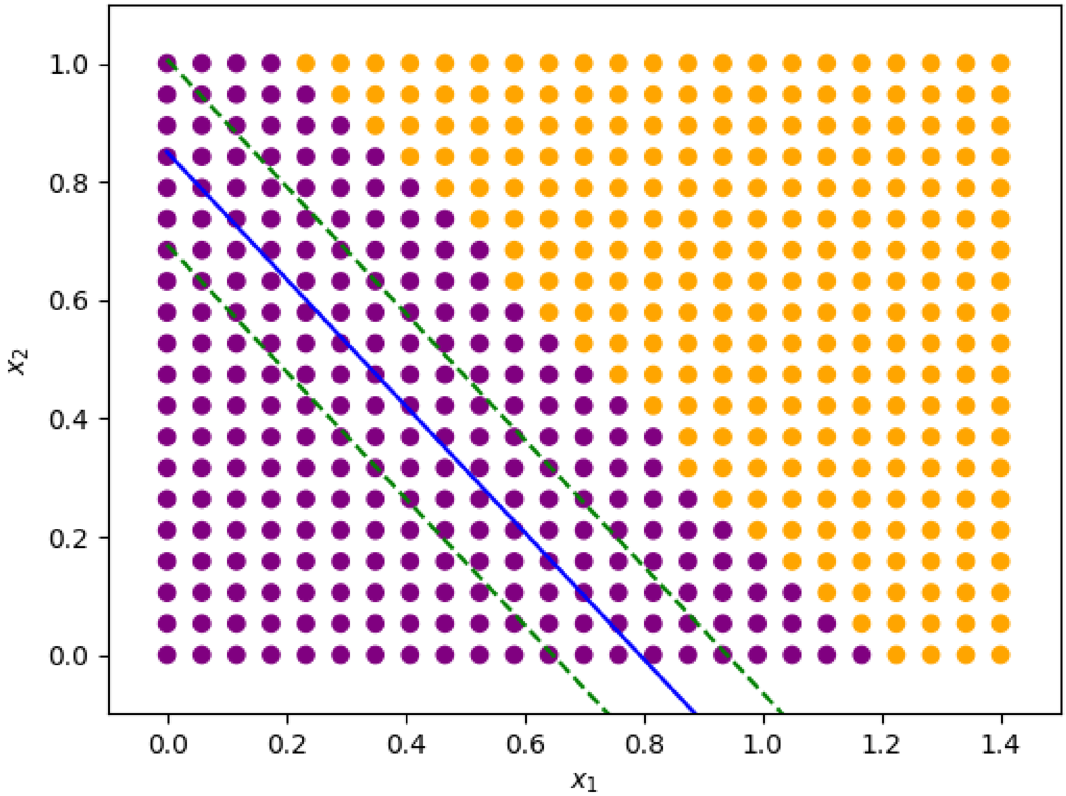
Finally, we will generate a grid of (x1,x2) points and evaluate our support vector classifier on each of these points. Given the array `X_grid`, determine `y_grid`, the class of each point in `X_grid` according to the support vector machine you trained.

[ ]:
```
x1vals = np.linspace(0,1.4,25)
x2vals = np.linspace(0,1,20)
x1s, x2s = np.meshgrid(x1vals, x2vals)
X_grid = np.vstack([x1s.flatten(),x2s.flatten()]).T

# YOUR CODE GOES HERE
print(np.ones(len(X_grid)).shape)
print(X_grid.T.shape)
y_grid = np.array([w1, w2, b]) @ np.concatenate([X_grid, np.ones(len(X_grid)).
 ↪reshape(-1,1)], axis=1).T

plot_boundary(X_grid, y_grid, w1, w2, b)
```

```
(500,)
(2, 500)
```



1.0
0.8
0.6
0.4
0.2
0.0
x_2
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
x_1