**Problem 1:**

1. 2

**Problem 2:**

1. 3

**Problem 3:**

$$\bar{a} = \frac{8 + 4 + 0 - 4}{4} = 2$$
$$\bar{b} = \frac{-16 - 12 - 10 + 2}{4} = -9$$

$$a_{ms} = (a_i - \bar{a}) = [6, 2, -2, -6]$$

$$b_{ms} = \left(b_i - \bar{b}\right) = [-7, -3, -1, 11]$$

$$r = \frac{\sum(a_i - \bar{a})(b_i - \bar{b})}{\sqrt{\sum(a_i - \bar{a})^2\left(b_i - \bar{b}\right)^2}} = \frac{a_{ms} * b_{ms}}{\sqrt{(a_{ms}^T a_{ms}) * (b_{ms}^T b_{ms})}}$$

$$= \frac{-112}{120}$$
$$\boldsymbol{r = -0.9333}$$

**Problem 4:**

1. 2 because 1 has a higher correlation with the output y

# M6-HW1

October 16, 2023

## 1 Problem 6 (30 Points)

During the lecture you worked with pipelines in SciKit-Learn to perform feature transformation before classification/regression using a pipeline. In this problem, you will look at another scaling method in a 2D regression context.

*You are welcome to use any of the code provided in the lecture activities.*

**Summary of deliverables:** Sklearn Models (no scaling): Print Train and Test MSE - Linear Regression (input degree 8 features) - SVR, C = 1000 - KNN, K = 4 - Random Forest, 100 estimators of max depth 10

Sklearn Pipeline (scaling + model): Print Train and Test MSE - Linear Regression (input degree 8 features) - SVR, C = 1000 - KNN, K = 4 - Random Forest, 100 estimators of max depth 10

Plots - 1x5 subplot showing model predictions on unscaled features, next to ground truth - 1x5 subplot showing pipeline predictions with features scaled, next to ground truth

Questions - Respond to the prompts at the end

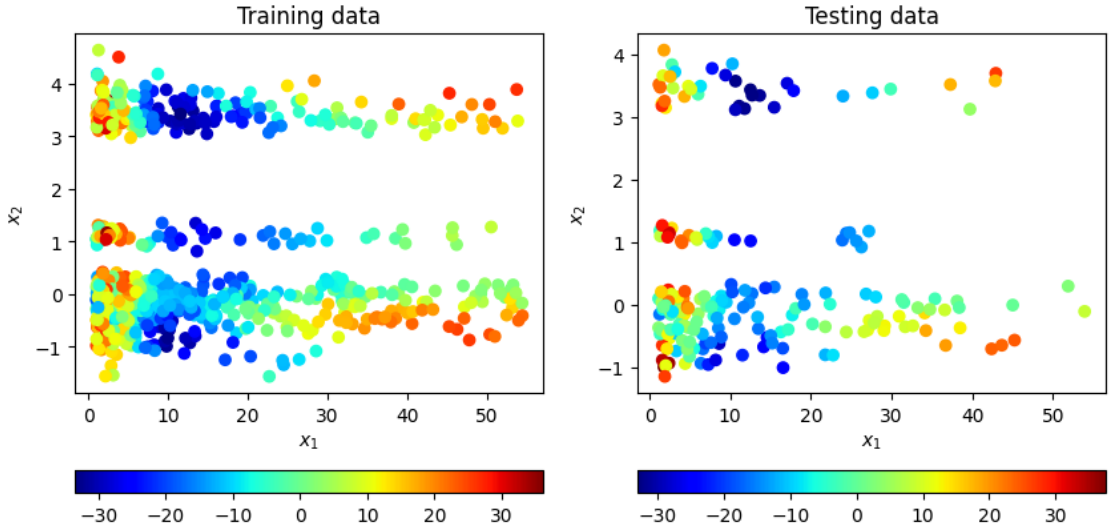
[ ]:
```
import numpy as np
import matplotlib.pyplot as plt



from sklearn.linear_model import LinearRegression
from sklearn.svm import SVR
from sklearn.ensemble import RandomForestRegressor
from sklearn.neighbors import KNeighborsRegressor
from sklearn.preprocessing import PolynomialFeatures, QuantileTransformer
from sklearn.pipeline import Pipeline
from sklearn.metrics import mean_squared_error
from sklearn.model_selection import train_test_split

def plot(X, y, title=""):
    plt.scatter(X[:,0],X[:,1],c=y,cmap="jet")
    plt.colorbar(orientation="horizontal")
    plt.xlabel("$x_1$")
    plt.ylabel("$x_2$")
    plt.title(title)
```

### 1.1 Load the data

Complete the loading process below by inputting the path to the data file "w6-p1-data.npy"

Training data is in `X_train` and `y_train`. Testing data is in `X_test` and `y_test`.

[ ]:
```
# Define path
data = np.load("data/w6-p1-data.npy")
X, y = data[:,:2], data[:,2]
X_train, X_test, y_train, y_test = train_test_split(X, y, train_size=int(0.
 ↪8*len(y)),random_state=0)

plt.figure(figsize=(10,5))
plt.subplot(1,2,1)
plot(X_train, y_train, "Training data")
plt.subplot(1,2,2)
plot(X_test, y_test, "Testing data")
plt.show()
```




### 1.2 Models (no input scaling)

Fit 4 models to the training data: - `LinearRegression()`. This should be a pipeline whose first step is `PolynomialFeatures()` with degree 7. - `SVR()` with C = 1000 and "rbf" kernel - `KNeighborsRegressor()` using 4 nearest neighbors - `RandomForestRegressor()` with 100 estimators of max depth 10

Print the Train and Test MSE for each

[ ]:
```
model_names = ["LSR", "SVR", "KNN", "RF"]
linear_regression_model = Pipeline([("Polynomial Features",␣
 ↪PolynomialFeatures(7)), ("Linear Regression", LinearRegression())])
```

```
svr_model = SVR(C=1000, kernel='rbf')
k_neighbors_regressor_model = KNeighborsRegressor(n_neighbors=4)
random_forest_regressor_model = RandomForestRegressor(n_estimators=100,␣
 ↪max_depth=10)
models = [linear_regression_model, svr_model, k_neighbors_regressor_model,␣
 ↪random_forest_regressor_model]

linear_regression_model.fit(X_train, y_train)
print(f"Model: {model_names[0]}")
print(f"\tTrain MSE: {mean_squared_error(y_train, linear_regression_model.
 ↪predict(X_train))}")
print(f"\tTest MSE: {mean_squared_error(y_test, linear_regression_model.
 ↪predict(X_test))}")
print()

svr_model.fit(X_train, y_train)
print(f"Model: {model_names[1]}")
print(f"\tTrain MSE: {mean_squared_error(y_train, svr_model.predict(X_train))}")
print(f"\tTest MSE: {mean_squared_error(y_test, svr_model.predict(X_test))}")
print()

k_neighbors_regressor_model.fit(X_train, y_train)
print(f"Model: {model_names[2]}")
print(f"\tTrain MSE: {mean_squared_error(y_train, k_neighbors_regressor_model.
 ↪predict(X_train))}")
print(f"\tTest MSE: {mean_squared_error(y_test, k_neighbors_regressor_model.
 ↪predict(X_test))}")
print()

random_forest_regressor_model.fit(X_train, y_train)
print(f"Model: {model_names[3]}")
print(f"\tTrain MSE: {mean_squared_error(y_train, random_forest_regressor_model.
 ↪predict(X_train))}")
print(f"\tTest MSE: {mean_squared_error(y_test, random_forest_regressor_model.
 ↪predict(X_test))}")
print()
```

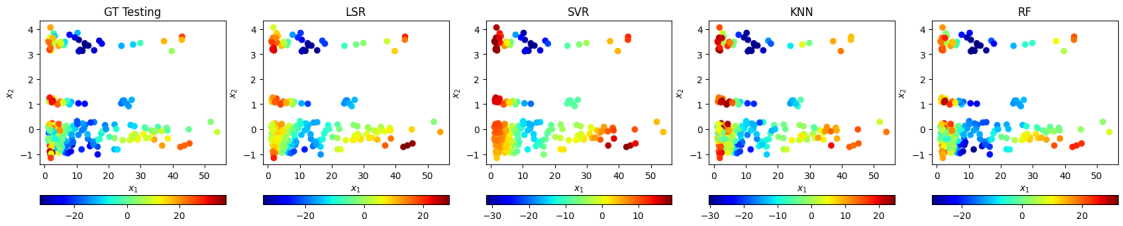
```
Model: LSR
        Train MSE: 50.86638993713532
        Test MSE: 57.28678105844514

Model: SVR
        Train MSE: 82.0435260356599
        Test MSE: 98.6331971940729

Model: KNN
        Train MSE: 26.856498566141628
```

```
        Test MSE: 47.63617328402055

Model: RF
        Train MSE: 5.835570260503268
        Test MSE: 25.12171532433398
```

### 1.3 Visualizing the predictions

Plot the predictions of each method on the testing data in a 1x5 subplot structure, with the ground truth values as the leftmost subplot.

[ ]:
```
plt.figure(figsize=(21,4))
plt.subplot(1,5,1)
plot(X_test, y_test, "GT Testing")

plt.subplot(1,5,2)
plot(X_test, linear_regression_model.predict(X_test), model_names[0])

plt.subplot(1,5,3)
plot(X_test, svr_model.predict(X_test), model_names[1])

plt.subplot(1,5,4)
plot(X_test, k_neighbors_regressor_model.predict(X_test), model_names[2])

plt.subplot(1,5,5)
plot(X_test, random_forest_regressor_model.predict(X_test), model_names[3])

plt.show()
```




### 1.4 Quantile Scaling

A `QuantileTransformer()` can transform the input data in a way that attempts to match a given distribution (uniform distribution by default).
- Create a quantile scaler with `n_quantiles = 800`. - Then, create a pipeline for each of the 4 types of models used earlier - Fit each pipeline to the training data, and again print the train and test MSE

[ ]:
```
pipeline_names = ["LSR, scaled", "SVR, scaled", "KNN, scaled", "RF, scaled"]
pipelines = []
models = [Pipeline([("Polynomial Features", PolynomialFeatures(7)), ("Linear␣
 ↪Regression", LinearRegression())]),
          SVR(C=1000, kernel='rbf'),
          KNeighborsRegressor(n_neighbors=4),
          RandomForestRegressor(n_estimators=100, max_depth=10)]

quantile_transformer = QuantileTransformer(n_quantiles=800)
for i in range(len(models)):
    pipelines.append(Pipeline([("Quantile Transformer",␣
 ↪quantile_transformer),(model_names[i], models[i])]))
    pipelines[i].fit(X_train, y_train)
    print(f"Pipeline: {pipeline_names[i]}")
    print(f"\tTrain MSE: {mean_squared_error(y_train, pipelines[i].
 ↪predict(X_train))}")
    print(f"\tTest MSE: {mean_squared_error(y_test, pipelines[i].
 ↪predict(X_test))}")
    print()
```

```
Pipeline: LSR, scaled
        Train MSE: 39.52893428670383
        Test MSE: 43.20363492250739

Pipeline: SVR, scaled
        Train MSE: 41.03425800595979
        Test MSE: 43.01791573789805

Pipeline: KNN, scaled
        Train MSE: 19.687691313922564
        Test MSE: 36.397038931930005

Pipeline: RF, scaled
        Train MSE: 6.0564178494409155
        Test MSE: 25.497032114929343
```

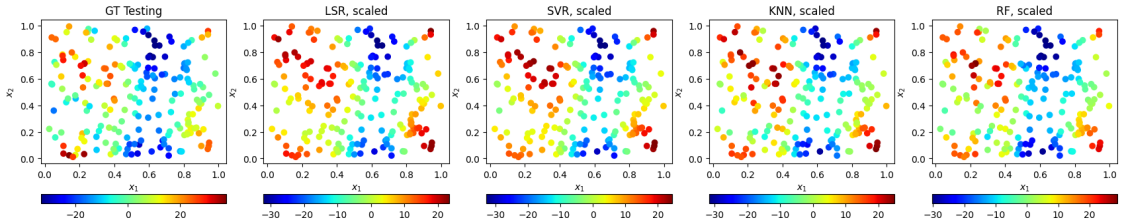
### 1.5 Visualization with scaled input

As before, plot the predictions of each *scaled* method on the testing data in a 1x5 subplot structure, with the ground truth values as the leftmost subplot.

This time, for each plot, show the scaled data points instead of the original data. You can do this by calling `.transform()` on your quantile scaler. The scaled points should appear to follow a uniform distribution.

[ ]:
```
plt.figure(figsize=(21,4))
plt.subplot(1,5,1)
```

```
plot(quantile_transformer.transform(X_test), y_test, "GT Testing")

i = 2
for pipeline in pipelines:
    plt.subplot(1,5,i)
    plot(quantile_transformer.transform(X_test), pipeline.predict(X_test),␣
 ↪pipeline_names[i-2])
    i += 1

plt.show()
```




## 1.6 Questions

1. Without transforming the input data, which model performed the best on test data? What about after scaling?

   The random forest model performed the best before scaling and after scaling but it performec better by a smaller margin after scaling.

2. For each method, say whether scaling the input improved or worsened, how extreme the change was, and why you think this is.

   LSR: Improved a little bit from scaling because the uniform distribution allows the final fitted result to better see patterns in the data that can't otherwise be seen with a clumped data.

   SVR: Improved a lot from scaling because SVR does better with more uniformly distributed data sets. This type of scaling reduces the impact of outlier data which can have a large effect on the final weightings of the SVR model. Escpecially with the relatively high C value, the impact of outlier data crossin boundaries can be very detrimental to the final fitted model.

   KNN: Improved a little bit from scaling because it allows spreads out the data points in a more uniform way allowing for better choosing between which points closest have the most influence. It doesn't improve a whole lot though because the data is already spread out a little bit in the x1 direction.

   RF: Improved by a very small margin but hardly noticeable. This is because this type of model already averages a number of of different models to random numbers of points thus incorporating features from the data that the change in distribution would normally allow other models to see.

# M6-HW2

October 16, 2023

## 1 Problem 7 (30 Points)

Data-driven field prediction models can be used as a substitute for performing expensive calculations/simulations in design loops. For example, after being trained on finite element solutions for many parts, they can be used to predict nodal von Mises stress for a new part by taking in a mesh representation of the part geometry.

Consider the plane-strain compression problem shown in "data/plane-strain.png".

In this problem you are given node features for 100 parts. These node features have been extracted by processing each part shape using a neural network. You will perform feature selection to determine which of these features are most relevant using feature selection tools in sklearn.

*You are welcome to use any of the code provided in the lecture activities.*

**Summary of deliverables:** SciKit-Learn Models: Print Train and Test MSE - `LinearRegression()` with all features - `DecisionTreeRegressor()` with all features - `LinearRegression()` with features selected by `RFE()` - `DecisionTreeRegressor()` with features selected by `RFE()`

Feature Importance/Coefficient Visualizations - Feature importance plot for Decision Tree using all features - Feature coefficient plot for Linear Regression using all features - Feature importance plot for DT showing which features RFE selected - Feature coefficient plot for LR showing which features RFE selected

Stress Field Visualizations: Ground Truth vs. Prediction - Test dataset shape index 8 for decision tree and linear regression with all features - Test dataset shape index 16 for decision tree and linear regression with RFE features

Questions - Respond to the 5 prompts at the end

**Imports and Utility Functions:**

[ ]:
```
import numpy as np
import matplotlib.pyplot as plt
from sklearn.tree import DecisionTreeRegressor
from sklearn.linear_model import LinearRegression
from sklearn.metrics import mean_squared_error
from sklearn.feature_selection import RFE

def plot_shape(dataset, index, model=None, lims=None):
```

```
    x = dataset["coordinates"][index][:,0]
    y = dataset["coordinates"][index][:,1]

    if model is None:
        c = dataset["stress"][index]
    else:
        c = model.predict(dataset["features"][index])

    if lims is None:
        lims = [min(c),max(c)]

    plt.scatter(x,y,s=5,c=c,cmap="jet",vmin=lims[0],vmax=lims[1])
    plt.colorbar(orientation="horizontal", shrink=.75, pad=0,ticks=lims)
    plt.axis("off")
    plt.axis("equal")

def plot_shape_comparison(dataset, index, model, title=""):
    plt.figure(figsize=[6,3.2], dpi=120)
    plt.subplot(1,2,1)
    plot_shape(dataset,index)
    plt.title("Ground Truth",fontsize=9,y=.96)
    plt.subplot(1,2,2)
    c = dataset["stress"][index]
    plot_shape(dataset, index, model, lims = [min(c), max(c)])
    plt.title("Prediction",fontsize=9,y=.96)
    plt.suptitle(title)
    plt.show()

def load_dataset(path):
    dataset = np.load(path)
    coordinates = []
    features = []
    stress = []
    N = np.max(dataset[:,0].astype(int)) + 1
    split = int(N*.8)
    for i in range(N):
        idx = dataset[:,0].astype(int) == i
        data = dataset[idx,:]
        coordinates.append(data[:,1:3])
        features.append(data[:,3:-1])
        stress.append(data[:,-1])
    dataset_train = dict(coordinates=coordinates[:split], features=features[:
 ↪split], stress=stress[:split])
    dataset_test = dict(coordinates=coordinates[split:],␣
 ↪features=features[split:], stress=stress[split:])
    X_train, X_test = np.concatenate(features[:split], axis=0), np.
 ↪concatenate(features[split:], axis=0)
```

```
    y_train, y_test = np.concatenate(stress[:split], axis=0), np.
 ↪concatenate(stress[split:], axis=0)
    return dataset_train, dataset_test, X_train, X_test, y_train, y_test

def get_shape(dataset,index):
    X = dataset["features"][index]
    y = dataset["stress"][index]
    return X, y

def plot_importances(model, selected = None, coef=False, title=""):
    plt.figure(figsize=(6,2),dpi=150)
    y = model.coef_ if coef else model.feature_importances_
    N = 1+len(y)
    x = np.arange(1,N)

    plt.bar(x,y)

    if selected is not None:
        plt.bar(x[selected],y[selected],color="red",label="Selected Features")
        plt.legend()

    plt.xlabel("Feature")

    plt.ylabel("Coefficient" if coef else "Importance")
    plt.xlim(0,N)
    plt.title(title)
    plt.show()
```

### 1.1 Loading the data

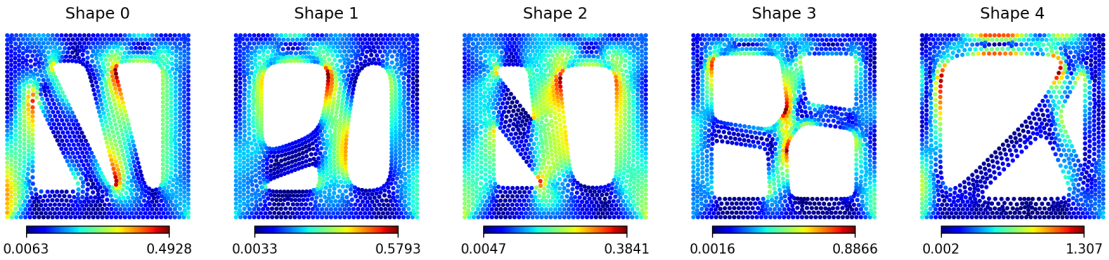
First, complete the code below to load the data and plot the von Mises stress fields for a few shapes. You'll need to input the path of the data file, the rest is done for you.

All training node features and outputs are in `X_train` and `y_train`, respectively. Testing nodes are in `X_test`, `y_test`.

`dataset_train` and `dataset_test` contain more detailed information such as node coordinates, and they are separated by shape.
Get features and outputs for a shape by calling `get_shape(dataset,index)`. `N_train` and `N_test` are the number of training and testing shapes in each of these datasets.

[ ]:
```
dataset_train, dataset_test, X_train, X_test, y_train, y_test =␣
 ↪load_dataset("data/stress_nodal_features.npy")
N_train = len(dataset_train["stress"])
N_test = len(dataset_test["stress"])

plt.figure(figsize=[15,3.2], dpi=150)
for i in range(5):
```

```
    plt.subplot(1,5,i+1)
    plot_shape(dataset_train,i)
    plt.title(f"Shape {i}")
plt.show()
```




## 1.2 Fitting models with all features

Create two models to fit the training data `X_train`, `y_train`: 1. A `LinearRegression()` model 2. A `DecisionTreeRegressor()` model with a `max_depth` of 20

Print the training and testing MSE for each.

[ ]:
```
linear_regression_model = LinearRegression()
linear_regression_model.fit(X_train, y_train)

decision_tree_regressor_model = DecisionTreeRegressor(max_depth=20)
decision_tree_regressor_model.fit(X_train, y_train)

print("Linear Regression Model")
print(f"\tTrain MSE: {mean_squared_error(y_train, linear_regression_model.
 ↪predict(X_train))}")
print(f"\tTest MSE: {mean_squared_error(y_test, linear_regression_model.
 ↪predict(X_test))}")
print()

print("Decision Tree Regressor Model")
print(f"\tTrain MSE: {mean_squared_error(y_train, decision_tree_regressor_model.
 ↪predict(X_train))}")
print(f"\tTest MSE: {mean_squared_error(y_test, decision_tree_regressor_model.
 ↪predict(X_test))}")
print()
```

```
Linear Regression Model
        Train MSE: 0.00811060145497322
        Test MSE: 0.009779482148587704

Decision Tree Regressor Model
        Train MSE: 0.0004944875978805109
```

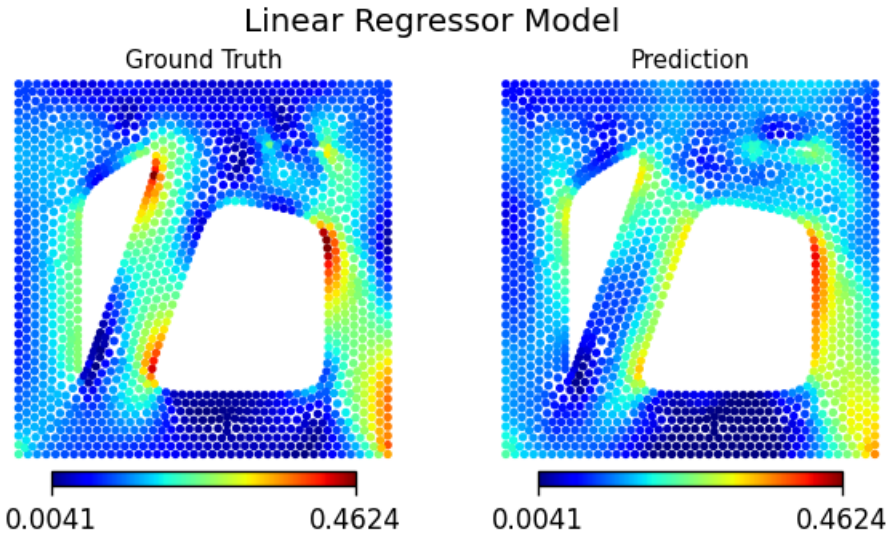
```
Test MSE: 0.00819731747363781
```

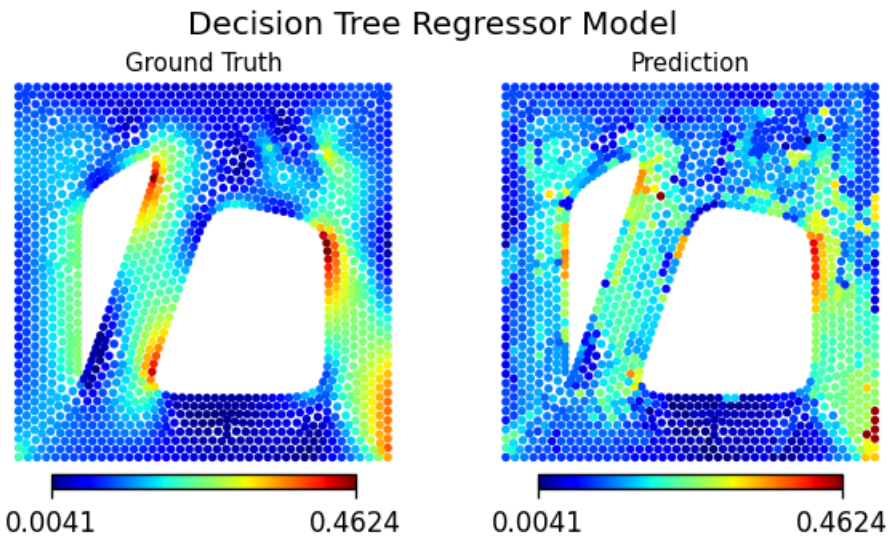
### 1.3 Visualization

Use the `plot_shape_comparison()` function to plot the index 8 shape results in `dataset_test` for each model.

Include titles to indicate which plot is which, using the `title` argument.

[ ]:
```
test_idx = 8

plot_shape_comparison(dataset_test, test_idx, linear_regression_model, "Linear␣
↪Regressor Model")

plot_shape_comparison(dataset_test, test_idx, decision_tree_regressor_model,␣
↪"Decision Tree Regressor Model")
```



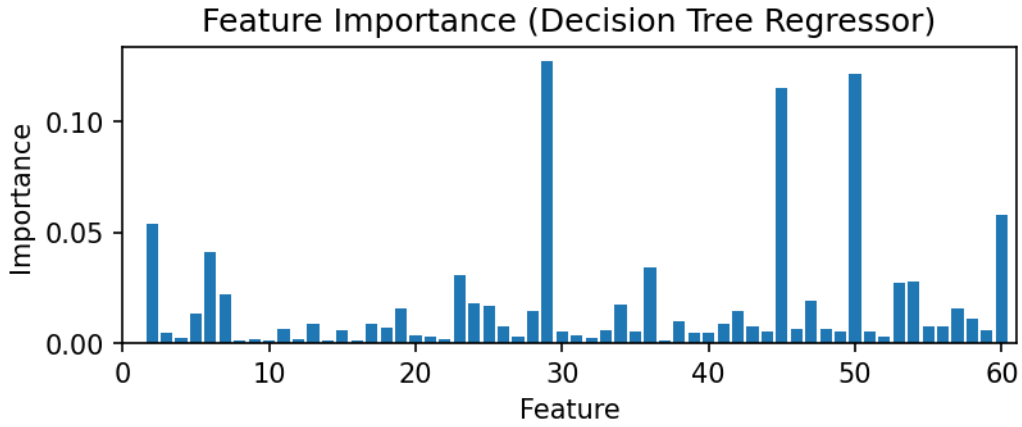


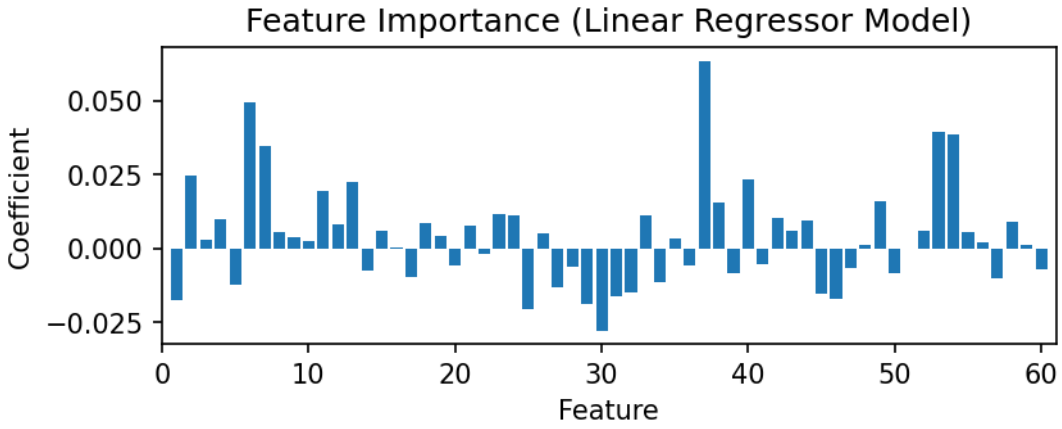

### 1.4 Feature importance

For a tree methods, "feature importance" can be computed, which can be done for an sklearn model using `.feature_importances_`.

Use the provided function `plot_importances()` to visualize which features are most important to the final decision tree prediction.
Then create another plot using the same function to visualize the linear regression coefficients by setting the "coef" argument to `True`.

[ ]:
```
plot_importances(decision_tree_regressor_model, title="Feature Importance␣
↪(Decision Tree Regressor)")
plot_importances(linear_regression_model, coef=True, title="Feature Importance␣
↪(Linear Regressor Model)")
```






### 1.5 Feature Selection by Recursive Feature Elimination

Using `RFE()` in sklearn, you can iteratively select a subset of only the most important features.

For both linear regression and decision tree (depth 20) models: 1. Create a new model. 2. Create an instance of `RFE()` with `n_features_to_select` set to 30. 3. Fit the RFE model as you would a normal sklearn model. 4. Report the train and test MSE.

Note that the decision tree RFE model may take a few minutes to train.
Visit https://scikit-learn.org/stable/modules/generated/sklearn.feature_selection.RFE.html for more information.

[ ]:
```
linear_regression_RFE = RFE(LinearRegression(), n_features_to_select=30)
decision_tree_RFE = RFE(DecisionTreeRegressor(max_depth=20),␣
  ↪n_features_to_select=30)

linear_regression_RFE.fit(X_train, y_train)
decision_tree_RFE.fit(X_train, y_train)

print("Linear Regression RFE")
print(f"\tTrain MSE: {mean_squared_error(y_train, linear_regression_RFE.
  ↪predict(X_train))}")
print(f"\tTest MSE: {mean_squared_error(y_test, linear_regression_RFE.
  ↪predict(X_test))}")
print()

print("Decision Tree RFE")
print(f"\tTrain MSE: {mean_squared_error(y_train, decision_tree_RFE.
  ↪predict(X_train))}")
print(f"\tTest MSE: {mean_squared_error(y_test, decision_tree_RFE.
  ↪predict(X_test))}")
```

```
print()
```

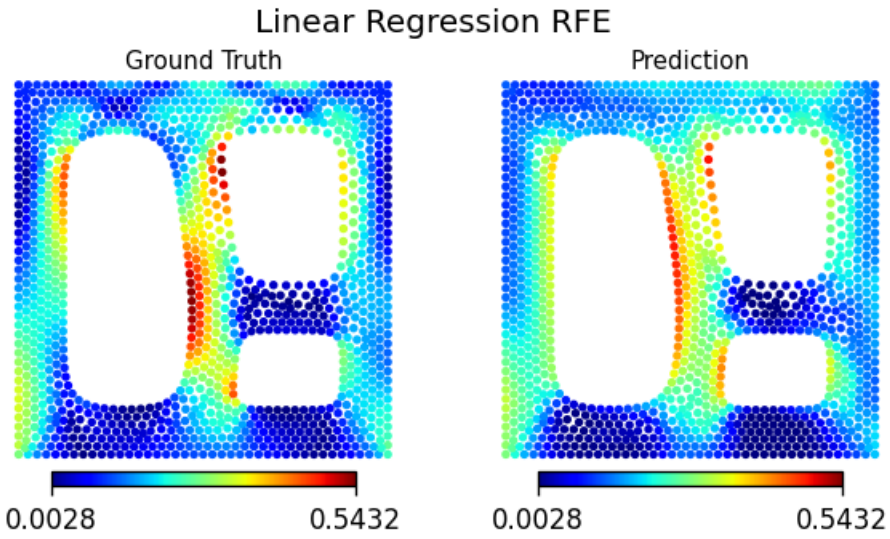
```
Linear Regression RFE
        Train MSE: 0.008508717641234398
        Test MSE: 0.01015037577599287

Decision Tree RFE
        Train MSE: 0.0005351821126843501
        Test MSE: 0.009160064414859462
```

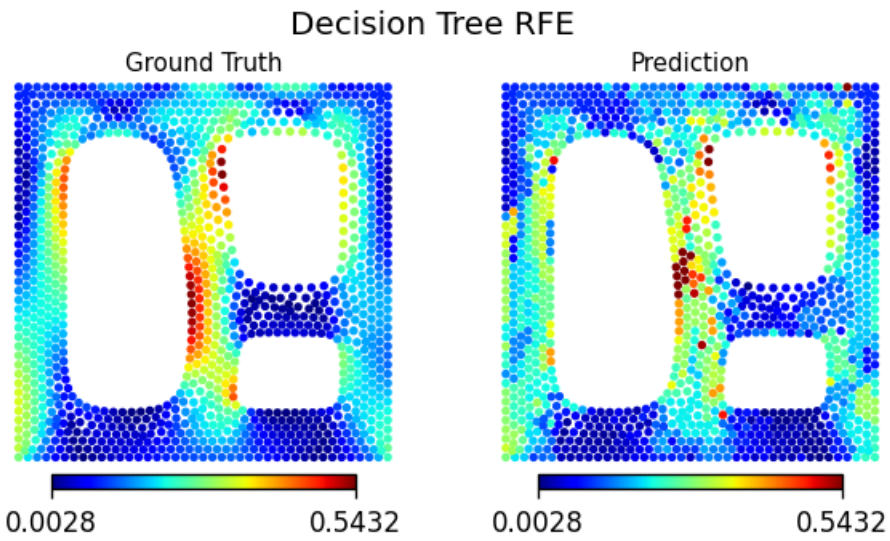
### 1.6 Visualization

Use the `plot_shape_comparison()` function to plot the index 16 shape results in `dataset_test` for each model.

As before, include titles to indicate which plot is which, using the `title` argument.

[ ]:
```
test_idx = 16

plot_shape_comparison(dataset_test, test_idx, linear_regression_RFE, "Linear␣
↪Regression RFE")
plot_shape_comparison(dataset_test, test_idx, decision_tree_RFE, "Decision Tree␣
↪RFE")
```



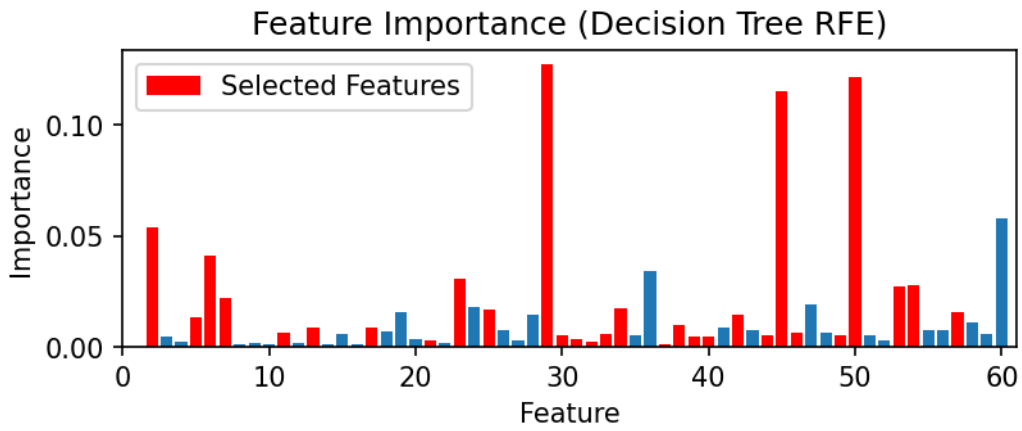


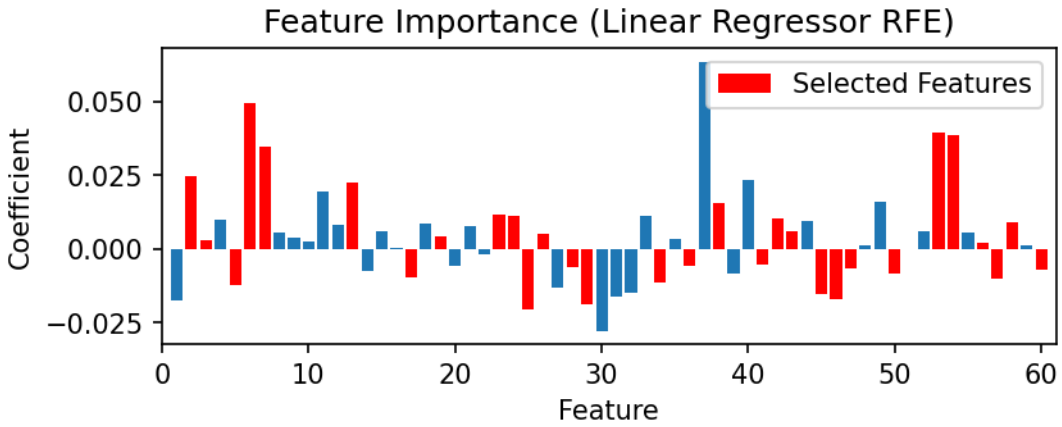

### 1.7 Feature importance with RFE

Recreate the 2 feature importance/coefficent plots from earlier, but this time highlight which features were ultimately selected after performing RFE by coloring those features red. You can do this by setting the `selected` argument equal to an array of selected indices.

For an RFE model `rfe`, the selected feature indices can be obtained via `rfe.get_support(indices=True)`.

[ ]:
```
linear_regression_indices = linear_regression_RFE.get_support(indices=True)
decision_tree_indices = decision_tree_RFE.get_support(indices=True)

plot_importances(decision_tree_regressor_model,␣
 ↪selected=linear_regression_indices, coef=False, title="Feature Importance␣
 ↪(Decision Tree RFE)")
plot_importances(linear_regression_model, selected=decision_tree_indices,␣
 ↪coef=True, title="Feature Importance (Linear Regressor RFE)")
```




### 1.8 Questions

1. Did the MSE increase or decrease on test data for the Linear Regression model after performing RFE?

   The MSE decreased after performing RFE but only by a marginal amount.

2. Did the MSE increase or decrease on test data for the Decision Tree model after performing RFE?

   The MSE decreased after performing RFE but only by a marginal amount.

3. Describe the qualitative differences between the Linear Regression and the Decision Tree predictions.

   The linear regression model has a much more smooth profile of stress concentration change across the surface whereas the decision tree model has very sharp changes in the stress concentration predictions. In addition, the decision tree model is able to predict higher stress

concentrations than the linear regressor model is able to do.

4. Describe how the importance of features that were selected by RFE compare to that of features that were eliminated (for the decision tree).

   In general it seems that the higher importance features had larger coefficients for the decision tree model. While this isn't a perfect match to the data, in general the model chose features with high coefficients as more important.

5. Describe how the coefficients that were selected by RFE compare to that of features that were eliminated (for linear regression).

   This model seemed to choose a variety of features with a range of coefficients seemingly around the average of the whole of the coefficients. This is much different to the decision tree feature importance because it doesn't seem to care too much about how large a feature's coefficient is in determining importance.

# M6-L1-P1

October 16, 2023

## 1 Problem 1 (6 Points)

### 1.1 MinMax Scaling

MinMax scaling scales the data such that the minimum in each column is transformed to 0, and the maximum in each column is transformed to 1, using the formula $X' = \frac{X - X_{min}}{X_{max} - X_{min}}$.

For example, for a training matrix X, MinMax scaling will give:

$$X = \begin{bmatrix} 10 & 2 \\ 9 & 0 \\ 3 & 8 \\ 4 & 9 \\ 0 & 8 \\ -3 & 2 \\ -4 & -2 \\ 5 & 6 \end{bmatrix} \xrightarrow{X} \begin{bmatrix} 1. & 0.364 \\ 0.929 & 0.182 \\ 0.5 & 0.909 \\ 0.571 & 1. \\ 0.286 & 0.909 \\ 0.071 & 0.364 \\ 0. & 0. \\ 0.643 & 0.727 \end{bmatrix}$$

Applying the same scaling to the given matrix of test data `A` should yield the following (notice we are scaling according to `X`, not `A`).

$$A = \begin{bmatrix} 2 & 1 \\ 5 & 5 \\ 11 & -1 \\ -3 & 2 \end{bmatrix} \xrightarrow{X} \begin{bmatrix} 0.429 & 0.273 \\ 0.643 & 0.636 \\ 1.071 & 0.091 \\ 0.071 & 0.364 \end{bmatrix}$$

### 1.2 Implementing MinMax Scaling

A function to compute the minimum and maximum of each column is provided. Complete the scaling function `MinMax_scaler(X, Min, Max)` below to implement $X' = \frac{X - X_{min}}{X_{max} - X_{min}}$.

Validate your results by comparing to the above.

[ ]:
```
import numpy as np
np.set_printoptions(precision=3)

def get_MinMax(X):
    Max = np.max(X,axis=0).reshape(1,-1)
    Min = np.min(X,axis=0).reshape(1,-1)
    return Min, Max
```

```python
def MinMax_scaler(X, Min, Max):
    # YOUR CODE GOES HERE
    # Scale values such that Max --> 1 and Min --> 0
    Xmin = np.min(X, axis=0)
    Xmax = np.max(X, axis=0)
    return (X - Xmin) / (Xmax - Xmin)

# Loading train data X and test data A:
X = np.array([[10,9,3,4,0,-3,-4,5],[2,0,8,9,8,2,-2,6]]).T
A = np.array([[2,5,11,-3],[1,5,-1,2]]).T

# Getting the scaling constants for each column of X:
Xmin, Xmax = get_MinMax(X)

# Scaling X and A, and printing the results
print("X =\n", X, " -->\n", MinMax_scaler(X,Xmin,Xmax))
print("A =\n", A, " -->\n", MinMax_scaler(A,Xmin,Xmax))
```

```
X =
 [[10  2]
 [ 9  0]
 [ 3  8]
 [ 4  9]
 [ 0  8]
 [-3  2]
 [-4 -2]
 [ 5  6]]  -->
 [[1.    0.364]
 [0.929 0.182]
 [0.5   0.909]
 [0.571 1.   ]
 [0.286 0.909]
 [0.071 0.364]
 [0.    0.   ]
 [0.643 0.727]]
A =
 [[ 2  1]
 [ 5  5]
 [11 -1]
 [-3  2]]  -->
 [[0.357 0.333]
 [0.571 1.   ]
 [1.    0.   ]
 [0.    0.5  ]]
```

### 1.3 Standard Scaling

Standard scaling scales the data according to the mean ($\mu$) and standard deviation ($\sigma$) of the training data. Scaling uses the formula $X' = \frac{X-\mu}{\sigma}$.

For example, for a training matrix X, Standard scaling will give:

$$X = \begin{bmatrix} 10 & 2 \\ 9 & 0 \\ 3 & 8 \\ 4 & 9 \\ 0 & 8 \\ -3 & 2 \\ -4 & -2 \\ 5 & 6 \end{bmatrix} \xrightarrow{X} \begin{bmatrix} 1.46 & -0.547 \\ 1.251 & -1.061 \\ 0. & 0.997 \\ 0.209 & 1.254 \\ -0.626 & 0.997 \\ -1.251 & -0.547 \\ -1.46 & -1.576 \\ 0.417 & 0.482 \end{bmatrix}$$

Applying the same scaling to the given matrix of test data `A` should yield the following (notice we are scaling according to `X`, not `A`).

$$A = \begin{bmatrix} 2 & 1 \\ 5 & 5 \\ 11 & -1 \\ -3 & 2 \end{bmatrix} \xrightarrow{X} \begin{bmatrix} -0.209 & -0.804 \\ 0.417 & 0.225 \\ 1.668 & -1.318 \\ -1.251 & -0.547 \end{bmatrix}$$

### 1.4 Implementing Standard Scaling

A function to compute the `mu` and `sigma` of each column is provided. Complete the scaling function `Standard_scaler(X, Min, Max)` below to implement $X' = \frac{X-\mu}{\sigma}$.

Validate your results by comparing to the above.

[ ]:
```
import numpy as np
np.set_printoptions(precision=3)

def get_MuSigma(X):
    mu = np.mean(X,axis=0).reshape(1,-1)
    sigma = np.std(X,axis=0).reshape(1,-1)
    return mu, sigma

def Standard_scaler(X, mu, sigma):
    return (X - mu) / sigma


# Loading train data X and test data A:
X = np.array([[10,9,3,4,0,-3,-4,5],[2,0,8,9,8,2,-2,6]]).T
A = np.array([[2,5,11,-3],[1,5,-1,2]]).T

# Getting the scaling constants for each column of X:
Xmu, Xsigma = get_MuSigma(X)
```

```
# Scaling X and A, and printing the results
print("X =\n", X, " -->\n", Standard_scaler(X,Xmu,Xsigma))
print("A =\n", A, " -->\n", Standard_scaler(A,Xmu,Xsigma))
```

```
X =
 [[10  2]
 [ 9  0]
 [ 3  8]
 [ 4  9]
 [ 0  8]
 [-3  2]
 [-4 -2]
 [ 5  6]]  -->
 [[ 1.46  -0.547]
 [ 1.251 -1.061]
 [ 0.     0.997]
 [ 0.209  1.254]
 [-0.626  0.997]
 [-1.251 -0.547]
 [-1.46  -1.576]
 [ 0.417  0.482]]
A =
 [[ 2  1]
 [ 5  5]
 [11 -1]
 [-3  2]]  -->
 [[-0.209 -0.804]
 [ 0.417  0.225]
 [ 1.668 -1.318]
 [-1.251 -0.547]]
```

# M6-L1-P2

October 16, 2023

## 1 Problem 2 (6 Points)

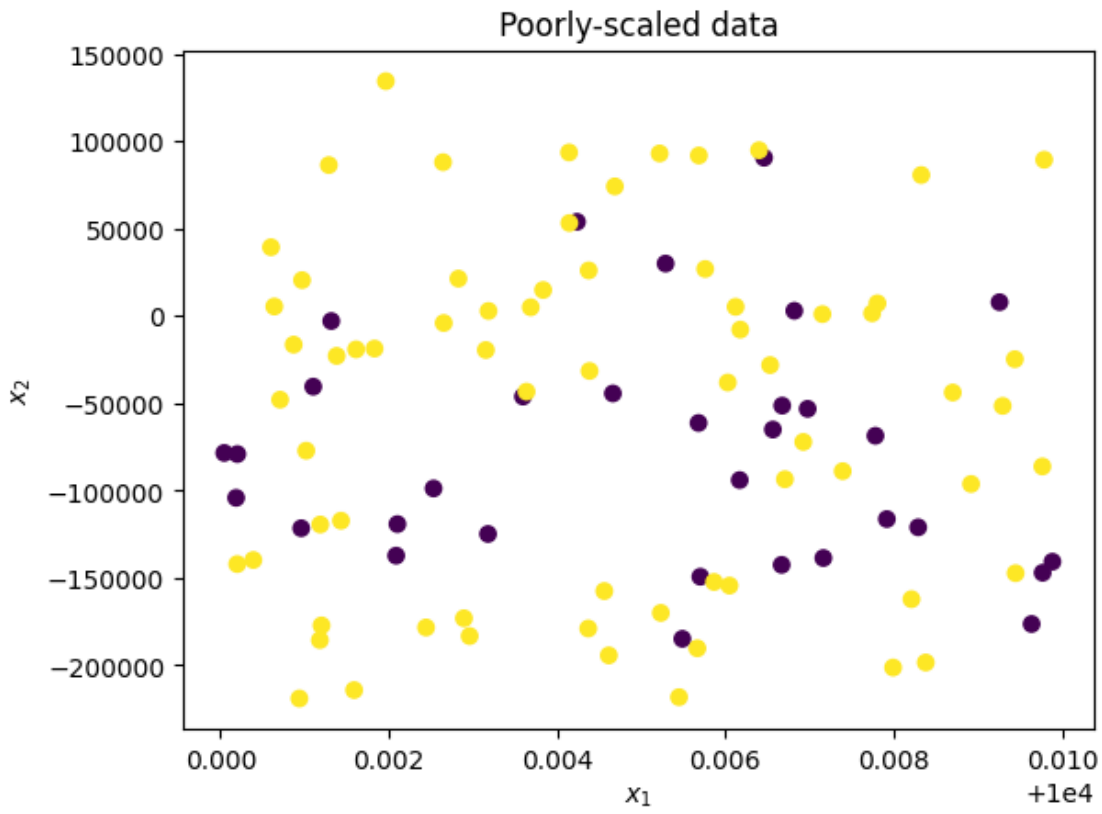
In this problem you'll learn how to make a 'pipeline' in SciKit-Learn. A pipeline chains together multiple sklearn modules and runs them in series. For example, you can create a pipeline to perform feature scaling and then regression. For more information see https://machinelearningmastery.com/standardscaler-and-minmaxscaler-transforms-in-python/

First, run the cell below to import modules and load data. Note the data axis scaling.

[ ]:
```
import numpy as np
import matplotlib.pyplot as plt
from sklearn.model_selection import train_test_split
from sklearn.metrics import accuracy_score
from sklearn.pipeline import Pipeline
from sklearn.preprocessing import StandardScaler, MinMaxScaler
from sklearn.neighbors import KNeighborsClassifier
from sklearn.svm import SVC
from sklearn.linear_model import LogisticRegression
```

```
x1 = np.array([10000.00548814, 10000.00715189, 10000.00602763, 10000.00544883,␣
 ↪10000.00423655, 10000.00645894, 10000.00437587, 10000.00891773, 10000.
 ↪00963663, 10000.00383442, 10000.00791725, 10000.00528895, 10000.00568045,␣
 ↪10000.00925597, 10000.00071036, 10000.00087129, 10000.00020218, 10000.
 ↪0083262 , 10000.00778157, 10000.00870012, 10000.00978618, 10000.00799159,␣
 ↪10000.00461479, 10000.00780529, 10000.00118274, 10000.00639921, 10000.
 ↪00143353, 10000.00944669, 10000.00521848, 10000.00414662, 10000.00264556,␣
 ↪10000.00774234, 10000.0045615 , 10000.00568434, 10000.0001879 , 10000.
 ↪00617635, 10000.00612096, 10000.00616934, 10000.00943748, 10000.0068182 ,␣
 ↪10000.00359508, 10000.00437032, 10000.00697631, 10000.00060225, 10000.
 ↪00666767, 10000.00670638, 10000.00210383, 10000.00128926, 10000.00315428,␣
 ↪10000.00363711, 10000.00570197, 10000.00438602, 10000.00988374, 10000.
 ↪00102045, 10000.00208877, 10000.0016131 , 10000.00653108, 10000.00253292,␣
 ↪10000.00466311, 10000.00244426, 10000.0015897 , 10000.00110375, 10000.
 ↪0065633 , 10000.00138183, 10000.00196582, 10000.00368725, 10000.00820993,␣
 ↪10000.00097101, 10000.00837945, 10000.00096098, 10000.00976459, 10000.
 ↪00468651, 10000.00976761, 10000.00604846, 10000.00739264, 10000.00039188,␣
 ↪10000.00282807, 10000.00120197, 10000.0029614 , 10000.00118728, 10000.
 ↪00317983, 10000.00414263, 10000.00064147, 10000.00692472, 10000.00566601,␣
 ↪10000.00265389, 10000.00523248, 10000.00093941, 10000.00575946, 10000.
 ↪00929296, 10000.00318569, 10000.0066741 , 10000.00131798, 10000.00716327,␣
 ↪10000.00289406, 10000.00183191, 10000.00586513, 10000.00020108, 10000.
 ↪0082894 , 10000.00004695])
```

```
x2 = np.array([-184863.4856705 ,    1074.38382588,  -38090.38042426, -218261.
 ↪93176495,   53942.6974416 ,   90630.02584275, 26090.16140437,  -96193.
 ↪23522311, -176367.73593595, 14900.6554238 , -116285.92522759,   30020.
 ↪05633442, -61255.25197308,    7897.51328353,  -47927.0242543 , -16408.
 ↪41486272,  -79054.99813513,   80728.34445153, -68577.91165667,  -43820.
 ↪95728998,   89483.56273506, -201298.31550282, -194343.64986372,    7245.
 ↪70373422, -185581.10646027,   94925.90670844, -117225.70826838, -147270.
 ↪93302967,   93064.78238323,   53246.3312291 , 88080.30643839,    1544.
 ↪01924478, -157510.31165492, 91905.84577891, -104120.30338562,   -7778.
 ↪92437832, 5252.67709964,  -93950.90837818,  -24732.85666885, 2998.60044099, ␣
 ↪-46121.70219599, -178946.07115258, -53158.56432145,   39374.73070183,␣
 ↪-142511.10737582, -93467.10862949, -119163.81965495,   86433.73556314,␣
 ↪-19493.47186888,  -43328.4347383 , -149292.44670008, -31467.57278374,␣
 ↪-140689.93945916,  -77135.24975531, -137226.1470541 ,  -19121.00345482, ␣
 ↪-28106.82650466, -98746.88800202,  -44359.39586045, -178375.53578575,␣
 ↪-214213.1833435 ,  -40454.74688619,  -64999.38541647, -22847.17067971, ␣
 ↪134483.02973775,    5003.15382914, -162154.00028997,   20531.46592863,␣
 ↪-198431.66694604, -121542.61443332,  -86141.74447922,   74200.84494844,␣
 ↪-147027.93398436, -154379.46847931,  -88860.72719829, -139713.04577259,  ␣
 ↪21397.23298959, -177193.83575271, -183272.178717  , -119403.804027  ,␣
 ↪-124822.92056231, 93657.88484353,    5447.87262332,  -72120.38827533,␣
 ↪-190289.19669472,   -4007.33212386, -170019.38126506, -219029.39870999,  ␣
 ↪26922.68131171,  -51475.16492676, 2877.29414027,  -51314.51123513,   -2885.
 ↪24492876, -138592.30339701, -173081.8557606 ,  -18656.49335465, -152306.
 ↪86977565, -142059.47999752, -120997.92531656, -78426.87568774])
X = np.vstack([x1,x2]).T
y = np.array([0, 1, 1, 1, 0, 0, 1, 1, 0, 1, 0, 0, 0, 0, 1, 1, 0, 1, 0, 1, 1, 1,␣
 ↪1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 0, 1, 1, 0, 1, 0, 0, 1, 0, 1, 0, 1, 0,␣
 ↪1, 1, 1, 0, 1, 0, 1, 0, 1, 1, 0, 0, 1, 1, 0, 0, 1, 1, 1, 1, 1, 1, 0, 1, 1,␣
 ↪0, 1, 1, 1, 1, 1, 1, 1, 0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 0, 0, 0, 1, 1, 1,␣
 ↪1, 0, 0])

X_train, X_test, y_train, y_test = train_test_split(X, y, random_state=0,␣
 ↪train_size=0.8)

plt.figure()
plt.scatter(x1,x2,c=y,cmap="viridis")
plt.xlabel("$x_1$")
plt.ylabel("$x_2$")
plt.title("Poorly-scaled data")
plt.show()
```




### 1.1 Creating a pipeline

In this section, code to set up a pipeline has been given. Make note of how each step works: 1. Create a scaler and classifier 2. Put the scaler and classifier into a new pipeline 3. Fit the pipeline to the training data 4. Make predictions with the pipeline

[ ]:
```
# Create a scaler and a classifier
scaler = MinMaxScaler()
model = KNeighborsClassifier()

# Put the scaler and classifier into a new pipeline
pipeline = Pipeline([("MinMax Scaler", scaler), ("KNN Classifier", model)])

# Fit the pipeline to the training data
pipeline.fit(X_train, y_train)

# Make predictions with the pipeline
pred_train = pipeline.predict(X_train)
pred_test = pipeline.predict(X_test)
print("Training accuracy:", accuracy_score(y_train, pred_train), "   Testing␣
 ↪accuracy:", accuracy_score(y_test, pred_test))
```

```
Training accuracy: 0.825    Testing accuracy: 0.6
```

### 1.2 Testing several pipelines

Now, complete the code to create a new pipeline for every combination of scalers and models below:

Scalers: - None - MinMax - Standard

Classifiers: - Logistic Regression - Support Vector Machine - KNN Classifier, 1 neighbor

Within the loop, a scaler and model are created. You will create a pipeline, fit it to the training data, and make predictions on testing and training data.

[ ]:
```
def get_scaler(i):
    if i == 0:
        return ("No Scaler", None)
    elif i == 1:
        return ("MinMax Scaler", MinMaxScaler())
    elif i == 2:
        return ("Standard Scaler", StandardScaler())

def get_model(i):
    if i == 0:
        return ("Logistic Regression", LogisticRegression())
    elif i == 1:
        return ("Support Vector Classifier", SVC())
    elif i == 2:
        return ("1-NN Classifier", KNeighborsClassifier(n_neighbors=1))

for scaler_index in range(3):
    for model_index in range(3):
        scaler = get_scaler(scaler_index)
        model = get_model(model_index)

        pipeline = Pipeline([scaler, model])
        pipeline.fit(X_train, y_train)
        acc_train = accuracy_score(y_train, pipeline.predict(X_train))
        acc_test = accuracy_score(y_test, pipeline.predict(X_test))

        print(f"{scaler[0]:>15},{model[0]:>26}:    Train Acc. = {100*acc_train:
 ↪5.1f}%    Test Acc. = {100*acc_test:5.1f}%")
```

```
      No Scaler,       Logistic Regression:    Train Acc. =  67.5%    Test Acc.
=  70.0%
      No Scaler, Support Vector Classifier:    Train Acc. =  78.8%    Test Acc.
=  65.0%
      No Scaler,           1-NN Classifier:    Train Acc. = 100.0%    Test Acc.
=  50.0%
  MinMax Scaler,       Logistic Regression:    Train Acc. =  67.5%    Test Acc.
=  70.0%
```

```
  MinMax Scaler, Support Vector Classifier:    Train Acc. =  67.5%    Test Acc.
=  70.0%
  MinMax Scaler,           1-NN Classifier:    Train Acc. = 100.0%    Test Acc.
=  85.0%
Standard Scaler,       Logistic Regression:    Train Acc. =  67.5%    Test Acc.
=  70.0%
Standard Scaler, Support Vector Classifier:    Train Acc. =  68.8%    Test Acc.
=  70.0%
Standard Scaler,           1-NN Classifier:    Train Acc. = 100.0%    Test Acc.
=  85.0%
```

### 1.3 Questions

Answer the following questions:

1. Which model's testing accuracy was improved the most by scaling data?

   The support vector classifier had marginal increase, but the 1-NN classifier improved the most with scaling.

2. Which performs better on this data: MinMax scaler, Standard scaler, or neither?

   Neither

# M6-L1-P3

October 16, 2023

## 1 Problem 3 (6 Points)

SciKit-Learn only offers a few built-in preprocessors, such as MinMax and Standard scaling. However, it also offers the ability to create custom data transformation functions, which can be integrated into your pipeline. In this problem, you will implement a log transform and observe how using it changes a regression result.

Start by running this cell to import modules and load data:

[ ]:
```
import numpy as np
import matplotlib.pyplot as plt
from sklearn.model_selection import train_test_split
from sklearn.metrics import mean_squared_error
from sklearn.pipeline import Pipeline
from sklearn.preprocessing import FunctionTransformer
from sklearn.svm import SVR

def plot(X_train, X_test, y_train, y_test, model=None,log = False):
    plt.figure(figsize=(5,5),dpi=200)
    if model is not None:
        X_fit = np.linspace(min(X_train)-0.15,max(X_train)+0.2)
        y_fit = model.predict(X_fit)
        plt.plot(np.log(X_fit+1) if log else X_fit,␣
 ↪y_fit,c="red",label="Prediction")
    if log:
        X_train = np.log(X_train+1)
        X_test = np.log(X_test+1)

    plt.
 ↪scatter(X_train,y_train,s=30,c="powderblue",edgecolors="navy",linewidths=.
 ↪5,label="Train")
    plt.scatter(X_test,y_test,s=30,c="orange",edgecolors="red",linewidths=.
 ↪5,label="Test")
    plt.legend()
    plt.xlabel("log(x+1)" if log else "x")
    plt.ylabel("y")
    plt.show()
```

```
x = np.array([ 5.83603919,  1.49205924,  2.66109578,  9.40172515,  6.47247125,␣
 ↪0.37633413,  2.58593829,  0.85954061,  0.90192956,  1.50771989, 1.15493443, ␣
 ↪4.28137195,  2.14049632,  1.12938701,  1.55871729, 1.3960884 ,  4.45523172, ␣
 ↪0.8145184 ,  1.36761412,  0.42566793, 0.07784856,  1.92248495,  2.37366743, ␣
 ↪0.47608207,  9.67702601, 0.23354846,  1.04682159,  0.82929126,  4.63102958, ␣
 ↪4.34644717, 1.16759657,  1.45960014,  0.41156606,  0.13795931,  0.70616091,␣
 ↪1.16923416,  3.42222417,  3.32802771,  0.67886919,  0.73911426, 0.35044449, ␣
 ↪0.24170968,  0.18154165,  7.0341397 ,  0.60070448, 0.64527784,  0.28570503, ␣
 ↪2.17600441,  0.19911   ,  0.80836606, 0.408417  ,  1.47241292,  0.60001229, ␣
 ↪0.30708454,  0.97221119, 1.53469532,  1.06877937,  1.35319965,  0.53029486, ␣
 ↪0.6957665 , 0.51045109,  0.69798814,  0.44346062,  0.17794467,  1.19413986,␣
 ↪0.66912731,  0.19589072,  1.58848742,  0.40361317,  1.05331823, 2.07319431, ␣
 ↪1.13767068,  3.12489501,  0.29088542,  1.49532211, 0.50418597,  0.41861772, ␣
 ↪0.56054281,  0.73230914,  1.05777256, 0.31187593,  2.46163678,  1.59306915, ␣
 ↪0.2151879 ,  4.42934711, 6.65846632,  3.25040489,  0.835333  ,  0.34275046, ␣
 ↪2.87040096, 0.66819385,  3.39547978,  1.23155177,  2.65551613,  1.42813072,␣
 ↪2.02703304,  1.01055534,  5.96476998,  1.13531721,  1.49479543, 6.57418553, ␣
 ↪0.25982185,  0.28069545,  2.63635349,  0.30939905, 6.98399558,  0.66125285, ␣
 ↪0.47357035,  6.84105546,  4.39520771, 6.47247753,  2.4745156 ,  0.42264374, ␣
 ↪6.75352745,  0.7649052 , 2.23101446,  2.5786138 ,  0.85640653,  1.84795453, ␣
 ↪2.51483368, 1.45706703,  0.3330706 ,  1.34748269,  3.76740297,  0.49929016,␣
 ↪0.86102259,  0.64716529,  6.35513869,  1.95872697,  1.50299808, 0.46305193, ␣
 ↪1.71471895,  0.50949631,  1.03234257,  0.52948731, 1.96685003,  1.77995987, ␣
 ↪0.81196442,  1.48587929,  0.33518874, 0.22508941,  1.551763  ,  1.18136848, ␣
 ↪1.88708146, 10.83893534, 2.57147454,  0.40138981,  3.05572319,  0.26823082, ␣
 ↪0.6302841 , 0.93403478,  5.54747418,  0.47485071,  0.43760503,  0.90623872,␣
 ↪0.5150567 ,  3.08525997,  0.33961879,  0.3174393 ,  0.64544192, 0.60772521, ␣
 ↪6.88628708,  2.58421247,  1.09149819,  0.29362979, 2.32649531,  0.36780023, ␣
 ↪0.2133607 ,  3.28061135,  1.37292378, 2.51144635,  1.37537669,  2.35568278, ␣
 ↪0.52151064,  0.35549545, 1.97702763,  0.44779951,  0.50180194,  0.63411021, ␣
 ↪1.01763281, 0.70187924,  0.25285191,  0.52538792,  0.10824012,  1.86867841,␣
 ↪0.20148151,  0.33141519,  1.05354965,  0.47732246,  4.67867334, 0.27448548, ␣
 ↪1.30610689,  0.96147875,  0.31095922,  1.68754812, 0.84236124,  2.16363689, ␣
 ↪2.27846997,  8.69924247,  3.80580659])
X = x.reshape(-1,1)
```

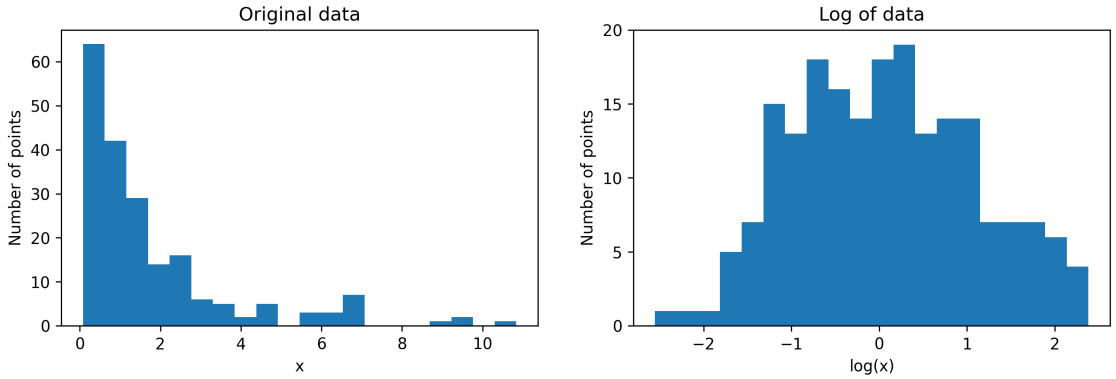
```
y = np.array([ 4.32538472e+00, -5.59312420e+00, -4.57455876e+00,  4.
 ↪23667057e+01, 1.04907251e+01, -4.16547735e+00, -6.27910380e+00, -4.
 ↪66593935e+00, -3.27628398e+00, -5.41260576e+00, -3.07553025e+00, -2.
 ↪60088666e+00, -4.94126516e+00, -5.07104868e+00, -6.78065624e+00, -5.
 ↪64645372e+00, -2.45259954e+00, -2.84042416e+00, -1.57873879e+00, -2.
 ↪01053220e+00, -1.81709993e+00, -6.43544903e+00, -6.92943404e+00, -1.
 ↪43153401e+00, 4.29485069e+01, -1.01830444e+00, -3.90351271e+00, -3.
 ↪11046074e+00, -2.60468704e+00, -3.19751543e+00, -6.61079247e+00, -5.
 ↪90754795e+00, -2.70273587e+00, -4.66887251e-02, -4.76641497e+00, -3.
 ↪30726512e+00, -3.15777577e+00, -8.66934765e+00, -2.29409449e+00, -2.
 ↪13391937e+00, -2.58556664e+00, -1.74603256e+00, -1.07407173e+00,  1.
 ↪38617365e+01, -3.10619598e+00, -5.32401140e+00,  3.81599556e-01, -4.
 ↪52559897e+00, -2.17595159e+00, -5.58801110e+00, -1.09368325e+00, -6.
 ↪05774675e+00, -2.42711696e+00, -1.92011443e+00, -2.87855321e+00, -4.
 ↪27606315e+00, -5.29000358e+00, -7.00989489e+00, -4.74466924e+00, -2.
 ↪07917240e+00, -4.07498403e+00, -3.76297780e+00, -2.91511682e+00, -9.
 ↪36910003e-01, -7.44914900e+00, -2.61473730e+00, -1.55243871e-01, -5.
 ↪28651169e+00, -2.32149151e+00, -4.01101159e+00, -5.46926738e+00, -8.
 ↪55294796e+00, -2.92563777e+00, -7.84672807e-01, -6.21923521e+00, -2.
 ↪85315642e+00, -1.17723512e+00, -2.66266171e+00, -6.17129572e+00, -1.
 ↪07324073e+00, -1.62792403e+00, -4.71826920e+00, -6.46555121e+00,  1.
 ↪27493192e+00, -2.09810420e+00,  1.19561079e+01, -7.25477255e+00, -1.
 ↪66216583e+00, -5.61547171e-01, -4.16003379e+00, -3.83661758e+00, -6.
 ↪16965664e+00, -1.18516405e+00, -7.81583847e+00, -5.30502079e+00, -4.
 ↪32096521e+00, -3.88496715e+00,  6.62906156e+00, -4.98681443e+00, -4.
 ↪68447995e+00, 8.38919748e+00,  1.25559415e+00, -1.50193339e+00, -7.
 ↪25167503e+00, -4.51692863e-01,  1.31651367e+01, -4.87039664e+00, -4.
 ↪16365912e+00, 1.36354222e+01, -2.89754788e+00,  9.33002536e+00, -4.
 ↪63273484e+00, -3.62482967e+00,  1.07464791e+01, -3.81676576e+00, -6.
 ↪03611939e+00, -6.30707705e+00, -3.97893131e+00, -5.91727631e+00, -6.
 ↪41073788e+00, -6.24169740e+00, -2.77390647e+00, -4.50992930e+00, -5.
 ↪98006234e+00, -3.98319304e+00, -4.03219142e+00, -3.05350405e+00,  1.
 ↪19971796e+01, -7.01407392e+00, -3.84109609e+00, -9.80060053e-01, -7.
 ↪41675111e+00, -1.12801561e+00, -5.87180262e+00, -6.35583810e+00, -5.
 ↪05627183e+00, -8.36537808e+00, -2.72413419e+00, -6.24757554e+00,  9.
 ↪25733994e-01, 5.27982307e-01, -6.03092529e+00, -5.54296733e+00, -7.
 ↪69544697e+00, 6.26264586e+01, -6.66542463e+00, -2.39287559e+00, -5.
 ↪70611595e+00, -4.01424069e-01, -2.22968078e+00, -4.94396881e+00,  8.
 ↪80411673e-01, -1.01972575e-01, -3.03070076e+00, -4.68836537e+00, -3.
 ↪09178407e+00, -8.67207510e+00, -2.29971402e+00, -2.20591252e+00, -1.
 ↪88007689e+00, -1.62161041e+00,  1.29951706e+01, -4.84513570e+00, -3.
 ↪75617518e+00, -1.40545367e+00, -5.97850387e+00, -1.98970437e+00, -1.
 ↪61355500e+00, -6.04224622e+00, -6.67171619e+00, -5.82920642e+00, -6.
 ↪47490784e+00, -4.96672564e+00, -2.70976774e+00, -1.57685774e+00, -5.
 ↪23574473e+00, -1.07350146e+00, -4.61313963e+00, -3.07881081e+00, -2.
 ↪73231916e+00, -5.56392046e+00, -6.19404753e-01, -5.73425346e+00, -2.
 ↪06324496e+00, -5.79348723e+00, -3.45188541e+00, -3.02550603e+00, -6.
 ↪36553389e+00, 3.13426823e-01, -1.25704084e+00, -4.46149712e-01, -5.
 ↪15863188e+00, -4.41309998e+00, -3.88281175e+00, -6.02767799e+00, -4.
 ↪64447206e+00, -4.84997397e+00, -4.48927165e+00,  3.43804753e+01, -3.
```

```
X_train, X_test, y_train, y_test = train_test_split(X, y, random_state=1,␣
↪train_size=0.8)
```

### 1.1 Distribution of x data

Let's visualize how the original input feature is distributed, alongside the log of the data – notice that performing this log transformation makes the data much closer to normally distributed.

[ ]:
```
plt.figure(figsize=(12,3.4),dpi=300)
plt.subplot(1,2,1)
plt.hist(x,bins=20)
plt.xlabel("x")
plt.ylabel("Number of points")
plt.title("Original data")

plt.subplot(1,2,2)
plt.hist(np.log(x),bins=20)
plt.xlabel("log(x)")
plt.ylabel("Number of points")
plt.title("Log of data")
plt.ylim(0,20)
plt.yticks([0,5,10,15,20])

plt.show()
```



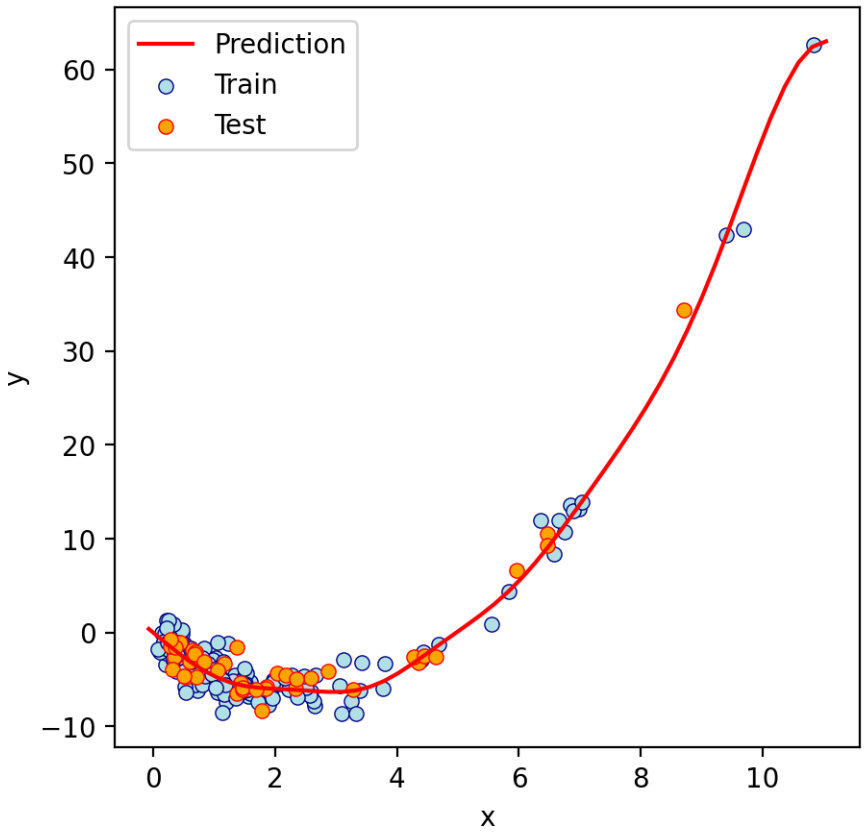

### 1.2 No log transform

First, we do support vector regression on the untransformed inputs. The code to do this has been provided below.

[ ]:
```
model = SVR(C=100)

pipeline = Pipeline([("SVR", model)])
```

```
# Fit the pipeline to the training data
pipeline.fit(X_train, y_train)

# Make predictions with the pipeline
pred_train = pipeline.predict(X_train)
pred_test = pipeline.predict(X_test)
print("Training MSE:", mean_squared_error(y_train, pred_train), "   Testing MSE:
 ↪", mean_squared_error(y_test, pred_test))

# Plot the predictions
plot(X_train, X_test, y_train, y_test, pipeline)
```

```
Training MSE: 2.0710061552336017    Testing MSE: 1.9453578716771627
```




### 1.3 With log transform

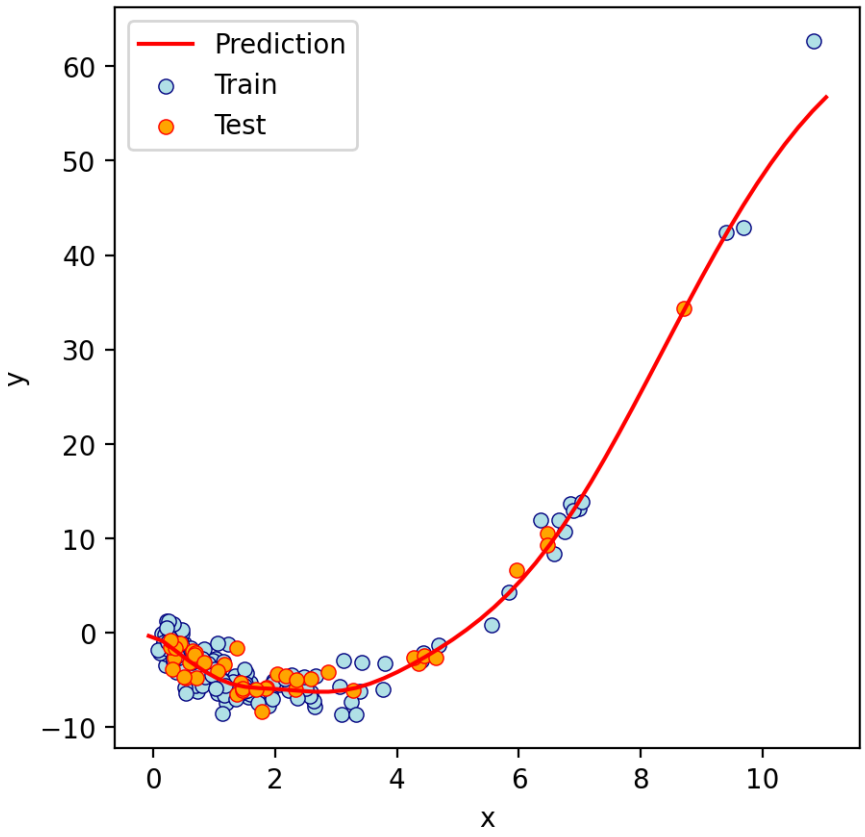
Notice that the data are not spread uniformly across the x axis. Instead, most input data points have low values – this is a roughly "log normal" distribution. If we take the log of the input, we saw it was more normally distributed, which can improve machine learning model results in some cases. The transform function has been given below. Add this to a new pipeline, train the pipeline, and

compute the train MSE and test MSE. Show a plot as above. Note the subtle change in behavior of the fitting curve.

Also, make another plot setting the `log` argument to `True`. This will show the scaling of the x-axis used by the model.

[ ]:
```
def log_transform(x):
    return np.log(x + 1.)

transform = FunctionTransformer(log_transform)
model = SVR(C=100)

pipeline = Pipeline([("transform", transform), ("SVR", model)])
pipeline.fit(X_train, y_train)

print("Training MSE:", mean_squared_error(y_train, pipeline.predict(X_train)),␣
  ↪"   Testing MSE:", mean_squared_error(y_test, pipeline.predict(X_test)))
plot(X_train, X_test, y_train, y_test, pipeline)
```

```
Training MSE: 2.3139214731606534    Testing MSE: 1.7200875366333022
```



# M6-L2-P1

October 16, 2023

## 1 Problem 4 (6 Points)

In this problem you will code a function to perform feature filtering using the Pearson's Correlation Coefficient method.

To start, run the following cell to load the mtcars dataset. Feature names are stored in `feature_names`, while the data is in `data`.

[ ]:
```
import numpy as np

feature_names =␣
 ↪["mpg","cyl","disp","hp","drat","wt","qsec","vs","am","gear","carb"]
data = np.array([[21,6,160,110,3.9,2.62,16.46,0,1,4,4], [21,6,160,110,3.9,2.
 ↪875,17.02,0,1,4,4], [22.8,4,108,93,3.85,2.32,18.61,1,1,4,1],[21.
 ↪4,6,258,110,3.08,3.215,19.44,1,0,3,1], [18.7,8,360,175,3.15,3.44,17.
 ↪02,0,0,3,2],
                [18.1,6,225,105,2.76,3.46,20.22,1,0,3,1], [14.3,8,360,245,3.
 ↪21,3.57,15.84,0,0,3,4], [24.4,4,146.7,62,3.69,3.19,20,1,0,4,2],[22.8,4,140.
 ↪8,95,3.92,3.15,22.9,1,0,4,2],[19.2,6,167.6,123,3.92,3.44,18.3,1,0,4,4],
                [17.8,6,167.6,123,3.92,3.44,18.9,1,0,4,4],[16.4,8,275.8,180,3.
 ↪07,4.07,17.4,0,0,3,3],[17.3,8,275.8,180,3.07,3.73,17.6,0,0,3,3],[15.2,8,275.
 ↪8,180,3.07,3.78,18,0,0,3,3],[10.4,8,472,205,2.93,5.25,17.98,0,0,3,4],
                [10.4,8,460,215,3,5.424,17.82,0,0,3,4],[14.7,8,440,230,3.23,5.
 ↪345,17.42,0,0,3,4],[32.4,4,78.7,66,4.08,2.2,19.47,1,1,4,1],[30.4,4,75.7,52,4.
 ↪93,1.615,18.52,1,1,4,2],[33.9,4,71.1,65,4.22,1.835,19.9,1,1,4,1],
                [21.5,4,120.1,97,3.7,2.465,20.01,1,0,3,1],[15.5,8,318,150,2.
 ↪76,3.52,16.87,0,0,3,2],[15.2,8,304,150,3.15,3.435,17.3,0,0,3,2],[13.
 ↪3,8,350,245,3.73,3.84,15.41,0,0,3,4],[19.2,8,400,175,3.08,3.845,17.
 ↪05,0,0,3,2],
                [27.3,4,79,66,4.08,1.935,18.9,1,1,4,1],[26,4,120.3,91,4.43,2.
 ↪14,16.7,0,1,5,2],[30.4,4,95.1,113,3.77,1.513,16.9,1,1,5,2],[15.8,8,351,264,4.
 ↪22,3.17,14.5,0,1,5,4],[19.7,6,145,175,3.62,2.77,15.5,0,1,5,6],
                [15,8,301,335,3.54,3.57,14.6,0,1,5,8],[21.4,4,121,109,4.11,2.
 ↪78,18.6,1,1,4,2]])
```

### 1.1 Filtering

Now define a function `find_redundant_features(data, target_index, threshold)`. Inputs: - data: input feature matrix - target_index: index of column in `data` to treat as the target feature - threshold: eliminate indices with pearson correlation coefficients greater than `threshold`

Return: - Array of the indices of features to remove.

Procedure: 1. Compute correlation coefficients with `np.corrcoeff(data.T)`, and take the absolute value. 2. Find off-diagonal entries greater than `threshold` which are not in the target_index row/column. 3. For each of these entries above `threshold`, determine which has a lower correlation with the target feature – add this index to the list of indices to filter out/remove. 4. Remove possible duplicate entries in the list of indices to remove.

[ ]:
```
def find_redundant_features(data, target_index, threshold):
    corrcoef = np.corrcoef(data.T)
    idx = np.where(abs(corrcoef) > threshold)
    idx = np.array([idx[0], idx[1]])
    idx = idx[:,np.where((idx[0,:] - idx[1,:] != 0) & (idx[0,:] !=␣
 ↪target_index) & (idx[1,:] != target_index))[0]]
    results = -1*np.ones(corrcoef.shape[0])
    for idx_pair in idx.T:
        idx1, idx2 = idx_pair
        corrcoef1 = abs(corrcoef[idx1, target_index])
        corrcoef2 = abs(corrcoef[idx2, target_index])
        remove_index = idx1 * (corrcoef1 < corrcoef2) + idx2 * (corrcoef1 >=␣
 ↪corrcoef2)
        results[remove_index] = remove_index

    return results[results != -1]
```

### 1.2 Testing your function

The following test cases should give the following results: | target_index | threshold | | Indices to remove | |—|—|—|—| | 0 | 0.9 | | [2] | | 2 | 0.7 | | [0, 3, 4, 5, 6, 7, 8, 9, 10] | | 10 | 0.8 | | [1, 2, 5] |

Try these out in the cell below and print the indices you get.

[ ]:
```
print(f"Target Index \t|\t Threshold \t|\t Indecies to Remove")
print(f"{0}          \t|\t    {0.9}   \t|\t {find_redundant_features(data, 0, 0.
 ↪9)}")
print(f"{2}          \t|\t    {0.7}   \t|\t {find_redundant_features(data, 2, 0.
 ↪7)}")
print(f"{10}         \t|\t    {0.8}   \t|\t {find_redundant_features(data, 10, 0.
 ↪8)}")
```

```
Target Index    |        Threshold      |        Indecies to Remove
0               |           0.9         |        [2.]
2               |           0.7         |        [ 0.  3.  4.  5.  6.  7.  8.
9. 10.]
```

```
10              |           0.8         |        [1. 2. 5.]
```

### 1.3 Using your function

Run these additional cases and print the results: | target_index | threshold | | Indices to remove | |—|—|—|—| | 4 | 0.9 | | ? | | 5 | 0.8 | | ? | | 6 | 0.95 | | ? |

[ ]:
```
print(f"Target Index \t|\t Threshold \t|\t Indecies to Remove")
print(f"{4}          \t|\t    {0.9}   \t|\t {find_redundant_features(data, 4, 0.
  ↪9)}")
print(f"{5}          \t|\t    {0.8}   \t|\t {find_redundant_features(data, 5, 0.
  ↪8)}")
print(f"{6}          \t|\t    {0.95}  \t|\t {find_redundant_features(data, 6, 0.
  ↪95)}")
```

```
Target Index    |        Threshold      |        Indecies to Remove
4               |           0.9         |        [1.]
5               |           0.8         |        [0. 1. 3. 7.]
6               |           0.95        |        []
```

# M6-L2-P2

October 16, 2023

## 1 Problem 5 (6 Points)

Now you will implement a wrapper method. This will iteratively determine which features should be most beneficial for predicting the output. Once more, we will use the MTCars dataset predicting `mpg`.

[ ]:
```
import numpy as np
np.set_printoptions(precision=3)
from sklearn.svm import SVR
from sklearn.metrics import mean_squared_error
from sklearn.model_selection import train_test_split
import itertools

feature_names =␣
 ↪["mpg","cyl","disp","hp","drat","wt","qsec","vs","am","gear","carb"]
data = np.array([[21,6,160,110,3.9,2.62,16.46,0,1,4,4], [21,6,160,110,3.9,2.
 ↪875,17.02,0,1,4,4], [22.8,4,108,93,3.85,2.32,18.61,1,1,4,1],[21.
 ↪4,6,258,110,3.08,3.215,19.44,1,0,3,1], [18.7,8,360,175,3.15,3.44,17.
 ↪02,0,0,3,2],
                [18.1,6,225,105,2.76,3.46,20.22,1,0,3,1], [14.3,8,360,245,3.
 ↪21,3.57,15.84,0,0,3,4], [24.4,4,146.7,62,3.69,3.19,20,1,0,4,2],[22.8,4,140.
 ↪8,95,3.92,3.15,22.9,1,0,4,2],[19.2,6,167.6,123,3.92,3.44,18.3,1,0,4,4],
                [17.8,6,167.6,123,3.92,3.44,18.9,1,0,4,4],[16.4,8,275.8,180,3.
 ↪07,4.07,17.4,0,0,3,3],[17.3,8,275.8,180,3.07,3.73,17.6,0,0,3,3],[15.2,8,275.
 ↪8,180,3.07,3.78,18,0,0,3,3],[10.4,8,472,205,2.93,5.25,17.98,0,0,3,4],
                [10.4,8,460,215,3,5.424,17.82,0,0,3,4],[14.7,8,440,230,3.23,5.
 ↪345,17.42,0,0,3,4],[32.4,4,78.7,66,4.08,2.2,19.47,1,1,4,1],[30.4,4,75.7,52,4.
 ↪93,1.615,18.52,1,1,4,2],[33.9,4,71.1,65,4.22,1.835,19.9,1,1,4,1],
                [21.5,4,120.1,97,3.7,2.465,20.01,1,0,3,1],[15.5,8,318,150,2.
 ↪76,3.52,16.87,0,0,3,2],[15.2,8,304,150,3.15,3.435,17.3,0,0,3,2],[13.
 ↪3,8,350,245,3.73,3.84,15.41,0,0,3,4],[19.2,8,400,175,3.08,3.845,17.
 ↪05,0,0,3,2],
                [27.3,4,79,66,4.08,1.935,18.9,1,1,4,1],[26,4,120.3,91,4.43,2.
 ↪14,16.7,0,1,5,2],[30.4,4,95.1,113,3.77,1.513,16.9,1,1,5,2],[15.8,8,351,264,4.
 ↪22,3.17,14.5,0,1,5,4],[19.7,6,145,175,3.62,2.77,15.5,0,1,5,6],
                [15,8,301,335,3.54,3.57,14.6,0,1,5,8],[21.4,4,121,109,4.11,2.
 ↪78,18.6,1,1,4,2]])
```

```
target_idx = 0
y = data[:,target_idx]
X = np.delete(data,target_idx,1)
```

### 1.1 Fitting a model

The following function is provided: `get_train_test_mse(X,y,feature_indices)`. This will train a model to fit the data, using only the features specified in `feature_indices`. A train and test MSE are computed and returned.

[ ]:
```
def get_train_test_mse(X, y, feature_indices=None):
    if feature_indices is not None:
        X = X[:,feature_indices]
    X_tr, X_te, y_tr, y_te =␣
 ↪train_test_split(X,y,random_state=12,train_size=int(len(y)*.8))
    model = SVR()
    model.fit(X_tr,y_tr)
    mse_train = mean_squared_error(y_tr,model.predict(X_tr))
    mse_test = mean_squared_error(y_te,model.predict(X_te))
    return mse_train, mse_test

mse_train, mse_test = get_train_test_mse(X, y, None)
print(f"Model using all features:    Train MSE={mse_train:.1f},  Test␣
 ↪MSE={mse_test:.1f}")
```

```
Model using all features:    Train MSE=16.1,  Test MSE=18.3
```

### 1.2 Wrapper method

Now your job is to write a function `get_next_pair(X, y, current_indices)` that considers all pairs of features to add to the model.
`X` and `y` contain the full input and output arrays. `current_indices` lists the indices currently used by your model and you want to determine the indices of the 2 features that best improve the model (gives the lowest test MSE). Return the indices as an array.

If you want to avoid a double for-loop, `itertools.combinations()` can help generate all pairs of indices from a given array.

[ ]:
```
def get_next_pair(X, y, current_indices):
    numFeatures = X.shape[1] #- len(current_indices)
    best_pair = (0,1)
    best_pair_MSE = 0
    existing_pairs = [pair for pair in itertools.combinations(current_indices.
 ↪flatten(),2)]
    for pair in itertools.combinations([i for i in range(numFeatures)],2):
        if (pair not in existing_pairs and pair[0] not in current_indices and␣
 ↪pair[1] not in current_indices):
            curr_pair_MSE = get_train_test_mse(X, y, pair)[0]
            if (best_pair_MSE < curr_pair_MSE):
```

```
                best_pair = pair
                best_pair_MSE = curr_pair_MSE
    return best_pair
```

### 1.3 Trying out the wrapper method

Now, let's start with an empty array of indices and add 2 features at a time to the model. Repeat this until there are 8 features considered. Each pair is printed as it is added.

The first few pairs should be: - (2, 5) - (0, 8)

[ ]:
```
indices = np.array([(2,5), (0,8)])
while len(indices) < 8:
    pair = get_next_pair(X, y, indices)
    print(f"Adding pair {pair}")
    indices = np.union1d(indices, pair)
print(indices)
```

```
Adding pair (3, 7)
Adding pair (6, 9)
[0 2 3 5 6 7 8 9]
```

### 1.4 Question

Which 2 feature indices were deemed "least important" by this wrapper method?

Features 1 and 4 were deemed least important