The Spatial Distribution of Shopping Areas, A Gravity Model Approach *

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Abstract

We modify a gravity model and simulate developments in an artificial shopping area. We show that a globally stable steady state exists given certain assumptions. Furthermore, we run numerical simulations an the system. In particular, we mimic the construction of a shopping center in a small rural area with three towns and report the major effects on the involved communities along a time-line. In general, we find that the shopping center can have positive and negative effects on the towns in the area. In the long run, the community that builds the shopping center does not necessarily benefit the most.

Keywords:
Numerical Computation, Gravity Model, Retail Market Areas

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1 Introduction

Do shopping centers destroy inner city precincts and why do communities have an incentive to attract a shopping center? We use a modified gravity model to investigate regional policy issues concerning the planning and coordination of shopping center (SC) sites in an artificial, computer-generated environment. Gravity models use Newton's Law and postulate that two physical masses attract each other with a certain force. Growing distances between the two masses weaken these gravitational forces.

In the economic literature gravity models have become successful tools for estimating bilateral trade relations. In traditional trade setups, supply factors of the export country, *i.e.* population and GDP, demand factors of the importing country (also population and GDP), and trade supporting, as well as, impeding determinants like transport costs, enter the model.[1] Gravity models have also been used to estimate border effects between countries. Another branch of the literature uses gravity models to describe population dynamics. (Bressler & Harsche 1998) present an introduction on gravity models in geodynamics on their website.[2]

In the following model we define as masses the "shopping quality" of a town. Quality should give an indication of the supply strength of a town. As a numeric proxy for quality one could think of the aggregate size in m^2 of retail outlets in a town, the product range, price levels, or service standards etc. In general, this measure describes the attractiveness of a shopping location.

Counter forces working against the attraction of a particular location are summarized in a distance variable. A shopping outlet with high quality factors loses some attraction if it is costly to access (*e.g.* geographical- or time distances). More indirect "distance" costs are caused by traffic jams due to bad roads, parking fees, road pricing, border crossings *etc.*

Our goal is to use a modified gravity model to describe purchasing-power streams over time, in an area with a given population and a given shopping outlet structure. We analytically show that given certain assumptions the long run steady state in our artificial environment supports an interior solution which is asymptotically stable. Furthermore, this solution is independent from initial quality distributions and depends only on population distribution and distances. We also show, that with an increase in mobility it is more likely that smaller towns will lose all of their quality (shops) and that border solutions become more prevalent.[3]

In the simulation section we investigate the effects of the construction of a new shopping center in an artificial region with threee towns. In our terms we call the construction of a shopping center a quality shock, since a town's quality increases sharply. We would like to identify the winners and losers of such a policy, or better, which towns can benefit from this investment decision. Finally, we are also interested in long-run effects on the attractiveness of towns in this environment and therefore pay attention to developments of the quality measure over time.

[1] See *e.g.* (Egger & Pfaffermayr 2001) for an econometric specification of trade gravity models.

[2] visit: www.uni-flensburg.de/geo/wisogeo

[3] Border solutions describe situations in which one town attracts all consumers and accumulates all shopping outlets in the region. All other towns lose all of their shops.

We also want to point out the fact that this model is a numerical simulation that uses very simplified assumptions about the underlying buying, consumption and investment behavior of the individuals involved. This allows us to simplify the complex processes involved and to isolate important effects that might go unnoticed if we relied entirely on econometric methods using, say, sales- and population data only.

In this sense, we are able to simulate demand changes after the opening of a new shopping center. The paper's goal is not to forecast exact numbers of the sales figures. However, future research might be able to estimate and quantify some of the effects from real data.

In the next section we briefly describe the role of the gravity model in the social sciences and why certain features of the traditional gravity model make it unsuited for our purposes. In section 3 we briefly describe the setup of the model and the main changes we implement in order to overcome the "weaknesses" of more traditional setups. assumptions. Section 4 contains an analytical analysis of the properties of the steady state of a 2 and 3 town system. We prove that only the population distribution and distances determine the steady state, independent of any starting values for qualities of the three towns. In section 5 we extend our initial example and run simulations under varying assumptions on a region with three towns and a partly fixed population distribution. The last section concludes our observations.

2 The Gravity Model in the Social Sciences

Gravity models, as used in the social sciences, borrow the established relationship between physical masses from Newton's law. The larger these masses are, the stronger are the attractive forces between them. The further apart these masses are, the weaker this attraction link becomes. Gravity models have been successfully employed in the social sciences. Trade models and models describing the dynamics of population migrations are a few to name.

In economic contexts "masses" are often interpreted as GDP or other relevant economic measures. The trade literature employs regression models of the form:

$$y_{home} = const + \beta_1 GDP_{home} + \beta_2 Pop_{home} + \beta_3 GDP_{foreign} + \beta_4 Pop_{foreign} + \varepsilon \tag{1}$$

where y_{home} is a measure for the trade stream between *home* and *foreign*, like exports. The parameter Pop denotes the population of the two countries.

The population dynamics literature replaced Newton's notion of physical mass with population pools or number of residents of a region. The attractive forces have then become migration flows or migration streams between regions and distance is sometimes re-interpreted as a variety of transactions costs which could be a mixture of geographical distance, transport costs, administrative or cultural distances, border effects, etc. The following equation describes these dependencies for a simple case:

$$F_{ij} = k \frac{M_i M_j}{d_{ij}^{\Theta}}, \tag{2}$$

where F_{ij} are the streams flowing (forces) between region i and region j, k is a gravity constant, M_i and M_j are the masses (or populations) of region i and region j, d is the distance between the regions and Θ is a constant determining the influence of distance on F_{ij}. These models suffer from a few deficiencies which make them particularly unsuited for our simulation purpose.

The first problem is, that the direction of the streams is not accounted for (undirected forces). In physics the aggregate "force" is enough to derive further results, in our setting, however, this is a severe disadvantage. In this sense, we can calculate the trade or purchasing stream between regions as an aggregate, but we cannot identify how many people of region 1, shop in region 2. This is in general not a problem for econometric estimations like (1), since we already have data on the dependent variable y (*e.g.* exports) and the regression run twice, once for *home* and once for *foreign* will result in differing parameters for GDP and Pop. However, we are dealing with a simulation and we therefore have to calculate the "trade streams" represented by y. Hence, we need a rule or an algorithm returning values for the parameters.

A further disadvantage of (2) is that gravitational forces are measured in units of $\frac{1}{distance}$ which is hard to interpret. The following example, out of (Bressler & Harsche 1998), illustrates this problem.

Example 1 *The population masses for two towns i and j are given in $M_i = 64$ and $M_j = 16$. Gravity constant $k = 1$, distance $d = 2$. Plugging this into (2) results in $F_{ij} = 512$. Since the population figures of both towns do not reach 512, an interpretation of how many residents of town i shop in town j is not possible.*

Finally, if we would like to employ the model to measure consumption streams in a geographical area with different towns , we would also like to account for purchases of consumers in their hometown. This self-attraction, however, is also excluded in the original model.

3 The Gravity-Flow Model

3.1 Basic Setup

In this section we introduce some modifications to the original gravity model to make it more suitable for analyzing consumption streams in an artificial geographical area with a finite number of shopping-towns. Accordingly, we extend the model in various directions, maintaining the main assumption: Shopping quality and distance are the decisive factors to determine the attractiveness of a shopping town. We start with the following basic definition.

Definition 2 *Towns.*
There are n towns. Town i is defined as a pair $(q_i, b_i) \in R_+ \times R_+$, where q_i denotes the quality (attractiveness) and b_i denotes the population of town i. Vector $\mathbf{q}(t) \in R_+^n$ is the quality vector of period t and vector $\mathbf{b} \in R_+^n$ is the population vector which is held constant over time.

Quality q summarizes a number of factors determining the attractiveness of a town. These factors include for instance, the range of goods (offered in a town), the quality of goods, the number of parking spaces, the presentation of the sales site (marketing) etc. We summarize all these factors into an index measure and call it the quality of a location.

Definition 3 *Distance.*
The distance between town i and town j is denoted as $d_{ij} \in \mathbf{D}$. The distance matrix $\mathbf{D} \in [1, \infty)^{n \times n}$ is symmetric, $d_{ij} = d_{ji}$ and its main diagonal contains the distance between a town and its own population.

Next we introduce directed attraction forces.[4] We thereby build on the following consideration. The attractiveness of any town $j \neq i$ is q_j. If we divide q_j measure by d_{ij}, the distance between town i and town j, we get a measure for the attractiveness of town j as seen through the eyes of consumers in town i. We calculate these attraction measures for every town where consumer i could go shopping. This naturally includes town i as well.

Definition 4 *Attraction.*
We denote the attraction $a_{ij} \in \mathbf{A}$ of shopping-town j on residents in (shopping-)town i as

$$a_{ij} = \frac{q_j}{d_{ij}^{\Theta}}, \tag{3}$$

where $\mathbf{A} \in R_+^{n \times n}$. Parameter Θ determines the influence of distance on the attractions between the two towns. If $\Theta = 1$, then the attraction is a linear function of distance. For $\Theta = 2$ this function is quadratic.

In this way, a large but remote shopping center in town j with high quality can become relatively unattractive for residents living in remote town i. Especially, if we assume that hometown shops in town i are more easily accessible, $d_{ii} < d_{ij}$. We therefore can calculate the absolute attractiveness of every town on all residents of all other towns, including its own population.

Intuitively, we would set d_{ii} (the distance customers have to cover if they shop in their home town) equal to a small number to explain the fact that shops in one's resident town are in general closer, or better accessible, than shops in neighboring towns (or large shopping areas outside of towns). However, this need not be true in general. Bad infrastructure in a consumer's hometown may make it more attractive to shop outside. This would then be reflected in a relatively large d_{ii} measure. In order to determine consumption streams of the whole region, we have to calculate the relative attractiveness of all towns.

Definition 5 *Relative Attractiveness.*
These relative attractiveness is defined as:

$$r_{ij} = \frac{a_{ij}}{\sum_{k=1}^{n} a_{ik}} \tag{4}$$

where $r_{ij} \in \mathbf{R}$ and matrix $\mathbf{R} \in R_+^{n \times n}$.

[4](Hentrop 2002) introduces such a directed constant k.

Relative attractiveness, r_{ij}, is the percentage attraction town j exerts on consumers living in town i. In other words, $r_{ij} \cdot b_i$ consumers living in town i will do their shopping in town j. We summarize the absolute shopping streams in the following matrix.

Definition 6 *Shopping-Streams.*
The number of people living in town i and doing their shopping in town j is denoted as $v_{ij} \in \mathbf{V}$ and calculated as:

$$v_{ij} = r_{ij} \cdot b_i \tag{5}$$

where r_{ij} is relative attractiveness of town j on people in town i and b_i is the number of residents in town i. Matrix $\mathbf{V} \in R_+^{n \times n}$ summarizes the absolute streams of purchases.

This matrix $\mathbf{V}$ describes where the respective consumers shop. Clearly, the entries of row i tell us where the inhabitants of town i do their shopping. So the sum of these entries is equal to b_i, the total population of town i. The entries of column j, on the other hand, tell us which people shop in town j. If we sum up the columns we get the total number of shoppers in town j. A numerical example illustrates how this works.

Example 7 *We assume three towns with the following attraction values for residents of town 1: $a_{11} = 47$, $a_{12} = 30$ and $a_{11} = 21$. The relative attraction of town 1 on its own residents is: $r_{11} = \frac{47}{47+30+21} = 0.4796$. Relative attractions for town 2 and 3 are given by: $r_{12} = \frac{30}{47+30+21} = 0.3061$ and $r_{13} = \frac{21}{47+30+21} = 0.2143$. These values sum up to 100%. The values of r_{1j} indicate the percentage of residents of town 1 shopping in towns $j = (1, 2, 3)$. If town 1 has, for example, 800 inhabitants, then $v_{11} = 0.4796 * 800 = 384$ will shop in their home town, $v_{12} = 0.3061 * 800 = 245$ residents go to town 2 and $v_{13} = 0.2143 * 800 = 171$ shop in town 3.*

We could, of course, implement further restrictions on consumer behavior (*e.g.* a home-bias, or an additional preference weight on large shopping areas, *etc.*). We could also alter the fact that in this model consumers do have linear preferences over distances. Setting $\Theta = 2$ would establish a quadratic influence of distances on attraction, enhancing their negative effects on consumers willingness to go to distant loactions. If distance is doubled, demand decreases to about a quarter of its initial value.

3.2 The Clearing Process and Shopping Streams over Time

How will these towns develop over time? At first, we would like to introduce a very simplified supply and demand schedule. We assume a constant total demand over all periods which is equal to the number of inhabitants of the system. By this we mean that we simply distribute total consumer demand over all towns according to the towns' attractiveness (quality and distance). The amount of consumers a town is able to attract within the period determines the town's next period quality. For a start we assume a one-to-one relationship and call it a lagged demand and supply schedule, since this periods demand (=the number of consumers showing up) is equal to next periods supply (= the quality of towns). The number of consumers, their distribution over all towns and the distance measures are constant over all periods. We do not model any pricing process is this setup. We give the formal definition of this schedule now and add some more intuition later.

Definition 8 *Demand in town i.*
We denote demand in town i in period t as, $m_i(t) \in \mathbf{m}(t)$, where $\mathbf{m}(t) \in R_+^n$ is the demand vector of period t of the whole region. Total demand is equal to total population, $\sum_{i=1}^n m_i(t) = \sum_{i=1}^n b_i$.

Since matrix V records all trade-streams of a period, there clearly must be a relationship between V and the demand vector. From definition (6) we see that this relationship fulfills

$$\mathbf{m}(t) = \mathbf{1V}(t)$$

where $\mathbf{1}$ is a summation vector of length n, summing up the respective columns of $\mathbf{V}$. This reduces our definition of demand to the number of people shopping in a certain town, attracted by the town's quality and "repelled" by the distances within the system. In other words, demand is a function of quality, distance and population distribution, where the last two are constant over time. Here is the formal definition:

Definition 9 *Demand Function.*
The vector function $\mathbf{m}(t)(\mathbf{q}(t), \mathbf{b}, \mathbf{D}) : \mathbf{R}_+^n \times \mathbf{R}_+^n \times \mathbf{R}_+^{n\times n} \rightarrow \mathbf{R}_+^n$ records the consumer distribution in period t.

Imposing our simplified law of motion of a one-to-one relationship between demand in period t and the quality in $t+1$ we get to the following rule.

Definition 10 *Lagged Demand-Supply Schedule.*

$$\mathbf{q}(t+1) = \mathbf{m}(t)$$

Supply tomorrow equals demand today.

Intuitively, if a town is doing badly in attracting consumers, this town will lose some of its shops.[5] Hence, this town's attractiveness q will decrease. We assume that the quality measure can adjust within one time period t without additional frictional costs. Like in a zero-sum game, the only thing that is happening here is that flows of consumption are re-distributed among towns, according to their qualities and distances. Total consumption is constant and equals total population.

4 Steady State Properties

4.1 The steady state in a 2 Town Model

How will this system behave in the long run? In this section we will proof for a simple two town case that this system has a steady state "sales distribution" where demand (= distribution of consumers' shopping locations) equals supply (= distribution of qualities) within a period. In the steady state the sum of residents is equal to the sum of qualities, since we postulated a one-to-one relationship with time-lag 1 between the

[5] They either downsize or go bankrupt. Both actions lead to diminished quality or attractiveness as a shopping location due to smaller variety etc.

two. The steady state also does only depend on the initial population distribution and the distances between the towns (the fixed factors), but not on the initial quality distribution. We give a proof of this simple result. This is an interesting outcome, since it basically says that no quality shock (*e.g.* the construction of a shopping center) will have an effect on the long-run sales distribution of the region. The starting situation for this scenario is summarized in the following definition.

Definition 11 *The 2 by 2 World.*
There are two towns denoted (q_1, b_1) and (q_2, b_2). The distance between the two towns is $d_{12} = d_{21} = \delta$. The inner town distance is the same for both towns and normalized to $d_{11} = d_{22} = 1$. For convenience we drop time indices. It should be clear however, that q changes over time.

The attraction matrix is defined as $\mathbf{A} = \begin{pmatrix} q_1 & \frac{q_2}{\delta} \\ \frac{q_1}{\delta} & q_2 \end{pmatrix}$ and the relative attractiveness is given by:

$$\mathbf{R} = \begin{pmatrix} \frac{q_1}{q_1+\frac{q_2}{\delta}} & \frac{\frac{q_2}{\delta}}{q_1+\frac{q_2}{\delta}} \\ \frac{\frac{q_1}{\delta}}{q_2+\frac{q_1}{\delta}} & \frac{q_2}{q_2+\frac{q_1}{\delta}} \end{pmatrix} = \begin{pmatrix} \frac{\delta q_1}{\delta q_1+q_2} & \frac{q_2}{\delta q_1+q_2} \\ \frac{q_1}{\delta q_2+q_1} & \frac{\delta q_2}{\delta q_2+q_1} \end{pmatrix}$$

Pre-multiplied with the population matrix we receive the following shopping-streams:

$$\mathbf{V} = \begin{pmatrix} b_1 & 0 \\ 0 & b_2 \end{pmatrix} \mathbf{R} = \begin{pmatrix} \frac{\delta q_1}{\delta q_1+q_2} b_1 & \frac{q_2}{\delta q_1+q_2} b_1 \\ \frac{q_1}{\delta q_2+q_1} b_2 & \frac{\delta q_2}{\delta q_2+q_1} b_2 \end{pmatrix}$$

From this we get the demand vector as:

$$\mathbf{m} = \mathbf{1V} = \begin{pmatrix} \frac{\delta q_1}{\delta q_1+q_2} b_1 + \frac{q_1}{\delta q_2+q_1} b_2 \\ \frac{q_2}{\delta q_1+q_2} b_1 + \frac{\delta q_2}{\delta q_2+q_1} b_2 \end{pmatrix}$$

Written as as system of autonomous nonlinear difference equations where next period's quality $q_i(t+1)$ is denoted as q_i' and equal to m_i we get:

$$\begin{aligned} q_1' &= \frac{\delta q_1}{\delta q_1 + q_2} b_1 + \frac{q_1}{\delta q_2 + q_1} b_2 \equiv \varphi(q_1, q_2) \\ q_2' &= \frac{q_2}{\delta q_1 + q_2} b_1 + \frac{\delta q_2}{\delta q_2 + q_1} b_2 \equiv \psi(q_1, q_2) \end{aligned}$$

or rewritten in difference form:

$$\begin{aligned} \Delta q_1 &= q_1' - q_1 = \varphi(q_1, q_2) - q_1 \\ \Delta q_2 &= q_2' - q_2 = \psi(q_1, q_2) - q_2 \end{aligned} \tag{6}$$

In the steady state

$$\begin{aligned} \Delta q_1 &= 0 \Rightarrow q_1 = \varphi(q_1, q_2) \\ \Delta q_2 &= 0 \Rightarrow q_2 = \psi(q_1, q_2) \end{aligned}$$

We can therefore write:

$$\begin{aligned}
q_1 &= \frac{\delta q_1}{\delta q_1 + q_2} b_1 + \frac{q_1}{\delta q_2 + q_1} b_2 \\
q_2 &= \frac{q_2}{\delta q_1 + q_2} b_1 + \frac{\delta q_2}{\delta q_2 + q_1} b_2
\end{aligned}$$

Solving for q_1 and q_2 we get the steady state values:

$$\bar{q}_1 = \frac{\delta b_1 - b_2}{\delta - 1}$$

$$\bar{q}_2 = \frac{\delta b_2 - b_1}{\delta - 1}$$

We conclude that in the steady state the qualities $\bar{\mathbf{q}} = (\bar{q}_1, \bar{q}_2)$ only depend on the population and on the distance, but not on the starting values for qualities, $(q_1^{t=0}, q_2^{t=0})$. This is an interesting result, since it implies that no quality shock (*e.g.* the construction of a shopping center) will have an effect on the long-run quality distribution of the region.

4.1.1 Stability of the Steady-State in the 2 Town Case

A general way to determine the stability of difference equation systems is via eigenvalue analysis. For this purpose we rewrite our system in vector notation:

$$\mathbf{q}(t+1) = F(\mathbf{q}(t), \mathbf{D}, \mathbf{b})$$

In order to derive stability properties, we linearize the system around the steady state using a Taylor approximation. The Jacobian for this system at the steady state $\bar{\mathbf{q}}$ is:

$$DF(\bar{\mathbf{q}}) = \begin{bmatrix} \frac{\partial\varphi(\bar{q}_1,\bar{q}_2)}{\partial q_1} & \frac{\partial\varphi(\bar{q}_1,\bar{q}_2)}{\partial q_2} \\ \frac{\partial\psi(\bar{q}_1,\bar{q}_2)}{\partial q_1} & \frac{\partial\psi(\bar{q}_1,\bar{q}_2)}{\partial q_2} \end{bmatrix}$$

Taking the respective derivatives we get the following Jacobian matrix:

$$DF(\cdot) = \begin{bmatrix} \left(\frac{\delta q_2 b_1}{(\delta q_1 + q_2)^2} + \frac{\delta q_2 b_2}{(\delta q_2 + q_1)^2}\right) & -\left(\frac{\delta q_1 b_1}{(\delta q_1 + q_2)^2} + \frac{\delta q_1 b_2}{(\delta q_2 + q_1)^2}\right) \\ -\left(\frac{\delta q_2 b_1}{(\delta q_1 + q_2)^2} + \frac{\delta q_2 b_2}{(\delta q_2 + q_1)^2}\right) & \left(\frac{\delta q_1 b_1}{(\delta q_1 + q_2)^2} + \frac{\delta q_1 b_2}{(\delta q_2 + q_1)^2}\right) \end{bmatrix} \tag{7}$$

Concerning the eigenvalues λ_1 and λ_2 of (7) we know that:

$$\begin{aligned}
trDF(\bar{\mathbf{q}}) &= \frac{\partial\varphi(\bar{q}_1,\bar{q}_2)}{\partial q_1} + \frac{\partial\psi(\bar{q}_1,\bar{q}_2)}{\partial q_2} = \lambda_1 + \lambda_2 \\
\det DF(\bar{\mathbf{q}}) &= \frac{\partial\varphi(\bar{q}_1,\bar{q}_2)}{\partial q_1}\frac{\partial\psi(\bar{q}_1,\bar{q}_2)}{\partial q_2} - \frac{\partial\psi(\bar{q}_1,\bar{q}_2)}{\partial q_1}\frac{\partial\varphi(\bar{q}_1,\bar{q}_2)}{\partial q_2} = \lambda_1\lambda_2
\end{aligned}$$

It follows that:

$$\begin{aligned}\lambda_1+\lambda_2 &= \frac{\delta\bar{q}_2 b_1+\delta\bar{q}_1 b_1}{\left(\delta\bar{q}_1+\bar{q}_2\right)^2}+\frac{\delta\bar{q}_2 b_2+\delta\bar{q}_1 b_2}{\left(\delta\bar{q}_2+\bar{q}_1\right)^2}\\ \lambda_1\lambda_2 &= 0\end{aligned}$$

From this we conclude that one of the eigenvalues is zero. Without loss of generality we set $\lambda_2 = 0$ then:

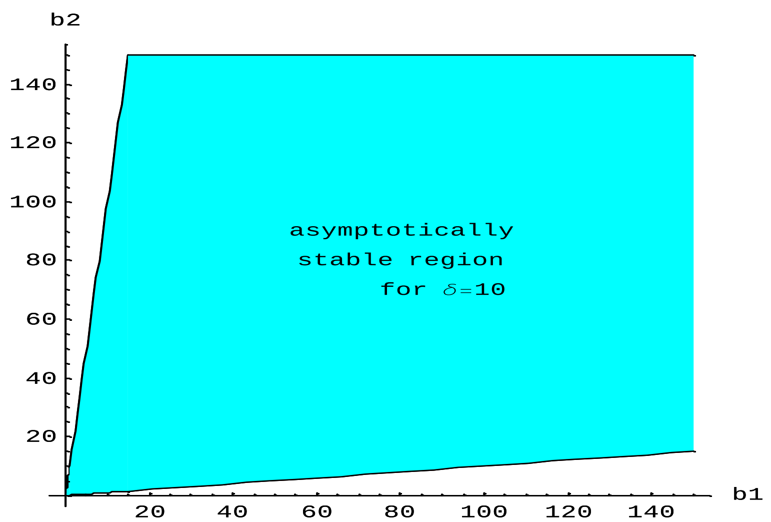
$$\lambda_1=\frac{\delta b_1\left(\bar{q}_1+\bar{q}_2\right)}{\left(\delta\bar{q}_1+\bar{q}_2\right)^2}+\frac{\delta b_2\left(\bar{q}_1+\bar{q}_2\right)}{\left(\delta\bar{q}_2+\bar{q}_1\right)^2} \tag{8}$$

If the eigenvalues of $DF\left(\bar{\mathbf{q}}\right)$ have moduli strictly less than 1, then $\bar{\mathbf{q}}$ is asymptotically stable. Accordingly we have to check whether $|\lambda_1| < 1$ once we plug $\bar{q}_1 = \frac{\delta b_1 - b_2}{\delta - 1}$ and $\bar{q}_2 = \frac{\delta b_2 - b_1}{\delta - 1}$ into (8) and get:

$$\begin{aligned}\lambda_1=\frac{\delta b_1\left(\frac{\delta b_1-b_2}{\delta-1}+\frac{\delta b_2-b_1}{\delta-1}\right)}{\left(\delta\frac{\delta b_1-b_2}{\delta-1}+\frac{\delta b_2-b_1}{\delta-1}\right)^2}+\frac{\delta b_2\left(\frac{\delta b_1-b_2}{\delta-1}+\frac{\delta b_2-b_1}{\delta-1}\right)}{\left(\delta\frac{\delta b_2-b_1}{\delta-1}+\frac{\delta b_1-b_2}{\delta-1}\right)^2} &\overset{?}{<} 1\\ \Rightarrow \frac{b_1^2\delta+2b_1b_2\delta+b_2^2\delta}{b_1b_2\left(1+\delta\right)^2} &\overset{?}{<} 1 \qquad (9)\end{aligned}$$

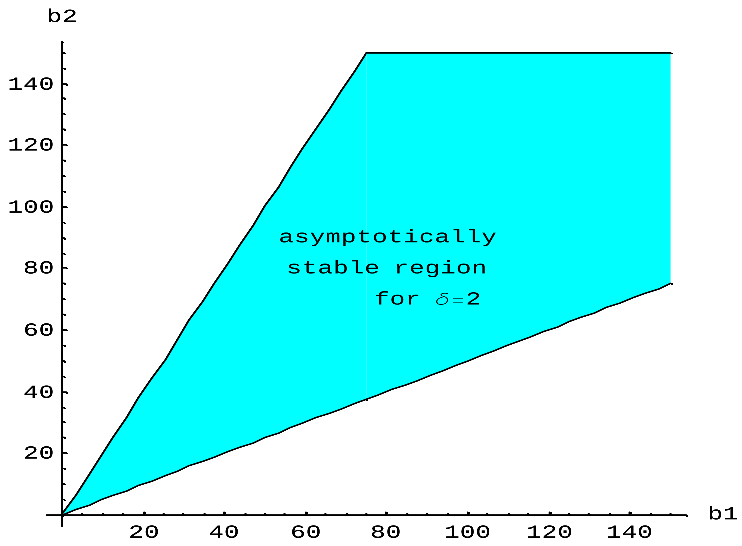
This allows us to plot a stable region of a combination of b_1 and b_2 for a given δ. Figure 1 depicts the stable combinations of b_1 and b_2 of expression 9 with $\delta = 10$.

Figure 1: The Stable Region with $\delta = 10$



With decreasing δ this stable region becomes smaller. Hence, with increasing mobility (δ decreases) the less likely the system ends up in an asymptotically stable (interior) solution. Figure 2 describes the stable region for $\delta = 2$.

Figure 2: The Stable Region with $\delta = 2$



We can also calculate the critical distance for a given distribution of residents that would result in a border solution.

$$\delta^2 - \left(\frac{b_1^2 + b_2^2}{b_1 b_2}\right)\delta + 1 > 0$$

The solution to this is

$$\delta > \delta^* \text{ with } \begin{cases} \delta^* = 1 \text{ if } b_1 = b_2 \\ \delta^* = \frac{b_1}{b_2} \text{ if } b_1 > b_2 \\ \delta^* = \frac{b_2}{b_1} \text{ if } b_2 > b_1 \end{cases}$$

As long as the distances are high enough ($\delta > \delta^*$) we end up with an asymptotically stable interior solution where both towns are able to maintain a certain positive quality level. Otherwise, the system runs towards a border solution that is, one town attracts all consumers and its quality rises whereas the other town loses all of its shops and its quality goes towards zero. The more uneven the population distribution is, the larger the required distance for stable interior solutions, which sounds reasonable. If towns are very different in size, the smaller town needs to be "protected" by distance in order to being able to maintain some of its shops. Hence, threshold δ^* increases with large differences in population numbers. If towns are equally sized ($b_1 = b_2$), then $\delta^* \to 1$. That is, even with the smallest distance possible, both towns are able to maintain some of their shops.

4.1.2 Numeric Determination of Stability

Another way to determine stability is to draw phase diagrams. In order to draw the phase lines we need some assumptions for the shopping region.

Definition 12 *The 2 by 2 Numeric World.*
There are two towns denoted $(q_1, b_1 = 100)$ *and* $(q_2, b_2 = 80)$. *The distance between the two towns is* $d_{12} = d_{21} = \delta = 10$. *The inner town distance is the same for both towns and normalized to* $d_{11} = d_{22} = 1$.

Since in the steady state it holds that:

$$\begin{aligned} \Delta q_1 &= 0 \Rightarrow q_1 = \varphi(q_1, q_2) \\ \Delta q_2 &= 0 \Rightarrow q_2 = \psi(q_1, q_2) \end{aligned}$$

and therefore also:

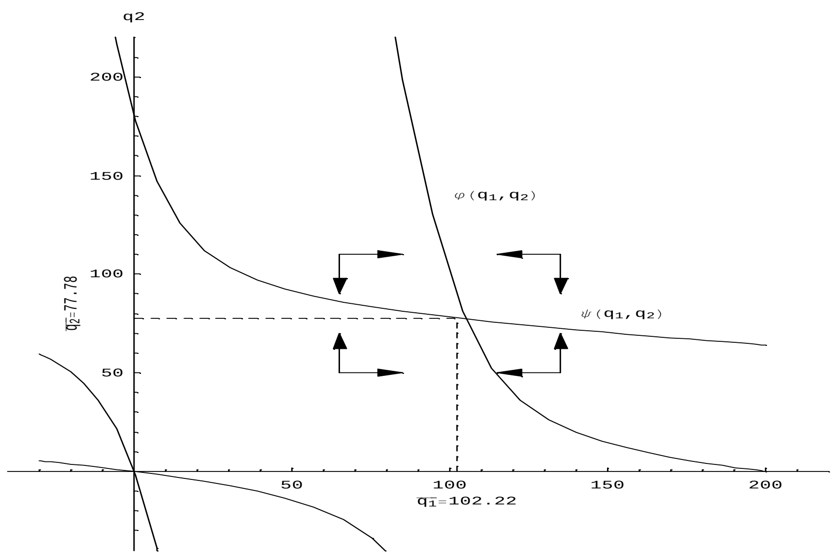
$$\begin{aligned} (\delta q_1 + q_2)(\delta q_2 + q_1) &= [(\delta q_2 + q_1)\delta] b_1 + (\delta q_1 + q_2) b_2 \\ (\delta q_1 + q_2)(\delta q_2 + q_1) &= (\delta q_2 + q_1) b_1 + [(\delta q_1 + q_2)\delta] b_2 \end{aligned}$$

we implicitly plot the solution of the following system:

$$\begin{aligned} 0 &= \delta q_1^2 + q_1\left(\delta^2 q_2 + q_2 - \delta b_1 - \delta b_2\right) - q_2\left(\delta^2 b_1 + b_2 - \delta q_2\right) \\ 0 &= \delta q_2^2 + q_2\left(\delta^2 q_1 + q_1 - \delta b_1 - \delta b_2\right) - q_1\left(b_2\delta^2 + b_1 - \delta q_1\right) \end{aligned}$$

We plot the two graphs in Figure 3 with the parameter specification of Definition 12.

Figure 3: The Phase Diagram



We get four steady states, three border solutions $(0, 0)$, $(180, 0)$ and $(0, 180)$ and one interior solution $(q_1 = 102.22, q_2 = 77.778)$. These phase lines divide the plane into four areas. Next we have to determine which steady states are stable. We proceed with the graphical analysis and determine the signs of either:

$$\frac{\partial \Delta q_1}{\partial q_1} = \frac{\partial \varphi(q_1, q_2)}{\partial q_1} - 1 \text{ or } \frac{\partial \Delta q_1}{\partial q_2} = \frac{\partial \varphi(q_1, q_2)}{\partial q_2}$$

to determine the pattern of the arrows of motion for $\varphi(q_1, q_2)$. The same analysis is applied to the second phase line determined by $\psi(q_1, q_2)$:

$$\frac{\partial \Delta q_2}{\partial q_2} = \frac{\partial \psi(q_1, q_2)}{\partial q_2} - 1 \text{ or } \frac{\partial \Delta q_2}{\partial q_1} = \frac{\partial \psi(q_1, q_2)}{\partial q_1}$$

Since both, $\frac{\partial \Delta q_1}{\partial q_2} = \frac{\partial \varphi(\cdot)}{\partial q_2}$ and $\frac{\partial \Delta q_2}{\partial q_1} = \frac{\partial \psi(\cdot)}{\partial q_1}$ are always negative (see entries in the Jacobian Matrix (7)), it is easy to determine the arrows of motion. They are always pointing towards the graphs. Hence, the steady states $(0, 0)$, $(180, 0)$ and $(0, 180)$ are unstable and steady state $(q_1 = 102.22, q_2 = 77.778)$ is stable. Figure 3 summarizes the information.

The respective eigenvalues for $(q_1 = 102.22, q_2 = 77.778)$ are $\lambda_1 = 0.33471$ and $\lambda_2 = 0$, which again confirms that the steady state is asymptotically stable. The eigenvalues for the unstable steady states are $\lambda_1 = 4.5$ and $\lambda_2 = 0$ for steady state $(180, 0)$ and $\lambda_1 = 5.6$ and $\lambda_2 = 0$ for steady state $(0, 180)$. For the $(0, 0)$ steady state an eigenvalue analysis is not possible. Anyway, this solution describes an absurd economic situation, where an arbitrarily high demand is ignored by both towns.

4.2 The steady state in a 3 Town Model

In this section we extend our analysis to a three town model. As before, we again start with the basic setup.

Definition 13 *The 3 by 3 Playground.*
There are three towns denoted (q_1, b_1), (q_2, b_2) and (q_3, b_3). The distances between towns are $\mathbf{D} = \begin{pmatrix} 1 & \delta_{12} & \delta_{13} \\ \delta_{21} & 1 & \delta_{23} \\ \delta_{31} & \delta_{32} & 1 \end{pmatrix}$. *For convenience we are again dropping time indexes.*

Therefore the attraction matrix A is given by $\mathbf{A} = \begin{pmatrix} q_1 & \frac{q_2}{\delta_{12}} & \frac{q_3}{\delta_{13}} \\ \frac{q_1}{\delta_{21}} & q_2 & \frac{q_3}{\delta_{23}} \\ \frac{q_1}{\delta_{31}} & \frac{q_2}{\delta_{32}} & q_3 \end{pmatrix}$ and the shopping stream matrix is the relative attractiveness $\mathbf{R}$ pre-multiplied with the population matrix:

$$\mathbf{V} = \begin{pmatrix} b_1 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & b_3 \end{pmatrix} \begin{pmatrix} \frac{q_1}{q_1 + \frac{q_2}{\delta_{12}} + \frac{q_3}{\delta_{13}}} & \frac{\frac{q_2}{\delta_{12}}}{q_1 + \frac{q_2}{\delta_{12}} + \frac{q_3}{\delta_{13}}} & \frac{\frac{q_3}{\delta_{13}}}{q_1 + \frac{q_2}{\delta_{12}} + \frac{q_3}{\delta_{13}}} \\ \frac{\frac{q_1}{\delta_{21}}}{q_2 + \frac{q_1}{\delta_{21}} + \frac{q_3}{\delta_{23}}} & \frac{q_2}{q_2 + \frac{q_1}{\delta_{21}} + \frac{q_3}{\delta_{23}}} & \frac{\frac{q_3}{\delta_{23}}}{q_2 + \frac{q_1}{\delta_{21}} + \frac{q_3}{\delta_{23}}} \\ \frac{\frac{q_1}{\delta_{31}}}{q_3 + \frac{q_1}{\delta_{31}} + \frac{q_2}{\delta_{32}}} & \frac{\frac{q_2}{\delta_{32}}}{q_3 + \frac{q_1}{\delta_{31}} + \frac{q_2}{\delta_{32}}} & \frac{q_3}{q_3 + \frac{q_1}{\delta_{31}} + \frac{q_2}{\delta_{32}}} \end{pmatrix}$$

Demand for this period is therefore

$$\mathbf{m} = \mathbf{1V} = \begin{pmatrix} b_1 \frac{q_1}{q_1 + \frac{q_2}{\delta_{12}} + \frac{q_3}{\delta_{13}}} + b_2 \frac{q_1}{\delta_{21}\left(q_2 + \frac{q_1}{\delta_{21}} + \frac{q_3}{\delta_{23}}\right)} + b_3 \frac{q_1}{\delta_{31}\left(q_3 + \frac{q_1}{\delta_{31}} + \frac{q_2}{\delta_{32}}\right)} \\ b_1 \frac{q_2}{\delta_{12}\left(q_1 + \frac{q_2}{\delta_{12}} + \frac{q_3}{\delta_{13}}\right)} + b_2 \frac{q_2}{q_2 + \frac{q_1}{\delta_{21}} + \frac{q_3}{\delta_{23}}} + b_3 \frac{q_2}{\delta_{32}\left(q_3 + \frac{q_1}{\delta_{31}} + \frac{q_2}{\delta_{32}}\right)} \\ b_1 \frac{q_3}{\delta_{13}\left(q_1 + \frac{q_2}{\delta_{12}} + \frac{q_3}{\delta_{13}}\right)} + b_2 \frac{q_3}{\delta_{23}\left(q_2 + \frac{q_1}{\delta_{21}} + \frac{q_3}{\delta_{23}}\right)} + b_3 \frac{q_3}{q_3 + \frac{q_1}{\delta_{31}} + \frac{q_2}{\delta_{32}}} \end{pmatrix}$$

Written as as system of autonomous nonlinear difference equations we get next period's quality:

$$\begin{aligned}
q_1' &= b_1 \frac{q_1}{q_1 + \frac{q_2}{\delta_{12}} + \frac{q_3}{\delta_{13}}} + b_2 \frac{q_1}{\delta_{21}\left(q_2 + \frac{q_1}{\delta_{21}} + \frac{q_3}{\delta_{23}}\right)} + b_3 \frac{q_1}{\delta_{31}\left(q_3 + \frac{q_1}{\delta_{31}} + \frac{q_2}{\delta_{32}}\right)} \equiv \varphi\left(q_1, q_2, q_3\right) \\
q_2' &= b_1 \frac{q_2}{\delta_{12}\left(q_1 + \frac{q_2}{\delta_{12}} + \frac{q_3}{\delta_{13}}\right)} + b_2 \frac{q_2}{q_2 + \frac{q_1}{\delta_{21}} + \frac{q_3}{\delta_{23}}} + b_3 \frac{q_2}{\delta_{32}\left(q_3 + \frac{q_1}{\delta_{31}} + \frac{q_2}{\delta_{32}}\right)} \equiv \psi\left(q_1, q_2, q_3\right) \\
q_3' &= b_1 \frac{q_3}{\delta_{13}\left(q_1 + \frac{q_2}{\delta_{12}} + \frac{q_3}{\delta_{13}}\right)} + b_2 \frac{q_3}{\delta_{23}\left(q_2 + \frac{q_1}{\delta_{21}} + \frac{q_3}{\delta_{23}}\right)} + b_3 \frac{q_3}{q_3 + \frac{q_1}{\delta_{31}} + \frac{q_2}{\delta_{32}}} \equiv \eta\left(q_1, q_2, q_3\right)
\end{aligned}$$

Which can be written in difference form:

$$\begin{aligned}
\Delta q_1 &= q_1' - q_1 = \varphi\left(q_1, q_2, q_3\right) - q_1 \\
\Delta q_2 &= q_2' - q_2 = \psi\left(q_1, q_2, q_3\right) - q_2 \\
\Delta q_3 &= q_3' - q_3 = \eta\left(q_1, q_2, q_3\right) - q_3
\end{aligned} \tag{10}$$

Due to the computational complexity we cannot get a closed form solution for the steady state. Solving the system numerically we need again parameter specifications for the population and distances.

Definition 14 *The 3 by 3 Numeric World.*
There are three towns denoted $(q_1, b_1 = 800)$, $(q_2, b_2 = 900)$ *and* $(q_3, b_3 = 1400)$. *The distances between towns are* $\mathbf{D} = \begin{pmatrix} 1 & 8 & 20 \\ 8 & 1 & 10 \\ 20 & 10 & 1 \end{pmatrix}$.

Going through the same steps as for the two town case we end up with 14 possible steady states. However, only one has positive results for all quantities and is asymptotically stable (all eigenvalues are smaller than unity). All other steady states have eigenvalues greater than one or negative values for quality.

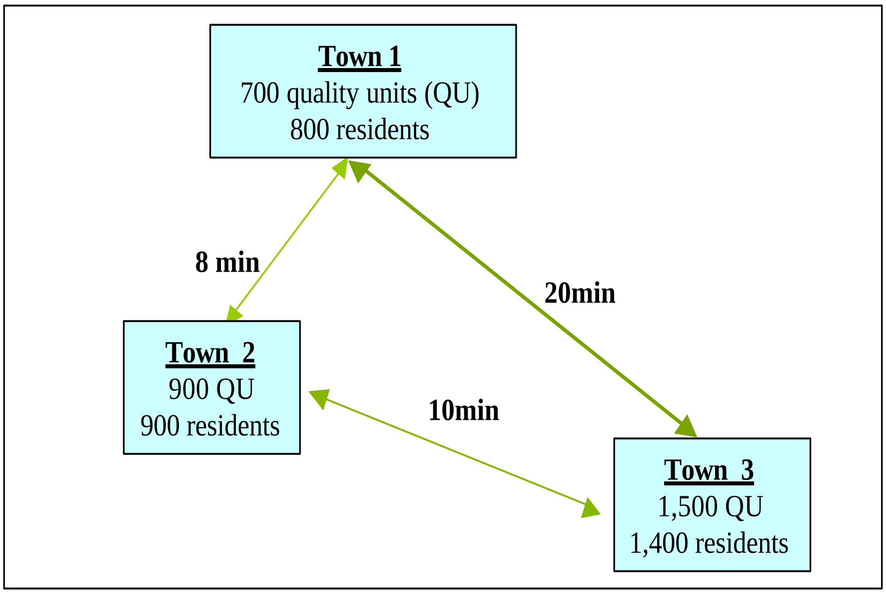
The interior steady state for the parameter specification of Definition 14 is $\bar{q}_1 = 754.606, \bar{q}_2 = 875.608$ and $\bar{q}_3 = 1471.47$ and its eigenvalues are $\lambda_1 = 0.51357, \lambda_2 = 0.30785$ and $\lambda_3 = 0.0116126$.

5 Numerical Simulations of the Gravity Model

In this section we run numerical simulations of the system. Our first concern is the development of the region with three towns in the absence of external shocks and without adaptation delays, adaptation costs *etc.* The starting situation for this scenario is presented in Figure 4.

In Figure 5 we give a first indication of how purchases in all towns evolve over time. After about 20 periods the system reaches a steady state. Keeping in mind that period t purchases in town i, are town i's next period quality, $q_i(t+1)$, we can treat them almost synonymic. One time period is interpreted as a time span, that allows towns

Figure 4: The Simulated Region



to expand or to downsize their shops. This directly translates into expanding/limiting a town's quality. In Figure 5, town 1's quality is too high, compared to the number of its residents. Also, for residents of other towns, town 1 is not very attractive because of its remote location relative to the other two towns. Town 1 therefore, gradually loses customers. Town 3, on the other hand, has the highest population percentage and gains demand. This leads town 3 to expand its quality.

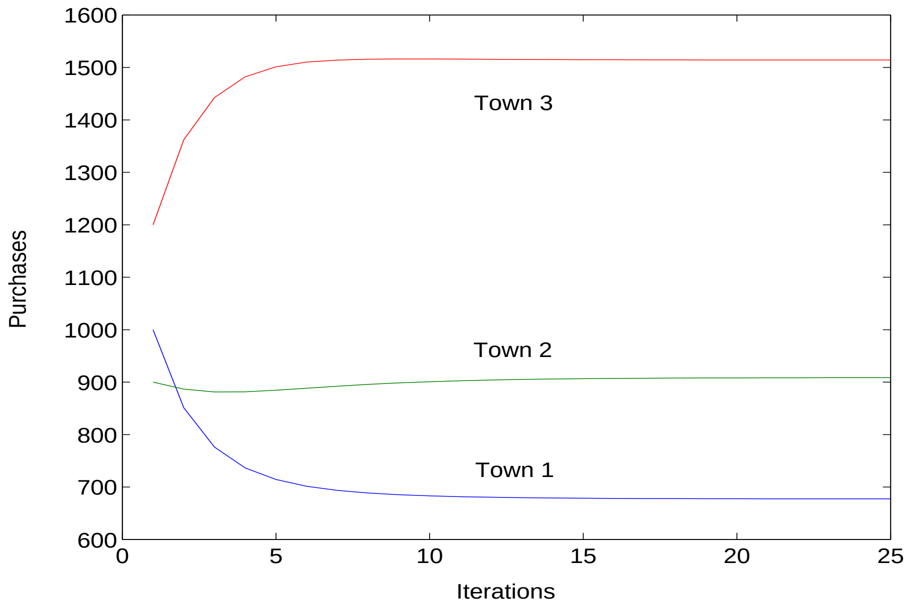
This base scenario is the simplest of a series. Naturally, we would assume that there is a certain adjustment lag, due to time and/or capacity constraints. Shops are not immediately expanded, just because demand is high in one period. This also does not happen in a gradual way. We would expect that demand must rise considerably and remain high for a certain period of time, before a shop-owner will decide to expand her facilities. We will present a scenario with capacity thresholds, that must be exceeded before any adjustment-action is taken in section 5.2. However, we abstract away from such details for a start and let demand and supply depend purely on population numbers and distance, the two main inputs in the gravity model.

The presented equilibrium is the result of a long-term dynamic development process and quality shocks do not have an influence on this steady state. Any measures to block expansion of qualities, or to increase qualities in a town must fail. The only measures that do matter in the long-run are measures influencing distance and consumer-residency.

5.1 Scenario 1: Constructing a Shopping Center

In this scenario we observe a quality shock. Town 1 constructs a shopping center (SC) in its district and thereby raises its quality. We also assume that the owners

Figure 5: Convergence in the Region without SC

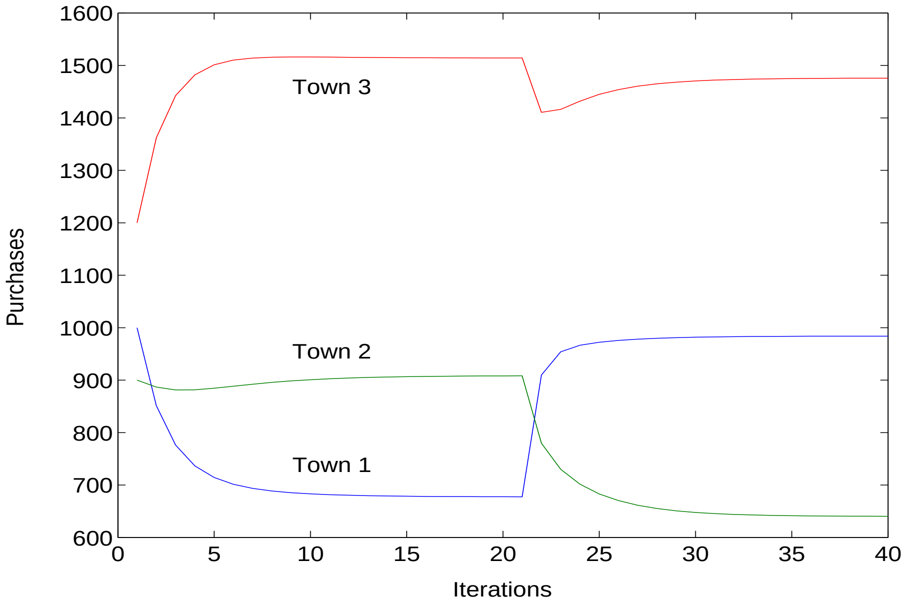


of the shopping center do not adjust immediately to demand changes but are able to maintain their quality for a few periods. This is a reasonable assumption once we take into account that larger shopping-malls are constructed by investor groups who can endure a certain period of losses, without having to give up on a location. In Figure 6 we see that again after 20 iterations a steady state is reached. Then town 1, or an investor group, decides to build a shopping center and increase its quality from 677 to 1300. Town 2 and town 3 have to decrease their capacities (qualities) in reaction to this. Because town 1 and town 2 are close to each other, town 2 suffers more from the quality shock in town 1, than does town 3 which is further away. Also, the total capacity of 1300 in town 1 is never fully met. Given the assumptions above, town 1 should react by immediately decreasing its quality. However, since we assumed in this scenario, that town 1 has the possibility to maintain its overall quality level for a given period, its quality becomes an exogenous variable which is held constant.[6] The system now reaches a new steady state.

What happens, if we lift the protective measure of stable quantity offer from the town with the shopping center? Capacity is far too high. Quality is at 1,300 whereas actual purchases not even reach 1,000 in Figure 6. This would cause shop-owners in town 1 to downsize their outlets. However, if they do this, then the quality of the shopping area is lower in the following period. Hence, less consumers than in the previous period are attracted and the problem of over-capacity arises again. At the same time, shops in town 2 and town 3 face an increasing demand and raise their capacity, hence quality for the following period. This process goes on, until the initial steady state of scenario 1 is reached again.

[6]Later we will treat town 1 and the shopping center as two separate identities, to identify the effects of the shopping center on town 1.

Figure 6: Construction of a SC



We reproduce the result of our basic approach with no external shocks. No capacity shock can change the steady state. The steady state is asymptotically stable. Policy measures, trying to increase town qualities, have only temporary effects. Only if we fix the quality of a town for the entire calculation horizon, we can reach a new steady state. Fixing qualities could be achieved by costly long term subsidies.

The reason for the arbitrarily close approximation of the fixed point lies in the absence of frictional costs. Adjustment of shop-spaces, *i.e.* town qualities, is costless, so the traders can adopt their shops perfectly even to extremely small changes of the demand.

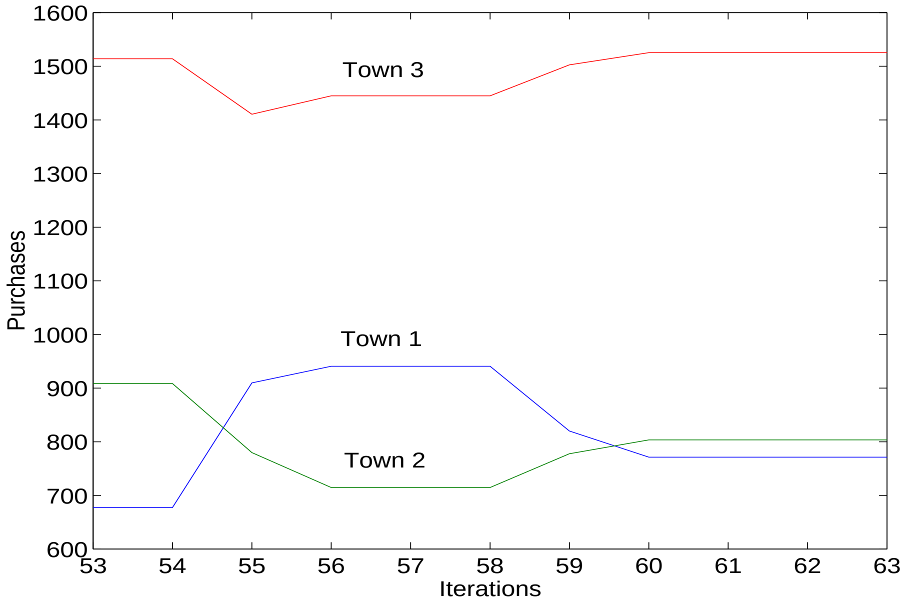
5.2 Scenario 2: Adaptation Costs

In this scenario we account for the fact that enlarging or downsizing of shopping facilities bears some risk and cost. Therefore, a shop-owner will only be willing to expand her shop if her town faces an excess demand of more than 30%. Also, shops are only downsized once demand falls below 90% (more than -10% decrease in demand) of the quality supplied. These thresholds are arbitrary and do not necessarily represent real data on retail shops. However, the effects are the same, only the numbers change. Because of these thresholds the long-term equilibrium depends on starting values of quality, since the thresholds prevent an arbitrary close approximation of the earlier equilibrium.

For this example we use as starting values, the steady state qualities of the previous example without adaptation costs, that is the quality of towns after iteration 20 in Figure 6. This is very close to the steady state. Now we assume, that town 1 increases its quality, $q_1 = 1,300$ via the construction of a new shopping center. Figure

7 presents the situation. Demand in town 1 is reacting immediately and increases in

Figure 7: SC in Inner City Precinct with Adaptation Costs



two steps from 677 to 940. Neighboring town 2 loses and downsizes capacities because its demand shock is larger than -10%. We again assume that town 1 can fix its quality for a certain number of periods. In this particular case we assume town 1 can fix $q_1 = 1,300$ from period 54 to 58 in Figure 7. After that we let the system react as specified above. Quality in town 1 decreases sharply, but because of the thresholds we introduced, the shopkeepers can keep their supplied quality high and quality is not moving back to the initial steady state. Town 2 increases its quantity, but can never fully regain its initial quality. Remember that demand has to be higher than 130% of capacity available. This restriction is binding here and does not allow for an arbitrarily close approach to the old steady state.

Town 3 faces an interesting situation. Since it is located farther away, it is less influenced by the construction of the shopping center and its quality reductions are comparatively small. After the subsidy period for the shopping center (period 54-58) when the system reacts again, the aggregate of town 1 and town 2 never fully regains its initial value and town 3 can grow above its starting value. This somewhat paradoxical situation not only allows town 1, the shopping center town, to increase its capacity in the long run, but also the remote town 3 can benefit from the shopping center in town 1. The reason for this is the high re-construction costs in town 2 that can never reach its initial strength.

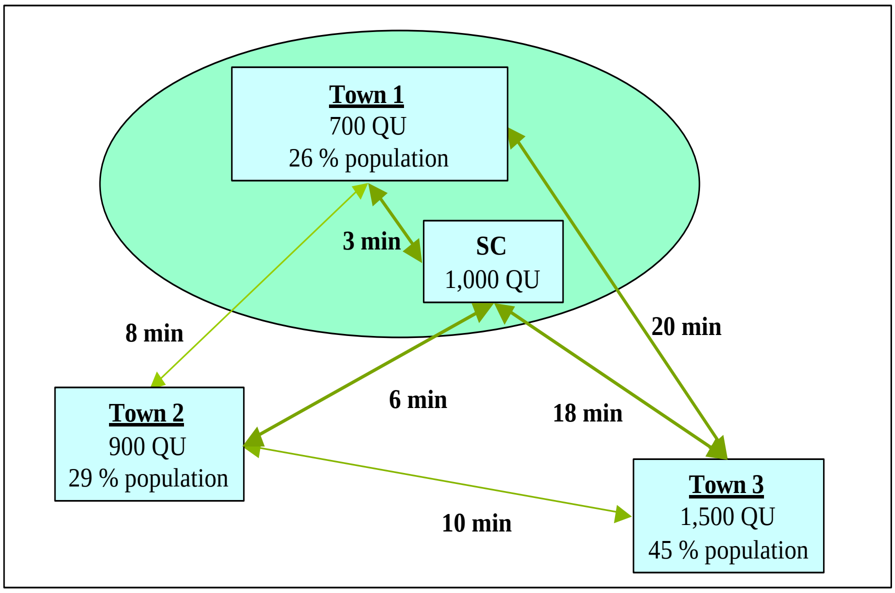
Differences in quality starting values as well as quality shocks do have an influence on the long-term outcome (steady state) in this situation. In this sense increases in town qualities can have long term effects. This new steady state also depends on the thresholds, or better, adjustment costs that we introduced in this section.

5.3 Scenario 3: Transactions Cost Effects

So far we did not distinguish between old shops of a town and the shopping center and ignored effects happening "inside" a town district. A quality shock was just added to the already existent quality units. However, we suspect that there must be interesting effects within a town since a new shopping center has an influence on existing outlets of a town. As a consequence we want to analyze such effects. Therefore, we assume that a shopping center is constructed at a certain distance to town 1. We also want to distinguish between two cases.

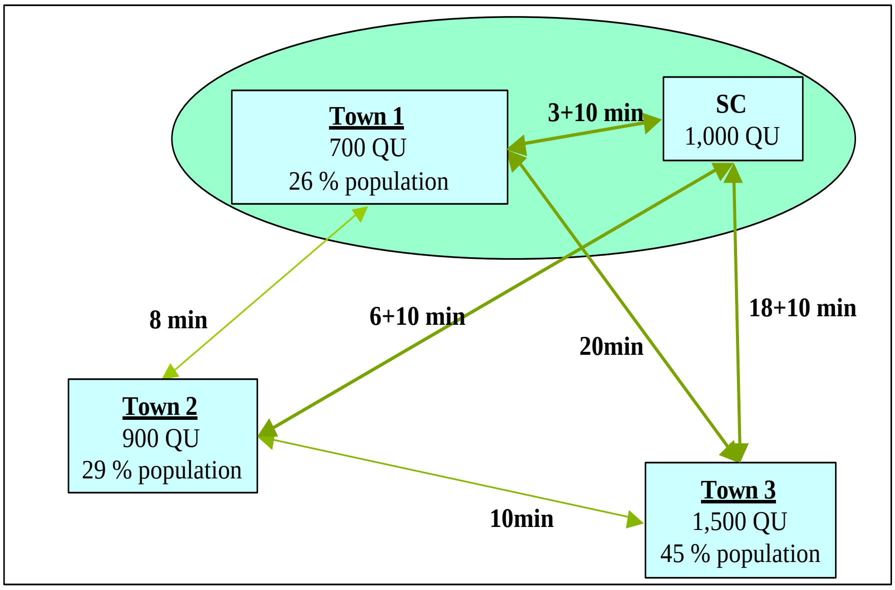
In the first we assume that the shopping center is "near" town 1. That means that transaction costs of purchasing in the shopping center are low. This could be the consequence of geographical closeness or the result of a well maintained and cheap-to-use infrastructure (no or low road pricing, no parking fees, no traffic jams, *etc.*). In our model we approximate this situation by postulating a "3 minute" distance from town 1 to the shopping center. Figure Figure 8 describes this case. In the second

Figure 8: Region with a Nearby SC



case, we assume a "remote" shopping center within the district of town 1. Remoteness is signalled by a "10 minute" distance and can be caused by to converse of the above: geographical remoteness, bad infrastructure, road-pricing, traffic jams, parking fees, border crossings, *etc.* Distance d allows us to model many interesting phenomena. Figures 8 and 9 summarize these cases. Figures 10 and 11 describe the long-term developments for the two cases just described. Figure 10 records the purchase and the quality development with a shopping center near town 1. Figure 11 describes the development of purchases and quality of a remote shopping center. In all calculations we assume adaptation costs, that is a 30% surplus in demand triggers an expansion of quality and a more than -10% decrease in demand causes towns to decrease quality.

Figure 9: Region with a Remote SC



In periods 50-85 we hold quality of town 1 constant. In period 50 town 1 constructs a shopping center.

The blue solid line is the sum of purchases in town 1 (old shops + shopping center). The blue dashed line describes the old shops in the town center and the light-blue dashed line represents the shopping center.

The first graph of Figure 10 is the development of qualities. In period 50 a shopping center with quality=1,000 is built in town 1. The blue solid line shows the aggregate quality of town 1. We can make the following statement. The city-center of town 1 loses the most. In the aggregate, town 1 can gain from the shopping center. It is able to not only raise its purchases but also its quality.

Remote town 3 is not very much influenced by the shopping center, although town 3 loses during the time when the shopping center is subsidized (quality of town 1 is held constant in periods 50-85). Afterwards, town 3 can somewhat regain its starting position. Because of the adjustment costs, town 3 stays below its starting capacity. Town 2 suffers most from the shopping center. Both, purchases as well as qualities drop considerably below the starting values and cannot be regained.

What would happen, if the distance to the shopping center is increased as in Figure 9. Policy measures like road-pricing, parking fees *etc.* would cause such a development. This would increase the transactions costs of buying in the shopping center and would make it less attractive for all consumers. We account for this case by increasing the respective distance measures by 10 and re-run the simulation with otherwise unchanged specifications.

We are now presented with the interesting case that town 1, the community that builds the shopping center, cannot benefit from it anymore. On the contrary, town 1 loses purchasing power and quality. The respective amount of this loss depends heavily on the adaptation-thresholds that we employ here. Because the shopping center is "so far away", town 2 and town 3 are hardly influenced by it anymore. Although, they lose some customers during the subsidy-time (fixed quality period in town 1), adaptation thresholds make sure that town 2 and town 3 do not react to this temporary decrease in purchases. Accordingly, quality in town 2 and town 3 remains unchanged for the entire period.

Town 1 can increase its total quality during the subsidy-period but soon thereafter, the shopping center cannot be maintained and the destroyed inner city capacity cannot be fully re-established. Therefore, in the long run, town 1 loses. One can see that in the long run the aggregate quality in the region decreases since town 1 loses some quality while the other towns hold their qualities constant.

5.4 Scenario 4: Feedback on Purchasing Power Respectively the Population of the Community

(Bodenhoefer, Grozea-Helmenstein & Kleissner 2002) describe indirect purchasing-power effects caused by the construction of shopping centers. We now include such multipliers into our model. We therefore assume that decreasing purchases in a town, lead to decreasing purchasing power available for the residents of that town, *e.g.* lower wages paid to shop-employees, more people unemployed because of fewer shops, *etc.* Since we do not assume that residents will only work in their hometown, we have to find a distributive key for such income effects. We decided to use again the qualities of towns and their distances to each other. So, for example, if town 1 loses 10% of its purchases, then this loss will have an effect not only on its own inhabitants, but also on those of other all towns. Again, this effect is distributed via distance to the other towns. The farther they are away, the less they lose. Since the number of residents in a town is a proxy of the purchasing power of this town, it seems reasonable to let this number vary, according to the above rule.

The modified model without adjustment costs is presented in Figure 12. The solid lines represent the number of purchases for each town and therefore represent also the quality development of the region. The number of residents is not fixed anymore. We replace the former number of residents with the purchasing-power of each town, which are then subject to change.

The dashed lines represent the development of the purchasing-power of each town. We assume again a one-to-one relationship between one unit of quality and one unit of purchasing power. The method of simulating trade stays the same, except for the variable number of residents, which we now call purchasing-power.

The feedback effect from purchases this year $m_i(t)$ on next years purchasing power $b_i(t+1)$ depends on the feedback parameter fe. The bold lines in Figure 12 represent cases where the income effects on a town (the purchases) have a 50% feedback effect. If purchases decrease by 30 units in town i, then next years purchasing-power decreases by 15 units. From this we see that town 1 looses quickly from its 800 units of purchasing power and decreases to about 650, while the number of purchases go to about 400. The thin lines in Figure 12 are 5% feedback effects. We therefore face an almost constant purchasing power development, similar to earlier specifications where the number of residents was constant.

If we increase the feedback effect above 50%, we get an interesting result in town 1. With $fe = 70\%$ town 1 loses all purchases and the purchasing power of its residents goes down to 400. The values for town 2 remain almost constant. If fe is increased to about 90%, then town 3 becomes so strong, that it attracts all customers in the region. Town 1 and town 2 lose all purchases and the purchasing power of their residents decreases considerably, and remain constant at a small but positive level although they do not have any shops (hence $q_1 = q_2 = 0$) anymore. Since the income of residents does not depend entirely on the retail trade-sector, a 5% feedback effect seems an appropriate number. However, we wanted to take a look at the more extreme cases to understand the dynamics underneath.

5.5 Scenario 5: Purchasing Power Feedback with Adaptation Costs

Here, we combine adjustment costs with the purchasing-power feedback effect from last section. We again assume asymmetric thresholds for quantity adjustments, 30% surplus for enlargements and -10% (oversupply) for quality reductions. The feedback factor is 50%. After the system has reached a steady state, town 1 opens a shopping center and sets its quality to 1,300. Figure 13 shows the results over time. We see again the paradoxical situation that town 3 benefits in the long-run, since the lost capacities of town 2 can not be re-established once the supporting period for the shopping center is over and the region can again freely adjust to the market forces. In the aggregate the region around town 1 and town 2 has lost when compared to town 3. However, town 1 itself could gain a little from the shopping center.

6 Conclusion

In this paper we presented a new application for gravity models, the explanation of the spatial distribution of shopping center areas. We introduced a formal model and proved important steady state properties such as the asymptotical stability of interior solutions. The region of stability is diminished once mobility increases. In the latter case, border solutions become more likely.

Furthermore, we presented a simulation framework for a small artificial region with three towns. We are able to describe the effects of opening a new shopping center on purchases, quality development and purchasing-power development. In a first step, we attributed the purchases of all residents in the region to the 3 towns according to the town's attractiveness to residents. This attractiveness depended only on the (shopping)-quality of the town and on the distances.

In the next step, we considered that town 1 opened a supermarket or shopping center of quality=1,000. Depending on how far away they live, consumers would decide to ignore their nearby hometown shops and drive the distance to the shopping center. We find the following results for this case.

Town 1's inner city precinct loses most. This does not necessarily need to have a negative effect on town 1, since it gets compensation from the new purchases in its shopping center. In the long-run, these effects can be positive or negative.

Neighboring town 2 is always the loser of the shopping center. Many of its residents will abandon their hometown shops and use the nearby shopping center to do their shopping. Unlike town 1, town 2 is not compensated for this loss of inner city shops.

Remote town 3 is relatively independent of these developments. Paradoxical outcomes have it, that town 3 can actually gain from a shopping center in town 1, via the losses of town 2.

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Figure 10: Purchases and Quality with Nearby SC

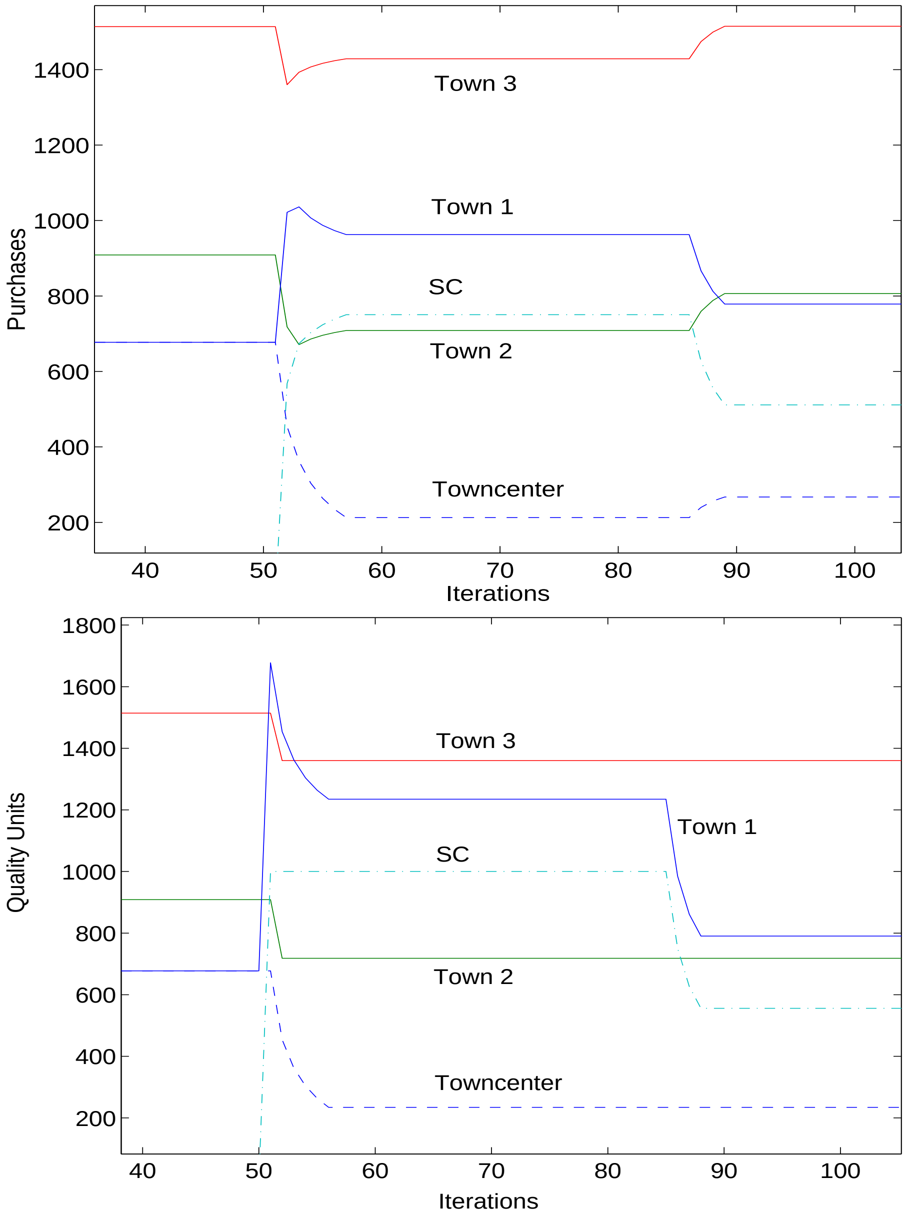

Figure 11: Purchases and Quality with Remote SC

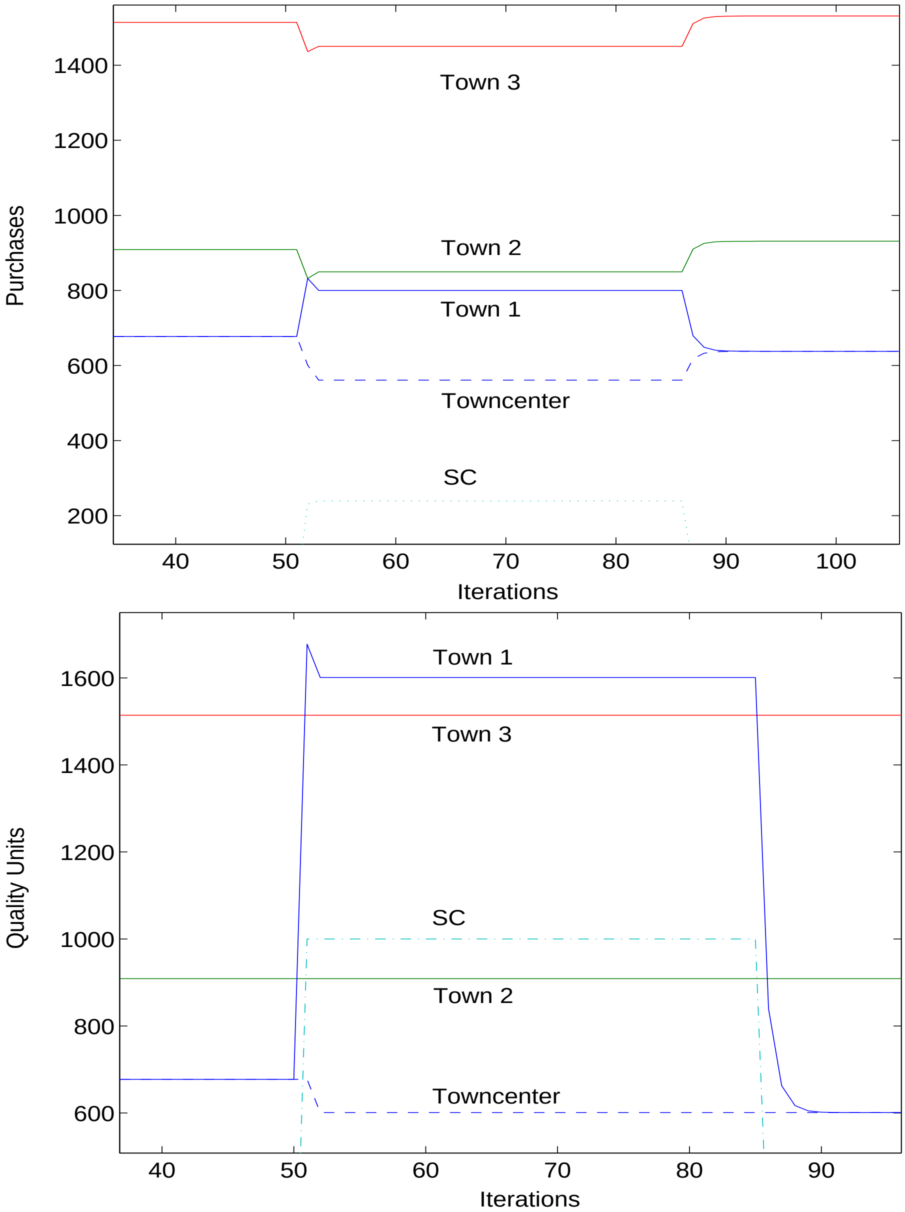

Figure 12: Income Effects on Communities - Evolution of Purchasing Power

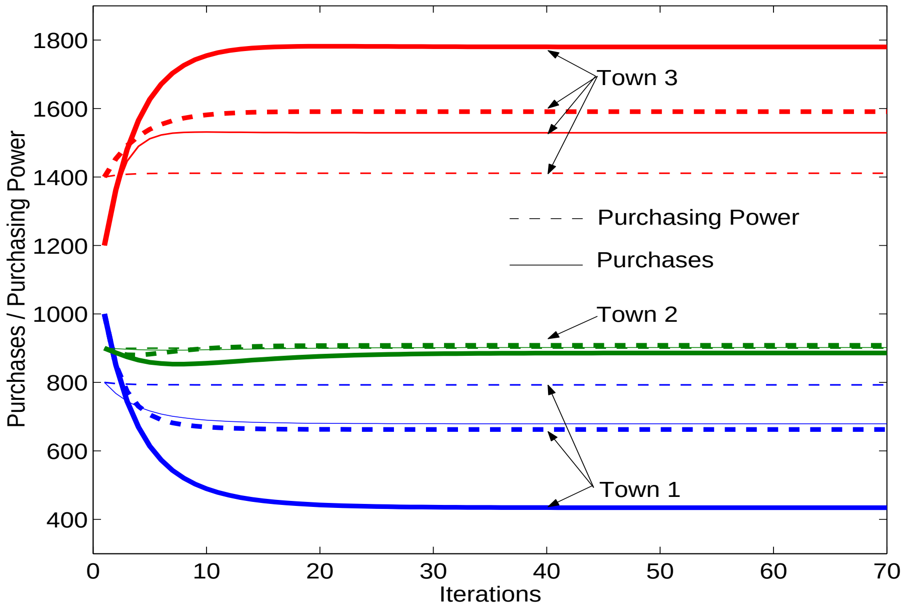

Figure 13: Income Effects with Adaptation Costs