Household Problem

Juergen Jung*
Department of Economics
Towson University

March 16, 2017

Abstract

This is a simple household problem in a one period economy.

*Department of Economics, Towson University, U.S.A., *E-mail:* jjung@towson.edu

1 Household Problem with 2 Goods

Household preferences are gives as

$$u\left(c_1, c_2\right) = \frac{c_1^{1-\sigma}}{1-\sigma} + \frac{c_2^{1-\sigma}}{1-\sigma}$$

The budget constraint is

$$p_1 c_1 + p_2 c_2 = I,$$

where income I is exogenously given (endowment income). The household maximization problem is:

$$max_{\{c_1, c_2\}} \left\{ \frac{c_1^{1-\sigma}}{1-\sigma} + \frac{c_2^{1-\sigma}}{1-\sigma} \right\}$$
$$s.t.$$
$$p_1 c_1 + p_2 c_2 = I.$$

The optimality condition of the household is

$$MRS \equiv \frac{MU_{c_1}}{MU_{c_2}} = \frac{p_1}{p_2},$$

where $MU_{c_1} = \frac{\partial u(c_1,c_2)}{\partial c_1}$is the marginal utility w.r.t. c_1 and $MU_{c_2} = \frac{\partial u(c_1,c_2)}{\partial c_2}$. With the functional form given above the MRS becomes:

$$MU_{c_1} = \frac{\partial u\left(c_1, c_2\right)}{\partial c_1} = \left(1-\sigma\right) \frac{c_1^{1-\sigma-1}}{\left(1-\sigma\right)} = c_1^{-\sigma},$$
$$MU_{c_2} = \frac{\partial u\left(c_1, c_2\right)}{\partial c_2} = \left(1-\sigma\right) \frac{c_2^{1-\sigma-1}}{\left(1-\sigma\right)} = c_2^{-\sigma},$$

and

$$MRS = \frac{c_1^{-\sigma}}{c_2^{-\sigma}} = \frac{c_2^{\sigma}}{c_1^{\sigma}} = \left(\frac{c_2}{c_1}\right)^{\sigma}.$$

The optimality condition says that this has to be equal to the price ratio so that

$$\left(\frac{c_2}{c_1}\right)^{\sigma} = \frac{p_1}{p_2}.$$

We can now express c_2 as a function of $c_1$and prices

$$
\begin{aligned}
\left(\frac{c_2}{c_1}\right)^{\sigma} &= \frac{p_1}{p_2}, \\
\rightarrow \left(\left(\frac{c_2}{c_1}\right)^{\sigma}\right)^{\frac{1}{\sigma}} &= \left(\frac{p_1}{p_2}\right)^{\frac{1}{\sigma}}, \\
\rightarrow \left(\frac{c_2}{c_1}\right)^{\sigma\times\frac{1}{\sigma}} &= \left(\frac{p_1}{p_2}\right)^{\frac{1}{\sigma}}, \\
\rightarrow \frac{c_2}{c_1} &= \left(\frac{p_1}{p_2}\right)^{\frac{1}{\sigma}}, \\
\rightarrow c_2 &= \left(\frac{p_1}{p_2}\right)^{\frac{1}{\sigma}} c_1.
\end{aligned}
$$

You can use this equation together with the household budget constraint to solve for $c_1$and c_2 because p_1, p_2, I and σ will all be given:

$$p_1 c_1 + p_2 c_2 = I, \tag{1}$$

$$c_2 = \left(\frac{p_1}{p_2}\right)^{\frac{1}{\sigma}} c_1. \tag{2}$$

Two equations in two unknowns. Solve it for $c_1^*$and c_2^*. Plug the second into the first equation

$$
\begin{aligned}
p_1 c_1 + p_2 \overbrace{\left[\left(\frac{p_1}{p_2}\right)^{\frac{1}{\sigma}} c_1\right]}^{c_2} &= I \\
\rightarrow p_1 c_1 + p_2 \left(\frac{p_1}{p_2}\right)^{\frac{1}{\sigma}} c_1 &= I \\
\rightarrow \left(p_1 + p_2 \left(\frac{p_1}{p_2}\right)^{\frac{1}{\sigma}}\right) c_1 &= I \\
\rightarrow c_1^* = \frac{I}{\left(p_1 + p_2 \left(\frac{p_1}{p_2}\right)^{\frac{1}{\sigma}}\right)}&.
\end{aligned}
$$

The solve for c_2^* using equation (2)

References