ECON 310
Problem Set 4 - Chapter 5: A Static Macroeconomic Model with Equilibrium

1 Problem Set 4

1.1 Question1

The profit function of the firm is:

$$\pi\left(N_d\right)=z\times K^{0.33}\times N_d^{0.67}-w\times N_d.$$

The maximization problem is:

$$\max_{N_d}\pi\left(N_d\right)=\max_{\dot{N}d}\left\{z\times K^{0.33}\times N_d^{0.67}-w\times N_d\right\}.$$

The firm optimality condition (first order condition is):

$$0.67\times z\times K^{0.33}\times N_d^{(0.67-1)}-w=0,$$

or

$$\overbrace{0.67\times z\times K^{0.33}\times N_d^{(0.67-1)}}^{MP_N}=w$$

Solving for optimal labor demand we get:

$$N_d^{(0.67-1)}=\frac{w}{0.67\times z\times K^{0.33}}\rightarrow$$

$$N_d^*=\left[\frac{w}{0.67\times z\times K^{0.33}}\right]^{\frac{1}{(0.67-1)}}$$

Assume $z=10, K=100$, and wage $w=\$20$ then solve for N_d^* :

$$N_d^*=\left(\frac{20}{0.67*10*100^{0.33}}\right)^{\frac{1}{(0.67-1)}}=3.6370$$

1.2 Question 2

The profit function of the firm is:

$$\pi\left(N_d\right)=z\times K^{0.33}\times N_d^{0.67}-w\times N_d.$$

Which a wage subsidy of \$4 per unit of labor, the firm receives \$4 for each quantity of N_d hired, so that the new profit function is

$$\pi\left(N_d\right)=z\times K^{0.33}\times N_d^{0.67}-w\times N_d+4\times N_d.$$

The firm optimality condition is (i.e., first order condition):

$$0.67 \times z \times K^{0.33} \times N_d^{(0.67-1)} - w + 4 = 0.$$

And optimal labor is

$$N_d^{(0.67-1)} = \frac{w-4}{0.67 \times z \times K^{0.33}} \rightarrow$$

$$N_d^* = \left[\frac{w-4}{0.67 \times z \times K^{0.33}}\right]^{\frac{1}{(0.67-1)}}$$

Which is

$$N_d^* = \left(\frac{20-4}{0.67 * 10 * 100^{0.33}}\right)^{\frac{1}{(0.67-1)}} = 7.1517$$

So that output becomes:

$$Y = 10 * 100^{0.33} * 7.1517^{0.67} = 170.79$$

1.3 Question 3

The marginal product of labor (MP_N or MPL) is

$$\frac{z \partial F(K, N_d)}{\partial N_d} = 0.67 \times z \times K^{0.33} \times N_d^{(0.67-1)}$$

So that the MP_N at point $N_d = 3$ is equal to $0.67 * 10 * 100^{0.33} * 3^{(0.67-1)} = 21.312$

1.4 Question 4

The marginal product of labor (MP_N or MPL) is

$$\frac{z \partial F(K, N_d)}{\partial N_d} = 0.67 \times z \times K^{0.33} \times N_d^{(0.67-1)}$$

So that the MP_N at point $N_d = N_d^* = 3.6370$ is equal to $0.67 * 10 * 100^{0.33} * 3.637^{(0.67-1)} = 20.000$ which is the wage rate w. You already see this in the optimality condition of question 1. So no need to calculate anything here.

1.5 Question 5

From the firm problem we have

$$\pi(N_d) = z * N_d - w * N_d, \tag{1}$$

so that the first order condition is

$$z = w.$$

Since $z = 10$, the wage rate in equilibrium is 10.

1.6 Question 6-11

The household maximization problem is

$$\begin{aligned} & \max_{c,l} u(c,l) \\ & s.t. \\ c &= (1-l)*w - T + \pi \end{aligned}$$

Method 1: The optimality condition is the marginal rate of substitution has to equal to the price ratio:

$$\frac{MU_l}{MU_c} = \frac{w}{1}.$$

Which given our utility function of

$$U(c,l) = \ln(c) + \ln(l)$$

is

$$\frac{c}{l} = \frac{w}{1}. \tag{2}$$

Use this together with the budget constraint and solve for c^* and l^*.

Method 2: You can get the same thing if you substitute out consumption using the budget constraint in the preferences

$$\max_l \ln((1-l)*w - T + \pi) + \ln(l).$$

Derive w.r.t. l to get the optimality condition (i.e., first order condition)

$$\frac{1}{(1-l)*w - T + \pi}(-w) + \frac{1}{l} = 0.$$

Solve this for l^* and then plug this l^* into the budget constraint to get c^*.

Solving you get

$$\begin{aligned} wl &= (1-l)w - T + \pi, \\ &\rightarrow l^* = \frac{w - T + \pi}{2w}. \end{aligned}$$

and from expression (2) we know that in equilibrium

$$c^* = w * l^*,$$

so that

$$c^* = w\frac{w - T + \pi}{2w}.$$

In addition we know that in equilibrium the government budget has to balance

$$G = T$$

and that firm profits are zero (you see this from equation (1). So that

$$\begin{aligned} l^* &= \frac{w-G}{2w}, \\ c^* &= w\frac{w-G}{2w}. \end{aligned}$$

Next assume $z = 10$ and $G = 6$. From above you know that $z = w = 10$. Therefore

$$\begin{aligned} l^* &= \frac{10-6}{2*10} = 0.2 \\ c^* &= 10*\frac{10-6}{2*10} = 2.0 \\ w^* &= 10 \end{aligned}$$

In equilibrium

$$(1 - l^*) = N_s^* = N_d^* = 0.8,$$

so that output is

$$Y^* = z * N_d^* = 10 * 0.8 = 8.0$$

and profits are

$$\pi\left(N_d^*\right) = z * N_d^* - w * N_d^* = 10 * 0.8 - 10 * 0.8 = 0.$$

And finally welfare/utility is

$$u\left(c^*, l^*\right) = u\left(2, 0.2\right) = \ln\left(2\right) + \ln\left(0.2\right) = -0.916\,29$$