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Agent-based modeling, public choice, and the legacy of Gordon Tullock

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Abstract The future of modeling in public choice may be glimpsed by examining its evolution in economics. For problems that are influenced by heterogeneity of actors, social networks, or *emergence*—the arising of a complex system from simple phenomena, such as Adam Smith's "invisible hand"—economists increasingly are turning to agent-based modeling. Agent-based modeling is a form of computational analysis that focuses on agents rather than on aggregates. In his long career as a founding scholar of public choice, Gordon Tullock repeatedly followed the agent-oriented approach of methodological individualism. Many of Tullock's models are thus highly amenable to further exploration using that method. As agent-based modeling becomes more and more popular, the importance of Tullock's work will continue to grow.

Keywords Agent-based modeling · Heterogeneity · Emergence

JEL Classification C63 · H10 · H30

1 Introduction

The field of public choice has been described as "the application of economics to political science" (Mueller 2003: 1), so it is not surprising that public choice theorists have adopted the economist's reliance on *models*. A model is an abstract, small-scale representation of some real-world phenomenon. Suppose, for example, that one asks an economist what will happen to the economy if the Federal Reserve raises banks' reserve requirements. The economist will respond by selecting a model, manipulating the reserve requirements variable in that model, and reporting what the model predicts. Like economists, public choice scholars create and manipulate models. The difference between an economist and a public choice scholar is a matter of domain: economists model economic phenomena, while public

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choice scholars model political phenomena. Given their common heritage and shared approach, it seems likely that public choice modeling will follow the same evolutionary path as economic modeling. This essay argues that this evolutionary path leads to agent-based modeling. It also argues that Gordon Tullock started down this path long ago, and thus that Tullock, 50 years later, remains at the forefront of the field he helped to found.

Agent-based modeling is a computational technique. Contrasted with other forms of modeling, agent-based modeling has two distinguishing features: support for arbitrarily heterogeneous actors (or "agents"), and support for adaptive behavior. These features may not seem impressive on their own, but when combined with a framework of rules guiding agents' interactions, they can produce almost startlingly complex results. The ability of a self-interested agent "to promote an end which was no part of his intention" was recognized by Adam Smith (1994 [1776]: 485) over two centuries ago; his original statement of the invisible hand theorem still confounds most of its readers. Hayek (1976: 33) calls it "spontaneous order": "the position of each individual is the resultant action of many other individuals, and nobody has the responsibility or the power to ensure that these separate actions of many will produce a particular result." It is for this reason that Vriend (2002: 811) describes Hayek as "an agent-based computational economist (ACE) *avant-la-lettre*." By describing economic systems in terms of agents and their interactions, Hayek followed the agent-based modeling approach implicitly. Wagner (2008) observes that the same can be said of Gordon Tullock. If Hayek can be called an ACE, so too can Tullock, whose "scholarly oeuvre ... reflects a deep understanding and appreciation of spontaneous order theorizing" (Wagner 2008: 56). Tullock anticipated the agent-based modeling approach. By continuing down that path, modern scholars can explore Tullock's work in new and deeper ways.

2 Economic modeling

Economic models have taken different forms at different times in history. An early form of economic modeling was mythology: in Roman times, for example, the goddess Ceres protected crops. The Roman model of grain production was essentially this: to obtain more grain, make Ceres happy (Spaeth 1996). This model may seem simplistic to the modern reader, yet it was compelling enough to the Romans to motivate the erection of temples dedicated to Ceres. The model was compelling because it provided both explanatory and predictive power: it explained the dynamics of grain production, and it offered a way of understanding seasonal output variations. But what this model lacked—and what makes the model unacceptable to the modern reader—was *falsifiability*. The absence of an *ex ante* definition of what precisely made Ceres happy meant that no test could be devised that would disprove the model.

A model's lack of falsifiability does not mean that the model is *wrong*; a model may be accurate, yet unprovable in any formal sense. But nonfalsifiable models are no longer regarded as useful in the scientific community (Popper 1959). By contrast, falsifiable models *are* useful. A falsifiable model is one that begins with assumptions, applies logical deductions to those assumptions, and concludes by stating empirically testable predictions. Such a model is disproved when the assumptions are granted but the predictions are not fulfilled. Falsifiable models are powerful because they split the problem being modeled into two distinct parts—the premises and the deductions—which can be analyzed separately. For this reason, among others, mathematical models have come to predominate the economics profession (Stigler et al. 1995). A mathematical model is the cleanest form of a falsifiable scientific model. Assumptions are expressed as mathematical identities; predictions are the algebraic consequences of those identities. Sociologist Vilfredo Pareto, writing in 1911, summarized the approach now followed by all orthodox economists: "given the mathematical laws according to which certain individuals usually behave, determine the consequences of these laws" (Pareto 1955 [1911]: 58).

It is easy to see why mathematical models have become ubiquitous in economics. Such models are crisp and precise. They make firm predictions that can, at least in theory (though not always in practice), be confirmed or refuted. The path from premises to predictions is undeniable: any disbelieving reader can double-check the algebra. Mathematical models thus offer significant benefits over the narrative models they displaced. But in order to reap these benefits, the model must, of course, be tractable. An intractable model cannot be solved, and thus yields no predictions. The use of mathematical models, then, hinges on tractability. There are two ways to transform a model from intractable to tractable. The first is to simplify it directly, by omitting the least relevant choice variables. The second is to simplify it indirectly, by assuming fully rational actors. The behaviors of fully rational actors can be predicted entirely by the current state of the model; therefore those behaviors can be eliminated as choice variables. Each of these approaches is widely practiced, but neither is entirely satisfying.

The first option—eliminating the least relevant sources of variation—often requires the modeler to make assumptions that clearly are untrue in the real world. For example, a modeler might assume, for purposes of the model, that all agents are identical and have infinite lifespans. The Ramsey-Cass-Koopmans model of economic growth employs this assumption (Romer 2006: 49). Households are not identical in the real world, but this does not mean that the Ramsey-Cass-Koopmans model is useless. In fact, the model is considered ideal for studying growth issues that are independent of generations. Growth issues that do depend on the existence of generations can be analyzed using the Diamond growth model instead. The Diamond model assumes two-period, overlapping generations instead of infinitely lived households (Romer 2006: 76). Once again, generations are not two-period entities in the real world, but this does not render the Diamond model useless. The fact that a model employs simplifying—and untrue—assumptions does not render the model invalid, at least in the domain in which the model is intended to serve. As Box and Draper (1987: 424) put it, "all models are wrong, but some are useful."

There is, however, something unsatisfying about an approach that requires two different models to describe a single phenomenon. And when those two models produce different predictions, the situation is worse than unsatisfying; it is unacceptable. For example, sustained economic growth is easily derived in the Ramsey-Cass-Koopmans model, but "may not be possible, or ... may depend on initial conditions" in the Diamond model (Box and Draper 1987: 48). When these two models produce conflicting results, one begins to wonder whether the "unnecessary" complexity that was removed for tractability was really unnecessary. How can the modeler be sure that the complexity being eliminated is irrelevant to the problem being modeled? How can the modeler know, for example, whether the assumption of infinitely lived agents affects a model's predictions? Given the choice between a model with obviously false assumptions and a model without them, the only reason to prefer the former is tractability.

The second approach to achieving tractability is the use of rational expectations. First introduced by Muth (1961), rational expectations can be described—among many other ways—as the use of the model to answer questions about the model. Actors in the model understand the model itself and can optimize their behavior based on the model's state. Given changes to the model's state—for example, changes in government policy—actors are able to "look ahead" to foresee the consequences of those changes, and adjust their own behaviors accordingly. Menzies and Zizzo (2005) offer an interesting perspective on rational expectations. It is certainly true that rational expectations can be defended on theoretical grounds: to deny that actors "look ahead" is to suggest they are not utility maximizers, and utility maximization is a cornerstone of economics. But the fact that rational expectations also make models more tractable cannot be denied. Is it possible that economists might believe that "the parsimony and tractability of rational expectations justifies their use" (Menzies and Zizzo 2005: 3)? The authors suggest that while parsimony and tractability are "unlikely on [their] own to be sufficient to accept rational expectations as scientifically valid, it may be more palatable if one can claim that lack of realism is an acceptable price when parsimony and tractability [are] matched by significant predictive power" (Menzies and Zizzo 2005: 3). In other words, it may be in the modeler's interests to assume rational expectations. The demand for tractability may drive, at least to some extent, the use of rational expectations.

Mathematical models are appealing, but the use of mathematical models implies tractability, and tractability implies assumptions that are either unrealistic or heroic. Mathematical models, therefore, rest on unrealistic or heroic assumptions. On the surface, this would seem to be an unstable foundation upon which to build a discipline. How, then, did economists come to rely on mathematical models? Boettke (1996: 31) suggests that this reliance was driven more by expediency than principle: an economist who concedes the shortcomings of mathematical modeling may nevertheless continue to practice it, because, in Boettke's words, "what is the alternative?" Even active practitioners of mathematical modeling are aware of this danger. Clement (2011: 38) quotes Ricardo Caballero, chair of MIT's economics department and a noted macroeconomist, as stating that "the current core of macroeconomics has become so mesmerized with its own internal logic that it [confuses] the precision it has achieved about its own world with the precision it has about the real one." Mathematical models "take on a life of their own ... the core of macroeconomics seems to transform things that may have been useful modeling short-cuts into a part of a new and artificial 'reality"' (Clement 2011: 39). Caballero also highlights the absence of public choice considerations in most macro models:

> Take, for example, the preferred "micro-foundation" of the supply of capital in the workhorse models of the core approach. A key parameter to calibrate in these models is the intertemporal substitution elasticity of a representative agent .... This parameter may be a reasonable estimate for an individual agent facing a specific micro decision, but what does it have to do with the aggregate? What happened with the role of Chinese bureaucrats, Gulf autocrats and the like in the supply of capital? A typical answer is not to worry about it (Clement 2011: 39).

Boettke and Caballero both recognize the need for alternative approaches to economic modeling. Caballero claims that "no one really knows with any certainty what we need to do next, and hence we need to allow for much more freedom of exploration" (Clement 2011: 39). He observes that "a key step is to embrace the complexity of the [economic] environment and what it does to economic agents and their decisions" (Clement 2011: 39). In the financial crisis of 2008, for example, the "main physical contagion mechanism [was] through network cascades" (Clement 2011: 34). Modeling the fact that "financial institutions ... have to worry about ... the financial situations of the neighbors of the neighbors of their neighbor" is "tremendously complex" (Clement 2011: 34). In the spirit of the exploration encouraged by Caballero, and in answer to the question posed by Boettke (1996: 31)—"what is the alternative?"—this paper argues that *agent-based modeling* is the alternative.

3 Agent-based modeling

An agent-based model is a computational problem in which the fundamental unit of analysis is the economic or political agent, rather than some aggregate or average of agents (Epstein and Axtell 1996). The designer of an agent-based model imbues each agent with preferences and behaviors, then allows the agents to interact. Those interactions result in changes in agents' endowments, and possibly in their future preferences and behaviors. Over repeated iterations, equilibria may or may not emerge (Tesfatsion 2006). The key advantage of the agent-based approach is that the modeler is no longer required to make broad claims about agents. An infinitely lived agent may be easily transformed into one with discrete generations with minimal effort. Such an agent may be further transformed into an agent with a deterministic or stochastic lifespan with equal ease. Some agents may form expectations rationally, others adaptively, and others naively. Agent-based modeling is the ideal mechanism for incorporating insights from behavioral economics into economic models (Macal and North 2010). Laboratory experiments in behavioral economics are useful, but they come with limitations: participants may have limited competence, may not understand complex interactions, or may allow their awareness of the experiment to influence their actions. Laboratory experiments are always limited in scope; an experiment with a million subjects is clearly not feasible. Agent-based models provide an opportunity to apply the results of laboratory experiments to larger sets of agents, in a controlled setting. Agents can be as heterogeneous, and as irrational, as their real-world, experimental counterparts.

It should be acknowledged that modern macroeconomics is not devoid of mathematical models incorporating and exploring the consequences of individual differences. For example, Shimer and Smith (2001) present a search model in which heterogeneity plays a key role. They contrast their findings with those of Mortensen (1982), who reached different conclusions by assuming homogeneity. However, the Shimer and Smith model still assumes that all agents "are risk-neutral, infinitely-lived, and discount the future at a [single] rate $r > 0$" (Mortensen 1982: 3). Agents are heterogeneous, but only along one dimension (high-versus low-productivity). Gomes and Michaelides (2008) present a more advanced model in which agents are heterogeneous in exposure to labor income shocks, stage of life, access to investment opportunities, risk aversion, and elasticity of intertemporal substitution. Yet even with this impressive degree of heterogeneity, the model still assumes that "[h]ouseholds are price takers and maximize utility given their expectations about future asset returns and aggregate wages" (Gomes and Michaelides 2008: 423). The model also assumes that the "social security system is balanced at all times" (Gomes and Michaelides 2008: 425). Observers of the contemporary American political scene might be interested in knowing what the model predicts if the social security system were to (as it has) become imbalanced; unfortunately, this model sheds no light on that question. Isaenko (2008) presents a highly detailed model that explores interest rate variations in a setting populated by heterogeneous investors; it presents strong conclusions in the face of arbitrary degrees of relative risk aversion and elasticities of intertemporal substitution. Isaenko concludes by recommending the introduction of "a new type of preferences that exhibit a habit formation" (Isaenko 2008: 477). But one suspects that if such an introduction were simple, he would have done it himself. In each of these cases, the agent-based approach would offer the distinct advantage of expanding the degree of heterogeneity allowed in the model. This heterogeneity would, of course, come at the cost of additional complexity; but the cost of introducing the same degree of heterogeneity to the mathematical models would, in effect, be infinite.

At this juncture it is useful to illustrate the agent-based approach by comparing a simple mathematical model to an agent-based counterpart. Consider the typical Economics 101 representation of supply and demand. In the mathematical version of this model, one equation represents the aggregate willingness to buy (the horizontal summation of buyers' demand curves), and another equation represents the aggregate willingness to sell (the horizontal summation of sellers' supply curves). For a single market with no out-of-equilibrium trading, the equilibrium obtains when these two equations are solved simultaneously. Given specific functional forms for supply and demand, this model predicts the quantity of units sold and the uniform price charged for every and all units. By contrast, an agent-based model of supply and demand focuses not on aggregates but on agents. Each buying-agent has his/her own calculus for bidding; each selling-agent has his/her own calculus for asking. Trades occur whenever a willing buyer encounters a willing seller. Whether an equilibrium price obtains—that is, whether all buyers will buy at the same price—is a function of the individuals and their preferences, and the market institutions that bring the individuals together.

The agent-based model can be constructed to produce the same results found by the mathematical model. For example, if agents are allowed to communicate with each other, and agents are designed to have identical preferences, the predicted equilibrium price will emerge. In fact it is perfectly acceptable for agent-based models to assume that agents are identical in respects that do not relate to the question at hand. But the power of the agent-based model is that it can make predictions for situations in which the mathematical model cannot: situations in which heterogeneity—in agent behavior or in circumstances—matters. Suppose that agents have different demand schedules; the result will be a shifting equilibrium price, as different buyers reach their price thresholds at different times. Suppose that buyers incur "search costs" while identifying sellers from whom to purchase; it is possible that no single equilibrium price will emerge. Suppose that some buyers are better than others at minimizing search costs; the result will be differing amounts of consumer surplus. None of these outcomes is captured by the standard equilibrium model of supply and demand.

The mathematical model of supply and demand became popular, and remains popular, for two good reasons: it is easy to understand, and it produces closed-form solutions. But it fails entirely to capture some extremely important features of the phenomenon that it seeks to explain. For example, Keynesians often deride this classical "market-clearing" model; they point to labor markets and claim, rightly, that these markets obviously do not clear in the manner predicted by neoclassical price theory. (If labor markets always cleared, there would be no unemployment.) This model does little more than tell us that "when the storm is past the ocean is flat again," as Keynes (1971 [1923]: 65) famously put it. But the academic descendents of Keynes did not, in consequence, abandon mathematical modeling. Instead they attempted to salvage the classical supply-demand model by introducing "nominal rigidities," such as long-term union contracts, sticky wages, and the costs involved in changing prices on menus (Snowden and Vane 2002: 552). It is fascinating to observe that this debate—do markets clear?; if not, why not?—continues to dominate popular macroeconomic discourse. It is perhaps even more fascinating to observe that the tool used by most practicing economists—the mathematical model—is not the best tool with which to address the question.

Consider the mathematical version of the model of supply and demand, with the usual upward-sloping supply curve and downward-sloping demand curve. Suppose that each seller's supply schedule is $Q_s = a + bP$, and that each buyer's demand schedule is $Q_d = c - dP$. Suppose further that there are S sellers and B buyers. In equilibrium, the quantity supplied equals the quantity demanded, and so the equilibrium price can be found by setting $SQ_s = BQ_d$ and solving for P. The equilibrium price is thus $(Bc - Sa)/(Sb + Bd)$, and the equilibrium quantity is $S(a + b(Bc - Sa)/(Sb + Bd))$. For example, when $a = 100$, $b = 10$, $c = 20$, and $d = 7$, a market with 1000 buyers and 50 sellers should observe sales of 6000 units at a price of 2 per unit. If the market then experiences a negative demand shock, and c drops from 20 to 17, the new equilibrium will be 5800 units at a price of 1.6 per unit.

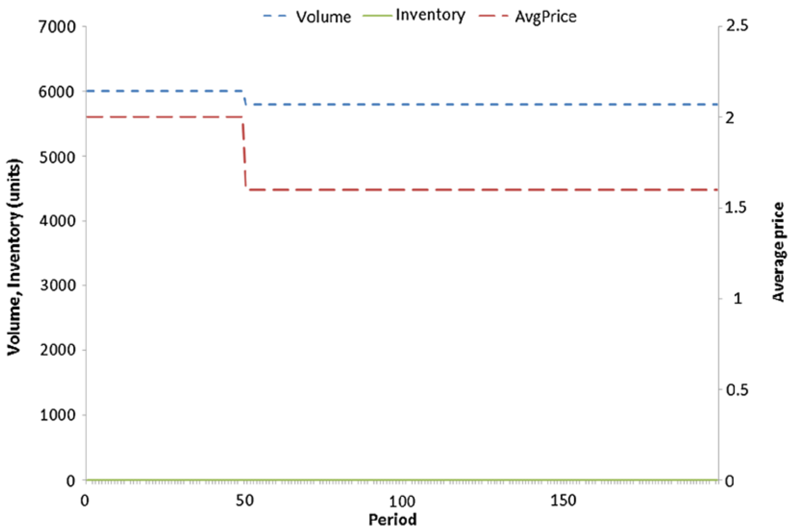

Fig. 1 Identical buyers, Walrasian auctioneer

Constructing an agent-based model to exhibit this behavior is straightforward. First, in order to demonstrate equivalence with its mathematical counterpart, consider an agent-based model in which each agent appeals to a Walrasian auctioneer (alternatively, the omniscient central planner) to determine the "correct" market price. Each period, the auctioneer examines each buyer and seller, and computes the surplus-maximizing price. All transactions then take place at that price. If each seller's supply schedule is $Q_s = a + bP$, and each buyer's demand schedule is $Q_d = c - dP$, the result predicted by the equilibrium model obtains (Fig. 1). Before the demand shock, each period sees 6000 units sold at a price of 2 per unit. After the demand shock in period 50, each period sees 5800 units sold at a price of 1.6 per unit. When the agent-based model assumes the existence of a Walrasian auctioneer, it generates the same predictions as the equilibrium model.

Notice in Fig. 1 that inventories are always zero. The Walrasian auctioneer is able to foresee the demand shock and adjust production accordingly; therefore suppliers are never left with unsold goods. But in actual markets, there is no Walrasian auctioneer; sellers are forced to discover optimal prices on their own. This is where the agent-based approach can provide insight. Consider a simple algorithm for price-setting: at the end of each period, a seller compares total sales with those of the previous period. If sales rose (fell) by $X\%$, the seller raises (lowers) prices by $X\%$ in the next period. The agent-based model easily is modified to support this type of seller. Figure 2 illustrates the effect of a demand shock when sellers must set their own prices, rather than rely on the auctioneer. (The figure depicts average outcomes over many iterations—a common practice among agent-based modelers.)

Sellers are unprepared for the demand shock, and inventories spike in the period of the shock. Sellers respond by slashing prices. Prices fall too far, partially offsetting the demand shock. Sellers then adjust in the other direction. Over time, average price converges to the expected equilibrium, as predicted by the mathematical model; but in the agent-based version, the dynamics of equilibration are clear. The mathematical model says nothing about inventories, implying that markets clear instantaneously; the agent-based model clearly shows that it takes time for the market to clear, and during that time the market price will sometimes be seen to *rise*, as well as to fall. The price-setting mechanism is explicit; there is no need to

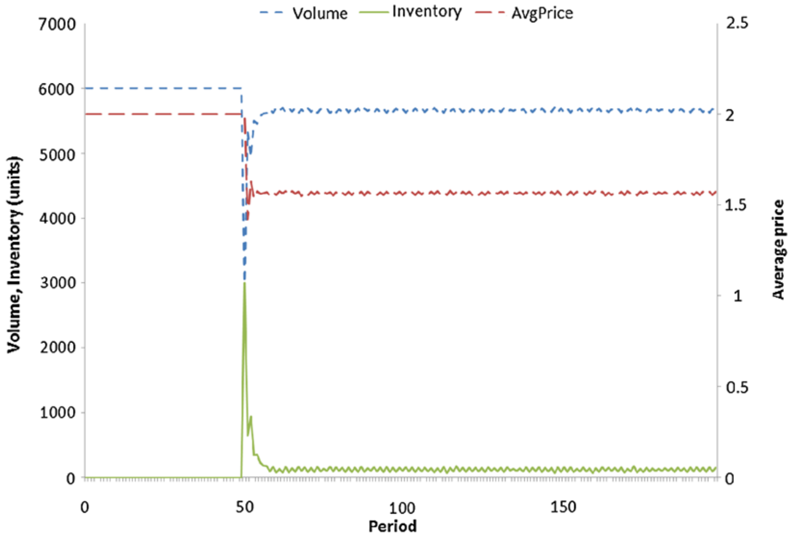

Fig. 2 Identical buyers, price-setting sellers

resort to a fictional auctioneer. And the model is easy to modify further. The modeler may wish to allow different demand schedules for different buyers, or to allow sellers to change their prices in different ways, or to add entry and exit by sellers or buyers to the model. The modeler may wish to acknowledge that in the presence of transaction and information costs, the "law of one price" may not hold, and exchanges may occur at non-equilibrium prices. All of these changes can be made without switching to a different model. Agent-based models cope with additional complexity more easily than their mathematical counterparts.

But perhaps most importantly for the social scientist, there is no question-begging in the agent-based version of the model. The mathematical model solves the supply and demand equations simultaneously—the modeler thereby assumes that a market equilibrium exists. The agent-based model is more subtle: an equilibrium may emerge, if circumstances permit, but there is no guarantee that this will happen. In other words, in the agent-based version of the model, equilibrium is an *emergent* phenomenon; in the mathematical version, equilibrium is a *fiat* phenomenon (Tesfatsion 2006). This is a crucial point: in the real world, economies continually are in flux. Contrary to the fiat equilibrium view, economies are not well described as "equilibrium, then shock, then equilibrium again." A better description of a modern economy would be "shock, shock, shock," and so on *ad infinitum*. Models in which equilibrium is assumed are not well suited to a world in which equilibrium never really arrives, except perhaps in the very long run. And readers do not need to be reminded what Keynes had to say about that.

The fact that agent-based modeling carries many advantages does not, however, make it immune from many of the same criticisms leveled against mathematical modeling. Perhaps the strongest criticism is the Lucas (1976) critique: models that assume policy invariance become useless when policies change. Lucas formulated his critique in an econometric context: attempting to infer behavioral patterns through econometric analysis of "nonstructural" variables assumes, implicitly, policy invariance. It assumes that actors will behave as they have always behaved. American audiences will understand a simple example: econometric analysis shows that National Football League teams frequently punt on fourth down, but almost never on third down. If the National Football League suddenly adopted Canadian rules—in which teams must gain ten yards in three downs, instead of four—we would expect the number of third-down punts to increase dramatically. But an econometric analysis that omitted the policy parameter—maximum number of downs to gain ten yards—would imply no change. The same criticism applies to an agent-based setting: if agent behaviors are not driven by "structural" factors such as preferences, agents will behave identically under different policy regimes. Such behavior is clearly unrealistic.

This criticism of agent-based modeling is an important one. The modeler may initially discount it, believing that agent-based models are immune from the Lucas critique. Yet it is all too easy to implement an agent in terms of *rules* rather than *preferences*. Rule-driven agents will obey those rules even when it is not in the agent's interests to do so; preference-driven agents attempt to maximize their own utility, and will alter their behaviors in response to policy changes. Markose (2005: F186) emphasizes that complex adaptive systems, such as agent-based models, "can throw light on ... endogenously arising innovative behavior of agents as best response strategies"; in this sense, agent-based modeling survives the Lucas critique. But Fagiolo et al. (2007: 208) note that the Lucas critique may still be relevant: agent-based models are frequently calibrated according to real-world conditions, and "calibration has a strongly conservative tendency." If preferences are influenced by calibration, agents are actually behaving in policy-invariant ways—adhering to behaviors that were more appropriate under a different policy regime—even though it *seems* they are reacting to policy changes. The subtle nature of this problem makes agent-based models even more susceptible to the Lucas critique than the econometric models against which the critique was first leveled.

Another valid criticism of agent-based models is that they are more susceptible than mathematical models to outright errors, simply because they are less transparent. Agent-based models are less contrived than their mathematical counterparts, but they are also more difficult to read, to verify, and to interpret. The agent-based model of supply and demand described earlier, for example, runs to four pages (see Appendix). A reader who disbelieves the predictions of an agent-based model is forced to wade through pages of computer code, looking for something to criticize. Such a model is more likely to be ignored than rejected. It is true that a similar charge could be made against many mathematical models, with appendices containing proofs of theorems and lemmas that span many pages. But people learn mathematics from their earliest days in school; demanding that the reader work through a few pages of proofs may be acceptable. Demanding that they learn computer programming is not.

Agent-based modelers have made many attempts to address this criticism. For example, as Tobias and Hofmann (2004: 1.3) note, "the use of already developed simulation frameworks ... increases the reliability and efficiency" of agent-based models. Such frameworks include AgentSheets, Ascape, Breve, Cormas, ECHO, JADE, Madkit, MAGSY, MASON, MIMOSE, NetLogo, Ps-i, Quicksilver, RePast, SimAgent, SimPack, StarLogo, Sugarscape, Swarm, TeamBots, and VSEit. The sheer number of frameworks, by itself, indicates a lack of consensus on the best approach to agent-based modeling. A key challenge for agent-based modelers is to converge upon a consensus approach that combines brevity with reliability. The emergence of such a consensus will constitute a major breakthrough in the field. Until that breakthrough arrives, modelers are left to their own devices.

4 Agent-based modeling and public choice

The motivation for such a breakthrough will continue to mount, however, as agent-based models show greater degrees of descriptive and predictive power. This breakthrough is likely to come in fields in which real-world complexity exceeds the capacity of mathematical modeling. Public choice is one such field: for as Buchanan and Tollison (1984: 14) put it, "[by] any comparison with politics, economic theory is *simple*." Economic interaction is impossible until "political exchange" has first occurred, and political exchange "necessarily involves *all* members of the relevant community rather than the two trading partners that characterize economic exchange" (Buchanan and Tollison 1984: 14). This combination of economic and political exchange occurs, for example, in the development of regulatory regimes: agents' trading behaviors are shaped by regulations over which they themselves, as voters, may have some degree of influence. The decision to trade is a private choice; the decision to regulate is a public choice. Yet the same agent is involved in both, and each impacts the other. As Tullock (2000: 5) observes, "the *same* people engage in market activities and in politics." Models of political behavior should recognize this.

That agent-based modeling is an ideal vehicle for exploring public choice is made clear by a closer examination of the entities that public choice scholars seek to understand. At the most elemental level, Buchanan and Tollison (1984: 13) criticize analysis of the political sector which "models the government as some sort of monolith, with a being of its own, somehow separate and apart from the individuals who actually participate in the process." The "government" is comprised of many different types of individuals—all of them self-interested, in the sense of *homo economicus*, but potentially with very different utility functions. In the legislative and executive realms, some pursue power for its own sake; some pursue power for monetary gain; some pursue power because they sincerely want to help others. Inevitably, too, some discover that doing good can also mean doing well, and that power carries unexpected charms. Utility functions change over time and with experience. Shifting utility functions are difficult to model mathematically; they are highly suited to an agent-based approach.

Modeling "government," however, is about more than modeling group formation; it is also about modeling group decision-making and group implementation. In each of these areas, agent-based modeling offers advantages. Wagner (2007: 71) notes, for example, that crafting a government budget is a process "of bottom-up emergence, in which the aggregate entity called a budget is generated" via competition among political agents. Modeling this process of political exchange is like modeling the process of economic exchange: a model that *assumes* equilibrium, rather than allowing equilibrium to emerge, is missing something important. (In real life there is no Walrasian auctioneer, and there is no philosopher king.) Even when a budget does emerge, its implementation is left to bureaucrats, whose interests may not coincide with those of the budget's writers. As Niskanen (1968: 293) observed, bureaucrats are not always best described as actors "who, for whatever reason, want to be efficient." Bureaucrats have their own utility functions, and bureaucrats are affected by the institutions within which they operate. "The theory of bureaucracy," wrote Tullock (1976: 27), "should be based upon the assumption that bureaucrats are as self-seeking as businessmen."

Agent-based modeling offers another key advantage for public choice scholars: the ability to merge related but distinct models. Consider two separate models, one of the budgeting process and another of the budget-maximizing bureaucrat. The bureaucrat may belong to a labor union that places pressure on the budgeters (Tullock 1987), but this connection between the two models is missing. Merging two mathematical models into one is usually not feasible, while merging two agent-based models is a much simpler task. This merging can be carried further, to bridge the gap between public choice and macroeconomics. In macro models that feature the accounting identity GDP $= C + I + G + NX$,[1] G is usually taken to be either exogenous or a function of C, I, and NX—thereby completely ignoring over 50 years of study into the determination of G. An agent-based treatment of G, when merged with agent-based macro models, makes public choice directly relevant to macroeconomists. Caballero bemoaned the lack of room for Chinese bureaucrats in macro models; agent-based modeling makes room for American, Chinese, and European bureaucrats.

5 Gordon Tullock: agent-based modeler

The roadmap for the incorporation of agent-based modeling into public choice was defined by Gordon Tullock himself. While exploring what he called "the General Irrelevance of [Arrow's] General Impossibility Theorem" (Tullock and Campbell 1970: 98), Tullock created—in 1970!—the first agent-based model in public choice. In "Computer simulation of a small voting system," Tullock and Campbell describe a computer program to explore cycling when a committee faces multiple motions, each of which has multiple dimensions. "Because most issues in the real world probably have more than one dimension," they write, a computer model "should give a more realistic measure of the importance of cycles in small voting bodies than has been made thus far" (Tullock and Campbell 1970: 99). They invoke their model with up to five dimensions, up to six motions, and up to 25 committee members. "The most interesting feature" of their model, they report, "is that the difference made by adding more dimensions is small" (Tullock and Campbell 1970: 101). They also find that in a two-dimensional issue space with three voters, there can be no cycles. "Although this is obvious when analyzed," they write, "it was at first unexpected" (Tullock and Campbell 1970: 103). This production of unexpected insights is one of the promises of agent-based modeling.

Despite its usefulness, this would be the last agent-based model Tullock would create. One can only speculate on the reasons he did not continue his foray into agent-based modeling. Perhaps the technology at the time proved too unwieldy for extensive use; their model was, after all, run on punch cards. Perhaps the computers of the time were just too slow. Perhaps no one asked for the computer code that they offered to provide upon request. Perhaps the prospect of continuing to implement models in Fortran and Algol simply did not appeal to Tullock, or to others.

It is certain, however, that Tullock did not abandon the approach for methodological reasons. A strict adherent of methodological individualism, Tullock's models are unfailingly expressed in terms of agents rather than aggregates. In "Information and logrolling," for example, Tullock (1983a: 33) creates several agent-based models "intended to be small-scale models of real-world situations." He creates models "with a very small number of people" because "such examples are easier to deal with" (Tullock 1983a: 33). But while using a small agent population "does not affect the conclusion," it nonetheless imparts an "aura of unreality" to the analysis, and this Tullock laments (Tullock 1983a: 33). It is regrettable that Tullock did not create an agent-based model in this instance. By doing so he would have dispensed with the "aura of unreality" and bolstered his claim that the small population "does not affect the conclusion" (Tullock 1983a: 33).

Tullock's recognition of the importance of agent heterogeneity is a recurring theme in his work. In "Hotelling and Downs in two dimensions," Tullock (1967a) extends Hotelling's and Downs's spatial model to multiple dimensions. He notes that in Downs's version of the

[1] That is: gross domestic product (GDP) equals consumption (C), plus investment (I), plus government expenditures (G), plus net exports (NX).

model, "the distribution of the voters becomes crucial" (Tullock 1967a: 56). Concerning that aspect, Tullock notes that the "distribution of the nonvoters would be quite different in different models, and this could be investigated quite easily" (Tullock 1967a: 57). Tullock then analyzes logrolling in a similarly multidimensional space, pausing to note with regret that, due to the limits of his geometric approach, "we will have to represent the preferences of interest groups, not individuals. Interest groups, of course, are built up out of individuals, and normally do not represent a group of people with identical preferences, but people who feel strongly on one issue and less strongly on others" (Tullock 1967a: 58). In this case Tullock has taken an approach common in economics: he has abstracted away from heterogeneity in favor of "an approximation of the interest group [average] preferences" (Tullock 1967a: 58). It is clear that Tullock regrets having to do this.

A few years later, in "A simple algebraic logrolling model," Tullock (1970) switches to an agent-based approach. Tullock considers "those situations in the real world in which we observe logrolling" and writes that "we observe immediate differences in the structure of the individual preferences ... individuals are assumed to have intense preferences on certain subjects" (Tullock 1970: 420). He then illustrates his model with three- and five-dimensional examples. In each version, Tullock demonstrates that in the presence of logrolling, "less than a majority of the voters ... may be able to control the outcome" (Tullock 1970: 423). The distribution of preferences, coupled with differences in district configurations, determines the degree to which a minority of voters can do this. Tullock concludes by stating that "this model ... provides a basis for future research by demonstrating that it is possible to obtain [Buchanan and Tullock's 1962] conclusions which differs from the widely used spatial models only by a minor change in parameters" (Tullock 1970: 424). A decade later he followed his own advice and explored the issue further in "Why so much stability?" (Tullock 1981). He extends his five-member legislative model to 25 members and attempts to explain why, in such a system, "not only is there no endless cycling, but acts are passed with reasonable dispatch and then remain unchanged for very long periods of time" (Tullock 1981: 189). An agent-based model of logrolling, calibrated with an agent population reflecting a country's actual population and the form of its legislature, would be the ideal vehicle to verify Tullock's answer: that "this stability [is not] a true equilibrium," but rather "a random member of a large set [of outcomes that] will be left unchanged for long periods of time" (Tullock 1981: 189).

Tullock will always be associated with the concept of rent seeking. In "Efficient rent seeking" (Tullock 1980), Tullock considers rent seeking as a game with a small number of heterogeneous players. He begins with an assumption that "the individuals can figure out the correct strategy" for optimal bidding for rents, and that "they assume that the other people will be able to figure it out" as well (Tullock 1980: 101). He builds an example of a rent-seeking game with eight different functional forms for probabilities of winning and different numbers of players (2, 4, 10, and 15). From a social welfare point of view, this game can have three different outcomes, which Tullock calls "zones." In the first zone, "expectancy of the players, if they all play, would be positive": the prize exceeds the cost of obtaining the prize (Tullock 1980: 102). In the second zone, "the sum of the payments made by the individual players is greater than the prize; in other words, it is a negative-sum game" (Tullock 1980: 104). In the third zone, "the individual players make payments that are higher than the prize. It might seem obvious that no one would play games of this sort, but, unfortunately, this is not true" (Tullock 1980: 104). The implications are obvious: "as a good social policy, we should try to avoid having games that are likely to lead" to zones two and three (Tullock 1980: 109). We should "attempt to lower the cost of rent seeking, and ... move ... into zone I" (Tullock 1980: 112). But how, in the presence of heterogeneous agents, does one structure the game to accomplish this result? Agent-based modeling provides a solution: by extending Tullock's 15-agent example to larger, more representative populations, one can realistically compare different versions of the game.

The creation of small, illustrative, example-oriented models continued throughout Tullock's professional life. In "A new and superior process for social choices" (Tideman and Tullock 1976), Tullock and coauthor T. Nicolaus Tideman turn their attention to demand revelation. They describe how "Vickrey showed that it would be possible to motivate individuals to reveal their true supply and demand schedules for a private good" and thus extend that concept to public goods (Tideman and Tullock 1976: 1146). They begin with "two alternatives, which may be conceived of as two policies or two candidates" (Tideman and Tullock 1976: 1147). They "then show how the process can be extended to more than two options" (Tideman and Tullock 1976: 1147). The important feature of their model is that individuals may have different preferences. In a reply, Riker (1979) takes issue with the conclusions reached by Tideman and Tullock. Riker offers an agent-based model of his own, with nine classes of voters. Tideman and Tullock (1981: 325) respond to "the last case given by Riker, in which there is in excess of three million voters, with a coalition of one hundred and ten of these" able to hold sway over the majority. Tideman and Tullock conclude that "Riker agrees with us that coalitions are possible in the demand revealing process, but apparently he does not agree with us that they are much less likely than in ordinary voting. It can be said, however, that the has not demonstrated that they would be more likely than they are with ordinary voting, only that they can exist" (Tideman and Tullock 1981: 328). In this exchange, the two sides are arguing about a set of questions that easily could be answered by an agent-based model, the workings of which are not in dispute.

Tullock again turns to small example models in "Income testing and politics" (Tullock 1982). He begins with a model that "has been designed ... to be extremely simple and straightforward and hence easy to follow" (Tullock 1982: 99). He had noted that "no direct transfer of the conclusions from the model into the real world is possible," because "the consequences we draw from this political model are very heavily affected by the detailed assumptions about things such as preference curves and number of people" (Tullock 1982: 99). In such a limited model, he suggests, it would not be possible to include enough real-world detail to produce real-world conclusions. Tullock nevertheless combines the model's results with "unfortunately not very sophisticated" empirical evidence to conclude that "in most of the cases in which income-tested programs have been converted into universal programs, the poor have been injured" (Tullock 1982: 116). This is a strong statement. In the 30 years that have passed since he made it, more empirical results have surely been obtained. Those results, in conjunction with a stronger, agent-based model, could strengthen or refute Tullock's conclusion.

Tullock explores government redistribution further in "Horizontal transfers" (Tullock 1983b). In an essay nearly a decade earlier, Tullock (1974: 7) had observed that governments transfers do not necessarily flow to the poor from the rich, but rather to parties with "sufficient political influence to initiate the transfer" from parties whose "political influence proved insufficient to stop it." In "Horizontal transfers" he constructs a model to explain why. He begins "with a simple model, which the reader may think unrealistic" (Tullock 1983b: 24). He agrees that the model has unrealistic elements, but that "the model is much discussed in the relevant public choice literature, and there are certain aspects of the model that can be observed in the real world" (Tullock 1983b: 24). His model examines transfers among five agents, each of whom votes on redistribution policy. "The net result," he suggests, is "that everybody has the same amount of money that they entered with" (Tullock 1983b: 28). In real life, this result "would not be expected with only five voters," but with "a larger number of voters it is not unrealistic" (Tullock 1983b: 28). An agent-based model would put Tullock's claim to the test.

An important element of agent-based modeling is emergence, another feature of Tullock's research. In "Proportional representation," Tullock (1967b) discusses how proportional representation arose in France. The French system "differs from the Anglo-Saxon system simply in that if no candidate gets a majority in the first balloting, a runoff is held in a few weeks in which only a plurality is necessary for election. These simple rules, together with the French talent for intrigue, have led to a functioning proportional representation system" (Tullock 1967b: 148). In other words, a system "approximating proportional representation" has emerged from the combination of agent behaviors and institutional rules (Tullock 1967b: 148). Would such a system produce similar results in England? An agent-based model would allow exploration of the extent to which this system is applicable to other (non-French) distributions of agent behavior. In "The politics of persuasion," Tullock (1967c) examines the emergence of consensus. Tullock famously never admitted to have voted in national elections; the benefit was simply not worth the cost. But his extensive writings show that he did engage in persuasion. The ability to persuade others "is more likely to affect the outcome of [an] election than is voting," Tullock states, with the caveat that "[t]here will be very great variation" in individuals' abilities to persuade (Tullock 1967c: 124). To quantify the effects of this, he creates a ten-agent, ten-opinion model, and shows how majority opinion emerges. In this model, Tullock demonstrates two virtues of agent-based modeling. First, different distributions of agents can produce different patterns of outcomes. Second, the persuasion model can be combined with the earlier logrolling models to predict actual election results. Voters might, for example, be persuaded that ethanol subsidies are inefficient; yet these same voters elect representatives who support ethanol subsidies. The creation of a single model that explains both phenomena is a topic worthy of research.

Above all, Tullock never loses sight of the fact that agent behavior is the key to understanding any problem in public choice. When Tullock turns to the question of voter turnout, for example, he asks: "What is the payoff to the individual from voting?" (Tullock 1967d: 108). A voter for whom "the estimated [benefit] is less than the cost of becoming informed ... will not bother" (Tullock 1967d: 103). The resulting concept of rational ignorance explains how interest groups come to dominate the political process: "the politician, in making up programs to appeal to rationally ignorant voters, would be attracted by fairly complex programs which have a concentrated beneficial effect on a small group of voters and a highly dispersed injurious effect on a large group of voters" (Tullock 1967d: 103). Rational ignorance also explains why "charitable activity is likely to be badly designed and ineptly carried out" (Tullock 1966: 142). Donors "are apt to be exceptionally ill informed about the effects of their gifts" (Tullock 1966: 142) because "incentives for becoming well informed are extremely weak" (Tullock 1966: 146). Even the irrational can be rational in the right circumstance: in dangerous situations, "[y]ou might threaten your opponent with irrational behavior on your part and the threat is indeed rational" (Tullock 1972a: 66). Tullock then says that the "existence of this type of loss of temper then automatically produces a bargaining range" (Tullock 1972a: 66). (The behavior of Iraqi leader Saddam Hussein, prior to the US invasion of 2003, is a relevant case study: Hussein sought to produce, and indeed did produce for a period of time, a bargaining range.) In modeling revolutions, Tullock observes that "if we are attempting to study the dynamics of the revolution ... we should turn to examination of the utility calculus of the participants" (Tullock 1971: 94). Revolutions occur so infrequently, Tullock says, because a single participant's risk is likely to be greater than the potential reward. Yet revolutions do, in fact, occur—though, as Tullock suggests, most forced changes of government come through coups d'etat. In each of these instances, Tullock resorts to analysis of the individual. Tullock, it might be said, is a natural-born agent-based modeler.

6 Conclusion

The history of modeling in political science has paralleled the history of modeling in economics. In the beginning, economists and political scientists modeled by narrative. Then came the explosion of mathematical economics. During the postwar period, mathematical modeling came to dominate the economics profession; not long thereafter, the economic style of modeling "invaded" (Tullock 1972b: 317) other social sciences. The invasion of political science was led by Tullock, among others, who combined to produce "a sizeable literature by economists and use of recognizable economic methods in the field normally described as political science" (Tullock 1972b: 317). This approach to political science would come to be known—to Tullock's occasional dismay—as public choice. The world of economic modeling has, in the meantime, continued to evolve, and agent-based modeling offers the potential to address many problems in economics that hinge on heterogeneity, network effects, and emergence. The world of public choice modeling is not far behind, and one of its founders, Gordon Tullock, has already shown the way.

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Appendix: A model of supply and demand, in Python

```
#
# define number of buyers, sellers, periods, and iterations
#

B = 1000
S = 50
P = 200
I = 10


#
# consider Qs = a + bP, Qd = c - dP... give some
# concrete values for a, b, c, and d
#

a = 100
b = 10
c = 20
d = 7

#
# define a demand shock (a change to the value of c)
#
```

```
demand_shock = -3


#
# define the Walrasian auctioneer
#

class auctioneer:

  # calculate the Walrasian equilibrium price
  def calculate (self, buyers, sellers):

    # begin with price zero and change it incrementally
    price = 0.0
    increment = 100.0

    # see whether the equilibrium price is positive or negative
    supply = sum (map (lambda s: s.quantity (price), sellers))
    demand = sum (map (lambda b: b.quantity (price), buyers))
    positive = (supply < demand)

    # discover the equilibrium price
    while abs (supply - demand) > 0.0000001:
      if supply < demand:
        price += increment
        if not positive:
          increment /= 2
      else:
        price -= increment
        if positive:
          increment /= 2
      supply = sum (map (lambda s: s.quantity (price), sellers))
      demand = sum (map (lambda b: b.quantity (price), buyers))
    # save the equilibrium price for later
    self.equilibrium = price

  # return the Walrasian equilibrium price
  def price (self):
    return self.equilibrium


#
# create a Walrasian auctioneer
#

walras = auctioneer()

#
# run a model
#
```

```
def run (name, model):

  # execute the model multiple times
  results = [model() for i in range (I)]

# initialize totals
units = [0 for p in range (P)]
sales = [0 for p in range (P)]
inventory = [0 for p in range (P)]

# calculate totals
for r in results:
  for (p,u,s,i) in r:
    units[p] += u
    sales[p] += s
    inventory[p] += i

# calculate averages
for p in range (P):
  units[p] /= I
  sales[p] /= I
  inventory[p] /= I

# create the results file
file = open (name + ".csv", "w")

# write the header
file.write ("Period,Volume,AvgPrice,Inventory\n")

# write results
for p in range (P):
  file.write (%d,%f,%f,%f\n" %
    (p, units[p], units[p] and sales[p]/units[p], inventory[p]))

  # close the output file
  file.close()

#
# define the basic model
#

def model (buyer_factory, seller_factory):

  # we haven't seen any results yet
  results = []

  # create the buyers and sellers
  buyers = [buyer_factory() for b in range (B)]
  sellers = [seller_factory() for s in range (S)]

  # for each period...
```

```
  for period in range (P):

    # introduce a demand shock
    if period == int (P / 4):
      for b in buyers:
        b.shock (demand_shock)

    # allow the auctioneer to determine the equilibrium price
    walras.calculate (buyers, sellers)

    # engage in production
    for s in sellers:
      s.produce()

    # initialize running totals
    units = 0
    sales = 0

    # for each buyer...
    for b in buyers:

      # identify the seller offering the best deal
      quantity = 0
      for s in sellers:
        if min (b.quantity (s.price()), s.inventory) > quantity:
          quantity = min (b.quantity (s.price()), s.inventory)
          seller = s

      # buy from this seller
      if quantity:
        seller.sell (quantity)
        units += quantity
        sales += quantity * seller.price()

    # determine end-of-period inventory
    inventory = sum (map (lambda s: s.inventory, sellers))

    # save this period's results
      results.append ((period, units, sales, inventory))

    # return the results to the caller
    return results

#
# homogeneous buyers
#

class homogeneous_buyer:

    # initialize the buyer
    def __init__ (self):
```

```
      self.shift = 0

    # define demand quantity as a function of price
    def quantity (self, price):
      return max (c + self.shift - d * price, 0)

    # introduce a demand shock
    def shock (self, shock):
      self.shift = shock

#
# price-taking seller
#

class walrasian_seller:

    # initialize inventory to zero
    def __init__ (self):
      self.inventory = 0

    # define supply quantity as a function of price
    def quantity (self, price):
      return max (a + b * price, 0)

    # this seller offers the Walrasian price
    def price (self):
      return walras.price()

    # produce goods
    def produce (self):
      quantity = self.quantity (self.price())
      if self.inventory < quantity:
        self.inventory += (quantity - self.inventory)

    # sell goods
    def sell (self, quantity):
      self.inventory -= quantity

#
# learning seller
#

class learning_seller:

    # initialize inventory to zero and clear sales history
    def __init__ (self):
      self.inventory = 0
      self.prevsales = 0
      self.sales = 0

    # define supply quantity as a function of price
```

```
    def quantity (self, price):
      return max (a + b * price, 0)

    # offer the Walrasian price initially and then use history
    def price (self):
      return self.nextprice

    # produce goods
    def produce (self):
      if self.sales > 0 and self.prevsales > 0:
        self.nextprice *= (1.0 + (self.sales-self.prevsales) /
          self.prevsales)
      elif self.sales == 0 and self.prevsales > 0:
        self.nextprice *= 0.5
      else:
        self.nextprice = walras.price()
      quantity = self.quantity (self.nextprice)
      if self.inventory < quantity:
        self.inventory += (quantity - self.inventory)
      self.prevsales = self.sales
      self.sales = 0

    # sell goods
    def sell (self, quantity):
      self.inventory -= quantity
      self.sales += quantity

#
# invoke the models
#

run ("classical",
  lambda: model (
    lambda: homogeneous_buyer(),
    lambda: walrasian_seller()))

run ("learning",
  lambda: model (
    lambda: homogeneous_buyer(),
    lambda: learning_seller()))

# end of supply-demand.py
```

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