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Notes for: Mechanisms in Programs and Nature | Origins of Simple Behavior
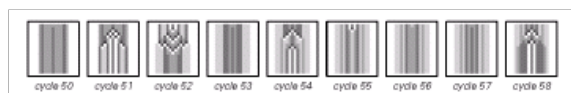
■ Self-organized criticality

The fact that in traditional statistical mechanics nesting had been encountered only at the precise locations of phase transitions led in the 1980s to the notion that despite its ubiquity in nature nesting must somehow require fine tuning of parameters. Already in the early 1980s, however, my studies on simple additive and other cellular automata (see page 26) had for example made it rather clear that this is not the case. But in the late 1980s it became popular to think that in many systems nesting (as well as the largely unrelated phenomenon of 1/f noise) might be the result of fine tuning of parameters achieved through some automatic process of self-regulation. Computer experiments on various cellular automata and related systems were given as examples of how this might work. But in most of these experiments mistakes and misinterpretations were found, and in the end little of value was learned about the origins of nesting (or 1/f noise). Nevertheless, a number of interesting systems did emerge, the best known being the idealized sandpile model from the 1987 work of Per Bak, Chao Tang and Kurt Wiesenfeld. This is a $k=8$ 2D cellular automaton in which toppling of sand above a critical slope is captured by updating an array of relative sand heights s according to the rule

$$SandStep[s\_]:= s + ListConvolve[\{\{0, 1, 0\}, \{1, -4, 1\}, \{0, 1, 0\}\},$$
$$UnitStep[s - 4], 2, 0]$$

Starting from any initial condition, the rule eventually yields a fixed configuration with all values less than 4, as in the picture below. (With an $n \times n$ initial block of 4's, stabilization typically takes about $0.4\, n^2$ steps.).

step 1   step 2   step 3   step 4   step 5   step 6   step 7   step 8   step 9   step 10

To model the pouring of sand into a pile one can consider a series of cycles, in which at each cycle one first adds 4 to the value of the center cell, then repeatedly applies the rule until a new fixed configuration *FixedPoint[SandStep, s]* is obtained. (The more usual version of the model adds to a random cell.) The picture below shows slices through the evolution at several successive cycles. Avalanches of different sizes occur, yielding activity that lasts for varying numbers of steps.

cycle 50   cycle 51   cycle 52   cycle 53   cycle 54   cycle 55   cycle 56   cycle 57   cycle 58

PAGE IMAGE

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- Nested lists
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The pictures at the top of the next page show some of the final fixed configurations, together with the number of steps needed to reach them. (The total value of s at cycle t is $4t$; the radius of the nonzero region is about $0.74\,Sqrt[t]$.) The behavior one sees is fairly complicated--a fact which in the past resulted in much confusion and some bizarre claims, but which in the light of the discoveries in this book no longer seems surprising.
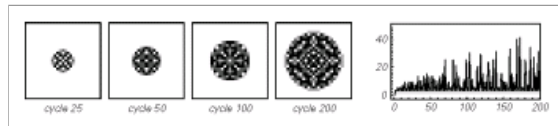
The system can be generalized to d dimensions as a $k=4d$ cellular automaton with $2d$ final values. The total value of s is always conserved. In 1D, the update rule is simply

    SandStep[s_] := s + ListConvolve[{1, -2, 1}, UnitStep[s - 2], 2, 0]

In this case the evolution obtained if one repeatedly adds to the center cell (as in the first picture below) is always quite simple. But as the pictures below illustrate, evolution from typical initial conditions yields behavior that often looks a little like rule 184. With a total initial s value of m, the number of steps before a fixed point is reached seems to increase roughly like m^2.
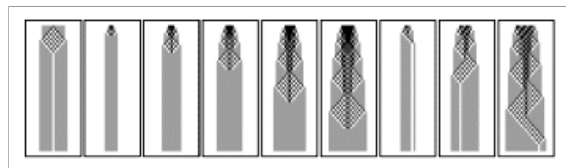
When $d>1$, more complicated behavior is seen for evolution from at least some initial conditions, as indicated above.