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MATH 4670
October 6, 2005

1 Section 2.4, Problem 7.a.

Using Newton's Method, we will find the solution accurate to within 10^{-5} for

$$\cos x + \sqrt{2} + x(\frac{x}{2} + \sqrt{2}) = 0$$

on the interval [-2, -1].

The derivative of the above function is

$$-\sin x + \sqrt{2} + x + \sqrt{2}$$

2 Source Code

The following C code can be used to find an accurate value:

```
#include <stdlib.h>
#include <math.h>

#define E 2.71828182846

double f(double x);
double fprime(double x);
void newton_method(double a, double b);

int main()
{
   newton_method(-2, -1);
   return 0;
}

double f(double x)
{
   return cos(x + sqrt(2)) + x * (x/2 + sqrt(2));
}

double fprime(double x)
{
   return -1 * sin(x + sqrt(2)) + x + sqrt(2);
}

void newton_method(double a, double b)
```

```
{
   double x[3];
   unsigned int k;
   x[0] = a;
   x[1] = b;

   printf("%d \t %5.20f \t %5.20f \n", 0, x[0], f(x[0]) );
   printf("%d \t %5.20f \t %5.20f \n", 1, x[1], f(x[1]) );

   for(k = 1; k < 30 && f(x[1]) > 0; k++ )
   {
        x[2] = x[1] - f(x[1])/fprime(x[1]);
        x[0] = x[1];
        x[1] = x[2];
        printf("%d \t %5.20f \t %5.20f \n", k+1, x[2], f(x[2]) );
   }
   printf("\n");
}
```

3 Results

	x	f(x)
0	-2.00000000000000000000	0.00485043299495568903
1	-1.00000000000000000000	0.00121955882844471120
2	-1.10385023330514142614	0.00038536837534329852
3	-1.18156580324967941387	0.00012184305074301082
4	-1.23978025051388041078	0.00003853594423272155
5	-1.28341070258907552493	0.00001219017769706179
6	-1.31612074463471984132	0.00000385654411772218
7	-1.34064788239584853358	0.00000122014495235613
8	-1.35904096139526520126	0.00000038604552672281
9	-1.37283481144235919302	0.00000012214437689409
10	-1.38317979437193305792	0.00000003864673868599
11	-1.39093836087224898002	0.00000001222797967797
12	-1.39675721371078265243	0.00000000386899319185
13	-1.40112132291876601009	0.00000000122417070084
14	-1.40439439171485758173	0.00000000038733473622
15	-1.40684918748150433743	0.00000000012255499399
16	-1.40869028068558432842	0.00000000003877713911
17	-1.41007109759894078138	0.00000000001226932927
18	-1.41110670697336448853	0.00000000000388205164
19	-1.41188340041647641954	0.00000000000122823165
20	-1.41246587010874602441	0.00000000000038860001
21	-1.41290264662980380450	0.00000000000012287453

22	-1.41322990353715871592	0.00000000000003885547
23	-1.41347484930564615269	0.00000000000001229030
24	-1.41365777988509377927	0.00000000000000383607
25	-1.41379184778032107594	0.00000000000000121290
26	-1.41388888125358191949	0.00000000000000031420
27	-1.41394396042770909006	0.00000000000000008191
28	-1.41396904007176194007	-0.00000000000000000314

4 Summary

The approximate value is -1.41394396042770909006.