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MATH 4670
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1 Section 2.4, Problem 7.a.

Using Newton's Method, we will find the solution accurate to within 10^{-5} for

$$e^{6x} + 3(\ln 2)^2 e^{2x} - (\ln 8)e^{4x} - (\ln 2)^3$$

on the interval [-1, 0].

The derivative of the above function is

$$6e^{6x} + 6(\ln 2)^2 e^{2x} - 4(\ln 8)e^{4x}$$

2 Source Code

The following C code can be used to an accurate value:

```
#include <stdlib.h>
#include <math.h>

#define E 2.71828182846

double f(double x);
double fprime(double x);
void newton_method(double a, double b);

int main()
{
   newton_method(-1, 0);
   return 0;
}

double f(double x)
{
   return pow(E, 6*x) + 3 * pow(log(2), 3) * pow(E, 2*x)
        - log(8) * pow(E, 4*x) - pow(log(2), 3) ;
}

double fprime(double x)
{
   return 6*pow(E, 6*x) + 6*pow(log(2), 2)*pow(E, 2*x) - 4*log(8)*pow(E, 4*x);
}
```

```
void newton_method(double a, double b)
{
   double x[3];
   unsigned int k;
   x[0] = a;
   x[1] = b;

   printf("%d \t %5.20f \t %5.20f \n", 0, x[0], f(x[0]) );
   printf("%d \t %5.20f \t %5.20f \n", 1, x[1], f(x[1]) );

   for(k = 1; k < 35 && f(x[1]) != f(x[0]); k++)
   {
        x[2] = x[1] - f(x[1])/fprime(x[1]);
        x[0] = x[1];
        x[1] = x[2];
        printf("%d \t %5.20f \t %5.20f \n", k+1, x[2], f(x[2]) );
   }
   printf("\n");
}
```

3 Results

	x	f(x)
0	-1.00000000000000000000	-0.23342224337486189301
1	0.00000000000000000000	-0.41339223770197691676
2	0.73172994978215033512	45.83165631529487882290
3	0.59740078551447139965	16.31375286902953547497
4	0.47653388692779696800	5.71735803639984574431
5	0.37481698992678053051	1.94597362843546006772
6	0.29913956941156655267	0.62250462172626019886
7	0.25413986586947495683	0.17537952558079544785
8	0.23542509541335426837	0.04087216141606431208
9	0.23026784555755044903	0.00820908566534783724
10	0.22918235949576021282	0.00156064307769926414
11	0.22897394396059850274	0.00029298160466423306
12	0.22893474337634733828	0.00005486645557528647
13	0.22892739968495590119	0.00001027002367937069
14	0.22892602498456340876	0.00000192219797395810
15	0.22892576768433359202	0.00000035976399265225
16	0.22892571952718404615	0.00000006733423768557
17	0.22892571051397969661	0.00000001260242016299
18	0.22892570882704893487	0.00000000235869651588
19	0.22892570851131927023	0.00000000044145842537
20	0.22892570845222667830	0.00000000008262390772

21	0.22892570844116683082	0.00000000001546379691
22	0.22892570843909687550	0.00000000000289462898
23	0.22892570843870940767	0.00000000000054128924
24	0.22892570843863696561	0.00000000000010230705
25	0.22892570843862328211	0.00000000000001898481
26	0.22892570843862072860	0.00000000000000416334
27	0.22892570843862017349	0.00000000000000016653
28	0.22892570843862014573	-0.00000000000000072164
29	0.22892570843862022900	0.00000000000000011102
30	0.22892570843862020125	-0.00000000000000077716
31	0.22892570843862031227	0.00000000000000094369
32	0.22892570843862017349	0.00000000000000016653
33	0.22892570843862014573	-0.00000000000000072164
34	0.22892570843862022900	0.00000000000000011102
35	0.22892570843862020125	-0.00000000000000077716

4 Summary

It is clearly shown to converge to approximately 0.22892570843862022900.