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MATH 4670
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1 Section 2.4, Problem 7.a.

Using Newton's Method, we will find the solution accurate to within 10^{-5} for

$$x^2 - 2xe^{-x} + e^{-2x} = 0$$

on the interval [0, 1].

The derivative of the above function is

$$2x + 2xe^{0x} - 2e^{-x} - 2e{-}2x$$

2 Source Code

The following C code can be used to an accurate value:

```
#include <stdlib.h>
#include <math.h>

#define E 2.71828182846

double f(double x);
double fprime(double x);
void newton_method(double a, double b);

int main()
{
   newton_method(0, 1);
   return 0;
}

double f(double x)
{
   return pow(x, 2) - 2 * x * pow(E, -x) + pow(E, -2 * x);
}

double fprime(double x)
{
   return 2 * x + 2 * x * pow(E, -x) - 2 * pow(E, -x) - 2 * pow(E, -2 * x);
}

void newton_method(double a, double b)
```

```
{
   double x[3];
   unsigned int k;
   x[0] = a;
   x[1] = b;

   printf("%d \t %5.20f \t %5.20f \n", 0, x[0], f(x[0]) );
   printf("%d \t %5.20f \t %5.20f \n", 1, x[1], f(x[1]) );

   for (k = 1; k < 30 && f(x[1]) != f(x[0]); k++ )
   {
        x[2] = x[1] - f(x[1])/fprime(x[1]);
        x[0] = x[1];
        x[1] = x[2];
        printf("%d \t %5.20f \t %5.20f \n", k+1, x[2], f(x[2]) );
   }
   printf("\n");
}
```

3 Results

```
           x                               f(x)
----------------------------------------------------------------
0        0.00000000000000000000          1.00000000000000000000
1        1.00000000000000000000          0.39957640089389145910
2        0.76894142136992604808          0.09329234606753285253
3        0.66458978660854051146          0.02253158719989264980
4        0.61503323767039574932          0.00553689448825028796
5        0.59088381393652311679          0.00137242352363842190
6        0.57896292997593168383          0.00034164377965878456
7        0.57304051621843821174          0.00008522899990143218
8        0.57008876253250717614          0.00002128455825967102
9        0.56861524224058546118          0.00000531830632283237
10       0.56787907038817608818          0.00000132922262713331
11       0.56751113142978193427          0.00000033226142520659
12       0.56732719867933179003          0.00000008305982807100
13       0.56723524148471804462          0.00000002076426608433
14       0.56718926518228773848          0.00000000519098014573
15       0.56716627760481508336          0.00000000129773419788
16       0.56715478395990337468          0.00000000032443220332
17       0.56714903717346765166          0.00000000008110795369
18       0.56714616378698512467          0.00000000002027700230
19       0.56714472709269925232          0.00000000000506922282
20       0.56714400874722503687          0.00000000000126731958
21       0.56714364956870455536          0.00000000000031674663
```

22	0.56714347002179665846	0.00000000000007926992
23	0.56714338017025789362	0.00000000000001981748
24	0.56714333522171944679	0.00000000000000494049
25	0.56714331277626317540	0.00000000000000116573
26	0.56714330216534181162	0.00000000000000033307
27	0.56714329639717031561	0.00000000000000005551
28	0.56714329450965572654	0.00000000000000000000
29	0.56714329450965572654	0.00000000000000000000

4 Summary

It is clearly shown to converge to approximately 0.56714329450965572654.