

EMET1001 Tutorial — Week 9.

Exercise 8.1. For the function $f(x, y) = 2xy^2$, find

$$(a) \quad \frac{f(x, y) - f(x, y_0)}{y - y_0}$$
$$(b) \quad \lim_{y \rightarrow y_0} \frac{f(x, y) - f(x, y_0)}{y - y_0}$$

The two expressions in (a) and (b) correspond to concepts from univariate calculus that you are already familiar with. What are those?

Exercise 8.2. Given the function with two-dimensional domain $f(x, y)$, the second-order partial derivatives are defined as follows:

$$f_{xx} = \frac{\partial^2 f(x, y)}{\partial x^2} = \frac{\partial}{\partial x} \left(\frac{\partial f(x, y)}{\partial x} \right) \quad f_{xy} = \frac{\partial^2 f(x, y)}{\partial y \partial x} = \frac{\partial}{\partial y} \left(\frac{\partial f(x, y)}{\partial x} \right)$$
$$f_{yx} = \frac{\partial^2 f(x, y)}{\partial x \partial y} = \frac{\partial}{\partial x} \left(\frac{\partial f(x, y)}{\partial y} \right) \quad f_{yy} = \frac{\partial^2 f(x, y)}{\partial y^2} = \frac{\partial}{\partial y} \left(\frac{\partial f(x, y)}{\partial y} \right)$$

Find the indicated second-order partial derivatives:

$$(a) \quad f(x, y) = \ln 2 \qquad f_{xx}(x, y)$$
$$(b) \quad f(x, y) = x^2 + 9y^2 - 4 \qquad f_{yy}(x, y)$$
$$(c) \quad f(x, y) = e^{xy^2} \qquad f_{xy}(x, y)$$
$$(d) \quad f(x, y) = \frac{3 \ln x}{y^2} \qquad f_{xx}(x, y)$$

Exercise 8.3. In economics, a Cobb–Douglas production function is a function of the form:

$$Q(L, K) = AL^\alpha K^\beta \qquad \text{with } \alpha + \beta = 1,$$

and A , α , and β are given constants. Economists use this function to describe the number of units Q produced from the utilization of L units of labor and K units of capital (e.g., tools, machinery, buildings). For a Cobb–Douglas production function, show that

$$(a) \quad \partial Q / \partial L = \alpha Q / L \qquad (b) \quad \partial Q / \partial K = \beta Q / K \qquad (c) \quad L \frac{\partial Q}{\partial L} + K \frac{\partial Q}{\partial K} = Q.$$

Related exercises in the textbook you should study, include (but are not limited to):

Exercises 15-1 — Problems 1-14, 25-30, 37-40

Exercises 15-2 — Problems 1-88

The tutors at the EMET1001 help desk are happy to help, if you have any questions.