

## EMET1001 Tutorial — Week 11.

**Exercise 10.1.** A firm's production function is  $Q = K^{0.5}L^{0.5}$ .

- (a) Find the marginal products of capital and labor. Are they always positive?
- (b) Find the equations of the isoquants for 10 units and for 100 units of output.
- (c) By implicit differentiation, find the slope of any isoquant and show that this slope is given by the ratio of marginal products of capital and labor. Is the slope always negative?
- (d) Suppose that, from any initial position, the labor input increases by a small amount  $dL$ . What change in the capital input is necessary to restore output to its initial level? Explain your answer in words.

**Exercise 10.2.** Use the Method of Lagrange multipliers (all 4 steps as discussed in the lecture) to find the maximum/minimum of  $f(x, y) = x^2 - y^2$  subject to  $x^2 + y^2 = 25$ .

**Exercise 10.3.** A firm's production function is  $Q = K^{0.4}L^{0.5}$ . The firm is perfectly competitive and factor prices are  $r = \$4.00$  (capital) per hour and  $w = \$5.00$  (labor) per hour. The market price is  $p = \$20$ . Total revenue equals  $p \cdot Q$ .

- (a) Show that the most profitable output is  $Q = 512$ . Find the profit at that level.
- (b) How would the most profitable output change if  $p$  rose from 20 to 22 (with input prices unchanged)? What does this tell you about the firm's supply function? Does supply appear to be elastic or inelastic?
- (c) Show that cost minimization requires  $K = L$ .
- (d) Using (b), find total cost as a function of output. (Hint: eliminate either  $K$  or  $L$  from the TC function, then substitute  $Q$  in place of  $K$  or  $L$ .) What can you deduce about marginal cost and average cost, as a function of output? How does marginal cost compare with marginal revenue?

*Related exercises in the textbook you should study, include (but are not limited to):*

*Exercises 15-4 — Problems 1-28*

*The tutors at the EMET1001 help desk are happy to help, if you have any questions.*