Jürgen Meinecke
Research School of Economics
Australian National University

Answer Key for Practice Final Exam

Note: Some solutions provided below are detailed and comprehensive. Many solutions, on the other hand, only give the end result. In the actual exam you will have to provide detailed and comprehensive solutions. You obtain no partial credit for merely stating the end result (even if it is correct). May contain errors and typos.

1. $-\frac{8}{x \cdot x_0}$

2. $\begin{bmatrix} -6 & 3 \\ -4 & 5 \end{bmatrix}$

3. Suppose the statement in the question is true. Then, by the definition of limits, there are positive numbers δ_1 and δ_2 such that

$$|f(x) - L_i| < \epsilon \qquad \text{if } 0 < |x - x_0| < \delta_i, \qquad i = 1, 2,$$

for $\epsilon > 0$. Letting $\delta = min(\delta_1, \delta_2)$,

$$\begin{aligned} |L_1 - L_2| &= |L_1 - f(x) + f(x) - L_2| \\ &\leq |L_1 - f(x)| + |f(x) - L_2| \\ &< 2\epsilon, \end{aligned}$$

if $0 < |x - x_0| < \delta$. We have now shown that $|L_1 - L_2| < 2\epsilon$ for any positive real number ϵ. Hence, $L_1 = L_2$.

4. (a) $G/Y = 0.4/0.5 = e^{0.01x}$ where x is time. Therefore, $x = \ln(1.25)/0.01$.
   (b) $G = G_0(1 + 0.03)^x$ and $Y = Y_0(1 + 0.02)^x$ with $G_0/Y_0 = 0.4$. Find x such that $G/Y = 0.5$ yields $x = \frac{\log 1.25}{\log 1.03 - \log 1.02}$

5. (a) $L = 50$ and $K = 30$
   (b) Modify Lagrangian and divide V_L by V_K to get $\frac{0.5K^{0.4}L^{-0.5}}{0.4K^{-0.6}L^{0.5}} = 3/4$. In the last equation, the left–hand side is equal to MPL/MPK (slope of isoquant) while the right–hand side is equal to w/r (slope of budget constraint).

6. See lecture notes.

7. (a) Minimum at zero. Decreasing on $(-\infty, 0)$, increasing on $(0, \infty)$.
   (b) Minimum at zero. Decreasing on $(-\infty, 0)$, increasing on $(0, \infty)$.

8. Saddle points at $(-3,1)$ and $(3,1)$. For the point $(0,0)$, no statement can be made regarding minimum, maximum or saddle point.

9. First, checking at zero. We have

$$|f(x)-f(0)| = |\sqrt{x}-0| = \sqrt{x} < \epsilon,$$

if $0 \leq x < \epsilon^2$, so that $\lim_{x\to 0+} = f(0)$ and thus the limit at $x = 0$ coincides with the function value and therefore the function is continuous at zero.

If $x_0 > 0$ and $x \geq 0$, then

$$\begin{aligned} |f(x)-f(x_0)| = |\sqrt{x}-\sqrt{x_0}| &= \frac{|\sqrt{x}-\sqrt{x_0}|\cdot|\sqrt{x}+\sqrt{x_0}|}{|\sqrt{x}+\sqrt{x_0}|} = \frac{|x-x_0|}{\sqrt{x}+\sqrt{x_0}} \\ &\leq \frac{|x-x_0|}{\sqrt{x_0}} < \epsilon, \end{aligned}$$

if $|x-x_0| < \epsilon\sqrt{x_0}$. Therefore, there exists a $\delta = \epsilon\sqrt{x_0} > 0$ such that $|f(x)-f(x_0)| < \epsilon$ whenever $|x-x_0| < \delta$ and hence the function is continuous for $x > 0$.

Alltogether, it follows that the function is continuous on $[0,\infty)$.

10. Minors: $|M_{11}| = 2, |M_{12}| = -2, |M_{13}| = -2, |M_{21}| = 11, |M_{22}| = 4, |M_{23}| = -6, |M_{31}| = 25, |M_{32}| = 10$, and $|M_{33}| = -10$
Cofactors: $C_{11} = 2, C_{12} = 2, C_{13} = -2, C_{21} = -11, C_{22} = 4, C_{23} = 6, C_{31} = 25, C_{32} = -10$, and $C_{33} = -10$
For the other matrix:
Minors: $|M_{11}| = 7, |M_{12}| = 1, |M_{13}| = -1, |M_{21}| = 3, |M_{22}| = 1, |M_{23}| = 0, |M_{31}| = -3, |M_{32}| = 0$, and $|M_{33}| = 1$
Cofactors: $C_{11} = 7, C_{12} = -1, C_{13} = -1, C_{21} = -3, C_{22} = 1, C_{23} = 0, C_{31} = -3, C_{32} = 0$, and $C_{33} = 1$

11. Nonsingular (determinant is 24)
For the other three matrices: Singular/ Singular/ Nonsingular (determinant is 80)

12. Not the same.

13. $0 = \alpha x^{\alpha-1}y^{\beta}dx + \beta y^{\beta-1}x^{\alpha}dy$ implies that $\frac{dy}{dx} = -\frac{\alpha y}{\beta x}$.

14. $C'(x) = -x^2 - 4x + 5$ and $C''(x) = -2x - 4$.
Extrema: At $x = 1$ it's a maximum because $C''(1) < 0$. At $x = -5$ it's a minimum because $C''(-5) > 0$.
Increasing/decreasing: Rewrite $C'(x) = -(x-1)(x+5)$ and consider two cases

   (i) $x - 1 > 0$ and $x + 5 > 0$ which implies that $x > 1$.

   (ii) $x - 1 < 0$ and $x + 5 < 0$ which implies that $x < -5$.

   Therefore, decreasing over $(1,\infty)$ and $(-\infty,-5)$. Increasing over $(-5,1)$.

15. We get $\lambda = -1/4$ and $y = 1/4$ and $x = 0$ and $z = 5/8$.