EMET1001 Tutorial — Week 10.

Exercise 9.1. An Economic Application of Multivariate Functions

A firm has the Cobb–Douglas production function

$$Q = 100K^{0.25}L^{0.75}, \qquad K, L \geq 0$$

where Q is weekly production output (in tonnes) and K and L are weekly inputs of machine–hours and worker–hours.

(a) (i) Show that the equation of the isoquant for $Q = 100$ is given by $K = 1/L^3$.

(ii) By differentiation, show that the isoquant slopes down and is convex.

(iii) Sketch a graph of the isoquant.

(iv) Give an economic interpretation to the isoquant's slope and curvature.

(v) Would other isoquants for this production function have similar shape?

(b) Treat output Q as variable, with $Q > 0$.

(i) By differentiation, find the marginal products of labor and capital.

(ii) Show that they are positive at every level of output.

(iii) Would you expect this to be true for every production function?

(iv) Examining the signs of the direct second derivatives, show that the marginal product of labor diminishes as the labor input increases (holding capital input constant); and that the marginal product of capital diminishes as the capital input increases (holding labor input constant).

(v) Sketch the graphs of the marginal products of labor and capital.

(c) (i) How would you explain the law of diminishing marginal productivity (DMP) implied by part (b) above? Would you expect it to be true for all production functions and at every level of output?

(ii) What does DMP imply about the graph of the average product of labor (APL) and of capital (APK)?

(iii) Check your answers to (ii) by deriving the equations of the APL and APK and sketching their graphs.

(d) (i) Examining the second order cross partial derivatives of the production function, show that an increase in the capital input increases the marginal product of labor, and vice versa.

(ii) Provide economic intuition. Do you expect this to be true in general?

(e) (i) Derive the equation of the short–run production function when the capital input is fixed at 160,000 and sketch its graph.
(ii) How does the shape of this graph relate to your findings in (b) above?
(iii) Repeat for capital input fixed at 810,000 and sketch graph.
(iv) How does your answer to (d) above reveal itself in these graphs?

(f) With the labor input fixed at 160,000, derive the relationship between capital input and output, and sketch its graph. (This is the quasi–short–run production function and is sometimes called the total product of capital.)

Related exercises in the textbook you should study, include (but are not limited to):
Exercises 15-3 — Problems 1-32

The tutors at the EMET1001 help desk are happy to help, if you have any questions.