EMET1001 Tutorial — Week 12.

Digression: Number of solutions for systems of linear equations. In the lecture we said that a system of linear equations has either one solution, no solution, or infinitely many solutions. Here is how those cases occur in Gauss–Jordan elimination:

(i) If you arrive at a contradiction in the Gauss–Jordan elimination process than the system has no solutions.

(ii) If the number of leftmost 1's in a reduced augmented coefficient matrix (reduced form) is less than the number of variables in the system and there are no contradictions, then the system is dependent and has infinitely many solutions.

(iii) If cases (i) and (ii) do not apply then the system has one solution.

Exercise 11.1. For the following two–equation systems, write down the augmented coefficient matrix. Then by doing only one of the three elementary row operations show for which value/s of k the system has either no solution or infinitely many solutions.

(a)
$$\begin{aligned} x_1 + kx_2 &= 3 \\ 2x_1 + 4x_2 &= 8 \end{aligned}$$

(b)
$$\begin{aligned} x_1 + 2x_2 &= 4 \\ -2x_1 + kx_2 &= -8 \end{aligned}$$

Exercise 11.2. The following equation system has the compact matrix representation $AX = B$, where $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$. Write down the augmented coefficient matrix, bring into reduced form and determine the solution for x_1, x_2, x_3, and x_4.

$$\begin{aligned} 2x_1 + 4x_2 + 5x_3 + 4x_4 &= 8 \\ x_1 + 2x_2 + 2x_3 + x_4 &= 3 \end{aligned}$$

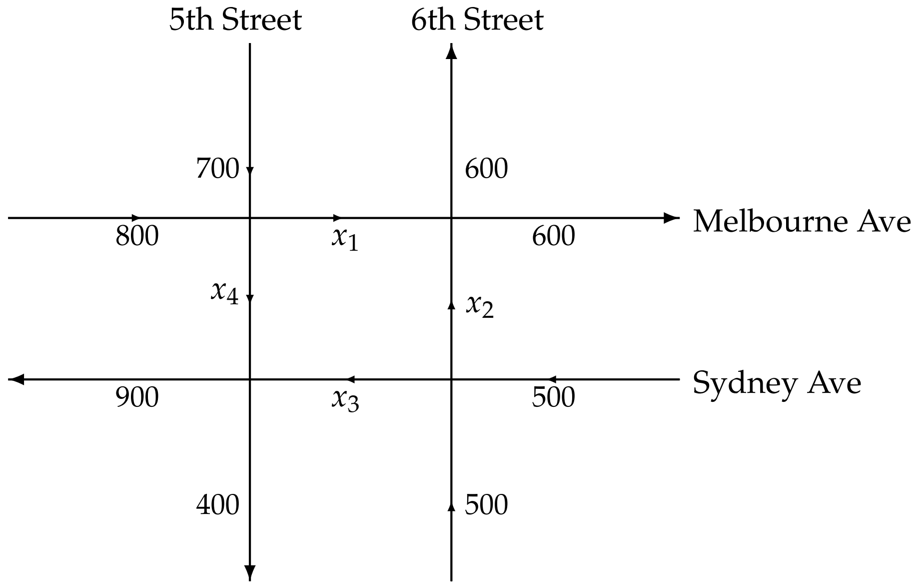
Exercise 11.3. The rush–hour traffic flow for a network of four one–way streets in a city is shown in the figure below. The numbers next to each street indicate the number of vehicles per hour that enter and leave the network on that street. The variables x_1, x_2, x_3 and x_4 represent the flow of traffic between the four intersections in the network.

(a) For a smooth traffic flow, the number of vehicles entering each intersection should always equal the number leaving. For example, since 1,500 vehicles enter the intersection of 5th Street and Melbourne Avenue each hour and $x_1 + x_4$ vehicles leave this intersection, we need $x_1 + x_4 = 1,500$ in order to have smooth traffic. Find the equations determined by the traffic flow at each of the other three intersections.

(b) Find the solution to the system in part (a). (How many solutions does this system have? Use the results from the 'Digression' at the beginning of this handout.)

(c) What is the maximum number of vehicles that can travel from Melbourne Avenue to Sydney Avenue on 5th Street? What is the minimum number?

(d) If traffic lights are adjusted so that 1,000 vehicles per hour travel from Melbourne Avenue to Sydney Avenue on 5th Street, determine the flow around the rest of the network.



Related exercises in the textbook you should study, include (but are not limited to):

Exercises 4.2 — Problems 13-74

Exercises 4.3 — Problems 1-90

The tutors at the EMET1001 help desk are happy to help, if you have any questions.