THE AUSTRALIAN NATIONAL UNIVERSITY

Second Semester PRACTICE Final Examination – November, 2013

Foundations of Economic and Financial Models

(EMET 1001/7001)

Reading Time: 10 Minutes
Writing Time: 90 Minutes
Permitted Materials: Non-programmable Calculator

Instructions:

- This handout of exam questions contains 4 pages (including cover page) with 15 exam questions. Make sure you are not missing any pages!
- Answer **ALL** questions of this handout in the script book provided to you.
- Show your work, provide full and mathematically rigorous derivations! No partial credit will be given for merely stating final results (even if they are correct).
- Unless otherwise stated, when determining local extrema, make sure to also show which ones are minima and which ones are maxima.
- You may not use L'Hospital's rule in solving questions.
- Cheat sheets are not permitted.
- Anything that is not a non-programmable calculator is not permitted. These include, but are not limited to: mobile phones, tablet computers, abacuses.
- Return this handout to the invigilators at the end of the exam.
- Total marks: 100. Good luck!

Note: The following set of questions covers material from weeks 1 through 13 of the lecture (weeks 2 through 13 of the tutorials). No particular emphasis is put on any particular question/topic for any particular purpose. No claim on comprehensiveness and exhaustiveness of questions/topics covered is made. You should be aware that the actual exam will contain different questions. The priority given to different questions/topics in the exam may be different to the priority these same questions/topics receive here.

1. [4 marks] For the following function, determine the difference quotient: $f(x) = \frac{x+8}{x}$. Simplify as much as possible.

2. [6 marks] Let

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 & 0 & -1 \\ 2 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}, \qquad C = \begin{bmatrix} 1 & 0 \\ 2 & -1 \\ 0 & 1 \end{bmatrix},$$

   and compute $A(B')^2C$.

3. [8 marks] Let $f : \mathbb{R} \to \mathbb{R}$ be a function such that $\lim_{x \to x_0} f(x)$ exists. Prove that the limit is unique; that is, if $\lim_{x \to x_0} f(x) = L_1$ and $\lim_{x \to x_0} f(x) = L_2$ then $L_1 = L_2$.

4. [6 marks] In a certain economy the ratio of government expenditure to GDP is currently 40%. In the future, government expenditure is expected to grow at 3% per year and GDP is expected to grow at 2% per year. Calculate in how many years from now the ratio of government expenditure to GDP will reach 50%, assuming

   (a) continuous growth.

   (b) growth in annual jumps.

5. [8 marks] A firm's production function is $Q = K^{0.4}L^{0.5}$, where Q is production output and K and L are inputs of machine–hours and worker–hours. The firm is perfectly competitive and factor prices are $r = \$4$ per hour (for K) and $w = \$3$ per hour (for L). Total cost per hour are defined as $TC(K, L) = r \cdot K + w \cdot L$.

   (a) The firm operates on a budget of $270 per hour, i.e. total cost equal $270 per hour. Using the Lagrange multiplier method, find the levels for K and L that maximize hourly output, given the budget constraint.

   (b) Show that to achieve the maximum output (at a budget constraint of $270 per hour) the firm must choose values for L and K such that the slope of the isoquant equals the slope of the budget constraint.

6. [6 marks] Give a mathematically rigorous definition of an arithmetic sequence. Using that definition, give a mathematically rigorous definition of an arithmetic series.

7. [6 marks] Let $f(x) = x^4 + kx^2$, where k is some constant. Discuss the extrema and curvature of the function for

   (a) $k > 0$

   (b) $k = 0$.

8. [6 marks] For the function $z = f(x, y) = 2x^2 - 2x^2y + 6y^3$, find all local extrema and saddle points!

9. [8 marks] Let $f(x) = \sqrt{x}$ for $0 \leq x < \infty$. Is this function continuous?

10. [8 marks] Find all the minors and cofactors (signed minors) of the following matrix. (Note: In the lecture during week 15 we learned about the cofactors of a 3×3 matrix. We derived the three cofactors A_{11}, A_{12}, A_{13} of the minors $|M_{11}|, |M_{12}|, |M_{13}|$ that correspond to the sub–matrices M_{11}, M_{12}, M_{13}. There are in fact six more cofactors: $A_{21}, A_{22}, A_{23}, A_{31}, A_{32}$, and A_{33} that are based on the respective minors $|M_{21}|, |M_{22}|, |M_{23}|, |M_{31}|, |M_{32}|$, and $|M_{33}|$. Make sure to derive all 9 minors and cofactors!)

$$\begin{bmatrix} 2 & 4 & 1 \\ 4 & 3 & 7 \\ 2 & 1 & 3 \end{bmatrix}.$$

(*Here's another matrix for you to practice with:*)

$$\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}.$$

11. [6 marks] Check whether the following matrix is singular:

$$\begin{bmatrix} 4 & 0 & 1 \\ 19 & 1 & -3 \\ 7 & 1 & 0 \end{bmatrix}.$$

(*Here are some more matrices for you to practice with:*)

$$\begin{bmatrix} 4 & -2 & 1 \\ -5 & 6 & 0 \\ 7 & 0 & 3 \end{bmatrix}, \quad \begin{bmatrix} 7 & -1 & 0 \\ 1 & 1 & 4 \\ 13 & -3 & -4 \end{bmatrix}, \quad \begin{bmatrix} -4 & 9 & 5 \\ 3 & 0 & 1 \\ 10 & 8 & 6 \end{bmatrix}.$$

12. [6 marks] Let

$$A = \begin{bmatrix} 2 & 1 \\ 5 & 1 \end{bmatrix} \qquad\qquad B = \begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix}.$$

Check whether $A^{-1}B^{-1} = (AB)^{-1}$.

13. [8 marks] Use the total differential to find the derivative $\frac{dy}{dx}$ of the following implicit function:

$$100 = x^{\alpha}y^{\beta}.$$

14. [6 marks] Determine for what values of x the following function is increasing or decreasing and where maxima or minima occur.

$$C(x) = -\frac{x^3}{3} - 2x^2 + 5x - 2.$$

15. [8 marks] Find the values of x, y, and z for which the following function has an extremum (minimum/maximum). Use the Lagrange multiplier method.

$$f(x,y,z) = 2x^2 + xy + y^2 + z \qquad\qquad \text{such that } x + 2y + 4z = 3.$$