Homework #4

Digital System Design 2022 Spring

DUE : 2022-05-02

total 104 pts = maximum 100 pts + 4 bonus pts

Extension limit = { `png` , `jpg` , `heic` , `zip` , `pdf` }

You must hand your answer in at the board before due time (2022-05-02 11:00 AM KST).

The problems start from the next page.

Name

김 주 은

In Korean

Student ID

20210774

8 digits

1. Obtain the 1's and 2's complements of the following binary numbers: (18pts = 6pts each)

   (a) 11100010

   (b) 10111101

   (c) 11000011

(a) 1's complement : 00011101

    2's complement : 00011110

(b) 1's complement : 01000010

    2's complement : 01000011

(c) 1's complement : 00111100

    2's complement : 00111101

2. Find the 9's and the 10's complements of the following decimal numbers: (16pts = 8pts each)

(a) 65,234,035

(b) 87,000,367

(a) 9's complement : 34765964

10's complement : 34765965

```
  99999999
- 65234035
----------
  34765964
```

(b) 9's complement : 12999632

10's complement : 12999633

```
  99999999
  87000367
----------
  12999632
```

3. Add the following numbers in binary using 2's complement to represent negative numbers. Use a word length of 5 bits including sign and indicate if an overflow occurs.
(15pts = 5pts each)

(a) $(-9) + (-11)$

(b) $(-9) + (-7)$

(c) $(-9) + 11$

9: 01001
-9: 10111
11: 01011
-11: 10101
7: 00111
-7: 11001

(a) -9 / -11 / -20

overflow!
01111
10111
10101
101100 ⇒ +12 ≠ −20.

(b) -9 / -7 / -16

overflow 발생 X.
11111
10111
11001
10000 ⇒ −16

(c) -9 / 11 / $+2$

overflow 발생 X
11111
10111
01011
00010 ⇒ +2

4. Repeat question 3 using 1's complement to represent negative numbers. (15pts = 5pts each)

9: 01001
-9: 10110
11: 01011
-11: 10100
7: 00111
-7: 11000

(a) -9, -11, -20

overflow!

$$\begin{array}{r} 10110 \\ 10100 \\ \hline 101010 \\ \curvearrowright\ 1 \\ \hline 01011 \end{array}$$

8 4 2 1

$01011 \Rightarrow \boxed{+11} \neq -20$

(b) -9, -7, -16

overflow!

$$\begin{array}{r} 10110 \\ 11000 \\ \hline 101110 \\ \curvearrowright\ 1 \\ \hline 01111 \end{array}$$

$01111 \Rightarrow \boxed{15} \neq -16$

(c) -9, 1, $+2$

overflow 상관X

$$\begin{array}{r} 10110 \\ 01011 \\ \hline 100001 \\ \curvearrowright\ 1 \\ \hline 00010 \end{array}$$

$00010 \Rightarrow \boxed{2}$

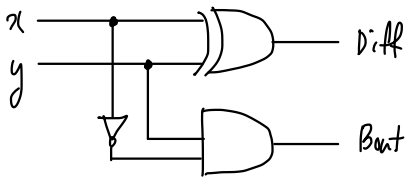
5. (a) Design a half-subtractor circuit with inputs x and y and outputs $Diff$ and B_{out}. The circuit subtracts the bits $x - y$ and places the difference in D and the borrow in B_{out}.
(b) Design a full-subtractor circuit with three inputs x, y, B_{in} and two outputs $Diff$ and B_{out}. The circuit subtracts $x - y - B_{in}$, where B_{in} is the input borrow, B_{out} is the output borrow, and $Diff$ is the difference. (20pts = 10pts each)

(a)

x	y	Diff	Bout
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

$Diff = x'y + xy' = x \oplus y$

$B_{out} = x'y$



(b)

x	y	Bin	Diff	Bout
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

<Diff>

x \ yBin	00	01	11	10
0	0	1	0	1
1	1	0	1	0

↳ $xy'Bin' + x'y'Bin + xyBin + x'yBin'$

$= (xy + x'y')Bin + (xy' + x'y)Bin' = (x \oplus y)'Bin + (x \oplus y)Bin'$

$= x \oplus y \oplus Bin$

$\Rightarrow Diff = x \oplus y \oplus Bin$

<Bout>

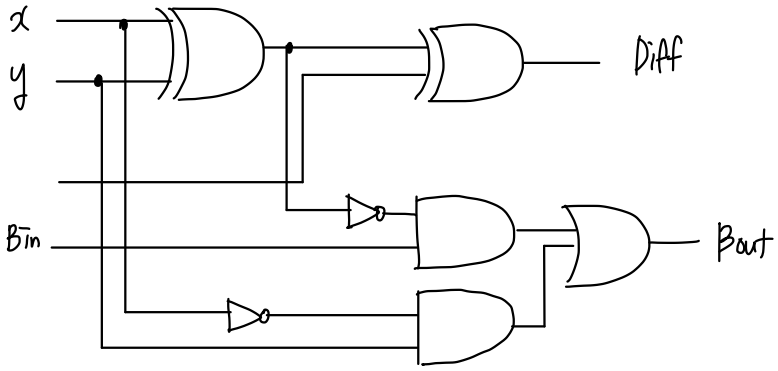
x \ yBin	00	01	11	10
0	0	1	1	1
1	0	0	1	0

$x'Bin + x'y + yBin$

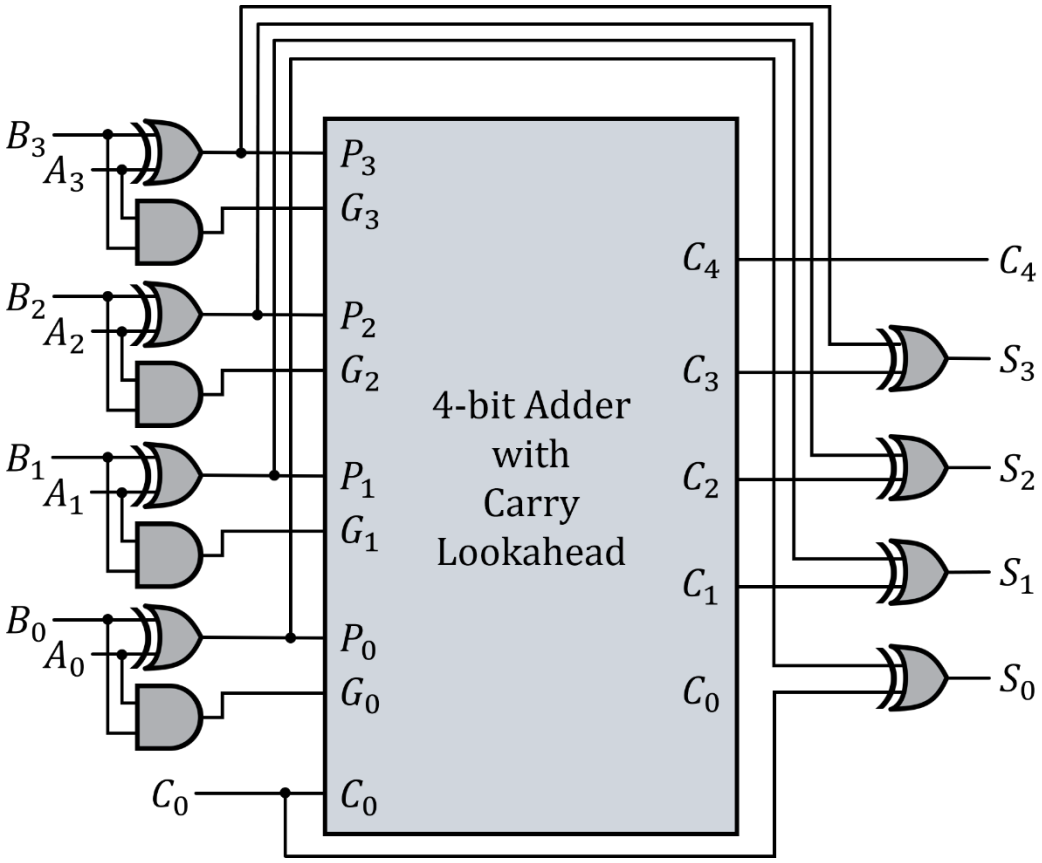
$= x'(y'+y)Bin + x'y + (x'+x)yBin$

$= Bin(x'y' + xy + x'y) + x'y = Bin(x'y' + xy) + x'y(1 + Bin)$

$= Bin(x \oplus y)' + x'y$



6. Derive the two-level Boolean expression for the output carry C_4 shown in the lookahead carry generator of below. (20pts)



$P_i = A_i \oplus B_i$

$G_i = A_i B_i$

$S_i = A_i \oplus B_i \oplus C_i = P_i \oplus C_i$

$C_{i+1} = A_i B_i + C_i(A_i \oplus B_i) = G_i + P_i C_i$

$C_1 = G_0 + P_0 C_0$

$C_2 = G_1 + P_1 C_1 = G_1 + P_1 G_0 + P_1 P_0 C_0$

$C_3 = G_2 + P_2 C_2 = G_2 + P_2 G_1 + P_2 P_1 G_0 + P_2 P_1 P_0 C_0$

$C_4 = G_3 + P_3 C_3 = \boxed{G_3 + P_3 G_2 + P_3 P_2 G_1 + P_3 P_2 P_1 G_0 + P_3 P_2 P_1 P_0 C_0}$

End of the Homework #4