Homework #2

Digital System Design 2022 Spring

DUE : 2022-04-04

total 103 pts = maximum 100 pts + 3 bonus pts

Extension limit = { png , jpg , heic , zip , pdf }

You must hand your answer in at the board before due time (2022-04-04 11:00 AM KST).

The problems start from the next page.

Name

김 주 은

In Korean

Student ID

20210914

8 digits

1. Obtain the truth table for the following four-variable functions, and express each function in sum-of-minterms and product-of-maxterms form: (8pts = 4pts each)
   (a) $(ac + b)(ab + d)$
   (b) $(b + c'd')(a + bc')$

(a)

a	b	c	d	(ac+b)(ab+d)	
0	0	0	0	0	M_0, M_0
0	0	0	1	0	m_1, M_1
0	0	1	0	0	m_2 M_2
0	0	1	1	0	m_3 M_3
0	1	0	0	0	m_4 M_4
0	1	0	1	1	m_5
0	1	1	0	0	m_6
0	1	1	1	1	m_7
1	0	0	0	0	m_8
1	0	0	1	0	m_9
1	0	1	0	0	m_{10}
1	0	1	1	1	m_{11}
1	1	0	0	1	m_{12}
1	1	0	1	1	m_{13}
1	1	1	0	1	m_{14}
1	1	1	1	1	m_{15}

$F(a.b.c.d)$
$= m_5 + m_7 + m_{11} + m_{12} + m_{13} + m_{14} + m_{15}$
$= \sum m(5.7.11.12.13.14.15)$
$= a'bc'd + a'bcd + ab'cd + abc'd'$
$+ abc'd + abcd' + abcd$

$F(a.b.c.d)$
$= M_0 M_1 M_2 M_3 M_4 M_6 M_8 M_9 M_{10}$
$= \Pi M(0.1.2.3.4.6.8.9.10)$
$= (a+b+c+d)(a+b+c+d')(a+b+c'+d)$
$(a+b+c'+d')(a+b'+c+d)(a+b'+c'+d)$
$(a'+b+c+d)(a'+b+c+d')(a'+b+c'+d)$

(b)

a	b	c	d	(b+c'd')(a+bc')	
0	0	0	0	0	M_0, M_0
0	0	0	1	0	m_1, M_1
0	0	1	0	0	m_2 M_2
0	0	1	1	0	m_3 M_3
0	1	0	0	1	m_4 M_4
0	1	0	1	1	m_5
0	1	1	0	0	m_6
0	1	1	1	0	m_7
1	0	0	0	1	m_8
1	0	0	1	0	m_9
1	0	1	0	0	m_{10}
1	0	1	1	0	m_{11}
1	1	0	0	1	m_{12}
1	1	0	1	1	m_{13}
1	1	1	0	1	m_{14}
1	1	1	1	1	m_{15}

$F(a.b.c.d) = m_4 + m_5 + m_8 + m_{12} + m_{13} + m_{14} + m_{15}$
$= \sum m(4.5.8.12.13.14.15)$
$= a'bc'd' + a'bc'd + ab'c'd' + abc'd' + abc'd$
$+ abcd' + abcd$

$F(a.b.c.d) = M_0 M_1 M_2 M_3 M_6 M_7 M_9 M_{10} M_{11}$
$= \Pi M(0.1.2.3.6.7.9.10.11)$
$= (a+b+c+d)(a+b+c+d')(a+b+c'+d)$
$(a+b+c'+d')(a+b'+c'+d)(a+b'+c'+d')$
$(a'+b+c+d')(a'+b+c'+d)(a'+b+c'+d')$

2. Express the complement of the following functions in sum-of-minterms form:
(8pts = 4pts each)

(a) $F(w,x,y,z) = \sum(1,5,7,11,12,14,15)$

(b) $F(x,y,z) = \Pi(2,4,5)$

(a) $F'(w,x,y,z) = \pi M(1.5.7.11.12.14.15)$

$= \boxed{\Sigma M(0.2.3.4.6.8.9.10.13)}$

(b) $F'(x,y,z) = \boxed{\Sigma m(2.4.5)}$

3. Show that a positive logic NAND gate is a negative logic NOR gate and vice versa. (6pts)

1) positive logic NAND gate = negative logic NOR gate.

positive logic : [high voltage = logic 1 / low voltage = logic 0] negative logic : [high voltage = logic 0 / low voltage = logic 1]

(먼저, positive logic NAND gate의 경우. L ⇒ 0, H ⇒ 1로 본다.)

			NAND	voltage
0 0	L	L	1	H
0 1	L	H	1	H
1 0	H	L	1	H
1 1	H	H	0	L

(negative logic NOR gate의 경우 L ⇒ 1, H ⇒ 0 으로 본다)

			NOR	voltage
1 1	L	L	0	H
1 0	L	H	0	H
0 1	H	L	0	H
0 0	H	H	1	L

positive logic NAND gate와 negative logic NOR gate가 (1.1.1.0), (00.0.1)로 0, 1이 반대로 나오지만 Positive과 negative이므로 High voltage와 low voltage에 똑같이 대응되므로 같은 gate라고 할수있다.

2) negative logic NAND gate = positive logic NOR gate.

negative logic NAND gate

			NAND	voltage
1 1	L	L	0	H
1 0	L	H	1	L
0 1	H	L	1	L
0 0	H	H	1	L

positive logic NOR gate.

			NOR	voltage
0 0	L	L	1	H
0 1	L	H	0	L
1 0	H	L	0	L
1 1	H	H	0	L

위 전자지로 voltage 값이 똑같이 나오기 때문에, gate 두개가 같다.

4. Simplify the following Boolean functions, using three-variable K-maps: (9pts = 3pts each)
   (a) $F(x,y,z) = \sum(0,1,2,4,5,6)$
   (b) $F(x,y,z) = \sum(1,2,3,5,6,7)$
   (c) $F(x,y,z) = \sum(1,3,5,7)$

0	1	3	2
4	5	7	6

(a) $\Sigma(0,1,2,4,5,6)$

x \ yz	00	01	11	10
0	1	1	0	1
1	1	1	0	1

y′ z′

$\Rightarrow y' + z'$

(b) $\Sigma(1,2,3,5,6,7)$

x \ yz	00	01	11	10
0	0	1	1	1
1	0	1	1	1

z y

$\Rightarrow y + z$

(c) $\Sigma(1,3,5,7)$

x \ yz	00	01	11	10
0	0	1	1	0
1	0	1	1	0

z

$\Rightarrow z$

5. Simplify the following Boolean expressions, using three-variable K-maps: (6pts = 3pts each)

(a) $F(x,y,z) = x'y'z + xyz' + x'yz + xy$

(b) $F(x,y,z) = xz' + x'z + x'y + xy'$

0	1	3	2
4	5	7	6

(a) $x'y'z + xyz' + x'yz + xy$

x	y	z	minterms
0	0	0	$x'y'z' = m_0$
0	0	1	$x'y'z = m_1$
0	1	0	$x'yz' = m_2$
0	1	1	$x'yz = m_3$
1	0	0	$xy'z' = m_4$
1	0	1	$xy'z = m_5$
1	1	0	$xyz' = m_6$
1	1	1	$xyz = m_7$

x \ yz	00	01	11	10
0	0	1	1	0
1	0	0	1	1

x'z, xy

$F(x,y,z) = x'z + xy$

(b) $xz' + x'z + x'y + xy'$

x	y	z	minterms
0	0	0	$x'y'z' = m_0$
0	0	1	$x'y'z = m_1$
0	1	0	$x'yz' = m_2$
0	1	1	$x'yz = m_3$
1	0	0	$xy'z' = m_4$
1	0	1	$xy'z = m_5$
1	1	0	$xyz' = m_6$
1	1	1	$xyz = m_7$

x \ yz	00	01	11	10
0	0	1	1	1
1	1	1	0	1

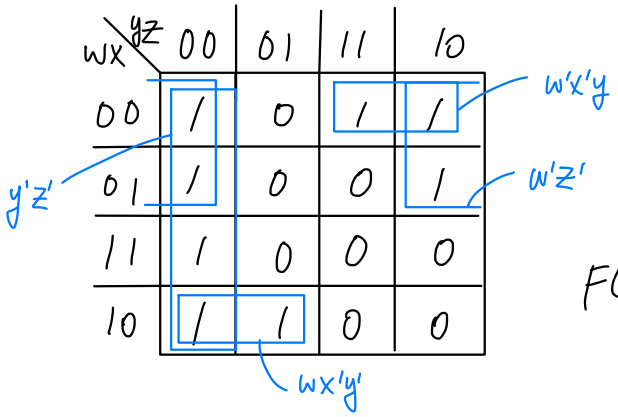
x'z, yz', xy'

$F(x,y,z) = xy' + x'z + yz'$

6. Simplify the following Boolean functions, using four-variable K-maps: (8pts = 4pts each)
   (a) $F(w,x,y,z) = \sum(0,2,3,4,6,8,9,12)$
   (b) $F(w,x,y,z) = \sum(2,3,5,7,11,13)$

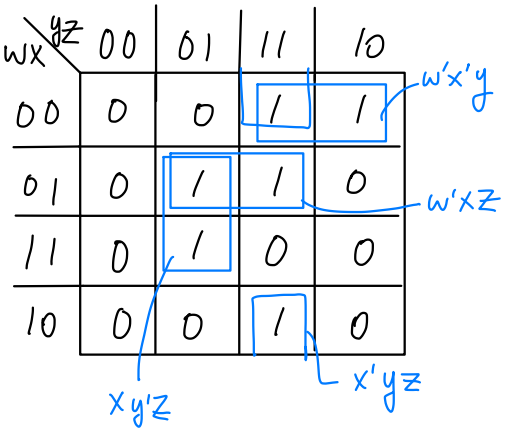
0	1	3	2
4	5	7	6
12	13	15	14
8	9	11	10

(a) $\Sigma(0,2,3,4,6,8,9,12)$



$F(w,x,y,z) = y'z' + w'x'y + w'z' + wx'y'$

(b) $\Sigma(2,3,5,7,11,13)$

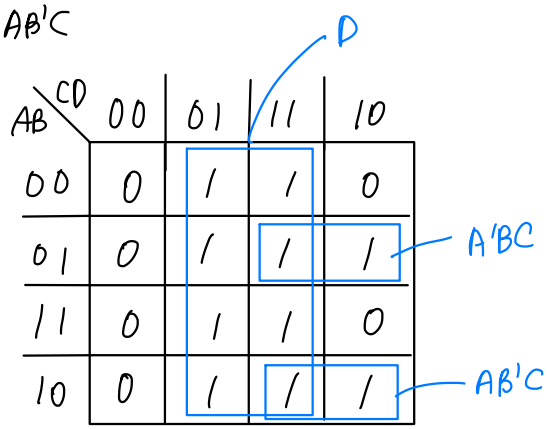


$F(w,x,y,z) = w'x'y + w'xz + x'yz + xy'z$

7. Simplify the following Boolean expressions, using <u>four-variable K-maps</u>: (8pts = 4pts each)
   (a) $A'B'CD + A'BC + C'D + ABCD + AB'C$
   (b) $A'B'C' + A'BD + A'BC'D' + BC'D + ABCD$

(a) $A'B'CD + A'BC + C'D + ABCD + AB'C$

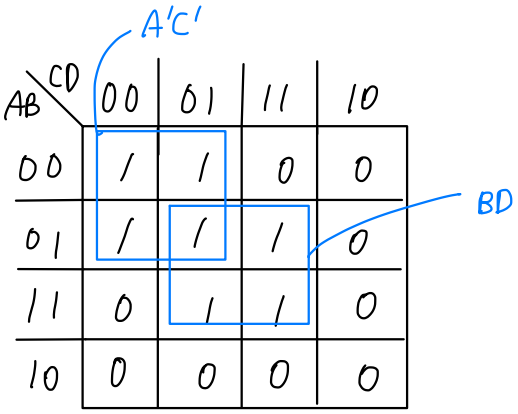
A	B	C	D	minterms
0	0	0	0	$A'B'C'D' = m_0$
0	0	0	1	$A'B'C'D = m_1$
0	0	1	0	$A'B'CD' = m_2$
0	0	1	1	$A'B'CD = m_3$
0	1	0	0	$A'BC'D' = m_4$
0	1	0	1	$A'BC'D = m_5$
0	1	1	0	$A'BCD' = m_6$
0	1	1	1	$A'BCD = m_7$
1	0	0	0	$AB'C'D' = m_8$
1	0	0	1	$AB'C'D = m_9$
1	0	1	0	$AB'CD' = m_{10}$
1	0	1	1	$AB'CD = m_{11}$
1	1	0	0	$ABC'D' = m_{12}$
1	1	0	1	$ABC'D = m_{13}$
1	1	1	0	$ABCD' = m_{14}$
1	1	1	1	$ABCD = m_{15}$



$F(A,B,C,D) = A'BC + AB'C + D$

(b) $A'B'C' + A'BD + A'BC'D' + BC'D + ABCD$

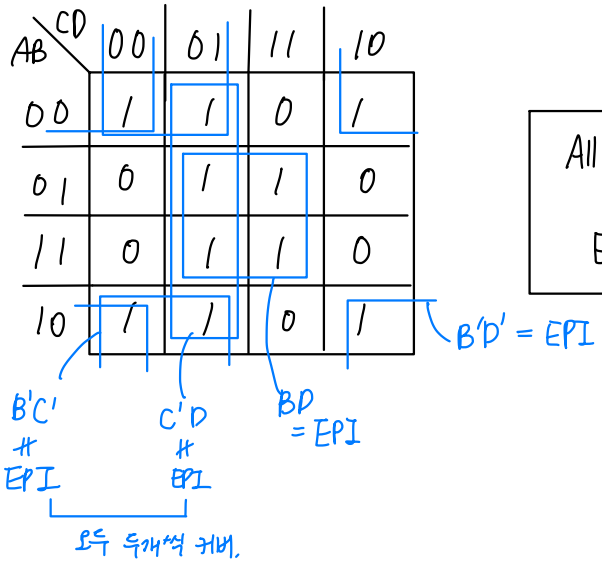
A	B	C	D	minterms
0	0	0	0	$A'B'C'D' = m_0$
0	0	0	1	$A'B'C'D = m_1$
0	0	1	0	$A'B'CD' = m_2$
0	0	1	1	$A'B'CD = m_3$
0	1	0	0	$A'BC'D' = m_4$
0	1	0	1	$A'BC'D = m_5$
0	1	1	0	$A'BCD' = m_6$
0	1	1	1	$A'BCD = m_7$
1	0	0	0	$AB'C'D' = m_8$
1	0	0	1	$AB'C'D = m_9$
1	0	1	0	$AB'CD' = m_{10}$
1	0	1	1	$AB'CD = m_{11}$
1	1	0	0	$ABC'D' = m_{12}$
1	1	0	1	$ABC'D = m_{13}$
1	1	1	0	$ABCD' = m_{14}$
1	1	1	1	$ABCD = m_{15}$



$F(A,B,C,D) = A'C' + BD$

8. Find <u>all the prime implicants</u> for the following Boolean functions, and determine which are essential: (6pts = 4pts each)
   (a) $F(A,B,C,D) = \sum(0,1,2,5,7,8,9,10,13,15)$
   (b) $F(w,x,y,z) = \sum(0,1,2,5,7,8,10,15)$

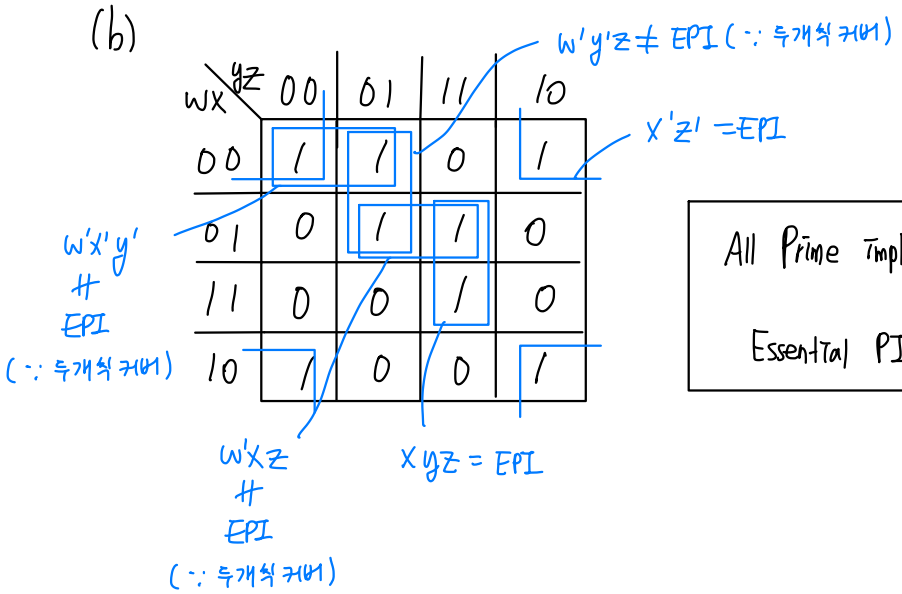
(a)



All Prime Implicants : B'D', BD, C'D, B'C'

Essential PI (EPI) : B'D', BD

(b)



All Prime Implicants : x'z', xyz, w'y'z, w'x'y', w'xz

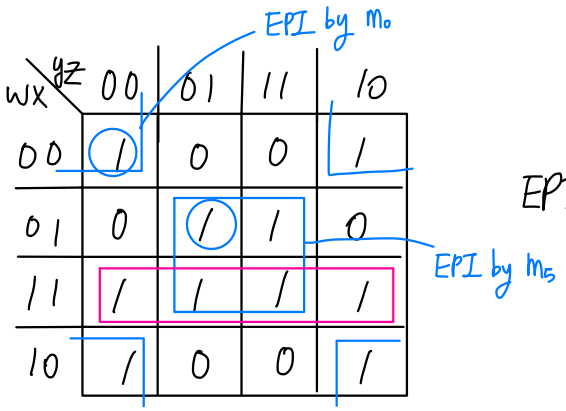
Essential PI (EPI) : x'z', xyz

9. Simplify the following Boolean functions by first finding the essential prime implicants:
(8pts = 4pts each)

(a) $F(w,x,y,z) = \sum(0,2,5,7,8,10,12,13,14,15)$

(b) $F(A,B,C,D) = \sum(0,2,3,5,7,8,10,11,14,15)$

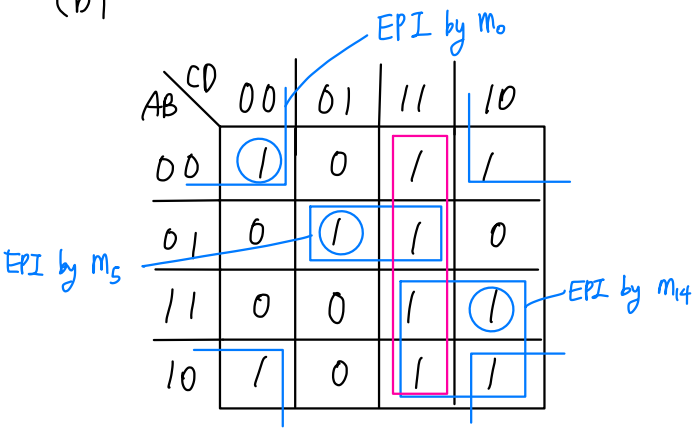
(a)



EPI : $x'z'$, xz

$\Rightarrow F(w,x,y,z) = \boxed{x'z' + xz + wx}$

(b)



EPI : $B'D'$, $A'BD$, AC

$F(A,B,C,D) = \boxed{B'D' + A'BD + AC + CD}$

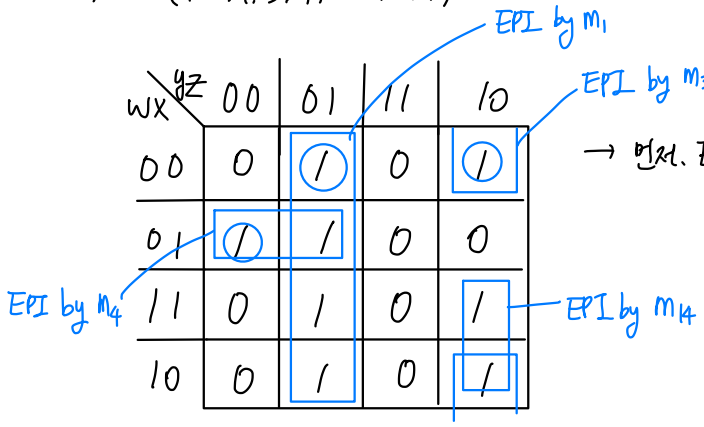
10. Using K-maps for F and F', convert the following Boolean function from a sum-of-products form to a simplified product-of-sums form. (7pts)

SOP POS

$$F(w,x,y,z) = \sum(1,2,4,5,9,10,13,14)$$

k-maps for F,

⇒ Σ(1.2.4.5.9.10.13.14)



→ 먼저, EPI를 찾는 방법으로 simplify된 Bool expression 구하기

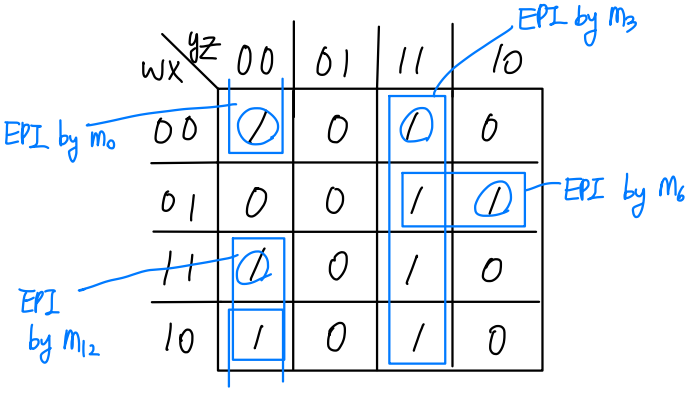
⇒ $F(w,x,y,z) = y'z + x'yz' + w'xy' + wyz'$

↳ 이는 sum of products form이다.

SOP에서 POS로 convert하기 위해서는 F'에 대해 SOP를 구해주어야 한다.

k-maps for F',

⇒ Σ(0,3,6,7,8,11,12,15)



⇒ $F'(w,x,y,z) = x'y'z' + yz + w'xy + wy'z'$

$F(w,x,y,z) = \{F'(w,x,y,z)\}'$

$= \{x'y'z' + yz + w'xy + wy'z'\}'$

$= \boxed{(x+y+z)(y'+z')(w+x'+y')(w'+y+z)}$

↳ 이것이 Product of sums form이다.

11. Simplify the following expressions to (1) sum-of-products and (2) products-of-sums: (6pts)

$$A'B + A'B'C + CD$$
$$0\ 0\ 1$$

(1) F의 k-map 22개

$\begin{pmatrix} A'B \Rightarrow 0.1\ XX \\ A'B'C \Rightarrow 0.0.1.X \\ CD \Rightarrow XX.1.1 \end{pmatrix} \Rightarrow$

AB \ CD	00	01	11	10
00	0	0	1	(1)
01	(1)	1	1	1
11	0	0	(1)	0
10	0	0	1	0

EPI by m_3

EPI by m_4

EPI by m_{15}

$\Rightarrow F(A,B,C,D) = \boxed{A'C + A'B + CD}$

$\Rightarrow$ SOP

(2) F'의 k-map — F의 k-map에서 1,0 바꾸기

AB \ CD	00	01	11	10
00	1	1	0	0
01	0	0	0	0
11	1	(1)	0	(1)
10	1	1	0	1

EPI by m_{13}

EPI by m_{14}

$F'(A,B,C,D) = B'C' + AC' + AD'$

$\downarrow$

$F(A,B,C,D) = (B'C' + AC' + AD')'$

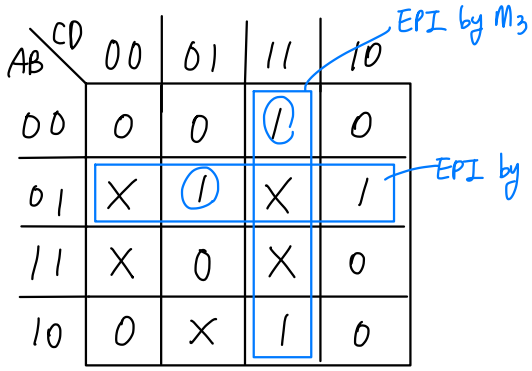
$= \boxed{(B+C)(A'+C)(A'+D)}$

$\Rightarrow$ POS

12. Simplify the following Boolean function F, together with the don't-care conditions d, and then express the simplified function in sum-of-minterms form: (6pts)

$$F(A,B,C,D) = \sum(3,5,6,11)$$
$$d(A,B,C,D) = \sum(4,7,9,12,15)$$

< k-map >



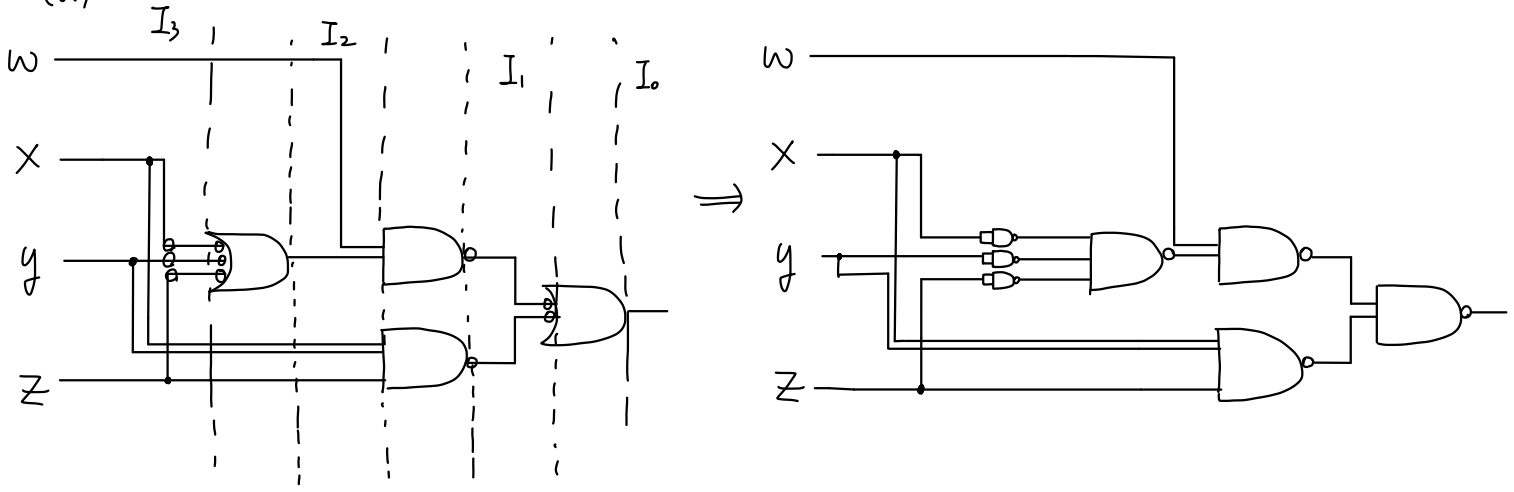
⇒ $CD + A'B = F$

13. Draw **(a)** the multiple-level NAND circuit for the following expression:

$$w(x + y + z) + xyz$$

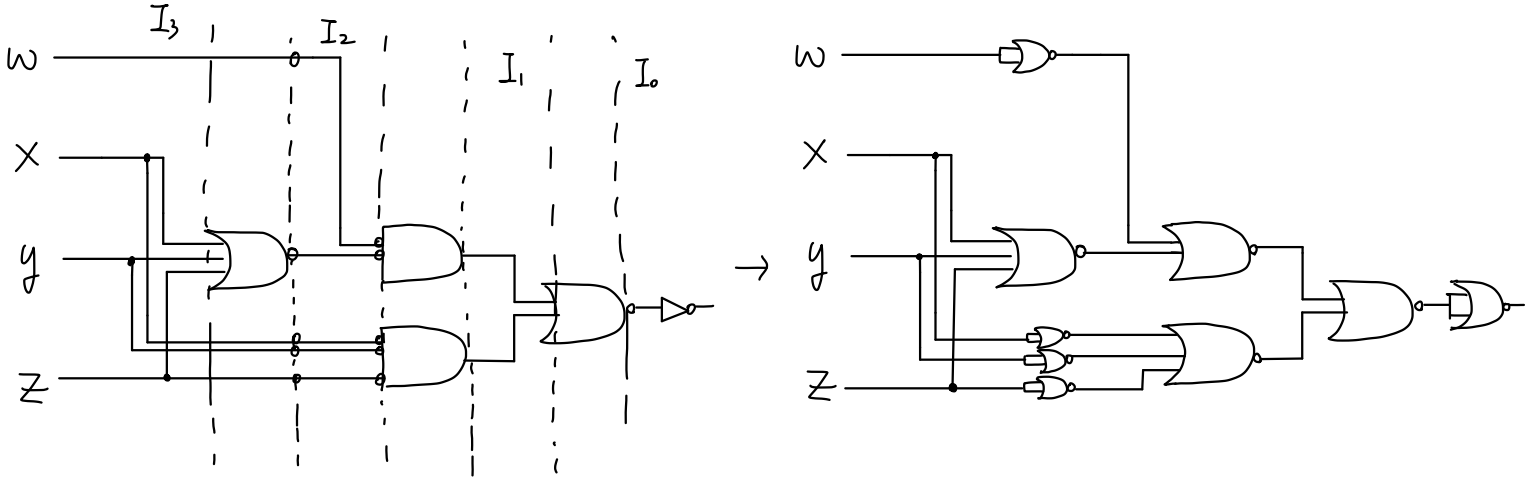
and **(b)** repeat (a) for a NOR circuit. (8pts = 4pts each)

(a)



output gate가 OR ⇒ I_{odd} 에 동그라미 그려주기 (NAND)

(b)



NOR → I_{even}에 동그라미!

14. Find the minimum SOP expression for the following Boolean function using the Q-M method. (9pts)

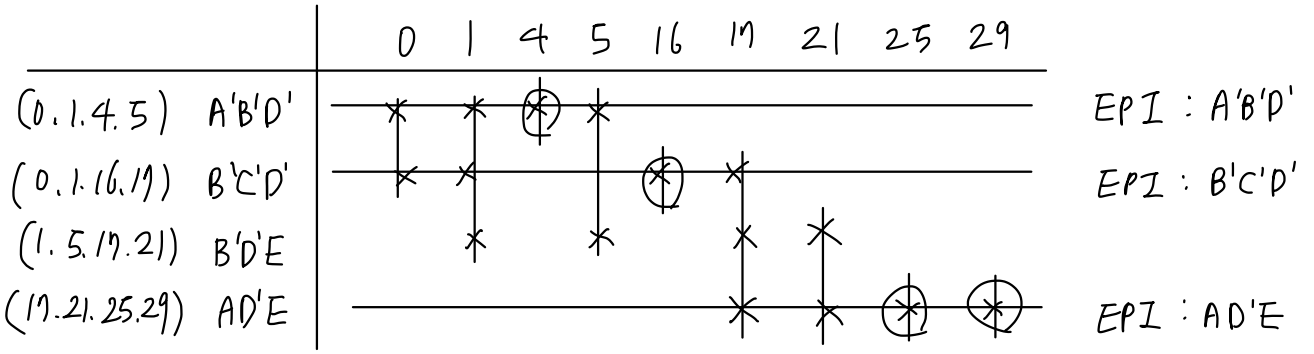
$$F(A, B, C, D, E) = \sum(0, 1, 4, 5, 16, 17, 21, 25, 29)$$

0	00000	✓
1	00001	✓
4	00100	✓
16	10000	✓
5	00101	✓
17	10001	✓
21	10101	✓
25	11001	✓
29	11101	✓

0, 1	0000-	✓
0, 4	00-00	✓
0, 16	-0000	✓
1, 5	00-01	✓
1, 17	-0001	✓
4, 5	0010-	✓
16, 17	1000-	✓
5, 21	-0101	✓
17, 21	10-01	✓
17, 25	1-001	✓
21, 29	1-101	✓
25, 29	11-01	✓

0, 1, 4, 5	00-0-	✓
0, 1, 16, 17	-00 0 -	✓
~~0, 4, 1, 5~~	~~00-0-~~	
~~0, 16, 1, 17~~	~~-000-~~	
1, 5, 17, 21	-0-01	✓
~~1, 17, 5, 21~~	~~-0-01~~	
17, 21, 25, 29	1--01	✓
~~17, 25, 21, 29~~	~~1--01~~	

PIs : A'B'D' (0, 1, 4, 5), B'C'D' (0, 1, 16, 17), B'D'E (1, 5, 17, 21), AD'E (17, 21, 25, 29)



$$\Rightarrow F(A, B, C, D, E) = \boxed{A'B'D' + B'C'D' + AD'E}$$

End of the Homework #2