Homework #3

Digital System Design 2022 Spring

DUE : 2022-04-09

- total 104 pts = maximum 100 pts + 4 bonus pts
- Extension limit = { `png` , `jpg` , `heic` , `zip` , `pdf` }
- You must hand your answer in at the board before due time (2022-04-09 11:59 PM KST).

The problems start from the next page.

Name

김주은

In Korean

Student ID

20210774

8 digits

1. Design a combinational circuit with three inputs and one output.

   (a) The output is 1 when the binary value of the inputs is more than 2. The output is 0 otherwise.

   (b) The output is 1 when the binary value of the inputs is not divisible by 3.

   $\Rightarrow$ divisible by 3 $\Rightarrow 0.$

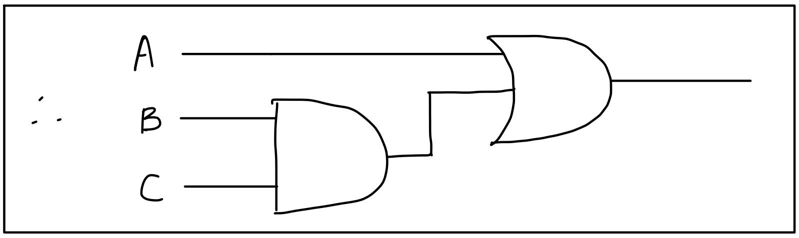
(a)

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

$\Rightarrow$

A \ BC	00	01	11	10
0	0	0	1	0
1	1	1	1	1

$\Rightarrow A + BC$



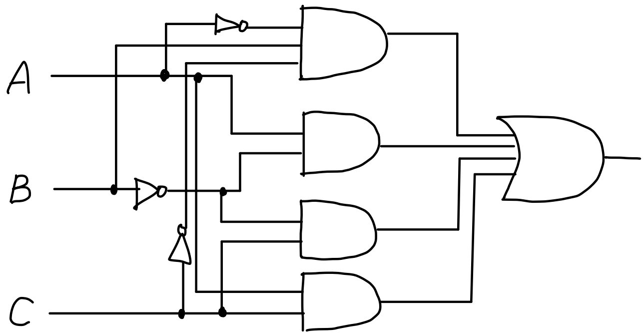
(b)

A	B	C	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

$\Rightarrow$

A \ BC	00	01	11	10
0	0	1	0	1
1	1	1	1	0

$F = B'C + A'BC' + AB' + AC$



2. A BCD-to-seven-segment decoder is a combinational circuit that converts a decimal digit in BCD to an appropriate code for the selection of segments in an indicator used to display the decimal digit in a familiar form. The seven outputs of the decoder (a, b, c, d, e, f, g) select the corresponding segments in the display, as shown in Fig. 1. The numeric display chosen to represent the decimal digit is shown in Fig. 2. Using a truth table and Karnaugh maps, design the BCD-to-seven-segment decoder using a minimum number of gates. The six invalid combinations should result in a blank display.

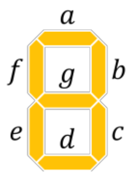


Fig. 1. Segment designation

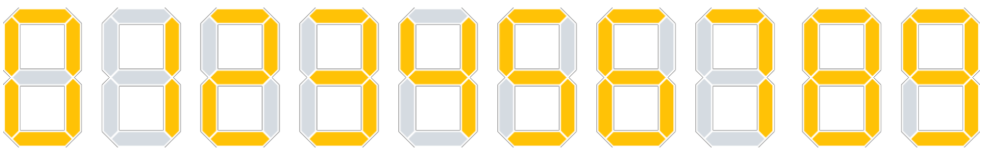


Fig. 2. Numerical designation for display

<truth table>

input				output						
A	B	C	D	a	b	c	d	e	f	g
0	0	0	0	1	1	1	1	1	1	0
0	0	0	1	0	1	1	0	0	0	0
0	0	1	0	1	1	0	1	1	0	1
0	0	1	1	1	1	1	1	0	0	1
0	1	0	0	0	1	1	0	0	1	1
0	1	0	1	1	0	1	1	0	1	1
0	1	1	0	1	0	1	1	1	1	1
0	1	1	1	1	1	1	0	0	0	0
1	0	0	0	1	1	1	1	1	1	1
1	0	0	1	1	1	1	1	0	1	1
1	0	1	0	0	0	0	0	0	0	0
1	0	1	1	0	0	0	0	0	0	0
1	1	0	0	0	0	0	0	0	0	0
1	1	0	1	0	0	0	0	0	0	0
1	1	1	0	0	0	0	0	0	0	0
1	1	1	1	0	0	0	0	0	0	0

<karnaugh map>

① output : a $\Rightarrow B'C'D' + A'BD + AB'C' + A'C$

AB \ CD	00	01	11	10
00	1	0	1	1
01	0	1	1	1
11	0	0	0	0
10	1	1	0	0

② output : b $\Rightarrow A'B' + B'C' + A'C'D' + A'CD$

AB \ CD	00	01	11	10
00	1	1	1	1
01	1	0	1	0
11	0	0	0	0
10	1	1	0	0

③ output : $C = A'B + B'C' + A'D$

AB \ CD	00	01	11	10
00	1	1	1	0
01	1	1	1	1
11	0	0	0	0
10	1	1	0	0

AB \ CD	00	01	11	10
00	1	0	1	1
01	0	1	0	1
11	0	0	0	0
10	1	1	0	0

④ output :

$d = AB'C' + A'BC'D + A'B'D'$
$+ A'B'C + A'CD'$

⑤ output : e $\rightarrow B'C'D' + A'CD'$

AB \ CD	00	01	11	10
00	1	0	0	1
01	0	0	0	1
11	0	0	0	0
10	1	0	0	0

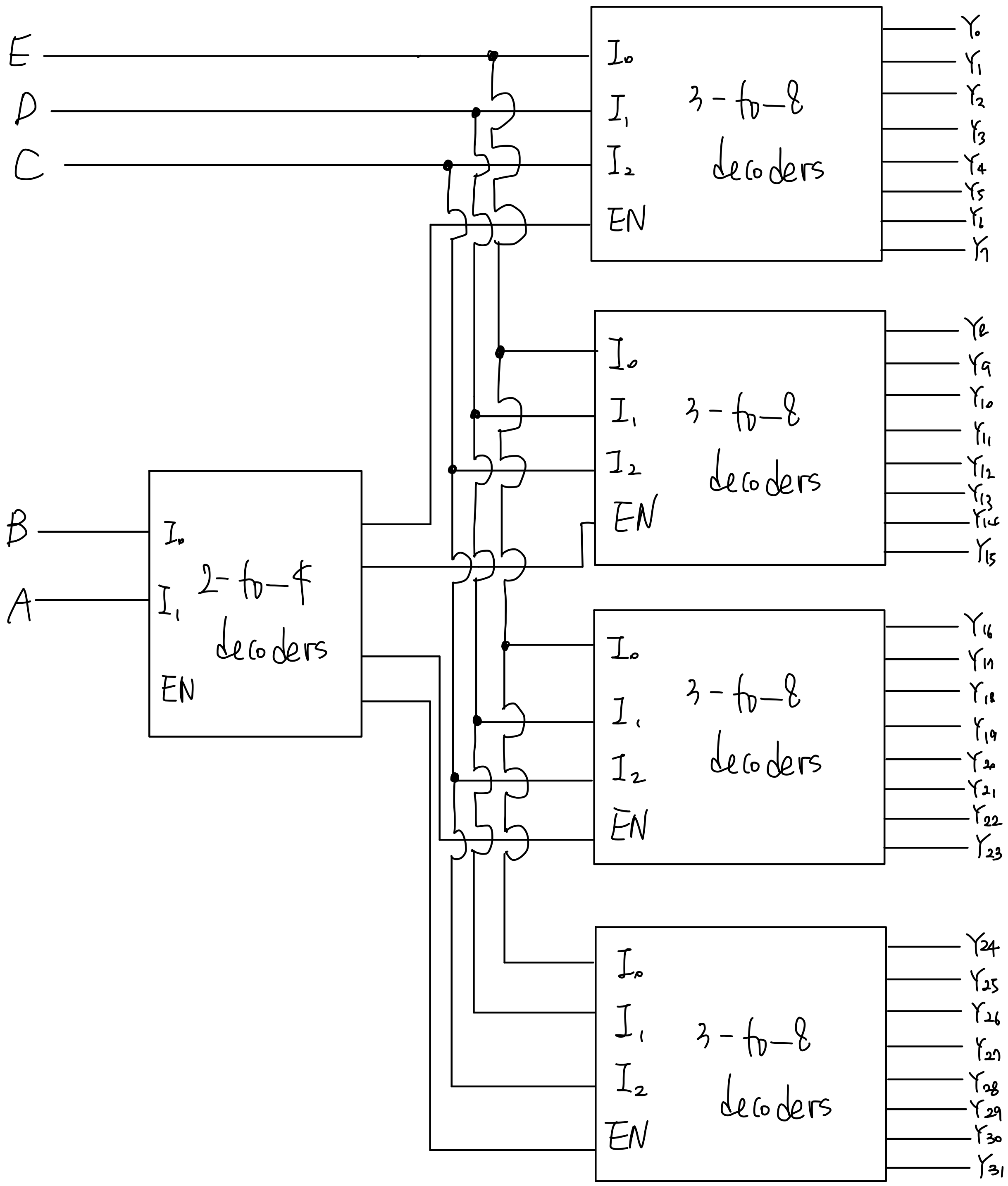
⑥ output : f $\rightarrow A'C'D' + A'BC' + A'BD' + AB'C'$

AB \ CD	00	01	11	10
00	1	0	0	0
01	1	1	0	1
11	0	0	0	0
10	1	1	0	0

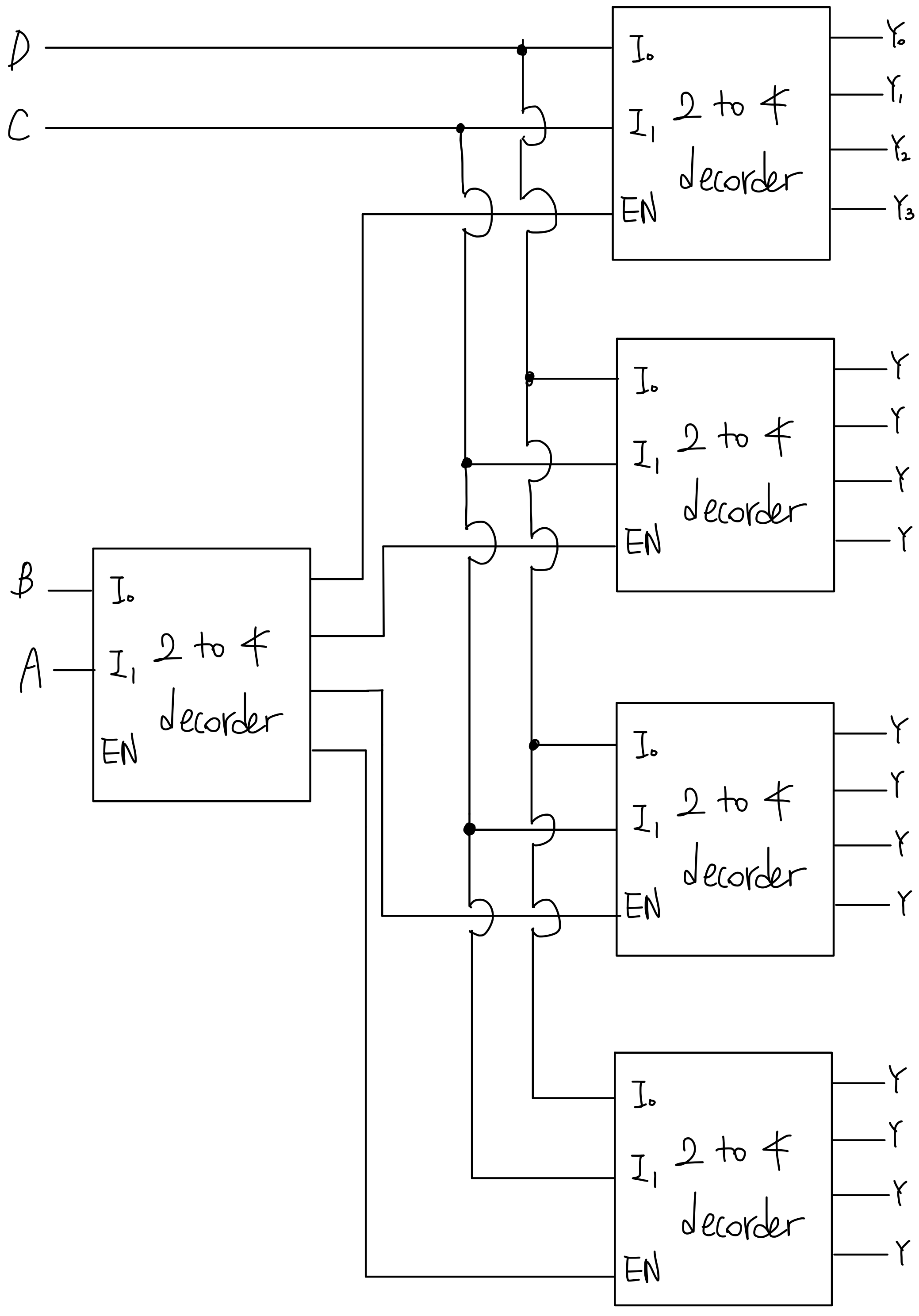
⑦ output : g $= A'B'C + A'CD' + A'BC' + AB'C'$

AB \ CD	00	01	11	10
00	0	0	1	1
01	1	1	0	1
11	0	0	0	0
10	1	1	0	0

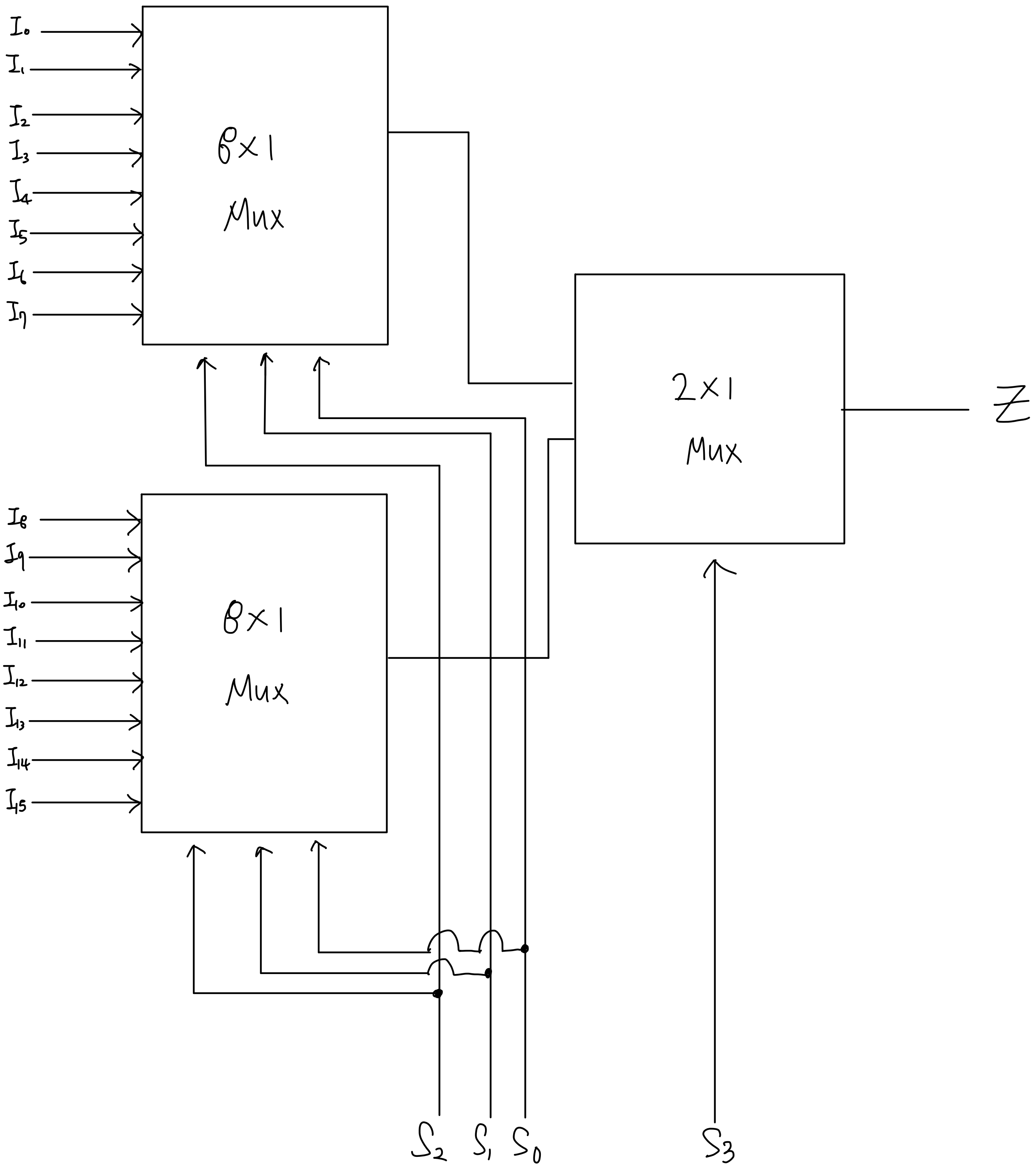
3. Construct a 5-to-32-line decoder with four 3-to-8-line decoders with enable and a 2-to-4-line decoder. Use block diagrams for the components.



4. Construct a 4-to-16-line decoder with five 2-to-4-line decoders with enable.



5. Construct a 16×1 multiplexer with ~~two~~ 8×1 and ~~one~~ 2×1 multiplexers. Use block diagrams.



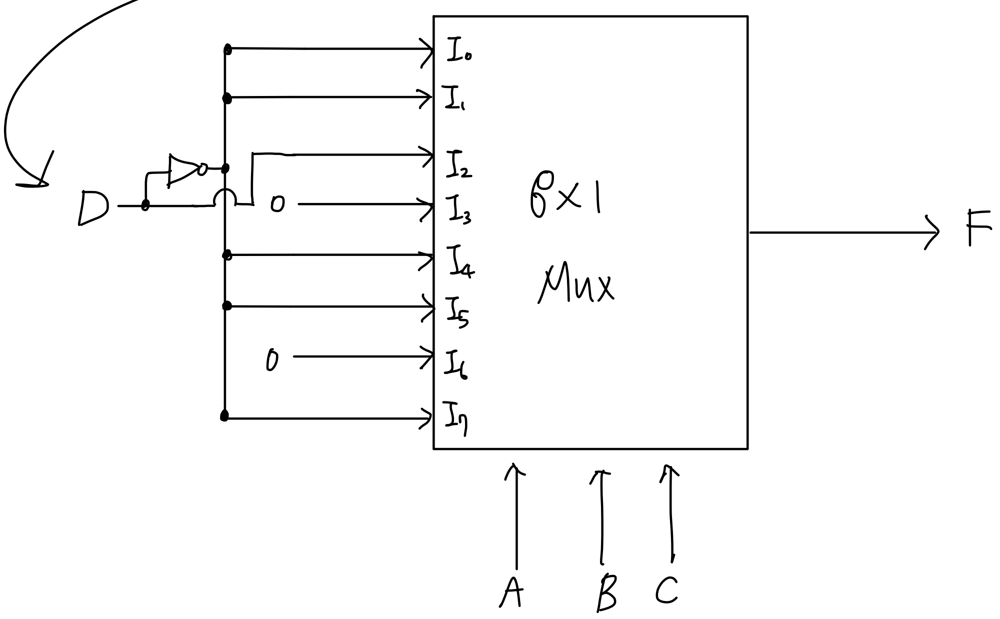
6. Implement the following Boolean function with a multiplexer:

$$F(A,B,C,D) = \sum(0,2,5,8,10,14)$$

A	B	C	D	F	
0	0	0	0	1	D'
0	0	0	1	0	
0	0	1	0	1	D'
0	0	1	1	0	
0	1	0	0	0	D
0	1	0	1	1	
0	1	1	0	0	0
0	1	1	1	0	
1	0	0	0	1	D'
1	0	0	1	0	
1	0	1	0	1	D'
1	0	1	1	0	
1	1	0	0	0	0
1	1	0	1	0	
1	1	1	0	1	D'
1	1	1	1	0	

A, B, C : control input

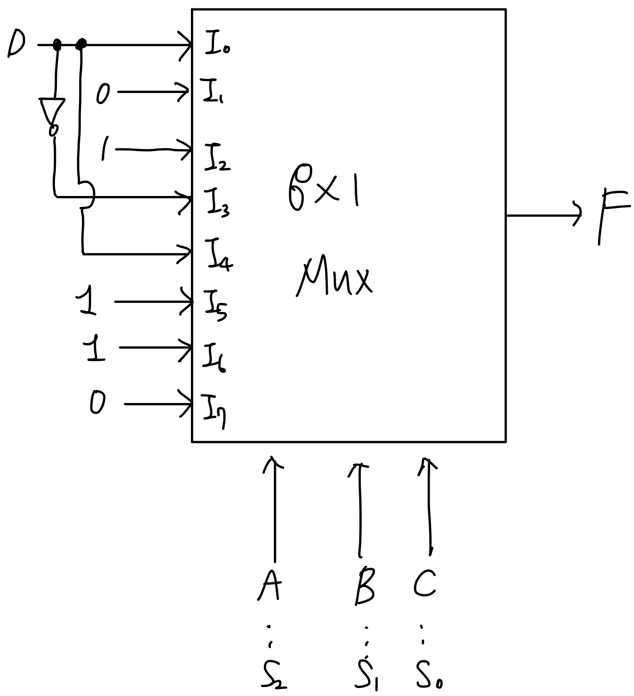
D : data input.



7. An 8×1 multiplexer has inputs A, B, and C connected to the selection inputs S_2, S_1, and S_0, respectively. The data inputs I_0 through I_7 are as follows:

$$I_1 = I_7 = 0;\ I_2 = I_5 = I_6 = 1;\ I_0 = I_4 = D;\ \text{and}\ I_3 = D'.$$

Determine the Boolean function that the multiplexer implements.



⇒

A	B	C	D	F	
0	0	0	0	0	D
0	0	0	1	1	
0	0	1	0	0	0
0	0	1	1	0	
0	1	0	0	1	1
0	1	0	1	1	
0	1	1	0	1	D′
0	1	1	1	0	
1	0	0	0	0	D
1	0	0	1	1	
1	0	1	0	1	1
1	0	1	1	1	
1	1	0	0	1	1
1	1	0	1	1	
1	1	1	0	0	0
1	1	1	1	0	

$F = \Sigma\, m(1, 4, 5, 6, 9, 10, 11, 12, 13)$ ⇒ 이를 simplify하면,

AB \ CD	00	01	11	10
00	0	1	0	0
01	1	1	0	1
11	1	1	0	0
10	0	1	1	1

$\Rightarrow\ F = C'D + BC' + A'BD' + AB'C$

8. Implement the following Boolean function with a 4×1 multiplexer and external gates.

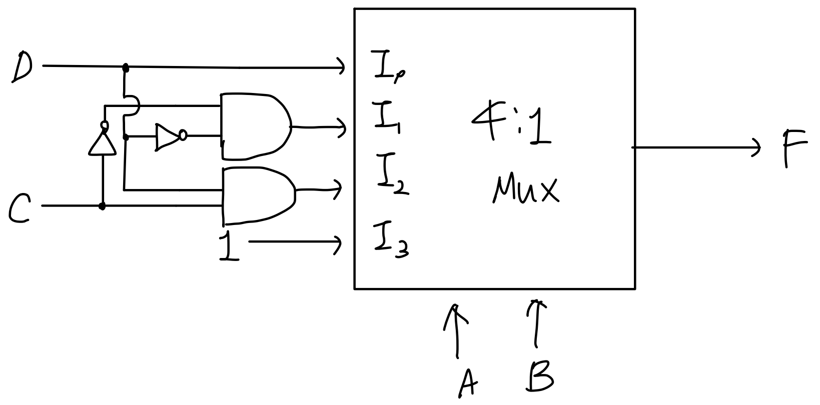
(a) $F_1(A,B,C,D) = \sum(1,3,4,11,12,13,14,15)$

(b) $F_2(A,B,C,D) = \sum(1,2,5,7,8,10,11,13,15)$

Connect inputs A and B to the selection lines. The input requirements for the four data lines will be a function of variables C and D. These values are obtained by expressing F as a function of C and D for each of the four cases when $AB = 00,\ 01,\ 10,$ and 11. These functions may have to be implemented with external gates.

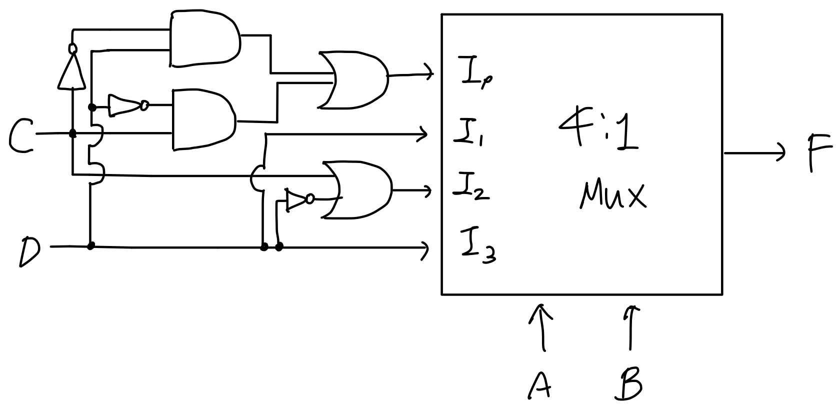
(a)

A	B	C	D	F	
0	0	0	0	0	D
0	0	0	1	1	
0	0	1	0	0	
0	0	1	1	1	
0	1	0	0	1	C'D'
0	1	0	1	0	
0	1	1	0	0	
0	1	1	1	0	
1	0	0	0	0	CD
1	0	0	1	0	
1	0	1	0	0	
1	0	1	1	1	
1	1	0	0	1	1
1	1	0	1	1	
1	1	1	0	1	
1	1	1	1	1	

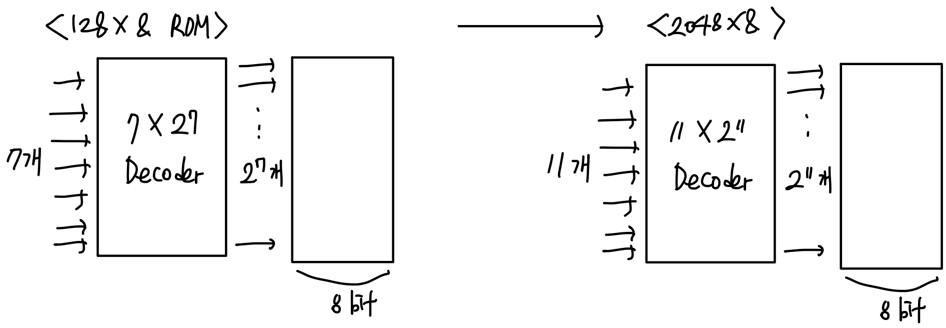


(b)

A	B	C	D	F	
0	0	0	0	0	C'D + CD'
0	0	0	1	1	
0	0	1	0	1	
0	0	1	1	0	
0	1	0	0	0	D
0	1	0	1	1	
0	1	1	0	0	
0	1	1	1	1	
1	0	0	0	1	D' + C
1	0	0	1	0	
1	0	1	0	1	
1	0	1	1	1	
1	1	0	0	0	D
1	1	0	1	1	
1	1	1	0	0	
1	1	1	1	1	



9. How many 128×8 ROM chips are required to construct a 2048×8 ROM chip? What is additionally needed and why?



$\Rightarrow 2^{11} \div 2^7 = 2^4 \Rightarrow$ 총 16개의 128×8 ROM chip이 필요하다.

즉, 2^7개의 output을 가지는 decoder들이 16개가 있어야 2048×8 ROM을 만들수 있다.

그리고, EN을 조절하여 output 2^{11} 개를 구현하기 위해 또 다른 decoder을 추가적으로 구현해주어야 한다.

4개의 input을 받아 16개의 output을 내놓음으로써 16개 decoder의 EN에 연결하도록, 4 to 16 decoder을

한개 만들어줘야 한다.

$\boxed{\therefore \text{16개, 4 to 16 decoder}}$

10. Tabulate the truth table for an 16×4 ROM that implements the Boolean functions

$$A(w, x, y, z) = \Sigma(0, 2, 5, 7, 8, 14)$$

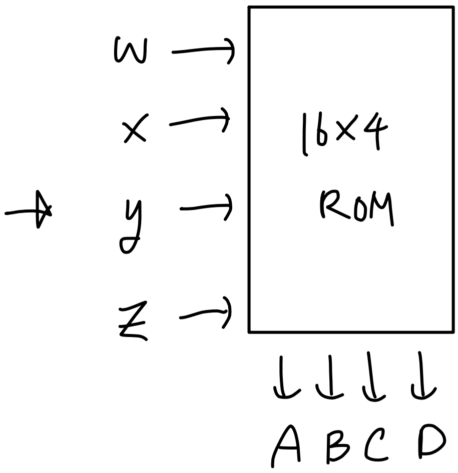
$$B(w, x, y, z) = \Sigma(3, 5, 7, 9, 11, 13, 15)$$

$$C(w, x, y, z) = \Sigma(0, 4, 8, 12)$$

$$D(w, x, y, z) = \Sigma(0, 1, 2, 4, 7, 9)$$

Consider now the ROM as a memory. Specify the memory contents at addresses 5 and 15.

w	x	y	z	A	B	C	D
0	0	0	0	1	0	1	1
0	0	0	1	0	0	0	1
0	0	1	0	1	0	0	1
0	0	1	1	0	1	0	0
0	1	0	0	0	0	1	1
0	1	0	1	1	1	0	0
0	1	1	0	0	0	0	0
0	1	1	1	1	1	0	1
1	0	0	0	1	0	1	0
1	0	0	1	0	1	0	1
1	0	1	0	0	0	0	0
1	0	1	1	0	1	0	0
1	1	0	0	0	0	1	0
1	1	0	1	0	1	0	0
1	1	1	0	1	0	0	0
1	1	1	1	0	1	0	0



address : memory contents

– 5 : 1100

– 15 : 0100

End of the Homework #3