Homework #1

Digital System Design 2022 Spring

DUE : 2022-03-28

total 104 pts (8 pts each) = maximum 100 pts + 4 bonus pts

Extension limit = { `png` , `jpg` , `heic` , `zip` , `pdf` }

You must hand your answer in at the board before due time (2022-03-28 11:00 AM KST).

The problems start from the next page.

Name

김 주 운

In Korean

Student ID

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8 digits

1. List the octal and hexadecimal numbers from 50_{10} to 64_{10}. Using A, B, C, and D for the last four digits, list the numbers from 11_{10} to 30_{10} in base 14.

8 | 50 .. 2 ⇒ 62
6 .. 6

16 | 50 .. 2 ⇒ 32
3 .. 3

<decimal>	<octal>	<hexadecimal>
50	62	32
51	63	33
52	64	34
53	65	35
54	66	36
55	67	37
56	70	38
57	71	39
58	72	3A
59	73	3B
60	74	3C
61	75	3D
62	76	3E
63	77	3F
64	100	40

$11_{10} \sim 30_{10}$

base 14
: 0~9, A. B. C. D
10 11 12 13

<decimal>	<base 14>
11	B
12	C
13	D
14	10
15	11
16	12
17	13
18	14
19	15
20	16
21	17
22	18
23	19
24	1A
25	1B
26	1C
27	1D
28	20
29	21
30	22

2. Convert the following numbers with the <u>indicated bases to decimal</u>: (4pts each)

(a) $(9922)_{14}$

(b) $(248)_{9}$

(a) $2\times1 + 2\times14 + 9\times14^2 + 9\times14^3 = (26490)_{10}$

↑ 계산

(b) $8\times1 + 4\times9 + 2\times9^2 = (206)_{10}$

↑ 계산

3. What is the largest binary number that can be expressed with 12 bits? What are the equivalent decimal, octal, and hexadecimal numbers?

가장 큰 숫자는 12bit가 전부 1일 때이다

$(1111\ 1111\ 1111)_2$

decimal : $1+2+2^2+2^3+\cdots 2^{11} = \frac{2^{12}-1}{2-1} = 4096-1 = 4095.$

$\therefore$ decimal $\Rightarrow (4095)_{10}$

octal : $\underbrace{111}_{4+2+1}\ \underbrace{111}_{4+2+1}\ \underbrace{111}_{''}\ \underbrace{111}_{''} = 7777$ $\therefore$ octal $\Rightarrow (7777)_8$

hexadecimal : $\underbrace{1111}_{2^3+2^2+2^1+2^0}\ \underbrace{1111}_{}\ \underbrace{1111}_{} = FFF$

$=8+4+2+1=15 \Rightarrow F$

$\therefore$ hexadecimal $\Rightarrow (FFF)_{16}$

4. Determine the base of the numbers in each case for the following operations to be correct: (4pts each)

(a) $12 \times 4 = 52$

(b) $123 + 120 = 303$

(a)
$$\begin{array}{r} {\scriptstyle 1} \\ 12 \\ \times\ 4 \\ \hline 52 \end{array}$$
인 경우라고 볼수 있다. 2×4는 원래 8이지만 12가 된것으로 보아서 base가 6임을 알수 있다. ($\because 6\times1+1\times2=8$)

$\therefore 6$

(b)
$$\begin{array}{r} {\scriptstyle 1} \\ 123 \\ +\ 120 \\ \hline 303 \end{array}$$
인 경우다 $2+2=4$이지만 10이 되므로 base는 4이다. ($\because 4\times1+1\times0=4$)

$\therefore 4$

5. Convert the decimal number 253 to binary in two ways: (a) convert directly to binary; (b) convert first to hexadecimal and then from hexadecimal to binary. Which method is faster?

(a)
$2\overline{)253}\ \cdots 1$
$2\overline{)126}\ \cdots 0$
$2\overline{)63}\ \cdots 1$
$2\overline{)31}\ \cdots 1$
$2\overline{)15}\ \cdots 1$
$2\overline{)7}\ \cdots 1$
$2\overline{)3}\ \cdots 1$
$1\ \cdots 1$

$\Rightarrow (11111101)_2$

(b)
$16\overline{)253}\ \cdots 13$
$15\ \cdots 15$

$\Rightarrow (FD)_{16} \Rightarrow (\underbrace{1111}_{F}\ \underbrace{1101}_{D})_2$

(b)가 더 빠르다. 16진수는 쉽게 바로 숫자를 나눌수 있고, 이 후 16진수에서 2진수로의 변환도 간단하다.

6. Express the following numbers in decimal. (4pts each)

(a) $(51DE.C)_{16}$

(b) $(110011.001)_2$

(a) $5\times16^3 + 1\times16^2 + 13\times16 + 14\times1 + 12\times16^{-1} = \boxed{(20958.75)_{10}}$

(b) $2^5 + 2^4 + 2^1 + 1 + 2^{-3} = \boxed{(51.125)_{10}}$

32
16
2
1
$\frac{1}{8}$

51

$2^3 2^2 2^1 0$

7. Represent the decimal number 6,428 in (a) BCD, (b) excess-3 code, (c) 2421 code, and (d) 6311 code. (2pts each)

(a) BCD — 6: 0110
4: 0100
2: 0010
8: 1000
⇒ 0110 0100 0010 1000

(b) excess-3 code — 6: 1001
↳ BCD + 0011 4: 0111
2: 0101
8: 1011
⇒ 1001 0111 0101 1011

(c) 2421 code - 6: 1100
4: 0100
2: 0010
8: 1110
⇒ 1100 0100 0010 1110

0 0000
1 0001
2 0010
3 0011
4 0100
5 1011
6 1100
7 1101
8 1110

(d) 6311 code — 6 : 1000
4 : 0101
2 : 0011
8 : 1011
⇒ 1000 0101 0011 1011

8. Simplify the following Boolean expressions to a minimum number of literals: (4pts each)

(a) $(a + b + c')(a'b' + c)$

(b) $a'bc + abc' + abc + a'bc'$

(a) $(a+b+c')(a'b'+c) = (a+b)c + a'b'c'$

consensus theorem

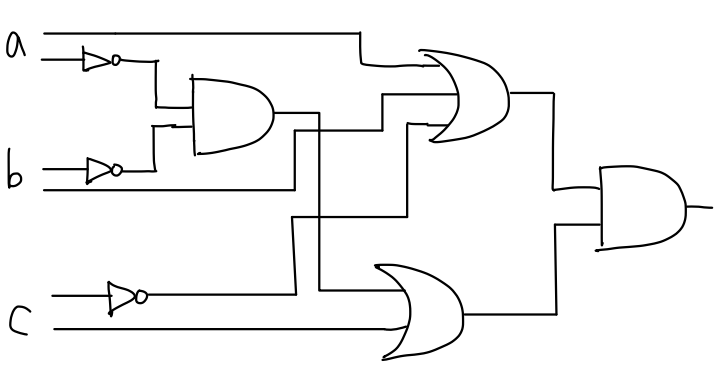
(b) $a'bc + abc' + abc + a'bc' = bc(a+a') + bc'(a+a') = b(c+c') = b$

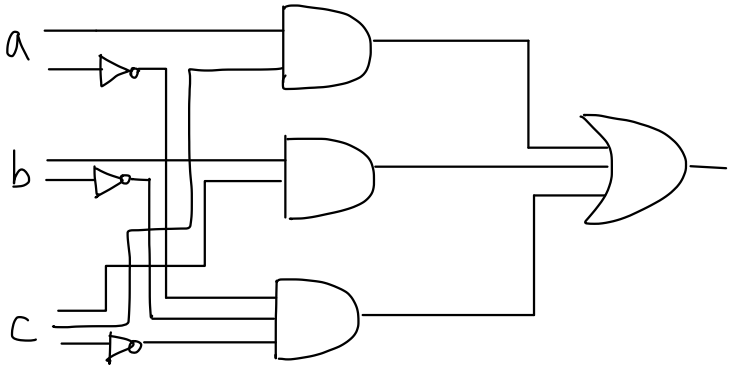
9. Draw logic diagrams of the circuits that implement the original and simplified expressions in Problem 8 (a) and (b). (4pts each)

8(a)

$(a+b+c')(a'b'+c) \longrightarrow ac+bc+a'b'c'$

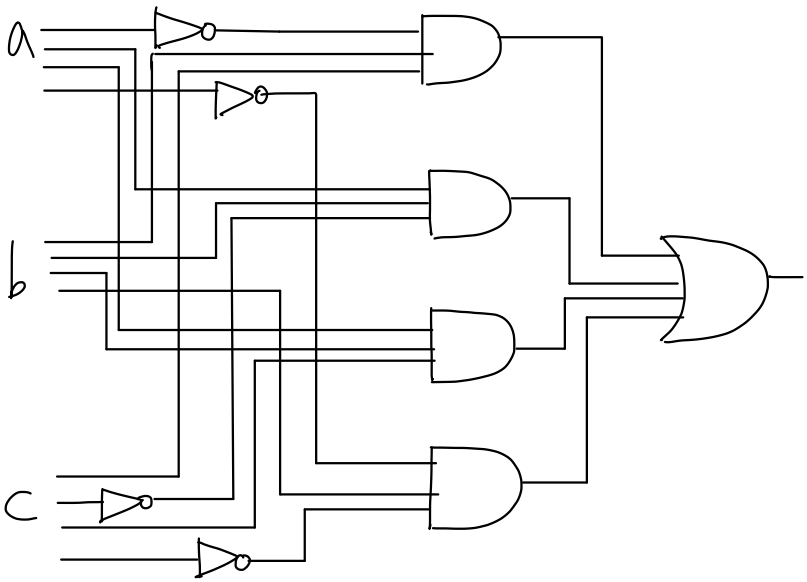
(original) (simplified)





8(b)

$a'bc+abc'+abc+a'bc' \longrightarrow b$



b ——— F

10. Find the complement of the following expressions: (4pts each)

(a) $(x + y')(y + z')(z + x')$

(b) $w'x' + w(x + y + z)$

(a) $\{(x+y')(y+z')(z+x')\}'$

$= (x+y')' + (y+z')' + (z+x')' = \boxed{x'y + y'z + z'x}$

(b) $\{w'x' + w(x+y+z)\}' = (w+x)\{w(x+y+z)\}' = (w+x)\{w' + (x+y+z)'\}$

$= (w+x)(w' + x'y'z')$

$= \boxed{wx'y'z' + xw'}$

↑ consensus theorem

11. List the truth table of the function: (4pts each)

(a) $F = x'z' + xy + yz$

(b) $F = a'b'c' + a'bc + ab'c' + abc$

(a) $F = x'z' + xy + yz$

x	y	z	x′	z′	x′z′	xy	yz	x′z′+xy+yz
0	0	0	1	1	1	0	0	1
0	0	1	1	0	0	0	0	0
0	1	0	1	1	1	0	0	1
0	1	1	1	0	0	0	1	1
1	0	0	0	1	0	0	0	0
1	0	1	0	0	0	0	0	0
1	1	0	0	1	0	1	0	1
1	1	1	0	0	0	1	1	1

(b) $F = a'b'c' + a'bc + ab'c' + abc$

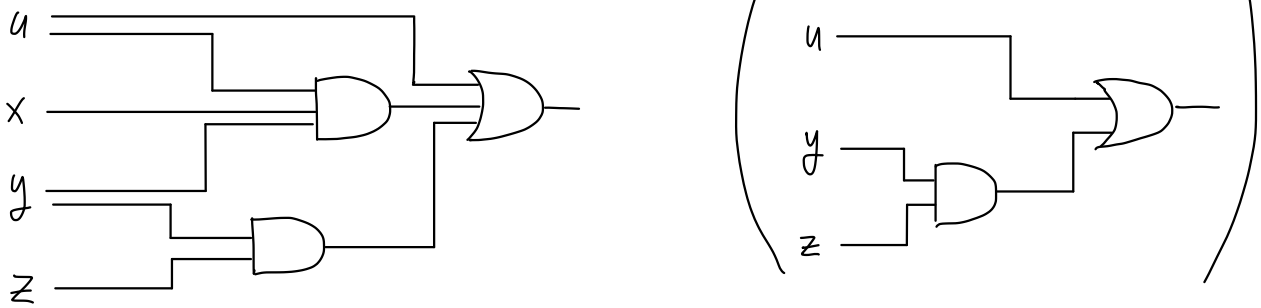
a	b	c	a′	b′	c′	a′b′c′	a′bc	ab′c′	abc	a′b′c′+a′bc+ab′c′+abc
0	0	0	1	1	1	1	0	0	0	1
0	0	1	1	1	0	0	0	0	0	0
0	1	0	1	0	1	0	0	0	0	0
0	1	1	1	0	0	0	1	0	0	1
1	0	0	0	1	1	0	0	1	0	1
1	0	1	0	1	0	0	0	0	0	0
1	1	0	0	0	1	0	0	0	0	0
1	1	1	0	0	0	0	0	0	1	1

12. Draw logic diagrams to implement the following Boolean expressions: (4pts each)

(a) $F = u + yz + uxy$

(b) $F = u + x + x'(u + y')$

(a) $F = u + yz + uxy$ → if simplify! → $u(1+xy) + yz = u + yz$

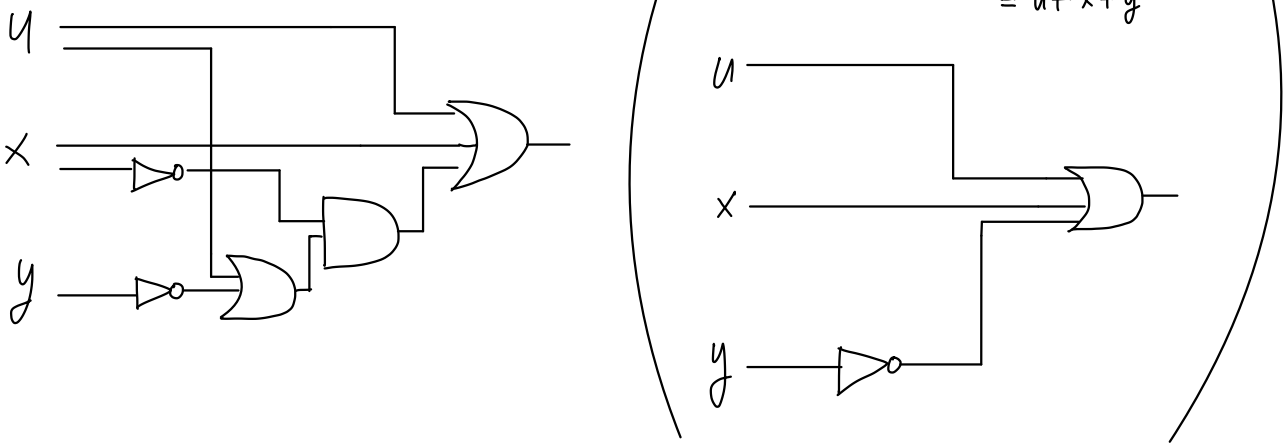


(b) $F = u + x + x'(u+y')$ → if simplify!

$F = u(1+x') + x + x'y'$

$= u + x + x'y' = u + (x+x')(x+y')$

$= u + x + y'$

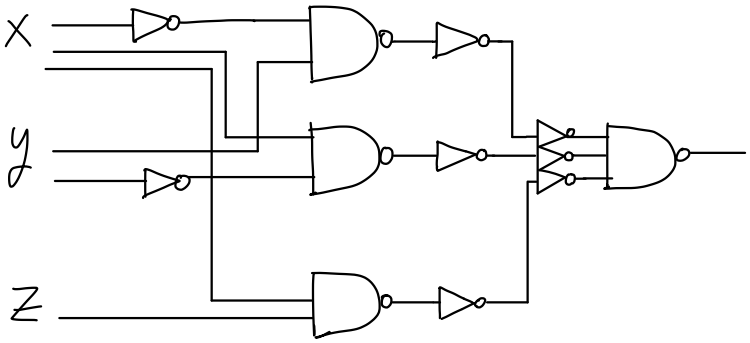


13. Implement the Boolean function $F = x'y + xy' + xz$

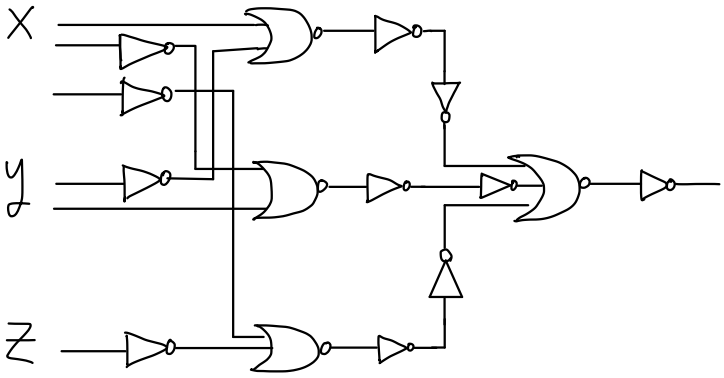
(a) With NAND and inverter gates. (4pts) ⇒ NAND, not

(b) With NOR and inverter gates. (4pts) ⇒ NOR, not

(a) $F = x'y + xy' + xz$



(b) $x'y + xy' + xz = \{(x'y + xy' + xz)'\}' = \{(x+y')(x'+y)(x'+z')\}'$



End of the Homework #1