Tabla 2.8: Características de algunas distribuciones para análisis de eventos extremos

Distribución	**Características**	**Parámetros**
Gumbel (max)	$f_X(x) = \frac{1}{\alpha}\exp\left[-\frac{x-b}{\alpha} - \exp\left(-\frac{x-b}{\alpha}\right)\right]$ $-\infty < x < \infty$ $F_X(x) = \exp\left[-\exp\left(-\frac{x-b}{\alpha}\right)\right]$ $\mu_X = b + 0,5772\alpha$ $\sigma_X^2 = \frac{\pi^2\alpha^2}{6}$ $\gamma_1 = 1,1396$ $x(F) = b - \alpha\ln\left[-\ln F\right]$	α: escala b: posición
Gumbel (min)	$f_Z(z) = \frac{1}{\alpha}\exp\left[\frac{z-b}{\alpha} - \exp\left(\frac{z-b}{\alpha}\right)\right]$ $-\infty < z < \infty$ $F_Z(z) = 1 - \exp\left[-\exp\left(\frac{z-b}{\alpha}\right)\right]$ $\mu_Z = b - 0,5772\alpha$ $\sigma_Z^2 = \frac{\pi^2\alpha^2}{6}$ $\gamma_1 = -1,1396$ $z(F) = b + \alpha\ln\left[-\ln(1-F)\right]$	α: escala b: posición
Pearson Tipo III	$f_X(x) = \frac{1}{a\Gamma(b)}\left(\frac{x-c}{a}\right)^{b-1}\exp\left[-\left(\frac{x-c}{a}\right)\right]$ $b > 0$ si $a > 0 \Rightarrow 0 < c < x$ si $a < 0 \Rightarrow x < c$ $\mu_X = c + ba$ $\sigma_X^2 = ba^2$ $\gamma_1 = \frac{2}{\sqrt{b}}$ si $a > 0$ $\gamma_1 = -\frac{2}{\sqrt{b}}$ si $a < 0$	a: escala b: forma c: posición

Tabla 2.9: Características de algunas distribuciones para análisis de eventos extremos

Distribución	Características	Parámetros
Gamma Invertida	$f_X(x) = \frac{\lambda^r}{\Gamma(r)} x^{-(r+1)} \exp\left(-\frac{\lambda}{x}\right)$ $x > 0$ $F_X(x) = \frac{\Gamma(r,\lambda/x)}{\Gamma(r)}$; $\Gamma(r, \lambda/x)$ función gamma incompleta $\mu_X = \frac{\lambda}{r-1}$ para $r > 1$ $\sigma_X^2 = \frac{\lambda^2}{(r-1)^2(r-2)}$ para $r > 2$ $\gamma_1 = \frac{4\sqrt{r-2}}{r-3}$ para $r > 3$	λ: escala r: forma
Log Pearson Tipo III	$f_X(x) = \frac{1}{ax\Gamma(b)} \left(\frac{\ln x - c}{a}\right)^{b-1} \exp\left[-\left(\frac{\ln x - c}{a}\right)\right]$ $b > 0$ si $a > 0 \Rightarrow 0 < c < \ln x$ si $a < 0 \Rightarrow \ln x < c$ $\mu_X = \frac{e^c}{(1-a)^b}$ $\sigma_X^2 = e^{2c}\left[\frac{1}{(1-2a)^b} - \frac{1}{(1-a)^{2b}}\right]$ $\gamma_1 = \frac{\frac{1}{(1-3a)^b} - \frac{3}{(1-a)^b(1-2a)^b} + \frac{2}{(1-a)^{3b}}}{\left[\frac{1}{(1-2a)^b} - \frac{1}{(1-a)^{2b}}\right]^{1,5}}$	a: escala b: forma c: posición
Fréchet	$f_X(x) = \frac{\theta}{a}\left(\frac{a}{x}\right)^{\theta+1} \exp\left[-\left(\frac{a}{x}\right)^{\theta}\right]$ con $a, \theta > 0$ $F_X(x) = \exp\left[-\left(\frac{a}{x}\right)^{\theta}\right]$ $\mu_X = a\Gamma\left(1 - \frac{1}{\theta}\right)$ para $\theta > 1$ $\sigma_X^2 = a^2\left[\Gamma\left(1 - \frac{2}{\theta}\right) - \Gamma^2\left(1 - \frac{1}{\theta}\right)\right]$ para $\theta > 2$ $V_X^2 = \frac{\Gamma(1-2/\theta)}{\Gamma^2(1-1/\theta)} - 1$ para $\theta > 2$ $\gamma_1 = \frac{\Gamma(1-3/\theta) - 3\Gamma(1-2/\theta)\Gamma(1-1/\theta) + 2\Gamma^3(1-1/\theta)}{[\Gamma(1-2/\theta) - \Gamma^2(1-1/\theta)]^{1,5}}$ para $\theta > 3$ $x(F) = a(-\ln F)^{-1/\theta}$ para $\theta > 2$	a: escala θ: forma

Tabla 2.10: Características de algunas distribuciones para análisis de eventos extremos

Distribución	Características	Parámetros
Generalizada de Valor Extremo (GEV)	$f_X(x) = \frac{1}{a}\left[1 - \frac{b}{a}(x-c)\right]^{(1-b)/b} \exp\left\{-\left[1-\frac{b}{a}(x-c)\right]^{1/b}\right\}$ $a > 0$ si $b > 0 \Rightarrow -\infty < x \leqslant c + (a/b)$ si $b < 0 \Rightarrow c + (a/b) \leqslant x < \infty$ $F_X(x) = \exp\left\{-\left[1-\frac{b}{a}(x-c)\right]^{1/b}\right\}$ $\mu_X = c + \frac{a}{b}\left[1 - \Gamma(1+b)\right]$ para $b > -1$ $\sigma_X^2 = \frac{a^2}{b^2}\left[\Gamma(1+2b) - \Gamma^2(1+b)\right]$ para $b > -0,5$ $\gamma_1 = \text{sgn}(b)\,\frac{-\Gamma(1+3b)-3\Gamma(1+b)\Gamma(1+2b)+2\Gamma^3(1+b)}{\left[\Gamma(1+2b)-\Gamma^2(1+b)\right]^{1,5}}$ $x(F) = c + a\left[1 - (-\ln F)^b\right]/b$	a: escala b: forma c: posición
Weibull	$f_Z(z) = \left(\frac{\kappa}{\alpha}\right)\left(\frac{z-b}{\alpha}\right)^{\kappa-1} \exp\left[-\left(\frac{z-b}{\alpha}\right)^{\kappa}\right]$ $z \geqslant b;\quad \alpha > b,\ \kappa > 0$ $F_Z(z) = 1 - \exp\left[-\left(\frac{z-b}{\alpha}\right)^{\kappa}\right]$ $\mu_Z = b + \alpha\Gamma\left(1+\frac{1}{\kappa}\right)$ $\sigma_Z^2 = \alpha^2\left[\Gamma\left(1+\frac{2}{\kappa}\right) - \Gamma^2\left(1+\frac{1}{\kappa}\right)\right]$ $\gamma_1 = \frac{\Gamma\left(1+\frac{3}{\kappa}\right)-3\Gamma\left(1+\frac{1}{\kappa}\right)\Gamma\left(1+\frac{2}{\kappa}\right)+2\Gamma^3\left(1+\frac{1}{\kappa}\right)}{\left[\Gamma\left(1+\frac{2}{\kappa}\right)-\Gamma^2\left(1+\frac{1}{\kappa}\right)\right]^{1,5}}$ $z(F) = b + \alpha\left[-\ln(1-F)\right]^{1/\kappa}$	α: escala κ: forma b: posición
Wakeby	$x(F) = m + \frac{a}{b}\left[1 - (1-F)^b\right] - \frac{c}{d}\left[1 - (1-F)^{-d}\right]$ $m \leq x \leq \infty$ si $d \geq 0$ y $c > 0$ $m \leq x \leq m + \frac{a}{b} - \frac{c}{d}$ si $d < 0$ o $c = 0$	$a \neq 0$ o $c \neq 0$ $b + d > 0$ o $b = c = d = 0$ Si $a = 0 \Rightarrow b = 0$ Si $c = 0 \Rightarrow d = 0$ $c \geq 0$ y $a + c \geq 0$

Tabla 2.11: Características de algunas distribuciones para análisis de eventos extremos

Distribución	Características	Parámetros
Generalizada de Pareto (GPD)	$f_X(x) = \frac{1}{b}\left(1 - a\frac{x-c}{b}\right)^{(1-a)/a}$ si $a \neq 0$ $f_X(x) = \frac{1}{b}\exp\left(-\frac{x-c}{b}\right)$ si $a = 0$ Si $a = 0 \Rightarrow$ distribución exponencial Si $a = 1 \Rightarrow$ distribución uniforme $[c, c+b]$ $F_X(x) = 1 - \left[1 - a\frac{x-c}{b}\right]^{1/a}$ si $a \neq 0$ $F_X(x) = 1 - \exp\left[-\frac{x-c}{b}\right]$ si $a = 0$ $\mu_X = c + \frac{b}{1+a}$ $\sigma_X^2 = \frac{b^2}{(1+a)^2(1+2a)}$ $\gamma_1 = \frac{2(1-a)(1+2a)^{0,5}}{1+3a}$ $x(F) = c + b\left[1 - (1-F)^a\right]/a$ si $a \neq 0$ $x(F) = c - b\ln(1-F)$ si $a = 0$	a: forma b: escala c: posición

Tabla 2.12: Estimación de parámetros de distribuciones usadas en el análisis de eventos extremos

Distribución	Momentos	Máxima verosimilitud
Gumbel (max)	Con n asintótico: $\widehat{\alpha} = \frac{\sqrt{6}s}{\pi}$ $\widehat{b} = \overline{x} - 0,4501s$ En función de n: $\widehat{\alpha} = \frac{s}{\sigma_Y^2}$ $\widehat{b} = \overline{x} - \widehat{\alpha}\mu_Y$ $\mu_Y = \frac{1}{n}\sum y_i$ $\sigma_Y^2 = \frac{1}{n}\sum (y_i - \mu_Y)^2$ $y_i = -\ln\left[-\ln\left(\frac{n+1-i}{n+1}\right)\right]$ con serie anual ordenada de mayor a menor	$\overline{x} = \widehat{\alpha} + \frac{\sum x_i \exp\left[-\frac{x_i}{\widehat{\alpha}}\right]}{\sum \exp\left[-\frac{x_i}{\widehat{\alpha}}\right]}$ $\widehat{b} = \widehat{\alpha}\left\{\ln n - \ln\left(\sum \exp\left[-\frac{x_i}{\widehat{\alpha}}\right]\right)\right\}$
Gumbel (min)	Con n asintótico: $\widehat{\alpha} = \frac{\sqrt{6}s}{\pi}$ $\widehat{b} = \overline{z} + 0,4501s$ En función de n: $\widehat{\alpha} = \frac{s}{\sigma_Y^2}$ $\widehat{b} = \overline{z} + \widehat{\alpha}\mu_Y$ $\mu_Y = \frac{1}{n}\sum y_i$ $\sigma_Y^2 = \frac{1}{n}\sum (y_i - \mu_Y)^2$ $y_i = \ln\left[-\ln\left(\frac{i}{n+1}\right)\right]$ con serie anual ordenada de menor a mayor	$\overline{z} = -\widehat{\alpha} + \frac{\sum z_i \exp\left[\frac{z_i}{\widehat{\alpha}}\right]}{\sum \exp\left[\frac{z_i}{\widehat{\alpha}}\right]}$ $\widehat{b} = -\widehat{\alpha}\left\{\ln n - \ln\left(\sum \exp\left[\frac{z_i}{\widehat{\alpha}}\right]\right)\right\}$

Tabla 2.13: Estimación de parámetros de distribuciones usadas en el análisis de eventos extremos

Distribución	Momentos	Máxima verosimilitud
Gamma Invertida	$\widehat{r}=\frac{\overline{x}^2}{s^2}+2$ $\widehat{\lambda}=\overline{x}\left(\frac{\overline{x}^2}{s^2}+1\right)$	Solución numérica para $\widehat{r}$ de: $\ln\widehat{r}-\Psi\left(\widehat{r}\right)=$ $\frac{1}{n}\sum_{i=1}^{n}\ln x_i+\ln\left[\sum_{i=1}^{n}x_i^{-1}\right]-\ln n$ con $\Psi\left(\cdot\right)$ la función Digamma $\widehat{\lambda}=n\widehat{r}\left[\sum_{i=1}^{n}x_i^{-1}\right]^{-1}$
Pearson Tipo III	$\widehat{a}=\frac{sg^*}{2}$ $b=\frac{4}{{g^*}^2}$ $c=\overline{x}-\frac{2s}{g^*}$ $g^*=g\left(\frac{1}{n-2}\right)\left[1+\frac{8{,}5}{n}\right]\left[n\left(n-1\right)\right]^{0{,}5}$ No siempre hay solución para valores pequeños de g^*. Además, $g^*<2$	Solución numérica de: $\frac{n\widehat{b}}{\widehat{a}}-\frac{1}{\widehat{a}^2}\sum\left(x_i-\widehat{c}\right)=0$ $n\Psi\left(\widehat{b}\right)-\sum\ln\left(x_i-\widehat{c}\right)+n\ln\widehat{a}=0$ $\left(\widehat{b}-1\right)\sum\left(\frac{1}{x_i-\widehat{c}}\right)-\frac{n}{\widehat{a}}=0$ siendo $\Psi\left(b\right)$ la función Digamma
Log Pearson Tipo III	Solución numérica de: $\overline{x}=\frac{e^{\widehat{c}}}{(1-\widehat{a})^{\widehat{b}}}$ $s^2=e^{2\widehat{c}}\left[\frac{1}{(1-2\widehat{a})^{\widehat{b}}}-\frac{1}{(1-\widehat{a})^{2\widehat{b}}}\right]$ $g=\frac{\frac{1}{(1-3\widehat{a})^{\widehat{b}}}-\frac{3}{(1-\widehat{a})^{\widehat{b}}(1-2\widehat{a})^{\widehat{b}}}+\frac{2}{(1-\widehat{a})^{3\widehat{b}}}}{\left[\frac{1}{(1-2\widehat{a})^{\widehat{b}}}-\frac{1}{(1-\widehat{a})^{2\widehat{b}}}\right]^{1{,}5}}$	Solución de optimización: Min $R=n\Psi\left(\widehat{b}\right)+\sum\left[\frac{\ln x_i-\widehat{c}}{\widehat{a}}\right]$ s.a. $\widehat{b}=\frac{s_1s_2}{(s_1s_2-n^2)}$ $\widehat{a}=\frac{s_1}{n\widehat{b}}$ $s_1=\sum\left(\ln x_i-\widehat{c}\right)$ $s_2=\sum\frac{1}{\ln x_i-\widehat{c}}$

Tabla 2.14: Estimación de parámetros de distribuciones usadas en el análisis de eventos extremos

Distribución	Momentos	Máxima verosimilitud
Fréchet	Solución numérica para $\widehat{\theta}$ de: $\frac{s^2}{\overline{x}^2} = \frac{\Gamma\left(1-2/\widehat{\theta}\right)}{\Gamma^2\left(1-1/\widehat{\theta}\right)} - 1$ $\widehat{a} = \frac{\overline{x}}{\Gamma\left(1-1/\widehat{\theta}\right)}$	Si $\widehat{\theta}$ es conocida: $\widehat{a} = \left[\frac{n}{t}\right]^{-\widehat{\theta}} \quad \text{con} \quad t = \sum_{i=1}^{n} (1/x_i)^{\widehat{\theta}}$ Si no, solución numérica de: $\frac{n}{\widehat{\theta}} + n \ln \widehat{a} - \sum_{i=1}^{n} \ln x_i - \sum_{i=1}^{n} \left(\frac{\widehat{a}}{x_i}\right)^{\widehat{\theta}} \ln\left(\frac{\widehat{a}}{x_i}\right) = 0$ $\frac{n\widehat{\theta}}{\widehat{a}} - \widehat{\theta}(\widehat{a}^{\widehat{\theta}-1}) \sum_{i=1}^{n} x_i^{-\widehat{\theta}} = 0$
Weibull	Si $b = 0 \Rightarrow 2$ ec. con 2 incog.: $\mu_Z = \overline{z};\, \sigma_Z^2 = s^2$ Alternativamente: resolver para κ $\frac{\overline{z}^2}{s^2+\overline{z}^2} = \frac{\Gamma^2(1+1/\kappa)}{\Gamma(1+2/\kappa)} \to \widehat{\kappa}$ $\widehat{\alpha} = \frac{\overline{z}}{\Gamma(1+1/\widehat{\kappa})}$ Si $b \neq 0 \Rightarrow 3$ ec. con 3 incog.: $\mu_Z = \overline{z};\, \sigma_Z^2 = s;\;\; \gamma_Z = g$	Si $b = 0$, resolver para κ: $\frac{\sum x_i^{\kappa} \ln x_i}{\sum x_i^{\kappa}} - \frac{1}{\kappa} - \frac{1}{n} \sum \ln x_i = 0 \to \widehat{\kappa}$ $\widehat{\alpha} = \left[\frac{1}{n} \sum x_i^{\widehat{\kappa}}\right]^{1/\widehat{\kappa}}$ Si $b \neq 0$, resolver: $\frac{n}{\kappa} + \sum \ln\left[\frac{x_i-b}{\alpha}\right] - \sum \left[\frac{x_i-b}{\alpha}\right]^{\kappa} \ln\left[\frac{x_i-b}{\alpha}\right] = 0$ $-\frac{n\kappa}{\alpha} + \frac{\kappa}{\alpha} \sum \left[\frac{x_i-b}{\alpha}\right]^{\kappa} = 0$ $-(\kappa-1) \sum (x_i-b)^{-1} + \frac{\kappa}{\alpha} \sum \left[\frac{x_i-b}{\alpha}\right]^{\kappa-1} = 0$
Generalizada de Pareto (GPD)	$g = \frac{2(1-\widehat{a})(1+2\widehat{a})^{0,5}}{1+3\widehat{a}} \to \widehat{a}$ $\widehat{b} = s\,(1+\widehat{a})\,(1+2\widehat{a})^{0,5}$ $\widehat{c} = \overline{x} - \frac{\widehat{b}}{1+\widehat{a}}$	$\widehat{c} = \text{mín}\,\{x_i\}$ Solución numérica de: $\sum_{i=1}^{n} \frac{(x_i-\widehat{c})/\widehat{b}}{1-\widehat{a}(x_i-\widehat{c})/\widehat{b}} = \frac{n}{1-\widehat{a}}$ $\sum_{i=1}^{n} \ln\left[1 - \widehat{a}\,(x_i-\widehat{c})\,/\widehat{b}\right] = -n\widehat{a}$

Tabla 2.15: Estimación de parámetros de distribuciones usadas en el análisis de eventos extremos

Distribución	Momentos	Máxima verosimilitud
Generalizada de Valor Extremo (GEV)	$g = \mathrm{sgn}\left(\widehat{b}\right) \frac{-\Gamma\left(1+3\widehat{b}\right)-3\Gamma\left(1+\widehat{b}\right)\Gamma\left(1+2\widehat{b}\right)+2\Gamma^3\left(1+\widehat{b}\right)}{\left[\Gamma\left(1+2\widehat{b}\right)-\Gamma^2\left(1+\widehat{b}\right)\right]^{1,5}} \rightarrow \widehat{b}$ $\widehat{a} = \frac{s\widehat{b}}{\left[\Gamma\left(1+2\widehat{b}\right)-\Gamma^2\left(1+\widehat{b}\right)\right]^{0,5}}$ $\widehat{c} = \overline{x} - \frac{\widehat{a}}{\widehat{b}}\left[1-\Gamma\left(1+\widehat{b}\right)\right]$	Solución numérica de: $\frac{Q}{\widehat{a}} = 0$ $\frac{1}{\widehat{a}}\frac{P+Q}{\widehat{b}} = 0$ $\frac{1}{\widehat{b}}\left(R - \frac{P+Q}{\widehat{b}}\right) = 0$ con $P = n - \sum_{i=1}^{n} e^{y_i}$ $Q = \sum_{i=1}^{n} e^{y_i+\widehat{b}y_i} - \left(1-\widehat{b}\right)\sum_{i=1}^{n} e^{y_i}$ $R = n - \sum_{i=1}^{n} y_i + \sum_{i=1}^{n} y_i e^{y_i}$ $y_i = -\frac{1}{\widehat{b}} \ln\left(1 - \frac{x_i - \widehat{c}}{\widehat{a}}\widehat{b}\right)$
$\mathrm{sgn}(k) = \begin{matrix} +1 \text{ si } k > 0 \\ -1 \text{ si } k < 0 \end{matrix}$		

Tabla 2.16: Estimación de parámetros de distribuciones usadas en el análisis de eventos extremos

Distribución	MPP	Momentos-L
Gumbel (max)	$\widehat{\alpha} = -\frac{\overline{x}-2\widehat{\beta}_1}{\ln(2)}$ $\widehat{b} = \overline{x} - 0,5772\widehat{\alpha}$	$\widehat{\alpha} = \frac{\widehat{\lambda}_2}{\ln(2)}$ $\widehat{b} = \overline{x} - 0,5772\widehat{\alpha}$
Gumbel (min)	$\widehat{\alpha} = -\frac{\overline{z}-2\widehat{\beta}_1}{\ln(2)}$ $\widehat{b} = \overline{z} + 0,5772\widehat{\alpha}$	$\widehat{\alpha} = \frac{\widehat{\lambda}_2}{\ln(2)}$ $\widehat{b} = \overline{z} + 0,5772\widehat{\alpha}$
Weibull	Si $b = 0$: $\widehat{\kappa} = \frac{-\ln(2)}{\ln\left(1-\frac{2\widehat{\beta}_1-\overline{x}}{\overline{x}}\right)}$ $\widehat{\alpha} = \frac{\overline{x}}{\Gamma(1+1/\widehat{\kappa})}$ Si $b \neq 0$: $\widehat{b} = \frac{4(a_3a_0-a_1^2)}{4a_3+a_0-4a_1}$ $\widehat{\alpha} = \frac{a_0-\widehat{b}}{\Gamma\{\ln[(a_0-2a_1)/(a_1-2a_3)]/\ln(2)\}}$ $\widehat{\kappa} = \ln(2)/\ln\{[a_0-2a_1]/[2(a_1-2a_3)]\}$	Si $b = 0$: $\widehat{\kappa} = \frac{-\ln(2)}{\ln(1-\widehat{\lambda}_2/\widehat{\lambda}_1)}$ $\widehat{\alpha} = \frac{\widehat{\lambda}_1}{\Gamma(1+1/\widehat{\kappa})}$ Si $b \neq 0 \rightarrow$ Ajustar a serie $\{-x_i\}$: $c = 2\widehat{\lambda}_2/(\widehat{\lambda}_3 - 3\widehat{\lambda}_2) - \ln(2)/\ln(3)$ $\widehat{\kappa} = \left[7{,}859c + 2{,}9554c^2\right]^{-1}$ $\widehat{b} = \frac{\widehat{\lambda}_2}{\widehat{\kappa}(1-2^{-1/\widehat{\kappa}})\Gamma(1+1/\widehat{\kappa})}$ $\widehat{\alpha} = -\widehat{\lambda}_1 - \widehat{b}\widehat{\kappa}/\left[\Gamma(1+1/\widehat{\kappa})+1\right]$
Pearson Tipo III		Si $\widehat{\tau}_3 \geq 1/3$, con $t = 1-\widehat{\tau}_3$: $\widehat{b} \approx \frac{0{,}36067t-0{,}59567t^2+0{,}25361t^3}{1-2{,}78861t+2{,}5096t^2-0{,}77045t^3}$ Si $\widehat{\tau}_3 < 1/3$, con $t = 3\pi\widehat{\tau}_3^2$: $\widehat{b} \approx \frac{1+0{,}2906t}{t+0{,}1882t^2+0{,}0442t^3}$ $\widehat{a} = \frac{\sqrt{\pi}\widehat{\lambda}_2\Gamma(\widehat{b})}{\Gamma(\widehat{b}+0{,}5)}$ $\widehat{c} = \widehat{\lambda}_1 - \widehat{a}\widehat{b}$
Log Pearson Tipo III		Usar Pearson Tipo III con $\ln x_i$ para estimar $\widehat{\lambda}_1$, $\widehat{\lambda}_2$ y $\widehat{\tau}_3$

Tabla 2.17: Estimación de parámetros de distribuciones usadas en el análisis de eventos extremos

Distribución	**MPP**	**Momentos-L**
Fréchet	$\widehat{\theta} = \frac{\ln(2)}{\ln\left(2\widehat{\beta}_1/\overline{x}\right)}$ $a = \frac{\overline{x}}{\Gamma\left(1-1/\widehat{\theta}\right)}$	
Generalizada de Valor Extremo (GEV)	$C = \frac{2\widehat{\beta}_1-\widehat{\beta}_0}{3\widehat{\beta}_2-\widehat{\beta}_0} - \frac{\ln(2)}{\ln(3)}$ $\widehat{b} = 7,859C + 2,9554C^2$ $\widehat{a} = \frac{\left(2\widehat{\beta}_1-\widehat{\beta}_0\right)\widehat{b}}{\Gamma\left(1+\widehat{b}\right)\left(1-2^{-\widehat{b}}\right)}$ $\widehat{c} = \widehat{\beta}_0 + \frac{\widehat{a}}{\widehat{b}}\left[\Gamma\left(1+\widehat{b}\right) - 1\right]$	$C = \frac{2}{3+\widehat{\tau}_3} - \frac{\ln(2)}{\ln(3)}$ $\widehat{b} = 7,859C + 2,9554C^2$ $\widehat{a} = \frac{\widehat{\lambda}_2\widehat{b}}{\Gamma\left(1+\widehat{b}\right)\left(1-2^{-\widehat{b}}\right)}$ $\widehat{c} = \widehat{\lambda}_1 + \frac{\widehat{a}}{\widehat{b}}\left[\Gamma\left(1+\widehat{b}\right) - 1\right]$
Generalizada de Pareto (GPD)	Definidos para $a > -1$ Si $\widehat{c}$ es conocido: $\widehat{a} = \frac{\overline{x}-\widehat{c}}{2\widehat{\beta}_1-\overline{x}-2}$ $\widehat{b} = (1+\widehat{a})(\overline{x}-\widehat{c})$ Si $\widehat{c}$ es desconocido: usar Momentos-L	Definidos para $a > -1$ Si $\widehat{c}$ es conocido: $\widehat{a} = \frac{\widehat{\lambda}_1-\widehat{c}}{\widehat{\lambda}_2-2}$ $\widehat{b} = (1+\widehat{a})\left(\widehat{\lambda}_1-\widehat{c}\right)$ Si $\widehat{c}$ es desconocido: $\widehat{a} = \frac{1-3\widehat{\tau}_3}{1+\widehat{\tau}_3}$ $\widehat{b} = (1+\widehat{a})(2+\widehat{a})\widehat{\lambda}_2$ $\widehat{c} = \widehat{\lambda}_1 - (2+\widehat{a})\widehat{\lambda}_2$

Tabla 2.18: Estimación de parámetros de distribuciones usadas en el análisis de eventos extremos

Distribución	MPP	Momentos-L
Wakeby	$N_1 = 64\widehat{\alpha}_3 - 81\widehat{\alpha}_2 + 24\widehat{\alpha}_1 - \widehat{\alpha}_0$ $N_2 = 16\widehat{\alpha}_3 - 27\widehat{\alpha}_2 + 124\widehat{\alpha}_1 - \widehat{\alpha}_0$ $N_3 = 4\widehat{\alpha}_3 - 9\widehat{\alpha}_2 + 6\widehat{\alpha}_1 - \widehat{\alpha}_0$ $C_1 = 125\widehat{\alpha}_4 - 192\widehat{\alpha}_3 + 81\widehat{\alpha}_2 - 8\widehat{\alpha}_1$ $C_2 = 25\widehat{\alpha}_4 - 48\widehat{\alpha}_3 + 27\widehat{\alpha}_2 - 4\widehat{\alpha}_1$ $C_3 = 5\widehat{\alpha}_4 - 12\widehat{\alpha}_3 + 9\widehat{\alpha}_2 - 2\widehat{\alpha}_1$ $\widehat{b} = \frac{(N_3C_1 - N_1C_3) + \left[(N_1C_3 - N_3C_1)^2 - 4(N_1C_2 - N_2C_1)(N_2C_3 - N_3C_2)\right]^{0,5}}{2(N_2C_3 - N_3C_2)}$ $\widehat{d} = \frac{\left(N_1 + \widehat{b}N_2\right)}{N_2 + \widehat{b}N_3}$ $\widehat{m} = \frac{4\left(4+\widehat{b}\right)\left(4-\widehat{d}\right)\widehat{\alpha}_3 - 3\left(3+\widehat{b}\right)\left(3-\widehat{d}\right)\widehat{\alpha}_2 - 2\left(2+\widehat{b}\right)\left(2-\widehat{d}\right)\widehat{\alpha}_1 + \left(1+\widehat{b}\right)\left(1-\widehat{d}\right)\widehat{\alpha}_0}{4}$ $\widehat{a} = \frac{\left(\widehat{b}+1\right)\left(\widehat{b}+2\right)}{\widehat{b}\left(\widehat{b}+\widehat{d}\right)}\left[\frac{2\left(2+\widehat{b}\right)\left(2-\widehat{d}\right)\widehat{\alpha}_1}{2+\widehat{b}} - \frac{\left(1+\widehat{b}\right)\left(1-\widehat{d}\right)\widehat{\alpha}_0}{1+\widehat{b}} - \widehat{m}\right]$ $\widehat{c} = \frac{\left(1-\widehat{d}\right)\left(2-\widehat{d}\right)}{\widehat{d}\left(\widehat{b}+\widehat{d}\right)}\left[-\frac{2\left(2+\widehat{b}\right)\left(2-\widehat{d}\right)\widehat{\alpha}_1}{2+\widehat{d}} - \frac{\left(1+\widehat{b}\right)\left(1-\widehat{d}\right)\widehat{\alpha}_0}{1+\widehat{d}} - \widehat{m}\right]$	Solución de: $\widehat{\lambda}_1 = \widehat{m} + \frac{\widehat{a}}{\left(1+\widehat{b}\right)} + \frac{\widehat{c}}{\left(1-\widehat{d}\right)}$ $\widehat{\lambda}_2 = \frac{\widehat{a}}{\left(1+\widehat{b}\right)\left(2+\widehat{b}\right)} + \frac{\widehat{c}}{\left(1-\widehat{d}\right)\left(2-\widehat{d}\right)}$ $\widehat{\lambda}_3 = \frac{\widehat{a}\left(1-\widehat{b}\right)}{\left(1+\widehat{b}\right)\left(2+\widehat{b}\right)\left(3+\widehat{b}\right)} + \frac{\widehat{c}\left(1+\widehat{d}\right)}{\left(1-\widehat{d}\right)\left(2-\widehat{d}\right)\left(3-\widehat{d}\right)}$ $\widehat{\lambda}_4 = \frac{\widehat{a}\left(1-\widehat{b}\right)\left(2-\widehat{b}\right)}{\left(1+\widehat{b}\right)\left(2+\widehat{b}\right)\left(3+\widehat{b}\right)\left(4+\widehat{b}\right)} + \frac{\widehat{c}\left(1+\widehat{d}\right)\left(2+\widehat{d}\right)}{\left(1-\widehat{d}\right)\left(2-\widehat{d}\right)\left(3-\widehat{d}\right)\left(4-\widehat{d}\right)}$ $\widehat{\lambda}_5 = \widehat{a}\frac{\Gamma\left[1+\widehat{b}\right]\Gamma\left[4-\widehat{b}\right]}{\Gamma\left[1-\widehat{b}\right]\Gamma\left[4+\widehat{b}\right]} + \widehat{c}\frac{\Gamma\left[1-\widehat{d}\right]\Gamma\left[4+\widehat{d}\right]}{\Gamma\left[1+\widehat{d}\right]\Gamma\left[4-\widehat{d}\right]}$

Tabla 2.19: Estimación del factor de frecuencia para varias distribuciones

Distribución	**Factor de Frecuencia, K_T**
Normal	$K_T = t$
Lognormal 2P	$K_T = \frac{\exp\left[s_y t - 0{,}5 s_y^2\right] - 1}{\left(\exp\left[s_y^2\right] - 1\right)^{0,5}}$ $\overline{y} = \frac{1}{n}\sum_{i=1}^{n} \ln x_i \qquad s_y^2 = \frac{1}{n}\sum_{i=1}^{n} \left(\ln x_i - \overline{y}\right)^2$
Lognormal 3P	$K_T = \frac{\exp\left[y_T - \overline{y} - 0{,}5 s_y^2\right] - 1}{\left(\exp\left[s_y^2\right] - 1\right)^{0,5}}$ $z_1 = \frac{s_X}{\overline{x}} \qquad w = 0{,}5\left[-g_X + \left(g_X^2 + 4\right)^{0,5}\right]$ $z_2 = \left(1 - w^{2/3}\right)/w^{1/3} \qquad a = \overline{x} - s_X/z_2$ $\overline{y} = \frac{1}{n}\sum_{i=1}^{n} \ln(x - a) \qquad s_y = \frac{1}{n}\sum_{i=1}^{n} \left[\ln(x - a) - \overline{y}\right]^2$
Gumbel	Max: $K_T = -\frac{\sqrt{6}}{\pi}\left\{0{,}5772 + \ln\left[\ln\left(1 - \frac{1}{T}\right)\right]\right\}$ si $n \to \infty$ $K_T = \frac{y_T - \overline{y}}{s_y}$ si $n = n$ con $y_T = -\ln\left[-\ln\left(1 - 1/T\right)\right]$ $\overline{y} = \frac{1}{n}\sum_{m=1}^{n} y_m \qquad s_y^2 = \frac{1}{n}\sum_{m=1}^{n} \left(y_m - \overline{y}\right)^2$ donde $y_m = -\ln\left[-\ln\left(\frac{n+1-m}{n+1}\right)\right]$ con serie ordenada de mayor $(m = 1)$ a menor $(m = n)$. Min: $K_T = \frac{\sqrt{6}}{\pi}\left\{0{,}5772 + \ln\left[\ln\left(\frac{1}{T}\right)\right]\right\}$ si $n \to \infty$ $K_T = -\frac{y_T - \overline{y}}{s_y}$ si $n = n$ ... con $y_m = \ln\left[-\ln\left(\frac{m}{n+1}\right)\right]$ con serie ordenada de menor $(m = 1)$ a mayor $(m = n)$.

Tabla 2.20: Estimación del factor de frecuencia para varias distribuciones. Cont.

Distribución	**Factor de Frecuencia, K_T**
Exponencial	$K_T = -\ln(1/T) - 1 \quad (\max)$ $K_T = -\ln(1 - 1/T) - 1 \quad (\min)$
Gamma	$K_T = \frac{2}{g_X}\left\{A^3 - 1\right\}$ $A = \left[\frac{g_X}{6}\left(t - \frac{g_X}{6}\right) + 1\right]$ $0 < g_X \leq 1$
Pearson Tipo III	$K_T = t + \left(t^2 - 1\right) d + \frac{1}{3}\left(t^3 - 6t\right) d^2$ $-\left(t^2 - 1\right) d^3 + t d^4 + \frac{1}{3} d^5$ con $d = \frac{\widetilde{g}_X}{6}$ y $\widetilde{g}_X = g_X \frac{[n(n-1)]^{0,5}}{n-2}\left(1 + \frac{8,5}{n}\right)$
Log Pearson Tipo III	Calcular K_T con los logaritmos de x usando Pearson Tipo III
Fréchet	$K_T = \frac{[-\ln(1-1/T)]^{-1/\theta} - \Gamma(1-1/\theta)}{[\Gamma(1-2/\theta) - \Gamma^2(1-1/\theta)]^{0,5}}$ con θ estimado con el respectivo método
Weibull	$K_T = A + B\left\{\left[-\ln\left(1 - \frac{1}{T}\right)\right]^{1/\kappa} - 1\right\}$ $A = \frac{\widehat{\alpha} + \widehat{b} - \widehat{\mu}_Z}{\widehat{\sigma}_Z} \qquad B = \frac{\widehat{\alpha}}{\widehat{\sigma}_Z}$ con $\widehat{\alpha}$, κ, $\widehat{b}$, $\widehat{\mu}_Z$ y $\widehat{\sigma}_Z$ estimados con el respectivo método
GEV	$K_T = \frac{\widehat{b}}{\lvert\widehat{b}\rvert} \frac{\Gamma\left[1+\widehat{b}\right] - [-\ln(1-1/T)]^{\widehat{b}}}{\left\{\Gamma\left[1+2\widehat{b}\right] - \Gamma^2\left[1+\widehat{b}\right]\right\}^{0,5}}$
GPD	$K_T = \frac{(1+2\widehat{a})^{0,5}}{\widehat{a}}\left[(1+\widehat{a})\left(1 - T^{-\widehat{a}}\right) - \widehat{a}\right]$ con $\widehat{a}$ estimado con el respectivo método

$$1 - \frac{1}{T} = \int_{-\infty}^{t} \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{x^2}{2}\right] dx$$

Kite (1977), Rao y Hamed (2000), Heo y Salas (1996)

Tabla 2.21: Desviación estándar con Momentos

Distribución	s_T **con Momentos**
Normal	$s_T = \left[1 + \frac{t^2}{2}\right]^{0,5} \frac{s}{\sqrt{n}}$
Lognormal 2P	$s_T = \left[1 + \frac{t^2}{2}\right]^{0,5} \frac{s_y}{\sqrt{n}}$
Lognormal 3P	$s_T = \left[1 + \frac{t^2}{2}\right]^{0,5} \frac{s_y}{\sqrt{n}}$
Gumbel	$s_T^2 = \frac{s_X^2}{n}\left\{1 + \gamma_1 K_T + \frac{K_T^2}{4}\left[\gamma_2 - 1\right]\right\}$ con $\gamma_1 = 1{,}1396$ y $\gamma_2 = 5{,}4002$
Exponencial	$s_T^2 = \frac{1}{\lambda^2 n}\left[1 + 2K_T + 2K_T^2\right]$ con λ estimado con Momentos
Gamma	$s_T^2 = \frac{r}{\lambda^2 n}\left[\left(1 + v_X K_T\right)^2 + \frac{1}{2}\left(K_T + 2v_X B\right)^2\left(1 + v_X^2\right)\right]$ $B = \frac{-2}{g_X^2}\left\{A^3 - 1\right\} + \frac{2}{g_X}\left\{3A^2\left(\frac{t}{6} - \frac{2g_X}{36}\right)\right\}$ con λ estimado con Momentos
Pearson Tipo III	$s_T^2 = \frac{a^2 b}{n}(1 + \widetilde{g}_X K_T + 0{,}5K_T^2\left(\frac{3}{4}\widetilde{g}_X^2 + 1\right)$ $+3K_T L\left(\widetilde{g}_X + 0{,}25\widetilde{g}_X^3\right) + 3L^2\left(2 + 3\widetilde{g}_X^2 + 5\widetilde{g}_X^4/8\right))$ $L = \frac{t^2-1}{6} + \frac{4\left(t^3-6t\right)}{6^3}\widetilde{g}_X - \frac{3\left(t^2-1\right)}{6^3}\widetilde{g}_X^2$ $+\frac{4t}{6^4}\widetilde{g}_X^3 - \frac{10}{6^6}\widetilde{g}_X^4$ con a y b estimados con Momentos
Log Pearson Tipo III	Calcular s_T usando Momentos de Pearson con los logaritmos de x, es decir $s_{T,y}$ y $s_T = \frac{x_T\left[\exp(s_{T,y}) - \exp(-s_{T,y})\right]}{2}$ $x_T = \exp\left[\left(\widehat{c} + \widehat{a}\widehat{b}\right) + \widehat{a}\widehat{b}^{0,5} K_T\right]$ con $\widehat{a}$, $\widehat{b}$ y $\widehat{c}$ estimados con Momentos
Fréchet	$s_T^2 = \frac{A}{n}\left[1 + \gamma_1 K_T + 0{,}25\left(\gamma_2 - 1\right)K_T^2\right]$ $A = a^2\left(D_2 - D_1^2\right)$ $\gamma_1 = \frac{D_3 - 3D_2 D_1 + 2D_1^3}{\left(D_2 - D_1^2\right)^{1,5}}$ $\gamma_2 = \frac{D_4 - 4D_3 D_1 + 6D_2 D_1^2 - 3D_1^4}{\left(D_2 - D_1^2\right)^2}$ $D_r = \Gamma\left(1 - r/\theta\right)$ con θ estimado con Momentos

$1 - \frac{1}{T} = \int_{-\infty}^{t} \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{x^2}{2}\right] dx$

Kite (1977), Rao y Hamed (2000), Heo y Salas (1996)

Tabla 2.21: Desviación estándar con Momentos

Distribución	s_T **con Momentos**
Normal	$s_T = \left[1 + \frac{t^2}{2}\right]^{0,5} \frac{s}{\sqrt{n}}$
Lognormal 2P	$s_T = \left[1 + \frac{t^2}{2}\right]^{0,5} \frac{s_y}{\sqrt{n}}$
Lognormal 3P	$s_T = \left[1 + \frac{t^2}{2}\right]^{0,5} \frac{s_y}{\sqrt{n}}$
Gumbel	$s_T^2 = \frac{s_X^2}{n}\left\{1 + \gamma_1 K_T + \frac{K_T^2}{4}\left[\gamma_2 - 1\right]\right\}$ con $\gamma_1 = 1{,}1396$ y $\gamma_2 = 5{,}4002$
Exponencial	$s_T^2 = \frac{1}{\lambda^2 n}\left[1 + 2K_T + 2K_T^2\right]$ con λ estimado con Momentos
Gamma	$s_T^2 = \frac{r}{\lambda^2 n}\left[\left(1 + v_X K_T\right)^2 + \frac{1}{2}\left(K_T + 2v_X B\right)^2\left(1 + v_X^2\right)\right]$ $B = \frac{-2}{g_X^2}\left\{A^3 - 1\right\} + \frac{2}{g_X}\left\{3A^2\left(\frac{t}{6} - \frac{2g_X}{36}\right)\right\}$ con λ estimado con Momentos
Pearson Tipo III	$s_T^2 = \frac{a^2 b}{n}(1 + \widetilde{g}_X K_T + 0{,}5K_T^2\left(\frac{3}{4}\widetilde{g}_X^2 + 1\right)$ $+3K_T L\left(\widetilde{g}_X + 0{,}25\widetilde{g}_X^3\right) + 3L^2\left(2 + 3\widetilde{g}_X^2 + 5\widetilde{g}_X^4/8\right))$ $L = \frac{t^2-1}{6} + \frac{4\left(t^3-6t\right)}{6^3}\widetilde{g}_X - \frac{3\left(t^2-1\right)}{6^3}\widetilde{g}_X^2$ $+\frac{4t}{6^4}\widetilde{g}_X^3 - \frac{10}{6^6}\widetilde{g}_X^4$ con a y b estimados con Momentos
Log Pearson Tipo III	Calcular s_T usando Momentos de Pearson con los logaritmos de x, es decir $s_{T,y}$ y $s_T = \frac{x_T[\exp(s_{T,y}) - \exp(-s_{T,y})]}{2}$ $x_T = \exp\left[\left(\widehat{c} + \widehat{a}\widehat{b}\right) + \widehat{a}\widehat{b}^{0,5} K_T\right]$ con $\widehat{a}$, $\widehat{b}$ y $\widehat{c}$ estimados con Momentos
Fréchet	$s_T^2 = \frac{A}{n}\left[1 + \gamma_1 K_T + 0{,}25\left(\gamma_2 - 1\right)K_T^2\right]$ $A = a^2\left(D_2 - D_1^2\right)$ $\gamma_1 = \frac{D_3 - 3D_2 D_1 + 2D_1^3}{\left(D_2 - D_1^2\right)^{1,5}}$ $\gamma_2 = \frac{D_4 - 4D_3 D_1 + 6D_2 D_1^2 - 3D_1^4}{\left(D_2 - D_1^2\right)^2}$ $D_r = \Gamma\left(1 - r/\theta\right)$ con θ estimado con Momentos

$1 - \frac{1}{T} = \int_{-\infty}^{t} \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{x^2}{2}\right] dx$

Kite (1977), Rao y Hamed (2000), Heo y Salas (1996)

Tabla 2.22: Desviación estándar con Momentos - Cont.

Distribución	s_T con Momentos
Weibull	$s_T^2 = \frac{C}{n} I$ $I = 1 + K_T H + \frac{K_T^2}{4}\left[\frac{D}{C^2} - 1\right]$ $+G\left[\frac{2D}{C^2} - 3H^2 - 6 + K_T\left(\frac{E}{C^{2,5}} - \frac{1{,}5DH}{C^2} - 2{,}5H\right)\right]$ $+G^2\left[\frac{F}{C^3} - \frac{3EH}{C^{2,5}} - \frac{6D}{C^2} + \frac{2{,}25DH^2}{C^2} + 8{,}75H^2 + 9\right]$ $C = \alpha^2\left\{\Gamma\left[1+\frac{2}{\kappa}\right] - \Gamma^2\left[1+\frac{1}{\kappa}\right]\right\}$ $D = \alpha^4 J \qquad E = \alpha^5 L \qquad F = \alpha^6 M$ $H = B^3\left\{\Gamma\left[1+\frac{3}{\kappa}\right] - 3\Gamma\left[1+\frac{2}{\kappa}\right]\Gamma\left[1+\frac{1}{\kappa}\right] + 2\Gamma^3\left[1+\frac{1}{\kappa}\right]\right\}$ $J = \Gamma\left[1+\frac{4}{\kappa}\right] - 4\Gamma\left[1+\frac{3}{\kappa}\right]\Gamma\left[1+\frac{1}{\kappa}\right]$ $+6\Gamma\left[1+\frac{2}{\kappa}\right]\Gamma^2\left[1+\frac{1}{\kappa}\right] - 3\Gamma^4\left[1+\frac{1}{\kappa}\right]$ $L = \Gamma\left[1+\frac{5}{\kappa}\right] - 5\Gamma\left[1+\frac{4}{\kappa}\right]\Gamma\left[1+\frac{1}{\kappa}\right] + 4\Gamma^5\left[1+\frac{1}{\kappa}\right]$ $+10\Gamma\left[1+\frac{3}{\kappa}\right]\Gamma^2\left[1+\frac{1}{\kappa}\right] - 10\Gamma\left[1+\frac{2}{\kappa}\right]\Gamma^3\left[1+\frac{1}{\kappa}\right]$ $M = \Gamma\left[1+\frac{6}{\kappa}\right] - 6\Gamma\left[1+\frac{5}{\kappa}\right]\Gamma\left[1+\frac{1}{\kappa}\right]$ $+15\Gamma\left[1+\frac{4}{\kappa}\right]\Gamma^2\left[1+\frac{1}{\kappa}\right] - 20\Gamma\left[1+\frac{3}{\kappa}\right]\Gamma^3\left[1+\frac{1}{\kappa}\right]$ $+15\Gamma\left[1+\frac{2}{\kappa}\right]\Gamma^4\left[1+\frac{1}{\kappa}\right] - 5\Gamma^6\left[1+\frac{1}{\kappa}\right]$ siendo α y κ estimados con Momentos $H^+ = H + 0{,}05 \qquad H^- = H - 0{,}05$ Encontrar valores de κ^+ y κ^- que satisfagan $H^+ = B^3\left\{\Gamma\left[1+\frac{3}{\kappa^+}\right] - 3\Gamma\left[1+\frac{2}{\kappa^+}\right]\Gamma\left[1+\frac{1}{\kappa^+}\right]\right.$ $\left.+2\Gamma^3\left[1+\frac{1}{\kappa^+}\right]\right\}$ $H^- = B^3\left\{\Gamma\left[1+\frac{3}{\kappa^-}\right] - 3\Gamma\left[1+\frac{2}{\kappa^-}\right]\Gamma\left[1+\frac{1}{\kappa^-}\right]\right.$ $\left.+2\Gamma^3\left[1+\frac{1}{\kappa^-}\right]\right\}$ Calcular K_T^+ y K_T^- con κ^+ y κ^- $G \approx \frac{K_T^+ - K_T^-}{0{,}1}$
GPD	$s_T^2 = A^2C + B^2D + ABE \qquad$ si $\quad c = 0$ $A = \frac{-\widehat{b}}{\widehat{a}^2}\left(1 - T^{-\widehat{a}}\right) + \frac{\widehat{b}}{\widehat{a}}T^{-\widehat{a}}\ln(T)$ $B = \frac{1}{\widehat{a}}\left(1 - T^{-\widehat{a}}\right)$ $C = \frac{1}{n}\frac{(1+\widehat{a})^2(1+2\widehat{a})^2\left(1+\widehat{a}+6\widehat{a}^2\right)}{(1+2\widehat{a})(1+3\widehat{a})(1+4\widehat{a})}$ $B = \frac{2\widehat{b}^2}{n}\frac{(1+\widehat{a})^2\left(1+6\widehat{a}+12\widehat{a}^2\right)}{(1+2\widehat{a})(1+3\widehat{a})(1+4\widehat{a})}$ $E = \frac{\widehat{b}}{n}\frac{(1+\widehat{a})^2(1+2\widehat{a})\left(1+4\widehat{a}+12\widehat{a}^2\right)}{(1+2\widehat{a})(1+3\widehat{a})(1+4\widehat{a})}$ válido para $\widehat{a} > -0{,}25$ con $\widehat{a}$ y $\widehat{b}$ estimados con Momentos

$1 - \frac{1}{T} = \int_{-\infty}^{t} \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{x^2}{2}\right] dx$

Kite (1977), Rao y Hamed (2000), Heo y Salas (1996)

Tabla 2.23: Desviación estándar con MV

Distribución	s_T con MV
Normal	Igual que por Momentos
Lognormal 2P	$s_T = \left[1+\frac{t^2}{2}\right]^{0,5}\left[\frac{s_y^2\exp(2\overline{y}+2ts_y)}{n}\right]^{0,5}$
Lognormal 3P	$s_T^2 = \frac{1}{2nD} + \frac{t^2w^2}{4nD}\left[\left(s_y^2+1\right)e^{2\left(s_y^2-\overline{y}\right)} - e^{s_y^2-2\overline{y}}\right]$ $+\frac{w^2s_y^2}{nD}\left[\frac{s_y^2+1}{2s_y^2}e^{2\left(s_y^2-\overline{y}\right)} - e^{s_y^2-2\overline{y}}\right]$ $+\frac{tws_y}{nD}e^{0,5s_y^2-\overline{y}} - \frac{w}{nD}e^{0,5s_y^2-\overline{y}} + \frac{tw^2s_y}{nD}e^{0,5s_y^2-2\overline{y}}$ $D = \frac{s_y^2+1}{2s_y^2}e^{2\left(s_y^2-\overline{y}\right)} - \frac{2s_y^2+1}{2s_y^2}e^{s_y^2-2\overline{y}}$
Gumbel	$s_T^2 = \frac{\widehat{\alpha}^2}{n}\left\{1+\frac{6}{\pi^2}\left[1-0,5772+y_T\right]^2\right\}$ siendo $\widehat{\alpha}$ el parámetro de la fdp estimado con MV
Exponencial	$s_T^2 = \frac{1}{\lambda^2 n}\left(1+2K_T+\frac{n}{n-1}K_T^2\right)^2$ con λ estimado con MV
Gamma	$s_T^2 = \frac{r}{\lambda^2 n(rC-1)}\left[(rC-1)\left(\frac{1+\lvert\lambda\rvert K_T/\lambda}{r^{0,5}}\right)^2+\frac{K_T^2}{4r}\right.$ $+\frac{B^2}{r^2}+\frac{\lvert\lambda\rvert K_T/\lambda}{r^{1,5}}B$ $C = \frac{1}{r}+\frac{1}{2r^2}+\frac{1}{6r^3}-\frac{1}{30r^5}+\frac{1}{42r^7}-\frac{1}{30r^9}+\frac{10}{132r^{11}}$ $C = \pi^2/6$ si $r=1$ con λ estimado con MV
Pearson Tipo III	$s_T^2 = P_1^2E+P_2^2F+I+2P_1P_2C_1+2P_1C_2+2P_2C_3$ $E = \frac{1}{na^2G}\left[\frac{H}{b-2}-\frac{1}{(b-1)^2}\right]$ $F = \frac{2}{na^4G(b-2)}$ $I = \frac{bH-1}{na^2G}$ $C_1 = \frac{-1}{na^3G}\left[\frac{1}{b-2}-\frac{1}{b-1}\right]$ $C_2 = \frac{1}{na^2G}\left[\frac{1}{b-1}-H\right]$ $C_3 = \frac{-1}{na^3G}\left[\frac{b}{b-1}-1\right]$ $G = \frac{1}{(b-2)a^4}\left[2H-\frac{2b-3}{(b-1)^2}\right]$ $H = \frac{1}{b}+\frac{1}{2b^2}+\frac{1}{6b^3}+\frac{1}{30b^5}+\frac{1}{42b^7}+\frac{1}{30b^9}+\frac{10}{132b^{11}}$ $P_1 = b+\mathrm{sgn}(a)K_Tb^{0,5}$ $P_2 = a+0{,}5aK_Tb^{-0,5}-ab^{-1}P_3$ $P_3 = \frac{-2}{\widetilde{g}_X^2}\left[P_4^3-1\right]+\frac{2}{\widetilde{g}_X}\left[3P_4^2\left\{\frac{t}{6}-\frac{2\widetilde{g}_X}{36}\right\}\right]$ $P_4 = \frac{\widetilde{g}_X}{6}\left[t-\frac{\widetilde{g}_X}{6}\right]+1$ con a y b estimados con MV

$$1-\frac{1}{T} = \int_{-\infty}^{t}\frac{1}{\sqrt{2\pi}}\exp\left[-\frac{x^2}{2}\right]dx$$

Kite (1977), Rao y Hamed (2000), Heo y Salas (1996)

Tabla 2.24: Desviación estándar con MV - Cont.

Distribución	s_T con MV
Log Pearson Tipo III	Calcular s_T usando MV de Pearson con los logaritmos de x, es decir $s_{T,y}$ y $s_T = \frac{x_T[\exp(s_{T,y})-\exp(-s_{T,y})]}{2}$ $x_T = \exp\left[\left(\widehat{c}+\widehat{a}\widehat{b}\right)+\widehat{a}\widehat{b}^{0,5}K_T\right]$ con $\widehat{a}$, $\widehat{b}$ y $\widehat{c}$ estimados con MV
Fréchet	$s_T^2 = \frac{X_T^2}{n\theta^2}\left[1+\frac{\{0,4228-\ln[-\ln(1-1/T)]\}^2}{\pi^2/6}\right]$ con θ estimado con MV
Weibull	Para $\kappa > 2$: $D = -\frac{\alpha}{\kappa^2}\left[\ln(T)\right]^{1/\kappa}\ln\left[\ln(T)\right]$; $E = \left[\ln(T)\right]^{1/\kappa}$ $K = 1{,}82368$; $C = \left\{\Gamma\left[1-\frac{2}{\kappa}\right]+\kappa\Gamma\left[2-\frac{2}{\kappa}\right]\right\}\left[\frac{\kappa-1}{\kappa^2}\right]$ $A = 1+\psi\left[2-\frac{1}{\kappa}\right]$; $J = \Gamma\left[1-\frac{1}{\kappa}\right]-A\Gamma\left[2-\frac{1}{\kappa}\right]$ $M = KC-0{,}84557J\Gamma\left[2-\frac{1}{\kappa}\right]-0{,}17875C$ $-K\Gamma^2\left[2-\frac{1}{\kappa}\right]-J^2$ $p_{11} = \frac{1{,}644934}{\kappa^2 n}$; $p_{22} = \frac{C-\Gamma^2[2-1/\kappa]}{M}$; $p_{33} = \frac{KC-J^2}{\kappa^2 M}$; $p_{12} = -\frac{J+0{,}422784\Gamma[2-1/\kappa]}{M}$; $p_{13} = -\frac{0{,}422784J+K\Gamma[2-1/\kappa]}{\kappa^2 M}$ $p_{23} = \frac{0{,}422784C+J\Gamma[2-1/\kappa]}{M}$; $v_1 = \frac{\alpha^2}{n}p_{11}$; $v_2 = \frac{\kappa^2}{n}p_{22}$; $v_3 = \frac{\alpha^2}{n}p_{33}$ $c_1 = \frac{\alpha}{n}p_{12}$; $c_2 = \frac{\alpha^2}{n}p_{13}$; $c_3 = \frac{\alpha}{n}p_{23}$ $s_T^2 = v_1+D^2v_2+E^2v_3+2Dc_1+2Ec_2+2DEc_3$
GPD	$s_T^2 = A^2C+B^2D+ABE$ si $c=0$ $A = \frac{-\widehat{b}}{\widehat{a}^2}\left(1-T^{-\widehat{a}}\right)+\frac{\widehat{b}}{\widehat{a}}T^{-\widehat{a}}\ln(T)$ $B = \frac{1}{\widehat{a}}\left(1-T^{-\widehat{a}}\right)$ $C = \frac{1}{n}(1-\widehat{a})^2$ $B = \frac{2\widehat{b}^2}{n}(1-\widehat{a})$ $E = \frac{\widehat{b}}{n}(1-\widehat{a})$ válido para $\widehat{a} < 0{,}50$ con $\widehat{a}$ y $\widehat{b}$ estimados con MV

$1-\frac{1}{T} = \int_{-\infty}^{t}\frac{1}{\sqrt{2\pi}}\exp\left[-\frac{x^2}{2}\right]dx$; $\psi[\cdot]$ = función Digamma

Kite (1977), Rao y Hamed (2000), Heo y Salas (1996)

Tabla 2.25: Desviación estándar con MPP

Distribución	s_T con MPP
Normal	$s_T = \left[1 + 0{,}5113 t^2\right]^{0,5} \frac{s}{\sqrt{n}}$
Gumbel	$s_T^2 = \frac{\widehat{\alpha}^2}{n}\left(1{,}1128 + 0{,}4574 y_T + 0{,}8046 y_T^2\right)$ siendo $\widehat{\alpha}$ el parámetro de la fdp estimado con MPP
Exponencial	$s_T^2 = \frac{1}{\lambda^2 n}\left[1 + 2K_T + \frac{4}{3}K_T^2\right]$ con λ estimado con MPP
Fréchet	$s_T^2 = A\left[1 - \left(2 - 2^{1+1/\theta}\right)B\right]\Gamma\left(1 - 2/\theta\right)$ $+A\left[1 - 2^{1+1/\theta} + 2H\right]B^2\Gamma\left(1 - 2/\theta\right)$ $-A\left[1 + \left(2 - 2^{1+1/\theta}\right)B + \left(1 + 2^{1+1/\theta}\right)B^2\right]\Gamma^2\left(1 - 1/\theta\right)$ $A = \frac{X_T^2}{n\Gamma^2(1-1/\theta)}$ $B = \left\{\psi\left(1 - 1/\theta\right) - \ln\left[-\ln\left(1 - 1/T\right)\right]\right\}/\ln\left(2\right)$ siendo ψ la función Digamma $H = \frac{\theta^2\left(1-2^{1/\theta}\right)^2}{3^{1/\theta}\left(1-2^{1/\theta}\right)\ln(3) - 2^{1/\theta}\left(1-3^{1/\theta}\right)\ln(2)}$ con θ estimado con MPP
Weibull	
GEV	$s_T^2 = \frac{1}{n}\left\{\widehat{a}^2 w_1 + \widehat{a}^2 w_2 A^2 + w_3 B^2\right\}$ $+\frac{1}{n}\left\{2\widehat{a}^2 w_4 A + 2\widehat{a} w_5 B + 2\widehat{a} w_6 AB\right\}$ $C = -\ln\left(1 - \frac{1}{T}\right)$ $A = \frac{1}{\widehat{b}}\left\{1 - C^{\widehat{b}}\right\}$ $B = -\frac{\widehat{a}}{\widehat{b}^2}\left\{1 - C^{\widehat{b}}\right\} - \frac{\widehat{a}}{\widehat{b}}\left\{C^{\widehat{b}}\ln\left[C\right]\right\}$ con valores de w_i dados en Tabla 2.26 y $\widehat{a}$ y $\widehat{b}$ estimados con MPP
GPD	$s_T^2 = A^2 C + B^2 D + ABE \qquad \text{si} \quad c = 0$ $A = \frac{-\widehat{b}}{\widehat{a}^2}\left(1 - T^{-\widehat{a}}\right) + \frac{\widehat{b}}{\widehat{a}}T^{-\widehat{a}}\ln\left(T\right)$ $B = \frac{1}{\widehat{a}}\left(1 - T^{-\widehat{a}}\right)$ $C = \frac{1}{n}\frac{(1+\widehat{a})(2+\widehat{a})^2\left(1+\widehat{a}+2\widehat{a}^2\right)}{(1+2\widehat{a})(3+2\widehat{a})}$ $B = \frac{\widehat{b}^2}{n}\frac{\left(7+18\widehat{a}+11\widehat{a}^2+2\widehat{a}^3\right)}{(1+2\widehat{a})(3+2\widehat{a})}$ $E = \frac{\widehat{b}}{n}\frac{(2+\widehat{a})\left(2+6\widehat{a}+7\widehat{a}^2+2\widehat{a}^3\right)}{(1+2\widehat{a})(3+2\widehat{a})}$ válido para $\widehat{a} > -0{,}50$ con $\widehat{a}$ y $\widehat{b}$ estimados con MPP

$1 - \frac{1}{T} = \int_{-\infty}^{t} \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{x^2}{2}\right] dx$

Kite (1977), Rao y Hamed (2000), Heo y Salas (1996)

Tabla 2.26: Valores de wi para estimar s_T con MPP para GEV

b	w_1	w_2	w_3	w_4	w_5	w_6
-0.4	1.6637	1.8461	2.9092	1.3355	1.1405	1.1628
-0.3	1.4153	1.2574	1.4090	0.8912	0.5640	0.4442
-0.2	1.3322	1.0013	0.9139	0.6727	0.3926	0.2697
-0.1	1.2915	0.8440	0.6815	0.5104	0.3245	0.2240
0.0	1.2686	0.7390	0.5633	0.3704	0.2992	0.2247
0.1	1.2551	0.6708	0.5103	0.2411	0.2966	0.2447
0.2	1.2474	0.6330	0.5021	0.1177	0.3081	0.2728
0.3	1.2438	0.6223	0.5294	-0.0023	0.3297	0.3033
0.4	1.2433	0.6368	0.5880	-0.1205	0.3592	0.3329

Tabla 2.27: Usos de distribuciones para diversas variables hidrológicas según algunos autores

Fdp	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**	**K**	**L**	**M**	**N**
Exp.	**b**					**g**								
Normal	**b**		**d** **e**			**b**								
Log-normal	**b**	**a**	**c** **d**	**f** **g**		**a** **g**	**g**		**a**	**a**				
Gamma			**e**			**a**								
Gumbel (max)	**g**	**a** **b** **c** **e**	**a**			**e** **g**	**a**	**c**	**g**					**c**
Gumbel (min)				**a** **e**	**f**									
Weibull				**a** **c** **e** **g**	**g**			**c**						
Fréchet		**a**				**a**					**a**			
Pearson	**b**		**b** **c** **d** **e**	**e** **f**		**c**				**g**				
Log Pearson		**b** **e**	**c** **d** **e**	**e**		**a** **c** **e** **g**								
Pareto			**b**										**c** **e**	
GEV	**c**		**b** **e**	**a**		**a** **c**			**a** **g**	**g**	**a**	**a**		
Wakeby			**b**											

(a) Kottegoda y Rosso (2008)
(b) Rao y Hamed (2000)
(c) Stedinger et al. (1993)
(d) McCuen (2003)
(e) Singh (1998)
(f) Joshi y St-Hilaire (2013)
(g) WMO (2009)

Tabla 2.28: Descripción de variables consideradas en la Tabla 2.27

Variable	Descripción
A	Caudal máximo anual
B	Caudal máximo
C	Caudal de creciente
D	Caudal mínimo
E	Caudal mínimo anual
F	Lluvia máxima
G	Lluvia máxima minutal, horaria, diaria
H	Lluvia máxima diaria
I	Variable meteorológica máxima
J	Variable hidrológica máxima
K	Viento máximo
L	Altura de ola máxima
M	Lluvia o caudal mayor que valor umbral
N	Concentración máxima contaminante

Tabla 2.29: Papeles de probabilidad para algunas distribuciones

Distribución	**PG sugeridas**	**Eje x**	**Eje y**
Gumbel	$p_i = \frac{i-0,44}{n+0,12}$ $p_i = \frac{i-0,35}{n}$	$-\ln\left[-\ln p_i\right]$	x_i
Weibull	$p_i = \frac{i-0,35}{n}$	$-\ln\left[-\ln\left(1-p_i\right)\right]$	$\ln\left[x_i\right]$
Pearson Tipo III	$p_i = \frac{i-0,375}{n+0,25}$	$\frac{2}{\gamma}\left[1+\frac{\gamma\Phi^{-1}(p_i)}{6}-\frac{\gamma^2}{36}\right]^3-\frac{2}{\gamma}$	x_i
Log Pearson Tipo III	$p_i = \frac{i-0,375}{n+0,25}$	$\frac{2}{\gamma}\left[1+\frac{\gamma\Phi^{-1}(p_i)}{6}-\frac{\gamma^2}{36}\right]^3-\frac{2}{\gamma}$	$\ln\left[x_i\right]$
GEV	$p_i = \frac{i-0,4}{n+0,2}$ $p_i = \frac{i-0,35}{n}$	$-\left[-\ln p_i\right]^\kappa$ con κ para máximo ajuste lineal	x_i