This lecture: Motion Analysis

- Remainder of segmentation lecture
- Motion Analysis
  - Optical Flow
  - Aperture Problem
  - Lucas-Kanade
  - Horn-Shunk & Variational Methods
  - Layered Models & Segmentation

  - By Prof. I. Kokkinos

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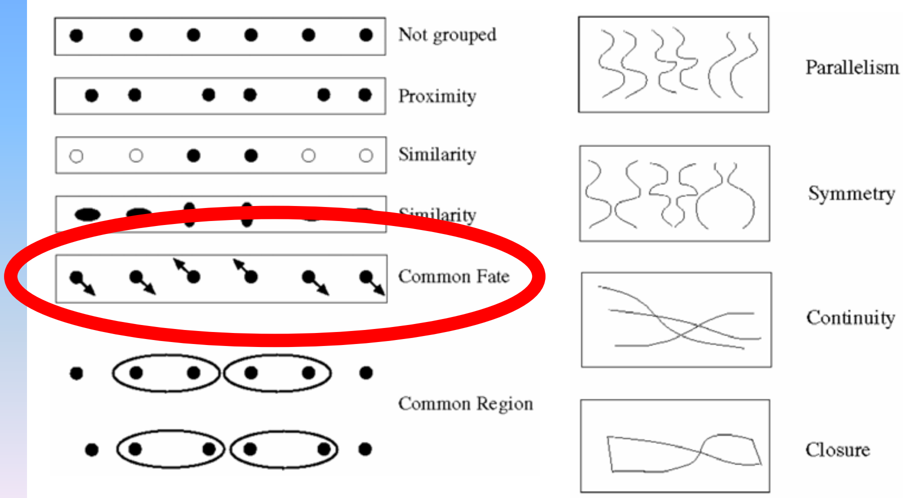
Motion and perceptual organization



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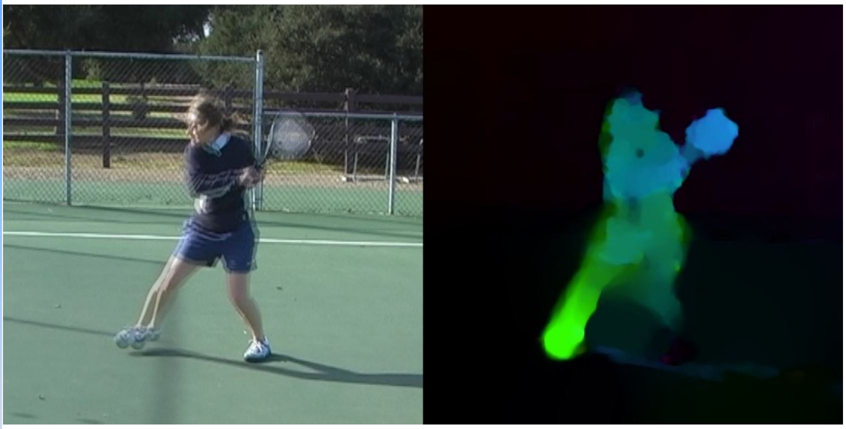
Motion Estimation



Brox et. al. PAMI 2011

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Why estimate visual motion?

- Recover dynamic information in a scene
  - Track objects and analyze trajectories
  - Action recognition
  - Segment objects based on motion
  - Applications: surveillance & security, car industry, robotics, multimedia, …
- Improve video quality
  - Camera instabilities cause jitter
  - Measure it and remove it (stabilize)
- Estimate 3D structure from motion pattern

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Action Recognition

- Even impoverished input can reveal action class

G. Johansson, “Visual Perception of Biological Motion and a Model For Its Analysis", *Perception and Psychophysics 14, 201-211, 1973.*

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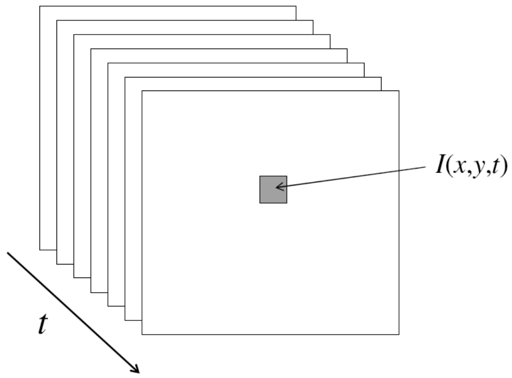
Video

- A video is a sequence of frames captured over time
- Now our image data is a function of space (x, y) and time (t)



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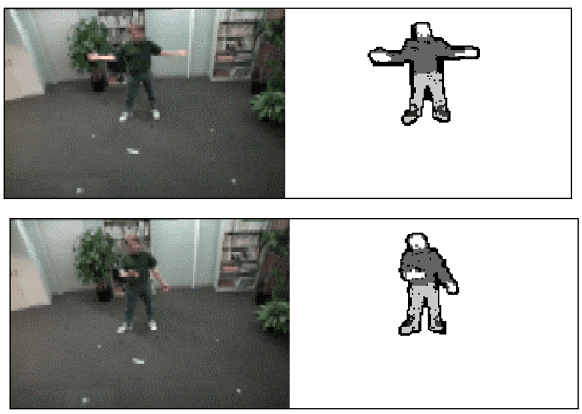
Motion Analysis & Video processing

- Background subtraction
  - A static camera is observing a scene.
  - Goal: separate the static *background* from the moving *foreground.*



How to come up with background frame estimate without access to "empty" scene?

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Motion Segmentation & Surveillance

- Static camera observing crowds
  - Example application: count number of people going in two directions and detect emergencies

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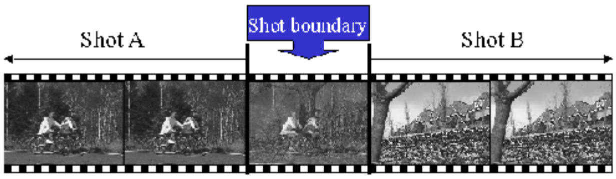
Motion Segmentation & Video Analysis

- Shot boundary detection
  - Commercial video is usually composed of *shots* or sequences showing the same objects or scene.
  - Goal: segment video into shots for summarization and browsing (each shot can be represented by a single keyframe).



- Customized delivery of multimedia
  - Detect beginnings of commercials/TV shows/weather/music
  - Deliver user-specific content (e.g. no R & B !)

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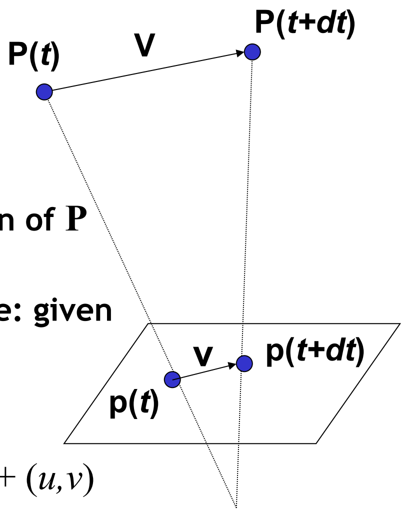
Motion Field

- $\mathbf{P}(t)$ is a moving 3D point
- Velocity of scene point: $\mathbf{V} = \mathrm{d}\mathbf{P}/\mathrm{d}t$
- $\mathbf{p}(t) = (x(t),y(t))$ is the projection of $\mathbf{P}$ in the image.
- Apparent velocity $\mathbf{v}$ in the image: given by components $u = \mathrm{d}x/\mathrm{d}t$ and $v = \mathrm{d}y/\mathrm{d}t$
- Considering $\mathrm{d}t = 1$: $\mathbf{p}(t+1) = \mathbf{p}(t) + (u,v)$
- Motion estimation task:
  - Estimate (u,v)



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Optical flow

- Optical flow is the *apparent* motion of brightness patterns in the image
- Optical flow should ideally equal the motion field
- But apparent motion can be caused by lighting changes without any actual motion

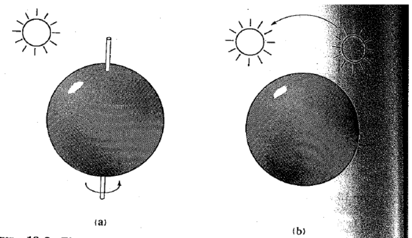

Figure 12-2. The optical flow is not always equal to the motion field. In (a) a smooth sphere is rotating under constant illumination—the image does not change, yet the motion field is nonzero. In (b) a fixed sphere is illuminated by a moving source—the shading in the image changes, yet the motion field is zero.

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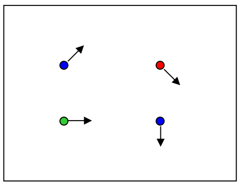
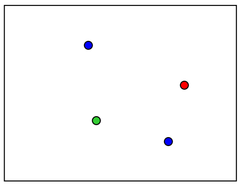
Estimating Optical Flow



$I(x,y,t-1)$



$I(x,y,t)$

- Given two subsequent frames, estimate the apparent motion field $u(x,y)$ and $v(x,y)$ between them.

- Consider 1D case first

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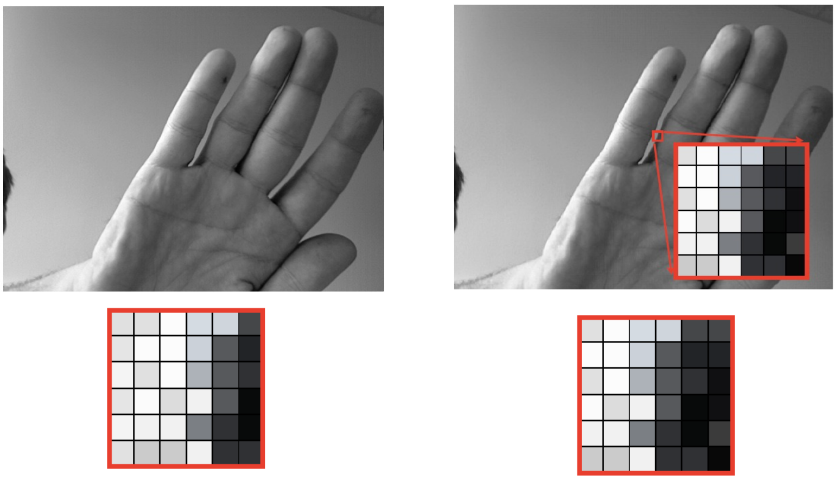
Optical flow estimation - 2D to 1D



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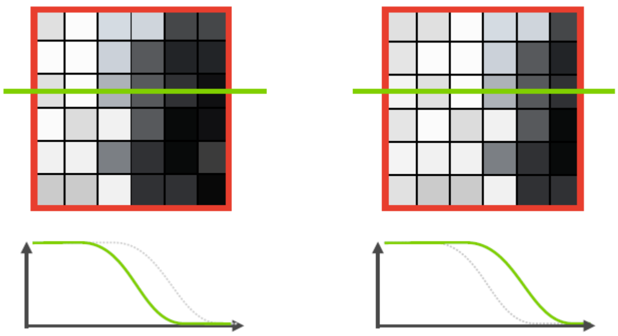
Optical flow estimation - 1D case

- How can we estimate the displacement?



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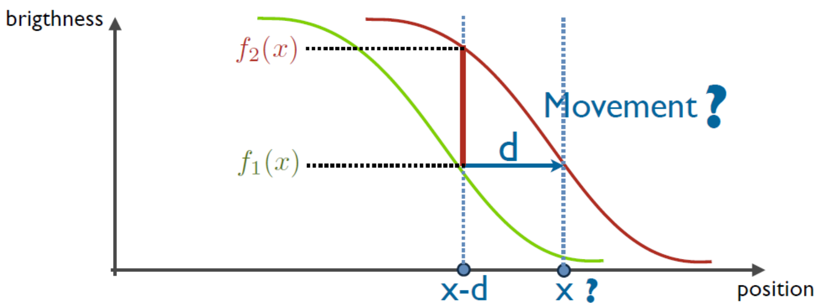
Optical Flow Estimation - 1D case

- Known: Gradient, Difference of intensities at x-d
- Unknown: d



$$d = \frac{f_1(x) - f_2(x)}{f_1'(x)}$$

How about 2D?

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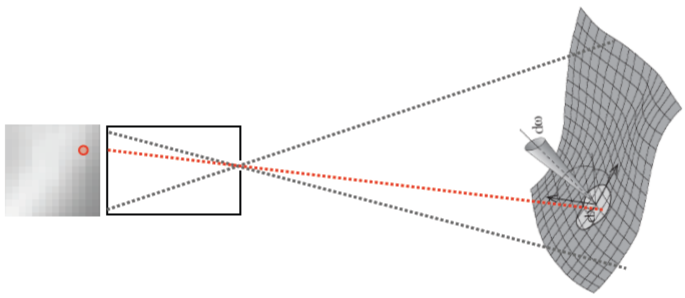
Brightness constancy Constraint

- Image is projection of 3D environment
- Each pixel: projection of a surface patch
- Pixel intensity influenced by 3D surface, incident light, camera …
- Assumption: intensity of surface patch remains constant



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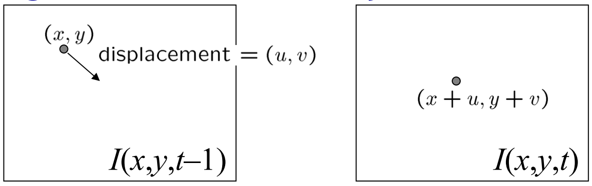
The Brightness Constancy Constraint



- **Brightness Constancy Equation:**

$$I(x, y, t-1) = I(x + u(x, y), y + v(x, y), t)$$

- **Linearizing the right hand side using Taylor expansion:**

$$I(x, y, t-1) \approx I(x, y, t) + I_x \cdot u(x, y) + I_y \cdot v(x, y)$$

- **Hence,** $I_x \cdot u + I_y \cdot v + I_t \approx 0$

The Brightness Constancy Constraint

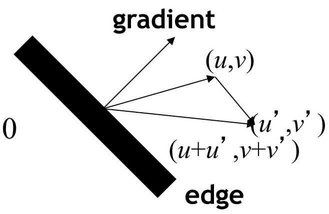
$$I_x \cdot u + I_y \cdot v + I_t = 0$$

- **How many equations and unknowns per pixel?**
  - One equation, two unknowns
- **Intuitively, what does this constraint mean?**

$$\nabla I \cdot (u, v) + I_t = 0$$

- **The component of the flow perpendicular to the gradient (i.e., parallel to the edge) is unknown**

If (u,v) satisfies the equation,
so does $(u+u', v+v')$ if $\nabla I \cdot (u', v') = 0$



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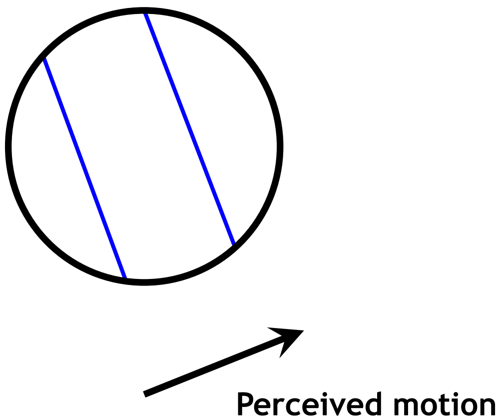
The Aperture Problem



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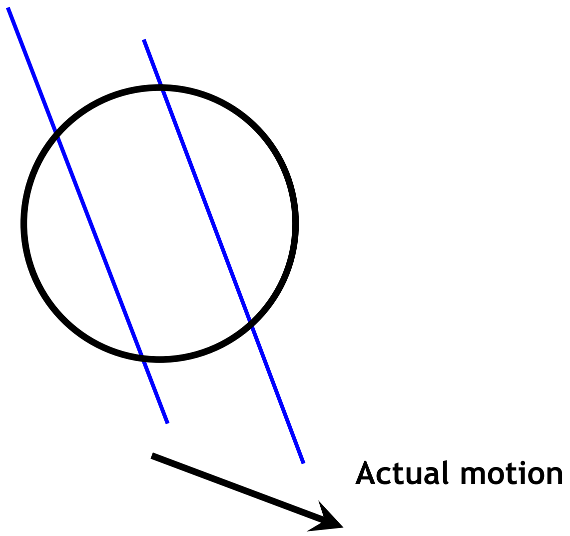
The Aperture Problem



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The Barber Pole Illusion

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The Barber Pole Illusion

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The Barber Pole Illusion

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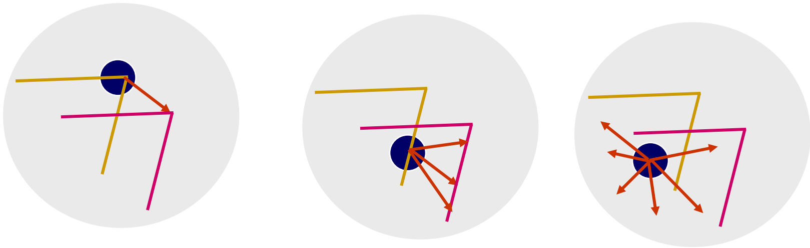
Local Patch Analysis

- How *certain* are the motion estimates?



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$$\begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}$$

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Solving the aperture problem

- How to get more equations for a pixel?
- Spatial coherence constraint: pretend the pixel's neighbors have the same (u,v)
  - If we use a 5x5 window, that gives us 25 equations per pixel

$$0 = I_t(\mathbf{p_i}) + \nabla I(\mathbf{p_i}) \cdot [u\ v]$$

$$\begin{bmatrix} I_x(\mathbf{p}_1) & I_y(\mathbf{p}_1) \\ I_x(\mathbf{p}_2) & I_y(\mathbf{p}_2) \\ \vdots & \vdots \\ I_x(\mathbf{p}_{25}) & I_y(\mathbf{p}_{25}) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} I_t(\mathbf{p}_1) \\ I_t(\mathbf{p}_2) \\ \vdots \\ I_t(\mathbf{p}_{25}) \end{bmatrix}$$

B. Lucas and T. Kanade. An iterative image registration technique with an application to stereo vision. In *Proceedings of the International Joint Conference on Artificial Intelligence*, pp. 674-679, 1981.

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Lucas-Kanade flow

- Least squares problem:

$$\begin{bmatrix} I_x(\mathbf{p}_1) & I_y(\mathbf{p}_1) \\ I_x(\mathbf{p}_2) & I_y(\mathbf{p}_2) \\ \vdots & \vdots \\ I_x(\mathbf{p}_{25}) & I_y(\mathbf{p}_{25}) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} I_t(\mathbf{p}_1) \\ I_t(\mathbf{p}_2) \\ \vdots \\ I_t(\mathbf{p}_{25}) \end{bmatrix} \qquad \underset{25\text{x}2}{A}\ \underset{2\text{x}1}{d} = \underset{25\text{x}1}{b}$$

- Minimum least squares solution given by solution of

$$\underset{2\text{x}2}{(A^T A)}\ \underset{2\text{x}1}{d} = \underset{2\text{x}1}{A^T b}$$

$$\underbrace{\begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix}}_{A^T A} \begin{bmatrix} u \\ v \end{bmatrix} = - \underbrace{\begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}}_{A^T b}$$

(The summations are over all pixels in the K x K window)

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Conditions for Solvability

- Optimal (u, v) satisfies Lucas-Kanade equation

$$\underbrace{\begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix}}_{A^T A} \begin{bmatrix} u \\ v \end{bmatrix} = - \underbrace{\begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}}_{A^T b}$$

- **When is this solvable?**
  - A^TA should be invertible.

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Conditions for Solvability

$$\underbrace{\begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix}}_{A^T A} \begin{bmatrix} u \\ v \end{bmatrix} = - \underbrace{\begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}}_{A^T b}$$

- Recall the Harris corner detector: $M = A^TA$ is the *second moment matrix*
- We can figure out whether the system is solvable by looking at the eigenvalues of the second moment matrix
  - The eigenvectors and eigenvalues of *M* relate to edge direction and magnitude
  - The eigenvector associated with the larger eigenvalue points in the direction of fastest intensity change, and the other eigenvector is orthogonal to it

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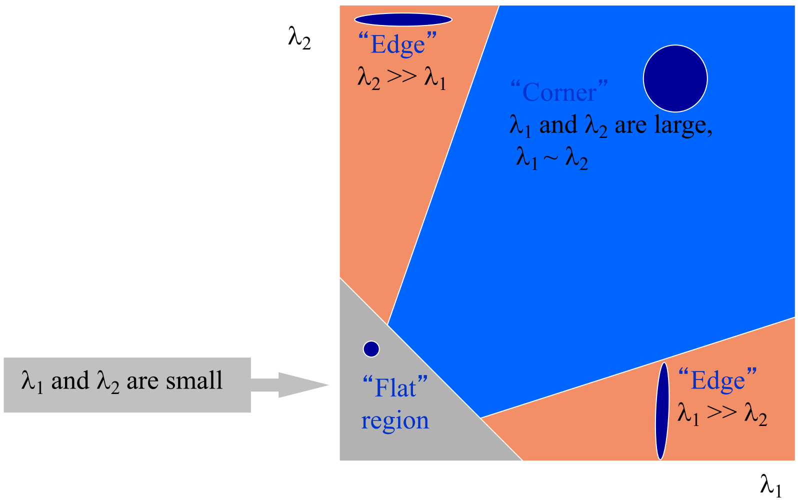
Interpreting the eigenvalues

Classification of image points using eigenvalues of the second moment matrix:



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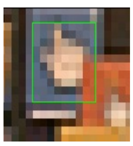
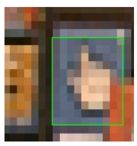
SSD measure reminder

- **Sum of squared differences**

$$E(x, y; d) = \sum_{(x', y') \in N(x,y)} [I_L(x'+d, y') - I_R(x', y')]^2$$

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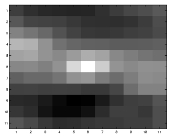
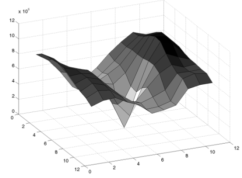
High-Texture Region





- Gradients are different, large magnitude
- Large λ_1 , large λ_2

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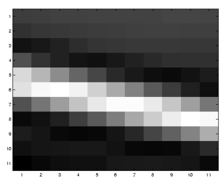
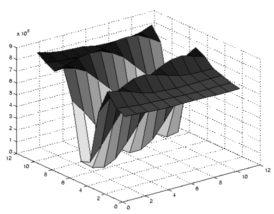
Edge





- Gradients very large or very small
- Large λ_1 , small λ_2

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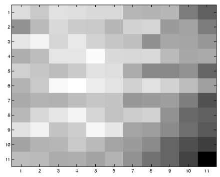
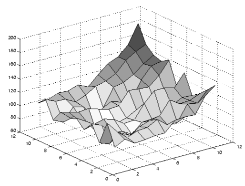
Low-Texture Region





- Gradients have small magnitude
- Small λ_1 , small λ_2

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Shi-Tomasi feature tracker

1. Find good features (min eigenvalue of 2×2 Hessian)
2. Use Lucas-Kanade to track with pure translation
3. Use affine registration with first feature patch
4. Terminate tracks whose dissimilarity gets too large
5. Start new tracks when needed

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Tracking example

Figure 1: Three frame details from Woody Allen's *Manhattan*. The details are from the 1st, 11th, and 21st frames of a subsequence from the movie.

Figure 2: The traffic sign windows from frames 1,6,11,16,21 as tracked (top), and warped by the computed deformation matrices (bottom).

J. Shi and C. Tomasi. Good Features to Track. CVPR 1994.

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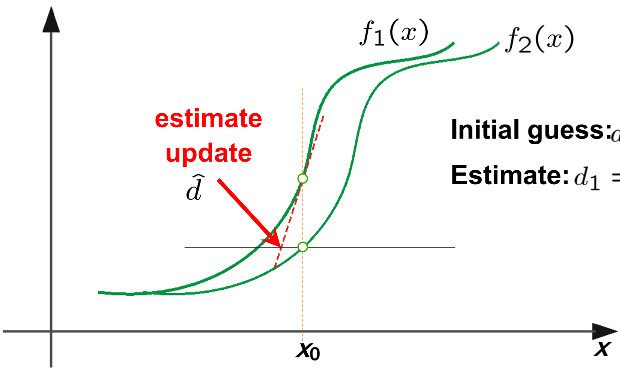
Optical Flow: Iterative Estimation



Initial guess: $d_0 = 0$

Estimate: $d_1 = d_0 + \hat{d}$

(using *d* for *displacement* here instead of *u*)

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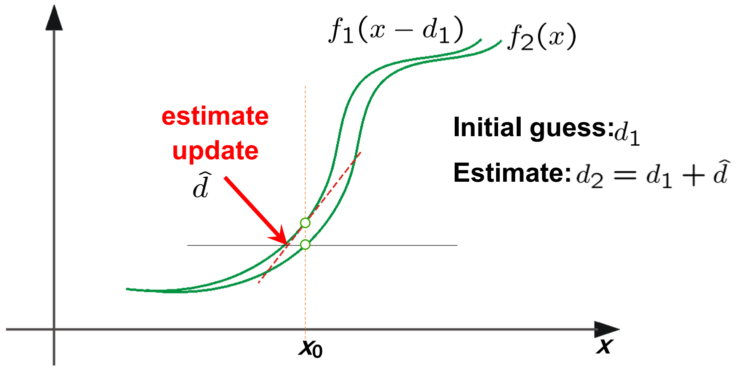
Optical Flow: Iterative Estimation



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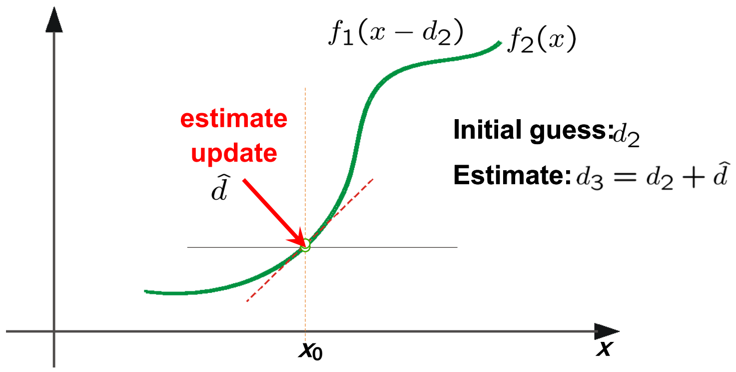
Optical Flow: Iterative Estimation



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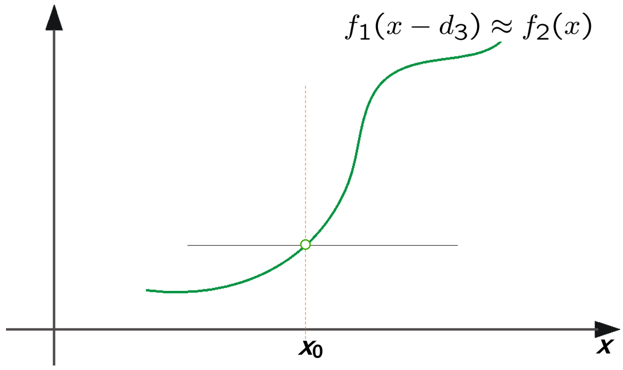
Optical Flow: Iterative Estimation



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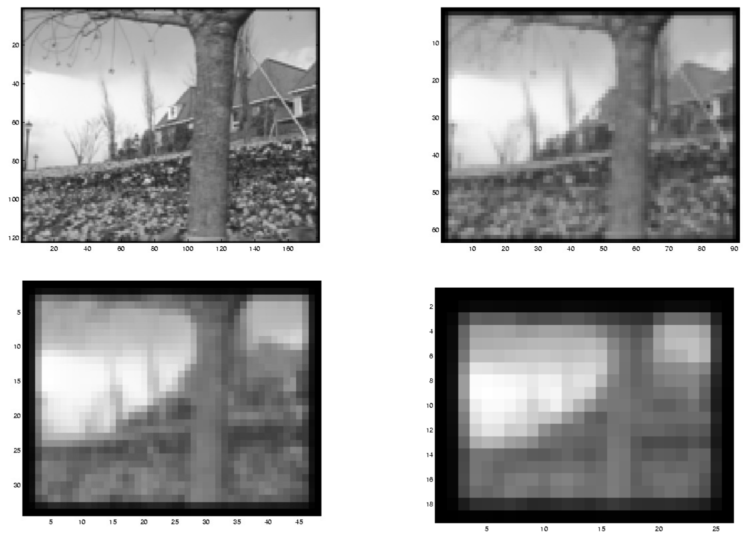
Large Displacements: Reduce Resolution!



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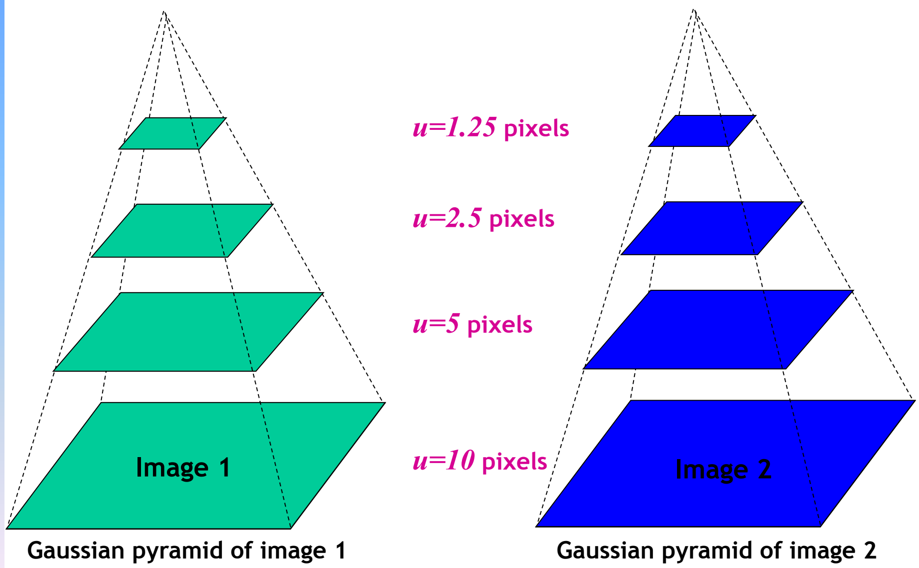
Coarse-to-fine Optical Flow Estimation



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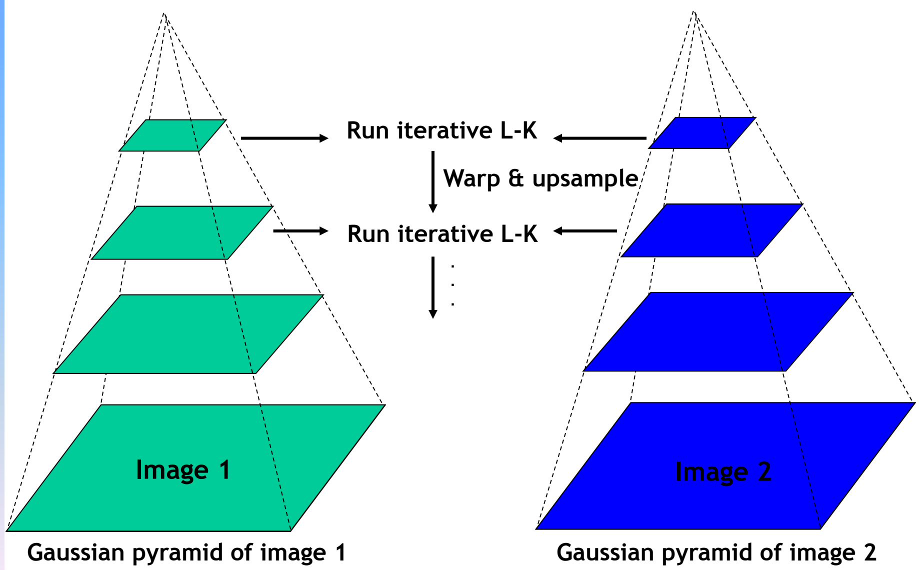
Coarse-to-fine Optical Flow Estimation



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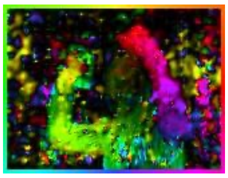

Lucas Kanade

Horn Schunk

Anisotropic

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Solving the aperture problem, revisted

- Lucas Kanade: pixel's neighbors have the same (u,v)
  - Brighntess constancy for a 5x5 window

$$0 = I_t(\mathbf{p_i}) + \nabla I(\mathbf{p_i}) \cdot [u \;\; v] \qquad \begin{bmatrix} I_x(\mathbf{p}_1) & I_y(\mathbf{p}_1) \\ I_x(\mathbf{p}_2) & I_y(\mathbf{p}_2) \\ \vdots & \vdots \\ I_x(\mathbf{p}_{25}) & I_y(\mathbf{p}_{25}) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} I_t(\mathbf{p}_1) \\ I_t(\mathbf{p}_2) \\ \vdots \\ I_t(\mathbf{p}_{25}) \end{bmatrix}$$

  - Solve for (u,v) separately at each pixel using least squares

- Horn-Schunk: neighboring pixels have similar (u,v)

$$J(u,v) = \frac{1}{2} \iint_\Omega \underbrace{[I_t + \nabla I \cdot (u,v)]^2}_{\text{Brightness Constancy}}$$

  - (u,v) estimates become coupled through Spatial Smoothness term
- Filling-in: smoothness term resolves aperture problem

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Variational Optical Flow-I

- Cost functional

$$J(u,v) = \frac{1}{2}\iint_{\Omega} \underbrace{\left[I_t + \nabla I \cdot (u,v)\right]^2}_{\text{Brightness Constancy}} + \lambda \underbrace{\left[(\nabla u)^2 + (\nabla v)^2\right]}_{\text{Spatial Smoothness}} \, dxdy$$

- This is a probabilistic graphical model!
- Can solve using least squares (as in Lucas-Kanade)
- Or using discrete optimization (e.g. belief propagation)

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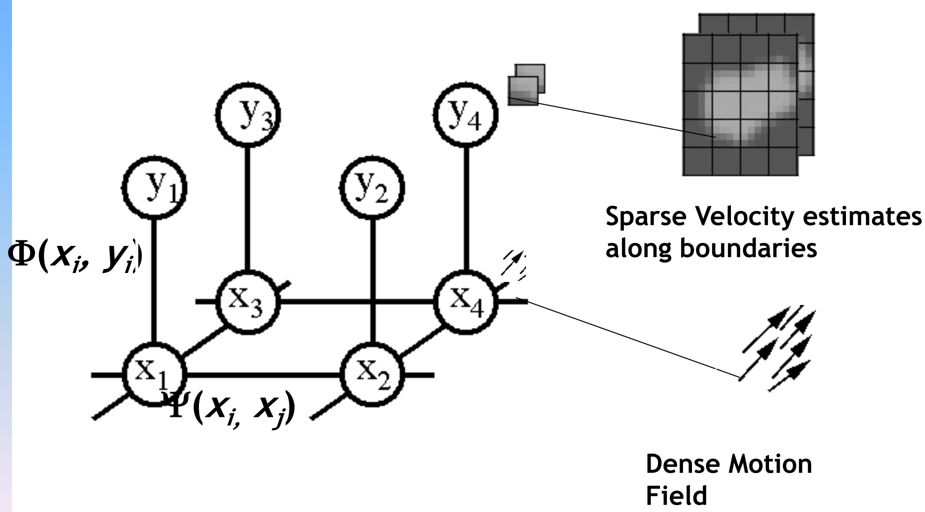
MRFs for Motion



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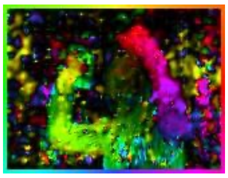
Pre-Deep Learning State-of-the-art

- Bruhn, Weickert et. al.



Local Method

Horn-Schunck
(isotropic)

Total Variation
(anisotropic,
image-based)

Proposed
(anisotropic, flow-based)

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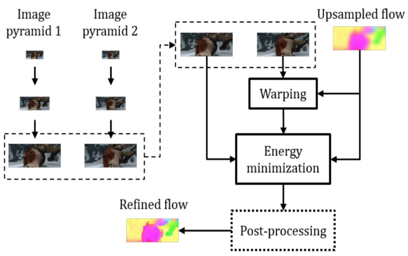
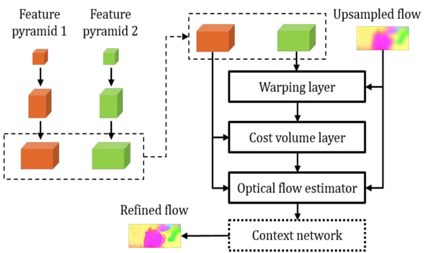
Flow Networks



Traditional coarse-to-fine flow



PWC-net

[Sun et al., “PWC-Net”, 2018]

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Fun with flow

- http://www.youtube.com/watch?v=TbJrc6QCeU0&feature=related

- http://www.youtube.com/watch?v=pckFacsIWg4

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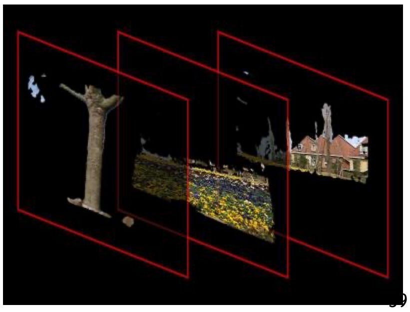
This lecture: Motion Analysis

- Remainder of segmentation lecture
- Motion Analysis
  - Optical Flow
  - Aperture Problem
  - Lucas-Kanade
  - Horn-Shunk & Variational Methods
  - Layered Models & Segmentation



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Motion Segmentation

- How do we represent the motion in this scene?

J. Wang and E. Adelson. Layered Representation for Motion Analysis. *CVPR 1993.* 60

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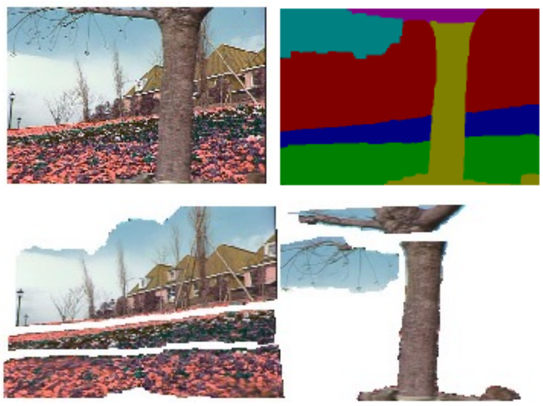
Layered Motion

- Break image sequence into "layers" each of which has a coherent motion



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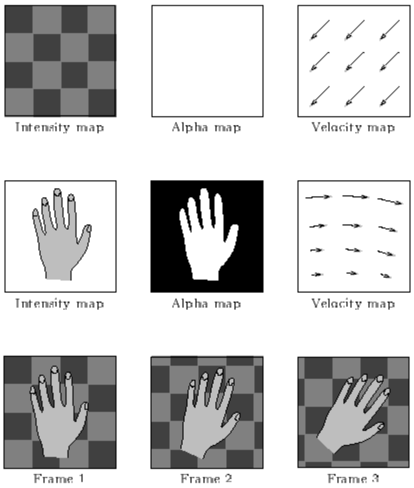
What is in each layer?

- **Intensities**
- **Alpha maps**
  - Transparency
- **Velocities**
  - Affine motion model

$$u(x, y) = a_1 + a_2 x + a_3 y$$
$$v(x, y) = a_4 + a_5 x + a_6 y$$



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Layered motion

- **Advantages:**
  - can represent occlusions / disocclusions
  - each layer's motion can be smooth
  - video segmentation for semantic processing
- **Difficulties:**
  - how do we determine the correct number?
  - how do we assign pixels?
  - how do we model the motion?

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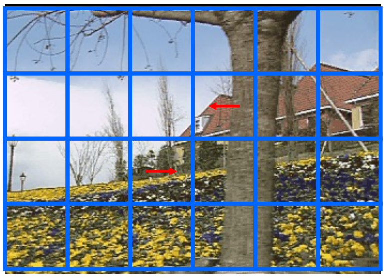
Block-based motion prediction

- Break image up into square blocks
- Estimate translation for each block
- Use this to predict next frame, code difference (MPEG-2)



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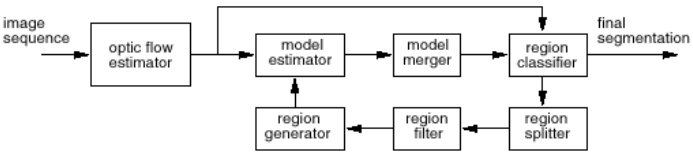
How do we estimate the layers?

1. compute coarse-to-fine flow
2. estimate affine motion in blocks (regression)
3. cluster with *k-means*
4. assign pixels to best fitting affine region
5. re-estimate affine motions in each region…



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Motion Segmentation with an Affine Model

$$\boxed{u(x,y)} = a_1 + a_2 x + a_3 y$$
$$\boxed{v(x,y)} = a_4 + a_5 x + a_6 y$$

Local flow estimates

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Motion Segmentation with an Affine Model

$$\boxed{u(x,y) = a_1 + a_2 x + a_3 y}$$
$$v(x,y) = a_4 + a_5 x + a_6 y$$

Equation of a plane (parameters a_1, a_2, a_3 can be found by least squares)

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Motion Segmentation with an Affine Model

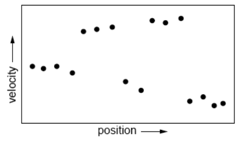
$$\boxed{u(x,y) = a_1 + a_2 x + a_3 y}$$

$$v(x,y) = a_4 + a_5 x + a_6 y$$

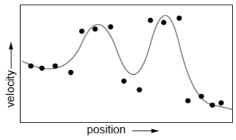
Equation of a plane (parameters a_1, a_2, a_3 can be found by least squares)

1D example

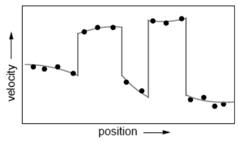
$u(x,y)$



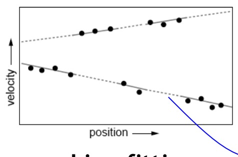
True flow



Local flow estimate



Segmented estimate



Line fitting

"Foreground"

"Background"

Occlusion

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Segmentation results

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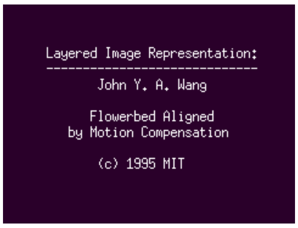
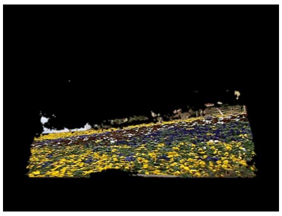
Once layered motion model is available

- Layer-based Stabilization



- Filling-in occlusions





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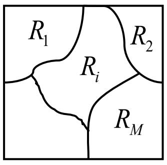
Motion Segmentation

- Probabilistic model for optical flow within each region
  - Affine motion model: special case
- Motion Competition: Region Competition for optical flow observations

$$J[\theta, \Gamma] = \sum_{i=1}^{K} \iint_{R_i} - \log \underset{\uparrow}{P(I; \theta_i)} + \frac{\beta}{2} \int_{\partial R_i} ds$$



Pixel Velocity estimates

$$\frac{\partial \Gamma_i}{\partial t} = \log \frac{P(I; \theta_i)}{P(I; \theta_j)} \mathcal{N} - \beta \kappa \mathcal{N} \qquad \Gamma_i = \partial R_i$$

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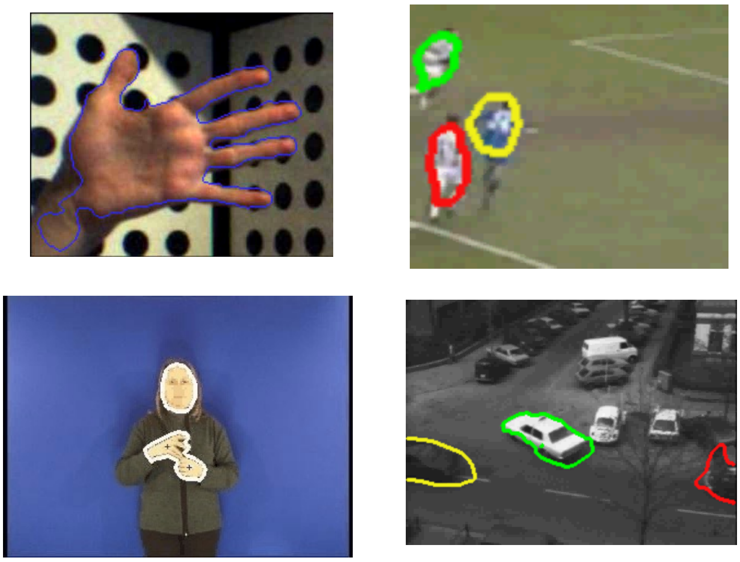
Variational Motion Segmentation Results



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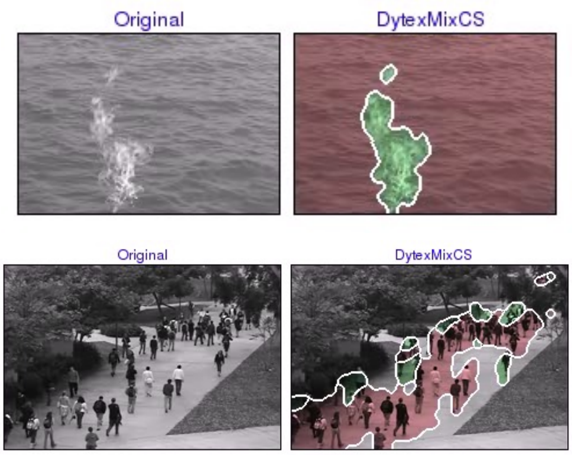
Dynamic Texture Segmentation

- Probabilistic model for texture within each region
  - Wait until next lecture for model



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Could/Should a computer see motion here?

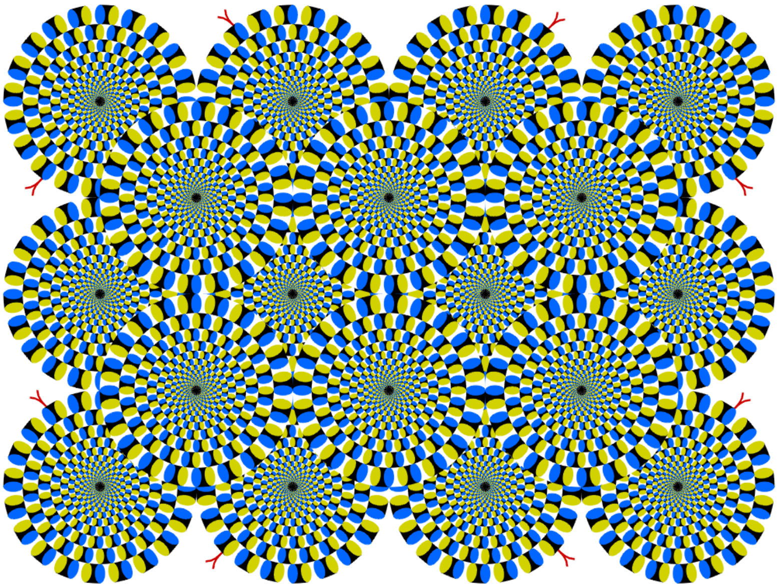

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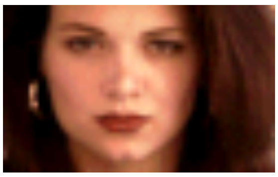
Image Morphing



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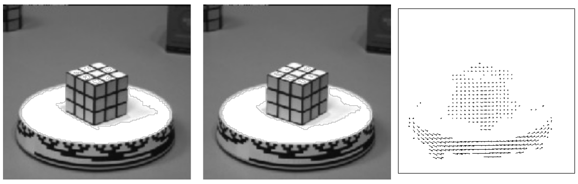
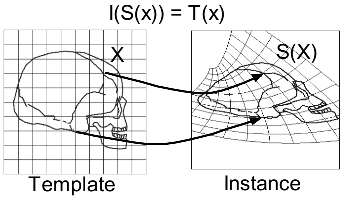
Motion Field & Deformations

- The motion field: projection of 3D scene motion into the image



- Image registration: a mapping from the domain of one image into that of another



- Different physical problems, common techniques

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Image Warping - non-parametric

- Specify more detailed warp function
- Examples:
  - splines
  - triangles
  - optical flow (per-pixel r

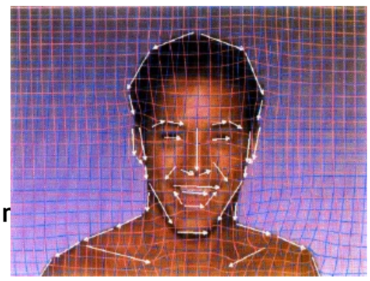

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Image Warping - non-parametric

- Move control points to specify spline warp

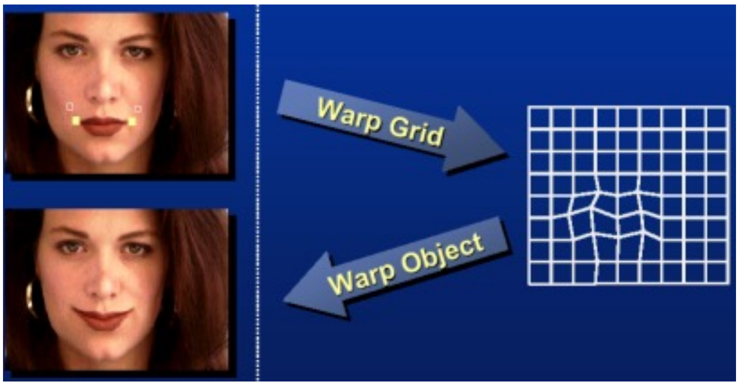


Image Morphing

- How can we *mix* two images?
  1. Cross-dissolve

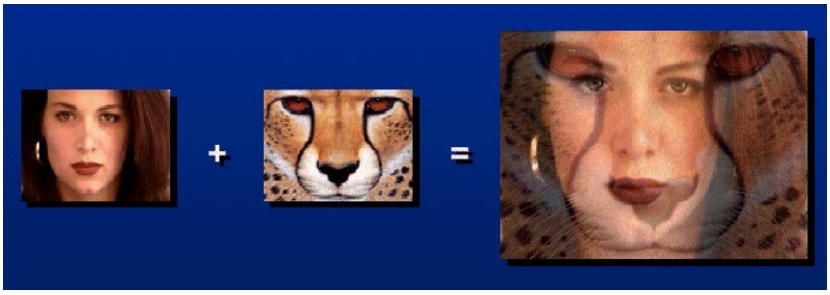
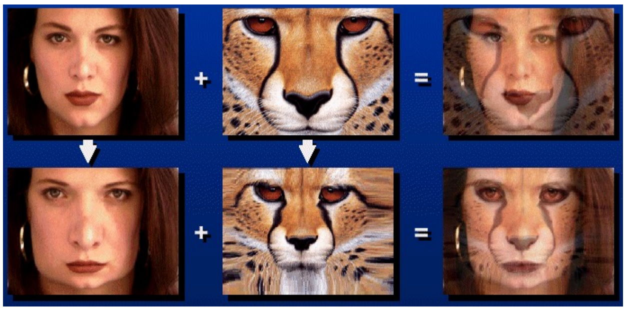


Image Morphing

- How can we *mix* two images?
  2. Warp then cross-dissolve = *morph*



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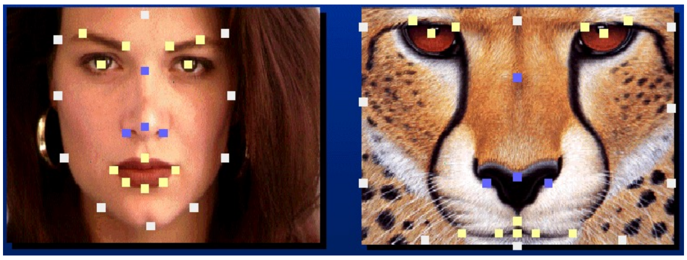
Warp specification

- How can we specify the warp?
  1. Specify corresponding *points*
     - *interpolate* to a complete warping function



Motion estimation

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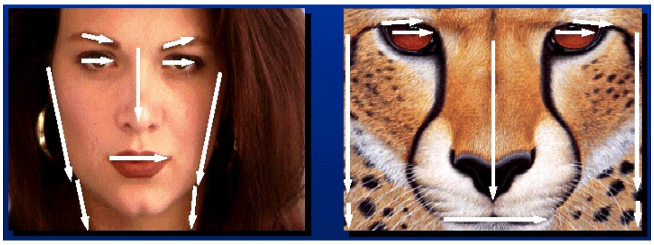
Warp specification

- How can we specify the warp?
  2. Specify corresponding *vectors*
     - *interpolate* to a complete warping function



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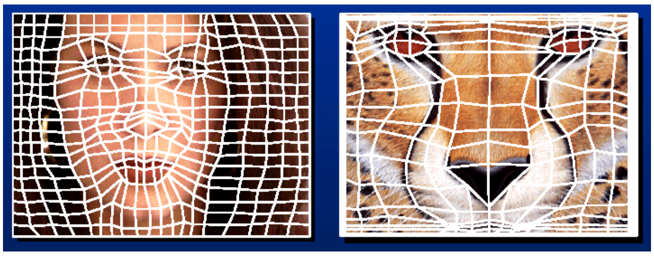
Warp specification

- How can we specify the warp?
  3. Specify corresponding *spline control points*
     - *interpolate* to a complete warping function



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Morphing Results

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