Homework 8

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Microeconomics-1

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Problem 1. . Let, x represent demand and q represent supply.

a.1 The price that consumers pay is $p^c + 1$ and the price the producer receive is p^c. Therefore, the equality of the (generic) demand and supply functions in the equilibrium of this competitive market when the consumer pays the tax is $x(p^c + 1) = q(p^c)$

a.2 The price that consumers pay is p^p and the price the producer receive is $p^p - 1$. Therefore, the equality of the (generic) demand and supply functions in the equilibrium of this competitive market when the producer pays the tax is $x(p^p) = q(p^p - 1)$

b. If the consumer pays the same price in both the cases i.e $p^p = p^c + 1 \implies$ the amount purchased in both the cases is equal i.e $x(p^p) = x(p^c + 1)$

c. If the ad valorem tax is collected from the consumers, then the price that consumers pay is $(1 + \tau)p^c$ and the price the producer receive is p^c. Therefore, the equality of the (generic) demand and supply functions in the equilibrium of this competitive market when the ad valorem tax is collected from the consumer is $x((1 + \tau)p^c) = q(p^c)$

On the other hand if the ad valorem tax is collected from the producers, then the price that consumers pay is p^p and the price the producer receive is $(1 - \tau)p^p$. Therefore, the equality of the (generic) demand and supply functions in the equilibrium of this competitive market when the ad valorem tax is collected from the consumer is $x(p^p) = q((1 - \tau)p^p)$

Next let the excess demand function be $z(p) = x(p) - q((1 - \tau)p)$. Further we know that $x'(p) \leq 0$ i.e non-increasing and $q'(p) \geq 0$ i.e non decreasing. Therefore we have the below:

$$
\begin{aligned}
z((1 + \tau)p^c) &= x((1 + \tau)p^c) - q((1 + \tau)(1 - \tau)p^c) \\
&= x((1 + \tau)p^c) - q((1 - \tau^2)p^c) \\
&\geq x((1 + \tau)p^c) - q(p^c) = 0
\end{aligned}
$$

Therefore, when we collect the ad valorem tax from the consumers the excess demand is greater than 0 while when collected from producer the producer the excess demand is equal to 0 which implies that when the ad valorem tax is collected from the consumers it leads to a higher cost to consumers.

d. In the case of supply being perfectly inelastic i.e $q(p) = \bar{q}$.

Therefore at equilibration $x((1+\tau)p^c) = x(p^p) = \bar{q} \implies (1+\tau)p^c = p^p \implies$ the consumers pay the same price no matter from who the tax is being collected.

While $(1-\tau)p^p = (1+\tau)(1-\tau)p^c < p^c \implies$ when the ad valorem tax is collected from the consumers, the producers receive p^c which is higher than $(1-\tau)p^p$ the amount that the producers would receive if the ad valorem tax is collected from the producers. In other words the producers are worse off if the ad valorem tax is collected from the producers. Therefore in any case collection method is not irrelevant with an ad valorem tax.

Problem 2. . The firms $i's$ profit maximization problem is below:

$$\max_{q_i} \frac{\theta q_i^{\beta-1}}{\sum_{j=1}^{N} g_j^{\beta}} q_i - (F + cq_i)$$

F.O.C with respect to q_i is below:

$$\theta\beta \frac{\left(\sum_{j=1}^{N} g_j^{\beta}\right) q_i^{\beta-1} - q_i^{2\beta-1}}{\left(\sum_{j=1}^{N} g_j^{\beta}\right)^2} = c$$

Further since in symmetric equilibrium each firms output is equal i.e $q_i^* = q^*$ we get the below

$$\theta\beta \frac{q^{2\beta-1}(N-1)}{q^{2\beta}N^2} = c$$

$$\iff q^* = \theta\beta \frac{(N-1)}{N^2 c}$$

Therefore, $q^* = q_i^* = \theta\beta \dfrac{(N-1)}{N^2 c}$ is the individual production level of every firm i, q_i as a function of β.

Further, since the parameter $\beta \in (0,1)$ captures the degree of substitutability and $\dfrac{\partial q_i^*}{\partial \beta} = \theta \dfrac{(N-1)}{N^2 c} > 0 \implies$ as goods become more differentiable the production level increases.

Finally, with respect to the number of firms, $\forall N > 1$, we have

$$\frac{\partial \left(\frac{\partial q_i^*}{\partial \beta}\right)}{\partial N} = \frac{\theta}{c}\left[\frac{(-1)}{N^2} + \frac{1}{N^3}\right] = \frac{\theta}{c}\frac{(1-N)}{N^3} < 0 \implies \text{as the number of firms increase,}$$

the production of each individual firm decreases.

Problem 3. .

a. The consumer utility maximization problem is below:

$$\max_{x,m} \left\{ \alpha + \beta \ln x + m \right\} \text{ s.t. } px + m \leq w_m$$

Which corresponds to the below Lagrangian.

$$L = \alpha + \beta \ln x + m - \lambda \big[px + m - w_m \big]$$

F.O.C is below:

$$\frac{dL}{dx} : \beta / x = \lambda p \tag{1}$$

$$\frac{dL}{dw} : 1 = \lambda \tag{2}$$

$$\frac{dL}{d\lambda} : px + m = w_m \tag{3}$$

Dividing equation 1 and 2 we get:

$x^* = \beta / p$

Next plugging x^* back into the equation 3 we get:

$w_m = \beta + m \iff m^* = w_m - \beta$

Further, the firms profit maximization problem is below:

$$\max_{q} \left\{ pq - \sigma q \right\}$$

Therefore taking the FOC with respect to q we get:

$p - \sigma = 0 \iff p^* = \sigma$

b. As shown in part (a) above by taking the F.O.C the competitive equilibrium is

$p^* = \sigma$, $m^* = w_m - \beta$ and $x^* = \beta / p$