Homework 9

Jugal Marfatia

Microeconomics-1

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Problem 1. .

a. The profit function for firm 1 is $\pi_1 = (34 - Q_1 - Q_2)Q_1 - 2Q_1^2 - 5Q_1 - Q_2$

Therefore taking the F.O.C with respect to Q_1 we get:

$34 - 2Q_1 - Q_2 - 4Q_1 - 5 = 0 \iff Q_1 = \dfrac{29 - Q_2}{6}$

On the other hand, the profit function for firm 2 is $\pi_2 = (34 - Q_1 - Q_2)Q_2 - Q_2^2 - 3Q_2 + 4Q_1$

Therefore taking the F.O.C with respect to Q_2 we get:

$34 - Q_1 - 2Q_2 - 2Q_2 - 3 = 0 \iff Q_2 = \dfrac{31 - Q_1}{4}$

Next plugging in Q_2 to solve for Q_1 we get:

$Q_1 = \dfrac{29}{6} - \dfrac{31 - Q_1}{4 * 6} \iff Q_1 = \dfrac{(29 * 4) - 31}{23} \approx 3.7$

Therefore $Q_2 \approx \dfrac{31 - 3.7}{4} \approx 6.825$

$\implies \pi_1 = (34 - Q_1 - Q_2)Q_1 - 2Q_1^2 - 5Q_1 - Q_2 = (34 - 3.7 - 6.825)3.7 - 2 * 3.7^2 - 5(3.7) - 6.825 \approx 34.15$

And $\pi_2 = (34 - Q_1 - Q_2)Q_2 - Q_2^2 - 3Q_2 - 4Q_1 = (34 - 3.7 - 6.825)6.825 - 6.825^2 - 3 * 6.825 + 4 * 3.7 \approx 107.96$

Further, the total quantity supplied if $Q^s = 3.7 + 6.825 = 10.525$, equilibrium price $p^* = 34 - 10.525 = 23.475$ and total profit $\pi \approx 107.96 + 34.15 = 142.11$

$CS = \displaystyle\int_0^{Q^s} (1 - Q)dQ - P * Q^s = Q^s - \frac{(Q^s)^2}{2} - (1 - Q^s) * Q^s = \frac{(Q^s)^2}{2} = \frac{(10.525)^2}{2} \approx 55.4$

$\implies SW = CS + \pi = 142.11 + 55.4 = 197.51$

b. In the case of merger total profit is:

$\pi = \pi_1 + \pi_2 = (34 - Q_1 - Q_2)(Q_1 + Q_2) - 2Q_1^2 - 5Q_1 - Q_2 - Q_2^2 - 3Q_2 + 4Q_1$

Therefore taking the F.O.C with respect to Q_1 we get:

$34 - 2Q_1 - 2Q_2 - 4Q_1 - 1 = 0 \iff Q_1 = \dfrac{33 - 2Q_2}{6}$

Taking the F.O.C with respect to Q_2 we get:

$34 - 2Q_1 - 2Q_2 - 2Q_2 - 4 = 0 \iff Q_2 = \dfrac{30 - 2Q_1}{4}$

Next plugging in Q_2 to solve for Q_1 we get:

$Q_1 = \dfrac{33}{6} - \dfrac{30 - 2Q_1}{2 * 6} \iff Q_1 = \dfrac{36}{10} \approx 3.6$

and $Q_2 = \dfrac{30 - 7.2}{4} \approx 5.7$

$\implies \pi_1 = (34 - Q_1 - Q_2)Q_1 - 2Q_1^2 - 5Q_1 - Q_2 = (34 - 3.6 - 5.7)3.6 - 2 * 3.6^2 - 5 * 3.6 - 5.7 \approx 39.3$

And $\pi_2 = (34 - Q_1 - Q_2)Q_2 - Q_2^2 - 3Q_2 - 4Q_1 = (34 - 3.6 - 5.7)5.7 - 5.7^2 - 3 * 5.7 - 4 * 3.6 \approx 76.8$

Since, firm 1's profit increased it has an incentive to merge whereas firm 2's profit decreased so it has no incentive to merge

The total quantity supplied is $Q^s = Q_1 + Q_2 = 9.3$, eq. price is $p^* = 34 - 9.3 = 24.7$ and total profit $\pi = 76.8 + 39.3 \approx 116.1$

$$CS = \int_0^{Q^s} (1 - Q)dQ - P * Q^s = Q^s - \frac{(Q^s)^2}{2} - (1 - Q^s) * Q^s = \frac{(Q^s)^2}{2} = \frac{(9.3)^2}{2} \approx 43.2451$$

$\implies SW = CS + \pi \approx 116.1 + 43.2451 \approx 159.3451$

c. The consumer surplus decreases with merger, the total profit decrease, however the profit for firm 1 increases so they would prefer the merger. The total welfare also decreases with merger.

No the merger does not ameliorate the negative externalities since the consumer surplus decreases with the merger. Further, there is in fact a decrease in SW from the merger.

Problem 2. .

a. For **period 1,** the monopolist firm's profit function is:

$\pi = (1-Q)Q - (c+t)Q$

Therefore taking the F.O.C with respect to Q we get:

$Q^m = \dfrac{1-c-t}{2}$

Next $CS = \displaystyle\int_0^Q (1-Q)dQ - P*Q = Q - \frac{Q^2}{2} - (1-Q)*Q = \frac{Q^2}{2}$

Therefore, $SW = PS + CS + T - ED = (1-Q)Q - (c+t)Q + \dfrac{Q^2}{2} + t*Q + d*Q^2$

Therefore taking the F.O.C with respect to Q_1 we get:

$Q + (1-C-t) - 2Q + t - 2dQ = 0 \iff Q^{SO} = \dfrac{1-c}{2d+1}$

Next setting $Q^{SO} = Q^m$ and solving for optimal commission fee in period 1 we get:

$\dfrac{1-c-t_1}{2} = \dfrac{1-c}{2d+1} \iff t_1 = \dfrac{(1-c)(1+2d) - 2(1-c)}{(1+2d)} = \dfrac{(1-c)(2d-1)}{(1+2d)}$

Next, for **period 2,** the incumbent firm's profit function is:

$\pi_1 = (1-Q_1-Q_2)Q_1 - (c+t)Q_1$

Therefore taking the F.O.C with respect to Q_1 we get:

$Q_1 = \dfrac{1-c-t-Q_2}{2}$

Next, the entrant firm profit function is:

$\pi_2 = (1-Q_1-Q_2)Q_1 - (c+t)Q_1$

Therefore taking the F.O.C with respect to Q_1 we get:

$Q_2 = \dfrac{1-c-t-Q_1}{2}$

Next plugging in Q_2 to solve for Q_1 we get:

$Q_1 = \frac{1-c-t}{2} - \frac{1-c-t-Q_1}{4} \iff Q_1 = \frac{1-c-t}{3}$

and similarly $Q_2 = \frac{1-c-t}{3}$

The aggregate output in period 2 with duopoly is $Q^d = \frac{1-c-t}{3} + \frac{1-c-t}{3} = 2\frac{(1-c-t)}{3}$

Since there is no change in PS, CS, T and ED, the socially optimal output is the same a period 1. i.e.

$Q^{SO} = \frac{1-c}{2d+1}$

Next setting $Q^{SO} = Q^d$ and solving for optimal commission fee in period 2 we get:

$2\frac{(1-c-t_2)}{3} = \frac{1-c}{2d+1} \iff t_2 = \frac{2(1-c)(1+2d)-3(1-c)}{2(1+2d)} = \frac{(1-c)(4d-1)}{2(1+2d)}$

b. With an inflexible policy the social planner needs to minimize the DWL is the two periods i.e.:

$$\min_t \left\{ DWL_1(t) + DWL_2(t) \right\}$$

Therefore we need to first evaluate the sum of two period DWL.

$$DWL_1(t) + DWL_2(t) = \int_{Q^m}^{Q^{SO}} \frac{\partial SW(Q)}{\partial Q} dQ + \int_{Q^d}^{Q^{SO}} \frac{\partial SW(Q)}{\partial Q} dQ$$

$$= \left[SW(Q) \right]_{Q^m}^{Q^{S0}} + \left[SW(Q) \right]_{Q^d}^{Q^{S0}} \text{(Using the FTC)}$$

$$= \left[(1-C)Q - \left(\frac{1}{2} + d \right) Q^2 \right]_{Q^m}^{Q^{S0}} + \left[(1-C)Q - \left(\frac{1}{2} + d \right) Q^2 \right]_{Q^d}^{Q^{S0}}$$

$$= 2\left[(1-C)Q^{S0} - \left(\frac{1}{2}+d\right)(Q^{S0})^2\right] - \left[(1-C)Q^{d} - \left(\frac{1}{2}+d\right)(Q^{d})^2\right] - \left[(1-C)Q^{m} - \left(\frac{1}{2}+d\right)(Q^{m})^2\right]$$

$$= \left[\frac{[(4d-1)c+2+2t-4d(1-t)]^2}{18(2d+1)}\right] + \left[\frac{[(2d-1)c+1+t-2d(1-t)]^2}{8(2d+1)}\right]$$

Therefore with inflexible policy the social planner needs to minimize the DWL is the two periods i.e.:

$$\min_t \left\{ \left[\frac{[(4d-1)c+2+2t-4d(1-t)]^2}{18(2d+1)}\right] + \left[\frac{[(2d-1)c+1+t-2d(1-t)]^2}{8(2d+1)}\right] \right\}$$

And taking the F.O.C with respect to t we get:

$$2\left[\frac{[(4d-1)c+2+2t-4d(1-t)]}{18(2d+1)}\right](2d+1) + 2\left[\frac{[(2d-1)c+1+t-2d(1-t)]^2}{8(2d+1)}\right](2d+1) = 0$$

$$\implies t^* = \frac{(1-c)[50d-17]}{25(1+2d)}$$

c. Now in order to compare from part a we from part a, flexible policy $t_1 = \dfrac{(2d-1)(1-c)}{(1+2d)}$

and $t_2 = \dfrac{(4d-1)(1-c)}{2(1+2d)} = \dfrac{(2d-0.5)(1-c)}{(1+2d)}$

And from part b, inflexible policy $t* = \dfrac{(1-c)[50d-17]}{25(1+2d)} = \dfrac{(2d-0.68)(1-c)}{(1+2d)}$

Since $(2d-0.5) > (2d-0.68) \implies \dfrac{(2d-0.5)(1-c)}{(1+2d)} > \dfrac{(2d-0.68)(1-c)}{(1+2d)} \implies t_2 > t^*$

And $(2d-1) < (2d-0.68) \implies \dfrac{(2d-1)(1-c)}{(1+2d)} < \dfrac{(2d-0.68)(1-c)}{(1+2d)} \implies t_1 < t^*$

This is consistent with what we learned during lectures that with higher level of output, which in this case occur with duopoly, the emission tax is higher.

Problem 3. .

a. The firm's objective function is below:

$$\max_{Q} \left\{ 10Q - Q^2 \right\}$$

Taking the F.O.Cwith respect to Q we get:

$$2Q = 10 \iff Q^E = 5$$

b. When setting a quota the social planner's objective function is below:

$$\max_{Q} \left\{ E_a[\pi(Q, a)] - 3Q^2 \right\}$$

Taking the F.O.C with respect to Q we get:

$$E_a \left[\frac{\partial \pi(Q, a)}{\partial Q} \right] - 6Q = 0$$

$$\iff E_a \left[10 - 2aQ \right] - 6Q = 0 \iff 10 - \left[(0.5)2Q + (0.5)Q \right] - 6Q = 0$$

$$\iff 7.5Q = 10 \iff Q = 4/3 \approx 1.333$$

Is the best quota that a social planner can select in order to maximize the expected value of aggregate surplus.

c. The social planner guesses the firm's best response given incomplete information and emission tax, from the below objective function :

$$\max_{Q} \left\{ E_a[\pi(Q, a)] - tQ \right\}$$

Taking the F.O.C with respect to Q we get:

$$E_a\left[\frac{\partial\pi(Q,a)}{\partial Q}\right] = t$$

$$\iff E_a\Big[10 - 2aQ\Big] = t \iff 10 - \Big[(0.5)2Q + (0.5)Q\Big] = t$$

$$\iff 1.5Q = 10 - t \iff Q^F = \frac{10-t}{1.5}$$

Is the best of the firm given incomplete information and emission tax t.

Further from above we know that for the social planner's point of view the social optimal output is $Q^{SO} = 4/3$

Therefore setting $Q^{SO} = Q^F \iff \dfrac{4}{3} = \dfrac{10-t}{1.5} \iff t^* = 10 - 2 = 8$

Is the best tax that the social planner can set under the context of incomplete information.

d. With emission tax and a = 1, the firms profit max objective is

$$\max_Q \left\{10Q - Q^2 - tQ\right\}$$

Taking the F.O.C with respect to Q we get:

$$Q^* = \frac{10-t}{2}$$

Therefore given $t^* = 8 \implies Q^* = 1$

Whereas for the social planner with emission tax and a = 1, the optimal quantity can be found by below

$$\frac{\partial d(Q^{SO})}{\partial Q} = \frac{\partial\pi(Q^{SO})}{\partial Q} \iff 6Q^{SO} = 10 - 2Q^{SO} \iff Q^{SO} = 1.25$$

Further the marginal dis-utility when $t^* = 8$ and $Q^* = 1$is $\dfrac{\partial d(Q^{SO})}{\partial Q} = 6Q^* = 6$

Therefore the DWL with tax is:

$$DWL_t = 0.5\Big(Q^{SO} - Q^*\Big)\Big(t^* - 6Q^*\Big) = 0.5(0.25)(2) = 0.25$$

On the other hand when the quota is set at $\bar{Q} = \frac{4}{3}$

The marginal profit is $10 - 2\bar{Q} = 10 - 2\frac{4}{3} = \frac{22}{3}$

And the expected marginal profit is:

$$10 - \left((0.5)2\bar{Q} + (0.5)^2 2\bar{Q}\right) = 10 - \left((0.5)2\frac{4}{3} + (0.5)^2 2\frac{4}{3}\right) = 8$$

Therefore the DWL with quota is:

$$DWL_q = 0.5\left(\bar{Q} - Q^{SO}\right)\left(8 - \frac{22}{3}\right) = 0.5\left(\frac{4}{3} - \frac{5}{4}\right)\left(8 - \frac{22}{3}\right) = \frac{1}{36}$$

Therefore, since $\frac{1}{36} < 0.25 \implies$ the dead weight loss from quota is lower. Thus the uninformed regulator will not prefer to chose emission fee.