Homework 4

Jugal Marfatia

Microeconomics-1

September 20, 2017

Problem 1. .

a.

$$L = p_1x_1 + p_2x_2 + \lambda\big[\bar{u} - x_1^{1/2}x_2^{1/2}\big]$$

F.O.C is below:

$$\frac{dL}{dx_1} : p_1 = \lambda\frac{1}{2}x_1^{-1/2}x_2^{1/2} \tag{1}$$

$$\frac{dL}{dx_2} : p_2 = \lambda\frac{1}{2}x_1^{1/2}x_2^{-1/2} \tag{2}$$

$$\frac{dL}{d\lambda} : \bar{u} = x_1^{1/2}x_2^{1/2} \tag{3}$$

Dividing equation 1 and 2 we get:

$$\frac{\lambda\frac{1}{2}x_1^{-1/2}x_2^{1/2}}{\lambda\frac{1}{2}x_1^{1/2}x_2^{-1/2}} = \frac{p_1}{p_2} \iff x_1 = x_2\frac{p_2}{p_1}$$

Next plugging x_1 back into the equation 3 we get:

$$x_2\left(\frac{p_2}{p_1}\right)^{\frac{1}{2}} = \bar{u} \iff x_2^H = \bar{u}\left(\frac{p_1}{p_2}\right)^{\frac{1}{2}}$$

Further using symmetry we get:

$$x_1^H = \bar{u}\left(\frac{p_2}{p_1}\right)^{\frac{1}{2}}$$

b. The expenditure function is,

$$e(p,u) = 2\bar{u}\ p_1^{1/2}\ p_2^{1/2}$$

c. The equivalent variation is,

$$EV = e(p_0, u_1) - e(p_1, u_1) = 200(2^{1/2} - 1) = 82.842$$

Problem 2. .

Since $u(x) = x_1 + \phi x_{-1} \iff x_1 = u(x) - \phi x_{-1}$

Further, the expenditure function:

$$e(p,u) = p_1 x_1 + \sum_{i=2}^{L} p_i x_i = x_1 + p_{-1} \cdot x_{-1} = u(x) - \phi x_{-1} + p_{-1} \cdot x_{-1} \text{ (Since } p_1 = 1).$$

(Note: $p_{-1} \cdot x_{-1}$ represents the dot product of the two vectors p_{-1} and x_{-1})

Therefore,

$$CV = e(p^1, u^1) - e(p^1, u^0) = u^1(x) - \phi x_{-1} + p_{-1} \cdot x_{-1} - u^0(x) + \phi x_{-1} - p_{-1} \cdot x_{-1} = u^1(x) - u^0(x)$$

$$EV = e(p^0, u^1) - e(p^0, u^0) = u^1(x) - \phi x_{-1} + p_{-1} \cdot x_{-1} - u^0(x) + \phi x_{-1} - p_{-1} \cdot x_{-1} = u^1(x) - u^0(x)$$

Thus, the compensating and the equivalent variation are equal when the utility function is quasilinear with respect to the first good.

Problem 3. .

a.

$$L = ax_1^{\alpha} + bx_2 + \lambda\big[w - p_1x_1 - p_2x_2\big]$$

F.O.C is below:

$$\frac{dL}{dx_1} : a\alpha x_1^{\alpha-1} = \lambda p_2 \tag{1}$$

$$\frac{dL}{dx_2} : b = \lambda p_2 \tag{2}$$

$$\frac{dL}{d\lambda} : w = p_1x_1 + p_2x_2 \tag{3}$$

Dividing equation 1 and 2 we get:

$\frac{a\alpha x_1^{\alpha-1}}{b} = \frac{p_1}{p_2} \iff x_1^w = \left(\frac{bp_1}{a\alpha p_2}\right)^{\frac{1}{\alpha-1}}$

Next plugging x_1 back into the equation 3 we get:

$x_2^w = \frac{w}{p_2} - \left(\frac{bp_1^{\alpha}}{a\alpha p_2^{\alpha}}\right)^{\frac{1}{\alpha-1}}$

b.

$$L = p_1 x_1 + p_2 x_2 + \lambda\big[\bar{u} - a x_1^{\alpha} - b x_2\big]$$

F.O.C is below:

$$\frac{dL}{dx_1} : \lambda a \alpha x_1^{\alpha-1} = p_2 \tag{1}$$

$$\frac{dL}{dx_2} : \lambda b = p_2 \tag{2}$$

$$\frac{dL}{d\lambda} : \bar{u} = a x_1^{\alpha} + b x_2 \tag{3}$$

Dividing equation 1 and 2 we get:

$\frac{a\alpha x_1^{\alpha-1}}{b} = \frac{p_1}{p_2} \iff x_1^H = \left(\frac{bp_1}{a\alpha p_2}\right)^{\frac{1}{\alpha-1}}$

Next plugging x_1 back into the equation 3 we get:

$x_2^H = \frac{\bar{u}}{p_2} - \left(\frac{bp_1^{\alpha}}{a\alpha p_2^{\alpha}}\right)^{\frac{1}{\alpha-1}}$

c. .

$\text{AV} = \displaystyle\int_1^2 \left(\frac{bp_1}{a\alpha p_2}\right)^{\frac{\alpha}{\alpha-1}} dp_1 = \int_1^2 \left(\frac{2}{p_1}\right)^{\frac{\alpha}{2}} dp_1 = \left[-\frac{4}{p_1}\right]_1^2 = 2$

$\text{CV} = \displaystyle\int_1^2 \left(\frac{bp_1}{a\alpha p_2}\right)^{\frac{\alpha}{\alpha-1}} dp_1 = \int_1^2 \left(\frac{2}{p_1}\right)^{\frac{\alpha}{2}} dp_1 = \left[-\frac{4}{p_1}\right]_1^2 = 2$

$\text{EV} = \displaystyle\int_1^2 \left(\frac{bp_1}{a\alpha p_2}\right)^{\frac{\alpha}{\alpha-1}} dp_1 = \int_1^2 \left(\frac{2}{p_1}\right)^{\frac{\alpha}{2}} dp_1 = \left[-\frac{4}{p_1}\right]_1^2 = 2$

Problem 4. .

a. From the utility function we know, that $q_2(q_1) = u + 1 - q_1^2$

Therefore, $q_2'(q_1) = -2q_1 < 0$ and $q_2''(q_1) = -2 < 0 \implies q_2(q_1)$ is strictly decreasing and concave. Which means the walrasian demand will be a corner solution.

Therefore we could have two cases:

Case 1.

$q_1 = \frac{w}{p_1}$ and $q_2 = 0$ if $\frac{w^2}{p_1^2} - 1 > \frac{w}{p_2} - 1 \iff \frac{w^2}{p_1^2} > \frac{w}{p_2}$

Case 2.

$q_1 = 0$ and $q_2 = \frac{w}{p_2}$ if $\frac{w^2}{p_1^2} - 1 \leq \frac{w}{p_2} - 1 \iff \frac{w^2}{p_1^2} \leq \frac{w}{p_2}$

Further, the corresponding indirect utility is:

From case 1,

$v(p, w) = \frac{w^2}{p_1^2} - 1$ if $\frac{w^2}{p_1^2} > \frac{w}{p_2}$

From case 2,

$v(p, w) = \frac{w}{p_2} - 1$ if $\frac{w^2}{p_1^2} \leq \frac{w}{p_2}$

b. Since the utility $q_2(q_1)$ is strictly decreasing and concave, the hicksian demand will also have corner solutions.

Therefore we could have two cases:

Case 1.

$q_1^H = \sqrt{\bar{u} + 1}$ and $q_2^H = 0$ if $p_1\sqrt{\bar{u} + 1} < p_2(\bar{u} + 1)$

Case 2.

$q_1^H = 0$ and $q_2^H = \bar{u} + 1$ if $p_1\sqrt{\bar{u} + 1} \geq p_2(\bar{u} + 1)$

Further, the corresponding expenditure function is:

From case 1,

$e(p, u) = p_1\sqrt{\bar{u}+1}$ if $p_1\sqrt{\bar{u}+1} < p_2(\bar{u}+1)$

From case 2,

$e(p, u) = p_2\bar{u} + 1$ if $p_1\sqrt{\bar{u}+1} \geq p_2(\bar{u}+1)$

c. For this part we initially have $w = 6, p_1 = 4, p_2 = 3$

Therefore, $\frac{w^2}{p_1^2} = 2.25 > \frac{w}{p_2} = 2$, which implies $u^0 = 2.25 - 1 = 1.25$

Further, $p_1\sqrt{\bar{u}+1} = 6 < p_2(\bar{u}+1) = 6.75$, which implies $e(p^0, u^0) = 6$

Next when prices increase by 50 %, $p_1\sqrt{\bar{u}+1} = 9 < p_2(\bar{u}+1) = 10.125$, which implies $e(p^1, u^0) = 9$

Thus, CV $= 9 - 6 = \$3$. Intuitively this tells us that the 50% increase in the prices should be compensated by \$3 increase in the wealth level, which is equivelant to 50% increase from the initial wealth level.

d. When the prices increased by 50%, then we reached $p_1 = 6, p_2 = 4.5$. Thus with the new prices $\frac{w^2}{p_1^2} = 1 \leq \frac{w}{p_2} = 1.33$. Which implies, the walrasian demand for the consumer is $q_1 = 0$ and $q_2 = \frac{w}{p_2}$.

Further, with the initial price of p_2, q_1 was preferred as long as below:

$$\frac{w^2}{p_1^2} > \frac{w}{p_2} \iff \frac{36}{p_1^2} > \frac{6}{3} \iff p_1 < \sqrt{18}.$$

Therefore, putting the above information together:

$$\text{AV} = \int_4^{\sqrt{18}} \left(\frac{6}{p_1}\right) dp_1 + \int_{\sqrt{18}}^{6} 0 dp_1 + \int_3^{4.5} \left(\frac{2}{p_2}\right) dp_2 = \Big[6\ln(p_1)\Big]_4^{\sqrt{18}} + \Big[6\ln(p_2)\Big]_3^{4.5}$$

Therefore AV $= 6\Big[\ln(\sqrt{18}) - \ln(4) + \ln(4.5) - \ln(3)\Big] \approx 2.786.$

The interpretation of the AV, is that in the first term we integrated walrasian demand of good 1 from 4 to $\sqrt{18}$ because the consumer will prefer good 1 given price of good 1 is below $\sqrt{18}$. Thus the second term is 0 because for price of good 1 greater than $\sqrt{18}$ the walrasian demand for good 1 = 0. Lastly in the third term we cacluate the area under the curve to evaluate the variance in the price of good 2.