Homework 6

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Microeconomics-1

October 18, 2017

Problem 1. .

a. $v(g) = \prod_{i=1}^{n}(1+a_i)^{p_i}$, is not a linear function there it cannot be a von Neumann-Morgenstern (vNM) utility function.

b. $\ln(v(g)) = \ln\left(\prod_{i=1}^{n}(1+a_i)^{p_i}\right) = \ln\left((1+a_1)^{p_1}(1+a_2)^{p_2}...(1+a_n)^{p_n}\right) = \sum_{i=1}^{n} p_i \ln(1+a_i)$

Further, $\frac{d}{dx}\ln(x) = \frac{1}{x} > 0 \implies \ln(x)$ is a monotone (increasing) transformation

And since $\sum_{i=1}^{n} p_i \ln(1+a_i)$, is a vNM utility function, the decision maker with a utility function $v(g) = \prod_{i=1}^{n}(1+a_i)^{p_i}$ has the same preference relation as an expected utility maximizer with vNM utility function.

In other words, the monotone (increasing) transformation will not change the ordinality of the preferences.

c. We have $u(w) = \ln(1+w)$ and $w > 0$. In order to check the risk attitude (concavity) we will check $u'(w)$ and $u''(w)$.

$u'(w) = \frac{1}{1+w} > 0$ and $u''(w) = \frac{-1}{(1+w)^2} < 0 \implies u(w)$ is concave and the decision maker is risk averse.

Further, $r_A(w,u) = -\frac{u''(w)}{u'(w)} = -\frac{\frac{-1}{(1+w)^2}}{\frac{1}{1+w}} = \frac{1}{1+w} \implies \frac{dr_A(w,u)}{dw} = \frac{-1}{(1+w)^2} < 0$

Therefore, the decision maker becomes less risk averse as income increases.

Problem 2. .

a. The expected utility from the two gambles are:

$$v(g_1) = \frac{1}{3}(0-2) + \frac{1}{3}(1-2) + \frac{1}{3}(2-2) = -1$$

$$v(g_2) = \frac{1}{2}(1-5) + \frac{1}{3}(4-5) + \frac{1}{6}(5-5) = -\frac{7}{3}$$

b. Let each a_i be all arbitrary payoffs with deterministic outcome (i.e $p_i = 1$), therefore for any a_i highest possible monetary payoff in the lottery $h(g) = a_i$.

Thus, $\forall a_i, v(a_i) = \sum_{i=1}^{n} p_i(a_i - h(g)) = \sum_{i=1}^{n} 1(a_i - a_i) = 0 \implies v(a_1) = v(a_2) = ... = v(a_n)$

c. Let two gambles with deterministic payoff be: $g_1 = (4, 3; 0, 1)$ and $g_2 = (6, 4; 0, 1)$

In this case we have that all the payoffs in g_2 are greater than g_1 i.e. $g_2 >> g_1$.

However $v(g_2) = 1(4-6) = -2 < v(g_1) = 1(3-4) = -1$. Therefore $g_1 \succsim g_2$ which implies preference relation does not satisfy monotonicity if outcomes are deterministic.

Problem 3. .

a. Since event A is the most preferred and event D is the least preferred, let utility level from A, $U_A = 1$ and utility level from D, $U_D = 0$.

Further utility level from event B, $U_B = p * U_A + (1-p)U_D = p * 1 + (1-p)0 = p$

Finally utility level from event C, $U_C = q * U_A + (1-q)U_D = q * 1 + (1-q)0 = q$

The expected utility $= P_A U_A + P_B U_B + P_C U_C + P_D U_D = P_A + P_B p + P_C q$

b. Let E be the event of evacuation and let F be the event of floods. From the question we know the probability of flooding, $P(F) = 0.99 \implies P(\bar{F}) = 0.01$
(Note: $\bar{F}$ represents the event not F)

Criterion 1. From question we know $P(E|F) = 0.9$ and $P(E|\bar{F}) = 0.1$

$P_A = P(\bar{F} \cap \bar{E}) = P(\bar{E}|\bar{F}) * P(\bar{F}) = 0.9 * 0.99 = 0.891$

$P_B = P(\bar{F} \cap E) = P(E|\bar{F}) * P(\bar{F}) = 0.1 * 0.99 = 0.099$

$P_C = P(F \cap E) = P(E|F) * P(F) = 0.9 * 0.01 = 0.009$

$P_D = P(F \cap \bar{E}) = P(\bar{E}|F)P(F) = 0.1 * 0.01 = 0.001$

Therefore the expected utility $u_1 = 0.891 + 0.099p + 0.009q$

Criterion 2. From question we know $P(E|F) = 0.95$ and $P(E|\bar{F}) = 0.15$

$P_A = P(\bar{F} \cap \bar{E}) = P(\bar{E}|\bar{F}) * P(\bar{F}) = 0.85 * 0.99 = 0.8415$

$P_B = P(\bar{F} \cap E) = P(E|\bar{F}) * P(\bar{F}) = 0.15 * 0.99 = 0.1485$

$P_C = P(F \cap E) = P(E|F) * P(F) = 0.95 * 0.01 = 0.0095$

$P_D = P(F \cap \bar{E}) = P(\bar{E}|F)P(F) = 0.05 * 0.01 = 0.0005$

Therefore the expected utility $U_2 = 0.8415 + 0.1485p + 0.0095q$

Finally Criterion 1 will be preferred if and only if $U_1 > U_2$
$\iff 0.891 + 0.099p + 0.009q > 0.891 + 0.099p + 0.009q \iff 0.0495 > 0.0495p + 0.0005q$

On the other hand, Criterion 2 will be preferred if and only if $U_1 < U_2$
$\iff 0.891 + 0.099p + 0.009q < 0.891 + 0.099p + 0.009q \iff 0.0495 < 0.0495p + 0.0005q$

Problem 4. .

a. If lotteries satisfy independent axiom, then we have

$\forall L, L'$, and $\lambda \in (0, 1) \implies \lambda L + (1-\lambda)L \sim \lambda L + (1-\lambda)L'$

Which is equivalent to $\forall L, L'$, and $\lambda \in (0, 1) \implies L \sim \lambda L + (1-\lambda)L'$ (betweenness axiom)

Therefore, if lotteries satisfy independent axiom, then they also satisfy betweenness axiom.

b.