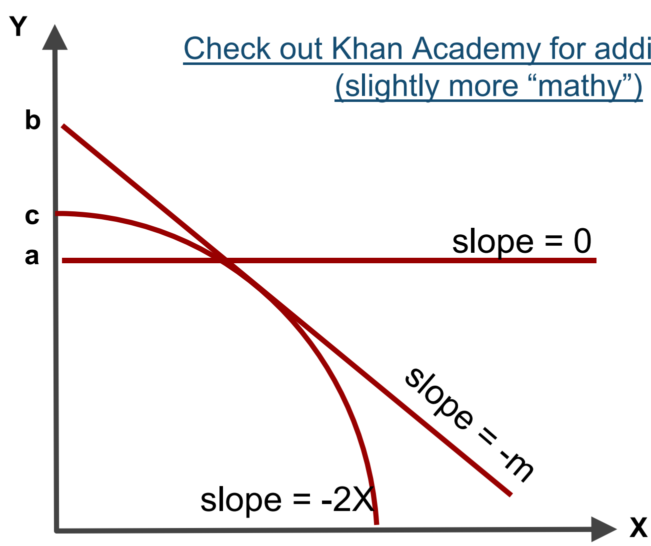
Math Review

Derivatives and Their Application in this Class

Slope of a curve – derivative of a curve
Answers the question: When X goes up by 1 unit, what is the change in Y?



[Check out Khan Academy for additional help (slightly more "mathy")]

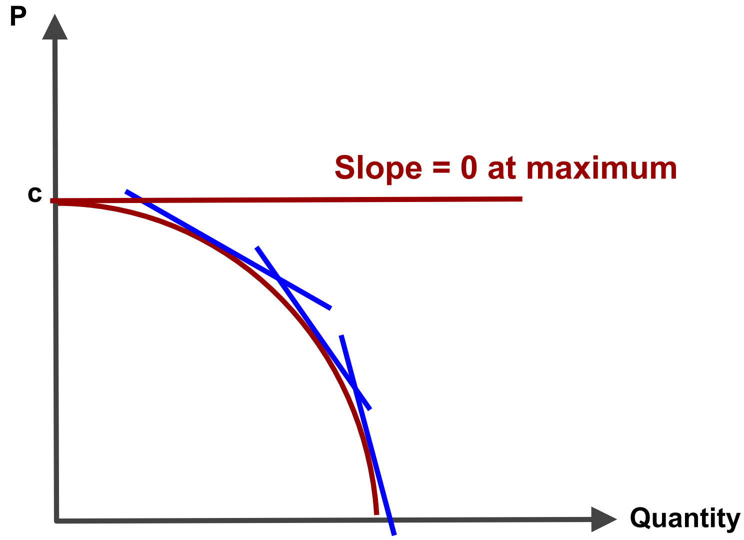
Slope:
$\Delta Y/\Delta X = dY/dX$
(derivative of Y wrt X)

What's the slope?
$Y = a$

What's the slope?
$Y = b - mX$

What's the slope?
$Y = -X^2 + c$

Derivative ≈ Slope



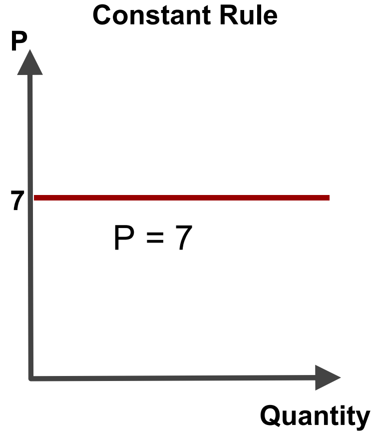
Slope:
Is not constant at every point

Each point on the curve has a different slope

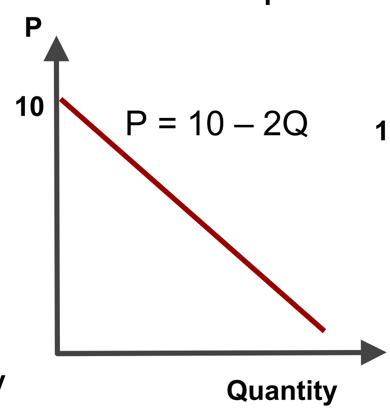
Only one interesting point on the entire curve

What Do You Mean slope = P'?

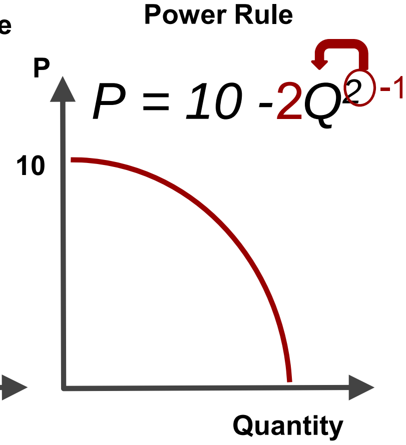
Constant Rule



Constant-Multiple Rule



Power Rule



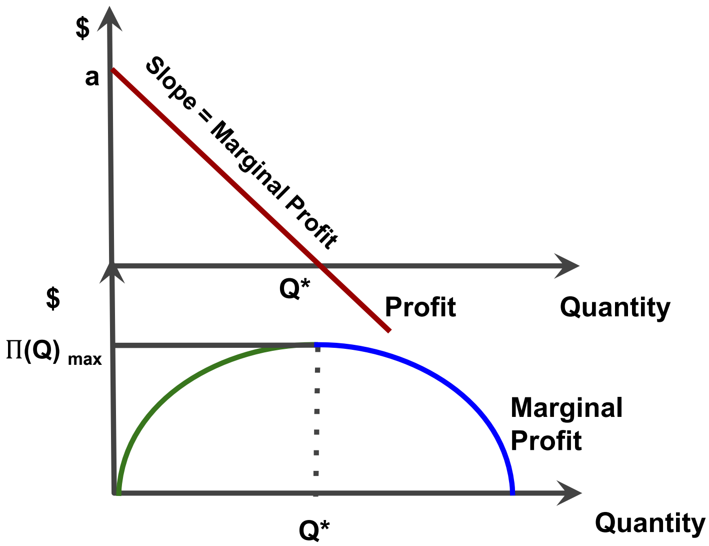
Derivative of P with respect to Q:

P' = 0

P' = -2

P' = -2Q

3 Types of Derivatives for this Course



Recall:

Profit:

$$\Pi(Q) = TR(Q) - TC(Q)$$

Marginal Profit:

$$\frac{d\Pi}{dQ} = \frac{dTR(Q)}{dQ} - \frac{dTC(Q)}{dQ}$$

$$M\Pi = MR(Q) - MC(Q)$$

Max Profit occurs where:

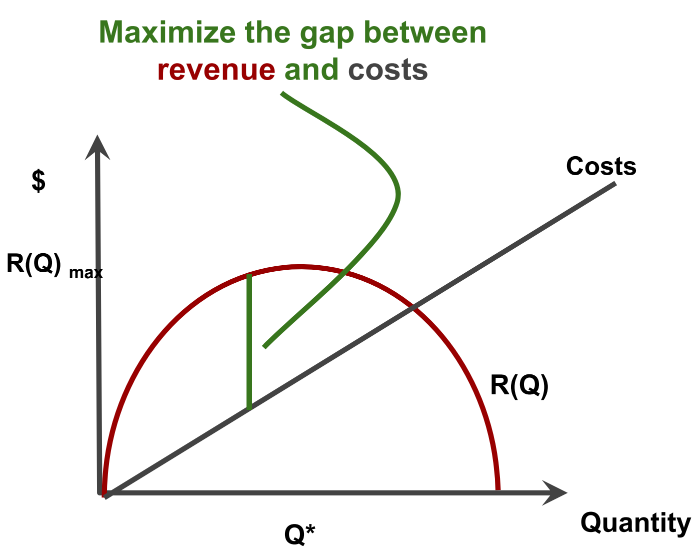
$$M\Pi = MR(Q) - MC(Q) = 0$$

$$MR(Q) = MC(Q)$$

Going forward:

Derivative ≈ Slope

Decision Making with Derivatives



Profit Maximizing Rule

Sell the good as long as additional revenue is more than additional cost

MR = Additional Revenue

MC = Additional Cost

MR ≥ MC

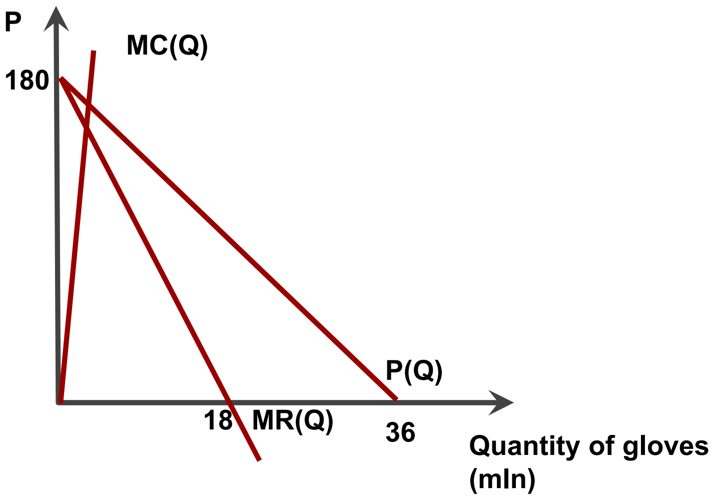
Profit = Revenue - Cost

Profit in Sports
(A Numerical Example)

A Numerical Example

Suppose you know that the demand function for Brett's Best baseball gloves is:
$P(Q) = 180 – 5Q$
The cost function of Brett's Best baseball gloves is:
$TC (Q) = 25Q^2$

Find the Total Revenue, Total Cost functions. Then find profit-maximizing Quantity, Price, and the maximum Profit



Demand for Brett's Best baseball gloves:
P(Q) = 180 – 5Q

Total Revenue:

$$TR(Q) = P(Q) \cdot Q = (180 - 5Q) \cdot Q = 180Q - 5Q^2$$

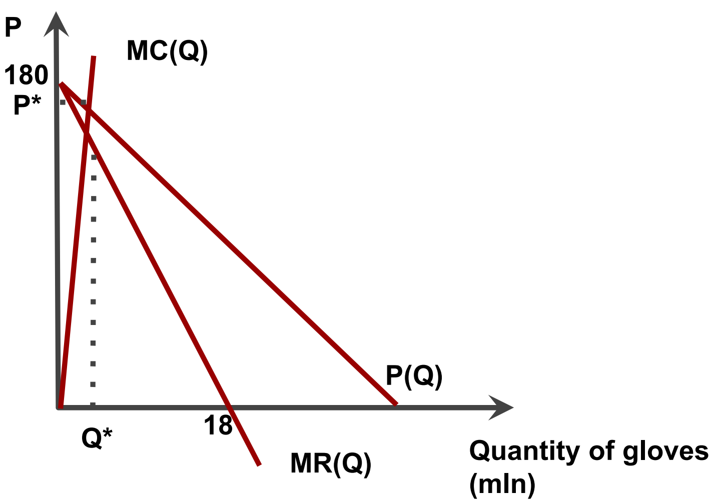
$$MR(Q) = \frac{dTR}{DQ} = 180 - 10Q$$

Total Cost:

$$TC(Q) = 25Q^2$$

$$MC(Q) = \frac{dTC}{DQ} = 50Q$$

A Numerical Example



Demand for Brett's Best baseball gloves:
P(Q) = 180 – 5Q

Profit-maximizing Quantity:

$$MR = MC$$
$$180 - 10Q = 50Q$$
$$Q^* = 3 \text{ mln}$$

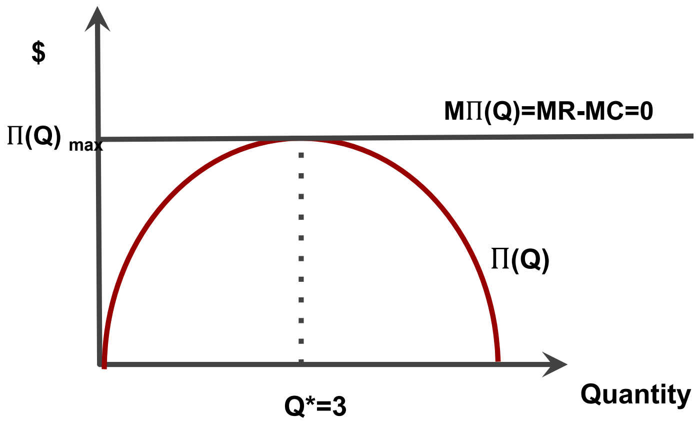
Profit-maximizing Price (plug Q* into the Demand function):

$$P^* = 180 - 5Q^* = \$\,165$$

Maximum Profit

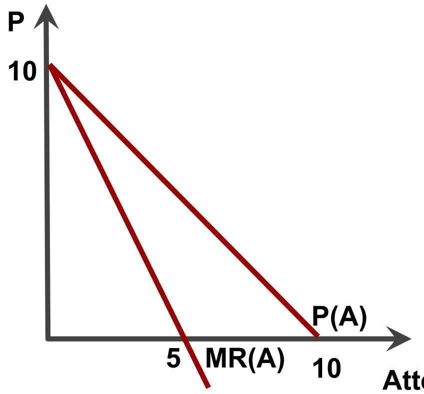
$$\Pi^* = TR - TC = 180Q - 5Q^2 - 25Q^2 = \$\,270 \text{ mln}$$

A Numerical Example



A Numerical Example

ELASTICITY OF DEMAND

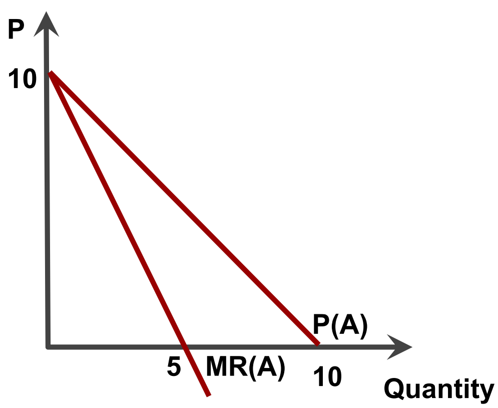


P(Q) = 10 – Q

MR(Q) = 10 – 2Q

Price	Atten.	Revenue
$1	9	$9
$2	8	$16
$3	7	$21
$4	6	$24
$5	5	$25
$6	4	$24
$7	3	$21

Mathematical Example of Revenue (Not in Lecture Notes)



P(Q) = 10 – Q

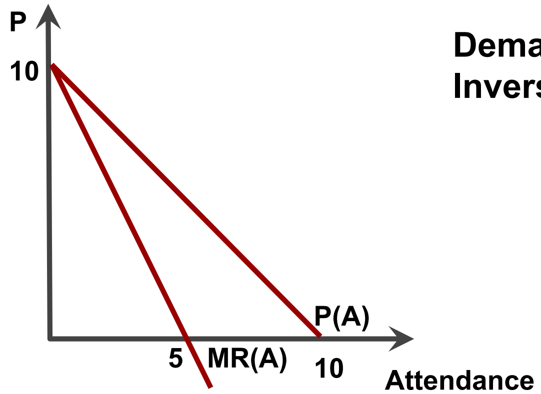
TR(Q) = P*Q= (10 - Q)*Q

$=10Q-Q^2$

How to find Marginal Revenue?

- **Take a derivative of Total Revenue with respect to Q**
- $MR(Q) = \frac{\partial TR}{\partial Q} = 10 - 2Q$

Mathematical Example of Revenue (Not in Lecture Notes)



Demand Function: Q = 10 - P
Inverse Demand Function: P(Q) = 10 – Q

How to find own-price Elasticity of Demand?

1) Take a derivative of Demand Function with respect to Price

$$\frac{\partial Q}{\partial P} = -1$$

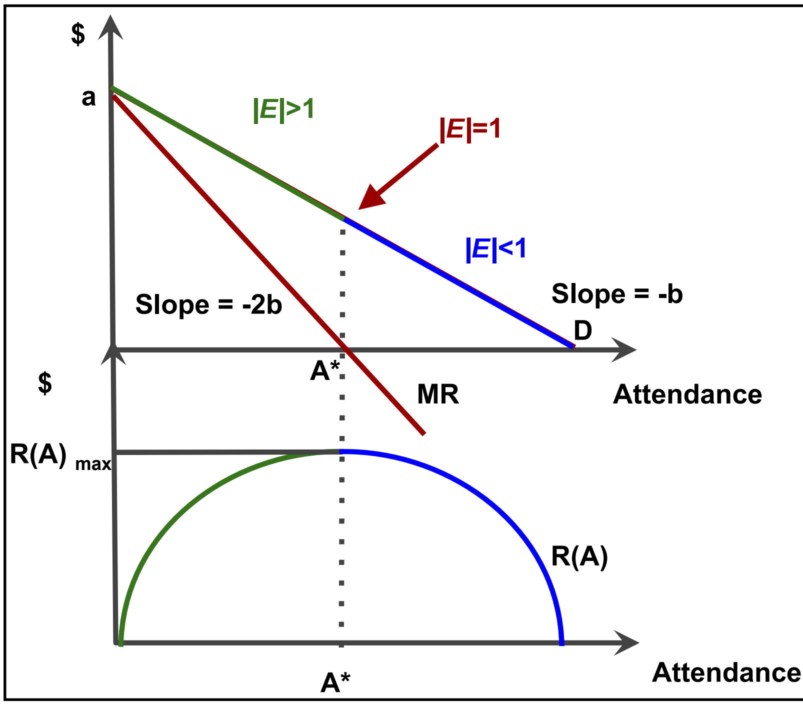
2) Elasticity at a point P, Q is:

$$E = \frac{\partial Q}{\partial P} \cdot \frac{P}{Q}$$

Examples: **Elasticity at a point P=3, Q=7 is:** $E = -1 \cdot \frac{3}{7} = -\frac{3}{7}$

Elasticity at a point P=5, Q=5 is: $E = -1 \cdot \frac{5}{5} = -1$

Mathematical Example of Revenue (Not in Lecture Notes)



3 Notes:

1. **Total Revenue, R(Q*) is maximized**
2. **MR(Q*) = 0**
3. **E = 1 (unit elastic)**

Calculating Future Value and Present Value

$1.00 received today is worth more in the future.

Consider a lender who lends $1.00 and will earn an interest rate of 10% on her loans.
At the end of year 1, they will have ($1.00)*(1.10) = $1.10
At the end of year 2, they will have ($1.00)*(1.10)*(1.10) = $1.21
….

Generalizing this: the Future Value of $1.00 in n years:

$FV = \$1.00 * (1 + i)^n$

where i is the interest rate, n is the number of years

Calculating Future Value

$1.00 received in the future is worth less today.

Consider a lender that will receive $1.00 in the future at 10% interest rate. What is the sum that the lender needs to invest today?

$1.00 to be received 1 year from now is worth $\frac{\$1.00}{1.10} = \$0.91$
$1.00 to be received 2 years from now is worth $\frac{\$1.00}{(1.10)(1.10)} = \$0.83$

Generalizing this: the Present Value of $1.00 received in n years:

PV = $1.00 / $(1 + i)^n$

where i is the interest rate, n is the number of years

Calculating Present Value

$$PV = \frac{FV}{(1+i)^n}$$

$$FV = PV(1+i)^n$$

Relationship between Future Value and Present Value

Calculating Future Value