Chapter 8: Insuring Player Talent

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February 27, 2019

Why insurance in sports?

Suppose a player has a contract for 5 years with an annual payment of 1 million to play for a team.

If the player gets injured in the first year and cannot compete anymore, does the player still receive the 1 million for the next 5 year?

Expected payoff

The expected payoff (for a gamble with 2 possible outcomes) is

$$E[V] = P(A) * w_A + P(B) * w_B$$

where P(A) and P(B) are the probabilities of outcome A and outcome B respectively.

w_A and w_A are the payoffs from outcome A and B.

Example: Suppose a person is faced with two possible gambles

Gamble A: 50% chance of winning \$ 100 and 50% chance of winning \$ 300

Gamble B: 50% chance of winning \$ 50 and 50% chance of winning \$ 350

Which Gamble will you choose?

What is the expected payoff from the two options?

$$E[V_A] = 0.5 * 100 + 0.5 * 300 = 200$$
$$E[V_B] = 0.5 * 50 + 0.5 * 350 = 200$$

Consider another game

A very boring fair game

- You pay an amount X to play the game
- If you play, the person flips a (fair) coin
- If it comes down heads you win $10
- If it comes down tails, you win $0

If you dont play the game you get $0 for sure

If you play the game, with 50% chance you get $10-X and with 50% chance you get X.

What is the most you would pay in order to play such a game? i.e. how big an x?

Maybe $ 5?

However there is a paradox

Here is a new game

- The fair person flips a coin
- If it comes down tails you get $2
- If it comes down heads, the coin gets flipped again
- If it comes down tails, you get $4
- If it comes down heads, the coin gets flipped again
- If it comes down tails you get $8
- If it come down heads, the coin gets flipped again
- Etc. etc

What is the expected payoff? How much would you pay to play the game?

$$E[V] = \frac{1}{2} * 2 + \frac{1}{4} * 4 + \frac{1}{8} * 8 + ...$$
$$\implies E[V] = 1 + 1 + 1 + 1 + ... = \infty$$

Expected utility

We need a model in which individuals can be risk averse (or risk neutral or risk lover), therefore we will compare expected utility rather than expected payoff.

Let, U(w) be the utility function of wealth (w).

Properties of U(w).

- $U'(w) > 0$, always since more wealth is always better.
- $U''(w) < 0 \implies$ risk averse.
- $U''(w) = 0 \implies$ risk neutral.
- $U''(w) > 0 \implies$ risk lover.

The expected utility (for a gamble with 2 possible outcomes) is

$$E[U(w)] = P(A) * U(w_A) + P(B) * U(w_B)$$

where P(A) and P(B) are the probabilities of outcome A and outcome B respectively.

$U(w_a)$ and $U(w_b)$ are the utility level from the payoff from outcome A and B (w_a and w_b) respectively.

So going back to the first example.

Expected utility from gamble A is

$$E[U(V_A)] = 0.5 * U(100) + 0.5 * U(300)$$

Expected utility from gamble B is

$$E[U(V_B)] = 0.5 * U(50) + 0.5 * U(350)$$

Lets consider a risk averse person with utility function $U(w) = ln(w)$(Natural log).

We know this person is risk averse since,

- $U'(w) = \dfrac{1}{w} > 0$
- $U''(w) = -\dfrac{1}{w^2} < 0 \implies$ risk averse.

So now the expected utility from gamble A is

$$\begin{aligned} E[U(V_A)] =& 0.5 * U(100) + 0.5 * U(300) \\ =& 0.5 * ln(100) + 0.5 * ln(300) \\ =& 5.15 \end{aligned}$$

And expected utility from gamble B is

$$\begin{aligned} E[U(V_B)] =& 0.5 * U(50) + 0.5 * U(350) \\ =& 0.5 * ln(50) + 0.5 * ln(350) \\ =& 4.88 \end{aligned}$$

So we see numerically that a risk averse person prefers gamble A, the safer gamble.

Lets consider a risk loving person with utility function $U(w) = w^2$

We know this person is risk averse since,

- $U'(w) = 2w > 0$
- $U''(w) = w \implies$ risk lover.

So now the expected utility from gamble A is

$$\begin{aligned} E[U(V_A)] =& 0.5 * U(100) + 0.5 * U(300) \\ =& 0.5 * (100)^2 + 0.5 * (300)^2 \\ =& 50,000 \end{aligned}$$

And expected utility from gamble B is

$$\begin{aligned} E[U(V_B)] =& 0.5 * U(50) + 0.5 * U(350) \\ =& 0.5 * (50)^2 + 0.5 * (350)^2 \\ =& 62,500 \end{aligned}$$

We see numerically that a risk loving person prefers gamble B, the riskier gamble.

So the question is how much will the risk averse pay for insurance to guarantee a payoff of 200?

Certainty Equivalence

Is the risk free income that will yield the same utility as the gamble. i.e. CE can be found using the below equation

- $U(CE) = E[U(w)]$

Where, E[U(w)] is the expected utility from the gamble.

In other words, person will be indifferent between taking CE with 100% probability and taking the gamble.

Lets consider an example from sports

Suppose, there is college player who expects to make $ 100 if he/ she is healthy (not injured) and drafted. However, if the player gets injured they will make $ 0. Lastly, data shows that this player has a 10% chance of being injured.

Also assume that the player is risk averse and has a utility function $U(w) = ln(1 + w)$. Where w is the player's payoff.

In this case the player's expected wealth is.

- $E[w] = 0.9 * 100 + 0.1 * 0 = 90$

Whereas, the player's expected utility is .

- $E[U(w)] = 0.9 * ln(1 + 100) + 0.1 * ln(1) = 4.15$

The certainty equivalence (CE) or the risk free payoff that will give the player the same utility as the risky scenario can be found via the below steps.

$$\begin{aligned} & U(CE) = E[U(w)] \\ \implies & ln(CE) = 4.15 \\ \implies & CE = e^{4.15} = 63.43 \end{aligned}$$

This means that a risk free payoff of \$ 63.43 will give the same uililty to the player as the risky scenario. i.e. since

- $ln(63.43) = 4.15 = E[U(w)]$

Now using the above information we can tell whether or not getting an insurance policy is good option. Or in other words whether or not the player should buy a certain type of insurance.

Suppose, this player has an insurance policy available that will pay the player \$ 100 in case he/ she gets injured. The cost of buying this insurance or the premium is \$30. This implies that the player will make \$ 70 (= 100 - 30) regardless of whether or not they are injured.

Should this player buy the insurance or simply take the risk of being injured?

Yes, this player should buy the insurance since the payoff of \$ 70 is higher than the CE.

Also since,

- $ln(1 + 70) = 4.26 > 4.15$

Real World Situation

In sports, players as well as teams buy such insurance policies in case a player get injured.

As seen in the above example, players will buy insurance in order to guarantee themselves a certain level of income in case they get injured.

- In NCAA, about 100 players have such insurance policies.
- These policies guarantee the player income of anywhere between 1 Million or 3 Million.
- The insurance premium on such policies is around \$ 10,000

Similarly, teams also buy insurance polices to protect themselves or give them money to recruit a players in order to replace an injured player.

The model used to analyze a player's decision making process to buy insurance can be applied to team's decision making process to buy insurance.

- Only difference is instead of comparing payoffs, teams will look at differences in profit (π) levels.