Chapter 6: Advertising in sports industry

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February 20, 2019

Optimal Advertising

Demand for the firms product is a function of price and advertising. Thus the inverse demand for function can be written as:

$$P = P(Q, A)$$

Price and quantity still have same inverse relationship Profit function is now:

$$\pi = P(Q, A)Q - C(Q) - wA$$

Example

Suppose a firm has a profit function like the one defined above. Solve for optimal Q and A given the below demand curve.:

$$P(Q, A) = 100 - 3Q + 4A^{1/2}$$

The Cost function is

$$C(Q) = 4Q^2 + 10Q$$

And the cost per unit of advertising is 1. i.e $w = 1$.

Using the above information we can write the profit function as below:

$$\pi = (100 - 3Q + 4A^{1/2})Q - 4Q^2 - 10Q - wA$$

$$\pi = 100Q - 3Q^2 + 4A^{1/2}Q - 4Q^2 - 10Q - A$$

Now in order to find the optimal level of Q and A we need to take the first order partial derivative of the π with respect to Q and A respectively and set the equal to 0. i.e.

$$\frac{d\pi}{dQ} = 0$$

$$\frac{d\pi}{dA} = 0$$

Therefore, we have

$$\frac{d\pi}{dQ} = 100 - 6Q + 4A^{1/2} - 8Q - 10 = 0$$

$$\implies 100 - 6Q + 4A^{1/2} = 8Q + 10$$

and,

$$\frac{d\pi}{dA} = 2QA^{1/2} - 1 = 0$$

$$\implies A^{1/2} = 2Q$$

This leaves us with two equations and two unknown. i.e.

$$100 - 6Q + 4A^{1/2} = 8Q + 10 \tag{1}$$

$$A^{1/2} = 2Q \tag{2}$$

Next, we plug in equation 2 into equation 1. i.e substitute $A^{1/2}$ for 2Q. Therefore, we have.

$$100 - 6Q + 4 * 2Q = 8Q + 10$$

$$\implies 6Q = 90$$

$$\implies Q^* = 15$$

And finally,

$$A^{1/2} = 2Q$$

$$\implies A^{1/2} = 2 * 15$$

$$\implies A^* = (30)^2 = 900$$

To get price, plug Q^* and A^* into the demand:

$$\begin{aligned} P(Q, A) =& 100 - 3Q + 4A^{1/2} \\ =& 100 - 3 * 15 + 4 * (900)^{1/2} \\ =& \$175 \end{aligned}$$

Now we can find the profit using the profit function:

$$\begin{aligned} \pi =& P(Q, A)Q - C(Q) - wA \\ =& 175 * 15 - 4 * (15)2 - 10 * 151 * 900 = \$675 \end{aligned}$$

Steps to find optimal Q (ticket sales) and A (Advertising)

1. Set up the profit function in terms of only Q and A. i.e. plug in inverse demand function in place of P.
2. Take first order derivative with respect to Q and A respectively and set it equal to 0.
3. You should have two equations. Equation 1 and equation 2.
4. Plug in equation 2 into equation 1 and solve for Q^*.
5. Finally, plug in Q^* in equation 2 to find A^*

Sports advertising by firms

Firms advertise in sports industry in several different ways:

1. Sports publication. Ads in different sports magazine.

2. Naming rights. Example Qualcomm stadium, Bank of America stadium. Sometimes even naming rights to practice stadium.

3. Sponsorship. For example official beverage, official clothing partner.

4. Endorsement. Individual players themselves also endorse certain brands that are different from the official sponsor. However, they should not be similar to the product from the official sponsor.

Present Value of Payments from a Ad contract.

$$PY[Payments] = \sum_{t=1}^{T} \frac{P_t}{(1+i)^t}$$

Where T is the time of the contract, i is the interest rate and P_t is the payments in the t year.

Example: Suppose there is contract for naming rights for 5 year, with a yearly payment of 5 Million and the interest rate is 10%. What is the total payment made by the firm? What is the present value of all the payments.

The Total payment is $5 * 5M = 25$ Million.

Whereas present value of all the payments is.

$$PY[Payments] = \sum_{t=1}^{5} \frac{5M}{(1+0.10)^t} = 18.95 Million.$$

Protection for official sponsor.
Most leagues have a policy regarding official sponsor.

For example the NFL has imposes the below fine if a player wears or somehow displays product from a competing brand to the official sponsor.

- $ 100,000 for violation during super bowl
- $ 50,000 for violation during the pro bowl
- $ 10,000 for violation during regular season

Short video on sponsorship in sports
https://www.youtube.com/watch?v=hGB7AN326Zc