# MS&E 135 Networks – Final Exam

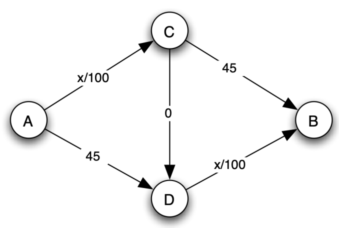
Stanford University, Winter 2021 (Pandemic Edition), Prof. Johan Ugander

**Instructions** Welcome to the 2021 **take home final**! This final is an open book, open note exam, but note that you are required to complete this exam alone, without collaborating or talking to any other persons, in person or virtually. Your final must be submitted online through Gradescope, and it will be due at 10:00 AM on March 17, 2021. You are required to tag your answers correctly on Gradescope. Please allow enough time prior to submission to ensure you are able to tag correctly.

The exam consists of three sections: True/False, short answer questions, and longer multi-part questions. For the short answer questions, follow the instructions there. For the longer questions, please explain your answers clearly, drawing illustrations where appropriate. Clear explanations of your reasoning are more important than flawless algebra or arithmetic. (On the True/False questions no reasoning is needed, nor will it be considered.)

**Part I: True/False (20 pts)** Label each statement as True or False. Write full words ("True" or "False") on your answer paper that you will upload, not just here on the exam sheet.

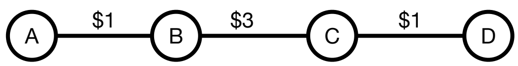
1. A player in a two-player game can have multiple strictly dominant strategies.
2. In a first-price auction, a bidder who bids their valuation is guaranteed a non-negative payoff.
3. For every integer $k > 1$, one can construct a (possibly very large) connected network whose diameter is at least $k$ times its average distance.
4. Under a VCG mechanism, the bidder with the highest valuation can increase her payoff by not bidding truthfully.
5. A cycle graph on $n > 3$ nodes is bipartite if and only if $n$ is even.
6. In a multi-item VCG auction, a *favorite seller* (someone for whom every buyer has the highest valuation for the item being sold) must receive the highest payment.
7. In the figure below, if 4,000 drivers leaving $A$ all destined for $B$ could coordinate, meaning that 2,000 were assigned to drive $A-C-B$ and 2000 were assigned to drive $A-D-B$, nobody would be incentivized to deviate from their assigned route.



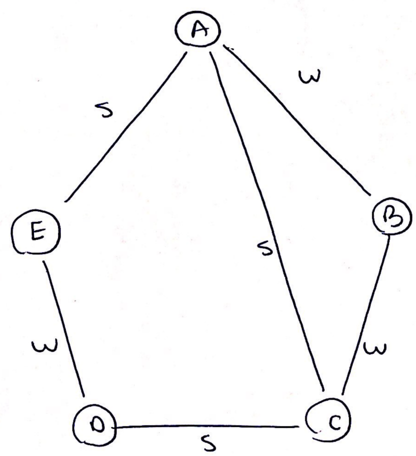

8. Consider an economy with network effects, where the reservation price is modeled by $r(x) = 1 - x$ and the network effect is modeled by $f(z) = z$. Suppose the price is $p^* = \frac{1}{2\pi}$. In this economy, there is exactly one stable self-fulfilling market equilibrium.
9. Stanley Milgram's "six degrees" experiment found that all chain letters completed with at most six intermediaries.
10. The MusicLab experiment provides empirical evidence that culture markets can be subject to significant herding.

## Part II: Short answers (30 pts)

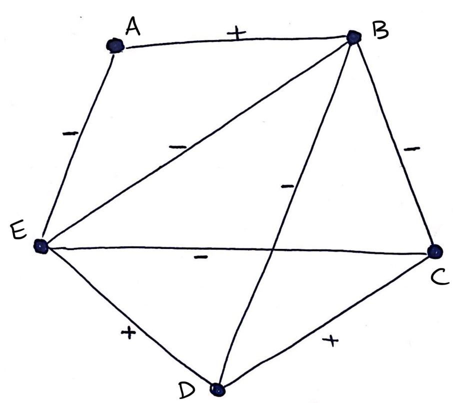
1. Consider a network exchange game played with a single-exchange rule but with heterogeneous amounts across edges, per the figure below. Which exchange do you think will happen (who will exchange, and how much will they each receive) and why?




2. In the following social network, which triplets of nodes violate the strong triadic closure property, if any? Briefly explain your answer.




3. In the signed social network below, find all the connected triads that do not satisfy strong structural balance.




4. (Web as a Network) Consider the network of 11 web pages drawn in Figure 1, with links forming a directed graph.

(a) Which nodes constitute the largest strongly connected component (SCC) in this graph?

(b) Taking your answer to (a) as the "giant" SCC, which nodes then belong to the sets IN and OUT?

(c) Add a node to the graph that is reachable from IN but does not reach SCC. Add a node that reaches OUT but is not reachable from SCC. Add a node that is both reachable from IN and reaches OUT but is not in the SCC.

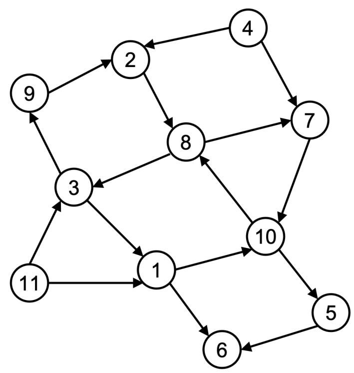
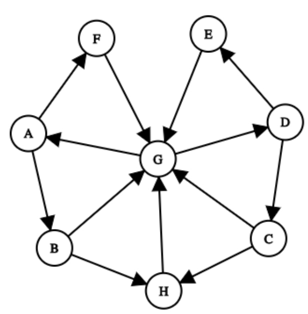



*Figure 1: Web network.*

5. Below is a network of 8 nodes. Perform two Basic PageRank updates, starting from $1/n$, showing the values for all nodes after each step.

   After these two steps, which three nodes have the highest PageRank values? Briefly explain why this aligns with the intuition that PageRank prioritizes "important" nodes in this case.



6. Table 1 summarizes buyer V, W, X, Y, and Z's valuations for a set of items, A, B, C, D, and E.

| | | Items | | | | |
|---|---|---|---|---|---|---|
| | | **A** | **B** | **C** | **D** | **E** |
| Buyers | **V** | 6 | 8 | 7 | 4 | 1 |
| | **W** | 3 | 10 | 5 | 10 | 7 |
| | **X** | 2 | 2 | 6 | 4 | 6 |
| | **Y** | 9 | 8 | 9 | 7 | 7 |
| | **Z** | 2 | 0 | 5 | 9 | 3 |

*Table 1: Buyer valuations of items.*

Assuming that the items are free, construct the preferred seller graph for this market. Does this bipartite network have a perfect matching? If so, return the matching, if not, identify the constricted set of buyers.

**Part III: Longer answers (50 pts)** Note: question components are demarcated (a), (b), (c), etc., to help make sure you answer each component, not because they necessarily carry equal weight.

## Question 1 (10 pts)

Companies $A$ and $B$ are developing marketing strategies for their new products, and they can either choose to have a low marketing budget or a high marketing budget, both of which will generate sales revenue.

Specifically, if B has a high marketing budget, A will generate $\$20,000$ from a low budget and $\$50,000$ from a high budget; and if B has a low budget, A will generate $\$50,000$ from a low budget and $\$40,000$ from a high budget. However, if A has a low budget, B will generate $\$20,000$ from a high budget and 0 from a low budget; and if A has a high budget, B will generate 0 from a high budget, and $\$40,000$ from a low budget.

(a) Set this up as a game with two players and two strategies, where the payoffs are in tens of thousands of dollars (i.e., $1 = \$10,000$). Find all (pure and mixed) Nash equilibria.

(b) An employee of $A$ argues that the company would be better off always choosing a high marketing budget because the worst possible outcome is better than the worst possible outcome from a low marketing budget. Should company $A$ follow this advice? Justify your answer with respect to part (a).

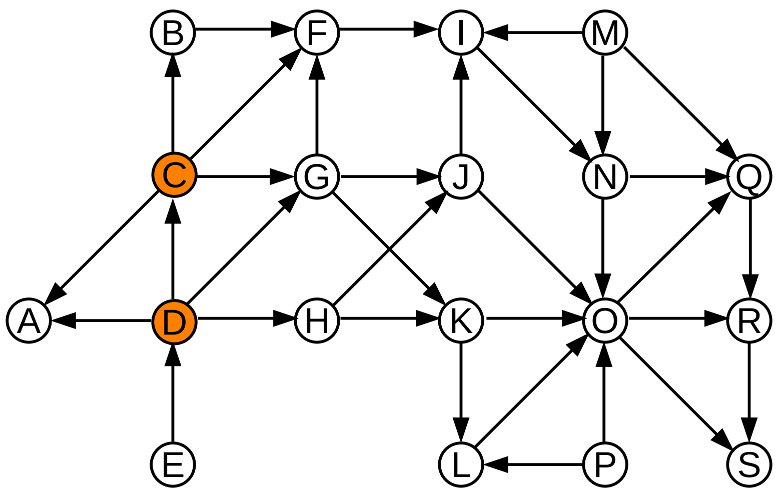
**Question 2 (10 pts)**

In an information cascades model, assume that the state of the world can be either Good or Bad with the same probability, 1/2. You'll observe a High signal given a Good state or a Low signal given a Bad state with probability $q = 3/4$. Suppose you are the $9th$ person to make a choice and you have observed that everyone before you chose *Accept*. What is the probability that this is an incorrect cascade?

**Question 3 (15 pts)**
We consider an SIR model for modeling the spread of interest in buying GameStop stock among users on Reddit. In this model, each node is a Reddit user, and nodes can be susceptible, infectious, or removed. If a node is infectious, we say that this node has "stonk fever". Importantly, we assume that stonk fever spreads like a biological infection, not a social contagion process, given the strong FOMO ("fear of missing out") that users experience.

In this network there is a directed edge from $x$ to $y$ if $y$ has a favorable opinion of $x$'s posts. The infection probability is $p = 0.5$ and the number of rounds that a user remains infectious is $t_I = 2$. As the SIR model dictates, we assume that removed nodes cannot become reinfected (because they're bankrupt, or they've retired).

Consider the following Reddit users in this network, at the very beginning of the spread of stonk fever (round $t = 0$). Initial infectious nodes $C$ and $D$ are orange.




(a) Suppose that at the conclusion of round $t = 1$, nodes $G$ and and $H$ are newly infected, and at the conclusion of round $t = 2$, nodes $B$, $F$, $J$, and $K$ are newly infected. Which nodes are in the susceptible set after round $t = 2$ concludes? Which nodes are in the removed set after round $t = 2$ concludes?

(b) What is the probability that, after the next round ($t = 3$) concludes, node $I$ is in the infected set?

(c) Continuing from the state in (a) (that is, the conclusion of round $t = 2$), find all nodes which can never be infected with stonk fever at any future time.

(d) We will crudely model the evolution of the price of GameStop stock as a function of the nodes that change states each round. Suppose that the price starts at \$2 at round $t = 0$, and moves up by \$1 for every newly infected node at the conclusion of each round $t = \tau$, and down by \$1 for every newly removed node at the conclusion of each round $t = \tau$. So, if there are 3 nodes newly infected and 1 node newly removed at the conclusion of round $t = \tau$, the GameStop price at the conclusion of round $t = \tau$ will be the price at the conclusion of round $t = \tau - 1$ plus \$2 (because \$3 - \$1 = \$2).

Forget what happened in part (a) and (b) above, and reset the model to its initial state, where $C$ and $D$ are infected at time $t = 0$, and the current time is $t = 0$. Assume that the new infections are (i) nodes $G$ and $H$ at the conclusion of round $t = 1$, (ii) nodes $B$, $F$, $J$, and $K$ at the conclusion of round $t = 2$, (iii) node $I$ at the conclusion of round $t = 3$, and (iv) node $O$ at the conclusion of round $t = 4$. What is the price of GameStop stock under this simple model, at the conclusion of rounds $t = 0, 1, 2, 3, 4$?

(e) Continuing from the conclusion of round $t = 4$ above, what is the expected value of the price of GameStop stock at the conclusion of round $t = 5$?

**Question 4 (15 pts)**
Suppose a search engine has two ad slots that it can sell. Slot $a$ has clickthrough rate of 200 and slot $b$ has a clickthrough rate of 100. Advertiser $x$ values clicks at \$10 per click, advertiser $y$ values clicks at \$4 per click, and advertiser $z$ values clicks at \$2 per click.

(a) Suppose that the search engine runs the VCG mechanism to allocate advertisers to slots. What assignment of slots will occur and what prices will the advertisers pay?

(b) Now, suppose that the search engine runs the Generalized Second-Price Auction (GSP) instead. Does truthful-bidding by all the advertisers form a Nash equilibrium? In other words, can any advertiser improve their payoff by changing his bid when others bid their values?

(c) Suppose that the search engine runs GSP again, but slot $b$ now has a clickthrough rate of 199 (instead of 100). Does truthful-bidding by all the advertisers form a Nash equilibrium?