Independence

P(cavity | raining) = P(cavity)

Cavity doesn't depend on weather, they are indep.

Bayes Rule

we observe P(effect | cause)

P(cause, effect$_1$)

But, if effect$_1$, effect$_2$, effect$_3$ ...

P(cause | e_1, e_2, e_3, e_4...)

P(cold | sneezing)

P(cold | sneeze, stomach pain, ...)

If indep. in variables identified, calc. simplified

Determine what's relevant in diagnosis of cause.

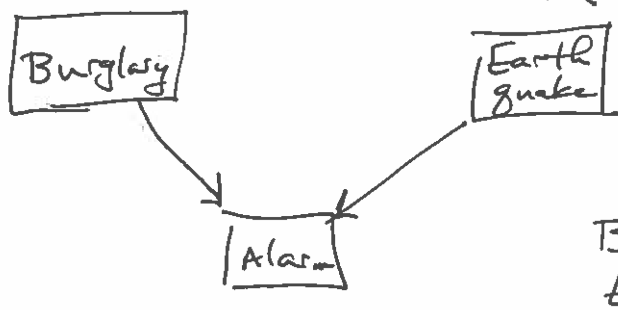
Bayes Net (Network)

Systematic way to represent dependence and indep. relationships in data

Directed graph. Used to make inference from data.

Properties

$P(B) = .001$ $\qquad$ $P(E) = .002$



B	E	P(a)
t	t	.95
t	f	.94
f	t	.29
f	f	.001

Calculate

$$P(a) = \sum_b \sum_e P(a,b,e) = \sum_b \sum_e P(a|b,e)\,P(b/e)P(e$$

$$= \sum_b \sum_e P(a|b,e)\,P(b)\,P(e)$$

$$= (.95 \times .001 \times .002) + (.94 \times .001 \times .998) + (.29 \times .999 \times .002) + (.001 \times .999 \times .998)$$

$$= .0025$$

$$P(a|b) = \frac{P(a,b)}{P(b)} = \frac{\sum_e P(a,b,e)}{P(b)} = \frac{\sum_e P(a|b,e)P(b)P(e}{P(b)}$$

$$= \frac{(.95 \times .001 \times .002) + (.94 \times .001 \times .998)}{.001} = .94$$

simplifies to $(.95 \times .002) + (.94 \times .998)$

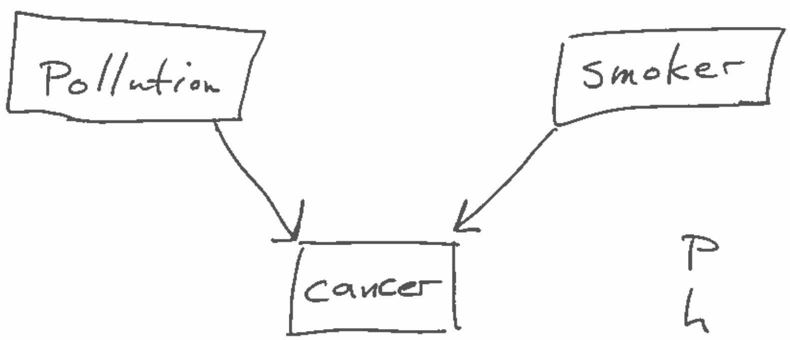
Bayes Rule

$$P(b|a) = \frac{P(a|b)\,P(b)}{P(a)}$$

$$= \frac{.94 \times .001}{.0025} = .376$$

Another example

$P(p) = .10$ $P(s) = .30$



p	s	P(c)
h	t	.05
h	f	.02
l	t	.03
l	f	.001

$$P(c) = \sum_p \sum_s P(c,p,s) = \sum_p \sum_s P(c|p,s)\,P(p)\,P(s)$$

$$= (.05 \times .10 \times .30) + (.02 \times .10 \times .70) +$$
$$(.03 \times .90 \times .30) + (.001 \times .90 \times .70)$$

$$= .01163$$

(3)

$$P(c|s) = \frac{\sum_{p} P(c,s,p)}{P(s)}$$

$$= \frac{P(c|s,p)P(s)P(p) + P(c|s,\neg p)P(s)P(\neg p)}{P(s)}$$

$$= P(c|s,p)P(p) + P(c|s,\neg p)P(\neg p)$$

$$= \underline{(.05 \times .10) + (.03 \times .90)} = .032$$

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$$P(s|c) = \frac{P(c|s)\,P(s)}{P(c)}$$

$$= \frac{.032 \times .30}{.01163}$$

$$= .825$$