

Unit-4: Vedik Mathematics Sutras

1. “Nikhilam Navataścaramam Daśataḥ” is a famous Sanskrit sutra from **Vedic Mathematics**.

Meaning (word by word)

- **Nikhilam** → All
- **Navataḥ** → From 9
- **Caramam** → The last
- **Daśataḥ** → From 10

Simple Translation

□ “All from 9 and the last from 10.”

What does it mean in practice?

This sutra is mainly used for **fast subtraction** (and also helps in multiplication and division techniques).

Rule

- Subtract **each digit from 9**
- Subtract the **last digit from 10**

Example 1: Subtract 123 from 1000

Step 1: Apply the rule to 123

- $1 \rightarrow 9 - 1 = 8$
- $2 \rightarrow 9 - 2 = 7$
- Last digit 3 $\rightarrow 10 - 3 = 7$

Result:

$$1000 - 123 = 877$$

Example 2: Subtract 458 from 1000

- $4 \rightarrow 9 - 4 = 5$
- $5 \rightarrow 9 - 5 = 4$
- Last digit 8 $\rightarrow 10 - 8 = 2$

Result:

$$1000 - 458 = 542$$

Why is this useful?

- ✓ Eliminates borrowing
- ✓ Very fast mental calculation
- ✓ Useful for numbers close to powers of 10 (10, 100, 1000, ...)

Key Idea to Remember

If the base is 10^n :

- Subtract **all digits from 9**
- Subtract the **last digit from 10**

2. Ekādhikena Pūrvēṇa: “By one more than the previous one.”

This is another important **sutra** from **Vedic Mathematics**, mainly used for **quick multiplication**, especially for numbers **ending in 5** and in some special algebraic cases.

Meaning (word by word)

- **Eka** → One
- **Adhikena** → More than
- **Pūrvēṇa** → The previous one

Literal Meaning

□ “**By one more than the previous one.**”

Main Application 1: Squaring numbers ending in 5

Rule

To square any number that ends in 5:

1. Take the digit(s) before 5
2. Multiply it by one more than itself
3. Write 25 at the end

Example 1: 35^2

- Previous number = 3

- One more than previous = 4
- Multiply: $3 \times 4 = 12$
- Append **25**

Result: $35^2 = 1225$

Example 2: 85^2

- Previous number = 8
- One more than previous = 9
- Multiply: $8 \times 9 = 72$
- Append **25**

Result:

$$85^2 = 7225$$

The sutra directly applies $n \times (n + 1) \rightarrow$ “one more than the previous”.

3. Udharan (Uddhāraṇa / Uddhāra): “The Extraction”

Udharan is a term used in **Vedic Mathematics** that refers to the **process of extracting** a value—most commonly **roots** (especially **square roots and cube roots**)—from a given number.

Meaning (word by word)

- **Udharan / Uddhāraṇa** → Extraction, drawing out, or taking out

Simple Meaning

□ “Extracting a value from a number.”

In mathematics, this usually means:

- Extracting square roots
- Extracting cube roots
- Occasionally, extracting factors or results hidden within a number

Main Application: Square Root Extraction

Example: Find $\sqrt{2025}$

Step-by-step idea (conceptual):

- We look for a number which, when squared, gives **2025**
- $45 \times 45 = 2025$

Result: $\sqrt{2025} = 45$

Here, **45** is extracted from 2025 → hence **Udharan (extraction)**.

Step 1: Group digits in pairs (from right to left)

20 25

Step 2: Find the nearest perfect square \leq first group

- First group = **20**
- Nearest square ≤ 20 is **16 = 4^2**

First digit of the root = **4**

Step 3: Subtract the square

$$20 - 16 = 4$$

Bring down the next group (**25**) → **425**

Step 4: Double the root obtained so far

$4 \times 2 = 8$. This acts as a trial divisor.

Step 5: Find the next digit

We need a digit **x** such that:

$$(80 + X) \times X \leq 425, \text{ we get } X = 5.$$

Next digit = 5 so answer is 45

4. Parāvartya (Parāvartya–Yojayet): “Transposition and Cancellation”

Parāvartya is an important concept/sutra in **Vedic Mathematics**, mainly used for **solving equations, algebraic simplification, and division**. It refers to **moving terms across an equation with sign change (transposition)** and **eliminating common terms (cancellation)** to simplify calculations quickly.

Meaning (word by word)

- **Parāvartya** → Transpose, shift, or change sides
- **Yojayet** → Apply or adjust
- **Cancellation** → Elimination of equal or common terms

Simple Meaning

“Transpose terms and cancel common factors to simplify.”

1 Parāvartya in Algebraic Equations (Transposition)

Example: Solve $3x+7=25$

Step-by-step:

- Transpose **+7** to RHS \rightarrow becomes **-7**

$$3x=25-7$$

$$3x=18$$

$$x=6$$

✓ Here, **Parāvartya** is used to shift terms across the equation.

5. Śūnyam Sāmyasamuccaye: "When the sum is the same, that sum is zero."

This is an important **Vedic Mathematics sutra**, mainly used for **solving algebraic equations**, especially **simultaneous equations** and equations where **common sums appear on both sides**.

Meaning (word by word)

- Śūnyam (Shunyam) \rightarrow Zero
- Sāmya \rightarrow Same / equal
- Samuccaye \rightarrow Collection or sum

Literal Meaning

□ "If the sums on both sides are equal, their difference is zero."

In practical math terms:

When identical expressions appear on both sides of an equation, they can be cancelled, leaving zero.

Core Idea (Very Simple)

If:

$$A+B=A+C$$

Then subtracting the common part A from both sides:

$$B = C$$

If $A-A=0$, This cancellation leads to **zero (Shunyam)**.

6. Anurūpyena: “Proportionately”

Anurūpyena is a Vedic Mathematics sutra used when **numbers change in proportion**. It helps simplify calculations by **adjusting values proportionally** instead of doing full, lengthy computations.

Meaning (word by word)

- **Anu** → According to / following
- **Rūpya** → Form, ratio, or pattern
- **Anurūpyena** → According to proportion

Simple Meaning

“Do it proportionately.”

Core Idea

When:

- Quantities increase or decrease in a **fixed ratio**
 - Or numbers are **not exactly suitable**, but close
- Adjust the calculation **proportionally** to get results **faster and mentally**.

1 Application in Multiplication (Common Use)

Example: Multiply 48×52

Observe:

- Both numbers are close to **50**

Step-by-step (Anurūpyena):

1. Take average base = **50**
2. Deviations:
 - $48 = 50 - 2$
 - $52 = 50 + 2$
3. Multiply deviations proportionately:

$$(50-2)(50+2) = 50^2 - 2^2 = 2500 - 4 = 2496$$

✓ Instead of long multiplication, we used **proportional adjustment**.

7. Sankalana–Vyavakalanābhyaṁ: “By addition and by subtraction.”

This is a fundamental **Vedic Mathematics sutra** used mainly for **solving simultaneous equations**, quick arithmetic, and mental math by cleverly using **addition and subtraction** instead of lengthy methods.

Meaning (word by word)

- **Sankalana** → Addition
- **Vyavakalana** → Subtraction
- **Ābhyaṁ** → By means of

Literal Meaning

□ “**By addition and by subtraction.**”

Core Idea

Instead of solving problems by complex algebra:

- **Add equations** to eliminate variables
- **Subtract equations** to eliminate variables

This leads to **quick simplification**.

1 Application in Simultaneous Linear Equations (Main Use)

Example:

Solve: $x+y=10$ & $x-y=2$

Step 1: Add the equations (Sankalana)

$$(x+y) + (x-y) = 10+2$$

$$2x=12 \Rightarrow x=6$$

Step 2: Subtract the equations (Vyavakalana)

$$(x+y) - (x-y) = 10-2$$

$$2y=8 \Rightarrow y=4$$

Final Answer: $x=6, y=4$

8. Pūraṇa–Apūraṇābhyaṁ

“By the completion or non-completion.”

This is a **Vedic Mathematics sutra** that deals with situations where a quantity is **complete (whole)** or **incomplete (short of a whole)**. It is especially useful in **fractions, decimals, complements, and adjustment-based calculations**.

Meaning (word by word)

- **Pūraṇa** → Completion / fullness
- **Apūraṇa** → Non-completion / deficiency
- **Ābhyaṁ** → By means of

Literal Meaning

□ “By completion or by non-completion.”

Core Idea (Intuition)

Instead of working directly with awkward numbers:

- **Complete the quantity to a convenient whole** (like 1, 10, 100)
- Or **work with the deficiency from that whole**

This makes calculations **simpler and faster**.

Application in Percentages

Example:

Find 97% of 250

Instead of direct calculation:

- 100% of 250 = 250
- Deficiency = 3% of 250 = 7.5

$$97\% = 250 - 7.5 = 242.5$$

✓ Used **Apūraṇa (deficiency)**.

Application in Multiplication

Example:

$$99 \times 46$$

99 is **1 less than 100**

$$\begin{aligned}99 \times 46 &= (100 \times 46) - (1 \times 46) \\&= 4600 - 46 \\&= 4554\end{aligned}$$

✓ Completed to 100, then subtracted the deficiency.

Application in Decimals

Example:

$$0.98 \times 50$$

0.98 is **0.02 less than 1**

$$(1 \times 50) - (0.02 \times 50)$$

$$50 - 1 = 49$$

9. Chalana–Kalana: “By motion or by applying a shift.”

Chalana–Kalana is a Vedic Mathematics sutra used when a problem can be solved by **shifting**, **moving**, or **sliding values** rather than recomputing everything from scratch. It is especially useful in **algebra**, **coordinate geometry**, **calculus-like reasoning**, and **mental math**.

Meaning (word by word)

- **Chalana** → Movement, shift, change of position
- **Kalana** → Calculation, computation

Literal Meaning

□ “**Solve by moving or shifting.**”

Core Idea (Intuition)

If a known value is slightly changed:

- **Shift** the value
- **Adjust** the result accordingly

Instead of full recalculation, we **track the effect of motion (change)**.

1 Application in Algebra (Shifting Values)

Example:

$$x^2 - 4x + 4 = 0$$

Recognize:

$$(x-2)^2=0$$

Here, **x is shifted by 2.**

$$x=2$$

✓ Instead of solving normally, we used a **shift**.

2 Application in Arithmetic Adjustment

Example:

Calculate:

$$101 \times 49$$

Shift 101 → 100 + 1

$$(100+1) \times 49$$

$$=4900 + 49$$

$$=4949$$

✓ Used **shift from base**.

4 Application in Sequences

Example:

Given:

$$2, 4, 6, 8, ?$$

Each term is obtained by **shifting +2**.

Next term:

$$10$$

10. Yāvadūnam: “Whatever is the deficiency.”

Yāvadūnam is a Vedic Mathematics sutra used when numbers are **less than a convenient base** (like 10, 100, 1000). The idea is to **identify the deficiency** from the base and use it directly to simplify calculations—especially **multiplication**.

Meaning (word by word)

- **Yāvat** → Whatever / as much as
- **Ūnam** → Deficiency / shortfall

Literal Meaning

- “**By whatever is the deficiency.**”

Core Idea (Very Simple)

When numbers are **below a base**:

1. Note how much each number is **less than the base**
2. Use those **deficiencies** to compute the answer quickly

This avoids long multiplication.

1 Main Application: Multiplication Near a Base

Example 1: $98 \times 9798 \times 9798 \times 97$

Base = 100

- $98 \rightarrow$ deficiency = **-2**
- $97 \rightarrow$ deficiency = **-3**

Step 1: Cross-subtract

$98 - 3 = 95$ or $97 - 2 = 95$

Step 2: Multiply deficiencies

$(-2) \times (-3) = 6$

Step 3: Combine

- Left part = **95**
- Right part = **06** (two digits because base = 100)
- Combined 9506

Final Answer:

$98 \times 97 = 9506$

11. Vyasti–Samastih: “The parts and the whole.”

Vyasti–Samastih is a Vedic Mathematics sutra that emphasizes understanding a problem by relating **individual parts (Vyasti)** to the **complete whole (Samasti)**. It is widely used in **algebra, arithmetic, averages, percentages, probability, and problem analysis**.

Meaning (word by word)

- **Vyasti** → Individual part, component
- **Samastih** → Whole, total, aggregate

Literal Meaning

- “The individual parts and the total whole.”

Core Idea (Intuition)

Instead of working blindly with formulas:

- Break the problem into **parts**
- Understand how they combine to form the **whole**
- Or derive parts when the **whole is known**

1 Application in Averages (Very Common)

Example:

Average marks of **5 students** is **60**.

Find total marks (Samasti):

$$60 \times 5 = 300$$

If one student scored **80**, find average of remaining **4**.

- Marks of remaining students (Vyasti):

$$300 - 80 = 220$$

- New average:

$$220 \div 4 = 55$$

✓ Used **parts ↔ whole** reasoning.

2 Application in Percentages

Example:

A class has **40 students**.

25% are girls.

- Girls (part):

$$25\% \text{ of } 40 = 10$$

- Boys (remaining part):

$$40 - 10 = 30$$

✓ Relates **parts to whole**.

12. Šeṣāṇyāṅkēna / Šeṣāṇyaṁ: “The remainder.”

(Often referred to simply as “Šeṣāṇya” or “Šeṣa” in Vedic Mathematics)

This sutra deals with **remainders** obtained after **division** and shows how the remainder can be used **directly and intelligently** to solve problems quickly.

Meaning (word by word)

- Šeṣa → Remainder

- **Aṅka / Aṅ** → Digit or value

Simple Meaning

- “**By the remainder.**”
- or
- “**Using the remainder.**”

Core Idea

When a number is divided:

- Instead of fully calculating the quotient,
- **Use the remainder** to draw conclusions, verify answers, or simplify further calculations.

1 Application in Divisibility Checks

Example:

Is **3456** divisible by **9**?

- Sum of digits:
 $3+4+5+6=18$
- Divide 18 by 9 → remainder **0**

✓ Since remainder is zero, **3456 is divisible by 9**

Here, the **remainder decides the result**.

2 Application in Finding Remainders of Large Numbers

Example:

Find the remainder when **1234** is divided by **9**

- Digit sum = $1+2+3+4=10$
- $10 \div 9 \rightarrow \text{remainder} = 1$

✓ Remainder = **1**

13. **Guṇita-Samuccayah**: “The product of the sum.”

Guṇita-Samuccayah is a Vedic Mathematics sutra used mainly for **checking results, simplifying algebraic expressions, and quick verification** of multiplication—especially using **digit sums** or **modular reasoning**.

Meaning (word by word)

- **Guṇita** → Multiplied / product
- **Samuccayah** → Sum / total / collection

Literal Meaning

□ “The product of the sum (is equal to the sum of the product).”
(Implied idea in Vedic usage)

In practice, it means:

If you take the sum (or digital root) of numbers and multiply, it should match the sum of the digits of their product (after reduction).

Core Idea (Very Simple)

This sutra is commonly used with **digit sums** (mod 9):

Digit sum of $(a \times b) \equiv (\text{digit sum of } a) \times (\text{digit sum of } b) \pmod{9}$

It is mainly a **verification sutra**, not a computation one.

1 Application: Checking Multiplication (Casting out 9s)

Example:

Check whether:

$$27 \times 43 = 1161 \quad 127 \times 43 = 1161$$

Step 1: Find digit sums

- $27 \rightarrow 2+7=9 \pmod{9} \rightarrow 0$
- $43 \rightarrow 4+3=7 \pmod{9} \rightarrow 7$

Multiply digit sums:

$$0 \times 7 = 0$$

Step 2: Digit sum of the result

- $1161 \rightarrow 1+1+6+1=9 \pmod{9} \rightarrow 0$

Step 3: Compare

✓ Both give 0 → Answer is correct

2 Another Example

Check:

$$58 \times 64 = 3712 \quad 58 \times 64 = 3712$$

- $58 \rightarrow 5+8=13 \rightarrow 4$
- $64 \rightarrow 6+4=10 \rightarrow 1$

Multiply:

$$4 \times 1 = 4$$

- $3712 \rightarrow 3+7+1+2=13 \pmod{9} \rightarrow 4$ hence ✓ Verified

14. Vistāraṇa (Vistaran): “Expansion.”

Vistāraṇa is a Vedic Mathematics sutra used to **expand expressions**—that is, to open brackets and express a compact mathematical form into a **detailed, expanded form**. It is especially useful in **algebra, multiplication, and polynomial operations**.

Meaning (word by word)

- Vistāraṇa → Expansion, spreading out, elaboration

Simple Meaning

- “Expand the expression.”

Core Idea

A compact expression can be:

- **Expanded** to show all its terms
- Making calculations, simplification, or comparison easier

This is the opposite of Saṅkalanā (factorization/condensation).

1 Application in Basic Algebra (Binomial Expansion)

Example:

$$(x+3)(x+5)$$

Step-by-step expansion (Vistāraṇa):

$$\begin{aligned} & x(x+5)+3(x+5) \\ & = x^2 + 5x + 3x + 15 \\ & = x^2 + 8x + 15 \end{aligned}$$

15. Rūpaṇa (Rupan): “Form.”

Rūpaṇa is a concept used in **Vedic Mathematics** to describe the **form or structure** in which a number or expression is written. The sutra emphasizes **choosing or transforming into the most convenient form** to make calculations easier and faster.

Meaning (word by word)

- Rūpa → Form, shape, appearance
- Rūpaṇa → Giving form, representation

Simple Meaning

- “Put the expression into a suitable form.”

Core Idea

Many problems are difficult **only because of their form.**

If we:

- **Change the form** (without changing the value),
- The problem often becomes **easy or mental.**

1 Application in Arithmetic (Changing Number Form)

Example:

Calculate: 99×37

Instead of direct multiplication, change the form:

$$\begin{aligned}(100-1) \times 37 \\ = 100 \times 37 - 1 \times 37 \\ = 3700 - 37 = 3663\end{aligned}$$

✓ Same value, **better form.**

2 Application in Algebra (Rewriting Expressions)

Example:

$$x^2 - 6x + 9$$

Rewrite in completed square form: $(x-3)^2$

✓ The **form** reveals the solution immediately.

Chidana (Chhedana / Chidana): “By splitting.”

Chidana is a Vedic Mathematics concept that means **splitting a number or expression into parts** so that each part becomes easier to handle. Instead of working with a number as a whole, we **divide it intelligently** and solve the problem step by step.

Meaning (word by word)

- **Chidana / Chhedana** → Splitting, cutting, dividing

Simple Meaning

- “**Solve by splitting.**”

Core Idea

A difficult calculation can be simplified by:

- **Breaking numbers into convenient parts**
- Solving each part separately
- Combining the results

This is a foundation of **mental arithmetic**.

1 Application in Multiplication

Example:

$$24 \times 13$$

Split 13:

$$\begin{aligned} &= 24 \times (10 + 3) \\ &= (24 \times 10) + (24 \times 3) \\ &= 240 + 72 = 312 \end{aligned}$$

✓ Used **splitting**.

2 Application in Addition

Example:

$$58 + 27$$

Split 27:

$$\begin{aligned} &58 + (20 + 7) \\ &= 78 + 7 = 85 \end{aligned}$$