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Team Reference Document

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1 Code

1.1 Build System Linux

```
{
  "cmd" : ["ulimit -s 268435456;g++ -std=c++20
           $file_name -o $file_base_name && timeout 4s
           ./ $file_base_name<input.txt>output.txt"],
  "selector" : "source.c",
  "shell" : true,
  "working_dir" : "$file_path"
}
```

15 1.2 Build System Windows

```
{
  "cmd": ["g++.exe", "-std=c++20", "${file}", "-o",
          "${file_base_name}.exe", "&&",
          "${file_base_name}.exe<input.txt>output.txt"],
  "selector": "source.cpp",
  "shell": true,
  "working_dir": "$file_path"
}
```

16 1.3 Stress Testing(check.sh)

```
// chmod u+x check.sh
// ./check.sh
set -e
g++ gen.cpp -o gen
g++ code.cpp -o code
g++ brute.cpp -o brute
for ((i = 1; ; ++i)); do
  echo "Passed on TestCase: " $i
  ./gen $i > in
  ./code < in > out1
  ./brute < in > out2
  diff -Z out1 out2 || break
done
echo -e "WA on the following test:"
cat in
echo -e "\nExpected:"
cat out2
echo -e "\nFound:"
cat out1
```

19 1.4 Stress Testing(gen.cpp)

```
#include <bits/stdc++.h>
using namespace std;
using ll = long long;
mt19937_64 rng(chrono::steady_clock::now().time_since_
               epoch().count());
inline ll gen_random(ll l, ll r) {
  return uniform_int_distribution<ll>(l, r)(rng);
}
inline double gen_random_real(double l, double r) {
  return uniform_real_distribution<double>(l, r)(rng);
}
int main(int argc, char* args[]) {
  int _ = atoi(args[1]);
  rng.seed(_);
  int n = gen_random(1, 5);
  vector<int> per;
  for (int i = 0; i < n; ++i) {
    per.push_back(i + 1);
  }
  shuffle(per.begin(), per.end(), rng);
  return 0;
}
```

1.5 dbg

```
#include <bits/stdc++.h>
using namespace std;
string to_string(const char c) {
    return "" + string(1, c) + "";
}
string to_string(const string& s) {
    return "" + s + "";
}
string to_string(const char* s) {
    return to_string((string) s);
}
string to_string(bool b) {
    return (b ? "true" : "false");
}
template <size_t N>
string to_string(bitset<N> v) {
    return v.to_string();
}
template <typename A, typename B>
string to_string(pair<A, B> p) {
    return "(" + to_string(p.first) + ", " +
        to_string(p.second) + ")";
}
template <typename A>
string to_string(A v) {
    bool first = true;
    string res = "{";
    for (const auto &x : v) {
        if (!first) {
            res += ", ";
        }
        first = false;
        res += to_string(x);
    }
    res += "}";
    return res;
}
void dbg_out() { cerr << endl; }
template <typename Head, typename... Tail>
void dbg_out(Head H, Tail... T) {
    cerr << " " << to_string(H);
    dbg_out(T...);
}
#define dbg(...) cerr << "Line " << __LINE__ << ": " <<
    << "[" << #__VA_ARGS__ << "]:", dbg_out(__VA_ARGS__)
/*
#include "dbg.h"
int main() {
    char c = 'a';
    int a = 2;
    string s = "diu";
    vector<int> v = {2, 1, 3};
    set<int> st = {2, 1, 3};
    map<int, int> cnt;
    cnt[0]++, cnt[1]++, cnt[0]++;
    dbg(c, a, s, v, st, cnt);
    dbg('c');
    dbg("diu");
    bitset<5> bs = 5;
    dbg(bs);
    dbg(int(bs[2]));
}
*/
```

1.6 pythonTemp

```
import math, sys
input = sys.stdin.buffer.readline
```

```
write = sys.stdout.write
tc = int(input())
for t in range(tc):
    h1, h2, b = map(int, input().split())
    h = math.log(h2 / h1)
    bb = math.log((b - 1) / b)
    ans = math.ceil(h / bb)
    print(ans)
```

1.7 2-SAT

```
struct _2SAT {
    int N;
    vector<bool> vis, value;
    vector<int> order, comp;
    vector<vector<int>> adj, adjT;
    _2SAT(int n) : N(n), adj(2 * n), adjT(2 * n), vis(2 * n), comp(2 * n), value(2 * n) {}
    void dfs1(int u) {
        vis[u] = true;
        for (auto v: adj[u]) {
            if (!vis[v]) {
                dfs1(v);
            }
        }
        order.push_back(u);
    }
    void dfs2(int u, int cnt) {
        comp[u] = cnt;
        for (auto v: adjT[u]) {
            if (!comp[v]) {
                dfs2(v, cnt);
            }
        }
    }
    void Kosaraju() {
        for (int i = 0; i < 2 * N; ++i) {
            if (!vis[i]) dfs1(i);
        }
        reverse(order.begin(), order.end());
        int cnt = 1;
        for (auto u: order) {
            if (!comp[u]) {
                dfs2(u, cnt++);
            }
        }
    }
    bool assignment() {
        Kosaraju();
        for (int i = 0; i < N; ++i) {
            if (comp[i] == comp[i + N]) {
                return false;
            }
            value[i] = comp[i] < comp[i + N] ? 0 : 1;
        }
        return true;
    }
    void addDisjunction(int a, bool pos_a, int b, bool
        pos_b) { // a V b
        int neg_a = a + N, neg_b = b + N;
        if (!pos_a) swap(a, neg_a);
        if (!pos_b) swap(b, neg_b);
        adj[neg_a].push_back(b);
        adj[neg_b].push_back(a);
        adjT[a].push_back(neg_b);
        adjT[b].push_back(neg_a);
    }
}
```

};

1.8 Aho Corasick

```
const int N = 1e6 + 3, A = 26;
int trie[N][A], node[N], dp[N];
int total = 0;
void add(string& s, int i) {
    int u = 0;
    for (char c: s) {
        int k = c - 'a';
        if (!trie[u][k]) {
            trie[u][k] = ++total;
        }
        u = trie[u][k];
    }
    node[i] = u;
}
vector<int> ord;
int slink[N];
void build() {
    queue<int> q;
    q.push(0);
    while (q.size()) {
        int p = q.front();
        q.pop();
        ord.push_back(p);
        for (int c = 0; c < A; ++c) {
            int u = trie[p][c];
            if (!u) continue;
            q.push(u);
            if (!p) continue;
            int v = slink[p];
            while (v and !trie[v][c]) v = slink[v];
            if (trie[v][c]) slink[u] = trie[v][c];
        }
    }
}
void solve() {
    build();
    int u = 0;
    for (char c: text) {
        c -= 'a';
        while (u and !trie[u][c]) u = slink[u];
        u = trie[u][c];
        dp[u]++;
    }
    reverse(ord.begin(), ord.end());
    for (int u: ord) {
        dp[slink[u]] += dp[u];
    }
}
```

1.9 Articulation Point and Bridges

```
// Articulation point
vector<vector<int>> adj;
vector<int> tin, low;
vector<bool> vis;
int timer;
void is_cutpoint(int v) {
    // process the cutpoint
}
void dfs(int v, int p = -1) {
    vis[v] = true;
    tin[v] = low[v] = timer++;
    int children = 0;
    for (int u: adj[v]) {
        if (u == p) continue;
        if (vis[u]) {
            low[v] = min(low[v], tin[u]);
        } else {
            dfs(u, v);
        }
    }
}
```

```

        low[v] = min(low[v], low[u]);
        if (low[u] >= tin[v] && p != -1) {
            is_cutpoint[v] = true;
        }
        ++children;
    }
}
if (p == -1 && children > 1) {
    is_cutpoint[v] = true;
}
}

void find_cutpoints(int n) {
    timer = 0;
    vis.assign(n + 1, false);
    is_cutpoint.assign(n + 1, false);
    tin.assign(n + 1, -1);
    low.assign(n + 1, -1);
    for (int i = 1; i <= n; ++i) {
        if (!vis[i]) {
            dfs(i);
        }
    }
}

// Bridges
vector<vector<int>> adj;
vector<int> tin, low;
vector<bool> vis;
int timer;
void is_bridge(int v, int to) {
    //process the found bridge
}

void dfs(int v, int p = -1) {
    vis[v] = true;
    tin[v] = low[v] = timer++;
    bool parent_skipped = false;
    for (int u : adj[v]) {
        if (u == p && !parent_skipped) {
            parent_skipped = true;
            continue;
        }
        if (vis[u]) {
            low[v] = min(low[v], tin[u]);
        } else {
            dfs(u, v);
            low[v] = min(low[v], low[u]);
            if (low[u] > tin[v]) {
                is_bridge(v, u);
            }
        }
    }
}

void find_bridges() {
    timer = 0;
    vis.assign(n, false);
    tin.assign(n, -1);
    low.assign(n, -1);
    for (int i = 0; i < n; ++i) {
        if (!vis[i]) {
            dfs(i);
        }
    }
}
}

```

1.10 Bellman Ford

```

const int INF = 1e9;
struct Edge {
    int u, v, w;
};
void solve() {
    int n, m;
    cin >> n >> m;
    vector<Edge> e(m);
    for (int i = 0; i < m; ++i) {

```

```

        cin >> e[i].u >> e[i].v >> e[i].w;
    }
    vector<int> d(n + 1, INF);
    d[1] = 0; // distance of source node
    vector<int> p(n + 1, -1); // parent vector
    int x;
    for (int i = 1; i <= n; ++i) {
        x = -1;
        for (auto [u, v, w]: e)
            if (d[u] < INF and d[u] + w < d[v]) {
                d[v] = d[u] + w;
                p[v] = u;
                x = v;
            }
    }
    if (x == -1) cout << "No negative cycle found\n";
    else { // Path Printing
        int y = x;
        for (int i = 0; i < n; ++i) y = p[y];
        vector<int> path;
        for (int cur = y; ; cur = p[cur]) {
            path.push_back(cur);
            if (cur == y && path.size() > 1) break;
        }
        reverse(path.begin(), path.end());
        cout << "Negative cycle: ";
        for (int u : path) cout << u << " ";
        cout << "\n";
    }
}

```

1.11 Big Integer

```

class BIG_INT {
private:
    string result;
public:
    string bigfinder(string a, string b) {
        if (a.size() < b.size()) swap(a, b);
        string d = b;
        reverse(full(b));
        while (b.size() < a.size()) b.pb('0');
        reverse(full(b));
        int i = 0;
        while (a[i]) {
            if (a[i] > b[i]) return a;
            else if (a[i] < b[i]) return d;
            i++;
        }
        return "same";
    }
    ll stringtonumber(string a) {
        ll n = 0;
        for (ll i = 0; a[i]; i++) n = (n * 10) + (a[i] - 48);
        return n;
    }
    string add(string a, string b) {
        result.clear();
        reverse(full(a));
        reverse(full(b));
        if (a.size() < b.size()) swap(a, b);
        while (b.size() < a.size()) b.pb('0');
        ll i = 0, carry = 0;
        while (a[i]) {
            carry = carry + a[i] - 48 + b[i] - 48;
            result.pb((carry % 10) + 48);
            carry = carry / 10;
            i++;
        }
        while (carry > 9) {
            result.pb((carry % 10) + 48);
            carry = carry / 10;
        }
        if (carry != 0) result.pb(carry + 48);
    }
}

```

```

        reverse(full(result));
        return result;
    }
    string subtraction(string a, string b) {
        result.clear();
        bool flag = true;
        if (bigfinder(a, b) == b) {
            swap(a, b);
            flag = false;
        }
        reverse(full(a));
        reverse(full(b));
        while (b.size() < a.size()) b.pb('0');
        int i = 0, carry = 0, x = 0;
        while (a[i]) {
            if (b[i] > a[i]) x = (a[i] - 48) + 10;
            else x = a[i] - 48;
            carry = x - (carry + (b[i] - 48));
            result.pb(carry + 48);
            carry = x / 10;
            i++;
        }
        while (result[result.size() - 1] == '0' and
            result.size() > 1)
            result.erase(result.size() - 1, 1);
        if (!flag) result.pb('-');
        reverse(full(result));
        return result;
    }
    string multiplication(string a, string b) {
        if (b.size() > a.size()) swap(a, b);
        reverse(full(a));
        reverse(full(b));
        while (a.size() > b.size()) b.pb('0');
        vector<string> x;
        for (ll i = 0; b[i]; i++) {
            ll carry = 0;
            string str;
            for (ll j = 0; a[j]; j++) {
                str += (((b[i] - 48) * (a[j] - 48)) + carry) % 10 + 48;
                carry = (((b[i] - 48) * (a[j] - 48)) + carry) / 10;
            }
            if (carry > 0) str += carry + 48;
            reverse(full(str));
            ll zero = i;
            while (zero--) str += '0';
            x.pb(str);
        }
        ll len = x.size();
        if (len == 1) result = x[0];
        else {
            for (ll i = 0; i < len - 1; i++) {
                x[i + 1] = add(x[i], x[i + 1]);
            }
            result = x[len - 1];
            while (result[0] == '0' and result.size() > 1)
                result.erase(result.begin() + 0);
            return result;
        }
    }
};

// Big Integer Division
void bigDivision() {
    string a = "50";
    ll b = 6;
    ll len = a.length(), mod = 0, d = Digit(b), lowest =
        0, i = 0;
    while (i < d or lowest < b) {
        lowest = (lowest * 10) + (a[i] - 48);
        i++;
    }
    while (i < len + 1) {

```

```

mod = lowest % b;
lowest = (mod * 10) + (a[i] - 48);
if (b > lowest) {
    lowest = (lowest * 10) + (a[i] - 48);
    i++;
}
i++;
}
cout << mod << endl;
}

```

1.12 Centroid Decomposition Struct

```

struct CentroidDecomposition {
    set<int> adj[N];
    map<int, int> dis[N];
    int sz[N], par[N], ans[N];
    void init(int n) {
        for(int i = 1; i <= n; ++i) {
            adj[i].clear(), dis[i].clear();
            ans[i] = inf;
        }
    }
    void addEdge(int u, int v) {
        adj[u].insert(v); adj[v].insert(u);
    }
    int dfs(int u, int p) {
        sz[u] = 1;
        for(auto v : adj[u]) if(v != p) {
            sz[u] += dfs(v, u);
        }
        return sz[u];
    }
    int centroid(int u, int p, int n) {
        for(auto v : adj[u]) if(v != p) {
            if(sz[v] > n / 2) return centroid(v, u, n);
        }
        return u;
    }
    void dfs2(int u, int p, int c, int d) {
        dis[c][u] = d;
        for(auto v : adj[u]) if(v != p) {
            dfs2(v, u, c, d + 1);
        }
    }
    void build(int u, int p) {
        int n = dfs(u, p);
        int c = centroid(u, p, n);
        if(p == -1) p = c;
        par[c] = p;
        dfs2(c, p, c, 0);
        vector<int> tmp(adj[c].begin(), adj[c].end());
        for(auto v : tmp) {
            adj[c].erase(v); adj[v].erase(c);
            build(v, c);
        }
    }
    void modify(int u) {
        for(int v = u; v != 0; v = par[v]) {
            ans[v] = min(ans[v], dis[v][u]);
        }
    }
    int query(int u) {
        int mn = inf;
        for(int v = u; v != 0; v = par[v]) {
            mn = min(mn, ans[v] + dis[v][u]);
        }
        return mn;
    }
} cd;

```

1.13 Centroid Decomposition

```

const int N = 2e5+5;
int n, k;
vector<int> adj[N];
int sz[N], cen[N];
ll ans = 0;
void dfs_sz(int u, int p) {
    sz[u] = 1;
    for(auto v: adj[u]) {
        if (v != p and !cen[v]) {
            dfs_sz(v, u);
            sz[u] += sz[v];
        }
    }
}
int get_cen(int u, int p, int s) {
    for(auto v: adj[u]) {
        if (v != p and !cen[v] and 2 * sz[v] > s) return
            get_cen(v, u, s);
    }
    return u;
}
int t, tin[N], tout[N], nodes[N], dep[N];
void dfs(int u, int p) {
    nodes[t] = u;
    tin[u] = t++;
    for(auto v: adj[u]) {
        if (v != p and !cen[v]) {
            dep[v] = dep[u] + 1;
            dfs(v, u);
        }
    }
    tout[u] = t - 1;
}
void go(int u) {
    dfs_sz(u, u);
    int c = get_cen(u, u, sz[u]);
    cen[c] = 1;
    t = 0;
    dep[c] = 0;
    dfs(c, c);
    int cnt[t][1];
    for(auto v: adj[c]) {
        if (!cen[v]) {
            for(int i = tin[v]; i <= tout[v]; ++i) {
                int w = nodes[i];
                int req = k - dep[w];
                if (req >= 0 and req < t) {
                    ans += cnt[req];
                }
            }
            for(int i = tin[v]; i <= tout[v]; ++i) {
                int w = nodes[i];
                cnt[dep[w]]++;
            }
        }
    }
    for(auto v: adj[c]) {
        if (!cen[v]) {
            go(v);
        }
    }
}
void solve () {
    cin >> n >> k;
    for(int e = 0; e < n - 1; ++e) {
        int u, v; cin >> u >> v; u--, v--;
        adj[u].push_back(v);
        adj[v].push_back(u);
    }
    go(0);
    cout << ans << "\n";
}

```

1.14 Chinese Remainder Theorem

```

ll extended_euclidean(ll a, ll b, ll& x, ll& y) {
    x = 1, y = 0;
    ll x1 = 0, y1 = 1, a1 = a, b1 = b;
    while (b1) {
        ll q = a1 / b1;
        tie(x, x1) = make_tuple(x1, x - q * x1);
        tie(y, y1) = make_tuple(y1, y - q * y1);
        tie(a1, b1) = make_tuple(b1, a1 - q * b1);
    }
    return a1;
}
pair<ll, ll> CRT(vector<ll> A, vector<ll> M) {
    ll n = A.size();
    ll a1 = A[0];
    ll m1 = M[0];
    for(ll i = 1; i < n; i++) {
        ll a2 = A[i];
        ll m2 = M[i];
        ll g = gcd(m1, m2);
        if (a1 % g != a2 % g) return {-1, -1};
        ll p, q;
        extended_euclidean(m1 / g, m2 / g, p, q);
        ll mod = m1 / g * m2;
        if (mod > 1e10) return {-1, -1};
        ll x = (a1 * (m2 / g) * q + a2 * (m1 / g) * p) %
            mod;
        a1 = x;
        if (a1 < 0) a1 += mod;
        m1 = mod;
    }
    return {a1, m1};
}

```

1.15 Convex Hull

```

struct Point {
    ll x, y;
    Point () {
        this->x = 0;
        this->y = 0;
    }
    Point (ll x, ll y) {
        this->x = x;
        this->y = y;
    }
    bool operator ==(const Point& p) const {
        return (this->x == p.x and this->y == p.y);
    }
    bool operator <(const Point& p) const {
        return make_pair(this->x, this->y) <
            make_pair(p.x, p.y); // with respect to x-axis
        // // with respect to angle from (0, 0)
        // if (*this * p == 0) {
        //     return dis() < p.dis();
        // }
        // return (*this * p < 0);
    }
    void operator -=(const Point& p) {
        this->x -= p.x;
        this->y -= p.y;
    }
    Point operator -(const Point& p) const {
        Point q;
        q.x = this->x - p.x;
        q.y = this->y - p.y;
        return q;
    }
    bool operator *(const Point& p) const {
        return x * p.y - y * p.x;
    }
    ll triangle(const Point& a, const Point& b) {

```



```

    return (a - *this) * (b - *this);
}
pair<double, double> rotate(double deg) {
    deg = deg * M_PI / 180.0;
    return {x * cos(deg) - y * sin(deg), x * sin(deg)
        + y * cos(deg)};
}
bool isInside(Point& a, Point& b) const { // if p is
    inside segment a-b
    if ((a - *this) * (b - *this) != 0) return false;
    bool d1 = this->x >= min(a.x, b.x) and this->x <=
        max(a.x, b.x);
    bool d2 = this->y >= min(a.y, b.y) and this->y <=
        max(a.y, b.y);
    return d1 and d2;
}
bool rayIntersect(Point a, Point b) {
    Point q(this->x, INT32_MAX); // if p-q ray
    intersects segment a-b
    for (int rep = 0; rep < 2; ++rep) {
        if ((a - *this) * (q - *this) <= 0 and (b -
            *this) * (q - *this) > 0 and (a - *this) *
            (b - *this) < 0) {
            return true;
        }
        swap(a, b);
    }
    return false;
}
friend istream& operator >>(istream& cin, Point& p) {
    cin >> p.x >> p.y;
    return cin;
}
friend ostream& operator <<(ostream& cout, const
    Point& p) {
    cout << p.x << " " << p.y;
    return cout;
}
};

// upper and lower part
void solve() {
    int n;
    cin >> n;
    vector<Point> v(n);
    for (int i = 0; i < n; ++i) {
        cin >> v[i];
    }
    sort(v.begin(), v.end());
    vector<Point> hull;
    for (int rep = 0; rep < 2; ++rep) {
        const int sz = hull.size();
        for (auto C: v) {
            while (hull.size() >= sz + 2) {
                Point A = hull.end()[-2];
                Point B = hull.end()[-1];
                if (((B - A) * (C - A)) <= 0) {
                    break;
                }
                hull.pop_back();
            }
            hull.push_back(C);
        }
        hull.pop_back();
        reverse(v.begin(), v.end());
    }
    cout << hull.size() << "\n";
    for (auto p: hull) {
        cout << p << "\n";
    }
}

// sorting by angle
void solve() {

```

```

    int n;
    cin >> n;
    vector<Point> v(n);
    for (int i = 0; i < n; ++i) {
        cin >> v[i];
        if (make_pair(v[i].x, v[i].y) < make_pair(v[0].x,
            v[0].y)) {
            swap(v[i], v[0]);
        }
    }
    for (int i = 1; i < n; ++i) {
        v[i] -= v[0];
    }
    sort(v.begin() + 1, v.end());
    int j = n - 1;
    while (j >= 2 and v[j] * v[j - 1] == 0) {
        --j;
    }
    reverse(v.begin() + j, v.end());
    vector<Point> hull;
    hull.push_back(Point{0, 0});
    for (int i = 1; i < n; ++i) {
        auto C = v[i];
        while (hull.size() >= 2) {
            Point A = hull.end()[-2];
            Point B = hull.end()[-1];
            if (((B - A) * (C - A)) <= 0) {
                break;
            }
            hull.pop_back();
        }
        hull.push_back(C);
    }
    cout << hull.size() << "\n";
    for (auto& p: hull) {
        p += v[0];
        cout << p << "\n";
    }
}

```

1.16 ConvexHullTrick

```

/*
 * Dynamic version of data structure
 * to be used in dynamic programming optimisation
 * called "Convex Hull trick"
 * 'Dynamic' means that there is no restriction on
 * adding lines order
 */
class ConvexHullDynamic
{
    typedef long long coef_t;
    typedef long long coord_t;
    typedef long long val_t;
    /*
     * Line 'y=a*x+b' represented by 2 coefficients 'a'
     * and 'b'
     * and 'xLeft' which is intersection with previous
     * line in hull (first line has -INF)
     */
private:
    struct Line
    {
        coef_t a, b;
        double xLeft;
        enum Type {line, maxQuery, minQuery} type;
        coord_t val;
        explicit Line(coef_t aa = 0, coef_t bb = 0) :
            a(aa), b(bb), xLeft(-INFINITY),
            type(Type::line), val(0) {}
        val_t valueAt(coord_t x) const { return a * x + b;
        }
    }
}

```

```

friend bool areParallel(const Line& l1, const
    Line& l2) { return l1.a == l2.a; }
friend double intersectX(const Line& l1, const
    Line& l2) { return areParallel(l1, l2) ?
    INFINITY : 1.0 * (l2.b - l1.b) / (l1.a -
    l2.a); }
bool operator<(const Line& l2) const
{
    if (l2.type == line)
        return this->a < l2.a;
    if (l2.type == maxQuery)
        return this->xLeft < l2.val;
    if (l2.type == minQuery)
        return this->xLeft > l2.val;
}
};

private:
    bool isMax; //whether or not saved
    envelope is top(search of max value)
    std::set<Line> hull; //envelope itself

private:
    /*
     * INFO: Check position in hull by iterator
     * COMPLEXITY: O(1)
    */
    bool hasPrev(std::set<Line>::iterator it) { return
        it != hull.begin(); }
    bool hasNext(std::set<Line>::iterator it) { return
        it != hull.end() && std::next(it) != hull.end();
    }

    /*
     * INFO: Check whether line l2 is irrelevant
     * NOTE: Following positioning in hull must be
     * true
     * l1 is next left to l2
     * l2 is right between l1 and l3
     * l3 is next right to l2
     * COMPLEXITY: O(1)
    */
    bool irrelevant(const Line& l1, const Line& l2,
        const Line& l3) { return intersectX(l1, l3) <=
        intersectX(l1, l2); }
    bool irrelevant(std::set<Line>::iterator it)
    {
        return hasPrev(it) && hasNext(it)
            && ( isMax && irrelevant(*std::prev(it),
                *it, *std::next(it))
                || !isMax &&
                    irrelevant(*std::next(it), *it,
                        *std::prev(it)) );
    }

    /*
     * INFO: Updates 'xValue' of line pointed by
     * iterator 'it'
     * COMPLEXITY: O(1)
    */
    std::set<Line>::iterator
        updateLeftBorder(std::set<Line>::iterator it)
    {
        if (isMax && !hasPrev(it) || !isMax &&
            !hasNext(it))
            return it;
        double val = intersectX(*it, isMax ?
            *std::prev(it) : *std::next(it));
        Line buf(*it);
        it = hull.erase(it);
        buf.xLeft = val;
        it = hull.insert(it, buf);
        return it;
    }
}

```

```

public:
explicit ConvexHullDynamic(bool isMax): isMax(isMax)
{ }
/*
* INFO:      Adding line to the envelope
*           Line is of type 'y=a*x+b' represented
*           by 2 coefficients 'a' and 'b'
* COMPLEXITY: Adding N lines(N calls of function)
*           takes O(N*log N) time
*/
void addLine(coef_t a, coef_t b)
{
//find the place where line will be inserted in set
Line l3 = Line(a, b);
auto it = hull.lower_bound(l3);
//if parallel line is already in set, one of them
// becomes irrelevant
if (it != hull.end() && areParallel(*it, l3))
{
if (isMax && it->b < b || !isMax && it->b > b)
it = hull.erase(it);
else
return;
}
//try to insert
it = hull.insert(it, l3);
if (irrelevant(it)) { hull.erase(it); return; }
//remove lines which became irrelevant after
// inserting line
while (hasPrev(it) && irrelevant(std::prev(it)))
hull.erase(std::prev(it));
while (hasNext(it) && irrelevant(std::next(it)))
hull.erase(std::next(it));
//refresh 'xLine'
it = updateLeftBorder(it);
if (hasPrev(it))
updateLeftBorder(std::prev(it));
if (hasNext(it))
updateLeftBorder(std::next(it));
}
/*
* INFO:      Query, which returns max/min(depends
*           on hull type - see more info above) value in
*           point with abscissa 'x'
* COMPLEXITY: O(log N), N-amount of lines in hull
*/
val_t getBest(coord_t x) const
{
Line q;
q.val = x;
q.type = isMax ? Line::Type::maxQuery :
Line::Type::minQuery;
auto bestLine = hull.lower_bound(q);
if (isMax) --bestLine;
return bestLine->valueAt(x);
}
};

```

1.17 Custom Hash

```

struct custom_hash {
static uint64_t splitmix64(uint64_t x) {
x += 0x9e3779b97f4a7c15;
x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
return x ^ (x >> 31);
}
size_t operator()(uint64_t x) const {
static const uint64_t FIXED_RANDOM = chrono::steady_
clock::now().time_since_epoch().count();
return splitmix64(x + FIXED_RANDOM);
}
};

```

```

};
unordered_map<long long int, int, custom_hash> mp; //
// this will work when the key is an int or long long
// int
// hash pair
struct hash_pair {
template <class T1, class T2>
size_t operator()(const pair<T1, T2> &p) const {
custom_hash hasher; // Create an instance of
custom_hash
size_t hash1 = hasher(p.first);
size_t hash2 = hasher(p.second);
// Combine hashes
if (hash1 != hash2) {
return hash1 ^ (hash2 << 1); // Better mixing
}
return hash1;
}
};

```

1.18 Custom Map(Pair Query)

```

// a1 <= a2 <= a3 <= a4.....
// b1 >= b2 >= b3 >= b4.....
map<ll, ll> mp;
void insert(ll a, ll b) {
auto it = mp.lower_bound(a);
if (it != mp.end() and it->second >= b) return;
it = mp.insert(it, {a, b});
it->second = b;
while (it != mp.begin() and prev(it)->second <= b) {
mp.erase(prev(it));
}
}
// returns the largest b among the a's that are greater
// than or equal to x
ll query(ll x) {
auto it = mp.lower_bound(x);
if (it == mp.end()) return 0;
return it->second;
}

```

1.19 DSU(weighted)

```

const int N = 2e5 + 3;
int par[N], sz[N];
long long w[N];
int find(int u) {
if (par[u] == u) return u;
int p = find(par[u]);
w[u] += w[par[u]];
return par[u] = p;
}
bool unite(int a, int b, ll d) {
int u = find(a), v = find(b);
if (u == v) return w[a] - w[b] == d;
if (sz[u] < sz[v]) {
w[u] += d + w[b] - w[a];
par[u] = v;
sz[v] += sz[u];
} else {
w[v] += w[a] - d - w[b];
par[v] = u;
sz[u] += sz[v];
}
return true;
}
void solve() {
int n, q;
cin >> n >> q;

```

```

for (int i = 1; i <= n; ++i) {
par[i] = i;
sz[i] = 1;
}
for (int i = 1; i <= q; ++i) {
int a, b, d;
cin >> a >> b >> d;
if (unite(a, b, d)) cout << i << " ";
}
cout << endl;
}

```

1.20 DSU

```

const int N = 1e5 + 9;
int parent[N], sz[N];
void make_set(int v) {
parent[v] = v;
sz[v] = 1;
}
int find_set(int v) {
if (v == parent[v]) return v;
return parent[v] = find_set(parent[v]);
}
void union_sets(int a, int b) {
a = find_set(a);
b = find_set(b);
if (a != b) {
if (sz[a] < sz[b]) swap(a, b);
parent[b] = a;
sz[a] += sz[b];
}
}

```

1.21 Discrete Log

```

// Returns minimum x for which a ^ x % m = b % m, a and
// m are coprime.
int solve(int a, int b, int m) {
a %= m, b %= m;
int n = sqrt(m) + 1;
int an = 1;
for (int i = 0; i < n; ++i)
an = (an * 1ll * a) % m;
unordered_map<int, int> vals;
for (int q = 0, cur = b; q <= n; ++q) {
vals[cur] = q;
cur = (cur * 1ll * a) % m;
}
for (int p = 1, cur = 1; p <= n; ++p) {
cur = (cur * 1ll * an) % m;
if (vals.count(cur)) {
int ans = n * p - vals[cur];
return ans;
}
}
return -1;
}
// a and m are not coprime:
// Returns minimum x for which a ^ x % m = b % m.
int solve(int a, int b, int m) {
a %= m, b %= m;
int k = 1, add = 0, g;
while ((g = gcd(a, m)) > 1) {
if (b == k)
return add;
if (b % g)
return -1;
b /= g, m /= g, ++add;
k = (k * 1ll * a / g) % m;
}

```

```

}
int n = sqrt(m) + 1;
int an = 1;
for (int i = 0; i < n; ++i)
    an = (an * 1ll * a) % m;

unordered_map<int, int> vals;
for (int q = 0, cur = b; q <= n; ++q) {
    vals[cur] = q;
    cur = (cur * 1ll * a) % m;
}

for (int p = 1, cur = k; p <= n; ++p) {
    cur = (cur * 1ll * an) % m;
    if (vals.count(cur)) {
        int ans = n * p - vals[cur] + add;
        return ans;
    }
}
return -1;
}

```

1.22 Euler Phi

```

1.  $\phi(n) = n * (p_1 - 1) / p_1 * (p_2 - 1) / p_2 \dots$ 
2. gcd d:  $\phi(n / d)$ 
3. Sum of coprime numbers of an integer =  $\phi(n) * n / 2$ 
4.  $N = \phi(d)$  where,  $d | N$ 
5. Code:
vector<int> phi(n + 1);
void prec(int n) { //nlogn
    phi[1] = 1;
    for (int i = 2; i <= n; i++)
        phi[i] = i - 1;
    for (int i = 2; i <= n; i++)
        for (int j = 2 * i; j <= n; j += i)
            phi[j] -= phi[i];
}

int phi(int n) { //sqrt(n)
    int result = n;
    for (int i = 2; i * i <= n; i++) {
        if (n % i == 0) {
            while (n % i == 0) n /= i;
            result -= result / i;
        }
    }
    if (n > 1) result -= result / n;
    return result;
}

```

1.23 Extended GCD

```

ll egcd(ll a, ll b, ll &x, ll &y) {
    if (b == 0) {
        x = 1; y = 0;
        return a;
    }
    ll x1, y1;
    ll d = egcd(b, a % b, x1, y1);
    x = y1; y = x1 - y1 * (a / b);
    return d;
}

```

1.24 Fenwick Tree

```

struct FenwickTree {
    vector<ll> bit;
    ll n;

    FenwickTree(int n) {
        this->n = n;
        bit.assign(n, 0);
    }

    FenwickTree(vector<ll> const& a) :
        FenwickTree(a.size()) {

```

```

        for (int i = 0; i < n; i++) {
            bit[i] += a[i];
            int r = i | (i + 1);
            if (r < n) bit[r] += bit[i];
        }

        ll sum(int r) {
            ll ret = 0;
            for (; r >= 0; r = (r & (r + 1)) - 1)
                ret += bit[r];
            return ret;
        }

        ll sum(int l, int r) {
            return sum(r) - sum(l - 1);
        }

        void add(int idx, ll delta) {
            for (; idx < n; idx = idx | (idx + 1))
                bit[idx] += delta;
        }
};

```

struct FenwickTree2D {

```

    vector<vector<int>> bit;
    int n, m;

    FenwickTree2D(int n, int m) {
        this->n = n;
        this->m = m;
        bit.assign(n, vector<int>(m, 0));
    }

    int sum(int x, int y) {
        int ret = 0;
        for (int i = x; i >= 0; i = (i & (i + 1)) - 1)
            for (int j = y; j >= 0; j = (j & (j + 1)) - 1)
                ret += bit[i][j];
        return ret;
    }

    void add(int x, int y, int delta) {
        for (int i = x; i < n; i = i | (i + 1))
            for (int j = y; j < m; j = j | (j + 1))
                bit[i][j] += delta;
    }
};

```

1.25 Floyd Warshall

```

const int N = 100, inf = 1e9 + 9;
int d[N][N], nextof[N][N];
int n;

void init() {
    for (int i = 1; i <= n; ++i) {
        for (int j = 1; j <= n; ++j) {
            nextof[i][j] = j;
            d[i][j] = inf;
            if (i == j) d[i][j] = 0;
        }
    }
}

void cal() {
    for (int k = 1; k <= n; ++k) {
        for (int i = 1; i <= n; ++i) {
            for (int j = 1; j <= n; ++j) {
                if (d[i][k] + d[k][j] < d[i][j]) {
                    d[i][j] = d[i][k] + d[k][j];
                    nextof[i][j] = nextof[i][k];
                }
            }
        }
    }
}

vector<int> findPath(int i, int j) {
    vector<int> path = {i};
    while (i != j) {

```

```

        i = nextof[i][j];
        path.push_back(i);
    }
    return path;
}

```

1.26 GP Hash Table

```

#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
const int RANDOM = chrono::high_resolution_clock::now().
    time_since_epoch().count();
struct custom_hash {
    int operator()(int x) const { return x ^ RANDOM; }
};
gp_hash_table<int, int, custom_hash> mp;

```

1.27 Geometric Sum

```

ll geometricSum() {
    ll a, r, n; cin >> a >> r >> n; //a = first value r =
    ratio, n = n-term
    ll res = BigMod(r, n);
    ll numara = (a * (1 - res)) % MOD;
    numara = (numara + MOD) % MOD;
    ll deno = ((1 - r) % MOD + MOD) % MOD;
    ll ans = (numara * BigMod(deno, MOD - 2)) % MOD;
    return ans;
}

// geometric sum for any MOD
const int mod = 1e9 + 7;
int geo_all(int n, int x) { //1 + x + x^2 + x^3 ....x^n
    = f(n, x)
    if (n == 0) return 0;
    int ret = 1ll * (1 + x) * geo_all(n / 2, 1ll * x * x
    % mod) % mod;
    if (n & 1) ret = (1 + 1ll * x * ret) % mod;
    return ret;
}

```

1.28 Heavy-Light Decomposition

```

vector<int> parent, depth, heavy, head, pos;
int cur_pos;

int dfs(int v, vector<vector<int>> const& adj) {
    int size = 1;
    int max_c_size = 0;
    for (int c : adj[v]) {
        if (c != parent[v]) {
            parent[c] = v, depth[c] = depth[v] + 1;
            int c_size = dfs(c, adj);
            size += c_size;
            if (c_size > max_c_size)
                max_c_size = c_size, heavy[v] = c;
        }
    }
    return size;
}

void decompose(int v, int h, vector<vector<int>>
    const& adj) {
    head[v] = h, pos[v] = cur_pos++;
    if (heavy[v] != -1)
        decompose(heavy[v], h, adj);
    for (int c : adj[v]) {
        if (c != parent[v] && c != heavy[v])
            decompose(c, c, adj);
    }
}

void init(vector<vector<int>> const& adj) {

```



```

    int n = adj.size();
    parent = vector<int>(n);
    depth = vector<int>(n);
    heavy = vector<int>(n, -1);
    head = vector<int>(n);
    pos = vector<int>(n);
    cur_pos = 0;

    dfs(0, adj);
    decompose(0, 0, adj);
}

int query(int a, int b) {
    int res = 0;
    for (; head[a] != head[b]; b = parent[head[b]]) {
        if (depth[head[a]] > depth[head[b]])
            swap(a, b);
        int cur_heavy_path_max =
            segment_tree_query(pos[head[b]], pos[b]);
        res = max(res, cur_heavy_path_max);
    }
    if (depth[a] > depth[b])
        swap(a, b);
    int last_heavy_path_max =
        segment_tree_query(pos[a], pos[b]);
    res = max(res, last_heavy_path_max);
    return res;
}

```

1.29 HopcroftKarp

```

struct HopcroftKarp {
    const int INF = 1e9 + 7;
    vector<vector<int>> g;
    vector<int> match, dist;
    int nodes;

    void init(int _nodes) {
        nodes = _nodes;
        g.resize(_nodes);
        match.resize(_nodes);
        dist.resize(_nodes);
    }

    void add_edge(int u, int v) {
        g[u].push_back(v);
    }

    bool bfs(int n) {
        queue<int> q;
        for (int i = 1; i <= n; ++i) {
            if (!match[i]) dist[i] = 0, q.emplace(i);
            else dist[i] = INF;
        }
        dist[0] = INF;
        while (!q.empty()) {
            int u = q.front(); q.pop();
            if (!u) continue;
            for (int v : g[u]) {
                if (dist[match[v]] == INF) {
                    dist[match[v]] = dist[u] + 1,
                    q.emplace(match[v]);
                }
            }
        }
        return dist[0] != INF;
    }

    bool dfs(int u) {
        if (!u) return 1;
        for (int v : g[u]) {
            if (dist[match[v]] == dist[u] + 1 and
                dfs(match[v])) {
                match[u] = v, match[v] = u;
                return 1;
            }
        }
    }
}

```

```

    }
    dist[u] = INF;
    return 0;
}

int hopcroftKarp() {
    int n = nodes - 1;
    int ret = 0;
    while (bfs(n)) {
        for (int i = 1; i <= n; ++i) {
            ret += !match[i] and dfs(i);
        }
    }
    return ret;
}
};

```

1.30 Interval Set

```

struct interval_set {
    set<array<ll, 2>> st;
    ll cnt = 0, d = 0;

    void init(ll _d) {
        d = _d;
    }

    void add(ll l, ll r, ll x) {
        cnt += x * (1 + (r - l) / d);
    }

    void insert(ll l, ll r) {
        auto it = st.lower_bound({l, INF});
        if (it != st.begin()) {
            it--;
            if (l <= (*it)[1]) {
                l = (*it)[0];
                r = max(r, (*it)[1]);
                add((*it)[0], (*it)[1], -1);
                st.erase(it);
            }
        }
        while (1) {
            auto it = st.lower_bound({l, -INF});
            if (it == st.end() || r < (*it)[0]) break;
            r = max(r, (*it)[1]);
            add((*it)[0], (*it)[1], -1);
            st.erase(it);
        }
        add(l, r, 1);
        st.insert({l, r});
    }

    ll count() {
        return cnt;
    }
};

```

1.31 KMP

```

// Longest Proper Prefix which is also a Suffix
vector<int> prefix_function(string &s) {
    int n = (int)s.length();
    vector<int> pi(n);
    for (int i = 1; i < n; i++) {
        int j = pi[i - 1];
        while (j > 0 && s[i] != s[j])
            j = pi[j - 1];
        if (s[i] == s[j])
            j++;
        pi[i] = j;
    }
    return pi;
}

```

1.32 LCA

```

const int N = 1e6 + 9, LOG = 21;
int up[N][LOG], depth[N];

```

```

vector<int> children[N];
void dfs(int a) {
    for (auto b: children[a]) {
        depth[b] = depth[a] + 1;
        up[b][0] = a; // a is parent of b
        for (int i = 1; i < LOG; ++i) {
            up[b][i] = up[up[b][i-1]][i-1];
        }
        dfs(b);
    }
}

int getKthAncestor(int node, int k) {
    if (depth[node] < k) return -1;
    for (int i = 0; i < LOG; ++i) {
        if (k & (1 << i)) {
            node = up[node][i];
        }
    }
    return node;
}

int getLCA(int u, int v) {
    if (depth[u] < depth[v]) swap(u, v);
    u = getKthAncestor(u, depth[u] - depth[v]);
    if (u == v) return v;
    for (int i = LOG - 1; i >= 0; --i) {
        if (up[u][i] != up[v][i]) {
            u = up[u][i];
            v = up[v][i];
        }
    }
    return up[u][0];
}

```

1.33 Linear Diophantine Equation

```

int gcd(int a, int b, int& x, int& y) {
    if (b == 0) {
        x = 1;
        y = 0;
        return a;
    }
    int x1, y1;
    int d = gcd(b, a % b, x1, y1);
    x = y1;
    y = x1 - y1 * (a / b);
    return d;
}

bool find_any_solution(int a, int b, int c, int &x0,
    int &y0, int &g) {
    g = gcd(abs(a), abs(b), x0, y0);
    if (c % g) {
        return false;
    }
    x0 *= c / g;
    y0 *= c / g;
    if (a < 0) x0 = -x0;
    if (b < 0) y0 = -y0;
    return true;
}

void shift_solution(int &x, int &y, int a, int b,
    int cnt) {
    x += cnt * b;
    y -= cnt * a;
}

int find_all_solutions(int a, int b, int c, int minx,
    int maxx, int miny, int maxy) {
    int x, y, g;
    if (!find_any_solution(a, b, c, x, y, g))
        return 0;
    a /= g;
    b /= g;
}

```

```

int sign_a = a > 0 ? +1 : -1;
int sign_b = b > 0 ? +1 : -1;
shift_solution(x, y, a, b, (minx - x) / b);
if (x < minx)
    shift_solution(x, y, a, b, sign_b);
if (x > maxx)
    return 0;
int lx1 = x;
shift_solution(x, y, a, b, (maxx - x) / b);
if (x > maxx)
    shift_solution(x, y, a, b, -sign_b);
int rx1 = x;
shift_solution(x, y, a, b, -(miny - y) / a);
if (y < miny)
    shift_solution(x, y, a, b, -sign_a);
if (y > maxy)
    return 0;
int lx2 = x;
shift_solution(x, y, a, b, -(maxy - y) / a);
if (y > maxy)
    shift_solution(x, y, a, b, sign_a);
int rx2 = x;
if (lx2 > rx2)
    swap(lx2, rx2);
int lx = max(lx1, lx2);
int rx = min(rx1, rx2);
if (lx > rx)
    return 0;
return (rx - lx) / abs(b) + 1;
}

```

1.34 Manacher

Description: pal[1][i] = longest odd (half rounded down) palindrome around pos i and starts at i - pal[1][i] and ends at i + pal[1][i]
 pal[0][i] = half length of longest even palindrome around pos i, i + 1 and starts at i - par[0][i] + 1 and ends at i + pal[0][i]

```

int pal[2][N];
void manacher(string &s) {
    int n = s.size(), idx = 2;
    while (idx-- > 0) {
        for (int l=-1, r=-1, i=0; i<n-1; ++i){
            if (i > r) l = r = i;
            else {
                int k = min(r-i, pal[idx][l+r-i]);
                l = i - k, r = i + k;
                while (l - idx >= 0 and r + 1 < n and s[l - idx] == s[r + 1]) l--, r++;
                pal[idx][i] = r - l;
                // [l - i + idx : r] palindrome
            }
        }
        idx--;
    }
}

```

1.35 Matrix Exponentiation

```

const int mod = 1e9 + 7;
struct Mat {
    int sz;
    vector<vector<int>> val;
    Mat(int sz) {
        this->sz = sz;
        val.resize(sz, vector<int>(sz, 0));
    }
    Mat(int sz, int v) {
        this->sz = sz;
        val.resize(sz, vector<int>(sz, 0));
        for (int i = 0; i < sz; ++i) {
            val[i][i] = v; // diagonal values
        }
    }
    Mat operator * (Mat m2) {

```

```

Mat ans(sz);
for (int i = 0; i < sz; ++i) {
    for (int j = 0; j < sz; ++j) {
        for (int k = 0; k < sz; ++k) {
            ans.val[i][j] = (ans.val[i][j] + (1LL *
                val[i][k] * m2.val[k][j]) % mod) % mod;
        }
    }
}
return ans;
};
Mat Mat_Expo(Mat a, long long n) {
    Mat ans(a.sz, 1); // identity matrix
    while (n) {
        if (n & 1) {
            ans = ans * a;
        }
        a = a * a;
        n >>= 1;
    }
    return ans;
}

```

1.36 Mint

```

struct Mint {
    int v;
    Mint(long long val = 0) {
        v = int(val % MOD);
        if (v < 0) v += MOD;
    }
    Mint operator+(const Mint &o) const { return Mint(v + o.v); }
    Mint operator-(const Mint &o) const { return Mint(v - o.v); }
    Mint operator*(const Mint &o) const { return Mint((1LL * v * o.v) % MOD); }
    Mint operator/(const Mint &o) const { return *this * o.inv(); }
    Mint& operator+=(const Mint &o) { v += o.v; if (v >= MOD) v -= MOD; return *this; }
    Mint& operator-=(const Mint &o) { v -= o.v; if (v < 0) v += MOD; return *this; }
    Mint& operator*=(const Mint &o) { v = int(1LL * v * o.v % MOD); return *this; }
    Mint pow(long long p) const {
        Mint a = *this, res = 1;
        while (p > 0) {
            if (p & 1) res *= a;
            a *= a;
            p >>= 1;
        }
        return res;
    }
    Mint inv() const { return pow(MOD - 2); }
    friend ostream& operator<<(ostream& os, const Mint& m) {
        os << m.v;
        return os;
    }
};

```

1.37 Mobius Function

```

const int N = 1E6 + 5;
int mu[N];
void pre() {
    mu[1] = 1;
    for (int i = 1; i < N; ++i) {
        for (int j = i + 1; j < N; j += i) {
            mu[j] -= mu[i];
        }
    }
}

```

```

}
}
}
1.38 N-th Permutation
vector<ll> fact(21, 1);
//does not handle if given ff-th permutation does not exist
string n_th_Permutation(string s, ll ff){
    int n = s.size();
    for(int i=0; i<n; i++){
        sort(s.begin()+i, s.end());
        int pos = i+ff/fact[n-1-i];
        ff %= fact[n-1-i];
        swap(s[i], s[pos]);
    }
    return s;
}

```

1.39 PBDS

Description: *x.find_by_order(k) : iterator to the k-th index
 x.order_of_key(k) : number of items smaller than k

```

#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;
template <typename T> using ordered_set = tree<T,
    null_type, less_equal<T>, rb_tree_tag,
    tree_order_statistics_node_update>;

```

1.40 Polynomial Interpolation

```

// P(x) = a0 + a1x + a2x^2 + ... + anx^n
// y[i] = P(i)
const int mod = 1e9 + 7;
ll BigMod(ll a, ll b) {
    ll res = 1;
    while (b) {
        if (b & 1) res = 1LL * res * a % mod;
        a = 1LL * a * a % mod;
        b >>= 1;
    }
    return res;
}
ll inv(ll x) {
    if (x < 0) x += mod;
    return BigMod(x, mod - 2);
}
ll add(ll &a, ll b) {
    a += b;
    if (a >= mod) a -= mod;
    return a;
}
ll eval (vector<ll> y, ll k) {
    int n = y.size() - 1;
    if (k <= n) {
        return y[k];
    }
    vector<ll> L(n + 1, 1);
    for (int x = 1; x <= n; ++x) {
        L[0] = L[0] * (k - x) % mod;
        L[0] = L[0] * inv(-x) % mod;
    }
    for (int x = 1; x <= n; ++x) {
        L[x] = L[x - 1] * inv(k - x) % mod * (k - (x - 1)) % mod;
        L[x] = L[x] * ((x - 1) - n + mod) % mod * inv(x) % mod;
    }
    ll yk = 0;
    for (int x = 0; x <= n; ++x) {
        yk = add(yk, L[x] * y[x] % mod);
    }
    return yk;
}

```

1.41 Prefix Sum 3D

```

pref[x][y][z] = pref[x - 1][y][z] + pref[x][y - 1][z]
+ pref[x][y][z - 1] - pref[x - 1][y - 1][z] -
pref[x - 1][y][z - 1] - pref[x][y - 1][z - 1] +
pref[x - 1][y - 1][z - 1] + arr[x][y][z];
// from x1 to x2, y1 to y2, z1 to z2
ans = pref[x2][y2][z2] - pref[x1 - 1][y2][z2] -
pref[x2][y1 - 1][z2] - pref[x2][y2][z1 - 1] +
pref[x1 - 1][y1 - 1][z] + pref[x1 - 1][y][z1 - 1] -
pref[x2][y1 - 1][z1 - 1] - pref[x1 - 1][y1 - 1][z1 - 1];

```

1.42 Segment Tree(Persistent)

```

//the code returns k-th number in a range l to r if the
// range were sorted
struct PST {
#define lc t[cur].l
#define rc t[cur].r
    struct node {
        int l = 0, r = 0, val = 0;
    } t[20 * N];
    int T = 0;
    int build(int b, int e) {
        int cur = ++T;
        if(b == e) return cur;
        int mid = b + e >> 1;
        lc = build(b, mid);
        rc = build(mid + 1, e);
        t[cur].val = t[lc].val + t[rc].val;
        return cur;
    }
    int upd(int pre, int b, int e, int i, int v) {
        int cur = ++T;
        t[cur] = t[pre];
        if(b == e) {
            t[cur].val += v;
            return cur;
        }
        int mid = b + e >> 1;
        if(i <= mid) {
            rc = t[pre].r;
            lc = upd(t[pre].l, b, mid, i, v);
        } else {
            lc = t[pre].l;
            rc = upd(t[pre].r, mid + 1, e, i, v);
        }
        t[cur].val = t[lc].val + t[rc].val;
        return cur;
    }
    int query(int pre, int cur, int b, int e, int k) {
        if(b == e) return b;
        int cnt = t[lc].val - t[t[pre].l].val;
        int mid = b + e >> 1;
        if(cnt >= k) return query(t[pre].l, lc, b, mid, k);
        else return query(t[pre].r, rc, mid + 1, e, k - cnt);
    }
} t;

int V[N], root[N], a[N];
int32_t main() {
    map<int, int> mp;
    int n, q;
    cin >> n >> q;
    for(int i = 1; i <= n; i++) cin >> a[i], mp[a[i]];
    int c = 0;
    for(auto x : mp) mp[x.first] = ++c, V[c] = x.first;
    root[0] = t.build(1, n);
    for(int i = 1; i <= n; i++) {
        root[i] = t.upd(root[i - 1], 1, n, mp[a[i]], 1);
    }
    while(q--) {

```

```

        int l, r, k;
        cin >> l >> r >> k;
        cout << V[t.query(root[l - 1], root[r], 1, n, k)]
            << '\n';
    }
    return 0;
}

```

1.43 Segment Tree(Special Variant)

```

// Range increment and decrement (increment before
// decrement) and number of positive elements in the
// whole array
const int N = 2e6 + 6;
int st[4 * N], cnt[4 * N];
void add(int l, int r, ll x, int u = 1, int s = 0, int
        e = N - 1) {
    if(s > r or e < l) return;
    int v = u << 1, w = v | 1, m = s + e >> 1;
    if(l <= s and e <= r) {
        cnt[u] += x;
        if(cnt[u]) st[u] = e - s + 1;
        else st[u] = (s == e) ? 0 : st[v] + st[w];
        return;
    }
    add(l, r, x, v, s, m);
    add(l, r, x, w, m + 1, e);
    if(!cnt[u]) st[u] = st[v] + st[w];
}

```

1.44 Segment Tree

```

struct ST {
#define lc (n << 1)
#define rc ((n << 1) + 1)
    long long t[4 * N], lazy[4 * N];
    ST() {
        memset(t, 0, sizeof t);
        memset(lazy, 0, sizeof lazy);
    }
    inline void push(int n, int b, int e) { // change
        this
        if(lazy[n] == 0) return;
        t[n] = t[n] + lazy[n] * (e - b + 1);
        if(b != e) {
            lazy[lc] = lazy[lc] + lazy[n];
            lazy[rc] = lazy[rc] + lazy[n];
        }
        lazy[n] = 0;
    }
    inline long long combine(long long a, long long b) {
        // change this
        return a + b;
    }
    inline void pull(int n) { // change this
        t[n] = t[lc] + t[rc];
    }
    void build(int n, int b, int e) {
        lazy[n] = 0; // change this
        if(b == e) {
            t[n] = a[b];
            return;
        }
        int mid = (b + e) >> 1;
        build(lc, b, mid);
        build(rc, mid + 1, e);
        pull(n);
    }
    void upd(int n, int b, int e, int i, int j, long
        long v) {
        push(n, b, e);
        if(j < b || e < i) return;
        if(i <= b && e <= j) {

```

```

            lazy[n] = v; //set lazy
            push(n, b, e);
            return;
        }
        int mid = (b + e) >> 1;
        upd(lc, b, mid, i, j, v);
        upd(rc, mid + 1, e, i, j, v);
        pull(n);
    }
    long long query(int n, int b, int e, int i, int j) {
        push(n, b, e);
        if(i > e || b > j) return 0; //return null
        if(i <= b && e <= j) return t[n];
        int mid = (b + e) >> 1;
        return combine(query(lc, b, mid, i, j), query(rc,
            mid + 1, e, i, j));
    }
} t;

```

1.45 Sieve upto 1e9

```

// credit: min 25
// takes 0.5s for n = 1e9
vector<int> sieve(const int N, const int Q = 17, const
    int L = 1 << 15) {
    static const int rs[] = {1, 7, 11, 13, 17, 19, 23,
        29};
    struct P {
        P(int p) : p(p) {}
        int p; int pos[8];
    };
    auto approx_prime_count = [] (const int N) -> int {
        return N > 60184 ? N / (log(N) - 1.1) : max(1., N
            / (log(N) - 1.11)) + 1;
    };
    const int v = sqrt(N), vv = sqrt(v);
    vector<bool> isp(v + 1, true);
    for(int i = 2; i <= vv; ++i) if(isp[i]) {
        for(int j = i * i; j <= v; j += i) isp[j] = false;
    }
    const int rsize = approx_prime_count(N + 30);
    vector<int> primes = {2, 3, 5}; int psize = 3;
    primes.resize(rsize);
    vector<P> sprimes; size_t pbeg = 0;
    int prod = 1;
    for(int p = 7; p <= v; ++p) {
        if(!isp[p]) continue;
        if(p <= Q) prod *= p, ++pbeg, primes[psize++] = p;
        auto pp = P(p);
        for(int t = 0; t < 8; ++t) {
            int j = (p <= Q) ? p : p * p;
            while(j % 30 != rs[t]) j += p << 1;
            pp.pos[t] = j / 30;
        }
        sprimes.push_back(pp);
    }
    vector<unsigned char> pre(prod, 0xFF);
    for(size_t pi = 0; pi < pbeg; ++pi) {
        auto pp = sprimes[pi]; const int p = pp.p;
        for(int t = 0; t < 8; ++t) {
            const unsigned char m = ~(1 << t);
            for(int i = pp.pos[t]; i < prod; i += p) pre[i]
                &= m;
        }
    }
    const int block_size = (L + prod - 1) / prod * prod;
    vector<unsigned char> block(block_size); unsigned
        char* pblock = block.data();
    const int M = (N + 29) / 30;
    for(int beg = 0; beg < M; beg += block_size, pblock
        += block_size) {

```

```

int end = min(M, beg + block size);
for (int i = beg; i < end; i += prod) {
    copy(pre.begin(), pre.end(), pblock + i);
}
if (beg == 0) pblock[0] &= 0xFE;
for (size_t pi = pbeg; pi < sprimes.size(); ++pi) {
    auto& pp = sprimes[pi];
    const int p = pp.p;
    for (int t = 0; t < 8; ++t) {
        int i = pp.pos[t]; const unsigned char m = ~(1
        << t);
        for (; i < end; i += p) pblock[i] &= m;
        pp.pos[t] = i;
    }
}
for (int i = beg; i < end; ++i) {
    for (int m = pblock[i]; m > 0; m &= m - 1) {
        primes[psize++] = i * 30 +
        < rs[__builtin_ctz(m)];
    }
}
assert(psize <= rsize);
while (psize > 0 && primes[psize - 1] > N) --psize;
primes.resize(psize);
return primes;
}

int32_t main() {
    ios_base::sync_with_stdio(0);
    cin.tie(0);
    int n, a, b; cin >> n >> a >> b;
    auto primes = sieve(n);
    vector<int> ans;
    for (int i = b; i < primes.size() && primes[i] <= n;
        < i += a) ans.push_back(primes[i]);
    cout << primes.size() << ' ' << ans.size() << '\n';
    for (auto x: ans) cout << x << ' '; cout << '\n';
    return 0;
}

```

1.46 Sieve(Linear)

```

const int N = 100000000;
vector<int> spf(N+1);
vector<int> pr;
for (int i=2; i <= N; ++i) {
    if (spf[i] == 0) {
        spf[i] = i;
        pr.push_back(i);
    }
    for (int j = 0; i * pr[j] <= N; ++j) {
        spf[i * pr[j]] = pr[j];
        if (pr[j] == spf[i]) {
            break;
        }
    }
}

```

1.47 Sieve(Segmented)

```

vector<char> segmentedSieve(long long L, long long R) {
    // generate all primes up to sqrt(R)
    long long lim = sqrt(R);
    vector<char> mark(lim + 1, false);
    vector<long long> primes;
    for (long long i = 2; i <= lim; ++i) {
        if (!mark[i]) {
            primes.emplace_back(i);
            for (long long j = i * i; j <= lim; j += i)
                mark[j] = true;
        }
    }
}

```

```

vector<char> isPrime(R - L + 1, true);
for (long long i : primes)
    for (long long j = max(i * i, (L + i - 1) / i
        * i); j <= R; j += i)
        isPrime[j - L] = false;
if (L == 1)
    isPrime[0] = false;
return isPrime;
}

int count_primes(int n) {
    const int S = 10000;
    vector<int> primes;
    int nsqrt = sqrt(n);
    vector<char> is_prime(nsqrt + 2, true);
    for (int i = 2; i <= nsqrt; i++) {
        if (is_prime[i]) {
            primes.push_back(i);
            for (int j = i * i; j <= nsqrt; j += i)
                is_prime[j] = false;
        }
    }
    int result = 0;
    vector<char> block(S);
    for (int k = 0; k * S <= n; k++) {
        fill(block.begin(), block.end(), true);
        int start = k * S;
        for (int p : primes) {
            int start_idx = (start + p - 1) / p;
            int j = max(start_idx, p) * p - start;
            for (; j < S; j += p)
                block[j] = false;
        }
        if (k == 0)
            block[0] = block[1] = false;
        for (int i = 0; i < S && start + i <= n; i++) {
            if (block[i])
                result++;
        }
    }
    return result;
}

```

1.48 Sieve

```

const int N = 1e6 + 3;
bitset<N> isPrime;
vector<int> prime;
void sieve() {
    isPrime[2] = 1;
    for (int i = 3; i <= N; i+=2) {
        isPrime[i] = 1;
    }
    for (int i = 3; i * i <= N; i += 2) {
        if (isPrime[i]) {
            for (int j = i * i; j <= N; j += (i + i)) {
                isPrime[j] = 0;
            }
        }
    }
    prime.push_back(2);
    for (int i = 3; i <= N; i+=2) {
        if (isPrime[i]) {
            prime.push_back(i);
        }
    }
}

```

1.49 Sparse Table

```

const int N = 2e5 + 3, M = __bit_width(N) + 1;
int maxTable[N][M], a[N];
void buildTable(int n) {
    for (int i = 0; i < n; ++i) {

```

```

        maxTable[i][0] = a[i];
    }
    for (int k = 1; k < M; ++k) {
        for (int i = 0; i + (1 << k) <= n; ++i) {
            maxTable[i][k] = max(maxTable[i][k - 1],
                < maxTable[i + (1 << (k - 1))][k - 1]);
        }
    }
}

int maxQuery(int i, int j, int n) {
    if (j < 0 or i >= n) return INT32_MIN;
    int k = __bit_width(j - i + 1) - 1;
    return max(maxTable[i][k], maxTable[j - (1 << k) +
        < 1][k]);
}

```

1.50 SquareRoot Decomposition

```

#include <bits/stdc++.h>
using namespace std;
struct Sqrt {
    int block_size;
    vector<int> nums;
    vector<long long> blocks;
    Sqrt(int sqn, vector<int> &arr) :
        < block_size(sqn), blocks(sqn, 0) {
        nums = arr;
        for (int i = 0; i < nums.size(); i++) { blocks[i /
            < block_size] += nums[i]; }
    }
    void update(int x, int v) {
        blocks[x / block_size] -= nums[x];
        nums[x] = v;
        blocks[x / block_size] += nums[x];
    }
    long long query(int r) {
        long long res = 0;
        for (int i = 0; i < r / block_size; i++) { res +=
            < blocks[i]; }
        for (int i = (r / block_size) * block_size; i < r;
            < i++) { res += nums[i]; }
        return res;
    }
    long long query(int l, int r) { return query(r) -
        < query(l - 1); }
};

int main() {
    int n, q;
    cin >> n >> q;
    vector<int> arr(n);
    for (int i = 0; i < n; i++) { cin >> arr[i]; }
    Sqrt sq((int)ceil(sqrt(n)), arr);
    for (int i = 0; i < q; i++) {
        int t, l, r;
        cin >> t >> l >> r;
        if (t == 1) {
            sq.update(l - 1, r);
        } else {
            cout << sq.query(l, r) << "\n";
        }
    }
}

#include <bits/stdc++.h>
using namespace std;
struct Query {
    int l, r, idx;
};

```



```
int main() {
    int n;
    cin >> n;
    vector<int> v(n);
    for (int i = 0; i < n; i++) { cin >> v[i]; }

    int q;
    cin >> q;
    vector<Query> queries;
    for (int i = 0; i < q; i++) {
        int x, y;
        cin >> x >> y;
        queries.push_back({--x, --y, i});
    }

    int block_size = (int)sqrt(n);
    auto mo_cmp = [&](Query a, Query b) {
        int block_a = a.l / block_size;
        int block_b = b.l / block_size;
        if (block_a == block_b) { return a.r < b.r; }
        return block_a < block_b;
    };
    sort(queries.begin(), queries.end(), mo_cmp);

    int different_values = 0;
    vector<int> values(VALMAX);
    auto remove = [&](int idx) {
        values[v[idx]]--;
        if (values[v[idx]] == 0) { different_values--; }
    };
    auto add = [&](int idx) {
        values[v[idx]]++;
        if (values[v[idx]] == 1) { different_values++; }
    };

    int mo_left = -1;
    int mo_right = -1;
    vector<int> ans(q);
    for (int i = 0; i < q; i++) {
        int left = queries[i].l;
        int right = queries[i].r;

        while (mo_left < left) { remove(mo_left++); }
        while (mo_left > left) { add(--mo_left); }
        while (mo_right < right) { add(++mo_right); }
        while (mo_right > right) { remove(mo_right--); }

        ans[queries[i].idx] = different_values;
    }

    for (int i = 0; i < q; i++) { cout << ans[i] <<
        '\n'; }
}
```

1.51 String Hashing

```
const int p1 = 137, mod1 = 127657753, p2 = 277, mod2 =
    987654319; // 911382323, 972663749
const int N = 1e6 + 3;
array<int, 2> pref[N], rev[N];
int pw1[N], pw2[N], ipw1[N], ipw2[N];
int power(int a, int n, int mod) {
    int ans = 1 % mod;
    while (n) {
        if (n & 1) ans = 1LL * ans * a % mod;
        a = 1LL * a * a % mod;
        n >>= 1;
    }
    return ans;
}
void prec() {
    pw1[0] = pw2[0] = ipw1[0] = ipw2[0] = 1;
    int ip1 = power(p1, mod1 - 2, mod1);
    int ip2 = power(p2, mod2 - 2, mod2);
    for (int i = 1; i < N; ++i) {
        pw1[i] = 1LL * pw1[i - 1] * p1 % mod1;
```

```
        pw2[i] = 1LL * pw2[i - 1] * p2 % mod2;
        ipw1[i] = 1LL * ipw1[i - 1] * ip1 % mod1;
        ipw2[i] = 1LL * ipw2[i - 1] * ip2 % mod2;
    }
}
void build(string& s) {
    int n = s.size();
    for (int i = 0; i < n; ++i) {
        pref[i][0] = 1LL * s[i] * pw1[i] % mod1;
        if (i) pref[i][0] = (pref[i][0] + pref[i - 1][0])
            % mod1;
        pref[i][1] = 1LL * s[i] * pw2[i] % mod2;
        if (i) pref[i][1] = (pref[i][1] + pref[i - 1][1])
            % mod2;
        rev[i][0] = 1LL * s[i] * ipw1[i] % mod1;
        if (i) rev[i][0] = (rev[i][0] + rev[i - 1][0]) %
            mod1;
        rev[i][1] = 1LL * s[i] * ipw2[i] % mod2;
        if (i) rev[i][1] = (rev[i][1] + rev[i - 1][1]) %
            mod2;
    }
}
array<int, 2> get_hash(int i, int j) {
    array<int, 2> ans = {0, 0};
    ans[0] = pref[j][0];
    if (i) ans[0] = (pref[j][0] - pref[i - 1][0] + mod1)
        % mod1;
    ans[1] = pref[j][1];
    if (i) ans[1] = (pref[j][1] - pref[i - 1][1] + mod2)
        % mod2;
    ans[0] = 1LL * ans[0] * ipw1[i] % mod1;
    ans[1] = 1LL * ans[1] * ipw2[i] % mod2;
    return ans;
}
array<int, 2> get_rev_hash(int i, int j) {
    array<int, 2> ans = {0, 0};
    ans[0] = rev[j][0];
    if (i) ans[0] = (rev[j][0] - rev[i - 1][0] + mod1) %
        mod1;
    ans[1] = rev[j][1];
    if (i) ans[1] = (rev[j][1] - rev[i - 1][1] + mod2) %
        mod2;
    ans[0] = 1LL * ans[0] * pw1[j] % mod1;
    ans[1] = 1LL * ans[1] * pw2[j] % mod2;
    return ans;
}
```

1.52 Strongly_Connected_Components(SCC)

```
const int N = 1e5 + 9;
int vis[N], id[N];
vector<int> adj[N], adj_t[N];
vector<int> order;
void dfs1(int v) {
    vis[v] = 1;
    for (int u: adj[v]) {
        if (!vis[u]) {
            dfs1(u);
        }
    }
    order.push_back(v);
}
void dfs2(int v, int cnt) {
    id[v] = cnt;
    for (int u: adj_t[v]) {
        if (!id[u]) {
            dfs2(u, cnt);
        }
    }
}
void solve() {
    int n, m;
```

```
    cin >> n >> m;
    for (int i = 0; i < m; ++i) {
        int u, v;
        cin >> u >> v;
        adj[u].push_back(v);
        adj_t[v].push_back(u);
    }
    for (int i = 1; i <= n; ++i) {
        if (!vis[i]) {
            dfs1(i);
        }
    }
    reverse(order.begin(), order.end());
    int cnt = 1;
    for (auto v: order) {
        if (!id[v]) {
            dfs2(v, cnt++);
        }
    }
}
```

1.53 Submask Enumeration

```
// Generate all submask of m
for (int s = m; ; s = (s - 1) & m) {
    // you can use s ...
    if (s == 0) break;
}
```

1.54 Suffix Array

Description: This function return two vectors (first vector is sorted suffix array position , second vector is longest common prefix with previous string)

```
array<vector<int>, 2> get_sa(string& s, int lim=128) {
    // for integer, just change string to vector<int>
    // and minimum value of vector must be >= 1
    int n = s.size() + 1, k = 0, a, b;
    vector<int> x(begin(s), end(s) + 1), y(n), sa(n),
        lcp(n), ws(max(n, lim)), rank(n);
    x.back() = 0;
    iota(begin(sa), end(sa), 0);
    for (int j = 0, p = 0; p < n; j = max(1, j * 2), lim
        = p) {
        p = j, iota(begin(y), end(y), n - j);
        for (int i = 0; i < n; ++i) if (sa[i] >= j) y[p++]
            = sa[i] - j;
        fill(begin(ws), end(ws), 0);
        for (int i = 0; i < n; ++i) ws[x[i]]++;
        for (int i = 1; i < lim; ++i) ws[i] += ws[i - 1];
        for (int i = n; i--;) sa[-ws[x[i]]] = y[i];
        swap(x, y), p = 1, x[sa[0]] = 0;
        for (int i = 1; i < n; ++i) a = sa[i - 1], b =
            sa[i], x[b] =
                (y[a] == y[b] && y[a + j] == y[b + j]) ? p - 1 :
                    p++;
    }
    for (int i = 1; i < n; ++i) rank[sa[i]] = i;
    for (int i = 0, j; i < n - 1; lcp[rank[i++]] = k)
        for (k && k--, j = sa[rank[i] - 1]; s[i + k] ==
            s[j + k]; k++);
    sa.erase(sa.begin(), lcp.erase(lcp.begin()));
    return {sa, lcp};
}
## Comparing Two Substrings
auto query = [&](int l1, int r1, int l2, int r2) {
    int len1 = r1 - l1 + 1, len2 = r2 - l2 + 1;
    int len = min(len1, len2);
    int i = pos[l1], j = pos[l2], x;
    if (l1 != l2) x = st.query(i, j);
```



```

else x = len;
if (x >= len) {
    if (len1 == len2) return 0;
    if (len1 < len2) return -1;
    return 1;
}
if (s[l1 + x] < s[l2 + x]) return -1;
return 1;
};
## Kth Unique Substring
auto kth = [&] (ll k) {
    int i = 0;
    while (i + 1 < n and k > n - sa[i] - lcp[i]) {
        k -= n - sa[i] - lcp[i];
        i++;
    }
    k = min(k, 0ll + n - sa[i] - lcp[i]);
    array<int, 2> ret = {sa[i], k + lcp[i]};
    return ret;
};
## Several Consecutive Identical Substrings
for (int i = 1; i < n; ++i) {
    for (int j = i; j < n; j += i) {
        // Block = [j-i...j-1]
        int e1 = rmq(0, pos[j - i], pos[j]), e2 = 0;
        if (i < j) {
            e2 = rmq(1, rev_pos[j - i - 1], rev_pos[j - 1]);
        }
        int k = (e1 + e2) / i + 1;
        // [j-i-e2 ... j-1+e1] is periodic with period
        // length = i
    }
}

```

1.55 Suffix Automaton

```

int len[2 * N], lnk[2 * N], last, sz = 1;
unordered_map<char, int> to[2 * N]; // Use map during
// finding kth substring
int deg[2 * N], focc[2 * N]; // First Occurrence
ll cnt[2 * N], dp[2 * N];
void init(int n) {
    fill(deg, deg + sz, 0);
    fill(cnt, cnt + sz, 0);
    while (sz) to[--sz].clear();
    lnk[0] = -1, last = 0, sz = 1;
}
void add(char c, int i) {
    int cur = sz++;
    len[cur] = len[last] + 1;
    cnt[cur] = 1; dp[cur] = i;
    focc[cur] = i;
    int u = last;
    last = cur;
    while (u != -1 and !to[u].count(c)) {
        to[u][c] = cur;
        u = lnk[u];
    }
    if (u == -1) {
        lnk[cur] = 0;
    }
    else {
        int v = to[u][c];
        if (len[u] + 1 == len[v]) {
            lnk[cur] = v;
        }
        else {
            int w = sz++;
            len[w] = len[u] + 1, lnk[w] = lnk[v], to[w] =
            // to[v];
            focc[w] = focc[v];
            while (u != -1 and to[u][c] == v) {
                to[u][c] = w, u = lnk[u];
            }
        }
    }
}

```

```

    lnk[cur] = lnk[v] = w;
}
}
bool exist(string &p) {
    int u = 0;
    for (auto c: p) {
        if (!to[u].count(c)) return false;
        u = to[u][c];
    }
    return true;
}
void build() {
    deg[0] = 1;
    for (int u = 1; u < sz; ++u) {
        deg[lnk[u]]++;
    }
    queue<int> q;
    for (int u = 0; u < sz; ++u) {
        if (!deg[u]) q.push(u);
    }
    while (!q.empty()) {
        int u = q.front(); q.pop();
        int v = lnk[u];
        cnt[v] += cnt[u]; // DP on suffix link tree
        for (auto [c, v]: to[u]) { // DP on DAG
            dp[u] = max(dp[u], dp[v]);
        }
        deg[v]--;
        if (!deg[v]) q.push(v);
    }
}
## Count number of occurrence for each k length
// substring of s in SA
ll count(string s, int k) {
    ll ret = 0;
    int u = 0, L = 0;
    for (auto c: s) {
        while (u and !to[u].count(c)) u = lnk[u], L =
        // len[u];
        if (!to[u].count(c)) continue;
        u = to[u][c], L++;
        while (len[lnk[u]] >= k) u = lnk[u], L = len[u];
        if (L >= k) ret += cnt[u];
    }
    return ret;
}
## Kth substring (not distinct)
ll dp[2 * N];
ll dfs(int u) {
    if (dp[u] != -1) return dp[u];
    dp[u] = cnt[u]; // For distinct dp[u] = 1
    for (auto [c, v]: to[u]) {
        dp[u] += dfs(v);
    }
    return dp[u];
}
void yo(int u, ll k, string &s) {
    if (k <= 0) return;
    for (auto [c, v]: to[u]) {
        if (k > dfs(v)) k -= dfs(v);
        else {
            s += c;
            k -= cnt[v]; // For distinct k -= 1
            yo(v, k, s);
            return;
        }
    }
}

```

1.56 Ternary Search

```

double ternary_search(double l, double r) {
    double eps = 1e-9; //set the error limit here
    while (r - l > eps) {

```

```

        double m1 = l + (r - l) / 3;
        double m2 = r - (r - l) / 3;
        double f1 = f(m1); //value of function at m1
        double f2 = f(m2); //value of function at m2
        if (f1 < f2)
            l = m1;
        else
            r = m2;
    }
    return f(l) //return the maximum of f(x) in [l,
    // r]
}

```

1.57 Topological Sorting

```

const int N = 1e5 + 9;
vector<int> g[N];
bool vis[N];
vector<int> ord;
void dfs(int u) {
    vis[u] = true;
    for (auto v: g[u]) {
        if (!vis[v]) {
            dfs(v);
        }
    }
    ord.push_back(u);
}
int32_t main() {
    ios_base::sync_with_stdio(0);
    cin.tie(0);
    int n, m; cin >> n >> m;
    while (m--) {
        int u, v; cin >> u >> v;
        g[u].push_back(v);
    }
    for (int i = 1; i <= n; i++) {
        if (!vis[i]) {
            dfs(i);
        }
    }
    reverse(ord.begin(), ord.end());
    // check if feasible
    vector<int> pos(n + 1);
    for (int i = 0; i < (int) ord.size(); i++) {
        pos[ord[i]] = i;
    }
    for (int u = 1; u <= n; u++) {
        for (auto v: g[u]) {
            if (pos[u] > pos[v]) {
                cout << "IMPOSSIBLE\n";
                return 0;
            }
        }
    }
    // print the order
    for (auto u: ord) cout << u << ' ';
    cout << '\n';
    return 0;
}

```

1.58 Trie

```

const int N = 1e6 + 3;
int nextof[N][26], cnt[N];
int tot = 1;
void add(string& s) {
    int u = 1;
    ++cnt[u];
    for (auto c: s) {
        int v = c - 'a';
        if (!nextof[u][v]) {
            nextof[u][v] = ++tot;

```

```

    }
    u = nextof[u][v];
    ++cnt[u];
}
}
int countPref(string& s) {
    int u = 1;
    for (auto c: s) {
        int v = c - 'a';
        if (!nextof[u][v]) return 0;
        u = nextof[u][v];
    }
    return cnt[u];
}

```

1.59 Z_algo

```

vector<int> z_function(string s) {
    int n = s.size();
    vector<int> z(n);
    int l = 0, r = 0;
    for (int i = 1; i < n; i++) {
        if (i < r) {
            z[i] = min(r - i, z[i - l]);
        }
        while (i + z[i] < n && s[z[i]] == s[i + z[i]]) {
            z[i]++;
        }
        if (i + z[i] > r) {
            l = i;
            r = i + z[i];
        }
    }
    return z;
}

```

1.60 int128

```

istream& operator >>(istream& cin, __int128& x) {
    string s;
    cin >> s;
    x = 0;
    for (int i = 0; i < s.size(); ++i) {
        x = x * 10 + (s[i] - '0');
    }
    return cin;
}
ostream& operator <<(ostream& cout, __int128 x) {
    if (x == 0) {
        cout << 0;
        return cout;
    }
    if (x < 0) {
        cout << "-";
        x *= -1;
    }
    string s;
    while (x) {
        s += (x % 10) + '0';
        x /= 10;
    }
    reverse(s.begin(), s.end());
    cout << s;
    return cout;
}

```

1.61 josephus problem

Description: Given natural numbers n and k , the numbers 1 to n are arranged in a circle. Starting from 1 , every k -th number is removed in a circular manner. This continues until only one number remains. Find the last remaining number.

```

int josephus(int n, int k) {
    if (n == 1)
        return 0;
}

```

```

    if (k == 1)
        return n - 1;
    if (k > n)
        return (josephus(n - 1, k) + k) % n;
    int cnt = n / k;
    int res = josephus(n - cnt, k);
    res -= n % k;
    if (res < 0)
        res += n;
    else
        res += res / (k - 1);
    return res;
}

```

1.62 nCr and nPr-1

```

int fact[N], ifact[N];
void prec() {
    fact[0] = 1;
    for (int i = 1; i < N; i++) {
        fact[i] = 1LL * fact[i - 1] * i % mod;
    }
    ifact[N - 1] = power(fact[N - 1], -1);
    for (int i = N - 2; i >= 0; i--) {
        ifact[i] = 1LL * ifact[i + 1] * (i + 1) % mod;
    }
}
int nPr(int n, int r) {
    if (n < r) return 0;
    return 1LL * fact[n] * ifact[n - r] % mod;
}
int nCr(int n, int r) {
    if (n < r) return 0;
    return 1LL * fact[n] * ifact[r] % mod * ifact[n - r] % mod;
}

```

1.63 nCr and nPr-2

```

const int N = 2005, mod = 1e9 + 7;
int C[N][N], fact[N];
void prec() { // O(n^2)
    for (int i = 0; i < N; i++) {
        C[i][0] = C[i][i] = 1;
        for (int j = 1; j < i; j++) {
            C[i][j] = (C[i - 1][j - 1] + C[i - 1][j]) % mod;
        }
    }
    fact[0] = 1;
    for (int i = 1; i < N; i++) {
        fact[i] = 1LL * fact[i - 1] * i % mod;
    }
}
int nCr(int n, int r) { // O(1)
    if (n < r) return 0;
    return C[n][r];
}
int nPr(int n, int r) { // O(1)
    if (n < r) return 0;
    return 1LL * nCr(n, r) * fact[r] % mod;
}

```

1.64 notes

Pick's Theorem:
 Given a certain lattice polygon with non-zero area. We denote its area by S , the number of points with integer coordinates lying strictly inside the polygon by I and the number of points lying on polygon sides by B .
 Then, the Pick's formula states: $S = I + (B / 2) - 1$

1.65 ntt

```

const int MOD = 1e9 + 7;
const int PRIMITIVE_ROOT = 3;
int mod_exp(int base, int exp, int mod) {
    int result = 1;
    while (exp > 0) {
        if (exp % 2 == 1) {
            result = (1LL * result * base) % mod;
        }
        base = (1LL * base * base) % mod;
        exp /= 2;
    }
    return result;
}
void ntt(vector<int> &a, bool invert) {
    int n = a.size();
    int log_n = log2(n);
    for (int i = 1; i < n; i++) {
        int bit = n / 2;
        while (j >= bit) {
            j -= bit;
            bit /= 2;
        }
        j += bit;
        if (i < j) swap(a[i], a[j]);
    }
    for (int len = 2; len <= n; len *= 2) {
        int wlen = mod_exp(PRIMITIVE_ROOT, (MOD - 1) / len, MOD);
        if (invert) wlen = mod_exp(wlen, MOD - 2, MOD);
        for (int i = 0; i < n; i += len) {
            int w = 1;
            for (int j = 0; j < len / 2; j++) {
                int u = a[i + j];
                int v = (1LL * a[i + j + len / 2] * w) % MOD;
                a[i + j] = (u + v) % MOD;
                a[i + j + len / 2] = (u - v + MOD) % MOD;
                w = (1LL * w * wlen) % MOD;
            }
        }
        if (invert) {
            int n_inv = mod_exp(n, MOD - 2, MOD);
            for (int &x: a) {
                x = (1LL * x * n_inv) % MOD;
            }
        }
    }
}
vector<int> multiply(vector<int> const &a, vector<int> const &b) {
    int n = 1;
    while (n < a.size() + b.size()) n *= 2;
    vector<int> fa(a.begin(), a.end()), fb(b.begin(), b.end());
    fa.resize(n);
    fb.resize(n);
    ntt(fa, false);
    ntt(fb, false);
    for (int i = 0; i < n; i++) {
        fa[i] = (1LL * fa[i] * fb[i]) % MOD;
    }
    ntt(fa, true);
    while (!fa.empty() && fa.back() == 0) fa.pop_back();
    return fa;
}

```

2 Geometry

2.1 Angular Sort

```
inline bool up (point p) {
    return p.y > 0 or (p.y == 0 and p.x >= 0);
}
sort(v.begin(), v.end(), [] (point a, point b) {
    return up(a) == up(b) ? a.x * b.y > a.y * b.x :
        up(a) < up(b);
});
inline int quad (point p) {
    if (p.y >= 0) return p.x < 0;
    return 2 + (p.x >= 0);
}
sort(pt.begin(), pt.end(), [] (point a, point b) {
    return quad(a) == quad(b) ? a.x * b.y > a.y * b.x :
        quad(a) < quad(b);
});
```

2.2 CircleCircleIntersection

Description: compute intersection of circle centered at a with radius r with circle centered at b with radius R .

```
vector<PT> CircleCircleIntersection(PT a, PT b, double
    r, double R) {
    vector<PT> ret;
    double d = sqrt(dist2(a, b));
    if (d > r+R || d+min(r, R) < max(r, R)) return ret;
    double x = (d*d-R*R+r*r)/(2*d);
    double y = sqrt(r*r-x*x);
    PT v = (b-a)/d;
    ret.push_back(a+v*x + RotateCCW90(v)*y);
    if (y > 0)
        ret.push_back(a+v*x - RotateCCW90(v)*y);
    return ret;
}
```

2.3 CircleLineIntersection

Description: Compute intersection of line through points a and b with circle centered at c with radius $r > 0$.

```
vector<PT> CircleLineIntersection(PT a, PT b, PT c,
    double r) {
    vector<PT> ret;
    b = b-a; a = a-c;
    double A = dot(b, b); double B = dot(a, b);
    double C = dot(a, a) - r*r;
    double D = B*B - A*C;
    if (D < -EPS) return ret;
    ret.push_back(c+a+b*(-B+sqrt(D+EPS))/A);
    if (D > EPS)
        ret.push_back(c+a+b*(-B-sqrt(D))/A);
    return ret;
}
```

2.4 Closest Pair of Points

```
ll min_dis(vector<array<int, 2>> &pts, int l, int r) {
    if (l + 1 >= r) return LLONG_MAX;
    int m = (l + r) / 2;
    ll my = pts[m][1];
    ll d = min(min_dis(pts, l, m), min_dis(pts, m, r));
    inplace_merge(pts.begin()+l, pts.begin()+m,
        pts.begin()+r);
    for (int i = l; i < r; ++i) {
        if ((pts[i][1] - my) * (pts[i][1] - my) < d) {
            for (int j = i + 1; j < r and (pts[i][0] -
                pts[j][0]) * (pts[i][0] - pts[j][0]) < d;
                ++j) {
                ll dx = pts[i][0] - pts[j][0], dy = pts[i][1]
                    - pts[j][1];
                d = min(d, dx * dx + dy * dy);
            }
        }
    }
```

```
    }
    }
    return d;
}
vector<array<int, 2>> pts(n);
sort(pts.begin(), pts.end(), [&] (array<int, 2> a,
    array<int, 2> b) {
    return make_pair(a[1], a[0]) < make_pair(b[1], b[0]);
});
```

2.5 ComputeCentroid

// centroid of a (possibly nonconvex) polygon.

```
PT ComputeCentroid(const vector<PT> &p) {
    PT c(0,0);
    double scale = 6.0 * ComputeSignedArea(p);
    for (int i = 0; i < p.size(); i++) {
        int j = (i+1) % p.size();
        c = c + (p[i]+p[j])*(p[i].x*p[j].y -
            p[j].x*p[i].y);
    }
    return c / scale;
}
```

2.6 ComputeCircleCenter

// compute center of circle passing through three points

```
PT ComputeCircleCenter(PT a, PT b, PT c) {
    b = (a+b)/2;
    c = (a+c)/2;
    return ComputeLineIntersection(b, b+RotateCW90(a-b),
        c, c+RotateCW90(a-c));
}
```

2.7 ComputeLineIntersection

Description: compute intersection of line passing through a and b with line passing through c and d , assuming that unique intersection exists; for segment intersection, check if segments intersect first.

```
PT ComputeLineIntersection(PT a, PT b, PT c, PT d) {
    b = b-a; d = d-c; c = c-a;
    assert(dot(b, b) > EPS && dot(d, d) > EPS);
    return a + b*cross(c, d)/cross(b, d);
}
```

2.8 ComputeSignedArea

Description: Computes the area of a (possibly nonconvex) polygon, assuming that the coordinates are listed in a clockwise or counter-clockwise fashion.

```
double ComputeSignedArea(const vector<PT> &p) {
    double area = 0;
    for (int i = 0; i < p.size(); i++) {
        int j = (i+1) % p.size();
        area += p[i].x*p[j].y - p[j].x*p[i].y;
    }
    return area / 2.0;
}
double ComputeArea(const vector<PT> &p) {
    return fabs(ComputeSignedArea(p));
}
// integer area
void computeIntArea() {
    int n; cin >> n;
    point arr[n];
    for (int i = 0; i < n; i++) {
        arr[i].read();
    }
    point a = {0, 0};
```

```
ll ans = 0;
for (int i = 0; i + 1 < n; i++) {
    ans += a.triangle(arr[i], arr[i + 1]);
}
ans += a.triangle(arr[n - 1], arr[0]);
cout << abs(ans) << "\n";
}
```

2.9 Convex Hull

```
vector<PT> convexHull(vector<PT> p) {
    int n = p.size(), m = 0;
    if (n < 3) return p;
    vector<PT> hull(n + n);
    sort(p.begin(), p.end(), [&] (PT a, PT b) {
        return a.x==b.x ? a.y<b.y : a.x<b.x;
    });
    for (int i = 0; i < n; ++i) {
        while (m > 1 and cross(hull[m - 2] - p[i], hull[m
            - 1] - p[i]) <= 0) --m;
        hull[m++] = p[i];
    }
    for (int i = n - 2, j = m + 1; i >= 0; --i) {
        while (m >= j and cross(hull[m - 2] - p[i], hull[m
            - 1] - p[i]) <= 0) --m;
        hull[m++] = p[i];
    }
    hull.resize(m - 1); return hull;
}
```

2.10 DistancePointPlane

Description: compute distance between point (x,y,z) and plane $ax+by+cz=d$

```
double DistancePointPlane(double x, double y, double
    z, double a, double b, double c, double d) {
    return fabs(a*x+b*y+c*z-d)/sqrt(a*a+b*b+c*c);
}
```

2.11 DistancePointSegment

// compute distance from c to segment between a and b

```
double DistancePointSegment(PT a, PT b, PT c) {
    return sqrt(dist2(c, ProjectPointSegment(a, b, c)));
}
```

2.12 Half Plane Intersection

Description: Calculates the intersection of halfplanes, assuming every half-plane allows the region to the left of its line.

```
struct Halfplane {
    PT p, pq; ld angle;
    Halfplane() {}
    // Two points on line
    Halfplane(const PT& a, const PT& b) : p(a), pq(b -
        a) {
        angle = atan2l(pq.y, pq.x);
    }
    bool out(const PT& r) {
        return cross(pq, r - p) < -EPS;
    }
    bool operator < (const Halfplane& e) const {
        return angle < e.angle;
    }
    friend PT inter(const Halfplane& s, const Halfplane&
        t) {
        ld alpha = cross((t.p - s.p), t.pq) / cross(s.pq,
            t.pq);
        return s.p + (s.pq * alpha);
    }
}
```

```
};
vector<PT> hp_intersect(vector<Halfplane>& H) {
    PT box[4] = { // Bounding box in CCW order
        PT(INF, INF), PT(-INF, INF),
        PT(-INF, -INF), PT(INF, -INF)
    };
    for(int i = 0; i < 4; i++) { // Add bounding box
        half-planes.
        Halfplane aux(box[i], box[(i+1) % 4]);
        H.push_back(aux);
    }
    sort(H.begin(), H.end());
    deque<Halfplane> dq; int len = 0;
    for(int i = 0; i < int(H.size()); i++) {
        while (len > 1 && H[i].out(inter(dq[len-1],
            dq[len-2]))) {
            dq.pop_back(); --len;
        }
        while (len > 1 && H[i].out(inter(dq[0], dq[1]))) {
            dq.pop_front(); --len;
        }
        if (len > 0 && fabs1(cross(H[i].pq, dq[len-1].pq))
            < EPS) {
            if (dot(H[i].pq, dq[len-1].pq) < 0.0)
                return vector<PT>();
            if (H[i].out(dq[len-1].p)) {
                dq.pop_back(); --len;
            }
            else continue;
        }
        dq.push_back(H[i]); ++len;
    }
    while (len > 2 && dq[0].out(inter(dq[len-1],
        dq[len-2]))) {
        dq.pop_back(); --len;
    }
    while (len > 2 && dq[len-1].out(inter(dq[0],
        dq[1]))) {
        dq.pop_front(); --len;
    }
    // Report empty intersection if necessary
    if (len < 3) return vector<PT>();
    // Reconstruct the convex polygon from the remaining
    half-planes.
    vector<PT> ret(len);
    for(int i = 0; i+1 < len; i++) {
        ret[i] = inter(dq[i], dq[i+1]);
    }
    ret.back() = inter(dq[len-1], dq[0]);
    return ret;
}
```

2.13 IsSimple

```
// tests whether or not a given polygon (in CW or CCW
// order) is simple
bool IsSimple(const vector<PT> &p) {
    for (int i = 0; i < p.size(); i++) {
        for (int k = i+1; k < p.size(); k++) {
            int j = (i+1) % p.size();
            int l = (k+1) % p.size();
            if (i == l || j == k) continue;
            if (SegmentsIntersect(p[i], p[j], p[k], p[l]))
                return false;
        }
    }
    return true;
}
```

2.14 LinesCollinear

```
bool LinesCollinear(PT a, PT b, PT c, PT d) {
    return LinesParallel(a, b, c, d)
        && fabs(cross(a-b, a-c)) < EPS
        && fabs(cross(c-d, c-a)) < EPS;
}
```

2.15 LinesParallel

```
// determine if lines from a to b and c to d are
// parallel or collinear
bool LinesParallel(PT a, PT b, PT c, PT d) {
    return fabs(cross(b-a, c-d)) < EPS;
}
```

2.16 Point

```
double INF = 1e100;
double EPS = 1e-12;
struct PT {
    double x, y;
    PT() {}
    PT(double x, double y) : x(x), y(y) {}
    PT(const PT &p) : x(p.x), y(p.y) {}
    PT operator + (const PT &p) const { return
        PT(x+p.x, y+p.y); }
    PT operator - (const PT &p) const { return
        PT(x-p.x, y-p.y); }
    PT operator * (double c) const { return PT(x*c,
        y*c ); }
    PT operator / (double c) const { return PT(x/c,
        y/c ); }
};
double dot(PT p, PT q) { return p.x*q.x+p.y*q.y; }
double dist2(PT p, PT q) { return dot(p-q, p-q); }
double abs(PT p) { return sqrt(p.x*p.x + p.y*p.y); }
double cross(PT p, PT q) { return p.x*q.y-p.y*q.x; }
ostream &operator<<(ostream &os, const PT &p) {
    return os << "(" << p.x << "," << p.y << ")";
}
// rotate a point CCW or CW around the origin
PT RotateCCW90(PT p) { return PT(-p.y, p.x); }
PT RotateCW90(PT p) { return PT(p.y, -p.x); }
PT RotateCCW(PT p, double t) {
    return PT(p.x*cos(t)-p.y*sin(t),
        p.x*sin(t)+p.y*cos(t));
}
// angle (range [0, pi]) between two vectors
double angle(PT v, PT w) {
    return acos(clamp(dot(v,w) / abs(v) / abs(w), -1.0,
        1.0));
}
```

2.17 PointInPolygon

Description: -1 = strictly inside, 0 = on, 1 = strictly outside.

```
int PointInPolygon(vector<PT> &P, PT a) {
    int cnt = 0, n = P.size();
    for(int i = 0; i < n; ++i) {
        PT q = P[(i+1) % n];
        if (onSegment(P[i], q, a)) return 0;
        cnt ^= ((a.y < P[i].y) - (a.y < q.y)) * cross(P[i]
            - a, q - a) > 0;
    }
    return cnt > 0 ? -1 : 1;
}
int PointInConvexPolygon(vector<PT> &P, const PT& q) {
    // O(log n)
    int n = P.size();
    ll a = cross(P[0] - q, P[1] - q), b = cross(P[0] -
        q, P[n-1] - q);
    if (a < 0 or b > 0) return 1;
}
```

```
int l = 1, r = n - 1;
while (l + 1 < r) {
    int mid = l + r >> 1;
    if (cross(P[0] - q, P[mid] - q) >= 0) l = mid;
    else r = mid;
}
ll k = cross(P[l] - q, P[r] - q);
if (k <= 0) return k < 0 ? 1 : 0;
if (l == 1 and a == 0) return 0;
if (r == n - 1 and b == 0) return 0;
return -1;
}
```

2.18 ProjectPointLine

```
// project point c onto line through a and b, assuming
// a != b
PT ProjectPointLine(PT a, PT b, PT c) {
    return a + (b-a)*dot(c-a, b-a)/dot(b-a, b-a);
}
```

2.19 ProjectPointSegment

```
// project point c onto line segment through a and b
PT ProjectPointSegment(PT a, PT b, PT c) {
    double r = dot(b-a, b-a);
    if (fabs(r) < EPS) return a;
    r = dot(c-a, b-a)/r;
    if (r < 0) return a;
    if (r > 1) return b;
    return a + (b-a)*r;
}
```

2.20 SegmentsIntersect

```
// determine if line segment from a to b intersects
// with line segment from c to d
bool SegmentsIntersect(PT a, PT b, PT c, PT d) {
    if (LinesCollinear(a, b, c, d)) {
        if (dist2(a, c) < EPS || dist2(a, d) < EPS ||
            dist2(b, c) < EPS || dist2(b, d) < EPS) return
            true;
        if (dot(c-a, c-b) > 0 && dot(d-a, d-b) > 0 &&
            dot(c-b, d-b) > 0)
            return false;
        return true;
    }
    if (cross(d-a, b-a) * cross(c-a, b-a) > 0) return
        false;
    if (cross(a-c, d-c) * cross(b-c, d-c) > 0) return
        false;
    return true;
}
```

3 Notes

3.1 Geometry

3.1.1 Triangles

Circumradius: $R = \frac{abc}{4A}$, Inradius: $r = \frac{A}{s}$

The area of a triangle using two sides and the included angle can be given as:

$$A = \frac{1}{2}ab\sin C$$

Length of median (divides triangle into two equal-area triangles):

$$m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$$

Length of bisector (divides angles in two): $s_a = \sqrt{bc \left[1 - \left(\frac{a}{b+c} \right)^2 \right]}$

Law of tangents: $\frac{a+b}{a-b} = \frac{\tan \frac{\alpha+\beta}{2}}{\tan \frac{\alpha-\beta}{2}}$

3.1.2 Quadrilaterals

With side lengths a, b, c, d , diagonals e, f , diagonals angle θ , area A and magic flux $F = b^2 + d^2 - a^2 - c^2$:

$$4A = 2ef \cdot \sin \theta = F \tan \theta = \sqrt{4e^2 f^2 - F^2}$$

For cyclic quadrilaterals the sum of opposite angles is 180° , $ef = ac + bd$, and $A = \sqrt{(p-a)(p-b)(p-c)(p-d)}$.

3.1.3 Spherical coordinates

$$\begin{aligned} x &= r \sin \theta \cos \phi & r &= \sqrt{x^2 + y^2 + z^2} \\ y &= r \sin \theta \sin \phi & \theta &= \arccos(z / \sqrt{x^2 + y^2 + z^2}) \\ z &= r \cos \theta & \phi &= \operatorname{atan2}(y, x) \end{aligned}$$

3.1.4 Pick's Theorem:

Given a lattice polygon with non-zero area, we define: S as the area of the polygon, I as the number of integer-coordinate points strictly inside the polygon, B as the number of integer-coordinate points on the boundary of the polygon. Then, Pick's Theorem states:

$$S = I + \frac{B}{2} - 1$$

The number of lattice points on segments (x1, y1) to (x2, y2) is: $\gcd(\text{abs}(x2 - x1), \text{abs}(y2 - y1)) + 1$

3.1.5 Polygon

For a regular polygon with n sides and side length a , the circumradius R is given by:

$$R = \frac{a}{2 \sin(\frac{\pi}{n})}$$

3.1.6 Area of a Circular Segment

The area of a circular segment, which is the region enclosed by a chord and the corresponding arc, can be calculated using the formula:

$$A = \frac{R^2}{2} (\theta - \sin \theta)$$

where: R is the radius of the circle, θ is the central angle subtended by the chord, in radians.

3.2 Binomial Coefficient

- Factoring in: $\binom{n}{k} = \frac{n}{k} \binom{n-1}{k-1}$
- Sum over k : $\sum_{k=0}^n \binom{n}{k} = 2^n$
- Alternating sum: $\sum_{k=0}^n (-1)^k \binom{n}{k} = 0$
- Even and odd sum: $\sum_{k=0}^n \binom{n}{2k} = \sum_{k=0}^n \binom{n}{2k+1} = 2^{n-1}$
- The Hockey Stick Identity
 - (Left to right) Sum over n and k : $\sum_{k=0}^m \binom{n+k}{k} = \binom{n+m-1}{m}$
 - (Right to left) Sum over n : $\sum_{m=0}^n \binom{m}{k} = \binom{n+1}{k+1}$
- Sum of the squares: $\sum_{k=0}^n \binom{n}{k}^2 = \binom{2n}{n}$
- Weighted sum: $\sum_{k=1}^n k \binom{n}{k} = n 2^{n-1}$
- Connection with the fibonacci numbers: $\sum_{k=0}^n \binom{n-k}{k} = F_{n+1}$
- Vandermonde's Identity: $\sum_{i=0}^k \binom{m}{i} \binom{n}{k-i} = \binom{m+n}{k}$

- If $f(n, k) = C(n, 0) + C(n, 1) + \dots + C(n, k)$, Then $f(n+1, k) = 2 * f(n, k) - C(n, k)$ [For multiple $f(n, k)$ queries, use Mo's algo]

Lucas Theorem

$$\binom{m}{n} \equiv \prod_{i=0}^k \binom{m_i}{n_i} \pmod{p}$$

- $\binom{m}{n}$ is divisible by p if and only if at least one of the base- p digits of n is greater than the corresponding base- p digit of m .
- The number of entries in the n th row of Pascal's triangle that are not divisible by $p = \prod_{i=0}^k (n_i + 1)$
- All entries in the $(p^k - 1)th$ row are not divisible by p .
- $\binom{n}{m} \equiv \lfloor \frac{n}{p} \rfloor \pmod{p}$

3.3 Fibonacci Number

- $k = A - B, F_A F_B = F_{k+1} F_{A-1}^2 + F_k F_A F_{A-1}$
- $\sum_{i=0}^n F_i^2 = F_{n+1} F_n$ **3.** $\sum_{i=0}^n F_i F_{i+1} = F_{n+1}^2 - (-1)^n$
- $\sum_{i=0}^n F_i F_{i+1} = F_{n+1}^2 - (-1)^n$ **5.** $\sum_{i=0}^n F_i F_{i-1} = \sum_{i=0}^{n-1} F_i F_{i+1}$
- $\gcd(F_m, F_n) = F_{\gcd(m, n)}$ **7.** $\sum_{0 \leq k \leq n} \binom{n-k}{k} = F_{n+1}$
- $\gcd(F_n, F_{n+1}) = \gcd(F_n, F_{n+2}) = \gcd(F_{n+1}, F_{n+2}) = 1$

3.4 Sums

$$\begin{aligned} 1^2 + 2^2 + 3^2 + \dots + n^2 &= \frac{n(2n+1)(n+1)}{6} \\ 1^3 + 2^3 + 3^3 + \dots + n^3 &= \frac{n^2(n+1)^2}{4} \\ 1^4 + 2^4 + 3^4 + \dots + n^4 &= \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30} \\ \sum_{i=1}^n i^m &= \frac{1}{m+1} \left[(n+1)^{m+1} - 1 - \sum_{i=1}^n ((i+1)^{m+1} - i^{m+1} - (m+1)i^m) \right] \\ \sum_{i=1}^{n-1} i^m &= \frac{1}{m+1} \sum_{k=0}^m \binom{m+1}{k} B_k n^{m+1-k} \end{aligned}$$

$$\sum_{k=0}^n k x^k = (x - (n+1)x^{n+1} + nx^{n+2}) / (x-1)^2$$

3.5 Series

$$\begin{aligned} e^x &= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots, (-\infty < x < \infty) \\ \ln(1+x) &= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots, (-1 < x \leq 1) \\ \sqrt{1+x} &= 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{2x^3}{92} - \frac{5x^4}{128} + \dots, (-1 \leq x \leq 1) \\ \sin x &= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots, (-\infty < x < \infty) \\ \cos x &= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots, (-\infty < x < \infty) \\ (x+a)^{-n} &= \sum_{k=0}^{\infty} (-1)^k \binom{n+k-1}{k} x^k a^{-n-k} \end{aligned}$$

Generating Function

$$\begin{aligned} 1/(1-x) &= 1 + x + x^2 + x^3 + \dots \\ 1/(1-ax) &= 1 + ax + (ax)^2 + (ax)^3 + \dots \\ 1/(1-x)^2 &= 1 + 2x + 3x^2 + 4x^3 + \dots \end{aligned}$$

$$1/(1-x)^3 = C(2,2) + C(3,2)x + C(4,2)x^2 + C(5,2)x^3 + \dots$$

$$1/(1-ax)^{(k+1)} = 1 + C(1+k,k)(ax) + C(2+k,k)(ax)^2 + C(3+k,k)(ax)^3 + \dots$$

$$x(x+1)(1-x)^{-3} = 1 + x + 4x^2 + 9x^3 + 16x^4 + 25x^5 + \dots$$

$$e^x = 1 + x + (x^2)/2! + (x^3)/3! + (x^4)/4! + \dots$$

3.6 Pythagorean Triples

The Pythagorean triples are uniquely generated by

$$a = k \cdot (m^2 - n^2), \quad b = k \cdot (2mn), \quad c = k \cdot (m^2 + n^2),$$

with $m > n > 0$, $k > 0$, $m \perp n$, and either m or n even.

3.7 Number Theory

- HCN: 1e6(240), 1e9(1344), 1e12(6720), 1e14(17280), 1e15(26880), 1e16(41472)

$$\gcd(a, b, c, d, \dots) = \gcd(a, b-a, c-b, d-c, \dots)$$

$$\gcd(a+k, b+k, c+k, d+k, \dots) = \gcd(a+k, b-a, c-b, d-c, \dots)$$

- Primitive root exists iff $n = 1, 2, 4, p^k, 2 \times p^k$, where p is an odd prime.

- If primitive root exists, there are $\phi(\phi(n))$ primitive roots of n .

- The numbers from 1 to n have in total $O(n \log \log n)$ unique prime factors.

- $x \equiv r_1 \pmod{m_1}$ and $x \equiv r_2 \pmod{m_2}$ has a solution iff $\gcd(m_1, m_2) | (r_1 - r_2)$ Solution of $x^2 \equiv a \pmod{p}$

$$ca \equiv cb \pmod{m} \iff a \equiv b \pmod{\frac{n}{\gcd(n, c)}}$$

$$ax \equiv b \pmod{m} \text{ has a solution } \iff \gcd(a, m) | b$$

- If $ax \equiv b \pmod{m}$ has a solution, then it has $\gcd(a, m)$ solutions and they are separated by $\frac{m}{\gcd(a, m)}$

$$ax \equiv 1 \pmod{m} \text{ has a solution or } a \text{ is invertible } \pmod{m} \iff \gcd(a, m) = 1$$

$$x^2 \equiv 1 \pmod{p} \text{ then } x \equiv \pm 1 \pmod{p}$$

$$\text{There are } \frac{p-1}{2} \text{ has no solution.}$$

$$\text{There are } \frac{p-1}{2} \text{ has exactly two solutions.}$$

$$\text{When } p \% 4 = 3, x \equiv \pm a^{\frac{p+1}{4}}$$

$$\text{When } p \% 8 = 5, x \equiv a^{\frac{p+3}{8}} \text{ or } x \equiv 2^{\frac{p-1}{4}} a^{\frac{p+3}{8}}$$

3.7.1 Primes

$p = 962592769$ is such that $2^{21} \mid p-1$, which may be useful. For hashing use 970592641 (31-bit number), 31443539979727 (45-bit), 3006703054056749 (52-bit). There are 78498 primes less than 1 000 000.

Primitive roots exist modulo any prime power p^a , except for $p = 2, a > 2$, and there are $\phi(\phi(p^a))$ many. For $p = 2, a > 2$, the group $\mathbb{Z}_{2^a}^\times$ is instead isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_{2^{a-2}}$.

3.7.2 Estimates

$\sum_{d|n} d = O(n \log \log n)$.

The number of divisors of n is at most around 100 for $n < 5e4$, 500 for $n < 1e7$, 2000 for $n < 1e10$, 200 000 for $n < 1e19$.

3.7.3 Perfect numbers

$n > 1$ is called perfect if it equals sum of its proper divisors and 1. Even n is perfect iff $n = 2^{p-1}(2^p - 1)$ and $2^p - 1$ is prime (Mersenne's). No odd perfect numbers are yet found.

3.7.4 Carmichael numbers

A positive composite n is a Carmichael number ($a^{n-1} \equiv 1 \pmod n$) for all a : $\gcd(a, n) = 1$, iff n is square-free, and for all prime divisors p of n , $p - 1$ divides $n - 1$.

3.7.5 Totient

- If p is a prime $(p^k) = p^k - p^{k-1}$
- If a, b are relatively prime, $\phi(ab) = \phi(a)\phi(b)$
- $\phi(n) = n(1 - \frac{1}{p_1})(1 - \frac{1}{p_2})(1 - \frac{1}{p_3}) \dots (1 - \frac{1}{p_k})$
- Sum of coprime to $n = n * \frac{\phi(n)}{2}$
- If $n = 2^k$, $\phi(n) = 2^{k-1} = \frac{n}{2}$
- For a, b , $\phi(ab) = \phi(a)\phi(b) \frac{d}{\phi(d)}$
- $\phi(ip) = p\phi(i)$ whenever p is a prime and it divides i
- The number of $a(1 < a < N)$ such that $\gcd(a, N) = d$ is $\phi(\frac{N}{d})$
- If $n > 2$, $\phi(n)$ is always even
- Sum of \gcd , $\sum_{i=1}^n \gcd(i, n) = \sum_{d|n} d \phi(\frac{n}{d})$
- Sum of lcm, $\sum_{i=1}^n n \text{ lcm}(i, n) = \frac{n}{2} (\sum_{d|n} (d \phi(d)) + 1)$
- $\phi(1) = 1$ and $\phi(2) = 1$ which two are only odd ϕ
- $\phi(3) = 2$ and $\phi(4) = 2$ and $\phi(6) = 2$ which three are only prime ϕ
- Find minimum n such that $\frac{\phi(n)}{n}$ is maximum- Multiple of small primes- $2 * 3 * 5 * 7 * 11 * 13 * \dots$

3.7.6 Mobius function

$\mu(1) = 1$. $\mu(n) = 0$, if n is not squarefree. $\mu(n) = (-1)^s$, if n is the product of s distinct primes. Let f, F be functions on positive integers. If for all $n \in N$, $F(n) = \sum_{d|n} f(d)$, then $f(n) = \sum_{d|n} \mu(d) F(\frac{n}{d})$, and vice versa. $\phi(n) = \sum_{d|n} \mu(d) \frac{n}{d}$. $\sum_{d|n} \mu(d) = 1$. If f is multiplicative, then $\sum_{d|n} \mu(d) f(d) = \prod_{p|n} (1 - f(p))$, $\sum_{d|n} \mu(d)^2 f(d) = \prod_{p|n} (1 + f(p))$.

$$\sum_{i=1}^n \sum_{j=1}^n [\gcd(i, j) = 1] = \sum_{k=1}^n \mu(k) \lfloor \frac{n}{k} \rfloor^2$$

$$\sum_{i=1}^n \sum_{j=1}^n \gcd(i, j) = \sum_{k=1}^n k \sum_{l=1}^{\lfloor \frac{n}{k} \rfloor} \mu(l) \lfloor \frac{n}{kl} \rfloor^2$$

$$\sum_{i=1}^n \sum_{j=1}^n \gcd(i, j) = \sum_{k=1}^n \left(\frac{\lfloor \frac{n}{k} \rfloor (1 + \lfloor \frac{n}{k} \rfloor)}{2} \right)^2 \sum_{d|k} \mu(d) k d$$

3.7.7 Legendre symbol

If p is an odd prime, $a \in \mathbb{Z}$, then $\left(\frac{a}{p}\right)$ equals 0, if $p|a$; 1 if a is a quadratic residue modulo p ; and -1 otherwise. Euler's criterion: $\left(\frac{a}{p}\right) = a^{\left(\frac{p-1}{2}\right)} \pmod p$.

3.7.8 Jacobi symbol

If $n = p_1^{a_1} \dots p_k^{a_k}$ is odd, then $\left(\frac{a}{n}\right) = \prod_{i=1}^k \left(\frac{a}{p_i}\right)^{a_i}$.

3.7.9 Primitive roots

If the order of g modulo m ($\min n > 0$: $g^n \equiv 1 \pmod m$) is $\phi(m)$, then g is called a primitive root. If Z_m has a primitive root, then it has $\phi(\phi(m))$ distinct primitive roots. Z_m has a primitive root iff m is one of $2, 4, p^k, 2p^k$, where p is an odd prime. If Z_m has a primitive root g , then for all a coprime to m , there exists unique integer $i = \text{ind}_g(a)$ modulo $\phi(m)$, such that $g^i \equiv a \pmod m$. $\text{ind}_g(a)$ has logarithm-like properties: $\text{ind}(1) = 0$, $\text{ind}(ab) = \text{ind}(a) + \text{ind}(b)$.

If p is prime and a is not divisible by p , then congruence $x^n \equiv a \pmod p$ has $\gcd(n, p-1)$ solutions if $a^{(p-1)/\gcd(n, p-1)} \equiv 1 \pmod p$, and no solutions otherwise. (Proof sketch: let g be a primitive root, and $g^i \equiv a \pmod p$, $g^u \equiv x \pmod p$. $x^n \equiv a \pmod p$ iff $g^{nu} \equiv g^i \pmod p$ iff $nu \equiv i \pmod p$.)

3.7.10 Discrete logarithm problem

Find x from $a^x \equiv b \pmod m$. Can be solved in $O(\sqrt{m})$ time and space with a meet-in-the-middle trick. Let $n = \lceil \sqrt{m} \rceil$, and $x = ny - z$. Equation becomes $a^{ny} \equiv ba^z \pmod m$. Precompute all values that the RHS can take for $z = 0, 1, \dots, n-1$, and brute force y on the LHS, each time checking whether there's a corresponding value for RHS.

3.7.11 Pythagorean triples

Integer solutions of $x^2 + y^2 = z^2$ All relatively prime triples are given by: $x = 2mn, y = m^2 - n^2, z = m^2 + n^2$ where $m > n, \gcd(m, n) = 1$ and $m \not\equiv n \pmod 2$. All other triples are multiples of these. Equation $x^2 + y^2 = 2z^2$ is equivalent to $(\frac{x+y}{2})^2 + (\frac{x-y}{2})^2 = z^2$.

3.7.12 Postage stamps/McNuggets problem

Let a, b be relatively-prime integers. There are exactly $\frac{1}{2}(a-1)(b-1)$ numbers *not* of form $ax + by$ ($x, y \geq 0$), and the largest is $(a-1)(b-1) - 1 = ab - a - b$.

3.7.13 Fermat's two-squares theorem

Odd prime p can be represented as a sum of two squares iff $p \equiv 1 \pmod 4$. A product of two sums of two squares is a sum of two squares. Thus, n is a sum of two squares iff every prime of form $p = 4k + 3$ occurs an even number of times in n 's factorization.

3.8 Permutations

3.8.1 Factorial

n	1	2	3	4	5	6	7	8	9	10
$n!$	1	2	6	24	120	720	5040	40320	362880	3628800
$\frac{n}{n!}$		$\frac{1}{1!}$	$\frac{1}{2!}$	$\frac{1}{3!}$	$\frac{1}{4!}$	$\frac{1}{5!}$	$\frac{1}{6!}$	$\frac{1}{7!}$	$\frac{1}{8!}$	$\frac{1}{9!}$
$\frac{n}{n!}$	4.0e7	4.8e8	6.2e9	8.7e10	1.3e12	2.1e13	3.6e14			
$\frac{n}{n!}$	20	25	30	40	50	100	150	171		
$\frac{n}{n!}$	2e18	2e25	3e32	8e47	3e64	9e157	6e262	>DBL_MAX		

3.8.2 Cycles

Let $g_S(n)$ be the number of n -permutations whose cycle lengths all belong to the set S . Then

$$\sum_{n=0}^{\infty} g_S(n) \frac{x^n}{n!} = \exp \left(\sum_{n \in S} \frac{x^n}{n} \right)$$

3.8.3 Derangements

Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n-1)(D(n-1) + D(n-2)) = nD(n-1) + (-1)^n = \left\lfloor \frac{n!}{e} \right\rfloor$$

3.8.4 Burnside's lemma

Given a group G of symmetries and a set X , the number of elements of X up to symmetry equals

$$\frac{1}{|G|} \sum_{g \in G} |X^g|,$$

where X^g are the elements fixed by g ($g.x = x$). If $f(n)$ counts "configurations" (of some sort) of length n , we can ignore rotational symmetry using $G = \mathbb{Z}_n$ to get

$$g(n) = \frac{1}{n} \sum_{k=0}^{n-1} f(\gcd(n, k)) = \frac{1}{n} \sum_{k|n} f(k) \phi(n/k)$$

3.9 Partitions and subsets

3.9.1 Partition function

Number of ways of writing n as a sum of positive integers, disregarding the order of the summands.

$$p(0) = 1, p(n) = \sum_{k \in \mathbb{Z} \setminus \{0\}} (-1)^{k+1} p(n - k(3k-1)/2)$$

$$p(n) \sim 0.145/n \cdot \exp(2.56\sqrt{n})$$

$\frac{n}{p(n)}$	0	1	2	3	4	5	6	7	8	9	20	50	100
	1	1	2	3	5	7	11	15	22	30	627	~2e5	~2e8

3.9.2 Partition Number

- Time Complexity: $O(n\sqrt{n})$

```
for (int i = 1; i <= n; ++i) {
    pent[2 * i - 1] = i * (3 * i - 1) / 2;
    pent[2 * i] = i * (3 * i + 1) / 2;
}
p[0] = 1;
for (int i = 1; i <= n; ++i) {
    p[i] = 0;
    for (int j = 1, k = 0; pent[j] <= i; ++j) {
        if (k < 2) p[i] = add(p[i], p[i - pent[j]]);
        else p[i] = sub(p[i], p[i - pent[j]]); ++k, k &= 3;
    }
}
```

- The number of partitions of a positive integer n into exactly k parts equals the number of partitions of n whose largest part equals k

$$p_k(n) = p_k(n-k) + p_{k-1}(n-1)$$

3.9.3 2nd Kaplansky's Lemma

The number of ways of selecting k objects, no two consecutive, from n labelled objects arrayed in a circle is $\frac{n}{k} \binom{n-k-1}{k-1} = \frac{n}{n-k} \binom{n-k}{k}$

3.9.4 Distinct Objects into Distinct Bins

- n distinct objects into r distinct bins $= r^n$
- Among n distinct objects, exactly k of them into r distinct bins $= \binom{n}{k} r^k$
- n distinct objects into r distinct bins such that each bin contains at least one object $= \sum_{i=0}^r (-1)^i \binom{r}{i} (r-i)^n$

3.10 Coloring

The number of labeled undirected graphs with n vertices, $G_n = 2^{\binom{n}{2}}$

The number of labeled directed graphs with n vertices, $G_n = 2^{n(n-1)}$

The number of connected labeled undirected graphs with n vertices, $C_n = 2^{\binom{n}{2}} - \frac{1}{n} \sum_{k=1}^{n-1} k \binom{n}{k} 2^{\binom{n-k}{2}} C_k = 2^{\binom{n}{2}} - \sum_{k=1}^{n-1} \binom{n-1}{k-1} 2^{\binom{n-k}{2}} C_k$

The number of k -connected labeled undirected graphs with n vertices, $D[n][k] = \sum_{s=1}^n \binom{n-1}{s-1} C_s D[n-s][k-1]$

Cayley's formula: the number of trees on n labeled vertices = the number of spanning trees of a complete graph with n labeled vertices $= n^{n-2}$

Number of ways to color a graph using k color such that no two adjacent nodes have same color
Complete graph = $k(k-1)(k-2)\dots(k-n+1)$

$$\text{Tree} = k(k-1)^{n-1}$$

$$\text{Cycle} = (k-1)^n + (-1)^n(k-1)$$

Number of trees with n labeled nodes: n^{n-2}

3.11 General purpose numbers

3.11.1 Eulerian numbers

Number of permutations $\pi \in S_n$ in which exactly k elements are greater than the previous element. k, j : s.t. $\pi(j) > \pi(j+1)$, $k+1, j$: s.t. $\pi(j) \geq j$, k, j : s.t. $\pi(j) > j$.

$$E(n, k) = (n-k)E(n-1, k-1) + (k+1)E(n-1, k)$$

$$E(n, 0) = E(n, n-1) = 1$$

$$E(n, k) = \sum_{j=0}^k (-1)^j \binom{n+1}{j} (k+1-j)^n$$

3.11.2 Bell numbers

Total number of partitions of n distinct elements. $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, \dots$. For p prime,

$$B(p^m + n) \equiv mB(n) + B(n+1) \pmod{p}$$

3.11.3 Bernoulli numbers

$$\sum_{j=0}^m \binom{m+1}{j} B_j = 0. \quad B_0 = 1, B_1 = -\frac{1}{2}, B_n = 0, \text{ for all odd } n \neq 1.$$

3.11.4 Catalan numbers

$$C_n = \frac{1}{n+1} \binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1, C_{n+1} = \frac{2(2n+1)}{n+2} C_n, C_{n+1} = \sum C_i C_{n-i}$$

- $C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \dots$
- sub-diagonal monotone paths in an $n \times n$ grid.
- strings with n pairs of parenthesis, correctly nested.
- binary trees with $n+1$ leaves (0 or 2 children).
- ordered trees with $n+1$ vertices.
- ways a convex polygon with $n+2$ sides can be cut into triangles by connecting vertices with straight lines.
- permutations of $[n]$ with no 3-term increasing subseq.
- Find the count of balanced parentheses sequences consisting of $n+k$ pairs of parentheses where the first k symbols are open brackets.

$$C_n^{(k)} = \frac{k+1}{n+k+1} \binom{2n+k}{n}$$

- Recursive formula of Catalan Numbers:

$$C_n^{(k)} = \frac{(2n+k-1) \cdot (2n+k)}{n \cdot (n+k+1)} C_{n-1}^{(k)}$$

3.11.5 Lucas Number

Number of edge cover of a cycle graph C_n is L_n

$$L(n) = L(n-1) + L(n-2); L(0) = 2, L(1) = 1$$

3.12 Ballot Theorem

Suppose that in an election, candidate A receives a votes and candidate B receives b votes, where $a > b$ for some positive integer k . Compute the number of ways the ballots can be ordered so that A maintains more than k times as many votes as B throughout the counting of the ballots.

The solution to the ballot problem is $\frac{a-kb}{a+b} \times C(a+b, a)$

3.13 Classical Problem

$F(n, k)$ = number of ways to color n objects using exactly k colors

Let $G(n, k)$ be the number of ways to color n objects using no more than k colors.

Then, $F(n, k) = G(n, k) - C(k, 1) * G(n, k-1) + C(k, 2) * G(n, k-2) - C(k, 3) * G(n, k-3) \dots$

Determining $G(n, k)$:

Suppose, we are given a $1 * n$ grid. Any two adjacent cells can not have same color. Then, $G(n, k) = k * ((k-1)^{n-1})$

If no such condition on adjacent cells. Then, $G(n, k) = k^n$

3.14 Matching Formula

3.14.1 Normal Graph

MM + MEC = n (excluding vertex), IS + VC = G, MIS + MVC = G

3.14.2 Bipartite Graph

MIS = n - MBM, MVC = MBM, MEC = n - MBM

3.15 Inequalities

3.15.1 Titu's Lemma

For positive reals a_1, a_2, \dots, a_n and b_1, b_2, \dots, b_n ,

$$\frac{a_1^2}{b_1} + \frac{a_2^2}{b_2} + \dots + \frac{a_n^2}{b_n} \geq \frac{a_1 + a_2 + \dots + a_n}{b_1 + b_2 + \dots + b_n}$$

Equality holds if and only if $a_i = kb_i$ for a non-zero real constant k .

3.16 Games

3.16.1 Grundy numbers

For a two-player, normal-play (last to move wins) game on a graph (V, E) : $G(x) = \text{mex}(\{G(y) : (x, y) \in E\})$, where $\text{mex}(S) = \min\{n \geq 0 : n \notin S\}$. x is losing iff $G(x) = 0$.

3.16.2 Sums of games

- Player chooses a game and makes a move in it Grundy number of a position is xor of grundy numbers of positions in summed games.

- Player chooses a non-empty subset of games (possibly, all) and makes moves in all of them A position is losing iff each game is in a losing position.

- Player chooses a proper subset of games (not empty and not all), and makes moves in all chosen ones. A position is losing iff grundy numbers of all games are equal.

- Player must move in all games, and loses if can't move in some game A position is losing if any of the games is in a losing position.

3.16.3 Misère Nim

A position with pile sizes $a_1, a_2, \dots, a_n \geq 1$, not all equal to 1, is losing iff $a_1 \oplus a_2 \oplus \dots \oplus a_n = 0$ (like in normal nim.) A position with n piles of size 1 is losing iff n is odd.

3.17 Tree Hashing

$f(u) = sz[u] * \sum_{i=0} f(v) * p^i$; $f(v)$ are sorted $f(child) = 1$

3.18 Permutation

To maximize the sum of adjacent differences of a permutation, it is necessary and sufficient to place the smallest half numbers in odd position and the greatest half numbers in even position. Or, vice versa.

3.19 String

- If the sum of length of some strings is N , there can be at most \sqrt{N} distinct length.

- A Text can have at most $O(N \times \sqrt{N})$ distinct substrings that match with given patterns where the sum of the length of the given patterns is N .

- Period = $n \% (n - \text{pi.back}() == 0) ? n - \text{pi.back}() : n$

- The first (*period*) cyclic rotations of a string are distinct. Further cyclic rotations repeat the previous strings.

- S is a palindrome if and only if its period is a palindrome.

- If S and T are palindromes, then the periods of $S \cdot T$ are same if and only if $S + T$ is a palindrome.

3.20 Bit

- $(a \text{ xor } b)$ and $(a + b)$ has the same parity

- $(a + b) = (a \text{ xor } b) + 2(a \text{ and } b)$

- $\text{gcd}(a, b) \leq a - b \leq \text{xor}(a, b)$

3.21 Convolution

- Hamming Distance: Replace 0 with -1 - SQRT Decomposition: Find block size, $B = \sqrt{8 * n}$

3.22 Matrix Rotation

3.22.1 Anti-Clockwise Rotation

$$\begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

3.22.2 Clockwise Rotation

$$\begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

3.23 Common Formulas

3.23.1 Permutation

$${}^n P_r = \frac{n!}{(n-r)!}$$

3.23.2 Combination

$${}^n C_r = \frac{n!}{r!(n-r)!}$$

3.24 Logarithms

3.24.1 Change of Base Formula

$$\log_a x = \frac{\log_b x}{\log_b a}$$

3.25 Common Series Sums

3.25.1 Sum of first n positive integers

$$1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

3.25.2 Sum of first n odd positive integers

$$1 + 3 + 5 + \dots + (2n-1) = n^2$$

3.25.3 Sum of first n even positive integers

$$2 + 4 + 6 + \dots + 2n = n(n+1)$$

3.25.4 Sum of first n squares

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

3.25.5 Sum of first n cubes

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \left[\frac{n(n+1)}{2} \right]^2$$

3.26 Progressions

3.26.1 Arithmetic Progression

- Sequence: $a, a+d, a+2d, \dots, a+(n-1)d$

- Sum of first n terms: $S_n = \frac{n}{2}[2a + (n-1)d]$

3.26.2 Geometric Progression

- Sequence: $a, ar, ar^2, \dots, ar^{n-1}$

- Sum (for $r > 1$): $S_n = \frac{a(r^n - 1)}{r - 1}$

- Sum (for $r < 1$): $S_n = \frac{a(1 - r^n)}{1 - r}$