National University
of Computer & Emerging Sciences
Islamabad

MT1003 – Calculus and Analytical Geometry

Assignment No: 01 | **Individual Assignment**

Section: CS (all sections) | **Semester:** Fall 25

Date: 11-09-2024 01:30pm | **Marks:** 160

Instructions:

1. Plagiarized work will result in zero marks.
2. No retake or late submission will be accepted.
3. Attach complete code, results, and screenshot for questions that require programming solution. Programs/codes should be typed.
4. **The complete assignment is to be submitted in soft copy as well as in hard copy. Submit the hardcopy before the deadline through CR, and softcopy on GCR.**
5. The softcopy should be a single PDF file of your complete assignment including programming and non-programming questions.
6. The PDF file should be according to the following format: id_section_A1 e.g., i25-123456_A_A1. A1 in the end denotes Assignment 1. **The title page must include complete student information, including name, section, id, course name, and assignment number.**
7. The images of by-hand solution should be properly scanned. You can use any mobile application such as CamScanner or Adobe Scan for scanning. Each of these applications allow you to export pdf or image files which you can use to combine with your programming solutions. Do not attach direct images from the camera application of your mobile phone, or screenshots.
8. Python is the only approved programming language.

Assignment CLO

CLO 1: Apply knowledge of mathematics, science and appropriate knowledge domain to mathematically model and conceptualize defined problems.

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Question 1

An oil company needs to run an oil pipeline from an oil rig 25 miles out to sea to a storage tank that is 5 miles inland. The shoreline runs east-west, and the tank is 8 miles east of the rig. Assume it costs \$50 thousand per mile to construct the pipeline under water and \$20 thousand per mile to construct the pipeline on land. The pipeline will be built in a straight line from the rig to a selected point on the shoreline, then in a straight line to the storage tank. **Interpret** the total cost function of the pipeline in terms of single variable and also state the domain.

Question 2

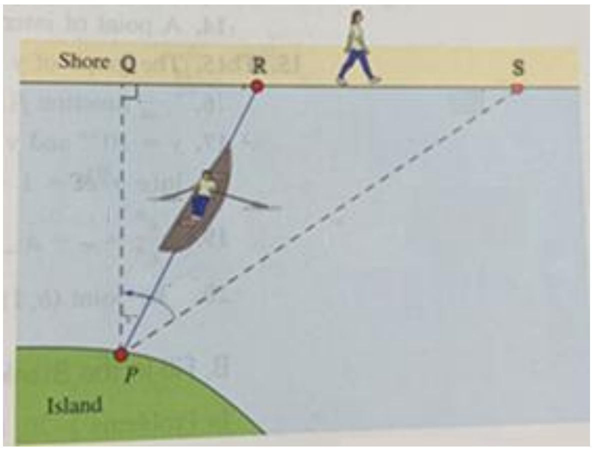
A running track is to be built around a rectangular field, with two straightaways and two semicircular curves at the ends. **interpret** the area of the enclosed rectangle as function of one variable and state the domain.

Question 3

A string of length L is cut into two pieces, and these pieces are shaped into a circle and a square. If x is the side of the square, **interpret** the total enclosed area as a function of x.

Question 4

A woman at point P on an island wishes to reach a village located at point S on a straight shore on the mainland. Point P is 9 miles from the closest point Q on the shore and the village at point S is 15 miles from point Q. If the woman rows a boat at a rate of 3 mi/hr to a point R on land, then walks the rest of the way to S at a rate of 5 mi/hr. Construct a function of single variable that **describes** the total time of the trip and also state the realistic domain.



Question 5

A tree is planted 30 ft from the base of a streetlamp that is 25 ft tall. **Interpret** the length of the tree's shadow as a function of its height. Also find the domain of the function.

Question 6

A 10 ft wall stands 5 ft from a building. A ladder, supported by the wall, is to reach from the ground to the building. **Describe** the length of the ladder in terms of the distance x between the base of the wall and the base of the ladder.

Question 7

During the drought, residents of Marin County, California, were faced with a severe water shortage. To discourage excessive use of water, the county water district initiated drastic rate increases. The monthly rate for a family of four was \$1.22 per 100 cubic feet of water for the first 1200 cubic feet, \$10 per 100 cubic feet for the next 1200 cubic feet, and \$50 per 100 cubic feet thereafter. **Interpret** the monthly water bill for a family of four as a function of the amount of water used.

Question 8

Sketch the graph not by plotting points but by starting from parent function and applying transformations. **Discuss** how each graph is obtained by applying transformations to the parent function.

a) $y = 2 - 2\sqrt{x - 3}$
b) $y = \cot\left(2\theta - \frac{\pi}{2}\right)$
c) $f(x) = 2(3^{x-1}) - 2$
d) $f(x) = 3\log(x - 2) + 1$

Question 9

A certain person's blood pressure oscillates between 140 and 80. If the heart beats once every second, **state** the person's blood pressure using a sine function.

Question 10

Part of a roller coaster track is a sinusoidal function. The high and low points are separated by 150 feet horizontally and 82 feet vertically. The low point is 6 feet above ground. **State** the roller coater's height above the ground using a sinusoidal function at a given horizontal distance x.

Question 11

State the domain of the following functions:

a) $f(x) = 5\log(x + 2)$
b) $f(x) = \log_4(2x - 3)$
c) $f(x) = \log(5 - 2x)$
d) $f(x) = \frac{x^2-4x}{x^2+4x-21}$
e) $f(x) = \frac{(\sqrt{1-2x})}{\sqrt{3x+5}}$

Question 12

State the limit of: $\lim_{x\to 0}\left[x^2\cos\left(\frac{1}{x}\right)\right]$

Question 13

State $\lim_{x\to 0} f(x)$, where f is defined by

$$f(x) = \begin{cases} x^2 + 2\cos x + 1, & for\ x < 0 \\ \sec x - 4, & for\ x \geq 0 \end{cases}$$

Question 14

Suppose a state's income tax code states that tax liability is 12% on the first \$20,000 of taxable earnings and 16% on the remainder. Find constants a and b for the tax function

$$T(x) = \begin{cases} a + 0.12x & if\ x \leq 20{,}000 \\ b + 0.16(x - 20{,}000) & if\ x > 20{,}000 \end{cases}$$

such that $\lim_{x \to 0^+} T(x) = 0$ and $\lim_{x \to 20,000} T(x)$ exists. **Why** is it important for these limits to exist?

Question 15

The number of cars passing through a toll plaza per minute can be modeled as:

$$f(t) = 50e^{-\frac{(t-30)^2}{200}}$$

where t is the time in minutes after 6:00am. Use Python programming to solve the following:

a) Plot $f(t)$ for $t = 0$ to $t = 60$.
b) A second toll lane opens 10 minutes later, which shifts the peak traffic 10 minutes to the right. Plot the transformed function and identify which transformation(s) caused this.
c) A promotional discount cuts the fee in half, doubling the number of cars. Plot the new function and identify which transformation(s) caused this.
d) Due to a change in schedule, the peak traffic now occurs earlier by 15 minutes and is less intense by 20%. Plot the new transformed graph and identify the transformation(s).
e) Reflect the original function over the horizontal axis $(-f(t))$ and explain in words what such a reflection would mean in this context (even if unrealistic).

Use Python to plot all these graphs on the same figure. The figure should be properly labeled with legends. **Discuss** all the transformations applied to obtain the functions.

Question 16

The height of a seat on a Ferris wheel can be modeled as:

$$H(t) = 30 + 25\sin\left(\frac{\pi}{15}t\right)$$

where $H(t)$ is the height (in meters) above the ground at time t minutes after the ride starts.

a) Find the amplitude and period (by-hand calculations).

b) Write a Python function $H(t)$ and graph it for one full revolution.

c) Find the height of the seat at $t = 7.5$ minutes (both, by-hand and using Python).

Compare the solution obtained manually and using Python code.