Question 1

An oil company needs to run an oil pipeline from an oil rig 25 miles out to sea to a storage tank that is 5 miles inland. The shoreline runs east-west, and the tank is 8 miles east of the rig. Assume it costs \$50 thousand per mile to construct the pipeline under water and \$20 thousand per mile to construct the pipeline on land. The pipeline will be built in a straight line from the rig to a selected point on the shoreline, then in a straight line to the storage tank. **Interpret** the total cost function of the pipeline in terms of single variable and also state the domain.

Question 2

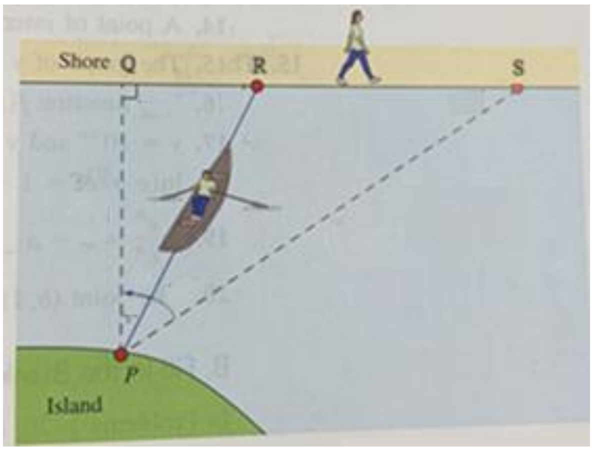
A running track is to be built around a rectangular field, with two straightaways and two semicircular curves at the ends. **interpret** the area of the enclosed rectangle as function of one variable and state the domain.

Question 3

A string of length L is cut into two pieces, and these pieces are shaped into a circle and a square. If x is the side of the square, **interpret** the total enclosed area as a function of x.

Question 4

A woman at point P on an island wishes to reach a village located at point S on a straight shore on the mainland. Point P is 9 miles from the closest point Q on the shore and the village at point S is 15 miles from point Q. If the woman rows a boat at a rate of 3 mi/hr to a point R on land, then walks the rest of the way to S at a rate of 5 mi/hr. Construct a function of single variable that **describes** the total time of the trip and also state the realistic domain.



Question 5

A tree is planted 30 ft from the base of a streetlamp that is 25 ft tall. **Interpret** the length of the tree's shadow as a function of its height. Also find the domain of the function.

Question 6

A 10 ft wall stands 5 ft from a building. A ladder, supported by the wall, is to reach from the ground to the building. **Describe** the length of the ladder in terms of the distance x between the base of the wall and the base of the ladder.

Question 7

During the drought, residents of Marin County, California, were faced with a severe water shortage. To discourage excessive use of water, the county water district initiated drastic rate increases. The monthly rate for a family of four was \$1.22 per 100 cubic feet of water for the first 1200 cubic feet, \$10 per 100 cubic feet for the next 1200 cubic feet, and \$50 per 100 cubic feet thereafter. **Interpret** the monthly water bill for a family of four as a function of the amount of water used.

Question 8

Sketch the graph not by plotting points but by starting from parent function and applying transformations. **Discuss** how each graph is obtained by applying transformations to the parent function.

a) $y = 2 - 2\sqrt{x-3}$
b) $y = \cot\left(2\theta - \frac{\pi}{2}\right)$
c) $f(x) = 2(3^{x-1}) - 2$
d) $f(x) = 3\log(x-2) + 1$

Question 9

A certain person's blood pressure oscillates between 140 and 80. If the heart beats once every second, **state** the person's blood pressure using a sine function.

Question 10

Part of a roller coaster track is a sinusoidal function. The high and low points are separated by 150 feet horizontally and 82 feet vertically. The low point is 6 feet above ground. **State** the roller coater's height above the ground using a sinusoidal function at a given horizontal distance x.

Question 11

State the domain of the following functions:

a) $f(x) = 5\log(x+2)$
b) $f(x) = \log_4(2x-3)$
c) $f(x) = \log(5-2x)$
d) $f(x) = \frac{x^2-4x}{x^2+4x-21}$
e) $f(x) = \frac{(\sqrt{1-2x})}{\sqrt{3x+5}}$

Question 12

State the limit of: $\lim_{x\to 0}\left[x^2\cos\left(\frac{1}{x}\right)\right]$

Question 13

State $\lim_{x\to 0} f(x)$, where f is defined by

$$f(x) = \begin{cases} x^2 + 2\cos x + 1, & for\ x < 0 \\ \sec x - 4, & for\ x \geq 0 \end{cases}$$

Question 14

Suppose a state's income tax code states that tax liability is 12% on the first \$20,000 of taxable earnings and 16% on the remainder. Find constants a and b for the tax function

$$T(x) = \begin{cases} a + 0.12x & if\ x \leq 20{,}000 \\ b + 0.16(x - 20{,}000) & if\ x > 20{,}000 \end{cases}$$

such that $\lim_{x \to 0^+} T(x) = 0$ and $\lim_{x \to 20{,}000} T(x)$ exists. **Why** is it important for these limits to exist?

Question 15

The number of cars passing through a toll plaza per minute can be modeled as:

$$f(t) = 50e^{-\frac{(t-30)^2}{200}}$$

where t is the time in minutes after 6:00am. Use Python programming to solve the following:

a) Plot $f(t)$ for $t = 0$ to $t = 60$.
b) A second toll lane opens 10 minutes later, which shifts the peak traffic 10 minutes to the right. Plot the transformed function and identify which transformation(s) caused this.
c) A promotional discount cuts the fee in half, doubling the number of cars. Plot the new function and identify which transformation(s) caused this.
d) Due to a change in schedule, the peak traffic now occurs earlier by 15 minutes and is less intense by 20%. Plot the new transformed graph and identify the transformation(s).
e) Reflect the original function over the horizontal axis $(-f(t))$ and explain in words what such a reflection would mean in this context (even if unrealistic).

Use Python to plot all these graphs on the same figure. The figure should be properly labeled with legends. **Discuss** all the transformations applied to obtain the functions.

Question 16

The height of a seat on a Ferris wheel can be modeled as:

$$H(t) = 30 + 25 \sin\left(\frac{\pi}{15} t\right)$$

where $H(t)$ is the height (in meters) above the ground at time t minutes after the ride starts.

a) Find the amplitude and period (by-hand calculations).

b) Write a Python function $H(t)$ and graph it for one full revolution.

c) Find the height of the seat at $t = 7.5$ minutes (both, by-hand and using Python).

Compare the solution obtained manually and using Python code.

Assignment # 1

Q1:- Let 'x' be the distance east from pt on shore closest to rig to the pt where pipeline hits the shore.

We need to construct cost function

$$C(x) = (C_1 \times \text{dist underwater}) + (C_2 \times \text{dist over land})$$

underwater pipeline cost $= C_1 \times$ dist underwater $= 50{,}000 \times$ dist underwater

Land pipeline cost $= C_2 \times$ dist overland $= 20{,}000 \times$ dist overland

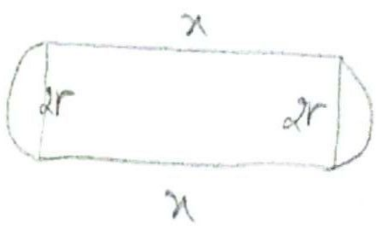
Dist underwater $= \sqrt{(25)^2+(x)^2} = \sqrt{(25)^2+x^2}$

Dist overland $= \sqrt{(8-x)^2+(5)^2}$

$$C(x) = 50{,}000\sqrt{(25)^2+(x)^2} + 20{,}000\sqrt{(8-x)^2+(5)^2}$$

Domain: $x \in [0, 8]$

Q2:- Let 'x' is the length of straightaways of the rectangle & 'r' is the radius of semicircular ends.



Total track length $= L$ m

$$L = 2x + 2\pi r \rightarrow (i)$$

Area of enclosed rectangle $= x \times 2r \rightarrow (ii)$

$$x = \frac{L - 2\pi r}{2}$$

$$x = \frac{L}{2} - \pi r$$

$\frac{L-2\pi r}{2} > 0$

Substituting 'x' in (ii) we get

$L - 2\pi r > 0$

$$A(r) = \left(\frac{L - 2\pi r}{2}\right) \cdot 2r$$

$L > 2\pi r$

$$A(r) = Lr - 2\pi r^2$$

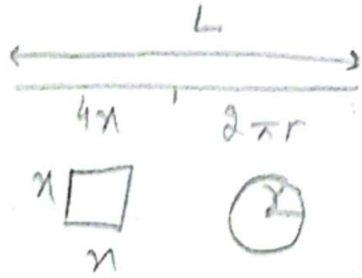
$r < \frac{L}{2\pi}$

Domain: $r \in \left(0, \frac{L}{2\pi}\right)$ (realistic domain)

$r \in \left[0, \frac{L}{2\pi}\right]$ (zero area domain)

Q3:- Let 'x' denotes the side of the square & 'r' is the radius of the circle. Sum of areas is

$$A = x^2 + \pi r^2 \longrightarrow (i)$$



where 'x' & 'r' are related by

$$4x + 2\pi r = L$$

$$2\pi r = L - 4x$$

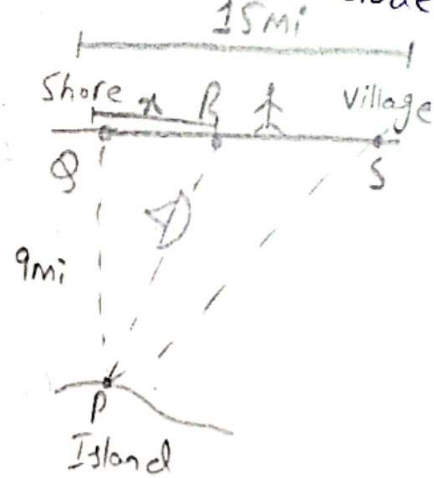
$$r = \frac{L - 4x}{2\pi}$$

Substitute 'r' in (i) we get

$$A = x^2 + \pi \left[\frac{L-4x}{2\pi}\right]^2$$

$$A = x^2 + \frac{1}{4\pi}[L - 4x]^2$$

Domain : $\left(0, \frac{L}{4}\right)$ To have both shapes we have to exclude the endpts.



Q4:- Let 'x' denotes the distance from pt 'Q' on the shore to pt 'R' where she lands on shore.

Total time = T = Rowing time + Walking time

distance = rate × time

$$\text{time} = \frac{\text{distance}}{\text{rate}}$$

Rowing rate = 3 mi/h

Walking rate = 5 mi/h

Distance she rows can be calculated by Pythagorean Theorem

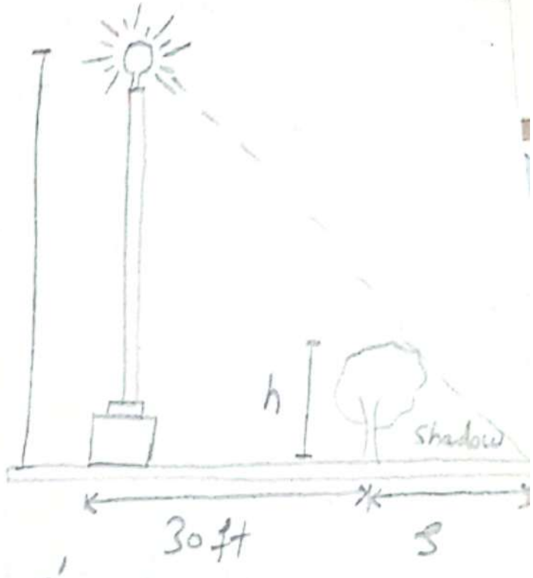
Distance she rows $= \sqrt{(9)^2 + (x)^2}$

" " walks $= 15 - x$

Total time $= T(x) = \dfrac{\sqrt{81 + x^2}}{3} + \dfrac{15 - x}{5}$

Domain : $[0, 15]$

Q5:- Let 'h' denotes the height of tree & 's' denotes the shadow respectively.



$$\frac{h}{s} = \frac{25}{s+30}$$ (concept of similar triangles)

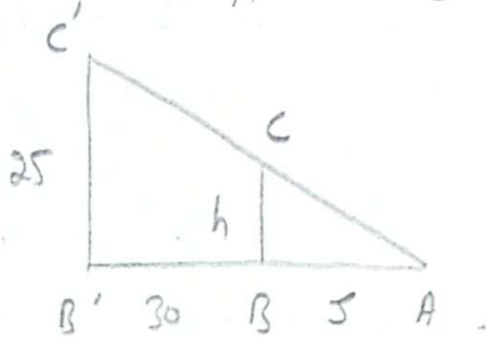
$$(s+30)h = 25s$$

$$sh + 30h = 25s$$

$$sh - 25s = -30h$$

$$(25-h)s = 30h$$

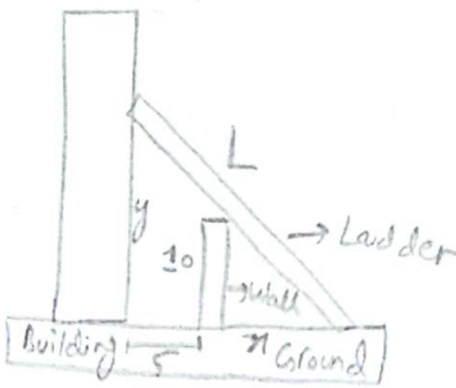
$$s(h) = \frac{30h}{25-h}$$



Domain : $[0, 25)$ If $h > 25$ then $s(h)$ is -ive, which makes no sense in physical content of problem

Q6:- Let 'L' denotes the length of ladder.

$$L^2 = (x+5)^2 + y^2 \rightarrow (i)$$ Ladder is hyp of larger triangle



Using the concept of similar triangles

$$\frac{y}{x+5} = \frac{10}{x}$$

$$y = \frac{10(x+5)}{x}$$

Substituting 'y' in (i) we get

$$L^2 = (x+5)^2 + \left[\frac{10(x+5)}{x}\right]^2$$

$$= (x+5)^2 \left[\frac{x^2 + 100}{x^2}\right]$$

CamScanner

$$L(x) = \frac{x+5}{x}\sqrt{x^2+100}$$

Q7:- Let 'x' denote the number of hundred-cubic-feet units of water used by the family during the month & $C(x)$ the corresponding cost in dollars. If $0 \le x \le 12$, the cost is the cost per unit times the no. of units used

$$C(x) = 1.22x$$

If $12 < x \le 24$, each of first 12 units costs \$1.22, & so the total cost of these 12 units is $1.22(12) = \$14.64$. Each of the remaining $(x-12)$ units costs \$10 & hence the total cost of these units is $10(x-12)$ dollars. The cost of all x units is the sum

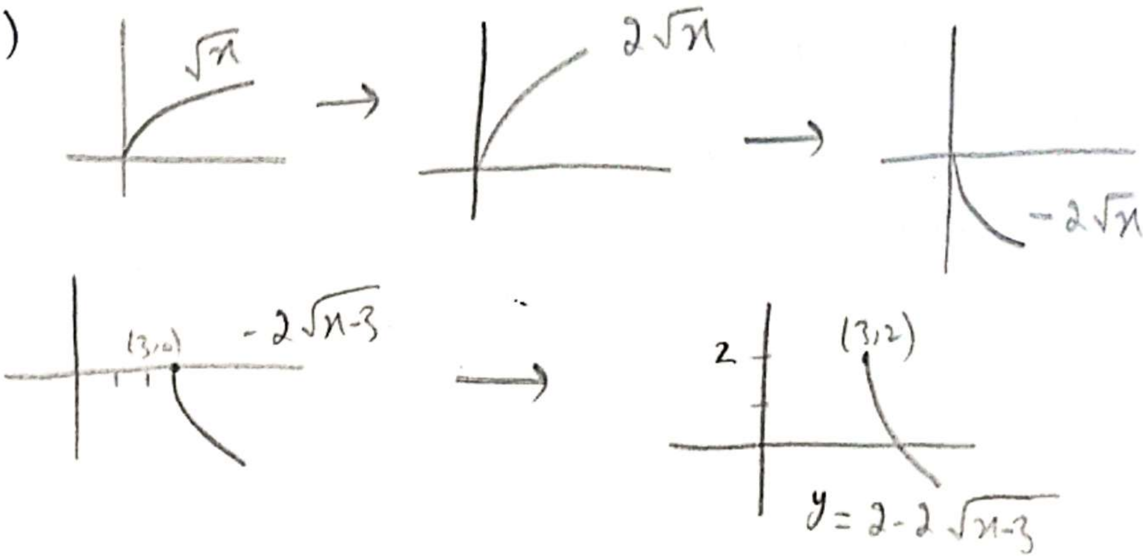
$$C(x) = 14.64 + 10(x-12) = 10x - 105.36$$

If $x > 24$, the cost of first 12 units is $1.22(12) = 14.64$ dollars, the cost of next 12 units is $10(12) = 120$ dollars & that of remaining $(x-24)$ units is $50(x-24)$ dollars. The cost of all x units is the sum

$$C(x) = 14.64 + 120 + 50(x-24) = 50x - 1065.36$$

$$C(x) = \begin{cases} 1.22x & 0 \le x \le 12 \\ 10x - 105.36 & 12 < x \le 24 \\ 50x - 1065.36 & x > 24. \end{cases}$$

Q8:- (a)



(b) $y = \cot\left(2\theta - \frac{\pi}{2}\right)$

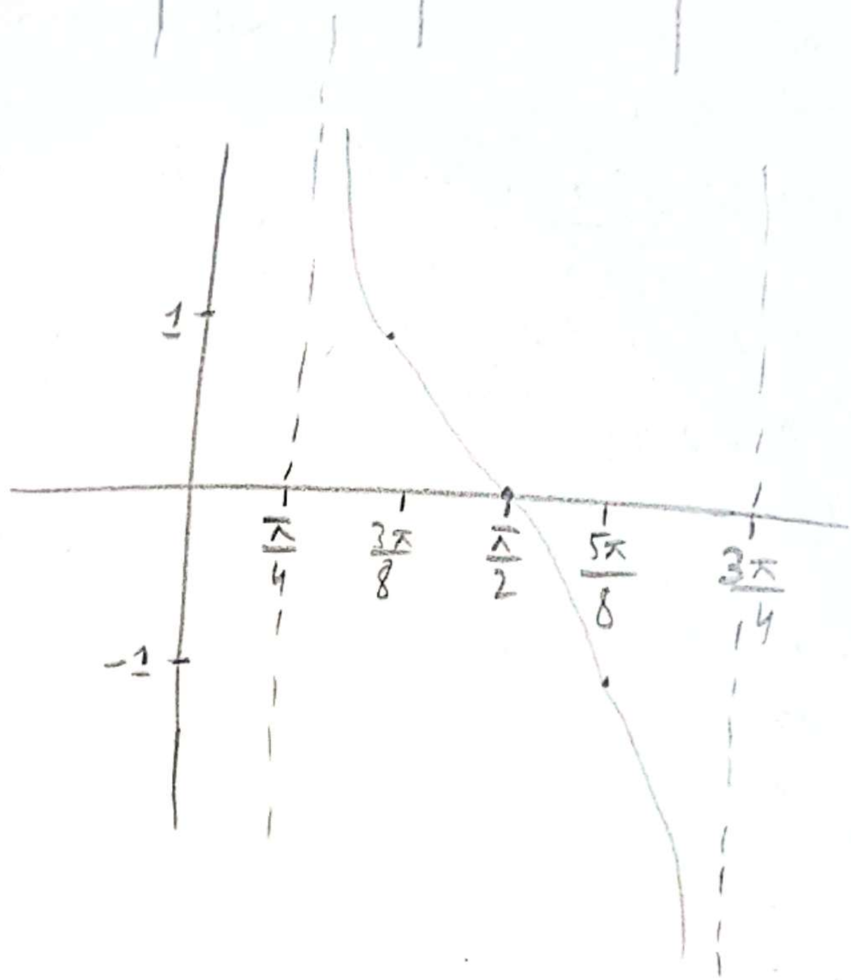
$y = \cot\left[2\left(\theta - \frac{\pi}{4}\right)\right]$

Vertical stretch : None

Period : π

Phase shift : right $\frac{\pi}{4}$

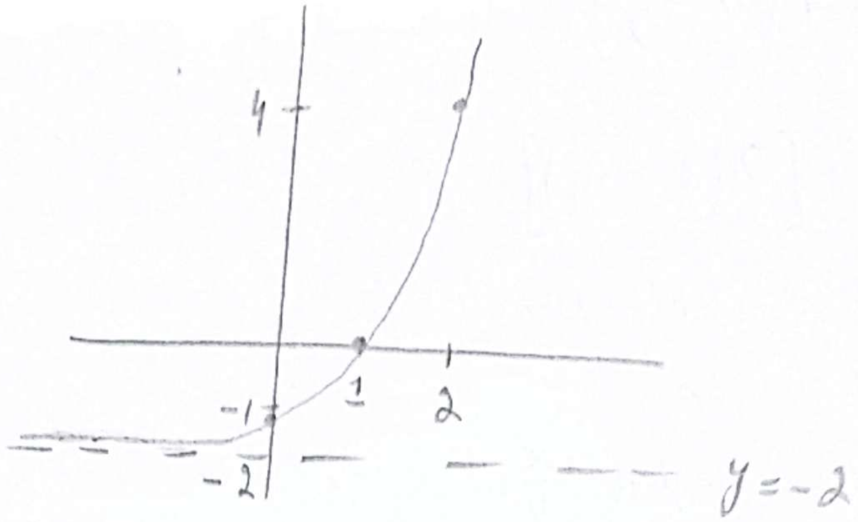
θ	$0+\frac{\pi}{4} = \pi/4$	$\frac{\pi}{8}+\frac{\pi}{4} = \frac{3\pi}{8}$	$\frac{2\pi}{8}+\frac{\pi}{4} = \pi/2$	$\frac{3\pi}{8}+\frac{\pi}{4} = \frac{5\pi}{8}$	$\frac{4\pi}{8}+\frac{\pi}{4} = \frac{3\pi}{4}$
$\cot(2\theta)$	undef	1	0	-1	undef



(c) $f(x) = 2(3^{x-1}) - 2$

x-intercept at $(1,0)$ A pt at $(2,4)$

y = = $(0,-4/3)$ HA at $y=-2$



(d) $f(x) = 3\log(x-2) + 1$

VA at $x = 2$

$(3, 1)$

For x-intercept

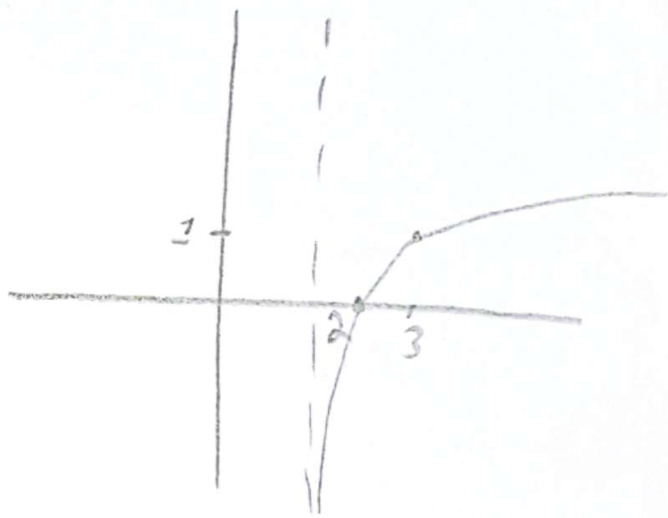
$3\log(x-2) + 1 = 0$

$\log(x-2) = -\frac{1}{3}$

$x - 2 = 10^{-1/3}$

$x \approx 2.46$

$(2.4, 0)$



Q9:- Period = 1

Period = $\frac{2\pi}{b}$

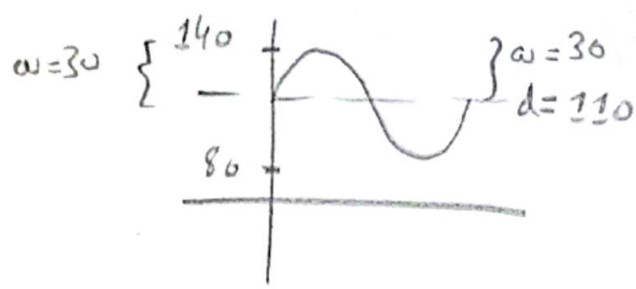
$1 = \frac{2\pi}{b}$

$b = 2\pi$

$a = 30$

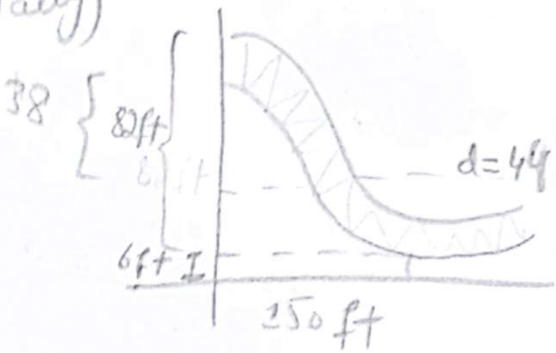
$c = 0$

$d = 110$



$y = 30\sin(2\pi x) + 110$

Q10:- 150ft is half the cycle (high & low pts are separated by 150ft horizontally)



$$\text{Period} = 300$$

$$\text{Period} = \frac{2\pi}{b}$$

$$300 = \frac{2\pi}{b}$$

$$300b = 2\pi$$

$$b = \frac{\pi}{150}$$

$$a = 38$$

$$c = 0$$

$$d = 44$$

$$y = 38\cos\left(\frac{\pi}{150}x\right) + 44$$

Q11:-(a) $(-2, \infty)$

(b) $(1.5, \infty)$ or $\left(\frac{3}{2}, \infty\right)$

(c) $(-\infty, 5/2)$

(d) $(-\infty, -7) \cup (-7, 3) \cup (3, \infty)$

(e) $1 - 2x \geq 0$ $\quad$ $3x + 5 > 0$

$x \leq \frac{1}{2}$ $\quad$ $x > -\frac{5}{3}$

$$\left(-\frac{5}{3}, \frac{1}{2}\right]$$

Q12:- Applying Squeeze Theorem

$$-1 \leq \cos\left(\frac{1}{x}\right) \leq 1$$

$$-x^2 \leq x^2\cos\left(\frac{1}{x}\right) \leq x^2$$

$$\lim_{x\to 0}(-x^2) \le \lim_{x\to 0} x^2\cos\left(\frac{1}{x}\right) \le \lim_{x\to 0}(x^2)$$

$$\lim_{x\to 0}(-x^2) = 0$$

$$\lim_{x\to 0}(x^2) = 0$$

So by S.T we say that

$$\lim_{x\to 0} x^2\cos\left(\frac{1}{x}\right) = 0$$

Q13:- LHL

$$\lim_{x\to 0^-}(x^2+2\cos x+1) = 2\cos 0+1 = 3$$

RHL

$$\lim_{x\to 0^+}(\sec x-4) = \sec 0 - 4 = 1-4 = -3$$

Since LHL $\ne$ RHL so $\lim_{x\to 0} f(x)$ DNE

Q14:- $\lim_{x\to 0^+} T(x) = \cancel{T(0)}$

$$\lim_{x\to 0^+} T(x) = a$$

$\cancel{T(0)}$

$$a = 0$$

For limit to exist at $x = 20{,}000$

$$\lim_{x\to 20{,}000^-} T(x) = \lim_{x\to 20{,}000^+} T(x)$$

LHL

$$\lim_{x\to 20{,}000^-} T(x) = a + 0.12(20{,}000)$$

$$= a + 2400$$

RHL

$$\lim_{x \to 20{,}000^+} T(x) = b$$

For limit to exist LHL = RHL

$$a + 2400 = b$$

As $a = 0$ so

$$b = 2400$$

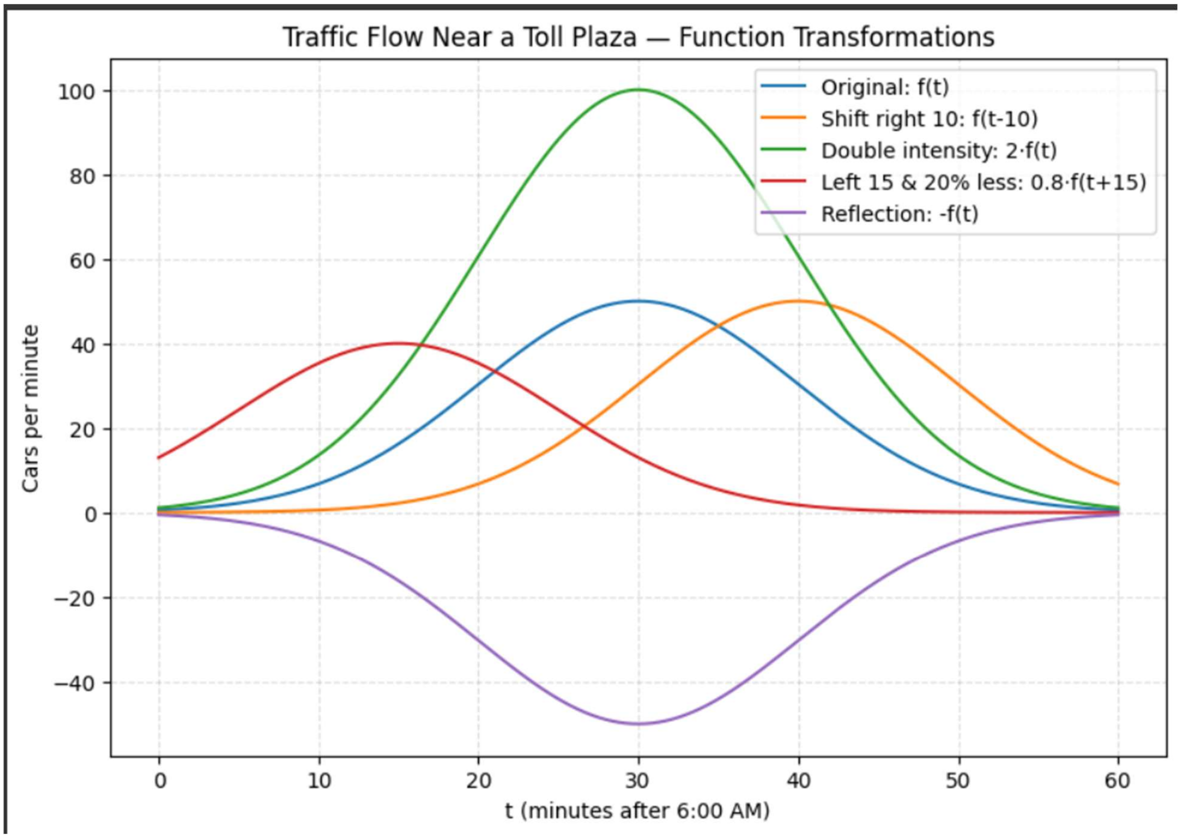
It is imp for these limits to exist to ensure the tax fn is continuous and fair. Continuity at $20,000 ensures that there is no sudden jump in tax owed as income increases slightly. Limit at '0' ensures that no tax is owed when there is no income.

CamScanner

Q15

Solution Code:

```
import numpy as np
import matplotlib.pyplot as plt

# Base function: traffic flow (cars per minute) after 6:00 AM
def f(t):
    return 50 * np.exp(-((t - 30) ** 2) / 200)

t = np.linspace(0, 60, 1000)

# 1) Original
f_orig = f(t)

# 2) Horizontal shift right by 10 minutes (peak moves from 30 -> 40)
#    g(t) = f(t - 10)
f_shift_right = f(t - 10)

# 3) Vertical scaling: discount doubles traffic
#    2·f(t)
f_vertical_scale = 2 * f_orig

# 4) Peak earlier by 15 minutes (left shift) and 20% less intense
#    0.8·f(t + 15)
f_combined = 0.8 * f(t + 15)

# 5) Reflection about the horizontal axis
#    -f(t)
f_reflect = -f_orig

plt.figure(figsize=(9, 6))
plt.plot(t, f_orig, label="Original: f(t)")
plt.plot(t, f_shift_right, label="Shift right 10: f(t-10)")
plt.plot(t, f_vertical_scale, label="Double intensity: 2·f(t)")
plt.plot(t, f_combined, label="Left 15 & 20% less: 0.8·f(t+15)")
plt.plot(t, f_reflect, label="Reflection: -f(t)")
plt.title("Traffic Flow Near a Toll Plaza — Function Transformations")
plt.xlabel("t (minutes after 6:00 AM)")
plt.ylabel("Cars per minute")
plt.legend(loc="best")
plt.grid(True, linestyle="--", alpha=0.4)
plt.show()
```



Traffic Flow Near a Toll Plaza — Function Transformations
Original: f(t)
Shift right 10: f(t-10)
Double intensity: 2·f(t)
Left 15 & 20% less: 0.8·f(t+15)
Reflection: -f(t)
Cars per minute
t (minutes after 6:00 AM)
100
80
60
40
20
0
−20
−40
0
10
20
30
40
50
60

Q16

(a) Amplitude & Period

General form: y = Asin (Bt)

So **Amplitude** = 25 meters and

Period = 2π/B and here B = π/15 so the answer will be 2π / (π / 15) = 30 minutes

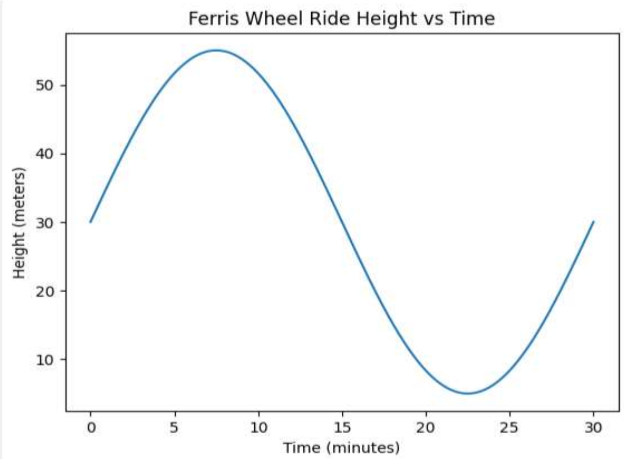
(b) Python Function + Graph

```
[1]: import numpy as np
     import matplotlib.pyplot as plt
```

```
[2]: def func(t):
         return 30 + 25 * np.sin((np.pi/15) * t)
```

```
[3]: t = np.linspace(0, 30, num = 1000)
     heights = func(t)
```

```
[4]: plt.plot(t, heights)
     plt.title("Ferris Wheel Ride Height vs Time")
     plt.xlabel("Time (minutes)")
     plt.ylabel("Height (meters)")
     plt.show()
```



c) Height by Hand and by Python

1. **By Hand**:

We substitute $t = 7.5$:

$$H(7.5) = 30 + 25\sin\left(\frac{\pi}{15} \cdot 7.5\right)$$

$$= 30 + 25\sin\left(\frac{\pi}{2}\right)$$

$$= 30 + 25(1) = 55 \text{ meters}$$

So at **7.5 minutes**, the seat is at its **maximum height (55 m)**.

By Python:

```
[5]: height_to_find = func(7.5)
     print("So the height at 7.5 minutes is: ", height_to_find)

     So the height at 7.5 minutes is:  55.0
```

:

So both values of height are found manually and by python matches so it verifies the answer.