

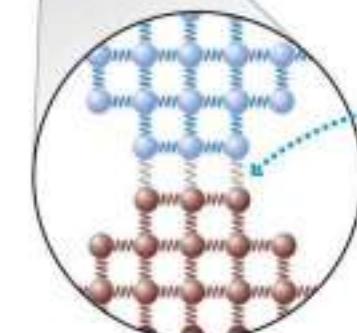
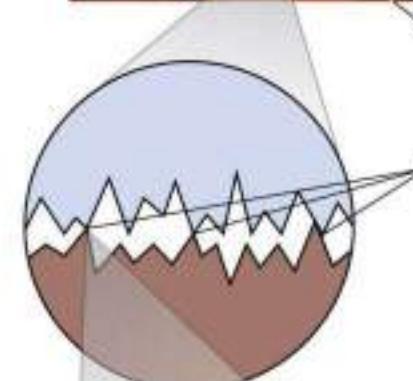
# Friction

- A contact force that resists sliding between surfaces.
- Slipping means that **surfaces slide with respect to each other--there is relative motion between the surfaces**. There may or may not be friction. If there is no slipping, then the surfaces do not slide at the point of contact. There may or may not be friction involved. An example would be a ball rolling without slipping.



# Static Friction

- A shoe pushes on a wooden floor but does not slip.
- On a microscopic scale, both surfaces are “rough” and high features on the two surfaces form molecular bonds.
- These bonds can produce a force *tangent* to the surface, called the **static friction** force.
- Static friction is a result of many molecular springs being compressed or stretched ever so slightly.



Two surfaces in contact

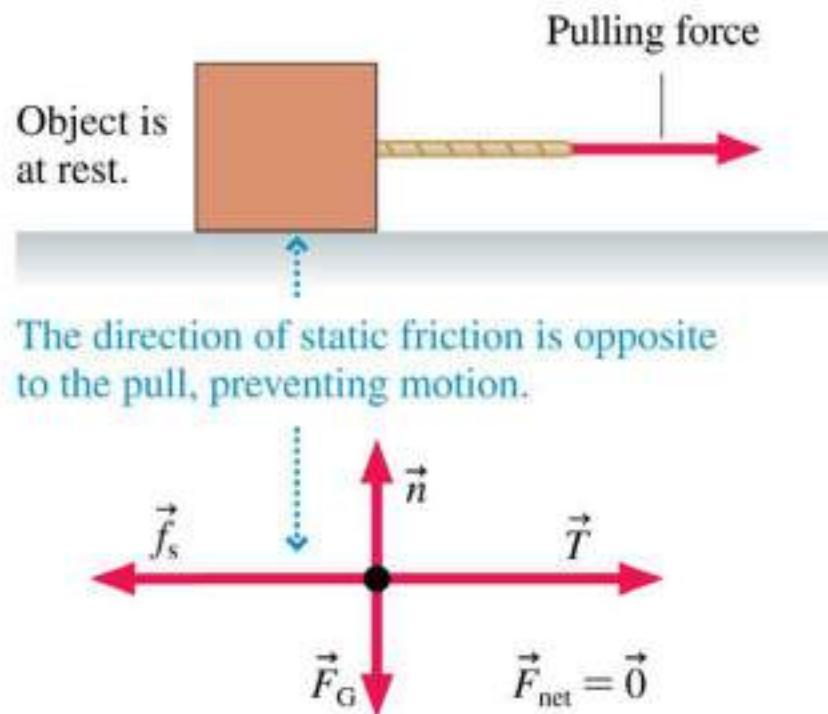
Very few points are actually in contact.

Molecular bonds form between the two materials. These bonds have to be broken as the object slides.

# Static Friction

- The figure shows a rope pulling on a box that, due to static friction, isn't moving.
- Looking at the free-body diagram, the  $x$ -component of Newton's first law requires that the static friction force must exactly balance the tension force:

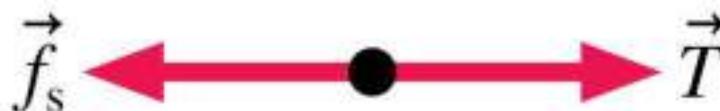
$$f_s = T$$



- $\vec{f}_s$  points in the direction *opposite* to the way the object would move if there were no static friction.

# Static Friction

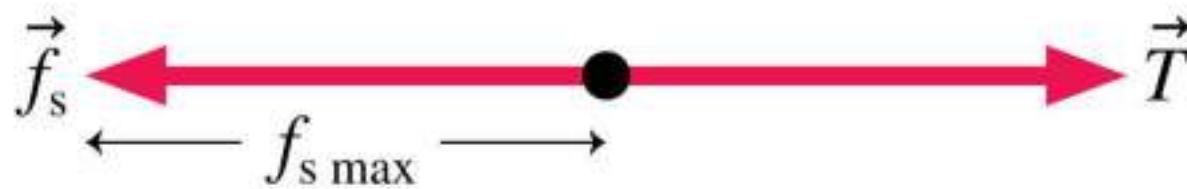
- Static friction acts in *response* to an applied force.



$\vec{T}$  is balanced by  $\vec{f}_s$  and the box does not move.



As  $T$  increases,  $f_s$  grows . . .



. . . until  $f_s$  reaches  $f_{s \text{ max}}$ . Now, if  $T$  gets any bigger, the object will start to move.

# Static Friction

To determine the direction of  $\vec{f}_s$ , decide which way the object would move if there were no friction. The static friction force  $\vec{f}_s$  points in the *opposite* direction to prevent the motion.

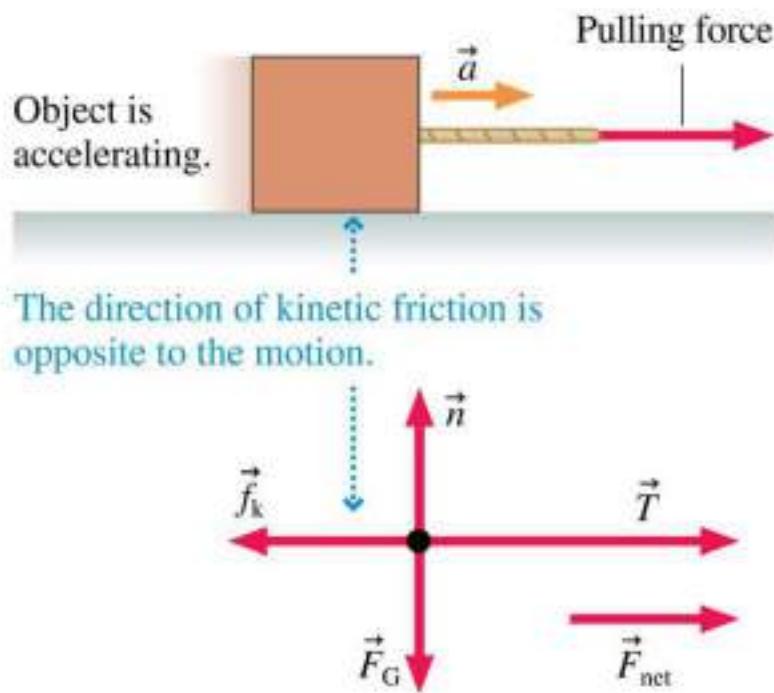
# Static Friction

- Static friction force has a *maximum* possible size  $f_{s\ max}$ .
- An object remains at rest as long as  $f_s < f_{s\ max}$ .
- The object just begins to slip when  $f_s = f_{s\ max}$ .
- A static friction force  $f_s > f_{s\ max}$  is not physically possible:

$$f_{s\ max} = \mu_s n$$

where the proportionality constant  $\mu_s$  is called the **coefficient of static friction**.

# Kinetic Friction



- The **kinetic friction** force is proportional to the magnitude of the normal force:

$$f_k = \mu_k n$$

where the proportionality constant  $\mu_k$  is called the **coefficient of kinetic friction**.

- The kinetic friction direction is opposite to the velocity of the object relative to the surface.
- For any particular pair of surfaces,  $\mu_k < \mu_s$ .

# Coefficients of Friction

TABLE 6.1 Coefficients of friction

Materials	Static $\mu_s$	Kinetic $\mu_k$	Rolling $\mu_r$
Rubber on dry concrete	1.00	0.80	0.02
Rubber on wet concrete	0.30	0.25	0.02
Steel on steel (dry)	0.80	0.60	0.002
Steel on steel (lubricated)	0.10	0.05	
Wood on wood	0.50	0.20	
Wood on snow	0.12	0.06	
Ice on ice	0.10	0.03	

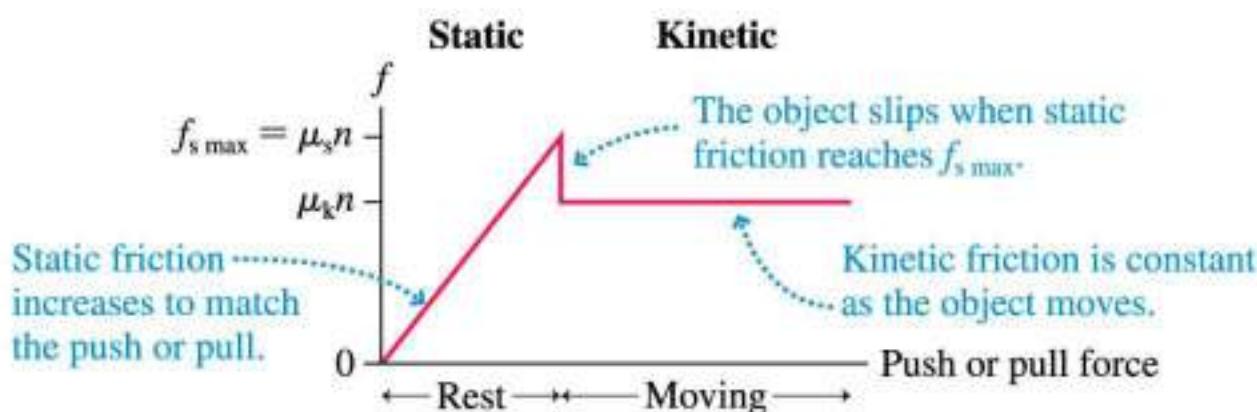
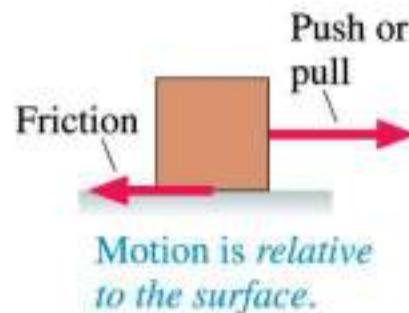
# A Model of Friction

## MODEL 6.3

### Friction

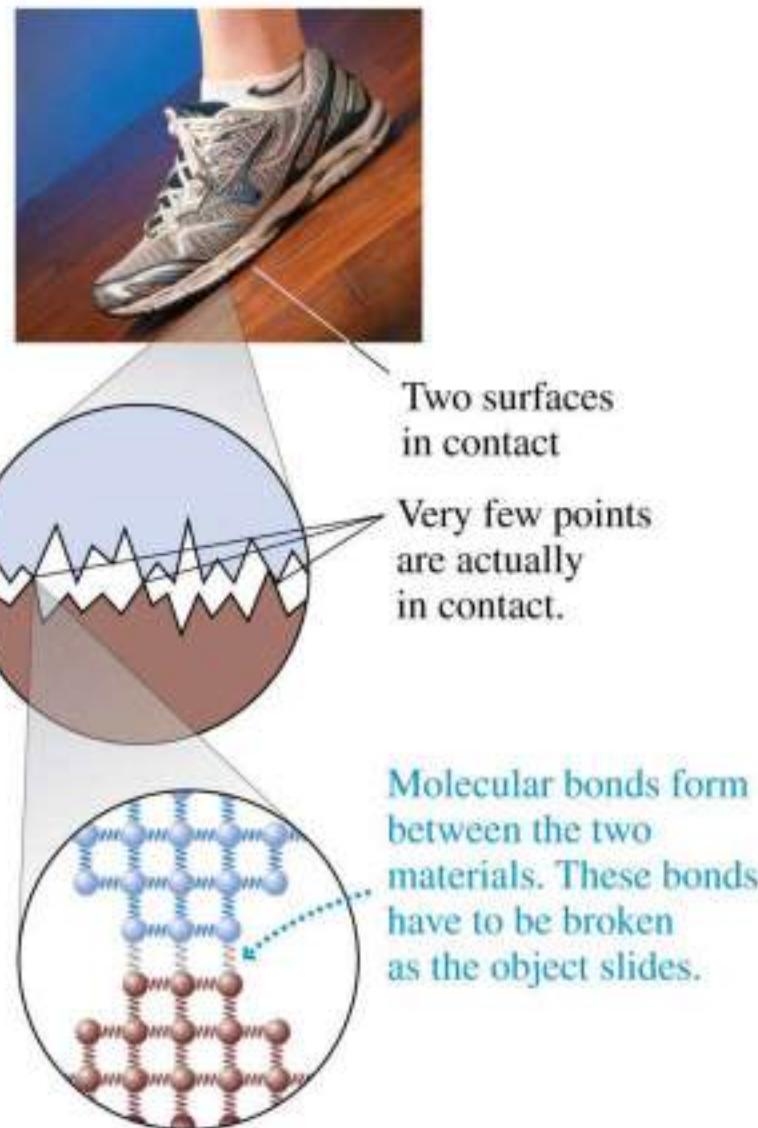
The friction force is *parallel* to the surface.

- Static friction: Acts as needed to prevent motion.  
Can have *any* magnitude up to  $f_{s\ max} = \mu_s n$ .
- Kinetic friction: Opposes motion with  $f_k = \mu_k n$ .
- Rolling friction: Opposes motion with  $f_r = \mu_r n$ .
- Graphically:



# Causes of Friction

- All surfaces are very rough on a microscopic scale.
- When two surfaces are pressed together, the high points on each side come into contact and form molecular bonds.
- The amount of contact depends on the normal force  $n$ .
- When the two surfaces are sliding against each other, the bonds don't form fully, but they do tend to slow the motion.

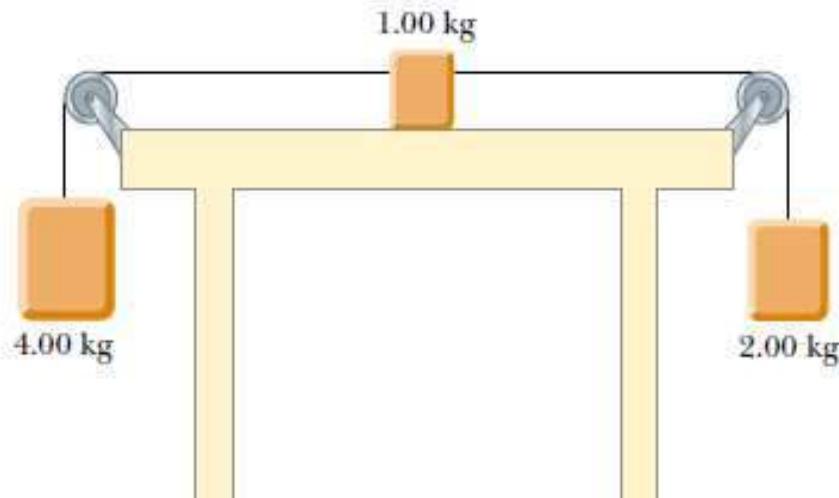


Ch 6 H/R/W

# **PROBLEMS WITH FRICTION**

# Problem

- Three objects are connected on the table as shown in Figure P5.44. The table is rough and has a coefficient of kinetic friction of 0.350. The objects have masses of 4.00 kg, 1.00 kg, and 2.00 kg, as shown, and the pulleys are frictionless. Draw free-body diagrams of each of the objects. (a) Determine the acceleration of each object and their directions. (b) Determine the tensions in the two cords.



# Sol.

Let  $a$  represent the positive magnitude of the acceleration  $-a\hat{j}$  of  $m_1$ , of the acceleration  $-a\hat{i}$  of  $m_2$ , and of the acceleration  $+a\hat{j}$  of  $m_3$ . Call  $T_{12}$  the tension in the left rope and  $T_{23}$  the tension in the cord on the right.

$$\text{For } m_1: \quad \sum F_y = ma_y \quad +T_{12} - m_1 g = -m_1 a$$

$$\text{For } m_2: \quad \sum F_x = ma_x \quad -T_{12} + \mu_k n + T_{23} = -m_2 a$$

$$\text{and} \quad \sum F_y = ma_y \quad n - m_2 g = 0$$

$$\text{for } m_3: \quad \sum F_y = ma_y \quad T_{23} - m_3 g = +m_3 a$$

we have three simultaneous equations

$$\begin{aligned} -T_{12} + 39.2 \text{ N} &= (4.00 \text{ kg})a \\ +T_{12} - 0.350(9.80 \text{ N}) - T_{23} &= (1.00 \text{ kg})a \\ +T_{23} - 19.6 \text{ N} &= (2.00 \text{ kg})a. \end{aligned}$$

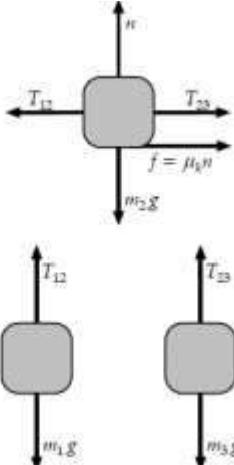


FIG. P5.44

- (a) Add them up:

$$+39.2 \text{ N} - 3.43 \text{ N} - 19.6 \text{ N} = (7.00 \text{ kg})a$$

$$a = \boxed{2.31 \text{ m/s}^2, \text{ down for } m_1, \text{ left for } m_2, \text{ and up for } m_3}$$

- (b) Now  $-T_{12} + 39.2 \text{ N} = (4.00 \text{ kg})(2.31 \text{ m/s}^2)$

$$\boxed{T_{12} = 30.0 \text{ N}}$$

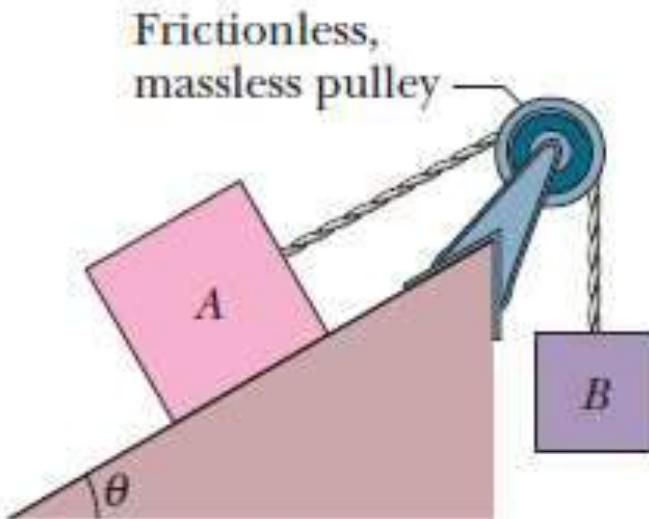
and  $T_{23} - 19.6 \text{ N} = (2.00 \text{ kg})(2.31 \text{ m/s}^2)$

$$\boxed{T_{23} = 24.2 \text{ N}}$$

# Problem

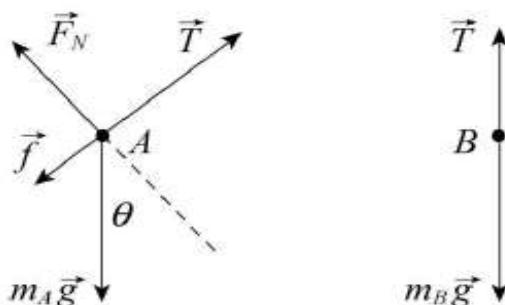
GOALS FOR THIS PROBLEM:

- 27**  Body *A* in Fig. 6-33 weighs 102 N, and body *B* weighs 32 N. The coefficients of friction between *A* and the incline are  $\mu_s = 0.56$  and  $\mu_k = 0.25$ . Angle  $\theta$  is  $40^\circ$ . Let the positive direction of an *x* axis be up the incline. In unit-vector notation, what is the acceleration of *A* if *A* is initially (a) at rest, (b) moving up the incline, and (c) moving down the incline?



**Figure 6-33**  
Problems 27 and 28.

27. First, we check to see if the bodies start to move. We assume they remain at rest and compute the force of (static) friction which holds them there, and compare its magnitude with the maximum value  $\mu_s F_N$ . The free-body diagrams are shown below.



$T$  is the magnitude of the tension force of the string,  $f$  is the magnitude of the force of friction on body  $A$ ,  $F_N$  is the magnitude of the normal force of the plane on body  $A$ ,  $m_A \vec{g}$  is the force of gravity on body  $A$  (with magnitude  $W_A = 102 \text{ N}$ ), and  $m_B \vec{g}$  is the force of gravity on body  $B$  (with magnitude  $W_B = 32 \text{ N}$ ).  $\theta = 40^\circ$  is the angle of incline. We are told the direction of  $\vec{f}$  but we assume it is downhill. If we obtain a negative result for  $f$ , then we know the force is actually up the plane.

(a) For  $A$  we take the  $+x$  to be uphill and  $+y$  to be in the direction of the normal force. The  $x$  and  $y$  components of Newton's second law become

$$\begin{aligned} T - f - W_A \sin \theta &= 0 \\ F_N - W_A \cos \theta &= 0. \end{aligned}$$

Taking the positive direction to be *downward* for body  $B$ , Newton's second law leads to  $W_B - T = 0$ . Solving these three equations leads to

$$f = W_B - W_A \sin \theta = 32 \text{ N} - (102 \text{ N}) \sin 40^\circ = -34 \text{ N}$$

(indicating that the force of friction is *uphill*) and to

$$F_N = W_A \cos \theta = (102 \text{ N}) \cos 40^\circ = 78 \text{ N}$$

which means that

$$f_{s,\max} = \mu_s F_N = (0.56) (78 \text{ N}) = 44 \text{ N}.$$

Since the magnitude  $f$  of the force of friction that holds the bodies motionless is less than  $f_{s,\max}$  the bodies remain at rest. The acceleration is zero.

- (b) Since  $A$  is moving up the incline, the force of friction is downhill with magnitude  $f_k = \mu_k F_N$ . Newton's second law, using the same coordinates as in part (a), leads to

$$\begin{aligned}T - f_k - W_A \sin \theta &= m_A a \\F_N - W_A \cos \theta &= 0 \\W_B - T &= m_B a\end{aligned}$$

for the two bodies. We solve for the acceleration:

$$\begin{aligned}a &= \frac{W_B - W_A \sin \theta - \mu_k W_A \cos \theta}{m_B + m_A} = \frac{32\text{N} - (102\text{N}) \sin 40^\circ - (0.25)(102\text{N}) \cos 40^\circ}{(32\text{N} + 102\text{N}) / (9.8 \text{ m/s}^2)} \\&= -3.9 \text{ m/s}^2.\end{aligned}$$

The acceleration is down the plane, i.e.,  $\vec{a} = (-3.9 \text{ m/s}^2) \hat{i}$ , which is to say (since the initial velocity was uphill) that the objects are slowing down. We note that  $m = W/g$  has been used to calculate the masses in the calculation above.

(c) Now body  $A$  is initially moving down the plane, so the force of friction is uphill with magnitude  $f_k = \mu_k F_N$ . The force equations become

$$\begin{aligned}T + f_k - W_A \sin \theta &= m_A a \\F_N - W_A \cos \theta &= 0 \\W_B - T &= m_B a\end{aligned}$$

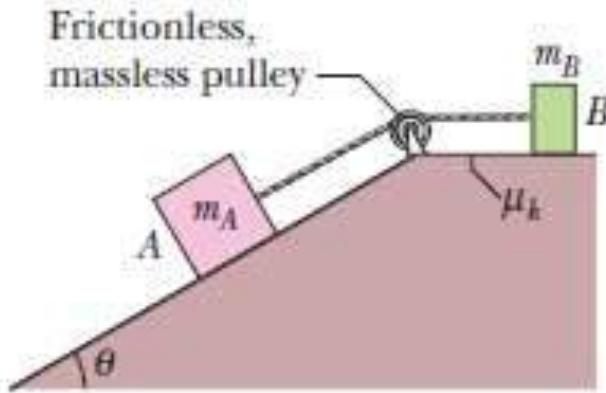
which we solve to obtain

$$\begin{aligned}a &= \frac{W_B - W_A \sin \theta + \mu_k W_A \cos \theta}{m_B + m_A} = \frac{32\text{N} - (102\text{N}) \sin 40^\circ + (0.25)(102\text{N}) \cos 40^\circ}{(32\text{N} + 102\text{N}) / (9.8 \text{ m/s}^2)} \\&= -1.0 \text{ m/s}^2.\end{aligned}$$

The acceleration is again downhill the plane, i.e.,  $\vec{a} = (-1.0 \text{ m/s}^2) \hat{i}$ . In this case, the objects are speeding up.

# Problem

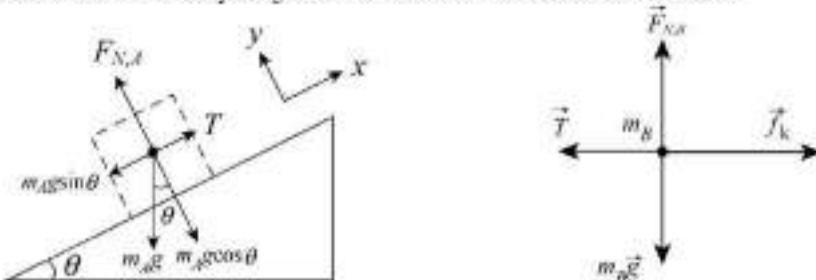
**79 SSM** Block  $A$  in Fig. 6-56 has mass  $m_A = 4.0 \text{ kg}$ , and block  $B$  has mass  $m_B = 2.0 \text{ kg}$ . The coefficient of kinetic friction between block  $B$  and the horizontal plane is  $\mu_k = 0.50$ . The inclined plane is frictionless and at angle  $\theta = 30^\circ$ . The pulley serves only to change the direction of the cord connecting the blocks. The cord has negligible mass. Find (a) the tension in the cord and (b) the magnitude of the acceleration of the blocks.



# Solution

79. **THINK** We have two blocks connected by a cord, as shown in Fig. 6-56. As block *A* slides down the frictionless inclined plane, it pulls block *B*, so there's a tension in the cord.

**EXPRESS** The free-body diagrams for blocks *A* and *B* are shown below:



Newton's law gives

$$m_A g \sin \theta - T = m_A a$$

for block *A* (where  $\theta = 30^\circ$ ). For block *B*, we have

$$T - f_k = m_B a$$

Now the frictional force is given by  $f_k = \mu_k F_{N,B} = \mu_k m_B g$ . The equations allow us to solve for the tension  $T$  and the acceleration  $a$ .

**ANALYZE** (a) Combining the above equations to solve for  $T$ , we obtain

$$T = \frac{m_A m_B}{m_A + m_B} (\sin \theta + \mu_k) g = \frac{(4.0 \text{ kg})(2.0 \text{ kg})}{4.0 \text{ kg} + 2.0 \text{ kg}} (\sin 30^\circ + 0.50)(9.80 \text{ m/s}^2) = 13 \text{ N}$$

## Solution cont.

(b) Similarly, the acceleration of the two-block system is

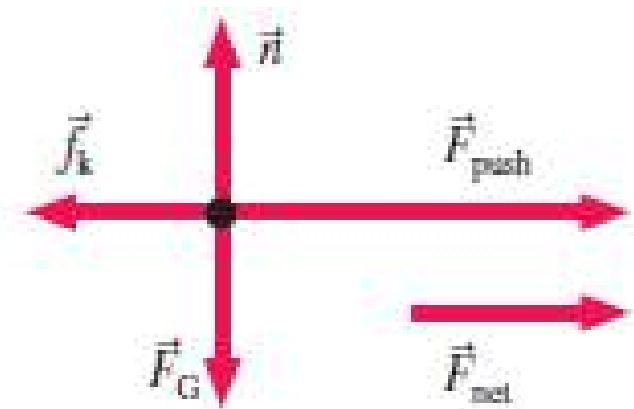
$$a = \left( \frac{m_A \sin \theta - \mu_k m_B}{m_A + m_B} \right) g = \frac{(4.0 \text{ kg}) \sin 30^\circ - (0.50)(2.0 \text{ kg})}{4.0 \text{ kg} + 2.0 \text{ kg}} (9.80 \text{ m/s}^2) = 1.6 \text{ m/s}^2.$$

- An object remains at rest as long as  $f_s < f_{s\max}$ .
- The object slips when  $f_s = f_{s\max}$ .
- A static friction force  $f_s > f_{s\max}$  is not physically possible.

To determine the direction of  $\vec{f}_s$ , decide which way the object would move were no friction. The static friction force  $\vec{f}_s$  points in the *opposite* direction to the motion.

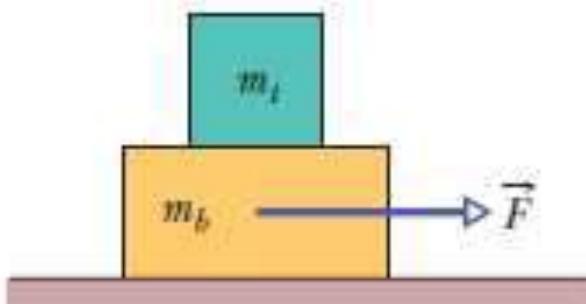
# Kinetic friction

- Friction when an object slides along a surface. Direction is opposite the object's sliding direction and is parallel to the contact surface.



# Problem

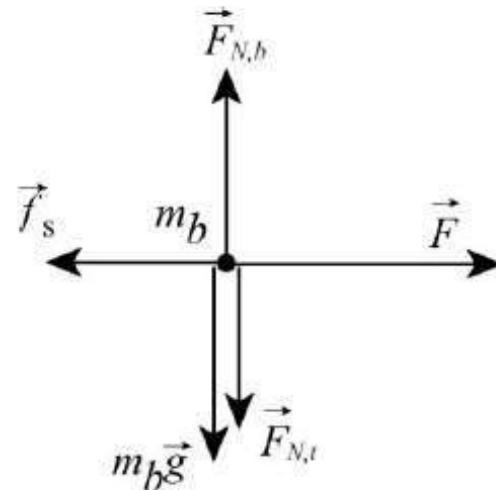
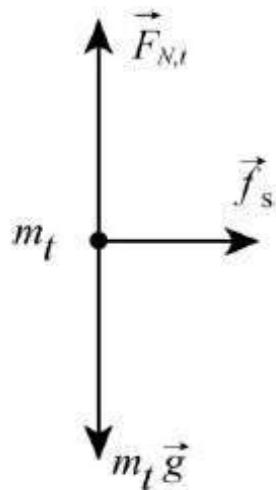
**61 SSM** A block of mass  $m_t = 4.0 \text{ kg}$  is put on top of a block of mass  $m_b = 5.0 \text{ kg}$ . To cause the top block to slip on the bottom one while the bottom one is held fixed, a horizontal force of at least  $12 \text{ N}$  must be applied to the top block. The assembly of blocks is now placed on a horizontal, frictionless table (Fig. 6-47). Find the magnitudes of (a) the maximum horizontal force  $\vec{F}$  that can be applied to the lower block so that the blocks will move together and (b) the resulting acceleration of the blocks.



# Solution

61. **THINK** Our system consists of two blocks, one on top of the other. If we pull the bottom block too hard, the top block will slip on the bottom one. We're interested in the maximum force that can be applied such that the two will move together.

**EXPRESS** The free-body diagrams for the two blocks are shown below.



## Solution cont.

We first calculate the coefficient of static friction for the surface between the two blocks. When the force applied is at a maximum, the frictional force between the two blocks must also be a maximum. Since  $F_t = 12 \text{ N}$  of force has to be applied to the top block for slipping to take place, using  $F_t = f_{s,\max} = \mu_s F_{N,t} = \mu_s m_t g$ , we have

$$\mu_s = \frac{F_t}{m_t g} = \frac{12 \text{ N}}{(4.0 \text{ kg})(9.8 \text{ m/s}^2)} = 0.31.$$

## Solut.cont.

Using the same reasoning, for the two masses to move together, the maximum applied force would be

$$F = \mu_s(m_t + m_b)g$$

**ANALYZE** (a) Substituting the value of  $\mu_s$  found above, the maximum horizontal force has a magnitude

$$F = \mu_s(m_t + m_b)g = (0.31)(4.0\text{ kg} + 5.0\text{ kg})(9.8\text{ m/s}^2) = 27\text{ N}$$

(b) The maximum acceleration is

$$a_{\max} = \frac{F}{m_t + m_b} = \mu_s g = (0.31)(9.8\text{ m/s}^2) = 3.0\text{ m/s}^2.$$

# Dynamics of Circular motion & NEWTON's 3rd law



Newton third law



Uniform and non-uniform circular motion



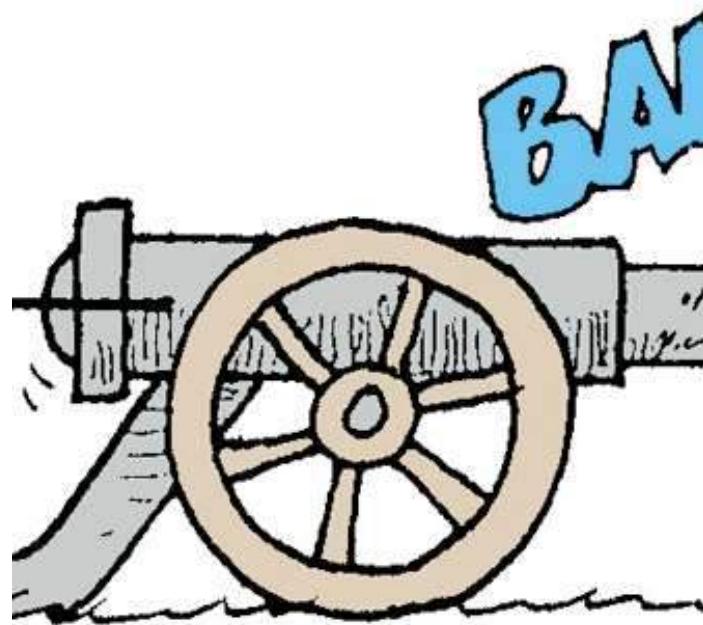
Centripetal acceleration



Problem solving with Newton's 2nd Law for circular motion

# Newton's 3<sup>rd</sup> Law of Motion

- Whenever one object exerts a force on a second object, the second object exerts an equal and oppositely directed force on the first.



# Action and Reaction

- According to Newton's third law (action and reaction are equal and opposite), the force that **the ball exerts on the racket is equal and opposite to that** which the racket exerts on the ball. Moreover, a second balanced action and reaction acts between player and racket.

# Problem

••55 SSM ILW WWW Two blocks are in contact on a frictionless table. A horizontal force is applied to the larger block, as shown in Fig. 5-50. (a) If  $m_1 = 2.3 \text{ kg}$ ,  $m_2 = 1.2 \text{ kg}$ , and  $F = 3.2 \text{ N}$ , find the magnitude of the force between the two blocks. (b) Show that if a force of the same magnitude  $F$  is applied to the smaller block but in the opposite direction, the magnitude of the force between the blocks is  $2.1 \text{ N}$ , which is not the same value calculated in (a). (c) Explain the difference.

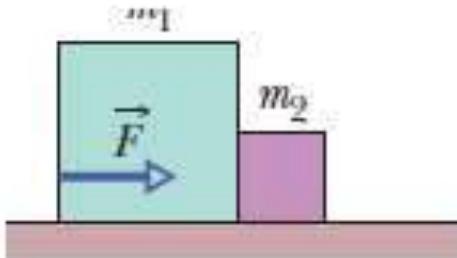
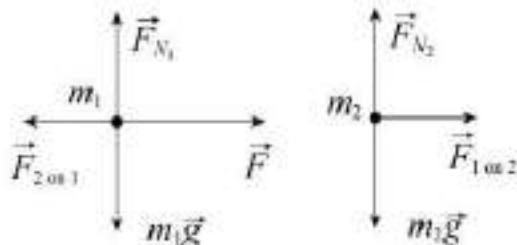


Figure 5-50  
Problem 55.

# Solution

**EXPRESS** The free-body diagrams for the two blocks in (a) are shown below.  $\vec{F}$  is the applied force and  $\vec{F}_{\text{int2}}$  is the force exerted by block 1 on block 2. We note that  $\vec{F}$  is applied directly to block 1 and that block 2 exerts a force  $\vec{F}_{\text{int1}} = -\vec{F}_{\text{int2}}$  on block 1 (taking Newton's third law into account).



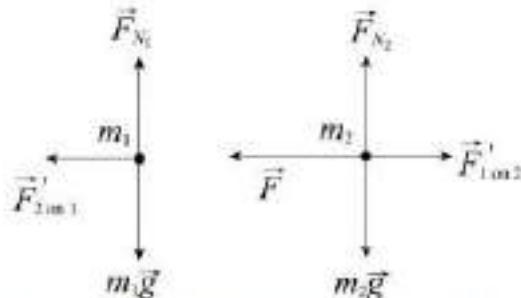
Newton's second law for block 1 is  $F - F_{\text{int1}} = m_1 a$ , where  $a$  is the acceleration. The second law for block 2 is  $F_{\text{int2}} - m_2 a$ . Since the blocks move together they have the same acceleration and the same symbol is used in both equations.

**ANALYZE** (a) From the second equation we obtain the expression  $a = F_{\text{int2}} / m_2$ , which we substitute into the first equation to get  $F - F_{\text{int1}} = m_1 F_{\text{int2}} / m_2$ . Since  $F_{\text{int1}} = F_{\text{int2}}$  (same magnitude for third-law force pair), we obtain

$$F_{\text{int1}} = F_{\text{int2}} = \frac{m_2}{m_1 + m_2} F = \frac{1.2 \text{ kg}}{2.3 \text{ kg} + 1.2 \text{ kg}} (3.2 \text{ N}) = 1.1 \text{ N}.$$

(b) If  $\vec{F}$  is applied to block 2 instead of block 1 (and in the opposite direction), the free-body diagrams would look like the following:

## Sol.contin..



The corresponding force of contact between the blocks would be

$$F'_{\text{int}1} = F'_{\text{int}2} = \frac{m_1}{m_1 + m_2} F = \frac{2.3 \text{ kg}}{2.3 \text{ kg} + 1.2 \text{ kg}} (3.2 \text{ N}) = 2.1 \text{ N}.$$

(c) We note that the acceleration of the blocks is the same in the two cases. In part (a), the force  $F_{\text{tot}1}$  is the only horizontal force on the block of mass  $m_1$  and in part (b)  $F'_{\text{tot}2}$  is the only horizontal force on the block with  $m_1 > m_2$ . Since  $F_{\text{tot}1} = m_1\alpha$  in part (a) and  $F'_{\text{tot}2} = m_2\alpha$  in part (b), then for the accelerations to be the same,  $F'_{\text{tot}1} > F'_{\text{tot}2}$ , i.e., force between blocks must be larger in part (b).

**LEARN** This problem demonstrates that when two blocks are being accelerated together under an external force, the contact force between the two blocks is greater if the smaller mass is pushing against the bigger one, as in part (b). In the special case where the two masses are equal,  $m_1 = m_2 = m$ ,  $F'_{\text{tot}1} = F'_{\text{tot}2} = F/2$ .

# DYNAMICS OF UCM

Problems



## Effect of force components

Components of force parallel and perpendicular to velocity have different effects.

$$d\vec{v} = \vec{a}dt = \frac{\vec{F}}{m}dt$$

$F_{||}$  causes change in magnitude of velocity vector (speed)

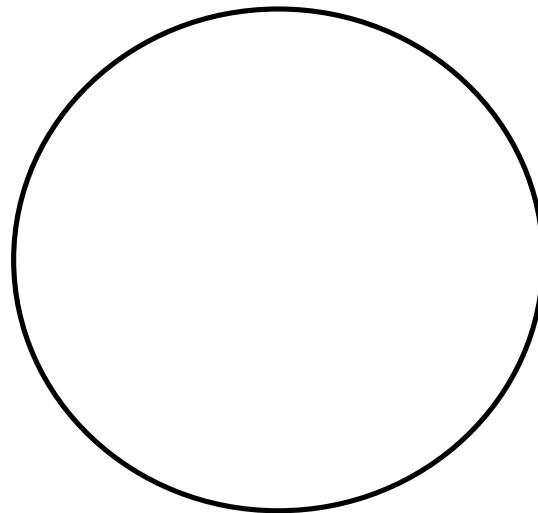
$F_{\perp}$  causes change in direction

# Uniform circular motion

Motion in a circle with constant speed

**Caution:**

velocity is a **vector** and has magnitude and direction  
⇒ constant *speed* does not mean constant *velocity*. There will be acceleration!



$$a_c = \frac{v^2}{R}$$

**Centripetal acceleration**

Directed **towards center** of the circle

# Non-uniform circular motion

Motion in a circle with non- constant speed

Centripetal acceleration

Towards the center  
changes direction

$$a_c = \frac{v^2}{R}$$

$v$  is speed at that instant, does not have to be constant

Tangential acceleration

tangential to circle,  
changes speed

$$a_{tan} = \frac{dv}{dt}$$

## Forces create centripetal acceleration

The acceleration towards the center must be created by a force that is acting towards the center.

$$\Sigma F_r = ma_c = m \frac{v^2}{R}$$

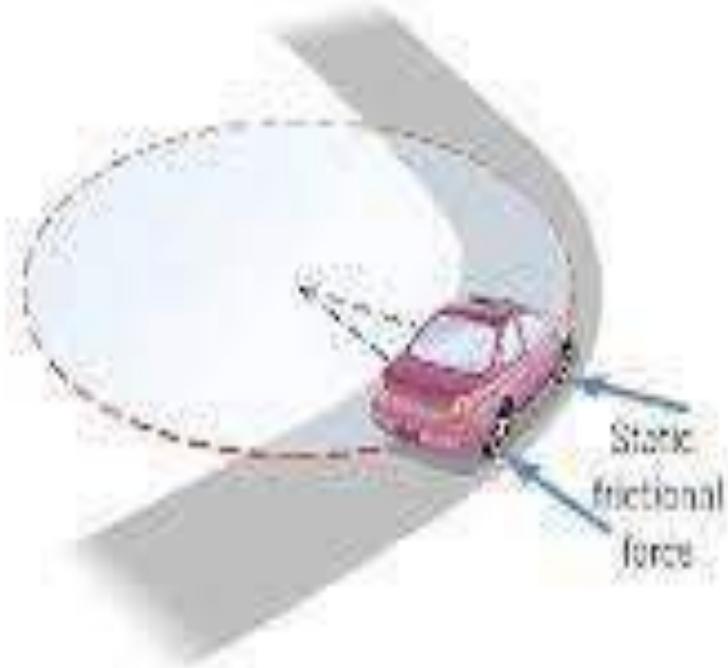
# Car in flat curve



# **Car in flat curve**

## Unbanked curve

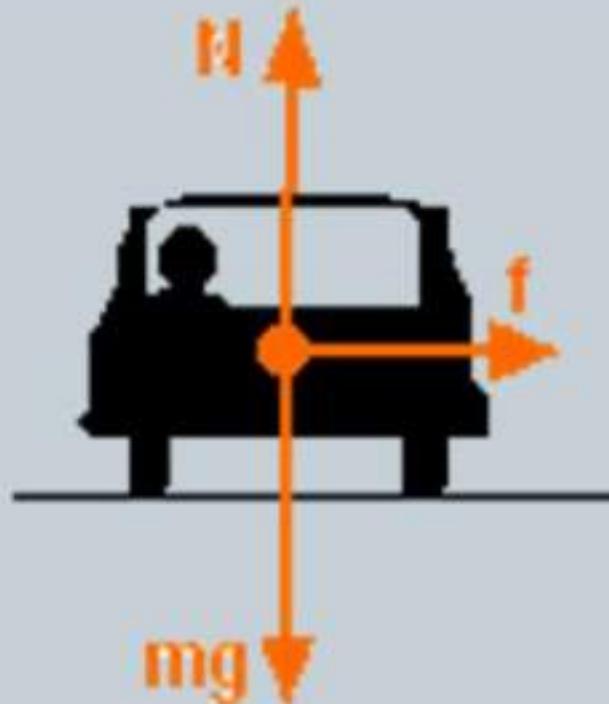
On an unbanked curve, the static frictional force provides the centripetal force.



## Unbanked Curve

- The car to the right is going around a circular curve of radius  $r$  that is flat; that is, unbanked. The static force of friction is acting towards the center of the circular path.

$$f_s = \frac{m v^2}{r}$$



## Car in flat curve worked out

$$\Sigma F_x = ma_x$$
$$f_s = m \frac{v^2}{R}$$

$$\Sigma F_y = ma_y$$
$$N + (-W) = 0$$
$$N = mg$$

Maximum speed if:  $f_s = f_{s\ max} = \mu N = \mu mg$

$$v_{max} = \sqrt{\mu g R}$$

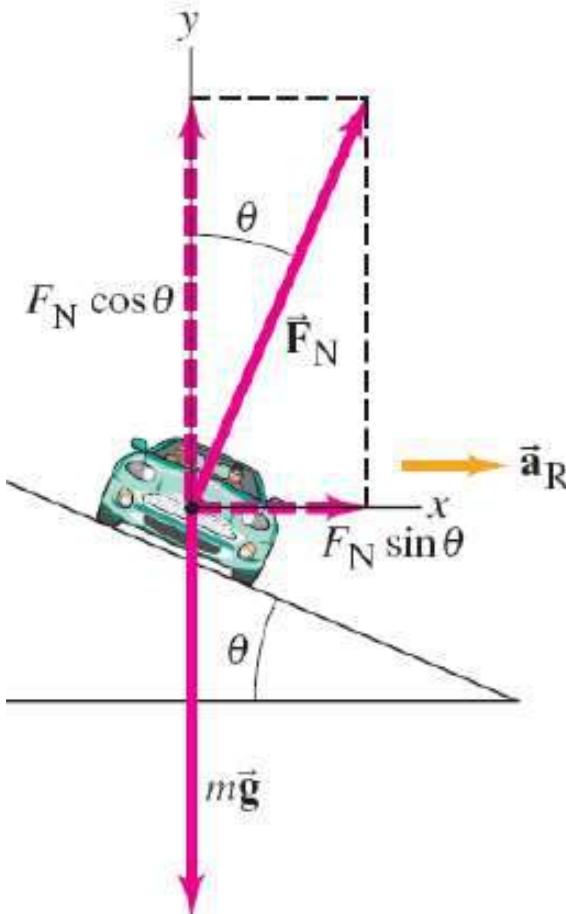
## Car in banked curve

Banking makes it possible to go around the curve even when the road is frictionless.

# Banked curves

- A banked curve is **a curve that has its surface at angle with respect to the ground on which the curve is positioned.** The reason for banking curves is to decrease the moving object's reliance on the force of friction.
- Banking the curve **can help keep cars from skidding.** When the curve is banked, the centripetal force can be supplied by the horizontal component of the normal force.

# Highway Curves: Banked and Unbanked



**Banking** the curve can help keep cars from skidding. When the curve is banked, the centripetal force can be supplied by the horizontal component of the **normal force**. In fact, for every banked curve, there is one speed at which the entire centripetal force is supplied by the horizontal component of the **normal force**, and no friction is required.

## Example: Banking angle.

- (a) For a car traveling with speed  $v$  around a curve of radius  $r$ , determine a formula for the angle at which a road should be banked so that no friction is required.  
(b) What is this angle for an expressway off-ramp curve of radius 50 m at a design speed of 50 km/h?

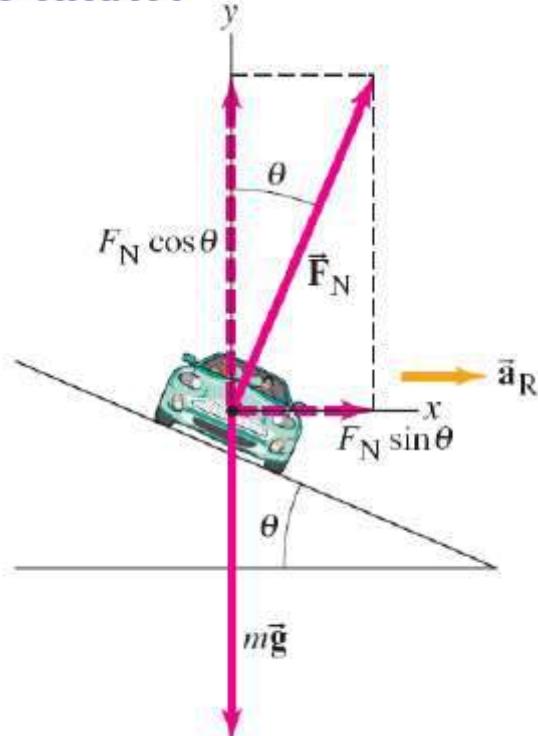
$$(a) F_N \sin \theta = m \frac{v^2}{R}$$

$$F_N \cos \theta - mg = 0$$

$$\tan \theta = \frac{v^2}{Rg}$$

$$(b) R = 50\text{m}, v = 50\text{km/h} = 13.89\text{m/s}$$

$$\theta = \tan^{-1} \frac{v^2}{Rg} = \tan^{-1} \frac{13.89^2}{50g} = 22^\circ$$



# **Car in banked curve: design speed**

# Car in banked curve with friction

Going **slower** than design speed

Find minimum  
speed in HW

# Car in banked curve with friction

Going **faster** than design speed

Find maximum  
speed in HW

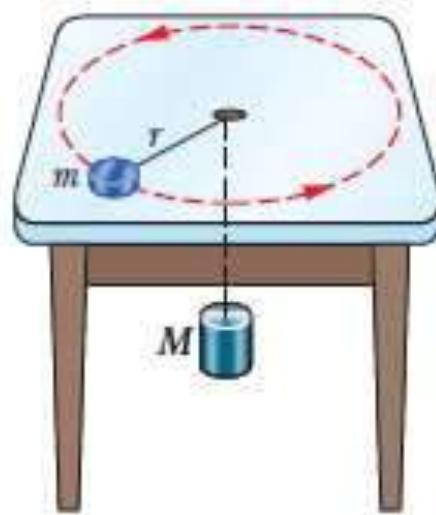
# Safe velocity on banked road

- Safe Velocity on Banked Road:
- The speed will be maximum when  $\tan \theta = 1$  i.e.  $\theta = 45^\circ$ . It means the vehicle can be driven with maximum safe speed only when the angle of banking =  $45^\circ$ .

# For reference

- [http://www.batesville.k12.in.us/physics/phynet/mechanics/circular%20motion/banked\\_with\\_friction.htm](http://www.batesville.k12.in.us/physics/phynet/mechanics/circular%20motion/banked_with_friction.htm)

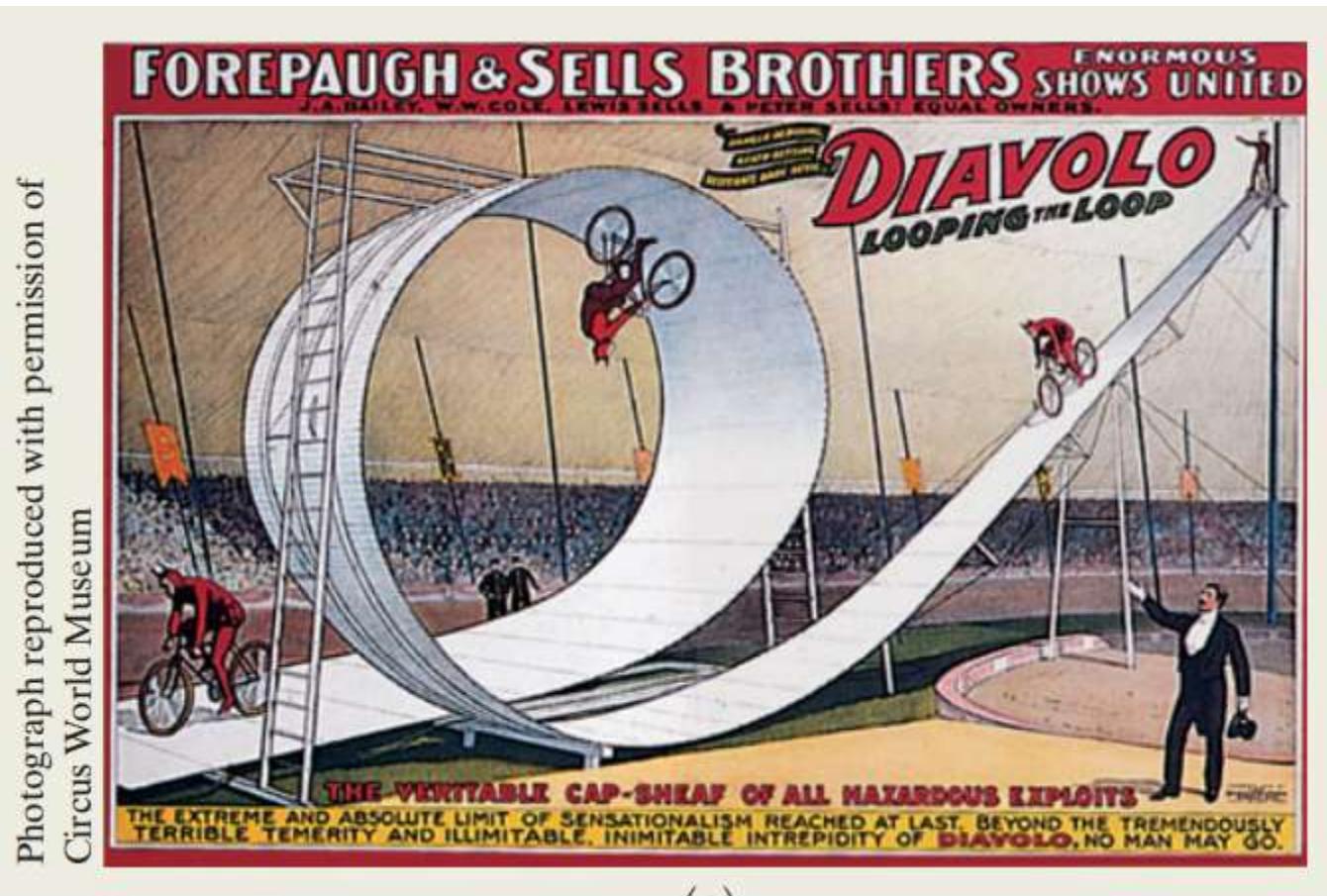
- \*\*57**  A puck of mass  $m = 1.50 \text{ kg}$  slides in a circle of radius  $r = 20.0 \text{ cm}$  on a frictionless table while attached to a hanging cylinder of mass  $M = 2.50 \text{ kg}$  by means of a cord that extends through a hole in the table (Fig. 6-43). What speed keeps the cylinder at rest?



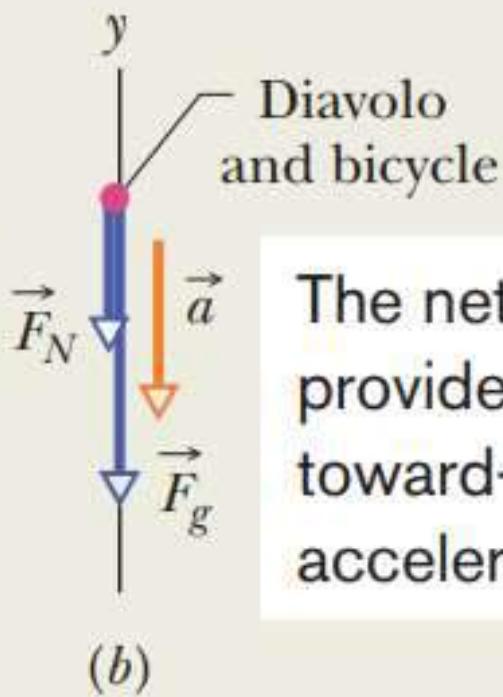
57. For the puck to remain at rest the magnitude of the tension force  $T$  of the cord must equal the gravitational force  $Mg$  on the cylinder. The tension force supplies the centripetal force that keeps the puck in its circular orbit, so  $T = mv^2/r$ . Thus  $Mg = mv^2/r$ . We solve for the speed:

$$v = \sqrt{\frac{Mgr}{m}} = \sqrt{\frac{(2.50 \text{ kg})(9.80 \text{ m/s}^2)(0.200 \text{ m})}{1.50 \text{ kg}}} = 1.81 \text{ m/s.}$$

Photograph reproduced with permission of  
Circus World Museum

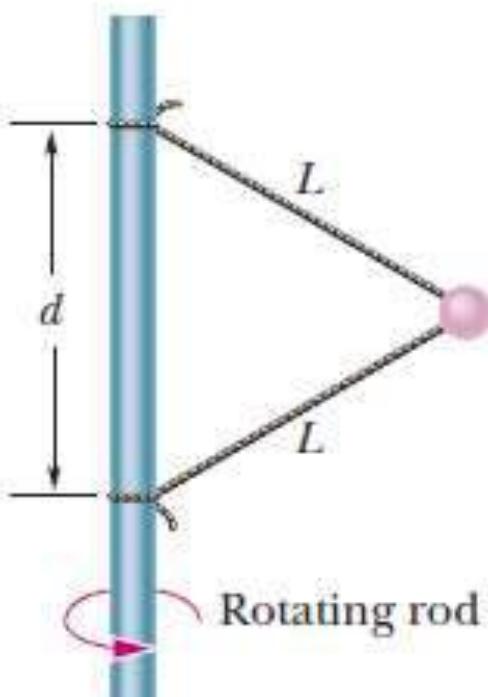


The normal force  
is from the  
overhead loop.



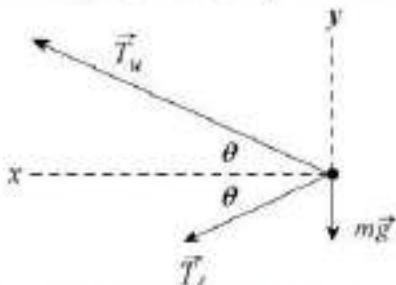
The net force  
provides the  
toward-the-center  
acceleration.

- ...59 SSM ILW** In Fig. 6-45, a 1.34 kg ball is connected by means of two massless strings, each of length  $L = 1.70\text{ m}$ , to a vertical, rotating rod. The strings are tied to the rod with separation  $d = 1.70\text{ m}$  and are taut. The tension in the upper string is 35 N. What are the (a) tension in the lower string, (b) magnitude of the net force  $\vec{F}_{\text{net}}$  on the ball, and (c) speed of the ball? (d) What is the direction of  $\vec{F}_{\text{net}}$ ?



59. **THINK** As illustrated in Fig. 6-45, our system consists of a ball connected by two strings to a rotating rod. The tensions in the strings provide the source of centripetal force.

**EXPRESS** The free-body diagram for the ball is shown below.  $\vec{T}_u$  is the tension exerted by the upper string on the ball,  $\vec{T}_l$  is the tension in the lower string, and  $m$  is the mass of the ball. Note that the tension in the upper string is greater than the tension in the lower string. It must balance the downward pull of gravity and the force of the lower string.



We take the  $+x$  direction to be leftward (toward the center of the circular orbit) and  $+y$  upward. Since the magnitude of the acceleration is  $a = v^2/R$ , the  $x$  component of Newton's second law is

$$T_u \cos \theta + T_l \cos \theta = \frac{mv^2}{R},$$

where  $v$  is the speed of the ball and  $R$  is the radius of its orbit. The  $y$  component is

$$T_u \sin \theta - T_l \sin \theta - mg = 0.$$

The second equation gives the tension in the lower string:  $T_l = T_u - mg / \sin \theta$ .

**ANALYZE** (a) Since the triangle is equilateral, the angle is  $\theta = 30.0^\circ$ . Thus

$$T_l = T_u - \frac{mg}{\sin \theta} = 35.0 \text{ N} - \frac{(1.34 \text{ kg})(9.80 \text{ m/s}^2)}{\sin 30.0^\circ} = 8.74 \text{ N}.$$

(b) The net force in the  $y$ -direction is zero. In the  $x$ -direction, the net force has magnitude

$$F_{\text{net,str}} = (T_u + T_\ell) \cos \theta = (35.0 \text{ N} + 8.74 \text{ N}) \cos 30.0^\circ = 37.9 \text{ N}.$$

(c) The radius of the path is

$$R = L \cos \theta = (1.70 \text{ m}) \cos 30^\circ = 1.47 \text{ m}.$$

Using  $F_{\text{net,str}} = mv^2/R$ , we find the speed of the ball to be

$$v = \sqrt{\frac{RF_{\text{net,str}}}{m}} = \sqrt{\frac{(1.47 \text{ m})(37.9 \text{ N})}{1.34 \text{ kg}}} = 6.45 \text{ m/s}.$$

(d) The direction of  $\vec{F}_{\text{net,str}}$  is leftward (“radially inward”).