Kinematics: Driving Realistic Motion in Computer Science

Kinematics bridges physics and computation. It creates dynamic virtual worlds. It is essential for animation, robotics, and simulation. This presentation explores how kinematics unlocks natural movement in digital spaces.

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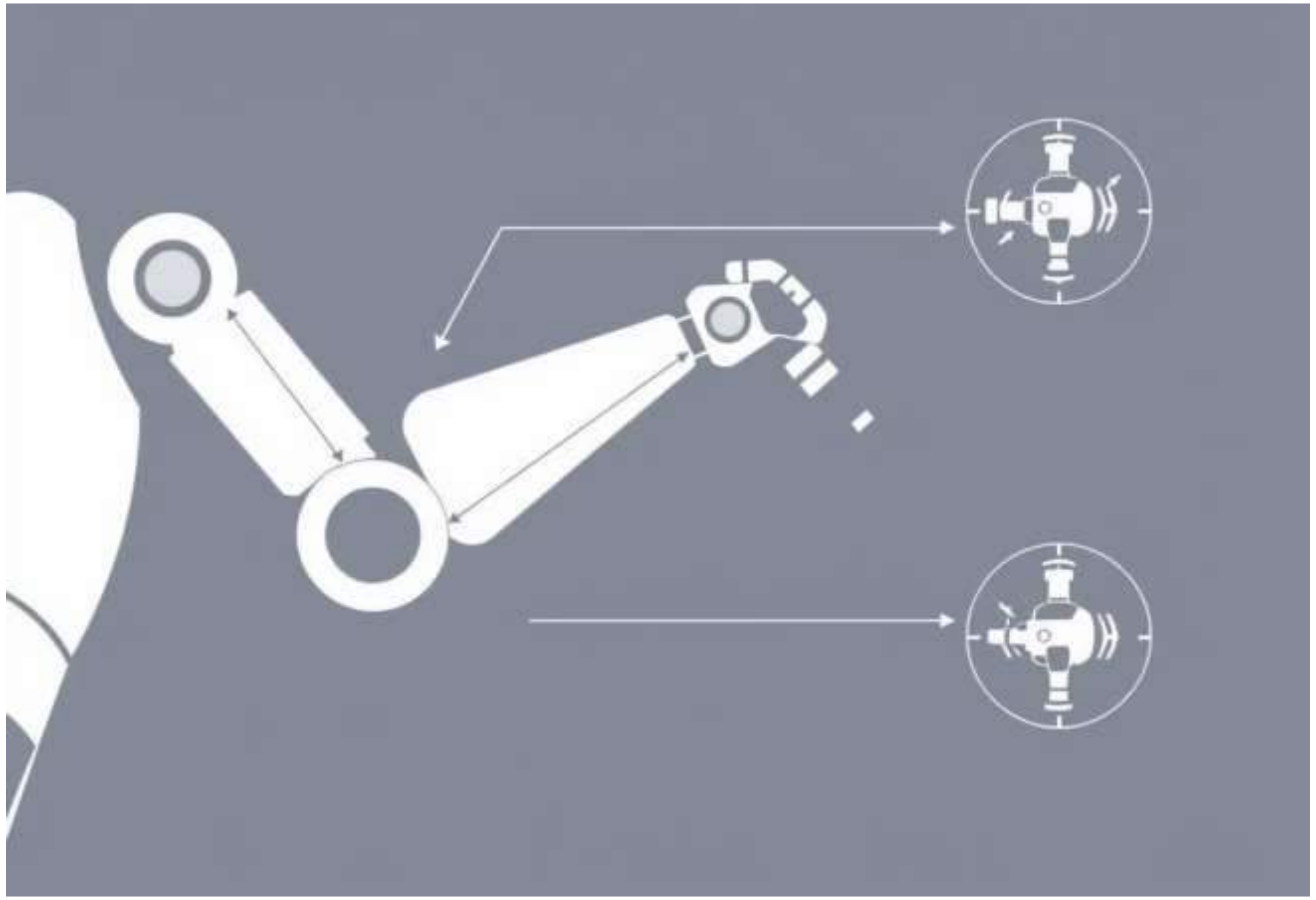
What is Kinematics?

Kinematics is the study of motion. It doesn't consider forces or mass. It focuses on position, velocity, and acceleration. This applies to points and rigid bodies.



Application of Linear Motion

- **In Physics:**

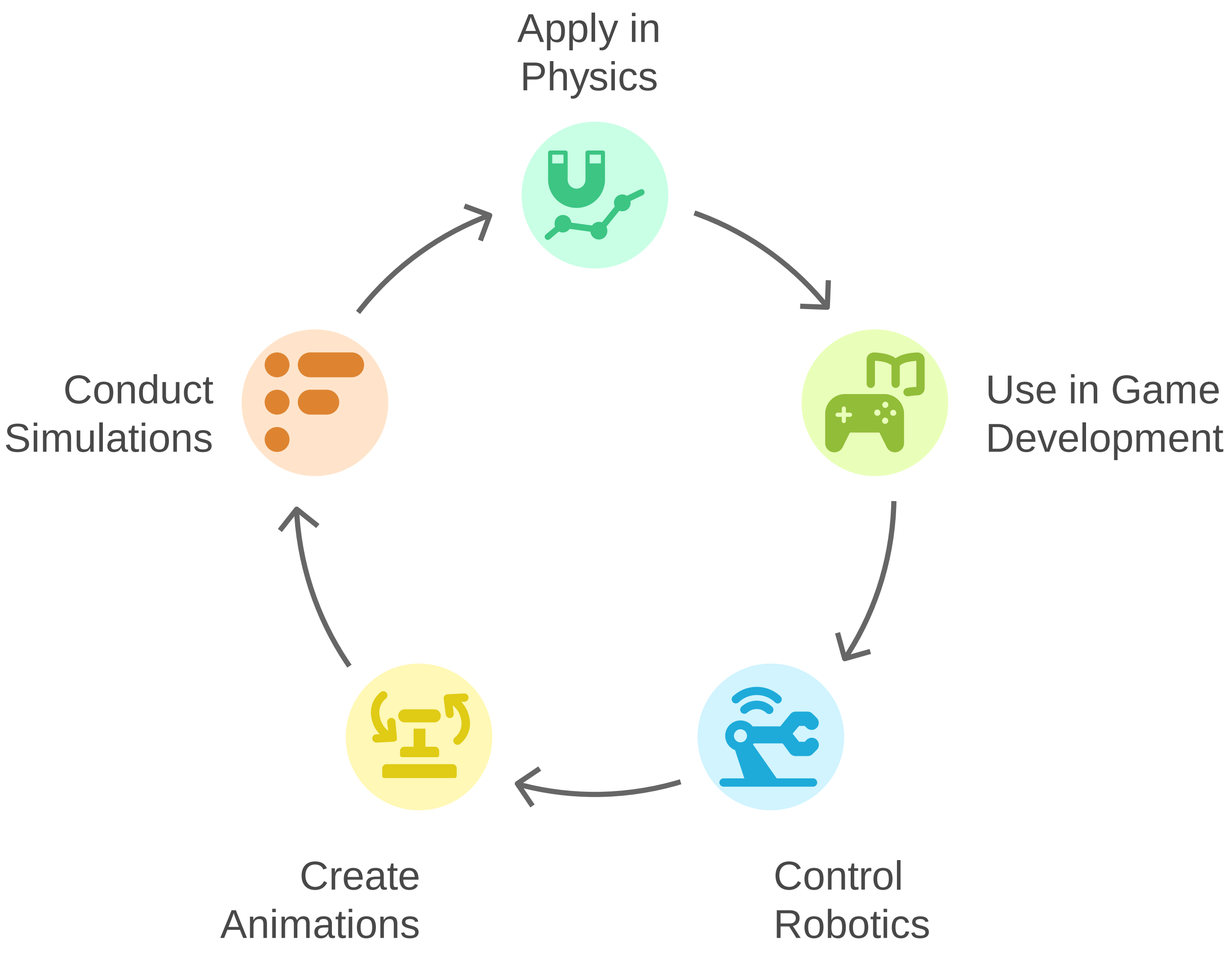
1. **Kinematics**: Describes the motion of objects with constant velocity or acceleration, such as cars moving on a straight road.
2. **Dynamics**: Analyzes forces and their effects on the motion of objects, such as calculating the forces acting on a projectile.
3. **Engineering**: Used in designing mechanisms and structures, like conveyor belts or sliding doors, where linear motion is essential.
4. **Astronomy**: Helps in understanding the motion of celestial bodies in straight-line trajectories, such as spacecraft traveling in space.

Application of Linear Motion

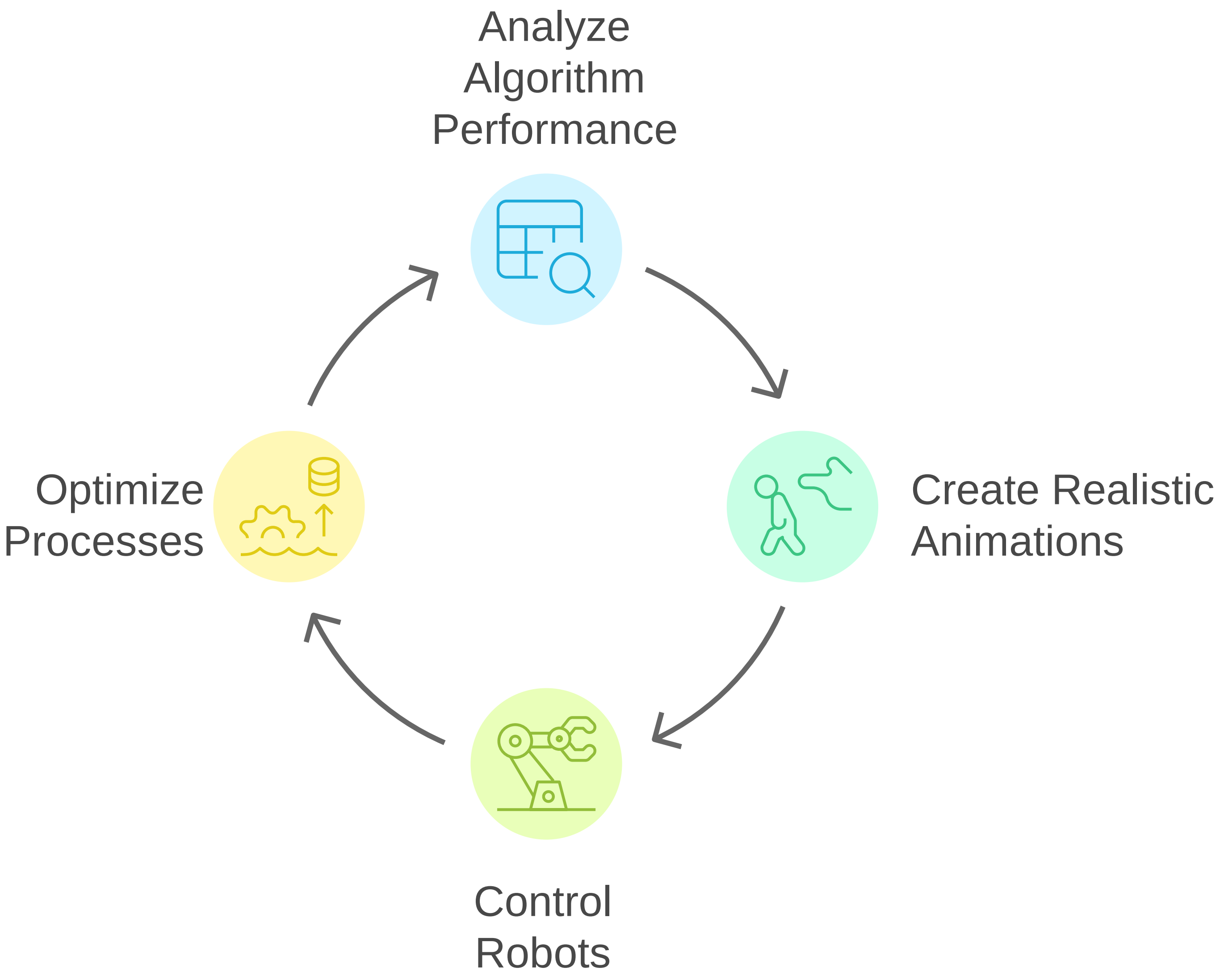
- **In Computer Science:**

1. **Animation**: Simulates linear motion in computer graphics and video games, creating realistic movement of characters and objects.
2. **Robotics**: Controls the linear movement of robotic arms and autonomous vehicles, allowing precise manipulation and navigation.
3. **Simulations**: Models physical systems and processes that involve linear motion for applications like virtual reality or scientific simulations.
4. **Algorithms**: Implements motion-related algorithms in simulations and data analysis, such as tracking moving objects in video surveillance.

Cycle of Linear Motion Applications



Motion Principles in Computer Science



Examples in daily life

- **Driving a Car**: When a car moves in a straight line on a road, it exhibits linear motion.
- **Elevators**: Elevators move vertically up and down, demonstrating linear motion.
- **Sliding Doors**: Automatic sliding doors move back and forth along a straight path.
- **Conveyor Belts**: Used in factories and airports, conveyor belts move items in a straight line.
- **Escalators**: Escalators in malls or subway stations move passengers up and down in a linear motion.
- **Walking**: When you walk in a straight line, your movement is a form of linear motion.

Motion

- **Motion** is the change in an object's position over time.
- **Types:**

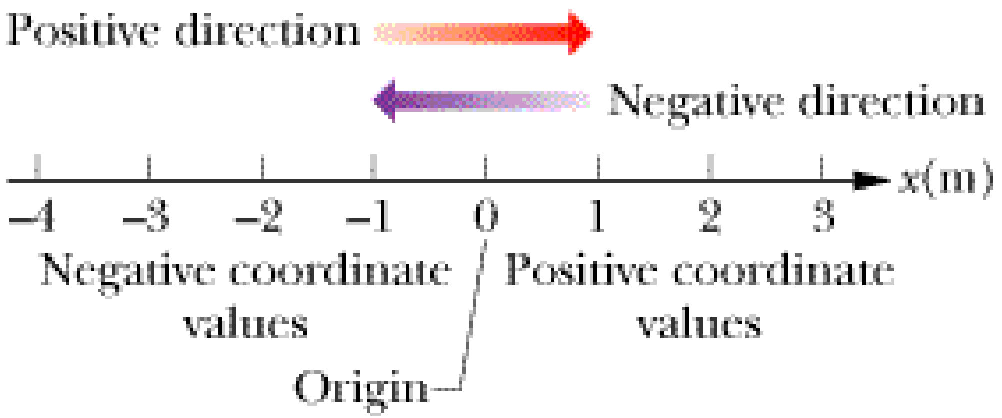
1. **Linear Motion**: Movement along a straight line (e.g., a car on a road).
2. **Rotational Motion**: Movement around an axis (e.g., a spinning wheel).
3. **Translational Motion**: Movement where all parts of an object move uniformly (e.g., a person walking forward).
4. **Periodic Motion**: Repeated movement at regular intervals (e.g., a swinging pendulum).
5. **Random Motion**: Erratic movement without a clear pattern (e.g., gas particles).
6. **Uniform Motion**: Constant speed in a straight line (e.g., a train on a track).
7. **Non-uniform Motion**: Varying speed or direction (e.g., a car accelerating).

CHAPTER 2
MOTION ALONG A STRAIGHT LINE

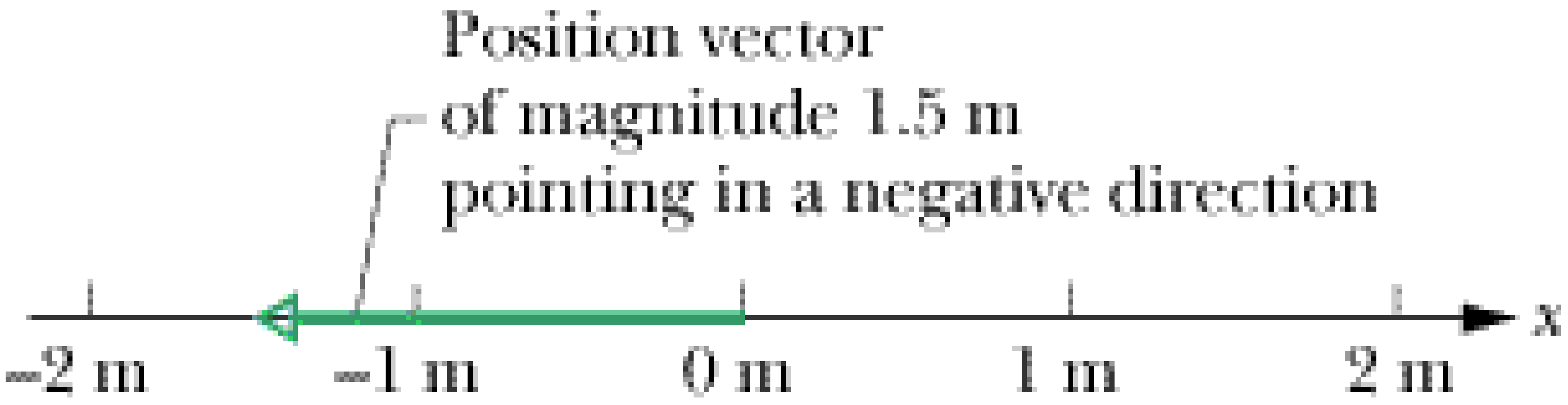
Defining a Coordinate System

One-dimensional **coordinate system** consists of:

- a *point of reference known as the origin (or zero point)*,
- a line that passes through the chosen origin called a *coordinate axis*, one direction along the coordinate axis, chosen as positive and the other direction as negative, and the units we use to measure a quantity



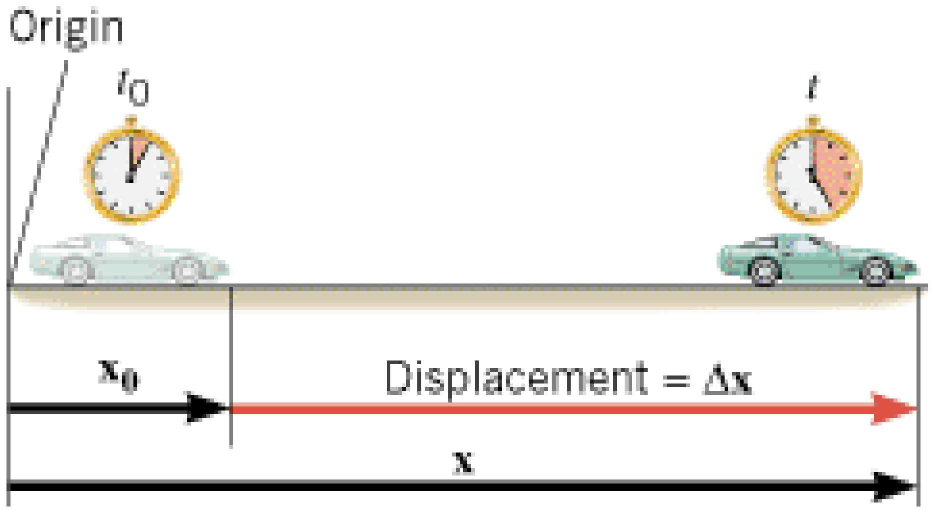
One-dimensional position vector



- The **magnitude** of the position vector is a scalar that denotes the distance between the object and the origin.

- The **direction** of the position vector is positive when the object is located to the positive side of axis from the origin and negative when the object is located to the negative side of axis from the origin.

Displacement

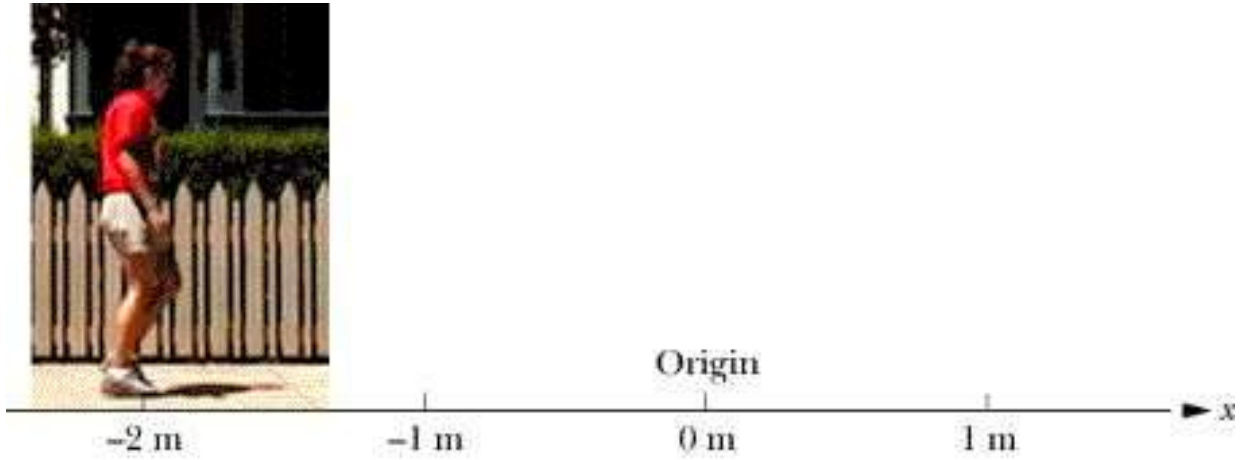


$$\Delta \vec{r} \equiv \vec{x}_2 - \vec{x}_1 = \Delta x\,\hat{i}\ (\text{displacement vector}),$$

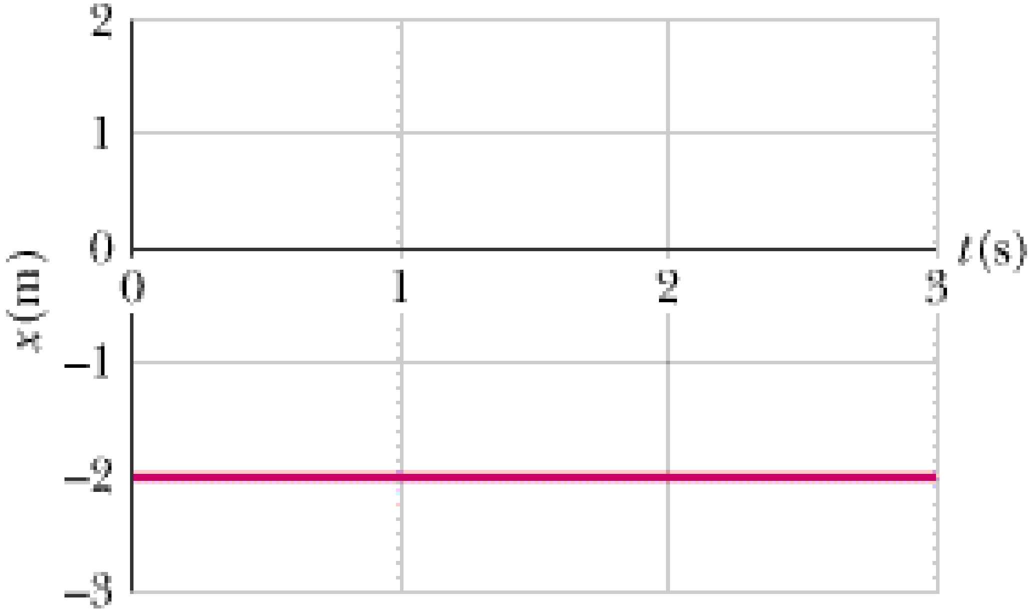
- **DISPLACEMENT** is defined as the change of an object's position that occurs during a period of time.

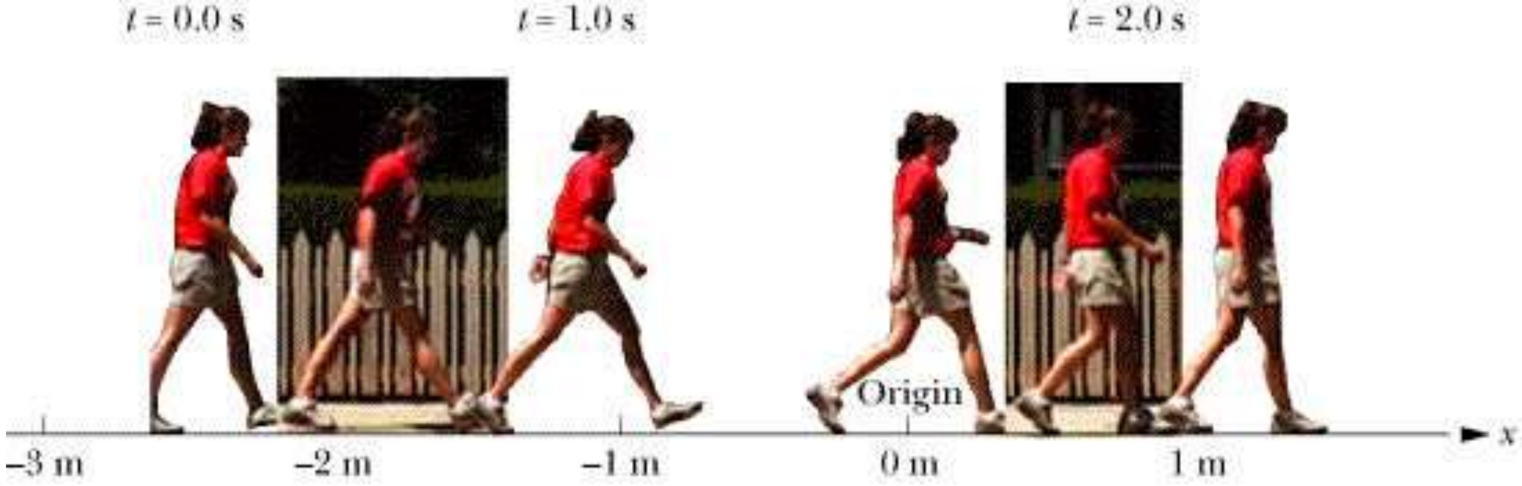
- The displacement is a vector that points from an object's initial position to its final position and has a magnitude that equals the shortest distance between the two positions.
- ***SI Unit of Displacement:*** meter (m)

Velocity and Speed

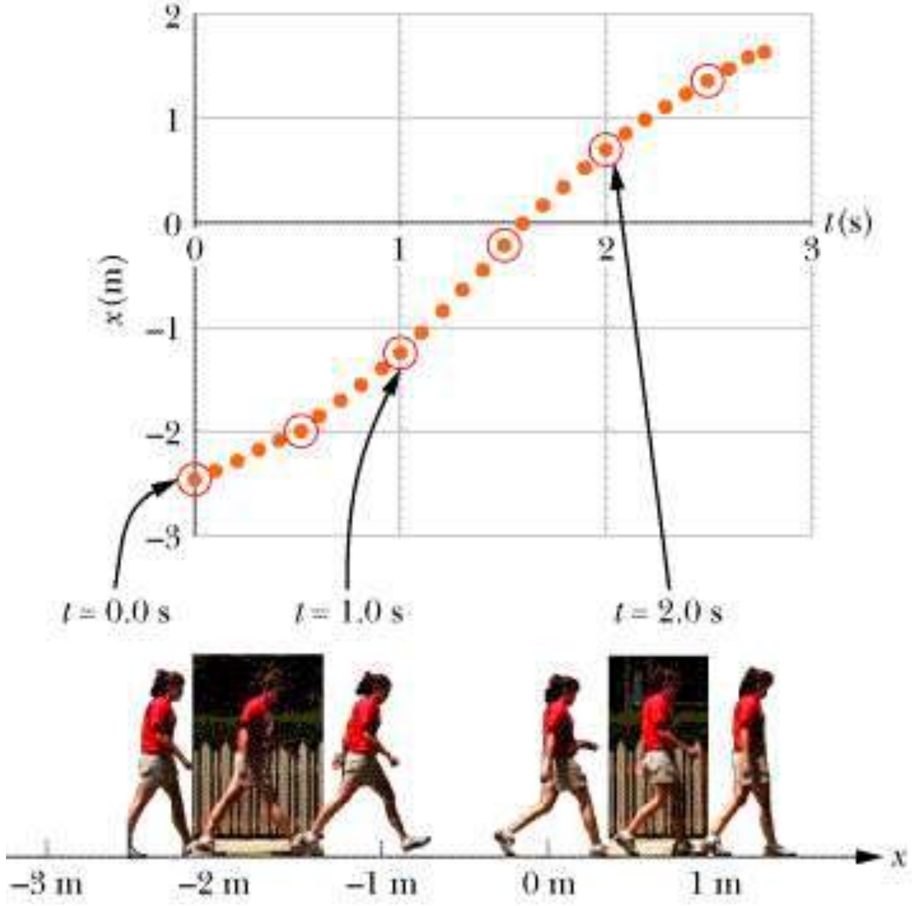


A student standing still with the back of her belt at a horizontal distance of 2.00 m to the left of a spot of the sidewalk designated as the origin.





A student starting to walk slowly. The horizontal position of the back of her belt starts at a horizontal distance of 2.47 m to the left of a spot designated as the origin. She is speeding up for a few seconds and then slowing down.



Average Velocity

$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Elapsed time}}$$

$$\langle v \rangle \equiv \frac{\Delta \vec{x}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{i} = \frac{x_2 - x_1}{t_2 - t_1}\hat{i}$$

- x_2 and x_1 are components of the position vectors at the final and initial times, and angle brackets denotes the average of a quantity.

- ***SI Unit of Average Velocity:*** meter per second (m/s)

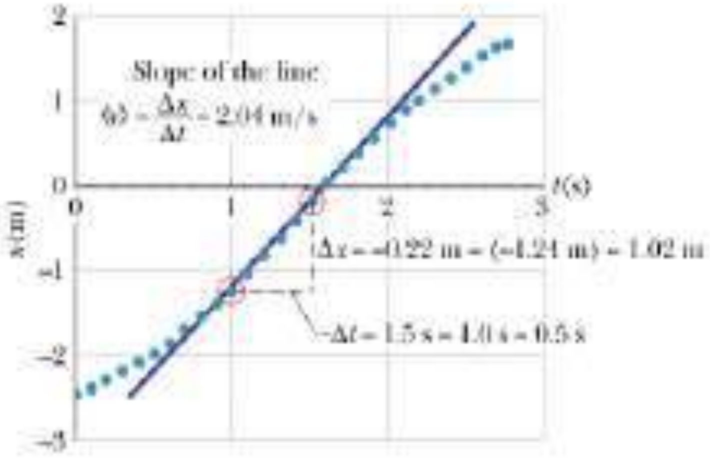
Average Speed

Average speed is defined as:

$$\langle s \rangle \equiv \frac{\text{total distance}}{\Delta t} \quad (\text{definition of average speed}).$$

Instantaneous Velocity and Speed

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{x}}{\Delta t} = \frac{d\vec{x}}{dt} = \frac{dx}{dt}\,\hat{i}$$

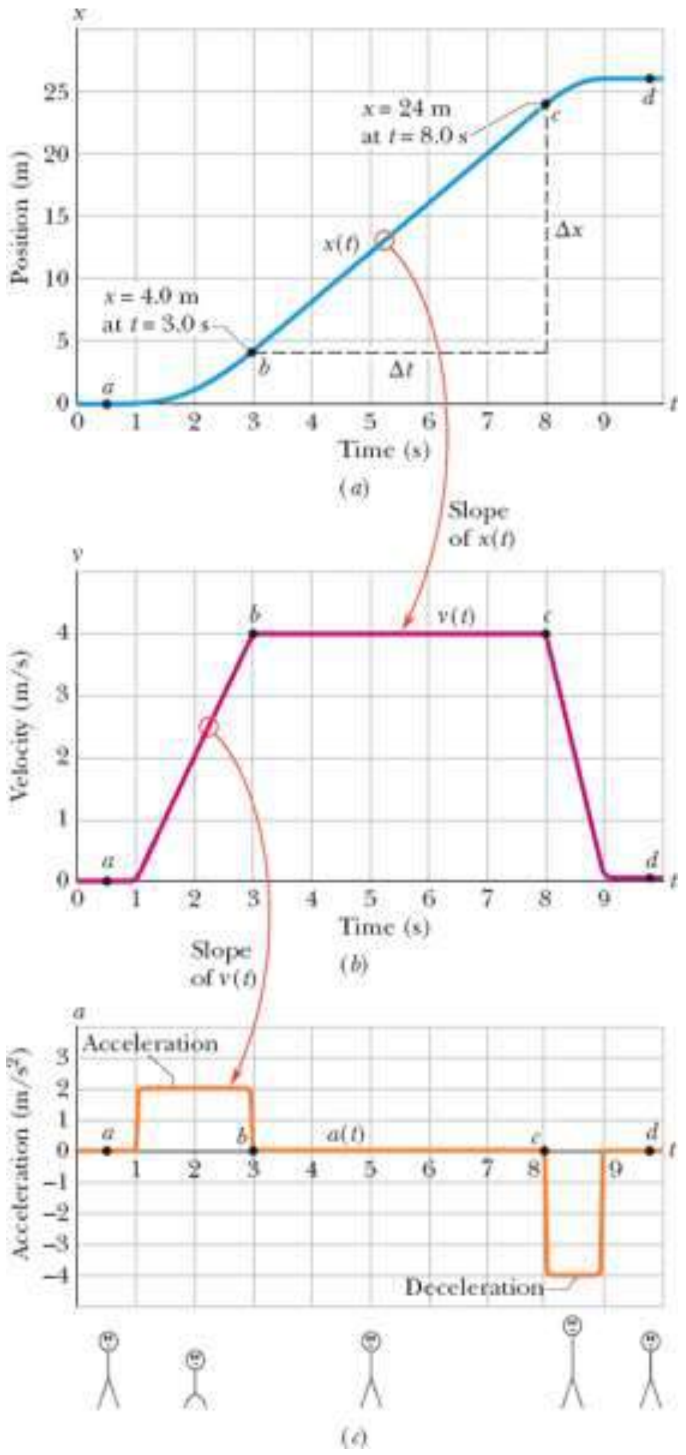


- The instantaneous velocity of an object can be obtained by taking the slope of a graph of the position component vs. time at the point associated with that moment in time

- The instantaneous velocity can be obtained by taking a derivative with respect to time of the object's position.

- **Instantaneous speed**, which is typically called simply **speed,** is just the magnitude of the instantaneous velocity vector,

How to Describe Change of Velocity？



Definition of Acceleration

$$\text{Average acceleration} = \frac{\text{Change in velocity}}{\text{Elapsed time}}$$

$$\langle \vec{a} \rangle = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta \vec{v}}{\Delta t}$$

SI Unit of Average Acceleration: meter per second squared (m/s^2)

Instantaneous acceleration:

$$\vec{a} \equiv \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} \quad \text{(definition of 1D instantaneous acceleration)}$$

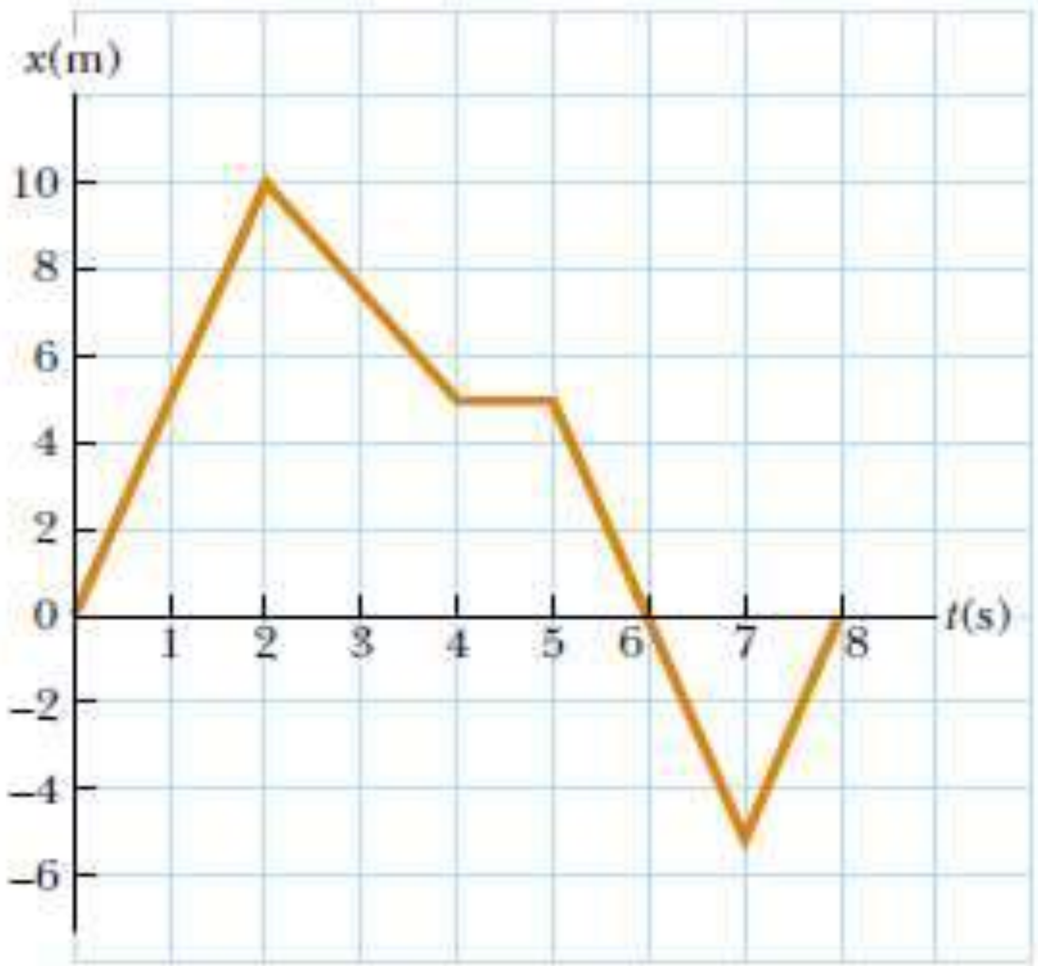
$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\frac{d\vec{x}}{dt}\right) = \frac{d^2\vec{x}}{dt^2}$$

MOTION IN 1-D

PROBLEMS SOLVING

Problem 1

]The position versus time for a certain particle moving along the x axis is shown in Figure P2.3. Find the average velocity in the time intervals (a) 0 to 2 s, (b) 0 to 4 s, (c) 2 s to 4 s, (d) 4 s to 7 s, (e) 0 to 8 s.



solution

(a) $$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{10\text{ m}}{2\text{ s}} = \boxed{5\text{ m/s}}$$

(b) $$\bar{v} = \frac{5\text{ m}}{4\text{ s}} = \boxed{1.2\text{ m/s}}$$

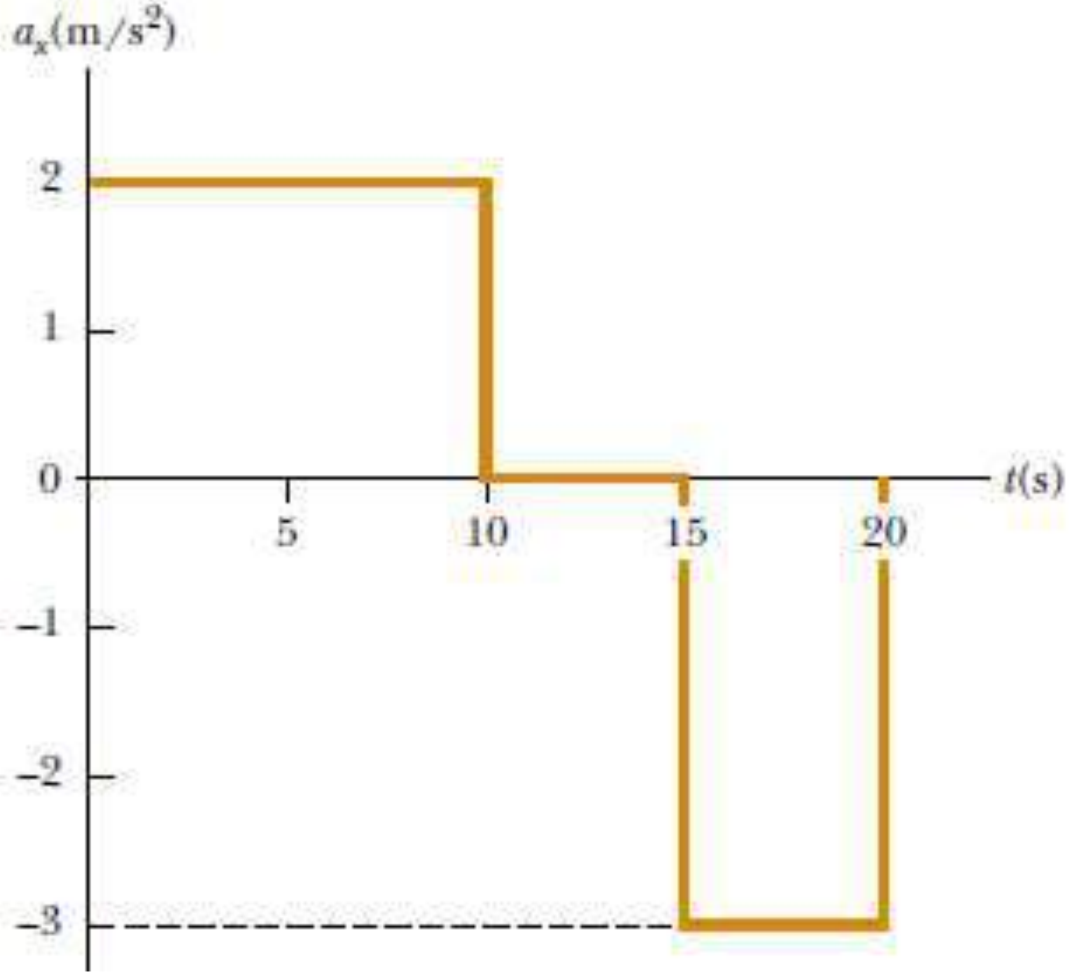
(c) $$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{5\text{ m} - 10\text{ m}}{4\text{ s} - 2\text{ s}} = \boxed{-2.5\text{ m/s}}$$

(d) $$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{-5\text{ m} - 5\text{ m}}{7\text{ s} - 4\text{ s}} = \boxed{-3.3\text{ m/s}}$$

(e) $$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{0 - 0}{8 - 0} = \boxed{0\text{ m/s}}$$

Problem 2

A particle starts from rest and accelerates as shown in Figure P2.12. Determine (a) the particle's speed at t = 10.0 s and at t = 20.0 s, and (b) the distance traveled in the first 20.0 s.



(a) Acceleration is constant over the first ten seconds, so at the end,

$$v_f = v_i + at = 0 + \left(2.00\ \text{m/s}^2\right)(10.0\ \text{s}) = \boxed{20.0\ \text{m/s}}.$$

Then $a = 0$ so v is constant from $t = 10.0\ \text{s}$ to $t = 15.0\ \text{s}$. And over the last five seconds the velocity changes to

$$v_f = v_i + at = 20.0\ \text{m/s} + \left(3.00\ \text{m/s}^2\right)(5.00\ \text{s}) = \boxed{5.00\ \text{m/s}}.$$

(b) In the first ten seconds,

$$x_f = x_i + v_i t + \frac{1}{2}at^2 = 0 + 0 + \frac{1}{2}\left(2.00\ \text{m/s}^2\right)(10.0\ \text{s})^2 = 100\ \text{m}.$$

Over the next five seconds the position changes to

$$x_f = x_i + v_i t + \frac{1}{2}at^2 = 100\ \text{m} + (20.0\ \text{m/s})(5.00\ \text{s}) + 0 = 200\ \text{m}.$$

And at $t = 20.0\ \text{s}$,

$$x_f = x_i + v_i t + \frac{1}{2}at^2 = 200\ \text{m} + (20.0\ \text{m/s})(5.00\ \text{s}) + \frac{1}{2}\left(-3.00\ \text{m/s}^2\right)(5.00\ \text{s})^2 = \boxed{262\ \text{m}}.$$

Equations of Motion with Constant Acceleration

Table 2-1 Equations for Motion with Constant Acceleration[a]

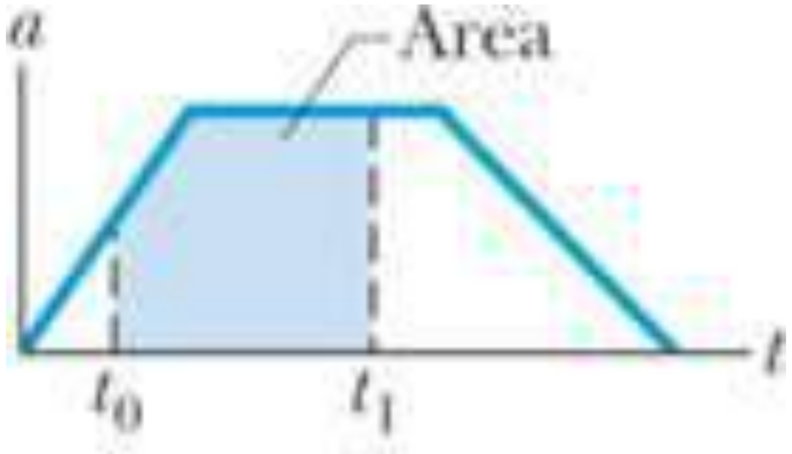
Equation Number	Equation	Missing Quantity
2-11	$v = v_0 + at$	$x - x_0$
2-15	$x - x_0 = v_0 t + \frac{1}{2}at^2$	v
2-16	$v^2 = v_0^2 + 2a(x - x_0)$	t
2-17	$x - x_0 = \frac{1}{2}(v_0 + v)t$	a
2-18	$x - x_0 = vt - \frac{1}{2}at^2$	v_0

Graphical Integration in Motion Analysis

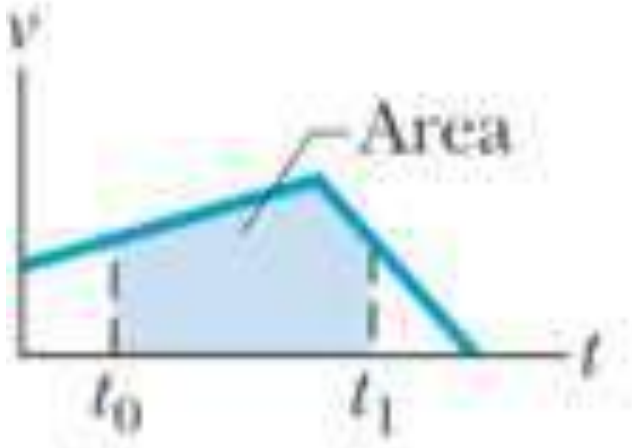
Integration in Motion

$$v_1 - v_0 = \int_{t_0}^{t_1} a\,dt\,.$$

$$x_1 - x_0 = \int_{t_0}^{t_1} v\,dt,$$



(a)

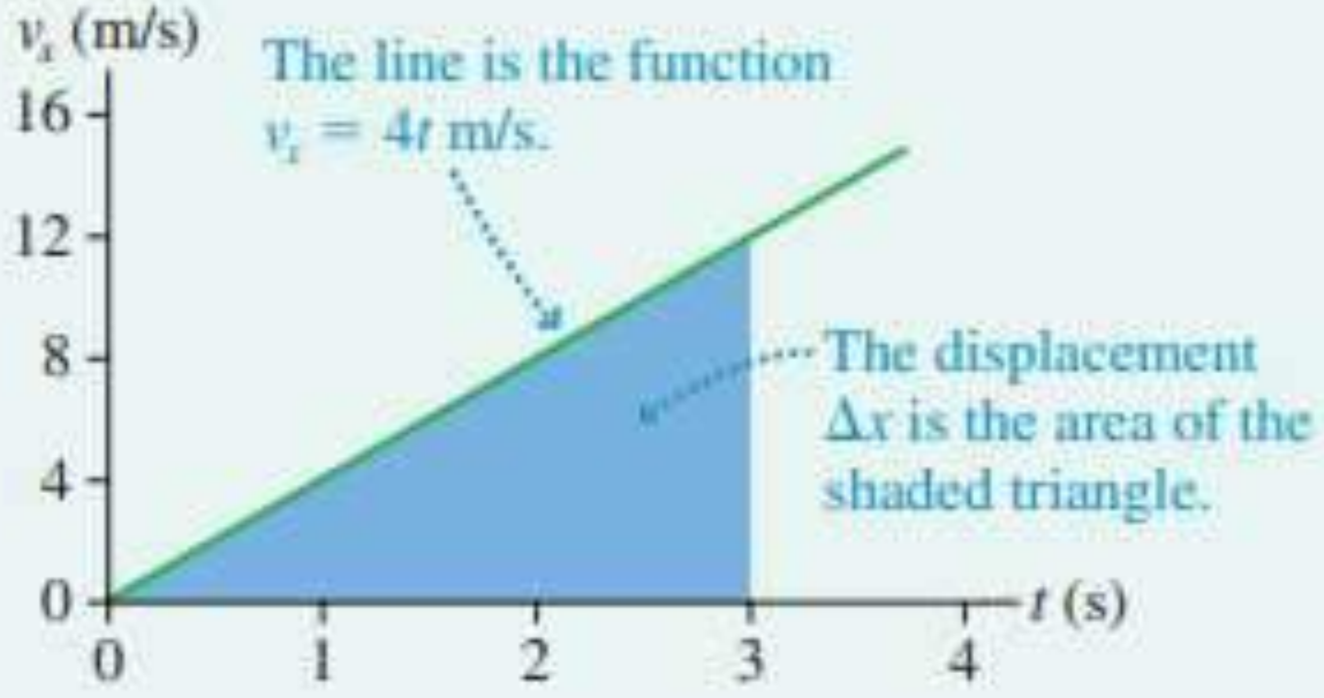


(b)

EXAMPLE 2.6 The displacement during a drag race

FIGURE 2.17 shows the velocity-versus-time graph of a drag racer. How far does the racer move during the first 3.0 s?

FIGURE 2.17 Velocity-versus-time graph for Example 2.6.



MODEL Represent the drag racer as a particle with a well-defined position at all times.

VISUALIZE Figure 2.17 is the graphical representation.

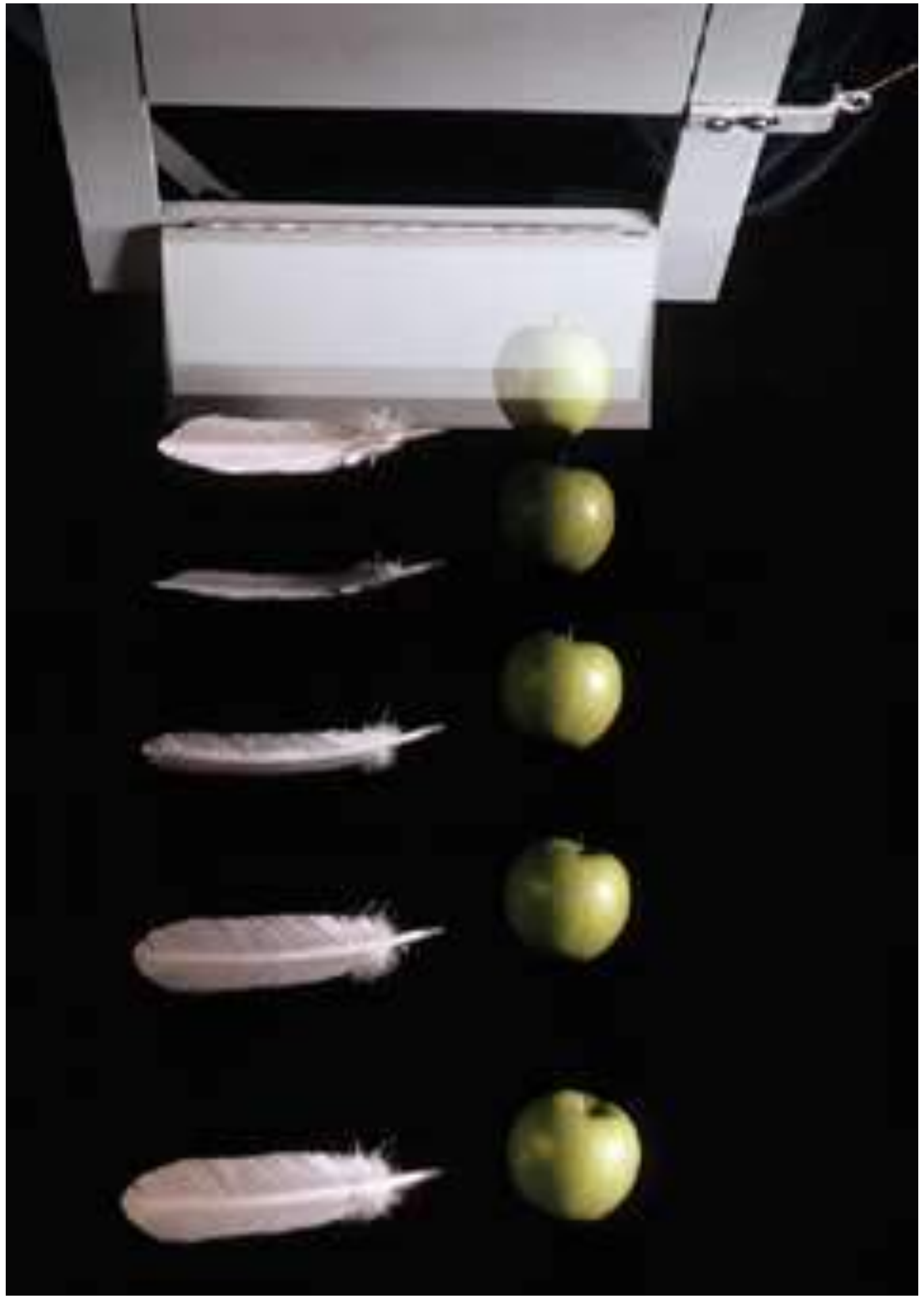
SOLVE The question "How far?" indicates that we need to find a displacement Δx rather than a position x. According to Equation 2.12, the car's displacement $\Delta x = x_f - x_i$ between $t = 0$ s and $t = 3$ s is the area under the curve from $t = 0$ s to $t = 3$ s. The curve in this case is an angled line, so the area is that of a triangle:

$$\begin{aligned}\Delta x &= \text{area of triangle between } t = 0 \text{ s and } t = 3 \text{ s}\\ &= \tfrac{1}{2} \times \text{base} \times \text{height}\\ &= \tfrac{1}{2} \times 3 \text{ s} \times 12 \text{ m/s} = 18 \text{ m}\end{aligned}$$

The drag racer moves 18 m during the first 3 seconds.

ASSESS The "area" is a product of s with m/s, so Δx has the proper units of m.

Free-Fall Acceleration



Objective

- Use the kinematic equations with the variables y and g to analyze free-fall motion.
- Describe how the values of the position, velocity, and acceleration change during a free fall.
- Solve for the position, velocity, and acceleration as functions of time when an object is in a free fall.

Free Fall

Free Fall

- **Free fall motion** is the motion of an object solely under the influence of gravity, without any other forces acting on it (like air resistance). During free fall, the only force acting on the object is gravity, causing it to accelerate downward at a constant rate.

Free Fall

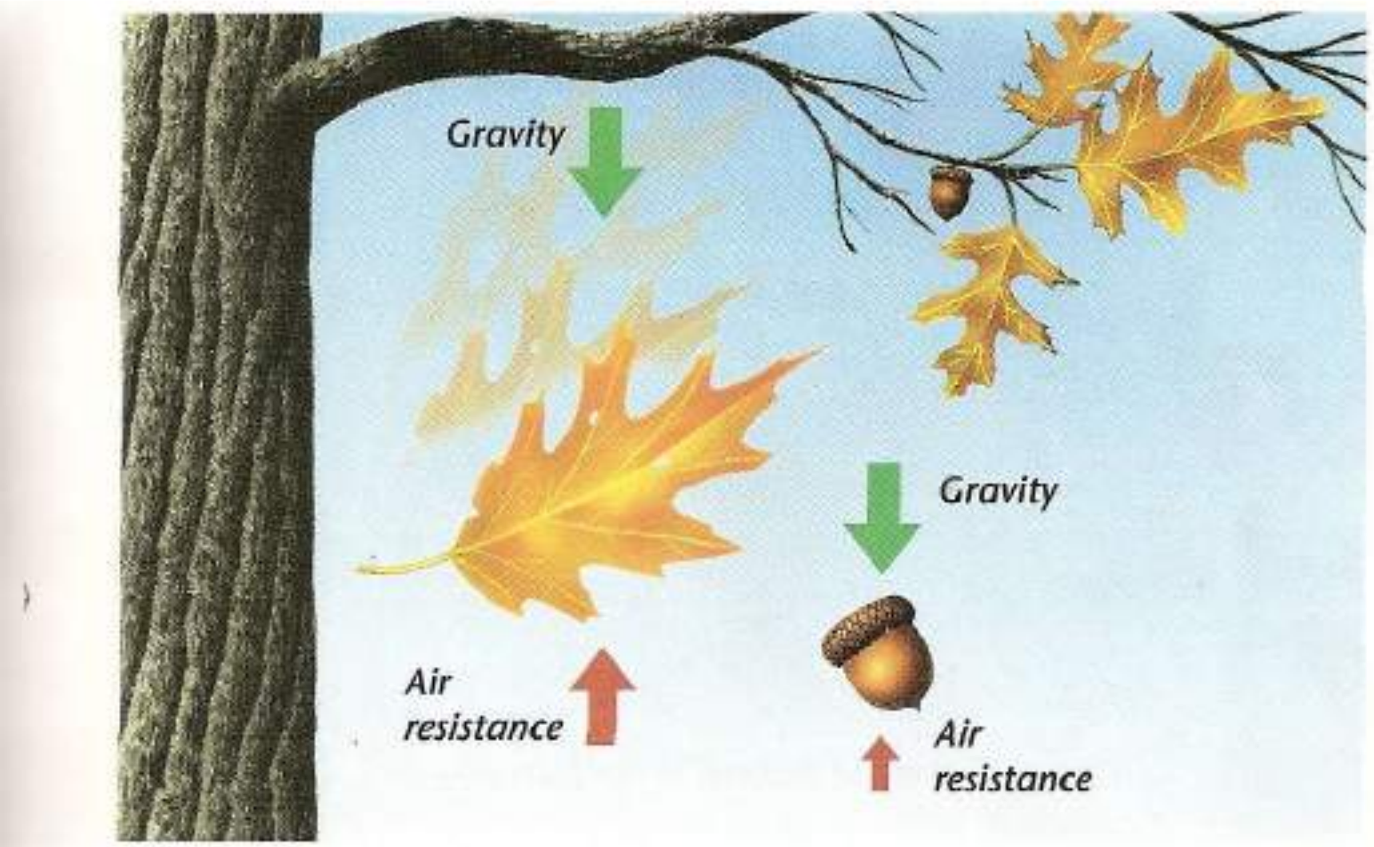
- When the only force acting on an object is gravity, the object is said to be in free fall

- *In a vacuum* (*no air)* all objects in free fall accelerate at the same rate, regardless of mass

Hammer

Feather

Free Fall



On Earth, gravity is not the only force on an object in free fall. The acorn falls faster than the leaf because it has less air resistance.

In a Vacuum acorn and leaf fall at same speed

Terminal Velocity

When sky divers jump, initially they accelerate rapidly.

Eventually, their combined weight due to gravity will be balanced by their upward air resistance.

They will stop accelerating and fall at a constant speed called their terminal velocity.

If they did this near the surface of the moon, there would be no air resistance.
Their speed would keep increasing until they crashed into the surface.

Free Fall Summary

- In a Vacuum, all objects fall at the same speed

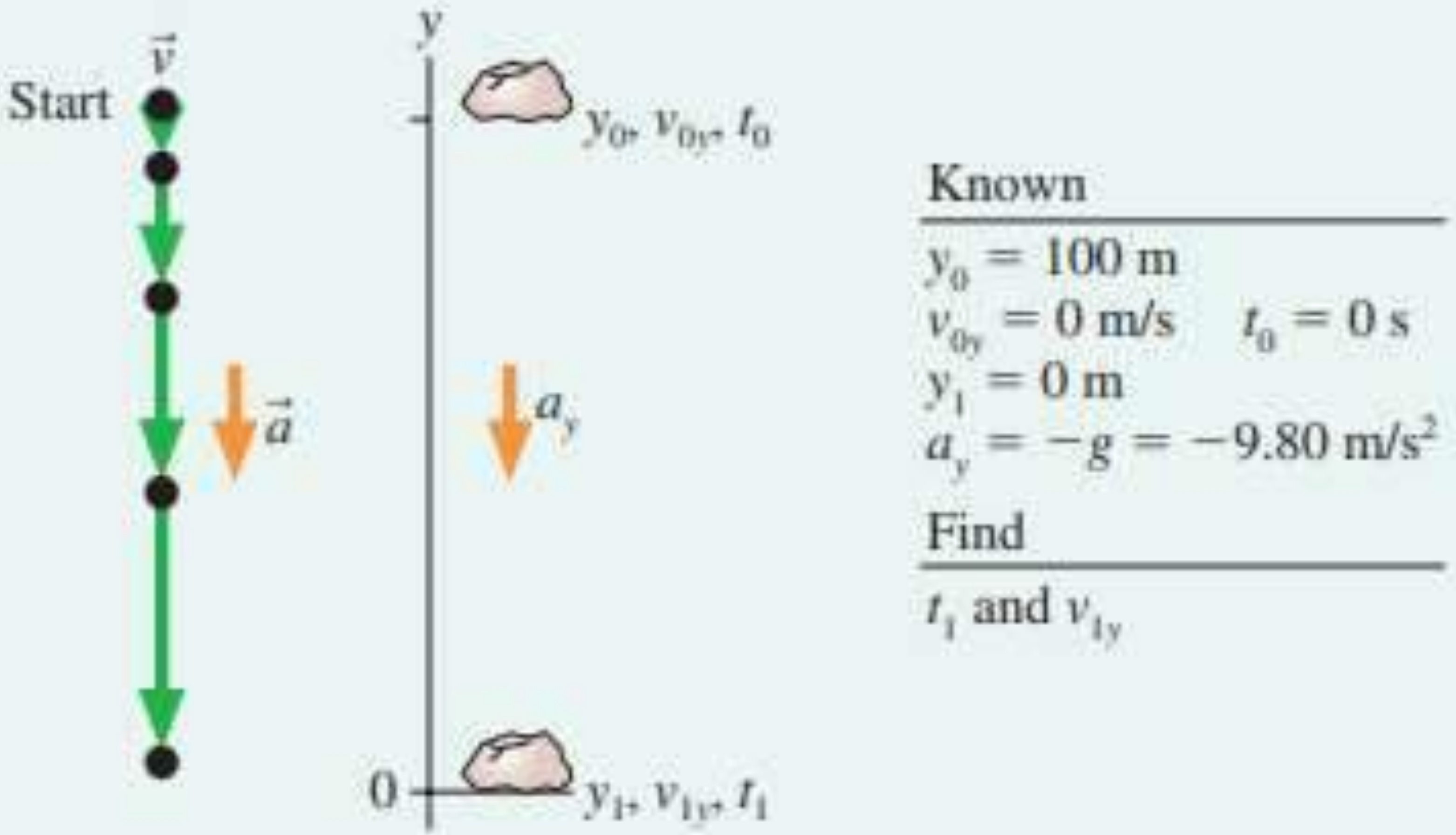
- In Air, heavier objects fall faster than lighter objects
  - If a heavy object and a light object are released at the same time
  - Initially they may appear to fall at same speed
  - When the light object reaches terminal velocity, the heavy object will still be accelerating

EXAMPLE 2.13 A falling rock

A rock is released from rest at the top of a 100-m-tall building. How long does the rock take to fall to the ground, and what is its impact velocity?

MODEL Represent the rock as a particle. Assume air resistance is negligible.

FIGURE 2.29 Pictorial representation of a falling rock.



SOLVE Free fall is motion with the specific constant acceleration $a_y = -g$. The first question involves a relation between time and distance, so only the second equation in Table 2.2 is relevant. Using $v_{0y} = 0$ m/s and $t_0 = 0$ s, we find

$$y_1 = y_0 + v_{0y}\,\Delta t + \tfrac{1}{2}a_y\,\Delta t^2 = y_0 + v_{0y}\,\Delta t - \tfrac{1}{2}g\Delta t^2 = y_0 - \tfrac{1}{2}gt_1^2$$

We can now solve for t_1, finding

$$t_1 = \sqrt{\frac{2(y_0 - y_1)}{g}} = \sqrt{\frac{2(100\text{ m} - 0\text{ m})}{9.80\text{ m/s}^2}} = \pm 4.52\text{ s}$$

$$v_{1y} = v_{0y} - g\Delta t = -gt_1 = -(9.80\text{ m/s}^2)(4.52\text{ s})$$
$$= -44.3\text{ m/s}$$

A baseball is hit so that it travels straight upward after being struck by the bat. A fan observes that it takes 3.00 s for the ball to reach its maximum height. Find (a) its initial velocity and (b) the height it reaches.

(a) $v_f = v_i - gt$: $v_f = 0$ when $t = 3.00$ s, $g = 9.80\ \text{m/s}^2$. Therefore,

$$v_i = gt = (9.80\ \text{m/s}^2)(3.00\ \text{s}) = \boxed{29.4\ \text{m/s}}.$$

(b) $y_f - y_i = \frac{1}{2}(v_f + v_i)t$

$$y_f - y_i = \frac{1}{2}(29.4\ \text{m/s})(3.00\ \text{s}) = \boxed{44.1\ \text{m}}$$

THANK YOU