

Projectile Motion Practice Problems

Case 2 – Angled Launch, Level Landing

1. A projectile is fired with an initial speed of 75 m/s at an angle of 35 degrees above the horizontal on a long flat firing range. Determine: maximum height reached by the projectile, total time in the air, and how far away the object hits the ground



$$v_{ix} = v_i \cos \theta = 75 \cos(35) = 61.44 \text{ m/s}$$

$$v_{iy} = v_i \sin \theta = 75 \sin(35) = 43.02 \text{ m/s}$$

Max Height ($v_y = 0$)

$$v_{fy}^2 = v_{iy}^2 + 2a_y \Delta y$$

$$0 = (43.02)^2 + 2(-9.8) \Delta y$$

$$\frac{-1850.72}{-19.6} = \Delta y$$

$$\Delta y = 94.42 \text{ m}$$

Time in the air

$$v_{fy} = v_{iy} + a_y t$$

0 @ max height

$$0 = 43.02 + (-9.8)t$$

$$\frac{-43.02}{-9.8} = t = 4.39 \text{ s}$$

Total = 2t

$$t = 8.78 \text{ s}$$

0 From launch to land.

$$\Delta y = v_{iy} t + \frac{1}{2} a_y t^2$$

$$0 = 43.02 t + \frac{1}{2} (-9.8) t^2$$

$$-43.02 t = -4.9 t^2$$

$$\frac{-43.02}{-4.9} = t$$

$$t = 8.78 \text{ s} \checkmark$$

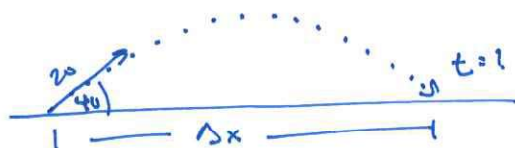
Range

$$v_{ix} = \frac{\Delta x}{t}$$

$$61.44 = \frac{\Delta x}{8.78}$$

$$\Delta x = 539.44 \text{ m}$$

2. A football is kicked at ground level with a speed of 20 m/s at a 40 degree angle to the horizontal. How much later does it hit the ground? What's the range of the football?



$$v_{ix} = v_i \cos \theta = 20 \cos 40 = 15.3 \text{ m/s}$$

$$v_{iy} = v_i \sin \theta = 20 \sin 40 = 12.86 \text{ m/s}$$

Time in the air.

$$v_{fy} = v_{iy} + a_y t$$

$$0 = 12.86 + (-9.8)t$$

$$\frac{-12.86}{-9.8} = t = 1.31 \text{ s}$$

Total = 2t

$$t = 2.62 \text{ s}$$

$$\Delta y = v_{iy} t + \frac{1}{2} a_y t^2$$

$$0 = 12.86 t + \frac{1}{2} (-9.8) t^2$$

$$-12.86 t = -4.9 t^2$$

$$\frac{-12.86}{-4.9} = t$$

$$t = 2.62 \text{ s} \checkmark$$

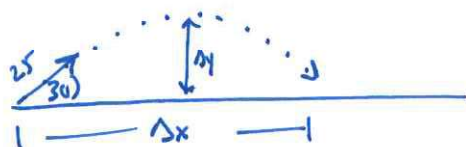
Range

$$v_{ix} = \frac{\Delta x}{t}$$

$$15.3 = \frac{\Delta x}{2.62}$$

$$\Delta x = 40.09 \text{ m}$$

3. Suppose Tiger Woods hits a golf ball 25 m/s at an angle of 30 degrees above the horizontal. Determine: maximum height reached by the ball, horizontal distance covered by the ball, and how long the ball is in the air.



$$v_{ix} = 25 \cos 30 = 21.65 \text{ m/s} \quad \Delta y = ?$$

$$v_{iy} = 25 \sin 30 = 12.5 \text{ m/s} \quad \Delta x = ?$$

$$t = ?$$

Max Height

$$v_{fy}^2 = v_{iy}^2 + 2a_y \Delta y$$

0 at max height

$$0 = (12.5)^2 + 2(-9.8)\Delta y$$

$$\frac{-156.25}{-19.6} = \Delta y$$

$$\Delta y = 7.97 \text{ m}$$

Time in Air

$$v_{fy} = v_{iy} + a_y t$$

$$0 = 12.5 + (-9.8)t$$

$$t = 1.28 \text{ s}$$

$$\text{Total} = 2t$$

$$t = 2.55 \text{ s}$$

0 from launch to land.

$$\Delta y = v_{iy} t + \frac{1}{2} a_y t^2$$

$$0 = 12.5 t + \frac{1}{2} (-9.8) t^2$$

$$-12.5 t = -4.9 t^2$$

$$\frac{-12.5}{-4.9} = t$$

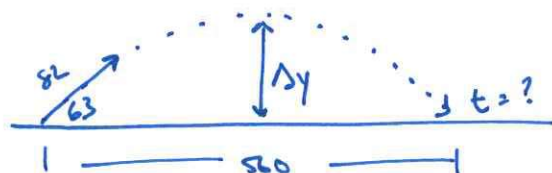
$$t = 2.55 \text{ s}$$

$$v_{ix} = \frac{\Delta x}{t}$$

$$21.65 = \frac{\Delta x}{2.55}$$

$$\Delta x = 55.23 \text{ m}$$

4. A pirate ship is 560 m away from a fort defending a harbor entrance. A defense cannon at sea level fires balls at an initial velocity of 82 m/s at an angle of 63 degrees. How long is the cannon ball in the air? What's the maximum height reached by the ball?



$$v_{ix} = 82 \cos 63 = 37.23 \text{ m/s}$$

$$v_{iy} = 82 \sin 63 = 73.06 \text{ m/s}$$

Max Height

$$v_{fy}^2 = v_{iy}^2 + 2a_y \Delta y$$

$$0 = (73.06)^2 + 2(-9.8)\Delta y$$

$$\frac{-5388.18}{-19.6} = \Delta y$$

$$\Delta y = 272.3 \text{ m}$$

Time in the Air

$$v_{fy} = v_{iy} + a_y t$$

$$0 = 73.06 + (-9.8)t$$

$$\frac{-73.06}{-9.8} = t = 7.45 \text{ s}$$

$$\text{Total} = 2t = 14.9 \text{ s}$$

0 from launch to land.

$$\Delta y = v_{iy} t + \frac{1}{2} a_y t^2$$

$$0 = 73.06 t + \frac{1}{2} (-9.8) t^2$$

$$-73.06 t = -4.9 t^2$$

$$\frac{-73.06}{-4.9} = t = 14.9 \text{ s}$$

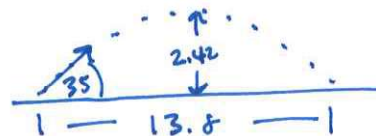
OR

$$v_{ix} = \frac{\Delta x}{t} \quad 37.23 = \frac{560}{t}$$

$$t = 15 \text{ s}$$

5. A soccer ball is kicked at 35 degrees above the horizontal, it lands 13.8 m away and reaches a maximum height of 2.42 m.

- What's the initial vertical velocity (v_{iy})
- How long does it take to hit the ground?
- What's the initial horizontal velocity (v_{ix})
- With what velocity was the ball initially kicked (\vec{v})



(A) $v_{iy} = ?$

$$v_f^2 = v_{iy}^2 + 2a_y \Delta y$$

0 @ max height

$$0 = v_{iy}^2 + 2(-9.8)(2.42)$$

$$47.43 = v_{iy}^2$$

$$v_{iy} = 6.89 \text{ m/s}$$

(B) $v_{fy} = v_{iy} + a_y t$

$$0 = 6.89 + (-9.8)t$$

$$\frac{-6.89}{-9.8} = t$$

$$t = 0.703 \text{ s}$$

$$\text{Total} = 2t$$

$$= 1.41 \text{ s}$$

(C) $v_{ix} = ?$

$$v_{ix} = \frac{\Delta x}{t}$$

$$v_{ix} = \frac{13.8}{1.41} = 9.79 \text{ m/s}$$

(D) $v_i = ?$

$$v_i^2 = (v_{ix})^2 + (v_{iy})^2$$

$$\tan \theta = \frac{v_{iy}}{v_{ix}}$$

$$v_i^2 = (9.79)^2 + (6.89)^2$$

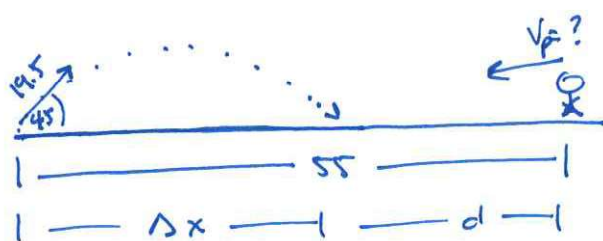
$$\theta = \tan^{-1}\left(\frac{6.89}{9.79}\right)$$

$$v_i^2 = 143.32 \text{ m/s}$$

$$v_i = 11.97 \text{ m/s}$$

$$\theta = 35.14^\circ \checkmark$$

6. A soccer ball is kicked from the ground with an initial speed of 19.5 m/s at an upward angle of 45 degrees. A player 55 m away in the direction of the kick starts running back to meet the ball at that instant. What must be his average speed if he is to meet the ball just before it hits the ground?



Need to know: how far player has to run and how much time they have to run it

$$v_{ix} = 19.5 \cos 45 = 13.79 \text{ m/s}$$

$$v_{iy} = 19.5 \sin 45 = 13.79 \text{ m/s}$$

Time

$$v_{fy} = v_{iy} + a_y t$$

$$0 = 13.79 + (-9.8)t$$

$$\frac{-13.79}{-9.8} = t = 1.41 \text{ s}$$

$$\text{Total} = 2t$$

$$= 2.82 \text{ s}$$

Range of Ball

$$v_{ix} = \frac{\Delta x}{t}$$

$$13.79 = \frac{\Delta x}{2.82}$$

$$\Delta x = 38.89 \text{ m}$$

Distance Player Needs to run

$$\Delta x + d = 55$$

$$38.89 + d = 55$$

$$d = 55 - 38.89$$

$$d = 16.11 \text{ m}$$

speed of player

$$v_p = \frac{d}{t} = \frac{16.11}{2.82}$$

$$v_p = 5.71 \text{ m/s}$$