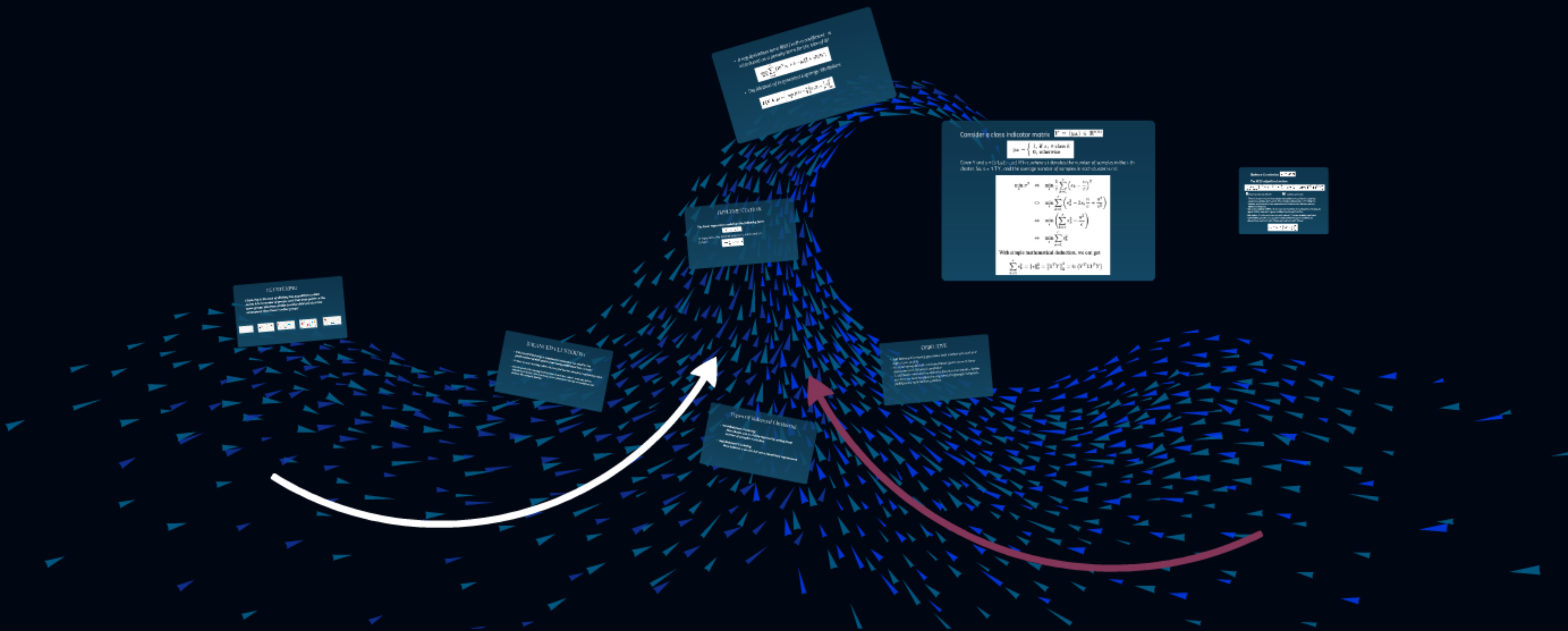
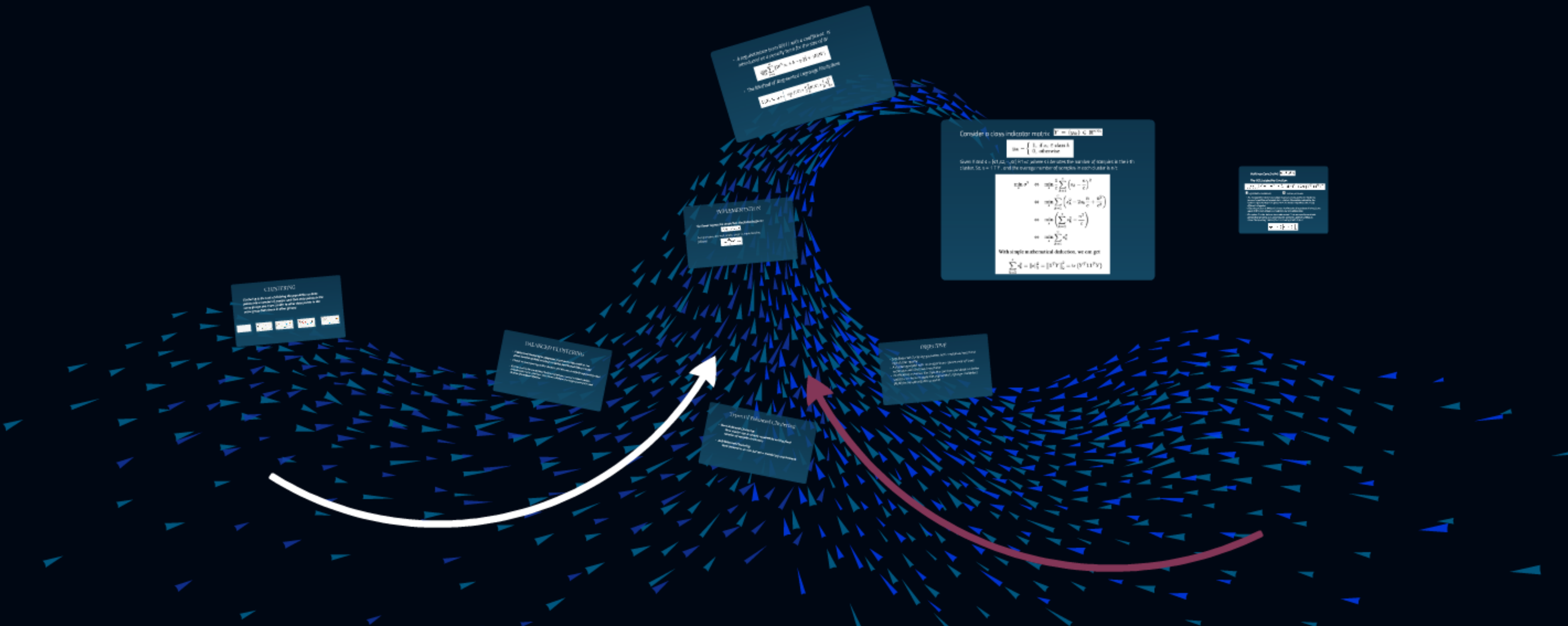


Balanced Clustering with Least Square Regression

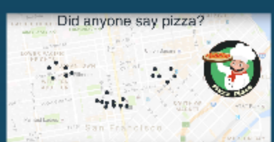


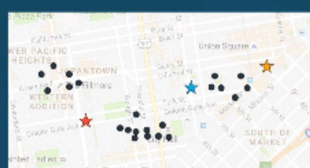
Balanced Clustering with Least Square Regression

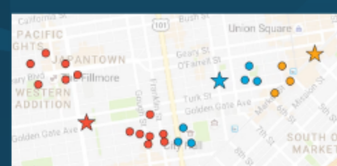
Prezi

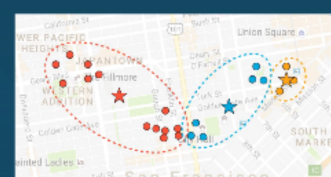
CLUSTERING

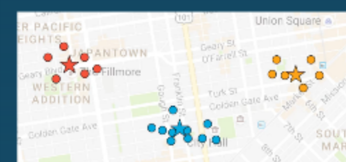
Clustering is the task of dividing the population or data points into a number of groups such that data points in the same groups are more similar to other data points in the same group than those in other groups











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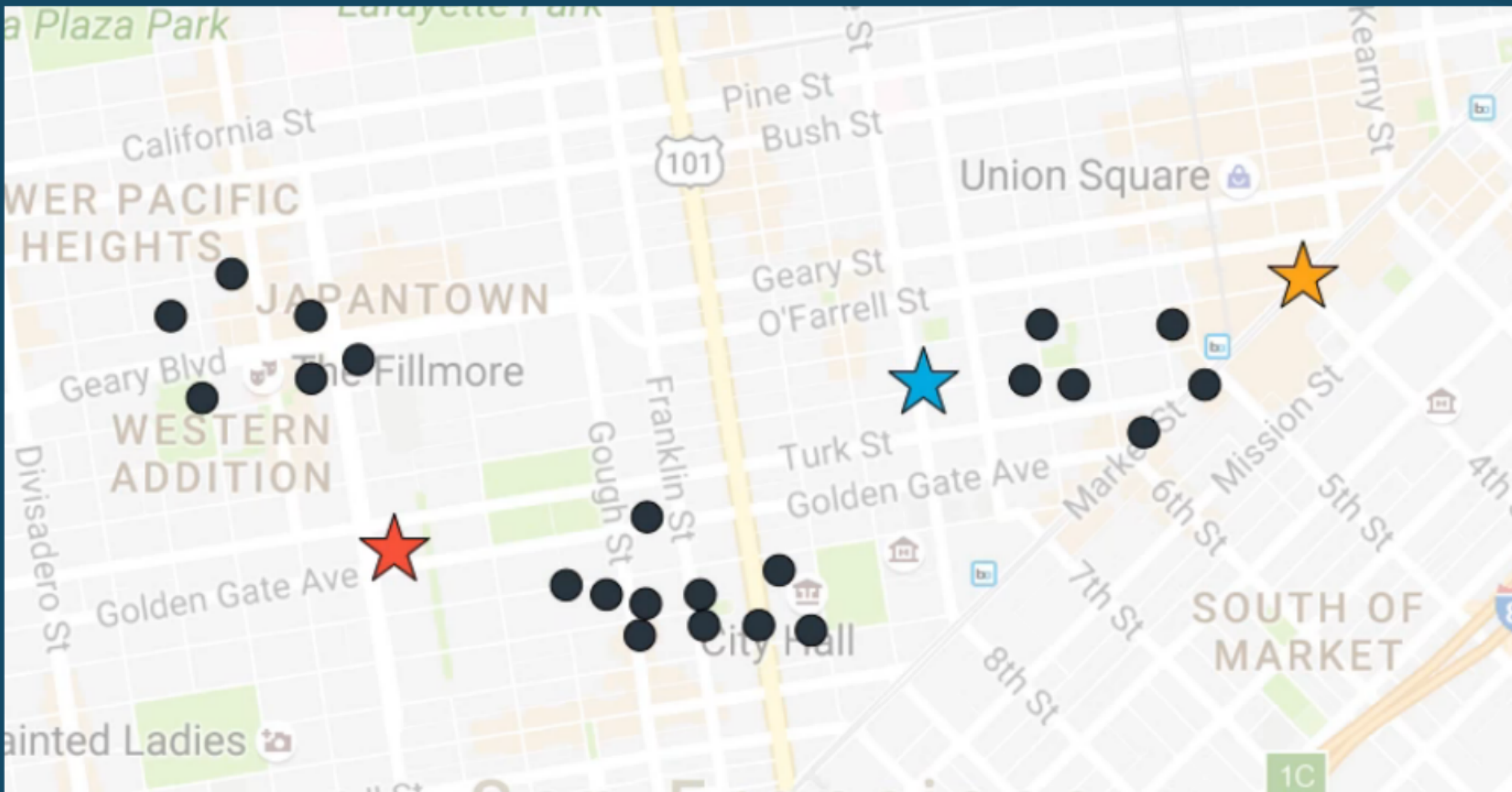
Did anyone say pizza?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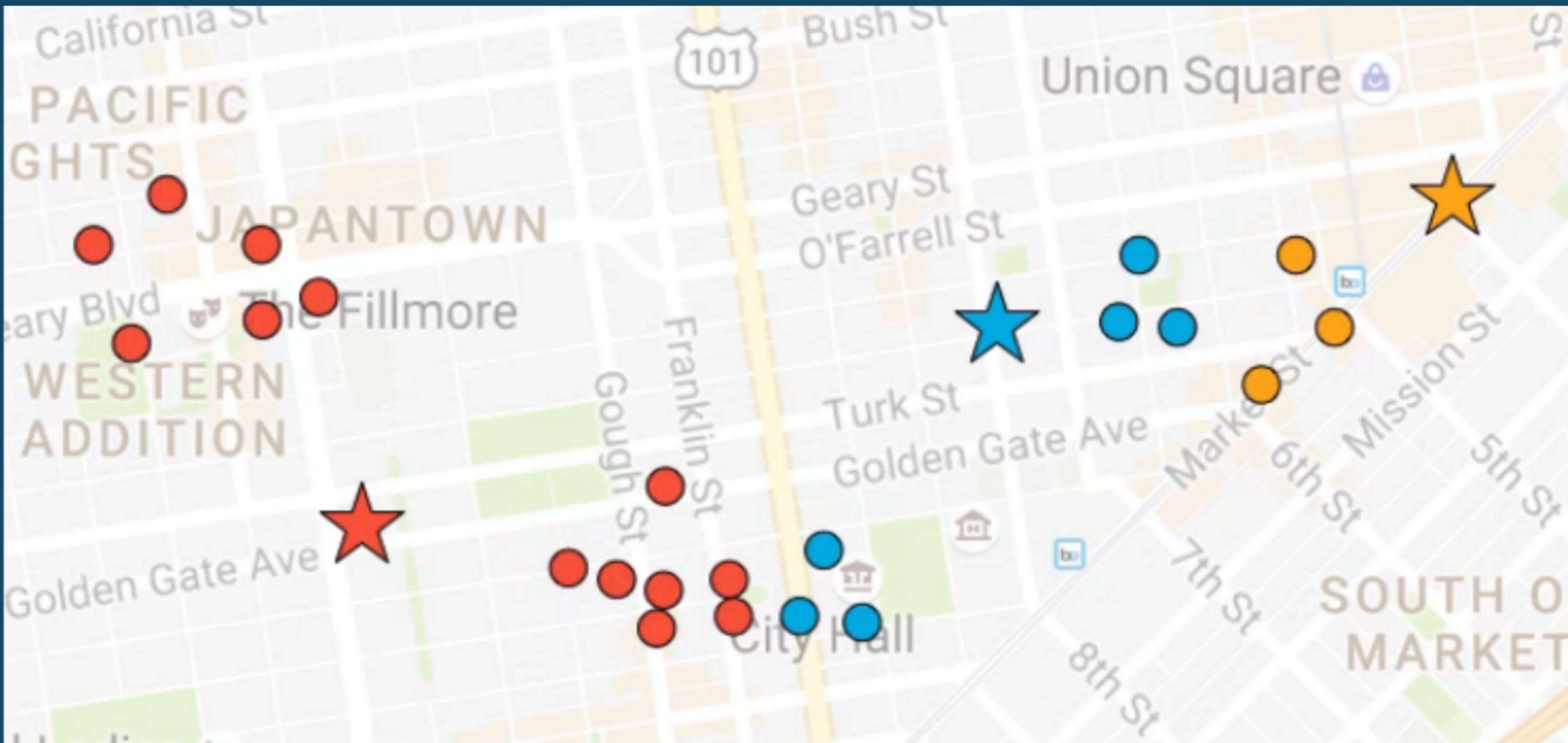
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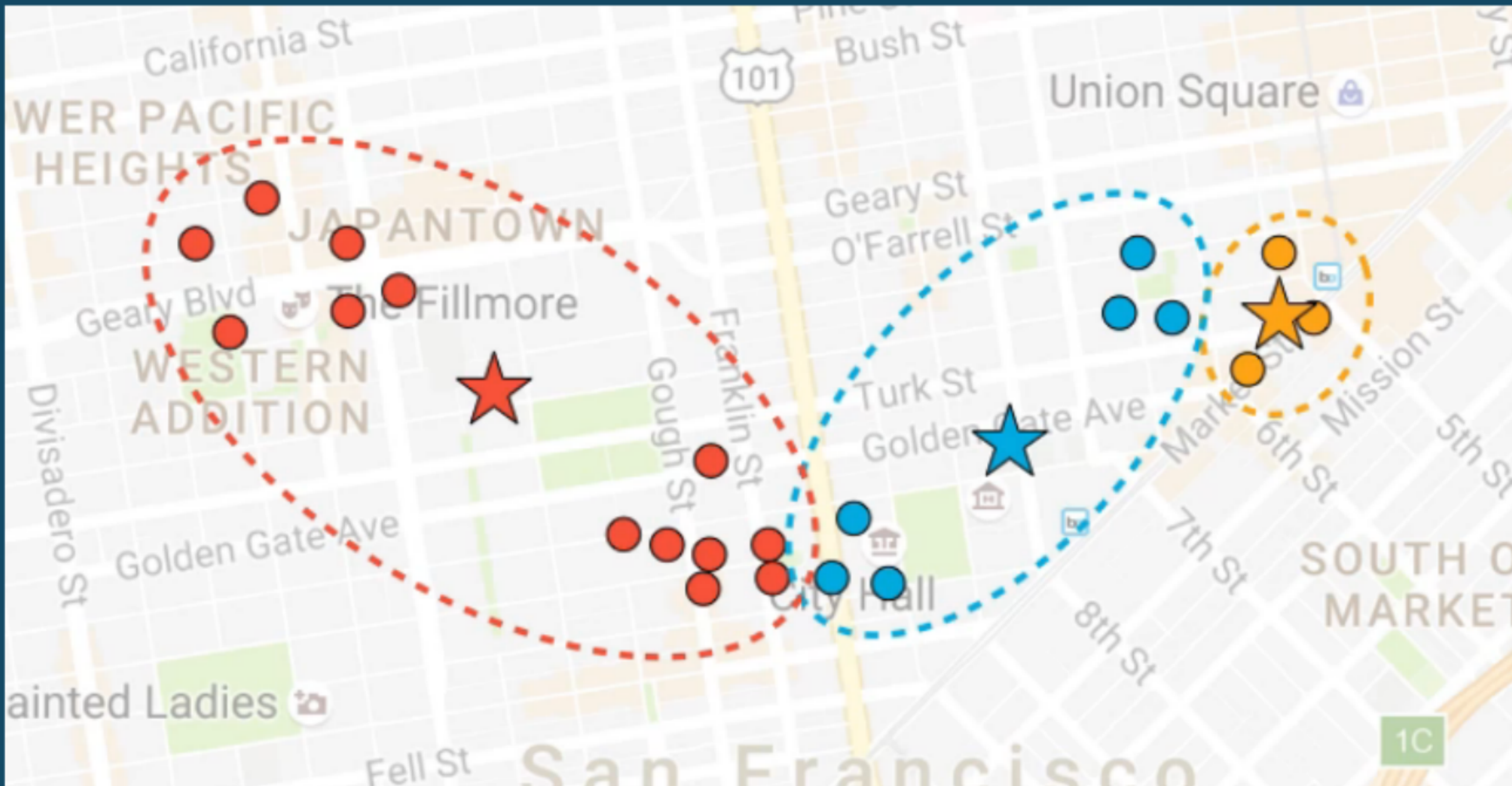
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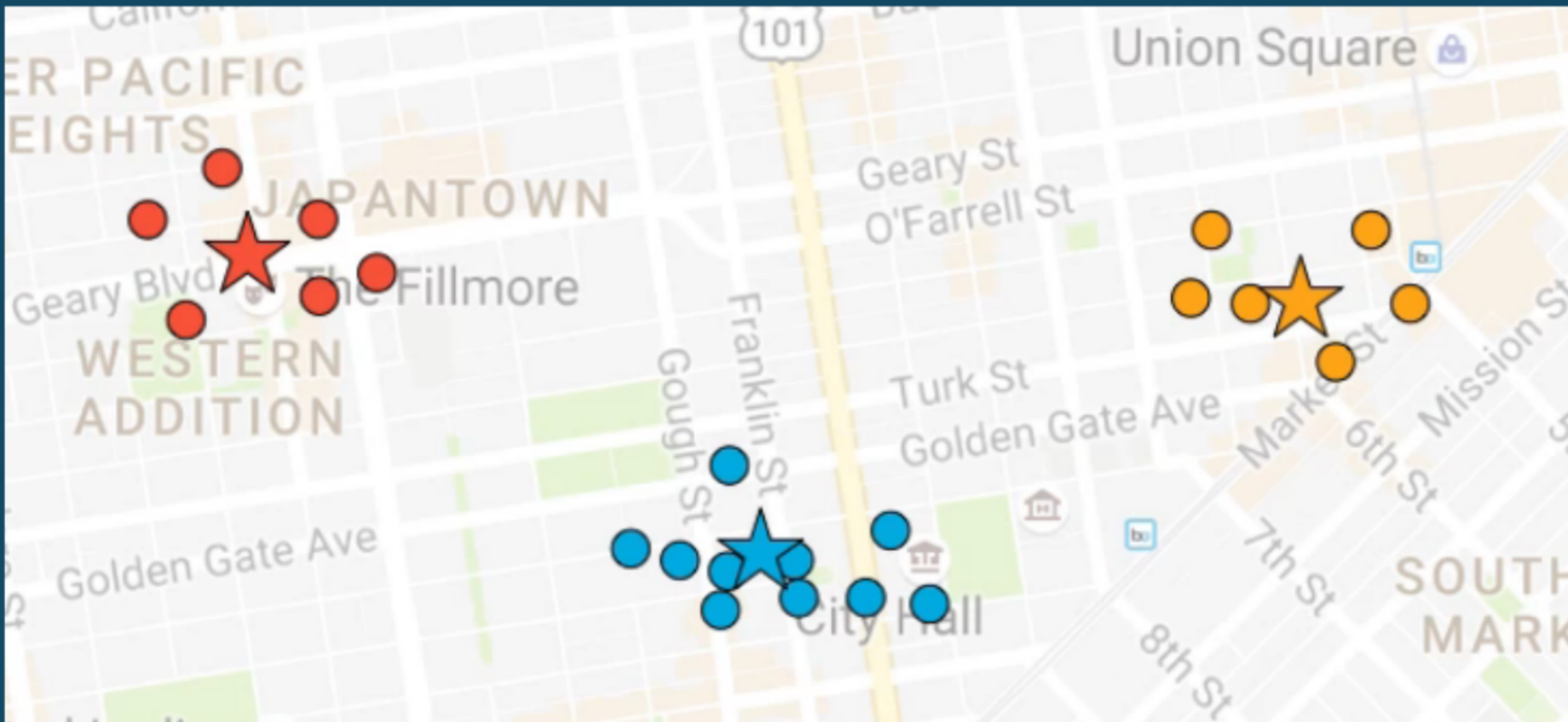
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Prezi

BALANCED CLUSTERING

- *A Balanced Clustering is supposed to prevent a too small or too great number of data points from being partitioned into a cluster*
- *It tends to avoid forming outlier clusters, and thus has beneficial regularizing effect*
- *Can be used in the energy load balance of wireless sensor networks where unbalanced cluster structure may cause unbalanced energy consumption and shorten the network lifetime*

Types Of Balanced Clustering

- ***Hard-Balanced Clustering:***
  Here cluster size is strictly required by setting fixed number of samples in clusters

- ***Soft-Balanced Clustering:***
  Here balance is an aim but not a mandatory requirement.

OBJECTIVE

- *Soft Balanced Clustering-guarantee both a balanced result and high cluster quality*
- *A clustering model with minimized least square error of linear regression with Balanced constraint*
- *To efficiently minimize the objective function and obtain a better solution, we have to apply the augmented Lagrange multipliers (ALM) for the optimization problem*

IMPLEMENTATION

The linear regression model has the following form:

$$f(x) = x^T w + b$$

In regression, the estimation error is minimized as follows:

$$\min \sum_{i=1}^{n} e(f(x_i), y_i)$$

- *A regularization term R(W) with a coefficient   is introduced as a penalty term for the size of W*

$$\min_{W,b} \sum_{i=1}^{n} \|W^T x_i + b - y_i\|_2^2 + \gamma R(W)$$

- *The Method of Augmented Lagrange Multipliers*

$$L(X, \Lambda, \mu) = \min_X f(X) + \frac{\mu}{2} \left\| H(X) + \frac{1}{\mu}\Lambda \right\|_F^2$$

Consider a class indicator matrix $Y = (y_{ik}) \in \mathbb{R}^{n\times c}$,

$$y_{ik} = \begin{cases} 1, & \text{if } x_i \in \text{class } k \\ 0, & \text{otherwise} \end{cases}$$

Given Y and s = [s1,s2,···,sc] R1×c ,where s i denotes the number of samples in the i-th cluster. So, s = 1 T Y , and the average number of samples in each cluster is n/c

$$\begin{aligned} \min_s \sigma^2 \quad &\Leftrightarrow \quad \min_s \frac{1}{c} \sum_{k=1}^{c} \left(s_k - \frac{n}{c}\right)^2 \\ &\Leftrightarrow \quad \min_s \sum_{k=1}^{c} \left(s_k^2 - 2s_k \frac{n}{c} + \frac{n^2}{c^2}\right) \\ &\Leftrightarrow \quad \min_s \left(\sum_{k=1}^{c} s_k^2 - \frac{n^2}{c}\right) \\ &\Leftrightarrow \quad \min_s \sum_{k=1}^{c} s_k^2 \end{aligned}$$

With simple mathematical deduction, we can get

$$\sum_{k=1}^{c} s_k^2 = \|s\|_2^2 = \left\|\mathbf{1}^T Y\right\|_2^2 = \operatorname{tr}\left(Y^T \mathbf{1}\mathbf{1}^T Y\right)$$

Balance Constraint: $\operatorname{tr}\left(Y^T \mathbf{1}\mathbf{1}^T Y\right)$

The BCLS objective function:

$$\min_{W,b,Y\in Ind} \left\|X^T W + \mathbf{1}b^T - Y\right\|_F^2 + \gamma \|W\|_F^2 + \lambda \operatorname{tr}\left(Y^T \mathbf{1}\mathbf{1}^T Y\right)$$

γ-regularization parameter

λ -balance parameter

- ***The minimization of the least square regression term guides the clustering process to partition data points into c clusters. Meanwhile, minimizing the balance regularization term guarantees the balanced partitioning among different categories.***
- ***Since the problem is NP-hard, it is very hard to solve it in polynomial time.So, we apply ALM to help obtain good solutions for the optimization***
- ***We replace Y in the balance term with matrix Z into an equality constraint optimization problem and approximately obtain the optimal solution by alternatively solving Y with Z fixed and solving Z with Y fixed.***

$$\min_X f(X) + \frac{\mu}{2}\left\|H(X) + \frac{1}{\mu}\Lambda\right\|_F^2$$

We get the final optimization problem for our BCLS method

$$\min_{W,b,Y\in Ind,Z} \left\|X^T W + \mathbf{1}b^T - Y\right\|_F^2 + \gamma\|W\|_F^2 \; + \lambda \operatorname{tr}\left(Z^T \mathbf{1}\mathbf{1}^T Z\right) + \frac{\mu}{2}\left\|Y - Z + \frac{1}{\mu}\Lambda\right\|_F^2$$

μ - *positive scalar and its value increases slightly after each iteration*

Λ -- *estimate of the Lagrange multiplier*

- ***The estimation accuracy improves at every step of iteration.***
- ***It can be solved by alternatively updating the four variables W , b, Y and Z***

1) With Y and Z fixed

$$\min_{W,b} \left\|X^T W + \mathbf{1}b^T - Y\right\|_F^2 + \gamma\|W\|_F^2$$

2) simply by setting the derivatives of the objective function with respect to W and b to zeros, we have

$$\begin{cases} W = \left(XX^T + \gamma I_d\right)^{-1} XY \\ b = \dfrac{1}{n} Y^T \mathbf{1} \end{cases}$$

3) With W , b and Y

$$\min_{Z} \lambda \operatorname{tr}\left(Z^T \mathbf{1}\mathbf{1}^T Z\right) + \frac{\mu}{2}\left\|Y - Z + \frac{1}{\mu}\Lambda\right\|_F^2$$

4) Z can be obtained by setting the derivative of the objective function with respect to Z to zero

$$Z = \left(\mu I_n + 2\lambda \mathbf{1}\mathbf{1}^T\right)^{-1}(\mu Y + \Lambda)$$

5) With W, b and Z fixed, we can transform BCLS eq into:

$$\min_{Y \in Ind} \|Y - V\|_F^2 + const.$$

where const. denotes a constant and V = (v ik) R n×c , and

$$V = \frac{2}{2+\mu}\left(X^T W + \mathbf{1}b^T\right) + \frac{1}{2+\mu}(\mu Z - \Lambda)$$

6) Each element of Y = (y ik) R n×c is binary and the sum of each row is 1

We can obtain the solution as follows:

$$y_{ik} = \begin{cases} 1, & \text{if } k = \arg\max_k \{v_{ik}\}_{k=1}^{c} \\ 0, & \text{otherwise} \end{cases}$$

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Algorithm of the BCLS method

Input: Centered dataset $X = [x_1, x_2, \cdots, x_n] \in \mathbb{R}^{d\times n}$, the number of clusters c, and parameters γ, λ, and $\rho(\rho > 0)$.
Output: Assignment matrix $Y \in \mathbb{R}^{n\times c}$.
Initialize Y randomly, initialize μ, and set $\Lambda = \mathbf{0} \in \mathbb{R}^{n\times c}$.
Repeat

1. Obtain W and b by solving the problem in Eq. (14) with the solution in Eq. (15);
2. Obtain Z by solving Eq. (16) with the solution in Eq. (17);
3. Obtain Y by Eq. (19) and Eq. (21);
4. **Update** Λ**:** $\Lambda^{(t+1)} = \Lambda^{(t)} + \mu^{(t)}(Y - Z)$;
5. **Update** μ**:** $\mu^{(t+1)} = \rho\mu^{(t)}$.

Until $Y - Z \to \mathbf{0}$ or Λ remains unchanged
Return Y.

$$\min_{W,b} \|X^T W + \mathbf{1}b^T - Y\|_F^2 + \gamma \|W\|_F^2 \quad (14)$$

$$\begin{cases} W = (XX^T + \gamma I_d)^{-1} XY \\ b = \frac{1}{n} Y^T \mathbf{1} \end{cases} \quad (15)$$

$$\min_{Z} \lambda \mathrm{tr}\left(Z^T \mathbf{1}\mathbf{1}^T Z\right) + \frac{\mu}{2} \left\| Y - Z + \frac{1}{\mu}\Lambda \right\|_F^2 \quad (16)$$

$$Z = \left(\mu I_n + 2\lambda \mathbf{1}\mathbf{1}^T\right)^{-1} (\mu Y + \Lambda) \quad (17)$$

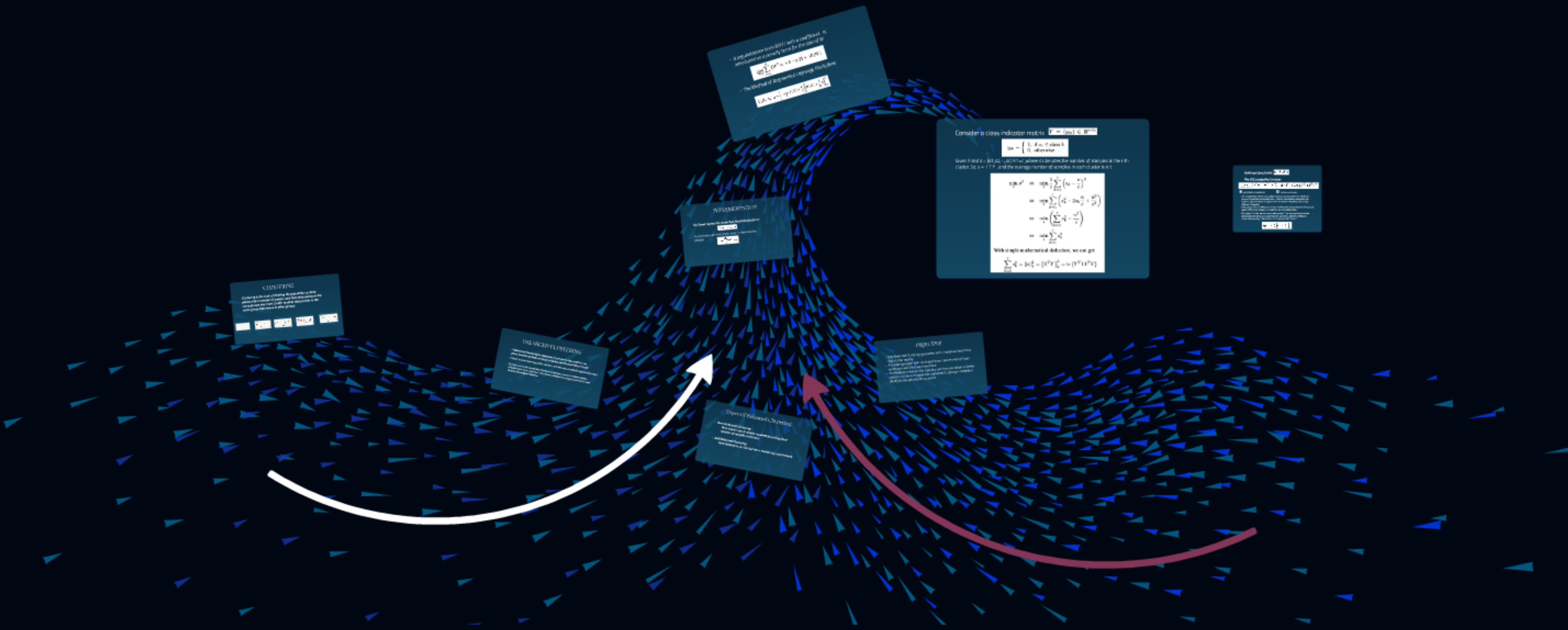
$$V = \frac{2}{2+\mu}(X^T W + \mathbf{1}b^T) + \frac{1}{2+\mu}(\mu Z - \Lambda) \quad (19)$$

$$y_{ik} = \begin{cases} 1, & \text{if } k = \arg\max_k \{v_{ik}\}_{k=1}^c \\ 0, & \text{otherwise} \end{cases} \quad (21)$$

THANK YOU!!!

https://github.com/juhi-09/SMAI-BCLS.git

Team:Breaking Bad
Juhi Gupta(20172031)
Nikita Agrawal(20172093)
Tanu Rathi(20172125)
Tejus Agarwal(20172066)



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