Questⁿ 2 $P(HIV) = 0.0005$

$P(\text{not } HIV) = 1 - 0.0005$

$= 0.9995$

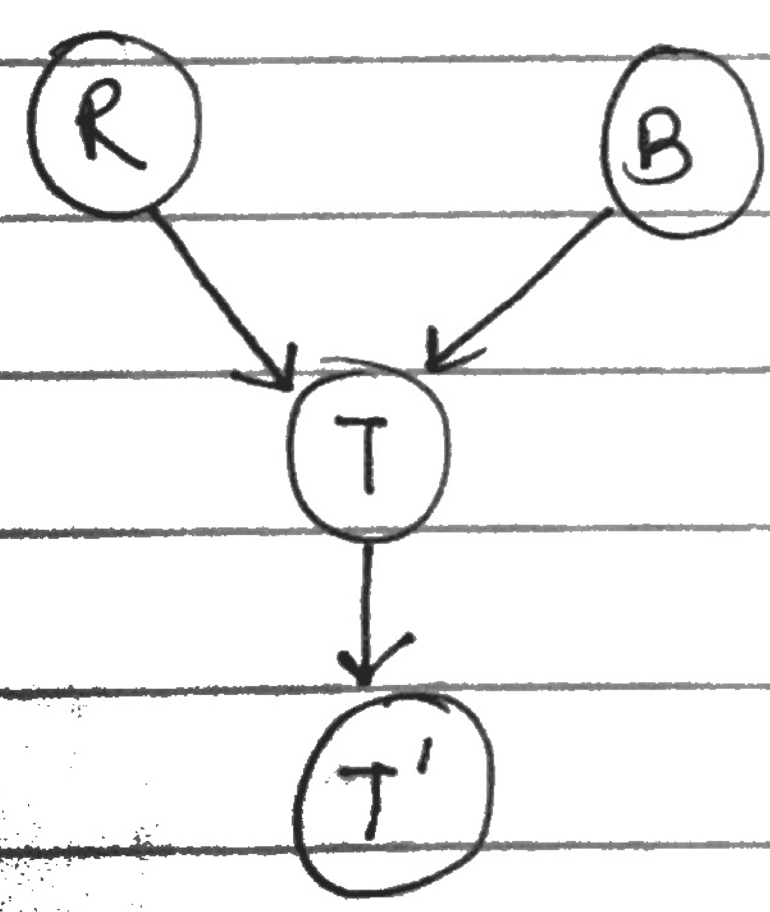
	HIV	¬HIV
+ve	0.98	0.03
−ve	0.02	0.97

To find: $P(HIV \mid +ve) = \dfrac{P(+ve \mid HIV)\, P(HIV)}{P(+ve)}$

$= \dfrac{0.98 \times 0.0005}{(0.98 \times 0.0005)\,(0.9995 \times 0.03)}$

$= 0.016.$

Questⁿ 3



(i) $R \perp\!\!\!\perp B$

Path $R \rightarrow T \rightarrow B$.

T is converging node & no evidence given

∴ BLOCK

∴ They are conditionally Independent

(ii) $R \perp\!\!\!\perp B \mid T$

Path $R \rightarrow T \rightarrow B$

T is converging node & also we have evidence about T ∴ Transmit

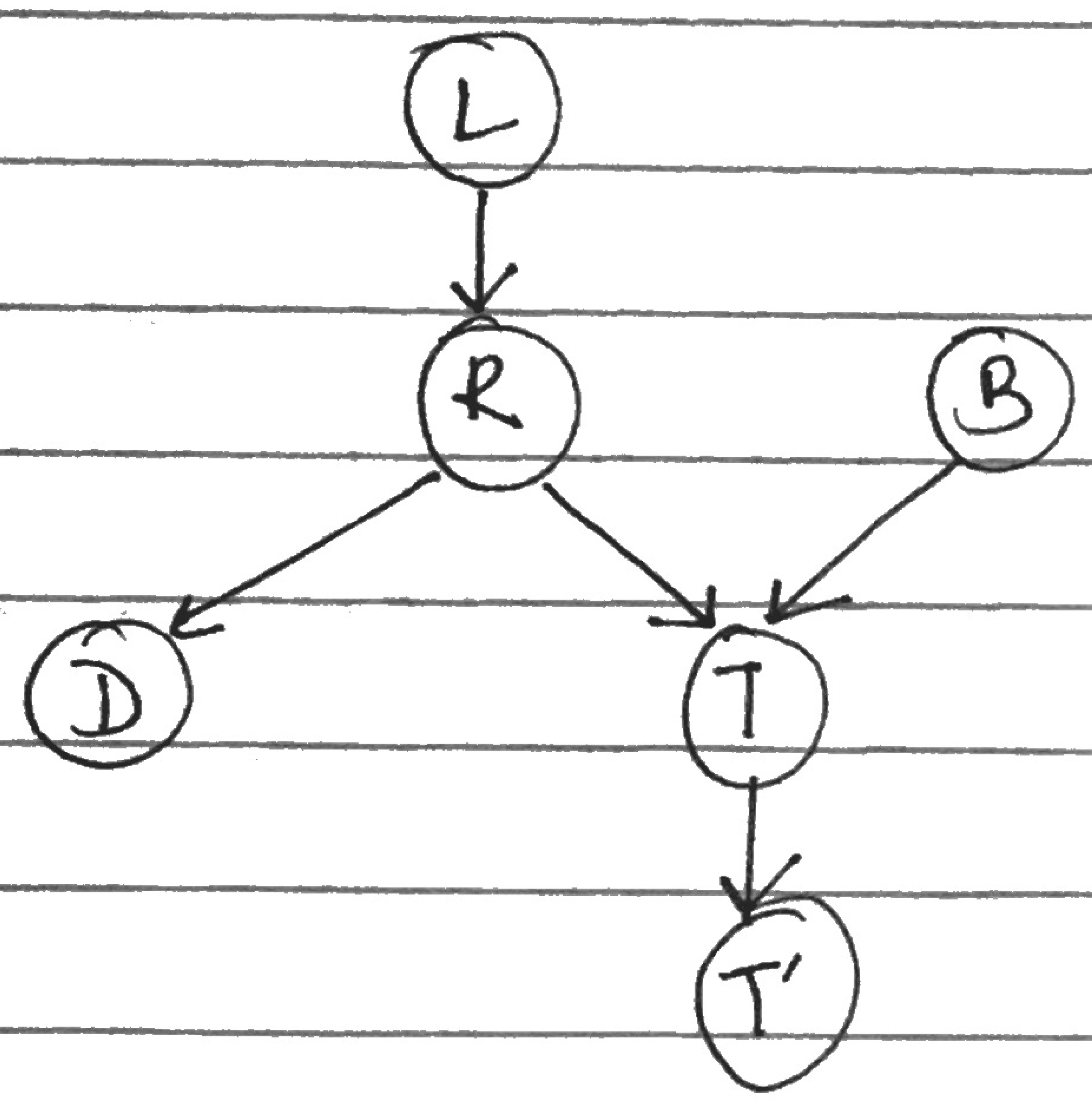
They are not conditionally independent

(iii) $R \perp\!\!\!\perp B \mid T'$

Path $R \rightarrow T \rightarrow B$

we have evidence about T' & T is converging
∴ Transmit & they are not conditionally independent

Questⁿ 4



i) $L \perp\!\!\!\perp T' \mid T$

Path $L \rightarrow R \rightarrow T \rightarrow T'$

R node - diverging & no evidence

∴ R is active

T node : converging and evidence given

∴ T is active

∴ They are not conditionally independent

(ii) $L \perp\!\!\!\perp B$

Path . $L \rightarrow R \rightarrow T \rightarrow B$

R: diverging and no evidence, thus active

T: converging & no evidence ∴ BLOCK

Thus, they are conditionally independent

(iii) $L \perp\!\!\!\perp B \mid T$

Path: $L \rightarrow R \rightarrow T \rightarrow B$

R is diverging & no evidence, thus active

T is converging and evidence give ∴ active

Thus, they are not conditionally independent.

(iv) $L \perp\!\!\!\perp B \mid T'$

Path $L \to R \to T \to B$

R node, diverging & no evidence given ∴ active.

T node, converging.

T' node - evidence given which reflects to T

Thus, they are not conditionally independent

(v) $L \perp\!\!\!\perp B \mid T, R$

Path, $L \to R \to T \to B$

R is divergent & in evidence ∴ Block.

T is convergent & in evidence ∴ non-blocking

Thus they are conditionally independent.

Questⁿ 5 a) $n=20$, $m=2$

Unconstrained fully joint distribution $O(2^n)$ probabilities

$= O(2^{20})$ enteries to store.

b) $n=20$, $m=5$

Unconstrained joint distⁿ $= 5^{20}$ enteries to store

c) $n=500$ & $m=10$.

unconstrained joint distribution $= 10^{500}$ enteries to store

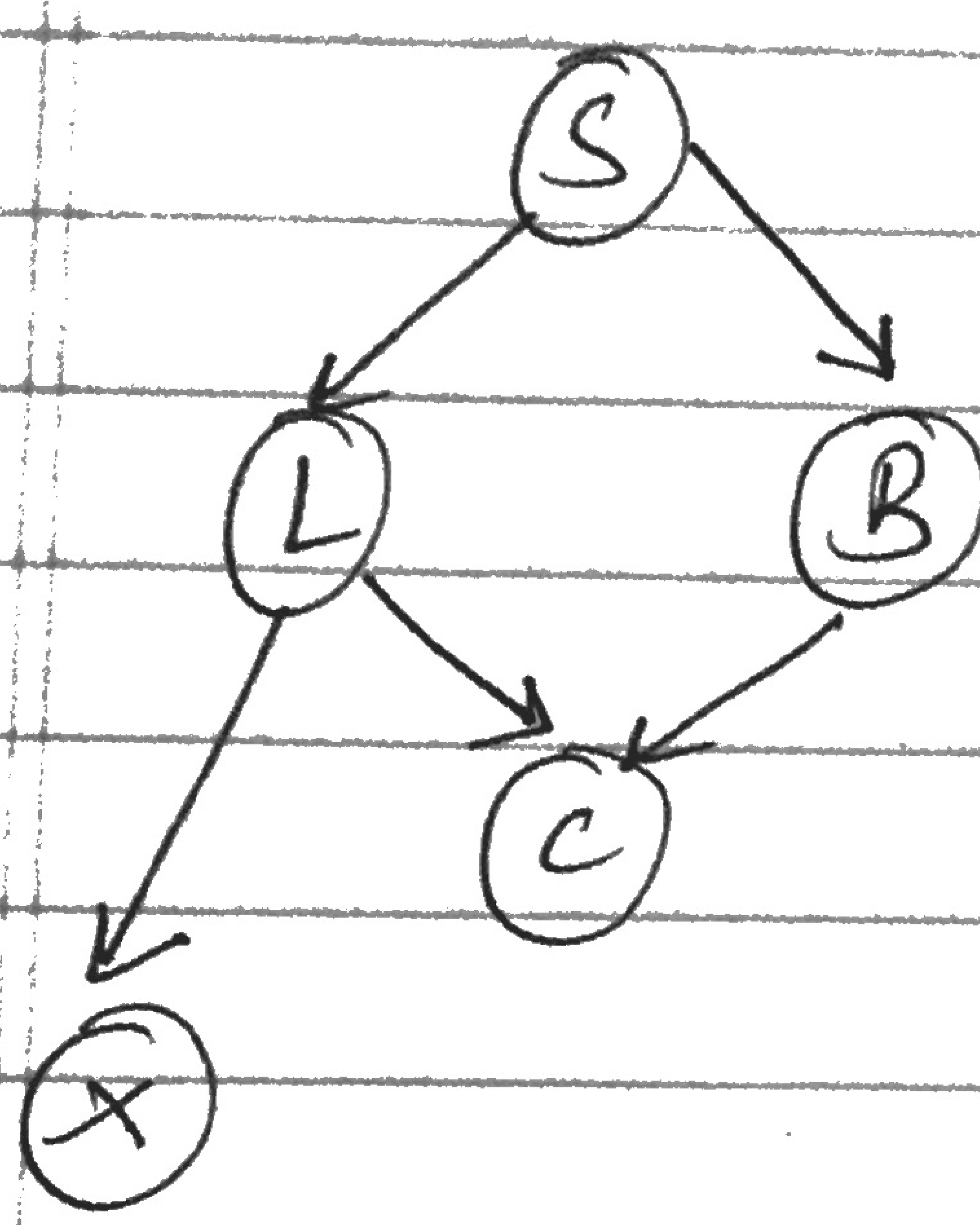
uest^n 5 b) for Bayes, no of enteries = m^{k+1}
k - no. of parents.

a) $n=20$, $m=2$
1 node with 0 parent's = $1 \times 2^{0+1} = 1 \times 2^1$
1 node with 1 parent = $1 \times 2^{1+1} = 1 \times 2^2$
2 nodes with 2 parents = 2×2^3
Remaining with 3 parent's = 16×2^4
Sum = 278
∴ Total table size is 278 enteries

b) $n=20$, $m=5$
1 node with 0 parent = 1×5^1
1 node with 1 parent = 1×5^2
2 nodes with 2 parents = 2×5^3
Remaining with 3 parents = 16×5^4
Sum = 10280
∴ Total table size = 10 280 enteries

c) $n=500$ $m=10$.
1 node with 0 parent = 1×10^1
1 node with 1 parent = 1×10^2
2 nodes with 2 parents = 2×10^3
Remaining with 3 parents = 469×10^4
Sum = 4962110
∴ Total table size = 4962110 enteries

Questⁿ 6.



a) S in evidence
Consider, $X \perp\!\!\!\perp B | S$
Path : X – L – S – B
S - diverging & in evidence
∴ Blocking.
L – diverging & not in evidence
Thus, non-blocking.
X & B are conditionally Independent.

$L \perp\!\!\!\perp B | S$
Path : L – S – B.
S - diverging & evidence ∴ blocking
Thus, L & B conditionally Independent.

When evidence about ~~X~~ S, conditionally independent pair's : (X, B) & (L, B)

b) Evidence about L.
(X, C) – path : X – L – C
L – divergent & in evidence, thus blocking
Thus, (X, C) - conditionally Independent

Consider (X, S)
Path : X – L – S
L - sequential & in evidence ∴ blocking
$X \perp\!\!\!\perp S | L$ conditionally independent.

Next consider $S \perp\!\!\!\perp C | L$
Path : S - L - C
L – sequential node & evidence ∴ Blocking
Thus $S \perp\!\!\!\perp C$ are conditionally independent

consider, $X \perp\!\!\!\perp B | L$
Path : X — L — S — B
L – diverging & evidence, thus blocking
S – diverging & not in evidence
∴ non-blocking
Thus ($X \perp\!\!\!\perp B$) conditionally independent.

when evidence about L, conditionally independent pair's : (X, S) (X, B) (X, C) (S, C)

c) {L, B}
first, (S, C) – Paths : S – L – C & S – B – C
L : sequential node & in evidence, ∴ blocking
B : sequential node & in evidence
hence will block
(S, C) are conditionally independent

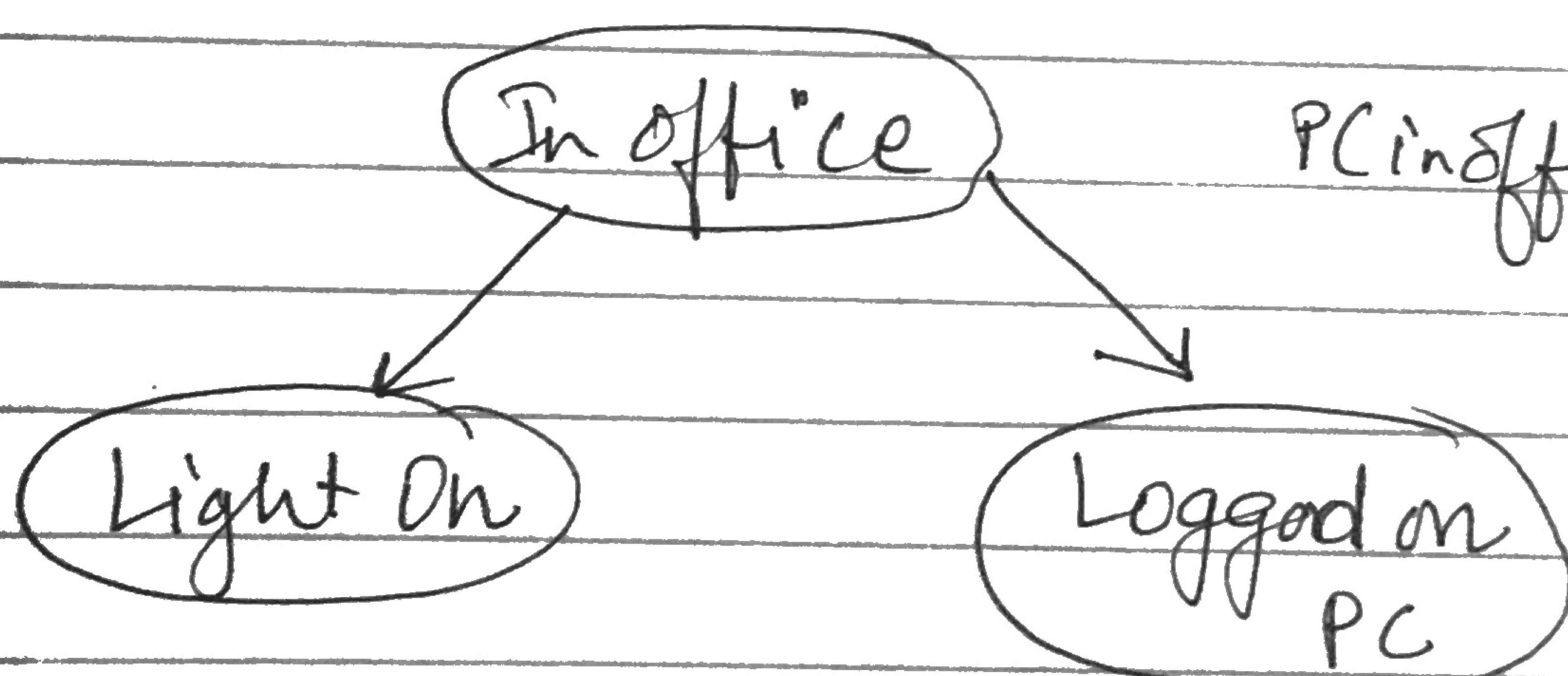
~~Net~~ Next consider, (X, B)
Path : X — L — S — B
X — L — C — B
L : diverging & in evidence, ∴ blocking
S : diverging & not in evidence ∴ non-blocking

C: converging node & no evidence
∴ blocking
Thus, (X, B) are conditionally independent

consider (X, S)
L - node is diverging & in evidence
hence it's blocking
(X, S) conditionally independent

when evidence about {L, B}
(S, C) (X, B) (X, S) are conditionally independent.

est" 7.



P(inoffice) = 0.60

P(Light on | Inoffice) = 0.50

P(logged on Pc | Inoffice) = 0.80

P(lights on | ¬ inoffice) = 0.05

P(logged on | ¬ inoffice) = 0.10

	In office	Not in office
Light ON	0.5	0.05
Logged on PC	0.8	0.1

$P(\text{prior}) = P(\text{in office}) = 0.60$

$P(\text{not in office}) = 0.40$

Questⁿ 7 b) To find,

$P(\text{light on} \mid \text{logged on comp}) = ?$

$\therefore P(\text{light on} \mid \text{logged on comp}) = \dfrac{P(\text{light on, logged on comp})}{P(\text{logged on comp})}$

$= P(\text{light ON, logged on comp} \mid \text{in office})\, P(\text{in office}) + P(\text{light ON, logged on comp} \mid \neg \text{in office})\, P(\neg \text{in office}) \,/\, P(\text{logged on comp})$

$= \dfrac{P(\text{light ON} \mid \text{in office})\, P(\text{logged on comp} \mid \neg \text{in office})\, P(\neg \text{in office})}{P(\text{logged on comp})}$

$= \dfrac{(0.5)(0.8)(0.6) + (0.05)(0.1)(0.4)}{0.52}$

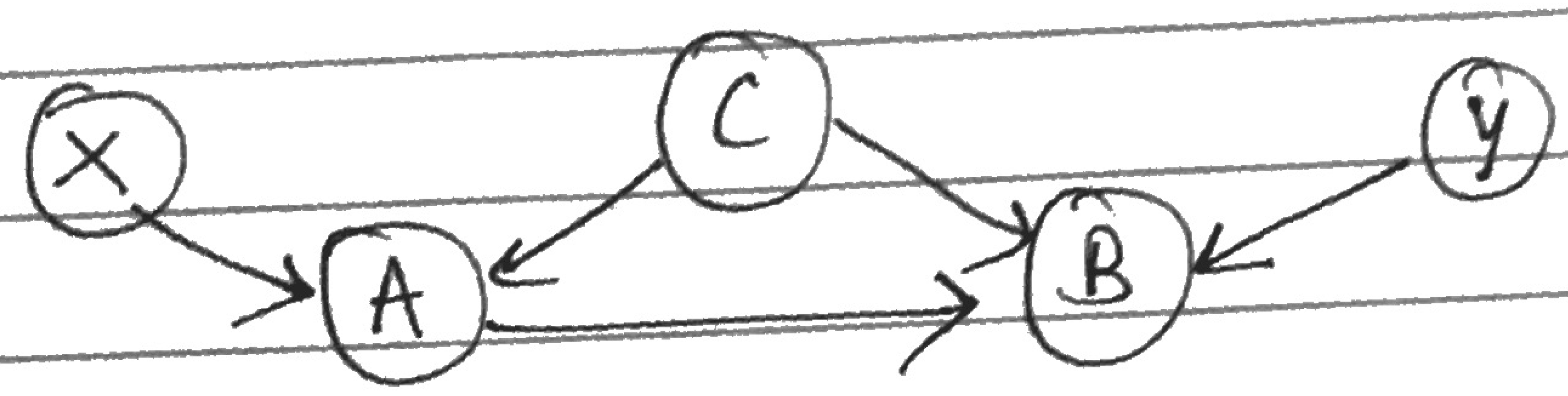
$= 0.465$

As, $P(\text{logged on comp}) = 0.8 \times 0.6 + 0.1 \times 0.4 = 0.52$

$\therefore P(\text{light ON} / \text{logged on comp}) = 0.465$

$= 46.5\%$

Questn - 8.



(i) Consider path X-A-C-B-Y from X to Y
X & Y are d-separate if
↳ we do not have information about A, OR
↳ we do not have information about B, OR
↳ we have information about C.

(ii) Consider path X-A-B-Y from X to Y
X & Y are d-separate if
↳ We have information about A, OR
↳ if we do not have information about B.

Thus, $X \perp\!\!\!\perp Y$ are d-separate.

i.e.
$X_1 = \{X\}$, $X_3 = \{Y\}$, $X_2 = \{C\}$ i.e. $X \perp\!\!\!\perp Y \mid C$
$X_1 = \{X\}$, $X_3 = \{Y\}$, $X_2 = \{A\}$ i.e. $X \perp\!\!\!\perp Y \mid A$.

Questn - 1. Given:
P(C) : Depends on A
P(D): Depends on B & C
$\therefore \Pr(C \wedge D) = P(C \mid A) \cdot P(D \mid B, C)$
$= 0.6 \times 0.8$
$= 0.48$