Question 3

1.

$$\text{Likelihood}(q) = \binom{N}{N_1} q^{N-N_1} (1-q)^{N_1}$$

N = number of students in the somple

N_1 = number saying yes

2.

$$\text{Likelihood}(q) = \prod_{i=1}^{N} q^{1-x_i}(1-q)^{x_i}$$

log likelihood

$$LL(q) = \sum_{i=1}^{N} \left[(1-x_i)\log(q) + x_i \log(1-q)\right]$$

$$= \left[N_0 \log(q) + N_1 \log(1-q)\right]$$

N_0 = Number of Students saying no

take derivative and set to 0

$$0 = \frac{N_0}{q} - \frac{N_1}{1-q}$$

$$\boxed{q = \frac{N_0}{N_0 + N_1} = \frac{N_0}{N}}$$

3.

$$q = \frac{13}{13+17} = 0.433$$

4. $$E(N_1) = \sum_{N_1=0}^{N} N_1 \binom{N}{N_1} q^{N-N_1} (1-q)^{N_1}$$

$$\Sigma(N_1) = N(1-q) \sum_{N_1=1}^{N} \binom{N-1}{N_1-1} q^{(N-1)(N_1-1)} (1-q)^{N_1-1}$$

change $r = N_1 - 1$

$$E(N_1) = N(1-q) \sum_{r=0}^{N-1} \binom{N-1}{r} q^{N-1-r} (1-q)^r$$

$$E(N_1) = N(1-q) [(1-q)+q]^{N-1}$$

students saying yes

$$E(N_1) = N(1-q)$$

Max likelihood estimate

$$E(q) = E\left(\frac{N_0}{N}\right)$$

$$= E\left(1 - \frac{N_1}{N}\right)$$

$$= 1 - E\left(\frac{N_1}{N}\right)$$

$$= 1 - \frac{1}{N} E(N_1)$$

$$= 1 - \frac{1}{N} N(1-q)$$

$$= 1 - (1-q)$$

$$= q$$