Quantum decoherence of excitons in a leaky cavity with quasimodes

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For the excitons in the quantum well placed within a leaky cavity, the quantum decoherence of a mesoscopically superposed states is investigated based on the factorization theory for quantum dissipation. It is found that the coherence of the exciton superposition states will decrease in an oscillating form when the cavity field interacting with the exciton is of the form of quasimode. The effect of the thermal cavity fields on the quantum decoherence of the superposition states of the exciton is studied and it is observed that the higher the temperature of the environment is, the shorter the decoherence characteristic time is.

I. INTRODUCTION

The quantum coherence of superposition states is a basic principle governing the quantum world. The question why macroscopic superposition states are not observable has been raised by Schrödinger in his famous cat paradox [1,2]. Recent experiments demonstrated that the coherent superposition and its losing process can be observed in the laboratory, at least in the mesoscopic domain. In an experiment performed by Wineland's group [3], a superposition of two different coherent states for an ion oscillating in a harmonic potential was created as the Schrödinger cat. In another experiment [4], two coherent states of a cavity mode are also superposed coherently by the atoms passing the cavity with large detuning.

In fact, the ideal coherent superposition state can only live in quantum world which are free from external influence. A real quantum system can rarely be completely isolated from its surrounding environment and is usually coupled to the external world (also called "heat bath") with a large number of degrees of freedom. Not only does the usual coupling allow for an exchange of energy between system and bath causing dissipation [5,6], there also exist different interactions leading to the so-called pure decoherence, the decoherence without energy dissipation. So the quantum mechanical interference effects are destroyed very rapidly due to the two influences of the environment, quantum dissipation and decoherence. When they happen, a superposition state of the system evolves into a statistical mixture state.

In passing years, quantum decoherence has been experimentally studied for various systems by using different techniques, such as the "which-way" experiments using atoms in Bragg scattering [7] and the Aharonov-Bohm electrons in mesoscopic system [8]. Recently, the decoherence of superposed motional states of a trapped single atom, which is induced by coupling the atom or ion to engineered reservoirs, was tested quantitatively in the experiments with cooling atoms and ion [9].

This paper is devoted to studying quantum decoherence in a solid-state-system, specially in the system of excitons. The motivation to investigate such kind of systems is to consider the practical realization of the quantum information processes, such as quantum computing and quantum communications [10]. The coherence is the essential requirement for the quantum information, but the decoherence will result in errors in the process of the computation and quantum communication. So the quantum decoherence is the biggest obstacle for its implement [11]. To overcome quantum decoherence, one should know its dynamic details theoretically and experimentally in various physical systems that are the potential carriers of quantum information. In solid-state system, maximally entangled states of Bell-type for exciton in two coupled quantum dots, and Greenberge-Horre-Zeilinger type for three coupled dots have been investigated [12]. The decoherence effects on the generation of entangled states of exciton have been investigated for the coupled quantum dot systems [13].

We will touch upon this problem for the system of exciton in the quantum well immersed in a leaky cavity with the dissipative cavity fields described by quasimodes [14].

In the case of low excitation, the collective behavior of many-molecule can be described by a bosonic exciton [15]. Therefore the total system can be modeled as a standard harmonic oscillator-bath (or environment) system. The coupling strength between the cavity fields and the excitons obeys the Lorentz spectral distribution around Ω. For such a typical system without any practical concerning, one of the authors (CPS) and his collab-

orators even systematically studied its quantum dissipation and decoherence in association with quantum measurement problem by consider the factorization [6,16–19] and partial factorization [20] of the wave function of the total system. This paper will generalize the partial factorizaton method to discuss quantum decoherence of excitons in a leaky cavity.

The paper is organized as follows. In section II, we first give a theoretical model and do our best to find an analytical solutions of the Heisenberg operators of the cavity fields and the exciton. In section III, by investigating the decoherence of the superposition state of the exciton, we find that, because of the effects of the environment on the system of exciton, the coherence of the superposition states of the exciton is suppressed in a oscillating-decaying way. In section IV, we study the quantum decoherence with the environment of thermal fields at finite temperature. Finally we give our conclusions and some necessary remarks.

II. THEORETICAL MODEL AND EXACT SOLUTIONS

We consider a quantum well placed within a leaky Fabry-Perot cavity [14]. The quantum well lies in the center of the cavity. It has a ideal cubic lattice with N lattice sites. It is so thin that it only contains one molecular layer. We assume that N identical two-level molecules distribute into these lattice sites. All these molecules have equivalent mode positions, so they have the same coupling constant. The density of excitation for the molecules is somewhat low and the inter-molecular interactions are neglected. It is also assumed that the direction of the dipole moment for molecules and wave vectors of the cavity fields are perpendicular to the surface of the quantum well. These molecules interact resonantly with a quasi-mode field with the frequency Ω. By using Dicke model [21], we write down the Hamiltonian for the quantum well and the cavity fields under the rotating wave approximation:

$$H = \hbar\Omega S_z + \hbar \sum_j \omega_j \hat{a}_j^\dagger \hat{a}_j + \hbar \sum_j g_j(\hat{a}_j^\dagger S_+ + \hat{a}_j^\dagger S_-) \quad (1)$$

with the collective operators

$$S_Z = \sum_{n=1}^N s_z(n), \quad S_\pm = \sum_{n=1}^N s_\pm(n), \quad (2)$$

where $s_z(n) = \frac{1}{2}(|e_n\rangle\langle e_n| - |g_n\rangle\langle g_n|)$, $s_+(n) = |e_n\rangle\langle g_n|$ and $s_-(n) = |g_n\rangle\langle e_n|$ are quasi-spin operators of the n-th molecule. Here $|e_n\rangle$ and $|g_n\rangle$ the excited state and the ground state of n-th molecule, Ω is a transition frequency of the isolate molecule. Operators $\hat{a}_j^\dagger(\hat{a}_j)$ are creation (annihilation) operators of the field modes which

labeled by continuous index j and the field frequency of each mode is denoted by ω_j. The coupling constant g_j between the molecules and the cavity fields takes a simple form which is proportional to a Lorentzian

$$g_j = \frac{\eta\Gamma}{\sqrt{(\omega_j - \Omega)^2 + \Gamma^2}}, \quad (3)$$

where η depends on the atomic dipole [22] and Γ is the decay rate of a quasi-mode of the cavity with a frequency Ω. In this paper we restrict our investigation to only one quasi-mode cavity. Because the excitation of the molecules is somewhat low, so we will make a bosonic approximation [15,23,24] $\hat{b} = \frac{S}{\sqrt{N}}$ and $\hat{b}^\dagger = \frac{S^\dagger}{\sqrt{N}}$ with $[\hat{b}, \hat{b}^\dagger] = 1$. Then the interaction between the cavity field and the quantum well occurs via excitons. The Hamiltonian (1) is changed into

$$H = \hbar\Omega\hat{b}^\dagger\hat{b} + \sum \hbar\omega_j \hat{a}_j^\dagger \hat{a}_j + \sum g(\omega_j)(\hat{a}_j^\dagger\hat{b} + \hat{b}^\dagger\hat{a}_j) \quad (4)$$

with $g(\omega_j) = \sqrt{N}g_j$. In the terms of Hamiltonian (4), we may write the Heisenberg equations of motion for the operators of the field modes $\hat{a}_k(\hat{a}_k^\dagger)$ and the excitons $\hat{b}^\dagger(\hat{b})$:

$$\frac{\partial\hat{b}}{\partial t} = -i\Omega\hat{b} - i\sum g(\omega_j)\hat{a}_j, \quad (5a)$$

$$\frac{\partial\hat{a}_j}{\partial t} = -i\omega_j\hat{a}_j - ig(\omega_j)\hat{b}. \quad (5b)$$

The integral equation of Eq.(5b) may be written as

$$\hat{a}_j(t) = \hat{a}_j(0)e^{-i\omega t} - ig(\omega_j)\int_0^t \hat{b}(t')e^{-i\omega_j(t-t')}dt'. \quad (6)$$

We firstly substitute Eq.(6) into Eq.(5a) and eliminate the field operators. Secondly, we let $\hat{b}(t) = \hat{B}(t)e^{-i\Omega t}$ to remove the high-frequency behavior from Eq.(5a). Then the equation of motion for the slowly varied exciton operator is

$$\frac{\partial\hat{B}}{\partial t} = -\int_0^t \hat{B}(t')K(t-t')dt' + \eta(t) \quad (7)$$

with the general memory kernal function $K(t - t') = \sum_j |g(\omega_j)|^2 e^{-i(\omega_j - \Omega)(t-t')}$ and $\eta(t) = -i\sum_j g(\omega_j)\hat{a}_j(0)e^{-i(\omega_j - \Omega)t}$. By use of the Laplace transformation, we could solve $\hat{B}(t)$ and find $\hat{b}(t)$

$$\hat{b}(t) = \hat{B}(t)e^{-i\Omega t} = [u(t)\hat{b}(0) + \sum u_j(t)\hat{a}_j(0)]e^{-i\Omega t} \quad (8a)$$

where the time-dependent coefficients are

$$u(t) = \mathcal{L}^{-1}\{\tilde{u}(p)\} \quad (8b)$$

$$\tilde{u}(p) = \left\{p + \tilde{K}(p)\right\}^{-1} \quad (8c)$$

and

$$u_j(t) = \mathcal{L}^{-1}\left\{\frac{g(\omega_j)}{p + i(\omega_j - \Omega)}\tilde{u}(p)\right\} \qquad (8d)$$

$\mathcal{L}^{-1}$ denotes the inverse Laplace transformation and $\tilde{K}(p)$ is the Laplace transformation of the general memory kernal function $K(t - t')$. Substituting Eqs.(8) into Eq.(6), we have

$$\hat{a}_j(t) = e^{-i\omega_j t}\hat{a}_j(0) + v_j(t)\hat{b}(0) + \sum v_{j,j'}(t)\hat{a}_{j'}(0) \quad (9)$$

here, the time-dependent coefficients $v_j(t)$ and $v_{j,j'}(t)$ are determined by

$$v_j(t) = -ig(\omega_j)e^{-i\omega t}\int_0^t u(t')e^{i(\omega_j - \Omega)t'}\mathrm{d}t', \qquad (10a)$$

$$v_{j,j'}(t) = -g(\omega_j)e^{-i\omega t}\int_0^t u_j(t')e^{i(\omega_j - \Omega)t'}\mathrm{d}t'. \qquad (10b)$$

In order to obtain above time-dependent coefficients , we begin to solve the function $K(t - t')$ by changing the sum $\sum_j$ into the integration $\frac{L}{\pi c}\int_0^\infty \mathrm{d}\omega_j$ which L is the length of the cavity and c is the speed of the light in the vacuum [25], that is

$$K(t - t') = \frac{\eta^2\Gamma^2 NL}{\pi c}\int_0^\infty \frac{e^{-i(\omega_j - \Omega)(t - t')}}{(\omega_j - \Omega)^2 + \Gamma^2}\mathrm{d}\omega_j. \qquad (11)$$

If we assume that Ω is much larger than all other quantities of the dimension of frequency, and Γ is small quantity. We may adopt to the standard approximation of extending the lower limit of the integral Eq.(11) to $-\infty$. By integrating eq.(11) we have:

$$K(t - t') = M\Gamma e^{-\Gamma|t - t'|} \qquad (12)$$

with $M = \frac{N\eta^2 L}{c}$. In the following calculation, we only need time-dependent coefficients $u(t)$ and $v_j(t)$. So after we give the Laplace form of the function $K(t - t')$, we will obtain $u(t)$ by use of Eqs.(8b-8c) as following

$$u(t) = [cos(\frac{\Theta}{2}t) + \frac{\Gamma}{\Theta}sin(\frac{\Theta}{2}t)]e^{-\frac{\Gamma}{2}t} \qquad (13)$$

where $\Theta = \sqrt{4M\Gamma - \Gamma^2}$. $v_j(t)$ also can be obtained by integral eq.(10a) as

$$v_j(t) = -ig(\omega_j)(\frac{1 - i\frac{\Gamma}{\Theta}}{2})\frac{e^{i(\frac{\Theta}{2} - \Omega)t - \frac{\Gamma}{2}t} - e^{-i\omega_j t}}{i(\frac{\Theta}{2} + \omega_j - \Omega) - \frac{\Gamma}{2}}$$
$$-ig(\omega_j)(\frac{1 + i\frac{\Gamma}{\Theta}}{2})\frac{e^{-i(\frac{\Theta}{2} + \Omega)t - \frac{\Gamma}{2}t} - e^{-i\omega_j t}}{i(\omega_j - \frac{\Theta}{2} - \Omega) - \frac{\Gamma}{2}} \qquad (14)$$

III. DECOHERENCE OF MESOSCOPIC SUPERPOSITION STATES OF EXCITON

If we prepare a superposition state for the system of the exciton, that is, the exciton initially is in the state

$C|\alpha_1\rangle + D|\alpha_2\rangle$ where α_1 or α_2 is a coherent state for the exciton, and the cavity fields are in the vacuum states $\prod_j |0\rangle_j$ (zero temperature). Thus the whole initial state for the exciton and the cavity fields is the product of the initial state of the exciton and the cavity fields.

$$|\Psi(0)\rangle = (C|\alpha_1\rangle + D|\alpha_2\rangle)\bigotimes\prod_j |0\rangle_j. \qquad (15)$$

In order to discuss the coherence properties for the system of excitons, we have to calculate the time evolution of the wave function. For this purpose, we firstly write a state for the whole system at any time t by virtue of the evolution operators of the whole system $U(t) = e^{-i\frac{Ht}{\hbar}}$:

$$|\Psi(t)\rangle = U(t)|\Psi(0)\rangle = U(t)(C|\alpha_1\rangle + D|\alpha_2\rangle)\bigotimes\prod_j |0\rangle_j. \qquad (16)$$

Because for any coherent state of the exciton we have

$$|\alpha\rangle = exp(\alpha b^\dagger(0) - \alpha^* b(0))|0\rangle \qquad (17)$$

So we have

$$|\Psi(t)\rangle = U(t)[Ce^{\alpha_1 b^\dagger(0) - \alpha_1^* b(0)} + De^{\alpha_2 b^\dagger(0) - \alpha_2^* b(0)}]|0\rangle$$
$$\bigotimes\prod_j |0\rangle_j \qquad (18)$$

Considering the role of the evolution operator $U(t)$, that is, $U^\dagger(t)OU(t) = O(t)$ and $U(t)|0\rangle = |0\rangle$ for any operator O. We interpolate operator $U^\dagger(t)U(t)$ into Eq.(18) and use the property of the evolution operator $U(t)$. we easily obtain the wave function of the whole system at any time t

$$|\Psi(t)\rangle = C|\alpha_1 u^*(t)\rangle\bigotimes\prod_j |\alpha_1 u_j^*(t)\rangle_j$$
$$+ D|\alpha_2 u^*(t)\rangle\bigotimes\prod_j |\alpha_2 u_j^*(t)\rangle_j \qquad (19)$$

We could calculate the reduced density matrix of the exciton system at any time t by $Tr_R(|\Psi(t)\rangle\langle\Psi(t)|)$, and obtain the decoherence factor by calculating one of the non-diagonal elements, such as

$$F(t) = \prod_j\langle\alpha_1^* u_j(t)|\alpha_2 u_j^*(t)\rangle$$
$$= e^{(-\frac{1}{2}|\alpha_1|^2 - \frac{1}{2}|\alpha_2|^2 + \alpha_1^*\alpha_2)\sum_j |u_j(t)|^2}. \qquad (20)$$

We know that $[\hat{b}(t), \hat{b}^\dagger(t)] = 1$. From Eq.(8a) and its complex conjugate, we have

$$\sum_j |u_j(t)|^2 = 1 - |u(t)|^2$$
$$= 1 - |cos(\frac{\Theta}{2}t) + \frac{\Gamma}{\Theta}sin(\frac{\Theta}{2}t)|^2 e^{-\Gamma t}. \qquad (21)$$

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Eq.(20) becomes into:

$$F(t) = e^{(-\frac{1}{2}|\alpha_1|^2 - \frac{1}{2}|\alpha_2|^2 + \alpha_1^*\alpha_2)(1-|cos(\frac{\Theta}{2}t)+\frac{\Gamma}{\Theta}sin(\frac{\Theta}{2}t)|^2 e^{-\Gamma t})} \tag{22}$$

so with the evolution of the time, the coherence of two coherent states of the excitons is suppressed. It is evident when the time $t \to \infty$, because of the environment effect, the energy of the exciton will be dissipated and states of the exciton will turn into vacuum states. We consider a behavior of the short time, that is $\gamma t, \frac{\Theta}{2}t \ll 1$, then the decoherence factor is

$$F(t) = e^{(-\frac{1}{2}|\alpha_1|^2 - \frac{1}{2}|\alpha_2|^2 + \alpha_1^*\alpha_2)\Gamma t} \tag{23}$$

The characteristic time t_d of the decoherence of the exciton superposition states is $[(\frac{1}{2}|\alpha_1|^2 + \frac{1}{2}|\alpha_2|^2 - \alpha_1^*\alpha_2)\Gamma]^{-1}$. The coherent properties of the exciton states is determined by their initial superposition states and the decay rate of the quasimode. The smaller the superposition of $|\alpha_1\rangle$ and $|\alpha_2\rangle$ is, The shorter the decoherence time t_d is.

Now we consider a special case in which many people are interested [26–29], that is, the system is initially in the odd or even coherent states of the exciton. We set $\alpha_1 = -\alpha_2 = \alpha$, then the decoherence factor is

$$F(t) = e^{-2|\alpha|^2(1-|cos(\frac{\Theta}{2}t)+\frac{\Gamma}{\Theta}sin(\frac{\Theta}{2}t)|^2 e^{-\Gamma t})}$$

In fig.1, we give a sketch of the decoherence factor as the function of the time t in a set of parameters. We find that the coherence of the exciton system will decrease in the oscillating decay form. It is similar to the case of the experiment [9]

In terms of the definition of the coherent state of the exciton, we know that $|\alpha|^2$ is the mean number of the excitons. Hence by adjusting the initial number of the excitons we could completely determined the decoherence time and. The smaller the mean number of the exciton is, the longer the decoherence time.
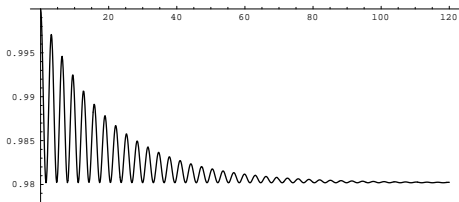

FIG. 1. $\hbar\Gamma$=0.05 mev, $|\alpha|^2$=0.01, $\hbar$M=20 mev

IV. TEMPERATURE EFFECT ON COHERENCE

In this section, we will discuss the temperature effect on the coherent properties of the exciton superposition states. For a single mode thermal cavity field of the frequency ω_j, its variables are in the thermal equilibrium mixture of states. Its density operator could be given using Fock states

$$\hat{\rho}_j = \frac{1}{\langle n_j \rangle + 1} \sum_{n_j} (\frac{\langle n_j \rangle}{\langle n_j \rangle + 1})^{n_j} |n_j\rangle\langle n_j|. \tag{24}$$

where the average thermal photon number $\langle n_j \rangle = [exp(\frac{\hbar\omega_j}{k_B T})-1]^{-1}$. k_B is Boltzman constant. T is temperature of the thermal fields. under the P representation eq.(19) is rewritten as

$$\hat{\rho}_j = \int \frac{d^2\alpha_j}{\pi\langle n_j \rangle} e^{-\frac{|\alpha_j|^2}{\langle n_j \rangle}} |\alpha_j\rangle\langle\alpha_j| \tag{25}$$

It is clear that the P-representation of the thermal fields is given by a Gaussian distribution. If we assumed that the cavity fields initially are in the thermal states, then the reduced density operator of the cavity fields is the multi-mode extension of the thermal field operators, namely, $\hat{\rho}_{bath} = \prod_j \hat{\rho}_j$. The whole density operator are

$$\hat{\rho} = \hat{\rho}_e \bigotimes \hat{\rho}_{bath} \tag{26}$$

with density operators of exciton

$$\hat{\rho}_e = (C|\alpha_1\rangle + D|\alpha_2\rangle)(\langle\alpha_1|C^* + \langle\alpha_2|D^*) \tag{27}$$

In order to investigate the effect of the thermal fields on coherence of the superposition states of the exciton system , we have to calculate the wave function of the whole system at any time t. So we set

$$|\psi(t)\rangle = U(t)(|\alpha\rangle \bigotimes |\alpha_j\rangle) \tag{28}$$

By using the same method as last section, we could obtain

$$|\psi(t)\rangle = |u^*(t)\alpha + \sum_j u^*(t)_j \lambda_j\rangle$$
$$\bigotimes |e^{i\omega_j t}\lambda_j + v_j^*(t)\alpha + \sum_{s \neq j} v^*(t)_{s,j}\lambda_s\rangle_j \tag{29}$$

By virtue of normalized condition

$$[\hat{b}(t), \hat{a}_j^\dagger(t)] = 0, \tag{30}$$

we could get the decoherence factor by calculating the reduced density matirx of the exciton system.

$$F(t) = Tr_R \hat{\rho}(t) = e^{(-\frac{1}{2}|\alpha_1|^2 - \frac{1}{2}|\alpha_2|^2 + \alpha_1^*\alpha_2)(1-|u(t)|^2} \times$$
$$\prod_j \int d^2\lambda_j \frac{1}{\pi\langle n_j \rangle} e^{(-\frac{|\lambda_j|^2}{\langle n_j \rangle} + \zeta\lambda_j^* + \zeta^*\lambda_j)} \tag{31}$$

with

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$$\zeta = \frac{1}{2}(\alpha_2 - \alpha_1)u(t)v_j^* \lambda_j^* \qquad (32)$$

Combining

$$\frac{1}{\pi}\int d^2\lambda_j e^{(-\eta|\lambda_j|^2 - \zeta\lambda_j^* + \zeta^*\lambda)} = \frac{1}{\eta}e^{-\frac{1}{\eta}|\zeta|^2} \quad \text{For } \mathrm{Re}(\eta)\rangle 0 \qquad (33)$$

we get the decoherence factor as following:

$$F(t) = exp[(-\frac{1}{2}|\alpha_1|^2 - \frac{1}{2}|\alpha_2|^2 + \alpha_1^*\alpha_2)(1 - |u(t)|^2]$$

$$\times exp[-\frac{1}{4}|\alpha_1 - \alpha_2|^2 |u(t)|^2 \beta(t,T)] \qquad (34a)$$

with

$$\beta(t,T) = \sum_j |v_j(t)|^2 \langle n_j \rangle \qquad (34b)$$

We may transform $\sum_j$ of eq.(34b) into $\frac{L}{\pi c}\int_0^\infty d\omega$ as reference [25]

$$\beta(t,T) = \frac{L}{\pi c}\int_0^\infty |v_j(t)|^2 \langle n_j \rangle d\omega_j \qquad (35)$$

We find that the integration in Eq.(35) is so strongly peaked at the near $\omega_j \approx \Omega$, so we may remove the slowly vary factor $\langle n_j \rangle$. Although we may calculate an exact integration of Eq.(35) (for the details please see the appendix A), we are more interested in the limit of the short time, we finish the integral of Eq.(35) and obtain

$$\beta(T,t) = \bar{n}\mathcal{D}\Gamma t \qquad (36)$$

with $\bar{n} = (e^{\frac{\hbar\Omega}{k_B T}} - 1)^{-1}$ and $\mathcal{D}$ is given in (A6). So in the case of the thermal fields, we only keep the first order terms of Γt and $\frac{\Theta}{2}t$, the decoherence factor of superposition states of the exciton system is

$$F(t) = exp[(-\frac{1}{2}|\alpha_1|^2 - \frac{1}{2}|\alpha_2|^2 + \alpha_1^*\alpha_2)\Gamma t] \times$$

$$\times exp[-\frac{\bar{n}\mathcal{D}}{4}|\alpha_1 - \alpha_2|^2\Gamma t] \qquad (37)$$

Under the condition of the high temperature, The decoherence factor is simplified

$$F(t) = exp[(-\frac{1}{2}|\alpha_1|^2 - \frac{1}{2}|\alpha_2|^2 + \alpha_1^*\alpha_2)\Gamma t]$$

$$\times exp[-\frac{\mathcal{D}}{4}|\alpha_1 - \alpha_2|^2\frac{Tk_B}{\hbar\Omega}\Gamma t] \qquad (38)$$

So the characteristic time t_d of the decoherence of the exciton system is

$$t_d = [(\frac{1}{2}|\alpha_1|^2 + \frac{1}{2}|\alpha_2|^2 - \alpha_1^*\alpha_2 + \frac{\mathcal{D}}{4}|\alpha_1 - \alpha_2|^2\frac{Tk_B}{\hbar\Omega})\Gamma]^{-1} \qquad (39)$$

It is shown that the higher the temperature of the environment is, the short the time of decoherence of the exciton system is. if the exciton system is initial in the odd or even coherent states, then the characteristic time is $t_d = [|\alpha_1|^2(2 + \frac{\mathcal{D}Tk_B}{\hbar\Omega})\Gamma]^{-1}$ So the decoherence time becomes more shorter when the cavity fields are initially in the thermal radiation.

V. CONCLUSIONS

The decoherence of the mesoscopic superposition states for the exciton in a quasimode cavity is investigated. We find that the coherence of the superposition states of exciton is reduced by the interaction between the cavity fields and the excitons. For long time we find that because of the environment effect, the energy of the exciton will dissipated and the states of the exciton will turn into turn into vacuum states. We find for a short time process, the smaller the superposition of two coherent states of exciton is, the shorter the decoherence time is. The temperature effect on coherence of the system of exciton also is studied by virtue of the P representation. We find that the more higher of the temperature of the environment is, the more shorter of the decoherence time is.

Appendix A   Solution of $\beta(T,t)$

In this appendix, we will give an integration of $\beta(T,t)$ in details. From Eq.(14) and Eq.(35), we have

$$\beta(T,t) = \left\{ \bar{n}\frac{L\eta^2\Gamma^2(\Theta - i\Gamma)^2}{4\pi\Theta^2 c}\int_0^\infty \frac{1 + e^{-\frac{\Gamma}{2}t + i\frac{\Theta}{2}t} - e^{i(\omega_j + \frac{\Theta}{2} - \Omega)t - \frac{\Gamma}{2}t} - e^{-i(\omega_j - \frac{\Theta}{2} - \Omega)t - \frac{\Gamma}{2}t}}{[(\omega_j - \Omega)^2 + \Gamma^2][(\omega_j + \frac{\Theta}{2} - \Omega) + i\frac{\Gamma}{2}][(\omega_j - \frac{\Theta}{2} - \Omega) - i\frac{\Gamma}{2}]}d\omega_j + h.c. \right\}$$

$$+ \frac{\bar{n}L\eta^2\Gamma^2[\Theta^2+\Gamma^2]}{4\pi\Theta^2 c}\int_0^\infty \frac{1+e^{-\Gamma t}-2e^{-\frac{\Gamma}{2}}cos(\omega_j+\frac{\Theta}{2}-\Omega)t}{[(\omega_j-\Omega)^2+\Gamma^2][(\omega_j+\frac{\Theta}{2}-\Omega)^2+\frac{\Gamma^2}{4}]}d\omega_j$$

$$+ \frac{\bar{n}L\eta^2\Gamma^2[\Theta^2+\Gamma^2]}{4\pi\Theta^2 c}\int_0^\infty \frac{[1+e^{-\Gamma t}-2e^{-\frac{\Gamma}{2}}cos(\omega_j-\frac{\Theta}{2}-\Omega)t]}{[(\omega_j-\Omega)^2+\Gamma^2][(\omega_j-\frac{\Theta}{2}-\Omega)^2+\frac{\Gamma^2}{4}]}d\omega_j \tag{1}$$

We set $x = \omega_j - \Omega$, and extend the lower limit of the integral Eq.(A1) to $-\infty$ as we do in the section **II**, then finish following some integral formulations

$$\int_{-\infty}^\infty \frac{dx}{[x^2+\Gamma^2][x+\frac{\Theta}{2}+i\frac{\Gamma}{2}][x-\frac{\Theta}{2}-i\frac{\Gamma}{2}]} = \frac{\pi}{\Gamma(\frac{\Theta}{2}+i\frac{3}{2}\Gamma)(i\frac{1}{2}\Gamma-\frac{\Theta}{2})} + \frac{i2\pi}{(2\frac{\Theta}{2}+i\Gamma)(\frac{\Theta}{2}+i\frac{3}{2}\Gamma)(-i\frac{1}{2}\Gamma+\frac{\Theta}{2})} \tag{2}$$

$$\int_{-\infty}^\infty \frac{e^{i(x+\frac{\Theta}{2})t-\frac{\Gamma}{2}t}dx}{[x^2+\Gamma^2][x+\frac{\Theta}{2}+i\frac{\Gamma}{2}][x-\frac{\Theta}{2}-i\frac{\Gamma}{2}]} = \frac{\pi e^{(i\frac{\Theta}{2}-\frac{3}{2}\Gamma)t}}{\Gamma(\frac{\Theta}{2}+i\frac{3}{2}\Gamma)(i\frac{1}{2}\Gamma-\frac{\Theta}{2})} + \frac{i2\pi e^{(i2\frac{\Theta}{2}-\Gamma)t}}{(2\frac{\Theta}{2}+i\Gamma)(\frac{\Theta}{2}+i\frac{3}{2}\Gamma)(-i\frac{1}{2}\Gamma+\frac{\Theta}{2})} \tag{3}$$

$$\int_{-\infty}^\infty \frac{cos(x\pm\frac{\Theta}{2})t]}{[x^2+\Gamma^2][(x\pm\frac{\Theta}{2})^2+\frac{\Gamma^2}{4}]}dx = \frac{\pi e^{-\Gamma t}[(\frac{\Theta}{2}^2-\frac{3}{4}\Gamma^2)cos(\frac{\Theta}{2}t)+2\Gamma\frac{\Theta}{2}sin(\frac{\Theta}{2}t)]+2\pi e^{-\frac{\Gamma}{2}t}(\frac{\Theta}{2}^2+\frac{3}{4}\Gamma^2)}{\Gamma(\frac{\Theta}{2}^2+\frac{9}{4}\Gamma^2)(\frac{\Theta}{2}^2+\frac{\Gamma^2}{4})} \tag{4}$$

$$\int_{-\infty}^\infty \frac{dx}{[x^2+\Gamma^2][(x\pm\frac{\Theta}{2})^2+\frac{\Gamma^2}{4}]} = \frac{\pi(3\frac{\Theta}{2}\pm i\frac{3}{2}\Gamma)}{\Gamma(\frac{\Theta}{2}\pm i\frac{\Gamma}{2})(\frac{\Theta}{2}^2+\frac{9}{4}\Gamma^2)} \tag{5}$$

from eqs.(A1-A5), we can obtain obtain an exact expression of $\beta(T,t)$, but we are only interested in the behavior of the short time, because after evolution of the long time, all states of the system will return to vacumm reduced by the eviornment. So in the condition of $\Gamma t \ll 1$ and $\frac{\Theta}{2}t \ll 1$, we only keep the first order small quantity, then we have

$$\beta(T,t) = \bar{n}\left[\frac{L\eta^2\Gamma(2\frac{\Theta}{2}^2+\Gamma^2)(\frac{\Theta}{2}^2+2\Gamma^2)}{2c(\frac{\Theta}{2}^2+\frac{\Gamma^2}{4})^2(\frac{\Theta}{2}^2+\frac{9}{4}\Gamma^2)}\right]\Gamma t = \bar{n}\mathcal{D}\Gamma t \tag{6}$$

which is needed in eq.(36).

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