Appendices of "Heat Transfer 2: Radiation and Exchangers"

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Appendices

A.1. Physical properties of some materials

	ρ kg m^{-3}	c_p J kg^{-1} K^{-1}	λ W m^{-1} K^{-1}		ρ kg m^{-3}	c_p J kg^{-1} K^{-1}	λ W m^{-1} K^{-1}
Metals and alloys				**Construction materials**			
Carbon steel	7,833	465	54	**Slate**	2,400	879	2.2
Stainless steel 15% Cr, 10% Ni	7,864	460	20	**Basalt**	2,850	881	1.6
Stainless steel 18% Cr, 8% Ni	7,816	460	16.3	**Cavernous concrete**	1,900	879	1.4
Stainless steel 25% Cr, 20% Ni	7,864	460	13	**Solid concrete**	2,300	878	1.75
Alumina			29	**Bitumen (cardboard)**	1,050	1,305	0.23
Aluminum	2,707	896	204	**Light hardwoods (dry)**	525	1,250	0.15
Silver	10,525	234	407	**Medium–heavy hardwoods (dry)**	675	1,250	0.23
Bronze 75% Cu, 25%Sn	8,800	377	188	**Very light hardwoods (dry)**	375	1,250	0.12
Bronze 92% Cu, 8% Al	7,900	377	71	**Light softwoods (dry)**	375	1,250	0.12
Graphite carbon	2,250	707	147	**Medium-weight softwoods (dry)**	500	1,250	0.15
Silicon carbide			13	**Very light softwoods (dry)**	375	1,250	0.12
Bronze 75% Cu, 25%Sn	2,118	7,160	449	**Terracotta brick**	1,800	878	1.15
Bronze 92% Cu, 8% Al	8,922	410	22.7	**Hard limestone**	2,450	882	2.4

	ρ (kg m^{-3})	c_p (J kg^{-1} K^{-1})	λ (W m^{-1} K^{-1})
Metals and alloys			
Copper	8,954	383	386
Cupronickel 70%Cu, 30% Ni	8,900	377	29.3
Duralumin	2,787	883	164
Tin	7,304	226	64
Iron	7,870	452	73
Melting	7,849	460	59
Brass 70%Cu, 30%Zn	8,522	385	111
Magnesium	1,740	1,004	151
Gold	19,300	128	312
Platinum	21,400	140	69
Lead	11,373	130	35
Liquid sodium	930	1,381	84.5
Titanium	4,500	523	20.9
Tungsten	19,350	134	163
Zinc	7,144	384	112
Miscellaneous materials			
Asbestos	575	1,046	0.15
Asphalt	2,115	920	0.062
Rubber (natural)	1,150		0.28
Rubber (vulcanized)	1,100	2,010	0.13
Cardboard	86	2,030	0.048
Leather	998		0.159
Ice	920	2,040	1.88
Plexiglass	1,190	1,465	0.19
Porcelain	2,400	1,088	1.035
Polyethylene	929	1,830	0.46
PVC	1,459	930	0.19
Sand	1,515	800	0.2-1.0
Teflon	2,170	1,004	0.25
Wet land	1,900	2,000	2
Dry land	1,500	1,900	1
Glass	2,300	837	1.05
Pyrex glass	2,220	728	1.13

	ρ (kg m^{-3})	c_p (J kg^{-1} K^{-1})	λ (W m^{-1} K^{-1})
Construction materials			
Cotta brick	1,650	879	1
Hard limestone	2,400	875	2.4
Soft limestone	400	3,000	0.12
Floor tile	500	3,000	0.15
Okoumé plywood	2,600	881	3
Pine plywood	1,800	889	0.7
Granite	2,500	880	2.6
Gravel (bulk)	2,350	881	1.1
Sandstone	2,700	881	2.5
Lava stone	1,440	840	0.48
Marble	2,400	879	2.2
Insulating materials			
Balsa	140		0.054
Cotton	80	1,300	0.06
Kapok			0.035
Rockwool	20	880	0.047
	55	880	0.038
	135	880	0.041
Glass wool	8	875	0.051
	10	880	0.045
	15	880	0.041
	40	880	0.035
Expanded cork	120	2,100	0.044
Carpet	200	1,300	0.06
Polyurethane (foam)	32	1,300	0.03
	50	1,360	0.035
	85	1,300	0.045
PVC (rigid foam)	30	1,300	0.031
	40	1,300	0.041
Expanded polystyrene	12	1,300	0.047
	14	1,300	0.043
	18	1,300	0.041
Styrofoam	30		0.032

A.2. Physical properties of air and water

Properties of water at saturation						
T	ρ	c_p	λ	10^4 µ	10^7 a	Pr
°C	kg m^{-3}	J kg^{-1} K^{-1}	W m^{-1} K^{-1}	Pa s^{-1}	m^2 s^{-1}	
0	1,002	4,218	0.552	17.90	1.31	13.06
20	1,001	4,182	0.597	10.10	1.43	7.02
40	995	4,178	0.628	6.55	1.51	4.34
60	985	4,184	0.651	4.71	1.55	3.02
80	974	4,196	0.668	3.55	1.64	2.22
100	960	4,216	0.680	2.82	1.68	1.74
120	945	4,250	0.685	2.33	1.71	1.45
140	928	4,283	0.684	1.99	1.72	1.24
160	910	4,342	0.680	1.73	1.73	1.10
180	889	4,417	0.675	1.54	1.72	1.00
200	867	4,505	0.665	1.39	1.71	0.94
220	842	4,610	0.652	1.26	1.68	0.89
240	816	4,756	0.635	1.17	1.64	0.88
260	786	4,949	0.611	1.08	1.58	0.87
280	753	5,208	0.580	1.02	1.48	0.91
300	714	5,728	0.540	0.96	1.32	1.02

Properties of air at 1 atm						
T	ρ	c_p	λ	10^4 µ	10^7 a	Pr
°C	kg m^{-3}	J kg^{-1} K^{-1}	W m^{-1} K^{-1}	Pa s^{-1}	m^2 s^{-1}	
0	1.292	1,006	0.0242	1.72	1.86	0.72
20	1.204	1,006	0.0257	1.81	2.12	0.71
40	1.127	1,007	0.0272	1.90	2.40	0.70
60	1.059	1,008	0.0287	1.99	2.69	0.70
80	0.999	1,010	0.0302	2.09	3.00	0.70
100	0.946	1,012	0.0318	2.18	3.32	0.69
120	0.898	1,014	0.0333	2.27	3.66	0.69
140	0.854	1,016	0.0345	2.34	3.98	0.69
160	0.815	1,019	0.0359	2.42	4.32	0.69
180	0.779	1,022	0.0372	2.50	4.67	0.69
200	0.746	1,025	0.0386	2.57	5.05	0.68
220	0.700	1,028	0.0399	2.64	5.43	0.68
240	0.688	1,032	0.0412	2.72	5.80	0.68
260	0.662	1,036	0.0425	2.79	6.20	0.68
280	0.638	1,040	0.0437	2.86	6.59	0.68
300	0.616	1,045	0.0450	2.93	6.99	0.68

Correlations between 0 and 100°C (T, temperature in °C).

For air:

$$\rho = \frac{353}{T+273} \text{ (kg m}^{-3}\text{)}$$

$$c_p = 1{,}008 \text{ (J kg}^{-1}\text{ K}^{-1}\text{)}$$

$$\lambda = 7.57 \times 10^{-5}\, T + 0.0242 \text{ (W m}^{-1}\text{ K}^{-1}\text{)}$$

$$\mu = 10^{-5}(0.0046\, T + 1.7176) \text{ (kg m}^{-1}\text{ s}^{-1}\text{)}$$

$$a = 10^{-5}(0.0146\, T + 1.8343) \text{ (m}^2\text{ s}^{-1}\text{)}$$

$$Pr = -2.54 \times 10^{-4}\, T + 0.7147$$

$$\beta \approx \frac{1}{T+273} \text{ (K}^{-1}\text{)}$$

For water:

$$\rho = -0.00380\,T^2 - 0.0505\,T + 1002.6 \ \ (\text{kg m}^{-3})$$

$$c_p = 4{,}180\ (\text{J kg}^{-1}\ \text{K}^{-1})$$

$$\lambda = -9.87 \times 10^{-6} T^2 + 2.238 \times 10^{-3} T + 0.5536\ \ (\text{W m}^{-1}\ \text{K}^{-1})$$

$$\mu = 10^{-4} \times \frac{0.0003354\,T^2 - 0.07377\,T + 17.9}{8.765 \times 10^{-5} T^2 + 0.03032\,T + 1}\ \ (\text{kg m}^{-1}\ \text{s}^{-1})$$

$$a = 10^{-7} \times (-0.00360\,T + 1.340)\ \ (\text{m}^2\ \text{s}^{-1})$$

$$Pr = \frac{-0.0037\,T^2 + 1.387\,T + 13.06}{0.005297\,T^2 + 0.1241\,T + 1}$$

$$\frac{g\beta\rho^2 c_p}{\lambda\mu} = (0.0105\,T^2 + 0.477\,T - 0.0363) \times 10^9\ \ (\text{K}^{-1}\ \text{m}^{-3})$$

$$\log_{10}[p_{sat}(T + 273)] = 20.3182 - \frac{2{,}795}{T + 273} - 3.868 \log_{10}(T + 273)$$
(mmHg) - 50°C < T < 200°C

$$L_v = 2{,}495 - 2.346\,T\ (\text{kJ kg}^{-1})\ 0°\text{C} < T < 100°\text{C}$$

A.3. Bessel equations and functions

A.3.1. *Particular Bessel equations and their solutions*

$$y'' + \frac{y'}{x} + m^2 y = 0 \Rightarrow y = k_1 J_0(mx) + k_2 Y_0(mx)$$

$$x^2 y'' + xy' + (x^2 - n^2)y = 0 \Rightarrow y = k_1 J_n(x) + k_2 Y_n(x)\ (n \text{ integer})$$

$$y'' + \frac{y'}{x} - m^2 y = 0 \Rightarrow y = k_1 I_0(mx) + k_2 K_0(mx)$$

$$x^2 y'' + xy' - (x^2 - n^2)y = 0 \Rightarrow y = k_1 I_n(x) + k_2 K_n(x)\ (n \text{ integer})$$

with:

– J_n, unmodified Bessel function of the first kind of order n.

– I_n, modified Bessel function of the first kind of order n.

– Y_n, unmodified Bessel function of the second kind of order n.

– K_n, modified Bessel function of the second kind of order n.

A.3.2. *Main properties of Bessel functions*

– Recurrence:

$$J_{n+1}(u) = -J_{n-1}(u) + \frac{2n}{u} J_n(u) Y_{n+1}(u) = -Y_{n-1}(u) + \frac{2n}{u} Y_n(u)$$

$$I_{n+1}(u) = I_{n-1}(u) - \frac{2n}{u} I_n(u) K_{n+1}(u) = K_{n-1}(u) - \frac{2n}{u} K_n(u)$$

– Derivative:

$$\frac{dJ_0(u)}{du} = -J_1(u); \quad \frac{dI_0(u)}{du} = I_1(u); \quad \frac{dK_0(u)}{du} = -K_1(u); \quad \frac{dY_0(u)}{du} = -Y_1(u);$$
$$\frac{d[uJ_1(u)]}{du} = uJ_0(u)$$

– Integrals:

$$\int x^v W_{v-1}(\alpha x)\, dx = \frac{1}{\alpha} x^v W_v(\alpha x) \text{ for } W = J, Y, I$$

$$\int x^{-v} W_{v+1}(\alpha x)\, dx = -\frac{1}{\alpha} x^{-v} W_v(\alpha x) \text{ for } W = J, Y, K$$

$$\int_0^R r[J_0(\alpha r)]^2 dr = \frac{R^2}{2}\{[J_0(\alpha R)]^2 + [J_1(\alpha R)]^2\}$$

A.3.3. *Asymptotic behavior of Bessel functions of order 0 and 1*

If $u \to 0$: $\quad J_0(u) \to 1; \quad J_1(u) \to \frac{u}{2}; \quad Y_0(u) \to \left(\frac{2}{\pi}\right)\left[ln\left(\frac{u}{2}\right) + \gamma\right]; \quad Y_1(u) \to \frac{2}{\pi u};$

$I_0(u) \to 1;\ I_1(u) \to \frac{u}{2};\ K_0(u) \to -ln\left(\frac{u}{2}\right) - \gamma;\ K_1(u) \to \frac{1}{u}$

If $u \to \infty$: $\quad J_0(u) \to \sqrt{\frac{2}{\pi u}} cos\left(u - \frac{\pi}{4}\right);\ J_1(u) \to \sqrt{\frac{2}{\pi u}} cos\left(\frac{u}{2} - \frac{\pi}{4}\right);$

$$Y_0(u) \to \sqrt{\frac{2}{\pi u}} sin\left(u - \frac{\pi}{4}\right);\ Y_1(u) \to \sqrt{\frac{2}{\pi u}} sin\left(\frac{u}{2} - \frac{\pi}{4}\right);$$

$$I_0(u), I_1(u) \to \sqrt{\frac{2}{\pi u}} exp(u);\ K_0(u), K_1(u) \to \sqrt{\frac{\pi}{2u}} exp(-u)$$

A.3.4. *Useful equations*

$$I_v(x)K_{v+1}(x) + I_{v+1}(x)K_v(x) = \frac{1}{x}$$

$$J_{v+1}(x)Y_v(x) - J_v(x)Y_{v+1}(x) = \frac{2}{\pi x}$$

A.4. Main integral transforms: Laplace, Fourier, Hankel

A.4.1. *Laplace transform*

– Definition:

$L[T(t)] = \theta(p) = \int_0^\infty exp(-pt)\,T(t)dt$ and: $L^{-1}[\theta(p)] = T(t)$ (inverse transform)

– Properties:

- linearity: $L[a_1T_1(t) + a_2T_2(t)] = a_1L[T_1(t)] + a_2L[T_2(t)]$, same for L^{-1};
- translation: $L[exp(at)T(t)] = \theta(p-a)$; $L^{-1}[\theta(p-a)] = exp(at)T(t)$:

$L^{-1}[exp(-ap)\theta(p)] = T(t-a)$ if $t > a$

$L^{-1}[exp(-ap)\theta(p)] = 0$ if $t \leq a$

– Change of scale:

$$L[T(at)] = \frac{1}{a}\theta\left(\frac{p}{a}\right) L^{-1}[\theta(ap)] = \frac{1}{a}T\left(\frac{t}{a}\right)$$

– Derivation:

$$L[T'(t)] = p\theta(p) - T(0)L^{-1}\left[\theta^{(n)}(p)\right] = (-1)^n t^n T(t)$$

$$L[T''(t)] = p^2\theta(p) - pT(0) - T'(0)$$

– Integration:

$$L\left[\int_0^t T(u)du\right] = \frac{\theta(p)}{p} L^{-1}\left[\int_p^\infty \theta(u)du\right] = \frac{T(t)}{t}$$

– Multiplication by t^n:

$$L[t^n T(t)] = (-1)^n \theta^{(n)}(p) L^{-1}[p\theta(p)] = T'(t) - T(0)\delta(t)$$

– Division by t:

$$L\left[\frac{T(t)}{t}\right] = \int_p^\infty \theta(u)du \, L^{-1}\left[\frac{\theta(p)}{p}\right] = \int_0^t T(u)du$$

– Periodic functions: $L[T(t)] = \frac{\int_0^P exp(-pt)T(t)dt}{1-exp(pP)}$.

(Period P).

A.4.2. *Complex Fourier transform*

– Definition:

$$F[T(x)] = \theta(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{i\omega x} T(x)dx$$

$$T(x) = F^{-1}[\theta(\omega)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{i\omega x} \theta(\omega)d\omega$$

– Properties:

$$F\left[\frac{\partial T}{\partial x}\right] = -i\omega\theta(\omega); \, F\left[\frac{\partial^2 T}{\partial x^2}\right] = -\omega^2\theta(\omega)$$

A.4.3. *Fourier transform in sine and cosine*

– Definitions:

- sine:

$$F_s[T(x)] = \theta_s(\omega) = \sqrt{\frac{2}{\pi}} \int_0^\infty T(x) sin(\omega x) \, dx$$

$$T(x) = F_s^{-1}[\theta_s(\omega)] = \sqrt{\frac{2}{\pi}} \int_0^{\infty} \theta_s(\omega) sin(\omega x)\, dx$$

- cosine:

$$F_c[T(x)] = \theta_c(\omega) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} T(x) cos(\omega x)\, dx$$

$$T(x) = F_c^{-1}[\theta_c(\omega)] = \sqrt{\frac{2}{\pi}} \int_0^{\infty} \theta_c(\omega) cos(\omega x)\, dx$$

– Properties:

$$F_s\left[\frac{\partial T}{\partial x}\right] = -\omega\theta_c(\omega); F_c\left[\frac{\partial T}{\partial x}\right] = -\omega\theta_s(\omega) - \sqrt{\frac{2}{\pi}}T(0)$$

$$F_s\left[\frac{\partial^2 T}{\partial x^2}\right] = -\omega^2\theta_s(\omega) + \omega\sqrt{\frac{2}{\pi}}T(0); F_c\left[\frac{\partial^2 T}{\partial x^2}\right] = -\omega^2\theta_c(\omega) - \sqrt{\frac{2}{\pi}}\left(\frac{\partial T}{\partial x}\right)_{x=0}$$

A.4.4. *Finite Fourier transform in sine and cosine*

– Definitions: if the temperature $T(x)$ is only defined on the interval $[0, L]$, we can use a finite Fourier transformation in sine or cosine:

$$F_s[T(x)] = \theta_s(n) = \int_0^L T(x) sin\left(\frac{n\pi x}{L}\right) dx$$

where $F_c[T(x)] = \theta_c(n) = \int_0^L T(x) cos\left(\frac{n\pi x}{L}\right) dx.$

$$T(x) = F_s^{-1}[\theta_s(n)] = \frac{2}{L}\sum_{n=1}^{\infty} \theta_s(n) sin\left(\frac{n\pi x}{L}\right)$$

where $T(x) = F_c^{-1}[\theta_c(n)] = \frac{1}{L}\theta_c(0) + \frac{2}{L}\sum_{n=1}^{\infty} \theta_c(n) cos\left(\frac{n\pi x}{L}\right).$

– Properties:

$$F_s\left[\frac{\partial^2 T}{\partial x^2}\right] = \frac{n\pi}{L}[T(0) - (-1)^n T(L)] - \frac{n^2\pi^2}{L^2}\theta_s(n)$$

$$F_c\left[\frac{\partial^2 T}{\partial x^2}\right] = (-1)^n\left(\frac{\partial T}{\partial x}\right)_{x=L} - \left(\frac{\partial T}{\partial x}\right)_{x=0} - \frac{n^2\pi^2}{L^2}\theta_c(n)$$

A.4.5. *Hankel transform of order v*

– Definition: for $\nu > -\frac{1}{2}$:

$$H_\nu[T(r)] = \theta_\nu(\sigma) = \int_0^\infty rJ_\nu(\sigma r)T(r)dr;\ T(r) = H_\nu{}^{-1}[\theta_\nu(\sigma)] = \int_0^\infty \sigma J_\nu(\sigma r)\theta_\nu(\sigma)d\sigma$$

– Property:

$$H_\nu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) - \frac{\nu^2 T}{r^2}\right] = -\sigma^2\theta_\nu(\sigma)$$

at order 0:

$$H_0[T(r) = T_i] = T_i\int_0^\infty rJ_0(\sigma r)dr = T_i\frac{r}{\sigma}J_1(\sigma r)$$

A.5. Inverse Laplace transform

A.5.1. *Analytical method*

The Laplace transform $\theta(p)$ of the function $T(t)$is given by: $L[T(t)] = \theta(p) = \int_0^\infty exp(-pt)\,T(t)dt$.

There is no general analytic formula for calculating $T(t)$ knowing $\theta(p)$. However, we know the exact expression of $T(t)$ for some particular functions $\theta(p)$, examples of which can be found in section A.5.4 (see Spiegel (1990) for more complete tables). The use of these tables, associated with the particular properties of the inverse Laplace transform recalled in section A.6, allows us to solve a certain number of cases. We will always try to decompose a complex function into a sum, product, series, etc., of simple functions that are more easily invertible.

A.5.2. *Numerical methods*

When an analytical solution cannot be found, we can use one of the following two numerical methods:

– Stehfest method: the inverse transform of the function $\theta(p)$ can be calculated by (Stehfest 1970):

$$T(t) = \frac{ln(2)}{t}\sum_{j=1}^{N} V_j\theta\left[\frac{jln(2)}{t}\right]$$

$N = 20$ (double precision):

$V_1 = -5.511463844797178 \times 10^{-6}$ $V_2 = 1.523864638447972 \times 10^{-1}$

$V_3 = -1.174654761904762 \times 10^{2}$ $V_4 = 1.734244933862434 \times 10^{4}$

$V_5 = -9.228069289021164 \times 10^{5}$ $V_6 = 2.377408778710318 \times 10^{7}$

$V_7 = -3.494211661953704 \times 10^{8}$ $V_8 = 3.241369852231879 \times 10^{9}$

$V_9 = -2.027694830723779 \times 10^{10}$ $V_{10} = 8.946482982379724 \times 10^{10}$

$V_{11} = -2.870209211471027 \times 10^{11}$ $V_{12} = 6.829920102815115 \times 10^{11}$

$V_{13} = -1.219082330054374 \times 10^{12}$ $V_{14} = 1.637573800842013 \times 10^{12}$

$V_{15} = -1.647177486836117 \times 10^{12}$ $V_{16} = 1.221924554444226 \times 10^{12}$

$V_{17} = -6.488065588175326 \times 10^{11}$ $V_{18} = 2.333166532137059 \times 10^{11}$

$V_{19} = -5.091380070546738 \times 10^{10}$ $V_{20} = 5.091380070546738 \times 10^{9}$

$N = 10$ (single precision):

$$V_1 = \frac{1}{12}; V_2 = -\frac{385}{12}; V_3 = 1{,}279; V_4 = -\frac{46{,}871}{3}; V_5 = \frac{505{,}465}{6};$$

$$V_6 = -\frac{473{,}915}{2}; V_7 = \frac{1{,}127{,}735}{3}; V_8 = -\frac{1{,}020{,}215}{3}; V_9 = \frac{328{,}125}{2}; V_{10} = -\frac{65{,}625}{2}$$

– Fourier method:

$$T(t) = \frac{exp(ct)}{t_{max}}$$

$$\langle\frac{\theta(c)}{2} + \sum_{k=1}^{\infty}\{Re[\theta(c+j\omega_k)]cos(\omega_k t) - Im[\theta(c+j\omega_k)]sin(\omega_k t)\}\rangle$$

with: $\omega_k = \frac{k\pi}{t_{max}}$.

The infinite sum is in practice calculated for a finite number N of terms; we will generally take $N > 100$. This method requires choosing two parameters: c and t_{max}. We must ensure a posteriori that $exp(-2ct_{max})(2t_{max}) \approx 0$.

A.5.3. *Choosing a method and checking the results*

The Stehfest method is simpler to implement, because it does not require choosing certain parameters. The Fourier method can lead to a better result in the case of inversion of certain functions such as periodic functions, for example Maillet et al. (2000), Den Iseger (2006), Toutain et al. (2011). We can also directly use the Matlab subroutine "Invlap" based on De Hoog's algorithm.

The study of the behavior of the function $\theta(p)$ at long times ($t \to \infty$ thus $p \to 0$) and at short times ($t \to 0$ thus $p \to \infty$) can lead to approximate formulas of $\theta(p)$ for which we can then find the inverse Laplace transform analytically. The comparison of these analytical solutions with the results of the numerical inversion gives an indication of the accuracy of the numerical inversion.

A.5.4. *Table of Laplace transforms*

From Spiegel (1990), $q = \sqrt{\frac{p}{a}}$.

$\theta(p) = [T(t)]$	$T(t)$
$\frac{1}{p}$	1
1	$\delta(t)$ Dirac
$\frac{1}{p^2}$	t
$\frac{1}{p^n}\, n = 1,2,3 \ldots$	$\frac{t^{n-1}}{(n-1)!}$
$\frac{ln(p)}{p}$	$-ln(t) - \gamma;\ \ \gamma = 0.57721$
$\frac{\omega}{p^2 - \omega^2}$	$sinh(\omega t)$
$\frac{p}{p^2 - \omega^2}$	$cosh(\omega t)$
$\frac{\omega^2}{p^2+\omega^2}$	$sin(\omega t)$
$\frac{p}{p^2 + \omega^2}$	$cos(\omega t)$

$\frac{1}{\sqrt{p}}$	$\frac{1}{\sqrt{\pi t}}$
$\frac{1}{p\sqrt{p}}$	$\frac{2}{\sqrt{\pi}}\sqrt{t}$
$\frac{b}{p(b+\sqrt{p})}$	$1-exp(b^2t)erfc(b\sqrt{t})$
$\frac{b}{\sqrt{p}(b+\sqrt{p})}$	$exp(b^2t)erf(b\sqrt{t})$
$\frac{1}{p+\alpha}$	$e^{-\alpha t}$
$\frac{1}{(p+\alpha)(p+\beta)}$	$\frac{e^{-\beta t}-e^{-\alpha t}}{\alpha-\beta}$
$\frac{1}{(p+\alpha)^2}$	$te^{-\alpha t}$
$\frac{p}{(p+\alpha)^2}$	$(1-\alpha t)e^{-\alpha t}$
$\frac{p}{(p+\alpha)(p+\beta)}$	$\frac{\alpha e^{-\beta t}-\beta e^{-\alpha t}}{\alpha-\beta}$
$\frac{1}{(p+\alpha)(p+\beta)(p+\gamma)}$	$\frac{(\gamma-\beta)exp(-\alpha t)+(\alpha-\gamma)exp(-\beta t)+(\beta-\alpha)exp(-\gamma t)}{(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)}$
$\frac{p}{(p+\alpha)(p+\beta)(p+\gamma)}$	$\frac{\alpha(\beta-\gamma)exp(-\alpha t)+\beta(\gamma-\alpha)exp(-\beta t)+\gamma(\alpha-\beta)exp(-\gamma t)}{(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)}$
e^{-qx}	$\frac{x}{2\sqrt{\pi a t^3}}exp\left(-\frac{x^2}{4at}\right)$
$\frac{e^{-qx}}{q}$	$\sqrt{\frac{a}{\pi t}}exp\left(-\frac{x^2}{4at}\right)$
$\frac{e^{-qx}}{p}$	$erfc\left(\frac{x}{2\sqrt{at}}\right)$
$\frac{e^{-qx}}{pq}$	$2\sqrt{\frac{a}{\pi t}}exp\left(-\frac{x^2}{4at}\right)-xerfc\left(\frac{x}{2\sqrt{at}}\right)$
$\frac{e^{-qx}}{p^2}$	$\left(t+\frac{x^2}{2a}\right)erfc\left(\frac{x}{2\sqrt{at}}\right)-x\sqrt{\frac{t}{\pi a}}exp\left(-\frac{x^2}{4at}\right)$

$\frac{e^{-qx}}{q+h}$	$\sqrt{\frac{a}{\pi t}} exp\left(-\frac{x^2}{4at}\right) - haexp(hx+ath^2)erfc\left(\frac{x}{2\sqrt{at}}+h\sqrt{at}\right)$
$\frac{e^{-qx}}{q(q+h)}$	$aexp(hx+ath^2)erfc\left(\frac{x}{2\sqrt{at}}+h\sqrt{at}\right)$
$\frac{e^{-qx}}{p(q+h)}$	$\frac{1}{h}erfc\left(\frac{x}{2\sqrt{at}}\right) - \frac{1}{h}exp(hx+ath^2)erfc\left(\frac{x}{2\sqrt{at}}+h\sqrt{at}\right)$
$\frac{e^{-qx}}{pq(q+h)}$	$\frac{2}{h}\sqrt{\frac{a}{\pi t}} exp\left(-\frac{x^2}{4at}\right) - \frac{1+hx}{h^2}erfc\left(\frac{x}{2\sqrt{at}}\right) + \frac{1}{h^2}exp(hx+ath^2)erfc\left(\frac{x}{2\sqrt{at}}+h\sqrt{at}\right)$
$\frac{e^{-qx}}{p-\alpha}$	$\frac{1}{2}exp(\alpha t)\left\{exp\left(-\frac{x\sqrt{\alpha}}{\sqrt{a}}\right)erfc\left(\frac{x}{2\sqrt{at}}-\sqrt{\alpha t}\right) + exp\left(\frac{x\sqrt{\alpha}}{\sqrt{a}}\right)erfc\left(\frac{x}{2\sqrt{at}}+\sqrt{\alpha t}\right)\right\}$
$\frac{e^{-qx}}{q(p-\alpha)}$	$\frac{1}{2}exp(\alpha t)\sqrt{\frac{a}{\alpha}}\left\{exp\left(-\frac{x\sqrt{\alpha}}{\sqrt{a}}\right)erfc\left(\frac{x}{2\sqrt{at}}-\sqrt{\alpha t}\right) - exp\left(\frac{x\sqrt{\alpha}}{\sqrt{a}}\right)erfc\left(\frac{x}{2\sqrt{at}}+\sqrt{\alpha t}\right)\right\}$
$\frac{e^{-qx}}{(q+h)^2}$	-2h $-2h\left(\frac{a^3t}{\pi}\right)exp\left(-\frac{x^2}{4at}\right) + a(1-hx-2ah^2t)$
$\frac{e^{-qx}}{p(q+h)^2}$	$\frac{1}{h^2}erfc\left(\frac{x}{2\sqrt{at}}\right) - \frac{2}{h}\sqrt{\frac{at}{\pi}}exp\left(-\frac{x^2}{4at}\right) - \frac{1}{h^2}(1+hx+2ah^2t)exp(hx+ah^2t) \times erfc\left(\frac{x}{2\sqrt{at}}+h\sqrt{at}\right)$
$K_0(qx)$	$\frac{1}{2t}exp\left(-\frac{x^2}{4at}\right)$
$\frac{1}{p^{1/2}}K_{2n}(qx)$	$\frac{1}{2\sqrt{\pi t}}exp\left(-\frac{x^2}{8at}\right)K_n\left(\frac{x^2}{8at}\right)$
$p^{n/2}K_n(qx)$	$\frac{x^n}{a^{n/2}(2t)^{n+1}}exp\left(-\frac{x^2}{4at}\right)$

A.6. erf function

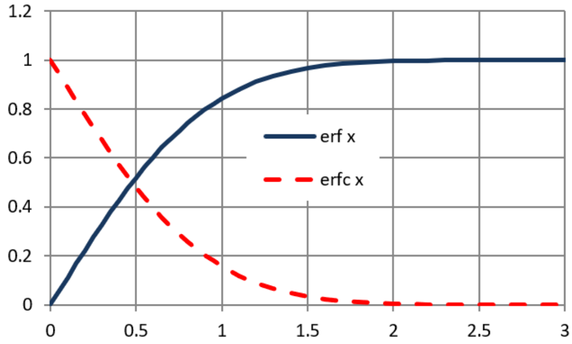
– Properties:

$$erf(u) = \frac{2}{\sqrt{\pi}}\int_0^u exp(-t^2)dt; \qquad erfc(u) = 1 - erf(u);$$

$$ierfc(u) = \frac{1}{\sqrt{\pi}} exp(-u^2) - u[1 - erf(u)]$$

$$\frac{d}{dx}[erf(x)] = \frac{2}{\sqrt{\pi}} exp(-x^2)$$

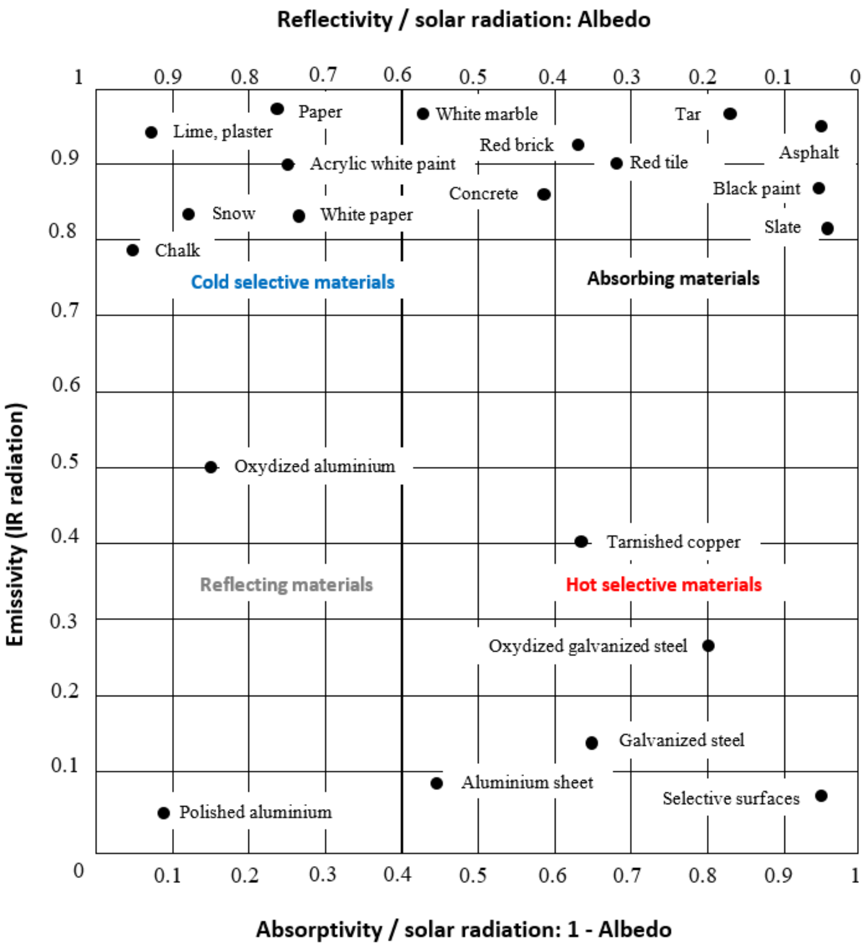
For $x \to \infty$: $erfc(x) = \frac{exp(-x^2)}{\sqrt{\pi}}\left[\frac{1}{x} - \frac{1}{2\,x^3} + \frac{1.3}{2^2 x^5} - \frac{1.3.5}{2^3 x^7} + \cdots\right]$.



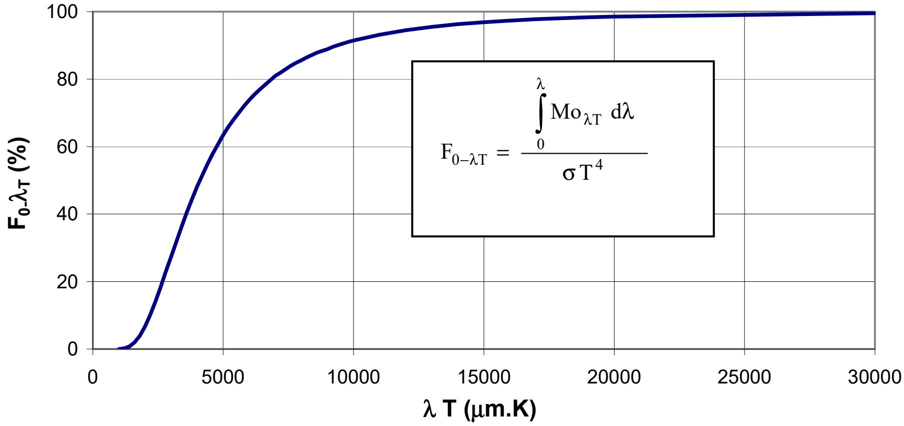
x	$erf(x)$	$erfc(x)$	$ierfc(x)$	x	$erf(x)$	$erfc(x)$	$ierfc(x)$
0	0.000000	1.000000	0.5641896	**1.1**	0.880205	0.11980	0.036465
0.05	0.056372	0.943628	0.5155995	**1.2**	0.910314	0.08969	0.026049
0.1	0.112463	0.887537	0.4698221	**1.3**	0.934008	0.06599	0.018314
0.15	0.167996	0.832004	0.4268365	**1.4**	0.952285	0.04772	0.012670
0.2	0.222703	0.777297	0.3866080	**1.5**	0.966105	0.03390	0.008623
0.25	0.276326	0.723674	0.3490886	**1.6**	0.976378	0.02362	0.005819
0.3	0.328627	0.671373	0.3142186	**1.7**	0.983790	0.01621	0.003799
0.35	0.379382	0.620618	0.2819256	**1.8**	0.989091	0.01091	0.002460
0.4	0.428392	0.571608	0.2521274	**1.9**	0.992790	0.00721	0.001563
0.45	0.475482	0.524518	0.2247329	**2**	0.995322	0.00468	0.000977
0.5	0.520500	0.479500	0.1996413	**2.1**	0.997021	0.00298	0.000602
0.55	0.563323	0.436677	0.1767460	**2.2**	0.998137	0.00186	0.000362
0.6	0.603856	0.396144	0.1559353	**2.3**	0.998857	0.00114	0.000216
0.65	0.642029	0.357971	0.1370922	**2.4**	0.999311	0.00069	0.000124
0.7	0.677801	0.322199	0.1200981	**2.5**	0.999593	0.00041	0.000072

0.75	0.711156	0.288844	0.1048325	**2.6**	0.999764	0.00024	0.000040
0.8	0.742101	0.257899	0.0911737	**2.7**	0.999866	0.00013	0.000023
0.85	0.770668	0.229332	0.0790027	**2.8**	0.999925	0.00008	0.000012
0.9	0.796908	0.203092	0.0682015	**2.9**	0.999959	0.00004	0.000007
0.95	0.820891	0.179109	0.0586561	**3**	0.999978	0.00002	0.000004
1	0.842701	0.157299	0.0502547				

A.7. Emissivity of some materials



A.8. Fraction $F_{0\text{-}\lambda T}$ of energy radiated by a black body between 0 and λ



a \ *b*	0	40	80	120	160
1,000	0.03	0.05	0.08	0.11	0.16
1,200	0.21	0.29	0.38	0.49	0.62
1,400	0.78	0.96	1.17	1.41	1.68
1,600	1.97	2.30	2.66	3.06	3.48
1,800	3.94	4.42	4.94	5.49	6.07
2,000	6.68	7.31	7.97	8.65	9.36
2,200	10.09	10.84	11.61	12.40	13.21
2,400	14.03	14.86	15.71	16.57	17.44
2,600	18.32	19.20	20.09	20.99	21.89
2,800	22.79	23.70	24.61	25.51	26.42
3,000	27.33	28.23	29.13	30.03	30.92
3,200	31.81	32.70	33.58	34.45	35.32

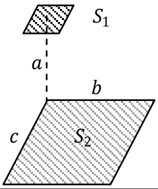
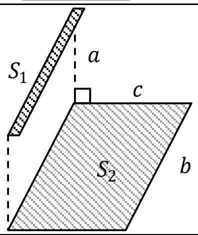
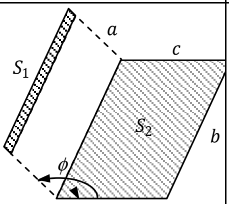
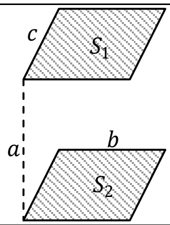
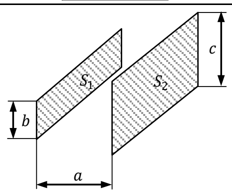
a \ b	0	40	80	120	160
7,800	84.80	84.97	85.14	85.30	85.47
8,000	85.63	85.78	85.94	86.10	86.25
8,200	86.40	86.55	86.69	86.83	86.98
8,400	87.12	87.25	87.39	87.52	87.66
8,600	87.80	87.92	88.04	88.17	88.29
8,800	88.41	88.53	88.65	88.77	88.88
9,000	88.89	89.11	89.22	89.33	89.44
9,200	89.55	89.65	89.76	89.86	89.96
9,400	90.06	90.16	90.26	90.35	90.45
9,600	90.54	90.63	90.72	90.81	90.90
9,800	90.99	91.08	91.16	91.25	91.33
10,000	91.42				

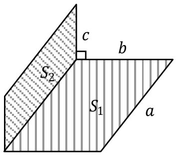
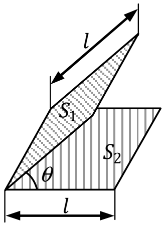
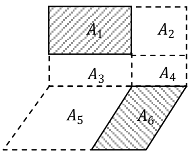
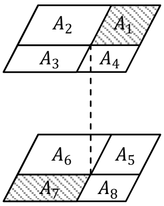
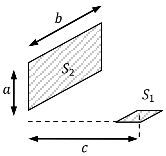
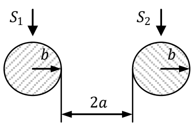
b / a	0	40	80	120	160
3,400	36.18	37.03	37.88	38.71	39.54
3,600	40.36	41.18	41.98	42.78	43.56
3,800	44.34	45.11	45.87	46.62	47.36
4,000	48.09	48.81	49.53	50.23	50.92
4,200	51.60	52.28	52.94	53.60	54.25
4,400	54.88	55.51	56.13	56.74	57.34
4,600	57.93	58.51	59.09	59.65	60.21
4,800	60.66	61.30	61.83	62.35	62.87
5,000	63.38	63.88	64.37	64.85	65.33
5,200	65.80	66.26	66.72	67.16	67.60
5,400	68.04	68.46	68.88	69.30	69.70
5,600	70.11	70.50	70.89	71.27	71.65
5,800	72.02	72.38	72.74	73.09	73.44
6,000	73.78	74.12	74.45	74.78	75.10
6,200	75.41	75.72	76.03	76.33	76.63
6,400	76.92	77.21	77.49	77.77	78.05
6,600	78.32	78.59	78.85	79.11	79.36
6,800	79.61	79.86	80.10	80.34	80.58
7,000	80.90	81.04	81.26	81.47	81.70
7,200	81.92	82.13	82.34	82.55	82.75
7,400	82.95	83.15	83.34	83.53	83.72
7,600	83.91	84.09	84.27	84.45	84.62

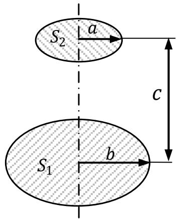
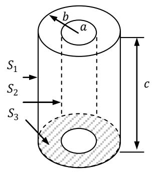
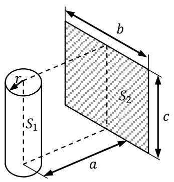
b / a	0	40	80	120	160
b / a	0	200	400	600	800
10,000	91.42	91.81	92.19	92.54	92.87
11,000	93.18	93.48	93.76	94.02	94.27
12,000	94.50	94.73	94.94	95.14	95.33
13,000	95.51	95.68	95.84	96.00	96.14
14,000	96.29	96.42	96.54	96.67	96.78
15,000	96.89	97.00	97.10	97.19	97.29
16,000	97.37	97.46	97.54	97.62	97.69
17,000	97.77	97.83	97.90	97.96	98.02
18,000	98.08	98.14	98.19	98.24	98.29
19,000	98.34	98.38	98.43	98.47	98.51
20,000	98.55				
30,000	99.53				
40,000	99.78	Use:			
50,000	99.89	$\lambda T = a + b$			
60,000	99.93				
70,000	99.96	Example: $\lambda T = 2{,}720$ μm K			
80,000	99.97	Reads at 2,600 + 120			
90,000	99.98	hence: $F_{0-\lambda T} = 20.99\%$			
100,000	99.98				

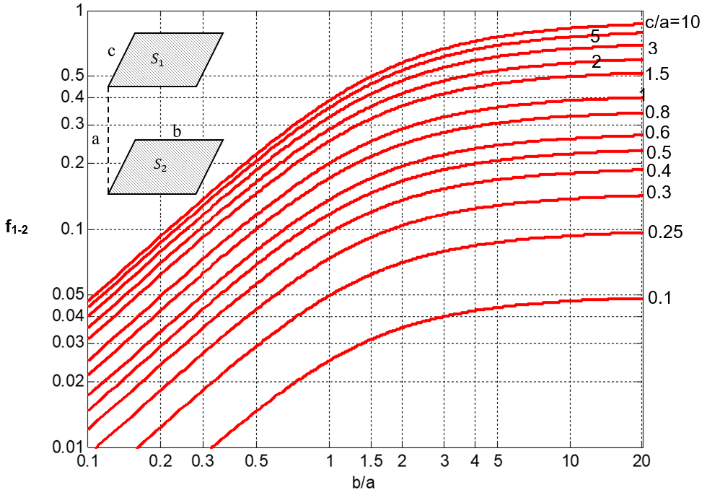
A.9. Radiation shape factors

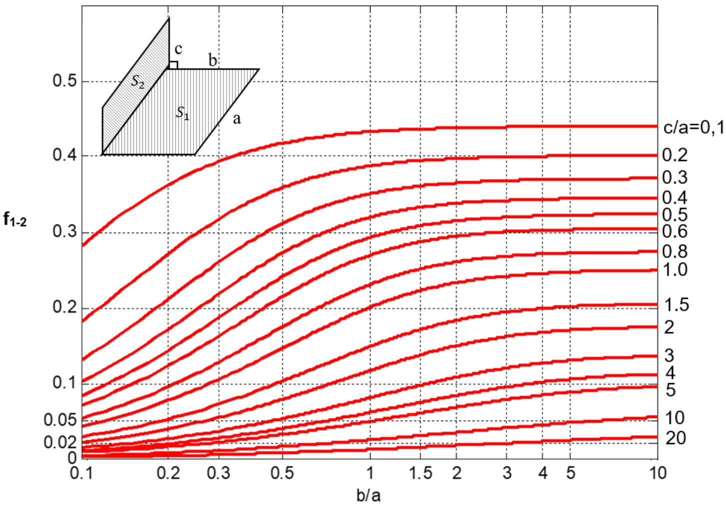
Adapted from Siegel and Howell (1992)

<table>
<tr><th>Configuration</th><th>Diagram</th><th>Shape factor</th></tr>
<tr><td>Elementary surface parallel to a rectangular plane</td><td></td><td>$$f_{1-2}=\frac{1}{2\pi}\left[\frac{B}{\sqrt{1+B^2}}tan^{-1}\left(\frac{C}{\sqrt{1+B^2}}\right)+\frac{C}{\sqrt{1+C^2}}tan^{-1}\left(\frac{B}{\sqrt{1+C^2}}\right)\right]$$ $$B=\frac{b}{a};\ C=\frac{c}{a}$$</td></tr>
<tr><td>Linear source parallel to a rectangular plane</td><td></td><td>$$f_{1-2}=\frac{1}{\pi B}\left[\sqrt{1+B^2}\,tan^{-1}\left(\frac{C}{\sqrt{1+B^2}}\right)-tan^{-1}(C)+\frac{BC}{\sqrt{1+C^2}}tan^{-1}\left(\frac{B}{\sqrt{1+C^2}}\right)\right]$$ $$B=\frac{b}{a};\ C=\frac{c}{a}$$</td></tr>
<tr><td>Linear source parallel to a rectangular plane, intersecting at an angle ϕ</td><td></td><td>$$f_{1-2}=\frac{1}{\pi}\times$$ $$\left\{tan^{-1}(B)+\frac{sin^2(\phi)}{2B}ln\left[\frac{B^2+X^2}{(1+B^2)X^2}\right]-\frac{sin(2\phi)}{2B}\left\{\frac{\pi}{2}-\phi+tan^{-1}\left[\frac{C-cos(\phi)}{sin(\phi)}\right]\right\}+\frac{Y}{B}\left[tan^{-1}\left(\frac{C-cos(\phi)}{sin(\phi)}\right)+tan^{-1}\left(\frac{cos(\phi)}{Y}\right)\right]cos(\phi)+\frac{Ccos(\phi)-1}{X}tan^{-1}\left(\frac{B}{X}\right)\right\}$$ $$B=\frac{b}{a};\ C=\frac{c}{a};\ X=\sqrt{C^2-2Ccos(\phi)+1};\ Y=\sqrt{B^2+sin^2(\phi)}$$</td></tr>
<tr><td>Two rectangular parallel planes with the same area</td><td></td><td>$$f_{1-2}=\frac{1}{\pi}\left[\frac{1}{BC}ln\left(\frac{XY}{X+Y-1}\right)+\frac{2\sqrt{X}}{B}tan^{-1}\left(\frac{C}{\sqrt{X}}\right)+\frac{2\sqrt{Y}}{C}tan^{-1}\left(\frac{B}{\sqrt{Y}}\right)-\frac{2}{C}tan^{-1}(B)-\frac{2}{B}tan^{-1}(C)\right]$$ $$B=\frac{b}{a};\ C=\frac{c}{a};\ X=1+B^2;\ Y=1+C^2$$</td></tr>
<tr><td>Two infinite parallel strips of different widths</td><td></td><td>$$f_{1-2}=\frac{1}{2B}\left[\sqrt{(B+C)^2+4}-\sqrt{(B-C)^2+4}\right]$$ $$f_{2-1}=\frac{1}{2C}\left[\sqrt{(B+C)^2+4}-\sqrt{(B-C)^2+4}\right]$$ $$f_{1-2}=f_{2-1}=\frac{1}{B}\left[\sqrt{B^2+1}-1\right]\text{ si } b=c$$ $$B=\frac{b}{a};\ C=\frac{c}{a}$$</td></tr>
</table>

Two perpendicular rectangular planes having a common side		$f_{1-2} = \frac{1}{\pi B}\left[\frac{1}{4}ln\left\{\left[\frac{(1+B^2)(1+C^2)}{1+B^2+C^2}\right]\left[\frac{B^2(1+B^2+C^2)}{(1+B^2)(B^2+C^2)}\right]^{B^2} \times \left[\frac{C^2(1+B^2+C^2)}{(1+C^2)(B^2+C^2)}\right]^{C^2}\right\} + Btan^{-1}\left(\frac{1}{B}\right) + Ctan^{-1}\left(\frac{1}{C}\right) - \sqrt{B^2+C^2}tan^{-1}\left(\frac{1}{B^2+C^2}\right)\right]$ $B = \frac{b}{a}; C = \frac{c}{a}; f_{1-2} = \frac{1}{2}\left[1 + \frac{c}{b} - \sqrt{1+\left(\frac{c}{b}\right)^2}\right]$ si $a \to \infty$
Two identical planes having a common side		$f_{1-2} = f_{2-1} = 1 - sin\left(\frac{\theta}{2}\right)$
Two perpendicular rectangles		$f_{1-6} = \frac{1}{2A_1}\left\{\begin{matrix}(A_1+A_2+A_3+A_4)f_{1234-56} - A_6f_{6-24} - A_5f_{5-13} \\ -(A_3+A_4)f_{34-56} - A_6f_{6-4} - A_5f_{5-3}\end{matrix}\right\}$
Two parallel rectangles		$f_{1-7} = \frac{1}{4A_1}\begin{pmatrix}A_{1234}f_{1234-5678} + A_1f_{1-5} + A_2f_{2-6} \\ +A_3f_{3-7} + A_4f_{4-8} - A_{12}f_{12-56} \\ -A_{14}f_{14-58} - A_{34}f_{34-78} - A_{23}f_{23-67}\end{pmatrix}$
Elementary surface perpendicular to a surface		$f_{1-2} = \frac{1}{2\pi}\left[tan^{-1}\left(\frac{1}{X}\right) - \frac{1}{\sqrt{1-\frac{Y}{X}}}tan^{-1}\left(\frac{1}{\sqrt{X^2+Y^2}}\right)\right]$ $X = \frac{c}{a}; Y = \frac{a}{b}$
Two infinite cylinders with parallel axes		$f_{1-2} = \frac{2}{\pi}\left[\sqrt{X^2-1} - X + \frac{\pi}{2} - cos^{-1}\left(\frac{1}{X}\right)\right]$ $X = 1 + \frac{a}{b}$

Two parallel disks		$f_{1-2}=\frac{1}{2}\left[Z-\sqrt{Z^2-4X^2Y^2}\right]$ $X=\frac{a}{c};\ Y=\frac{c}{b};\ Z=1+(1+X^2)Y^2$
Two finite coaxial cylinders		$f_{1-2}=$ $\frac{1}{X}-\frac{1}{\pi X}\left\{cos^{-1}\left(\frac{B}{A}\right)-\frac{1}{2Y}\left[\begin{matrix}\sqrt{(A+2)^2-4X^2}cos^{-1}\left(\frac{B}{XA}\right)\\+B sin^{-1}\left(\frac{1}{X}\right)-\frac{\pi A}{2}\end{matrix}\right]\right\}$ $f_{1-1}=1-\frac{1}{X}+\frac{2}{\pi X}tan^{-1}\left(\frac{2\sqrt{X^2-1}}{Y}\right)-$ $\frac{Y}{2\pi X}\left\{\begin{matrix}\frac{\sqrt{4X^2+Y^2}}{Y}sin^{-1}\left[\frac{4(X^2-1)+\frac{Y^2}{X^2}(X^2-2)}{Y^2+4(X^2-1)}\right]\\-sin^{-1}\left(\frac{X^2-2}{X^2}\right)+\frac{\pi}{2}\left(\frac{\sqrt{4X^2+Y^2}}{Y}-1\right)\end{matrix}\right\}$ $f_{1-3}=\frac{1}{2}(1-f_{1-1}-f_{1-2})$ $X=\frac{b}{a};\ Y=\frac{c}{a};A=X^2+Y^2-1;\ B=Y^2-X^2+1$
A plane rectangle and a cylinder with axis located in the middle plane rectangle		$f_{1-2}=$ $\frac{2}{Y}\int_0^{Y/2}\left\{\frac{X}{X^2+\beta^2}-\frac{X}{\pi(X^2+\beta^2)}\left[\begin{matrix}cos^{-1}\left(\frac{W}{V}\right)-\frac{1}{2Z}\sqrt{V^2+4Z}\\\times cos^{-1}\left(\frac{W}{V\sqrt{X^2+\beta^2}}\right)\\+W sin^{-1}\left(\frac{1}{\sqrt{X^2+\beta^2}}\right)-\frac{\pi V}{2}\end{matrix}\right]\right\}d\beta$ $X=\frac{a}{r};\ Y=\frac{b}{r};\ Z=\frac{c}{r};$ f $V=X^2+Z^2+\beta^2-1;\ W=Z^2-X^2-\beta^2+1$

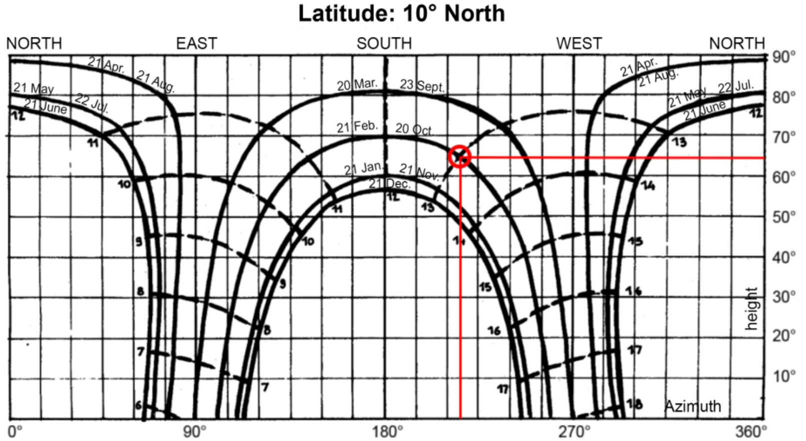


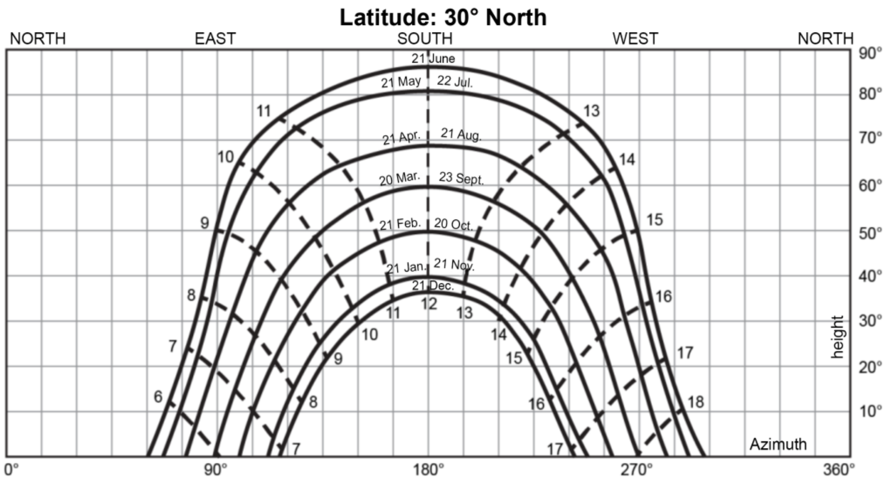


A.10. Cylindrical solar diagram

The diagram enables the calculation of the azimuth a and the height h of the Sun according to the latitude L.

Example: on October 20 at 1 p.m. in a place of latitude 10°N, the azimuth is $a = 217°$ and the height of the Sun is $h = 65°$.





A.11. Albedo values

Surface	Albedo	Surface (various materials)	Albedo
Earth (planet)	0.20–0.60	Polished silver	0.94
Ground		Oxidized silver	0.50
Cover of fresh snow	0.80–0.90	Polished aluminum	0.97
Aged, packed snow cover	0.50–0.70	Oxidized aluminum	0.85
Bare cultivated land	0.08–0.25	Concrete	0.50
Meadow and green grass	0.12–0.25	Carbon	0.15

Surface	Albedo	Surface (various materials)	Albedo
Sandy soil	0.15–0.25	Gravels	0.25
Light sand, dry or wet	0.25–0.45	Asphalt	0.18
Forests of deciduous trees in summer	0.10–0.20	White lime	0.75
Evergreen forests in summer	0.05–0.15	Papier blanc	0.85
Forest and snow	0.25–0.50	White paints	0.90
Grass and dry vegetation	0.28–0.33	Black matt paints	0.07
Bodies of water (seas, lakes)		Window glass	0.10
Perfectly calm water, h > 30°	0.06–0.02	Dry white plaster	0.90
Perfectly calm water, h < 10°	0.35–0.60	Asbestos cement	0.20
Seas and oceans, h > 30°	0.02–0.05	Clouds	
Seas and oceans, h < 10°	0.02–0.20	Stratiforms	0.40–0.75
Vast icy surfaces	0.25–0.40	Cumuliforms	0.60–0.85

A.12. Integral exponential functions

The integral exponential function of order n, with a positive argument x, is defined by:

$$E_n(x) = \int_0^1 \mu^{n-2} e^{-\frac{x}{\mu}} d\mu \qquad \text{(a)}$$

All the following properties are only valid for n integer.

– For $x = 0$, equation (a) becomes:

$$E_n(0) = \int_0^1 \mu^{n-2} d\mu$$

thus:

$$E_n(0) \to \infty \qquad \text{(b)}$$

for $n = 1$, and:

$$E_n(0) = \frac{1}{n-1} \text{ for } n \geq 2 \qquad \text{(c)}$$

– By differentiating relation (a), we obtain:

$$\frac{dE_n(x)}{dx} = -\frac{1}{x} e^{-x} \qquad \text{(d)}$$

for $n = 1$, and:

$$\frac{dE_n(x)}{dx} = -E_{n-1}(x) \quad \text{(e)}$$

For $n \geq 2$.

From where:

$$\int E_n(x)dx = -E_{n+1}(x) \quad \text{(f)}$$

– An integration by parts leads to the recurrence:

$$n\,E_{n+1}(x) = e^{-x} - x\,E_n(x) \quad \text{(g)}$$

– We frequently encounter the integral:

$$\int_0^x x^m E_n(x)dx \quad \text{(h)}$$

Integrating by parts and using relations (d) and (e), this integral can be written as:

$$\int_0^x x^m E_n(x)dx = \frac{x^{m+1}}{m+1}E_n(x) + \frac{x^{m+2}}{(m+1)(m+2)}E_{n-1}(x) + \cdots + \frac{x^{m+n}}{(m+1)(m+2)\ldots(m+n)}E_1(x) + \frac{1}{(m+1)(m+2)\ldots(m+n)}\int_0^1 x^{m+n-1}e^{-x}dx \quad \text{(i)}$$

– According to the recurrence relation (f), $E_1(x)$ can be put in the form:

$$E_1(x) = \int_1^\infty e^{-xt}\frac{dt}{t} = \int_x^\infty e^{-t}\frac{dt}{t} \quad \text{(j)}$$

– For large values of x, the expansion to infinity of $E_n(x)$ is:

$$E_n(x) = \frac{e^{-x}}{x}\left(1 - \frac{n}{x} + \frac{n(n+1)}{x^2} - \frac{n(n+1)(n+2)}{x^3} + \cdots\right) \quad \text{(k)}$$

and therefore: $E_n(x) = \frac{e^{-x}}{x}$ when $x \to \infty$.

– When $x \to 0$, $E_1(x)$ presents a logarithmic singularity point and can be developed in series, i.e.:

$$E_1(x) = -\gamma - log(x) + \sum_{n=1}^{\infty}(-1)^{n+1}\frac{x^n}{n!n} \quad \text{(l)}$$

where $\gamma = 0.5772156$ is Euler's constant.

Approximation of integral exponential functions:

We seek to approach the integral exponentials by classical exponentials, in particular:

$$E_2(x) \approx a\, exp(-bx)$$

$$E_3(x) \approx c\, exp(-dx)$$

There are several methods to determine the constants a, b, c and d; the most used is that of Lick:

– we keep the derivation property of the exponential integrals: $\frac{d}{dx}E_3(\alpha x) \approx -\alpha E_2(\alpha x)$, which imposes $E_3(x) \approx \frac{a}{b} exp(-bx)$;

– we impose the moments of order 0 and order 1 of the function E: $a = \frac{3}{4}$ and $b = \frac{1}{2}$.

From where:

$$E_2(x) \approx \frac{3}{4}\, exp\left(-\frac{1}{2}x\right)$$

$$E_3(x) \approx \frac{1}{2} exp\left(-\frac{3}{2}x\right)$$

A.13. Fredholm singular integral equation

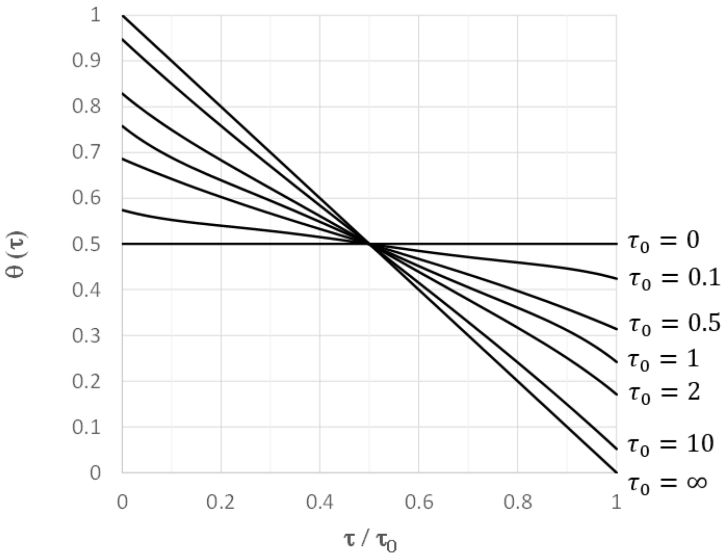
The reduced temperature is given by the equation:

$$\theta(\tau) = \frac{1}{2}\left[E_2(\tau) + \int_0^{\tau_0} \theta(\tau')E_1(|\tau - \tau'|)d\tau'\right]$$

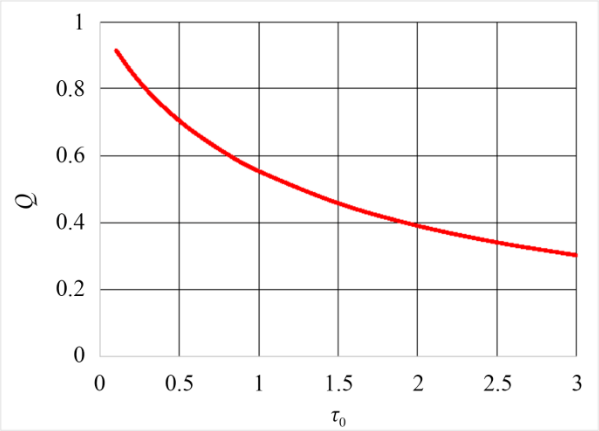
which is the singular Fredholm integral equation whose numerically calculated values for different values of τ are given by the following graph.

The reduced radiation flux is expressed by:

$$Q = 1 - 2\int_0^{\tau_0} \theta(\tau')\, E_2(\tau')d\tau'$$



τ_0	Q
0.1	0.9157
0.2	0.8491
0.3	0.7934
0.4	0.7458
0.5	0.7040
0.6	0.6672
0.8	0.6045
1.0	0.5532
1.5	0.4572
2.0	0.3900
2.5	0.3401
3.0	0.3016

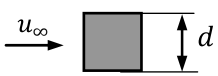


A.14. Correlations for heat transfer coefficients in forced convection

The characteristics of the fluid calculated at $T_f = \frac{T_p + T_\infty}{2}$.

<table>
<tr><th>Geometry</th><th>Correlation</th></tr>
<tr><td>Flow on a plane</td><td>
$Nu(x)$: Nu at the distance x from the edge of the plane
$\overline{Nu}_L$: average Nu over the length L of the plane
Turbulent flow: ($Re > 5.10^5$ and $Pr > 0.5$)
$Nu(x) = 0.0288\, Re(x)^{0.8} Pr^{1/3}$
$\overline{Nu}_L = 0.035\, Re_L{}^{0.8} Pr^{1/3}$
Laminar flow: ($Re < 5.10^5$ and $10 > Pr > 0.5$)
$Nu(x) = 0.324\, Re(x)^{0.5} Pr^{1/3}$
$\overline{Nu}_L = 0.628\, Re_L{}^{0.5} Pr^{1/3}$
</td></tr>
<tr><td>Flow in a tube</td><td>
Turbulent flow: ($Re > 5{,}000$ and $100 > Pr > 0.6$)
$Nu = 0.023\, Re^{0.8} Pr^n$
$n = 0.3$ if $T_{fluid} > T_{wall}$
$n = 0.4$ if $T_{fluid} < T_{wall}$
Re calculated for $D_h = \frac{4S}{P}$, where S is the fluid passage section and P is the fluid/wall contact perimeter
Laminar flow: $Nu = 1.86(RePr)^{1/3} \left(\frac{D}{L}\right)^{1/3} \left(\frac{\mu}{\mu_p}\right)^{0.14}$
Valid for $Pr\frac{D}{L} > 10$, μ_p calculated at T_p
</td></tr>
<tr><td>Flow perpendicular to a circular cylinder

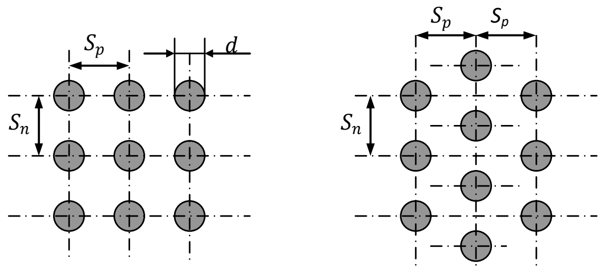
(Jakob 1949)</td><td>
$Nu = CRe^n Pr^{1/3}$, velocity u_∞ calculated upstream of the tube
<table>
<tr><th>Re</th><th>C</th><th>n</th></tr>
<tr><td>0.4–4</td><td>0.989</td><td>0.330</td></tr>
<tr><td>4–40</td><td>0.911</td><td>0.385</td></tr>
<tr><td>40–4,000</td><td>0.683</td><td>0.466</td></tr>
<tr><td>4,000–40,000</td><td>0.193</td><td>0.618</td></tr>
<tr><td>40,000–250,000</td><td>0.0266</td><td>0.805</td></tr>
</table>
</td></tr>
<tr><td>Flow perpendicular to a non-circular cylinder

(Jakob 1949)</td><td>
<table>
<tr><th>Geometry</th><th>Re</th><th>C</th><th>n</th></tr>
<tr><td rowspan="2">
</td><td>$5\ 10^3$–10^5</td><td>0.102</td><td>0.675</td></tr>
<tr><td>$4\ 10^3$–$1.5\ 10^4$</td><td>0.228</td><td>0.731</td></tr>
</table>
</td></tr>
</table>

Geometry	Correlation
Flow perpendicular to a bundle of 10 tubes (Grimson 1937)	

$\frac{S_p}{d}$	$\frac{S_n}{d}$ = 1.25		1.5		2.0		3.0	
	C	n	C	n	C	n	C	n
In-line arrangement								
1.25	0.386	0.592	0.305	0.608	0.111	0.704	0.070	0.752
1.5	0.407	0.586	0.278	0.620	0.112	0.702	0.075	0.744
2.0	0.464	0.570	0.332	0.602	0.254	0.632	0.220	0.648
3.0	0.322	0.601	0.396	0.584	0.415	0.581	0.317	0.608
Staggered arrangement								
0.16	-	-	-	-	-	-	0.236	0.636
0.9	-	-	-	-	0.495	0.571	0.445	0.581
1.0	-	-	0.552	0.558	-	-	-	-
1.125	-	-	-	-	0.531	0.565	0.575	0.560
1.25	0.575	0.556	0.561	0.554	0.576	0.556	0.579	0.562
1.5	0.501	0.568	0.511	0.562	0.502	0.568	0.542	0.568
2.0	0.448	0.572	0.462	0.568	0.535	0.556	0.498	0.570
3.0	0.344	0.592	0.395	0.580	0.488	0.562	0.467	0.574

$Nu = CRe^n Pr^{1/3}$, velocity u_∞ calculated upstream of the tube



In-line arrangement   Staggered arrangement

Geometry	Correlation
Flow perpendicular to a bundle of *n* rows of tubes ($n \leq 10$) (Grimson 1937)	$N = \dfrac{h_n}{h_{10}}$

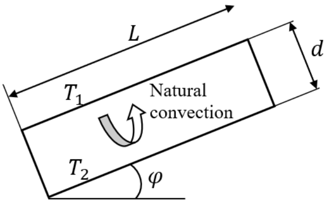
Number of rows *n*	1	2	3	4	5	6	7	8	9	10
***N* in-line**	0.64	0.80	0.87	0.90	0.92	0.94	0.96	0.98	0.99	1.0
***N* staggered**	0.68	0.75	0.83	0.89	0.92	0.95	0.97	0.98	0.99	1.0

A.15. Correlations for heat transfer coefficients in natural convection

According to Holman (1990). $Ra = GrPr = \frac{c\beta g \Delta T \rho^2 d^3}{\lambda\mu}$

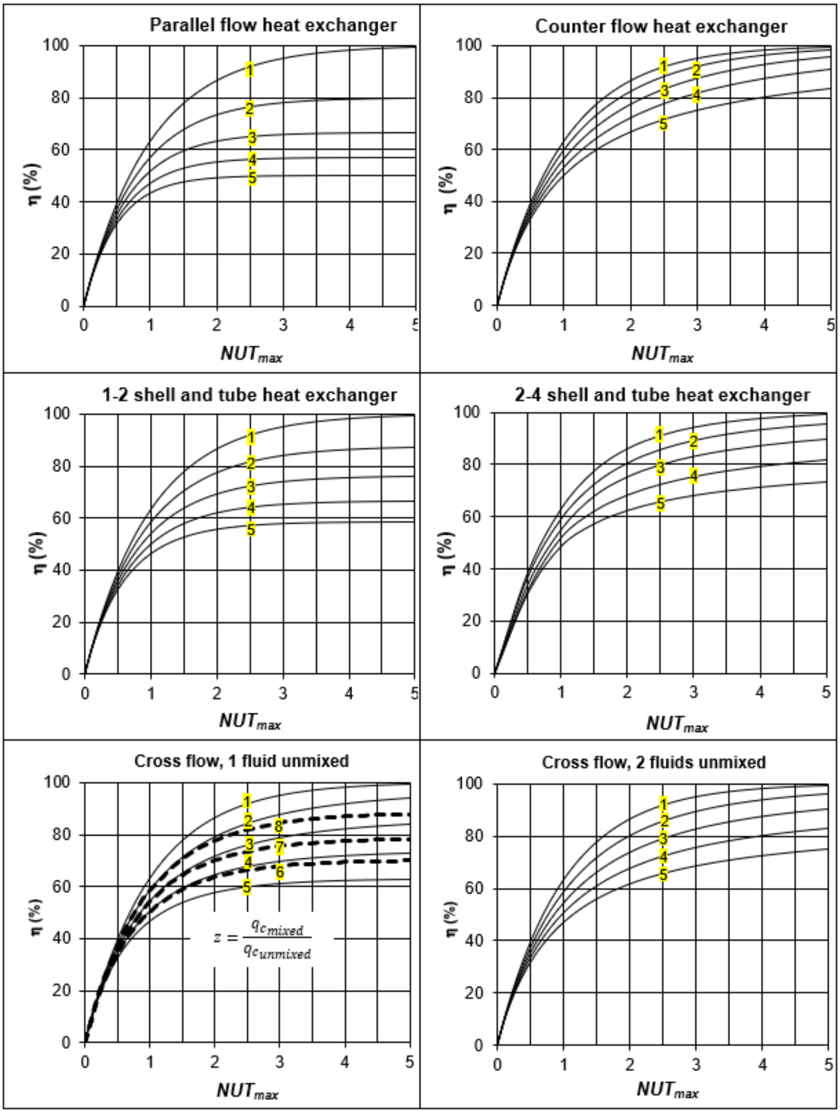
Correlations valid for all fluids: $\boldsymbol{Nu = C(GrPr)^m}$			
Geometry	$\boldsymbol{GrPr}$	$\boldsymbol{C}$	$\boldsymbol{m}$
Vertical plates and cylinders	10^4–10^9 10^9–10^{13}	0.59 0.021	1/4 2/5
Horizontal cylinders	10^{-10}–10^{-2} 10^{-2}–10^2 10^2–10^4 10^4–10^7 10^7–10^{12}	0.675 1.02 0.850 0.480 0.125	0.058 0.148 0.188 0.25 0.33
Upper face of a hot plate or lower face of a cold plate	2×10^4–8×10^6 8×10^6–10^{11}	0.54 0.15	0.25 0.33
Lower face of a hot plate or upper side of a cold plate	10^5–10^{11}	0.27	0.25
Simplified relations for air at atmospheric pressure and at $\boldsymbol{T \approx 20°C}$			
Geometry	Laminar $10^4 < GrPr < 10^9$		Turbulent $GrPr > 10^9$
Vertical plate or cylinder	$h = 1.54\left(\frac{\Delta T}{L}\right)^{1/4}$		$h = 1.22(\Delta T)^{1/3}$
Horizontal cylinder	$h = 1.26\left(\frac{\Delta T}{D}\right)^{1/4}$		$h = 1.52(\Delta T)^{1/3}$
Upper face of a hot horizontal plate or lower face of a cold plate	$h = 1.41\left(\frac{\Delta T}{L}\right)^{1/4}$		$h = 1.83(\Delta T)^{1/3}$

Lower face of a hot plate or upper face of a cold plate	$h = 0.71\left(\frac{\Delta T}{L}\right)^{1/4}$	h $= 0.71\left(\frac{\Delta T}{L}\right)^{1/4}$

<table>
<tr><th colspan="2">Enclosed rectangular cell</th></tr>
<tr><td rowspan="2">
</td>
<td>Correlation of Hollands et al. (1976)
$$Nu = 1 + 1.44\left[1 - \frac{1{,}708}{Ra_d cos(\varphi)}\right]^{*}\left\{1 - \frac{1{,}708[sin(1.8\varphi)^{1.6}]}{Ra_d cos(\varphi)}\right\} + \left\{\left[\frac{Ra_d cos(\varphi)}{5{,}830}\right]^{1/3} - 1\right\}^{*}$$
If $0 < \varphi < tan^{-1}(4{,}800\ Pr), \frac{L}{d} > 12$
The quantities * are taken equal to 0 if the result of their calculation leads to a negative number.
$0 < \varphi < 90°$ if the hot plate is down.</td></tr>
<tr><td>Correlation of El Sherbiny et al. (1982):
Valid for $\frac{L}{d} < 100$ and $Ra_d < 10^6$
$$Nu = max[Nu_1, Nu_2, Nu_3]$$
with:
$$Nu_1 = 0.0605\ Ra_d^{\ 0.33}$$
$$Nu_2 = \left\{1 + \left[\frac{0.104\ Ra_d^{\ 0.293}}{1+\left(\frac{6310}{Ra_d}\right)^{1.36}}\right]^3\right\}^{\frac{1}{3}}$$
$$Nu_3 = 0.242\left(\frac{Ra_d d}{L}\right)^{0.272}$$</td></tr>
</table>

A.16. NUT charts = f(η) for heat exchangers

$z = \frac{q_{c_{min}}}{q_{c_{max}}}$	0	0.25	0.5	0.75	1	1.33	2	4
Point	1	2	3	4	5	6	7	8



A.17. Calculation formulas for singular pressure drops

A.17.1. *Pressure drops by change of direction*

$$\Delta P = R\frac{\rho u^2}{2}$$

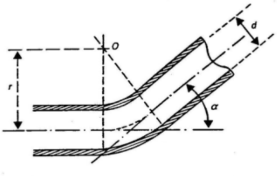
with:

– ρ, fluid density;

– u, fluid velocity;

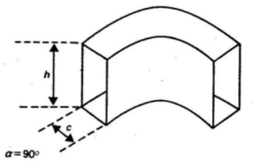
– R, coefficient given in the tables below.

– Circular pipes:



r/d \ α	10°	20°	45°	60°	90°	120°
0.5	0.18	0.33	0.59	0.70	0.86	0.97
0.6	0.13	0.26	0.48	0.55	0.68	0.72
0.8	0.10	0.19	0.33	0.39	0.48	0.52
1	0.07	0.15	0.27	0.32	0.38	0.43
2	0.04	0.09	0.17	0.19	0.23	0.26
5	0.03	0.05	0.12	0.13	0.14	0.16
10	0.03	0.05	0.09	0.10	0.12	0.14

– Rectangular pipes:



h/c \ *r/d*	0.5	0.6	0.7	0.8	1.0	1.5
0.5	1.3	0.8	0.55	0.41	0.3	0.2
1	1	0.65	0.44	0.35	0.25	0.16
2	0.8	0.5	0.35	0.28	0.2	0.13

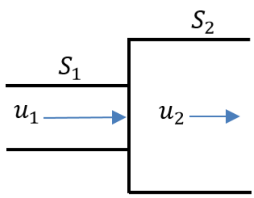
Multiplier coefficient k to be applied to R for $\alpha \neq 90°$

α	15°	30°	45°	60°	90°	135°	180°
k	0.31	0.53	0.69	0.81	1.00	1.21	1.34

A.17.2. *Pressure drops by sudden change of cross-section*

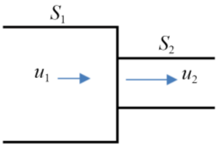
– Abrupt enlargement:

$$\Delta P = \rho \frac{(u_1 - u_2)^2}{2} = \rho \left(1 - \frac{S_1}{S_2}\right) \frac{{u_1}^2}{2}$$



– Abrupt narrowing:

$$\Delta P = R\frac{\rho {u_1}^2}{2}$$



S_1/S_2	0.01	0.1	0.2	0.4	0.5	0.6	0.8	1
R	1.5	1.45	1.35	1.2	1.1	0.9	0.5	0