English consonants

p	pie	pea		lowercase *p*
t	tie	tea		lowercase *t*
k	kye	key		lowercase *k*
b	by	bee		lowercase *b*
d	dye	D		lowercase *d*
g	guy			lowercase *g*
m	my	me	ra*m*	lowercase *m*
n	nigh	knee	ra*n*	lowercase *n*
ŋ			ra*ng*	eng (or angma)
f	fie	fee		lowercase *f*
v	vie	V		lowercase *v*
θ	thigh			theta
ð	thy	thee		eth
s	sigh	sea	listen	lowercase *s*
z		Z	mizzen	lowercase *z*
ʃ (š)	shy	she	mission	esh (or long *s*)
ʒ (ž)			vision	long *z* (or yogh)
l	lie	lee		lowercase *l*
w	why	we		lowercase *w*
r	rye			lowercase *r*
j (y)		ye		lowercase *j*
h	high	he		lowercase *h*

Note also the following:

tʃ (tš)	chi(me)	chea(p)
dʒ (dž)	ji(ve)	G

English consonants

p	pie	pea		lowercase *p*
t	tie	tea		lowercase *t*
k	kye	key		lowercase *k*
b	by	bee		lowercase *b*
d	dye	D		lowercase *d*
g	guy			lowercase *g*
m	my	me	ra*m*	lowercase *m*
n	nigh	knee	ra*n*	lowercase *n*
ŋ			ra*ng*	eng (or angma)
f	fie	fee		lowercase *f*
v	vie	V		lowercase *v*
θ	thigh			theta
ð	thy	thee		eth
s	sigh	sea	listen	lowercase *s*
z		Z	mizzen	lowercase *z*
ʃ (š)	shy	she	mission	esh (or long *s*)
ʒ (ž)			vision	long *z* (or yogh)
l	lie	lee		lowercase *l*
w	why	we		lowercase *w*
r	rye			lowercase *r*
j (y)		ye		lowercase *j*
h	high	he		lowercase *h*

Note also the following:

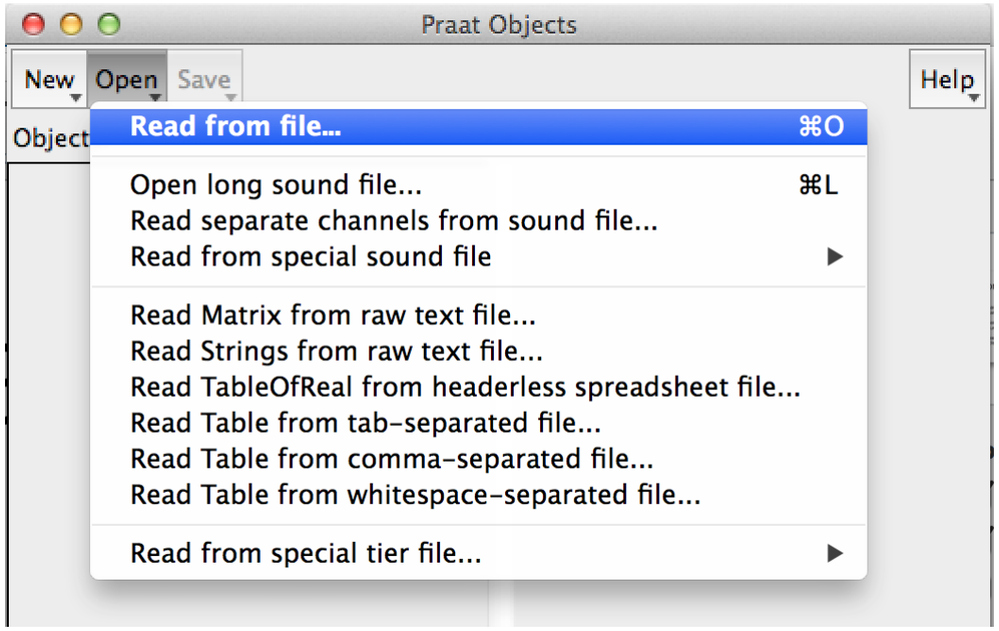
tʃ (tš)	chi(me)	chea(p)
dʒ (dž)	ji(ve)	G

English vowels

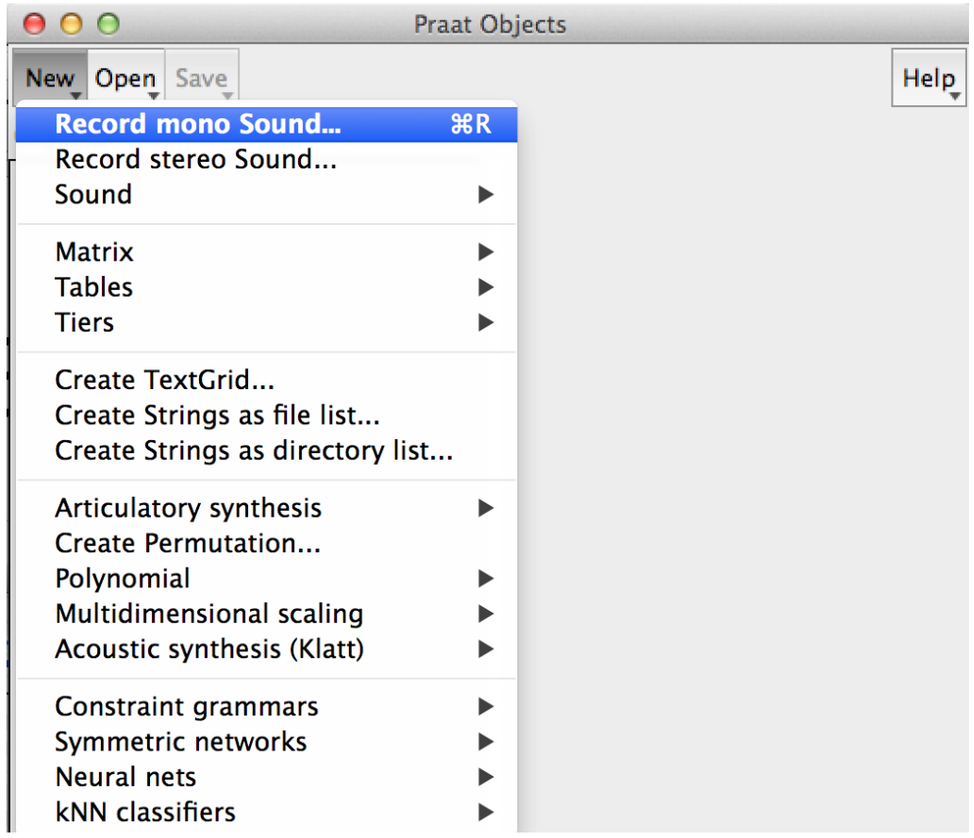
	1	2						
monophthongs	i	i	heed	he	bead	heat	keyed	lowercase *i*
	ɪ	ɪ	hid		bid	hit	kid	small capital *I*
	eɪ	eɪ	hayed	hay	bayed	hate	Cade	lowercase *e*
	ɛ	ɛ	head		bed			epsilon
	æ	æ	had		bad	hat	cad	ash
	ɑ	ɑ	hard		bard	heart	card	script *a*
	ɑ	ɒ	hod		bod	hot	cod	turned script *a*
	ɔ	ɔ	hawed	haw	bawd		cawed	open *o*
	ʊ	ʊ	hood				could	upsilon
	oʊ	əʊ	hoed	hoe	bode		code	lowercase *o*
	u	u	who'd	who	booed	hoot	cooed	lowercase *u*
	ʌ	ʌ	Hudd		bud	hut	cud	turned *v*
	ɝ	ɜ	herd	her	bird	hurt	curd	reversed epsilon
diphthongs	aɪ	aɪ	hide	high	bide	height		lowercase a (+I)
	aʊ	aʊ		how	bowed		cowed	(as noted above)
	ɔɪ	ɔɪ		(a)hoy	Boyd			(as noted above)
	ɪr	ɪə		here	beard			(as noted above)
	ɛr	ɛə		hair	bared		cared	(as noted above)
	aɪr	aə	hired	hire				(as noted above)
	Note also:							
	ju	ju	hued	hue	Bude		cued	(as noted above)

Praat

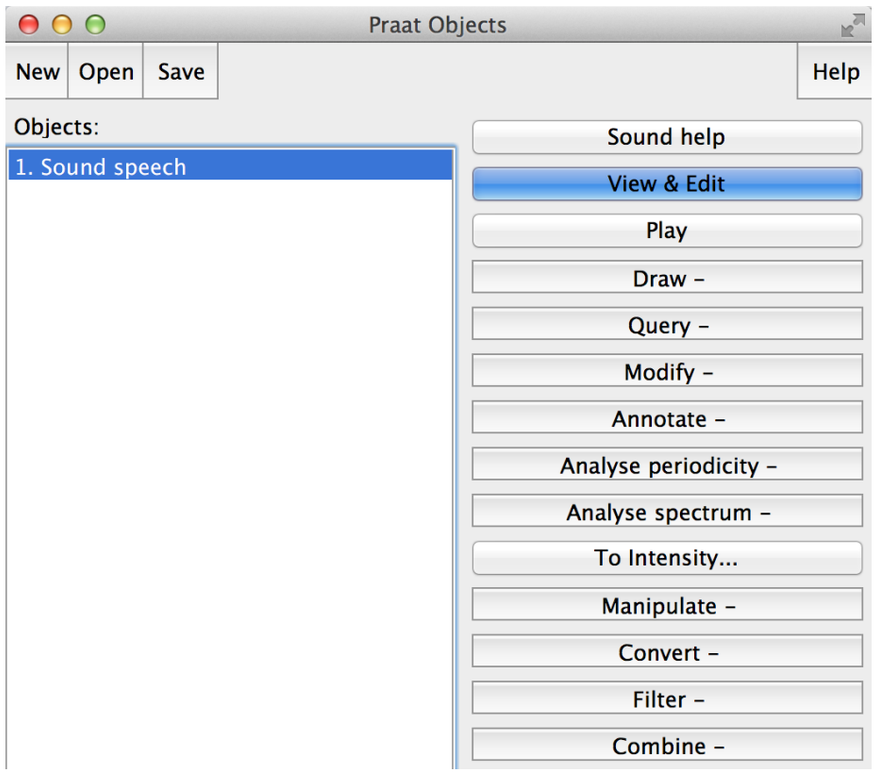
open sound file

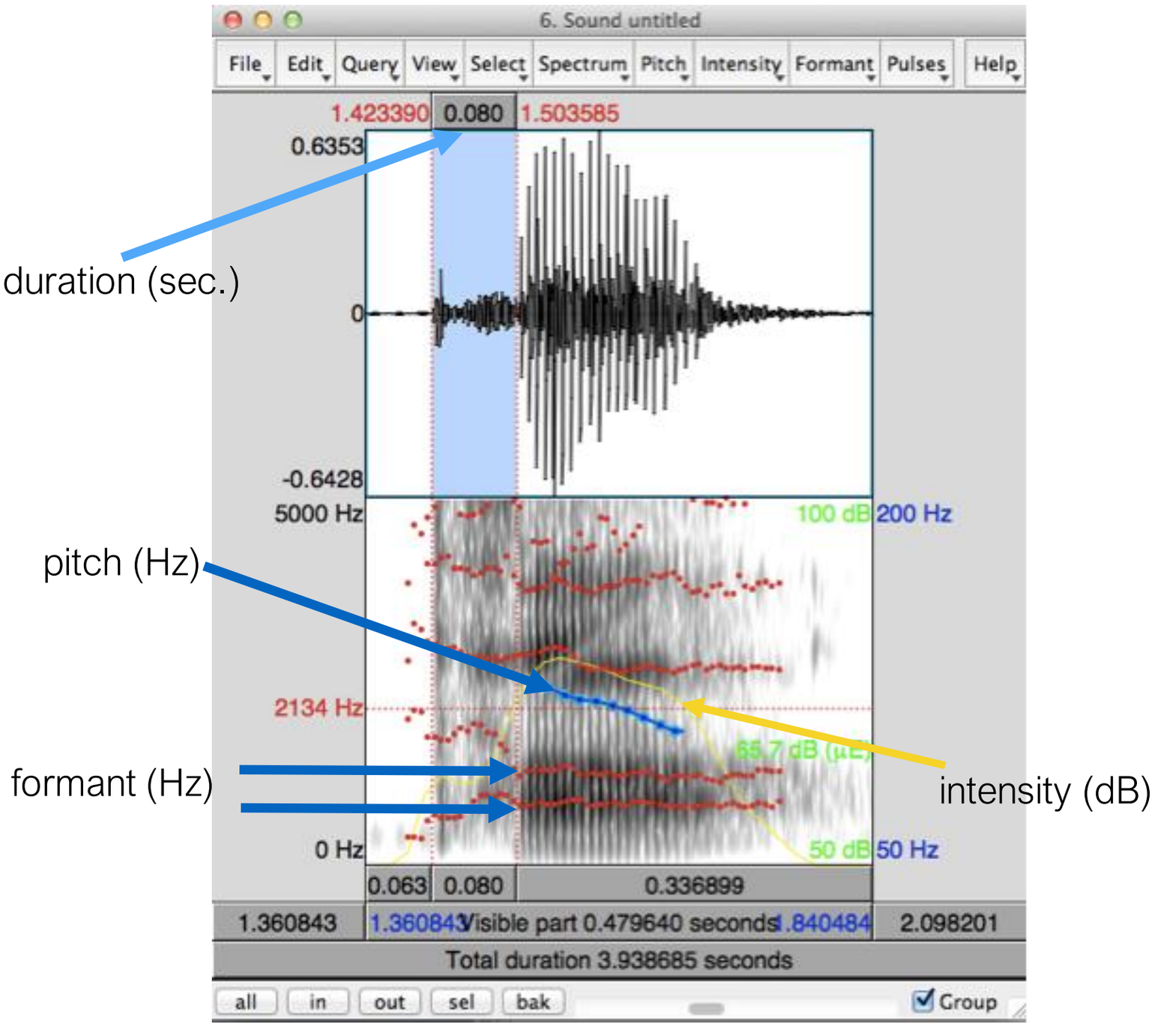


record sound



Duration, pitch, intensity, formant





Hands-on experiments with Praat

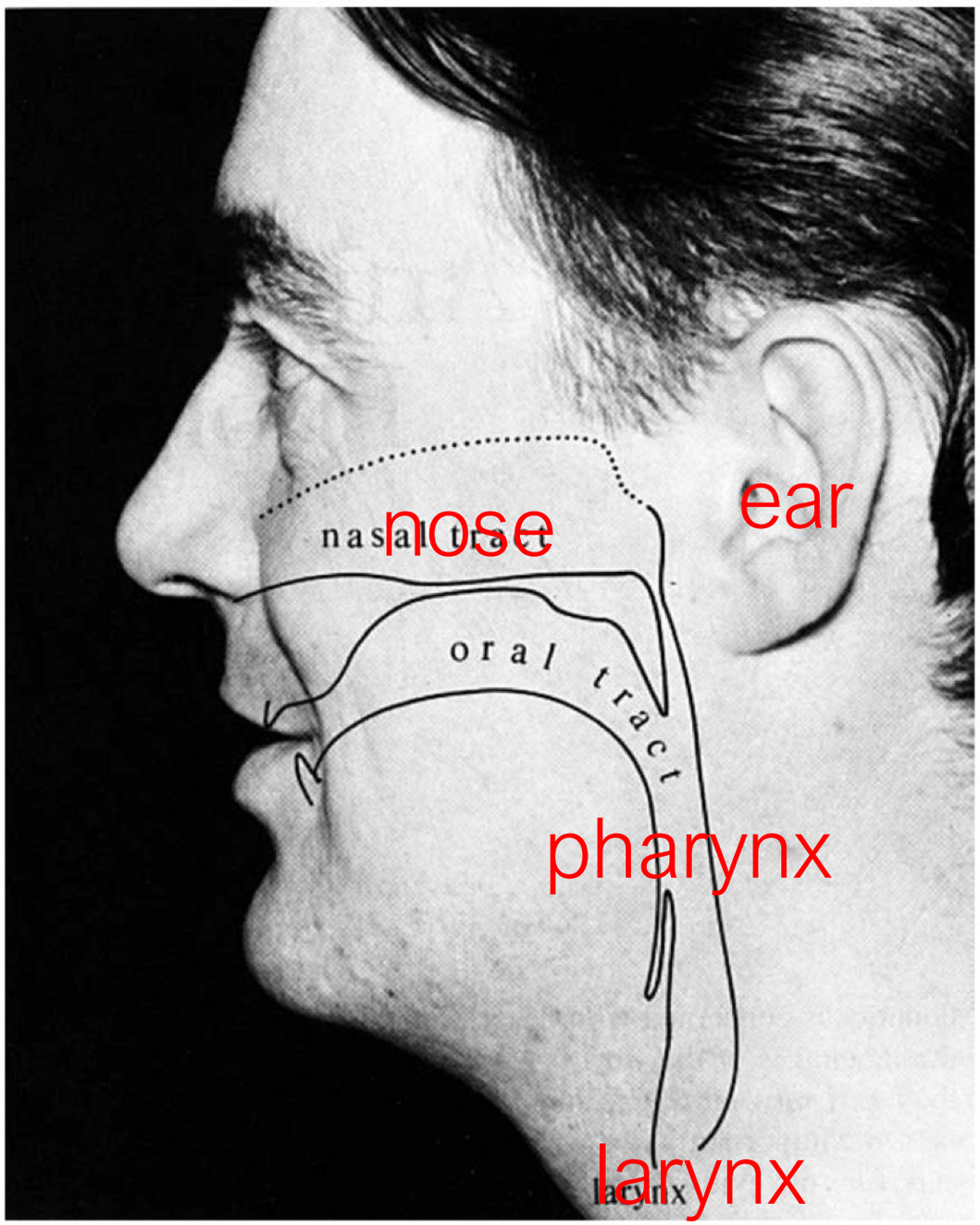
- Record yourself with the sentence:
  - “Your good pants look great”
- Pitch
- Labeling
- Segmentation

Phonetics

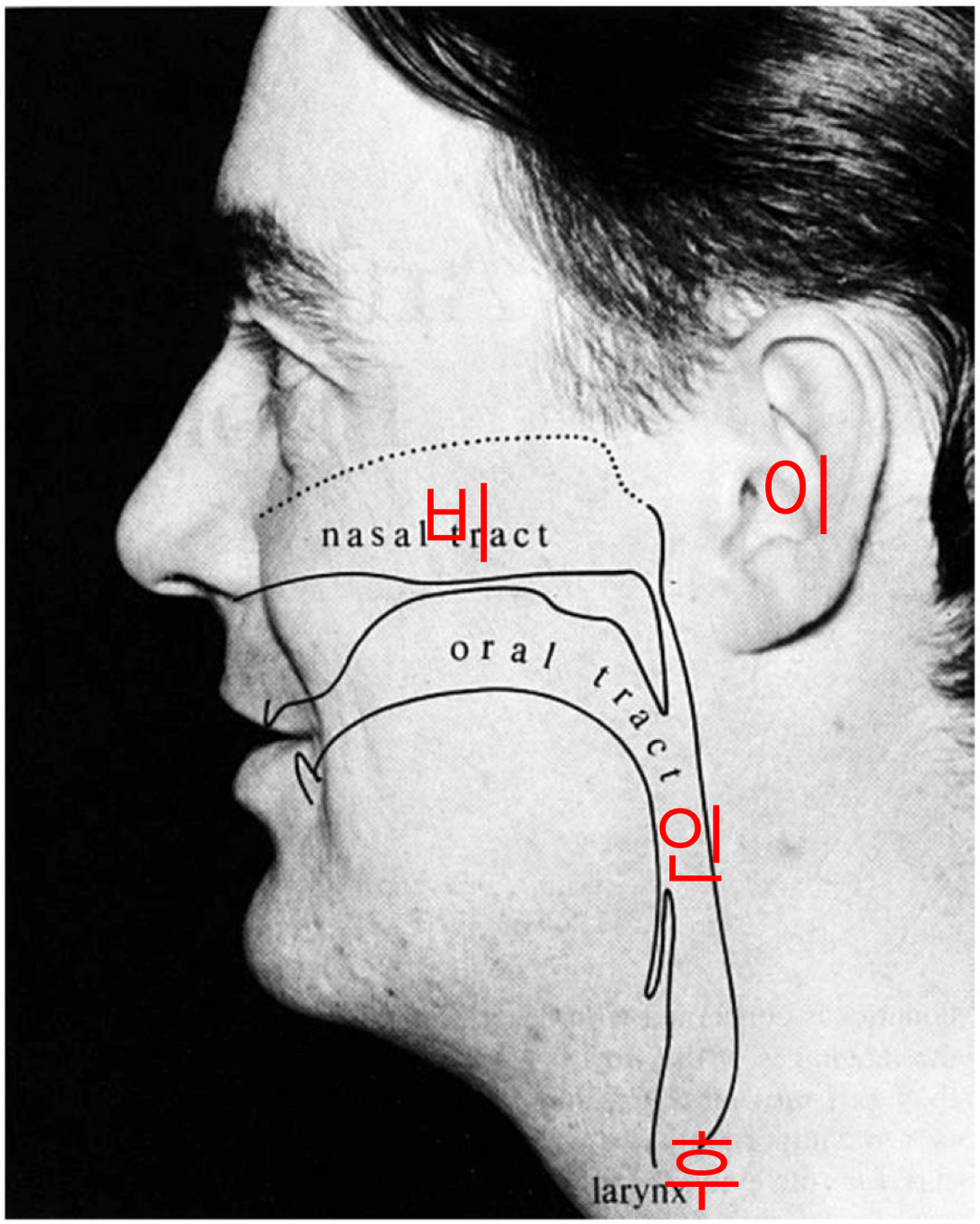
- A study on speech
- How speech is described
  - Articulatory phonetics (*from mouth*) ← the most primitive
    - How to produce speech
  - Acoustic phonetics (*through air*)
    - How to transmit speech
  - Auditory phonetics (*to ear*)
    - How to hear speech

Articulation

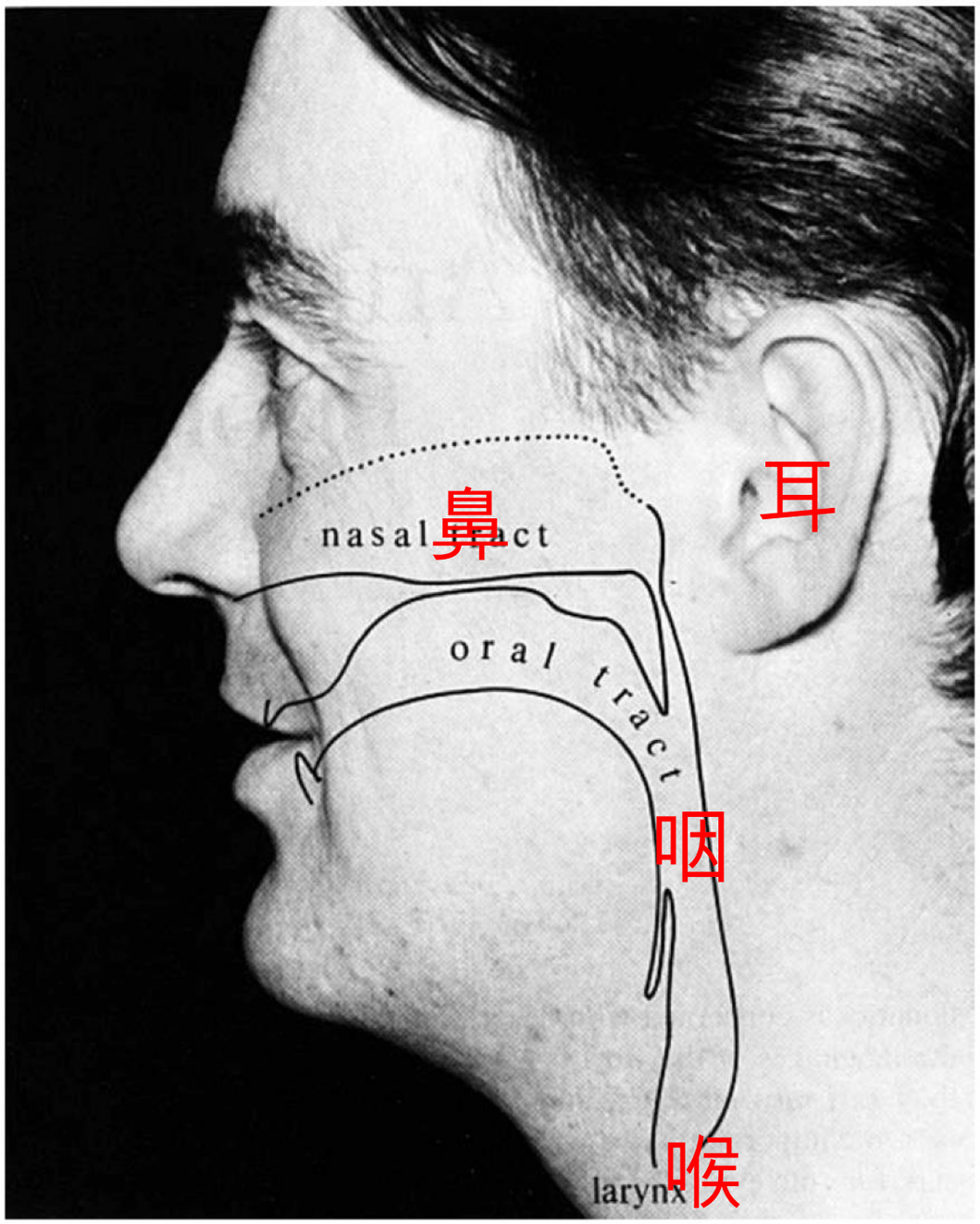
The vocal tract



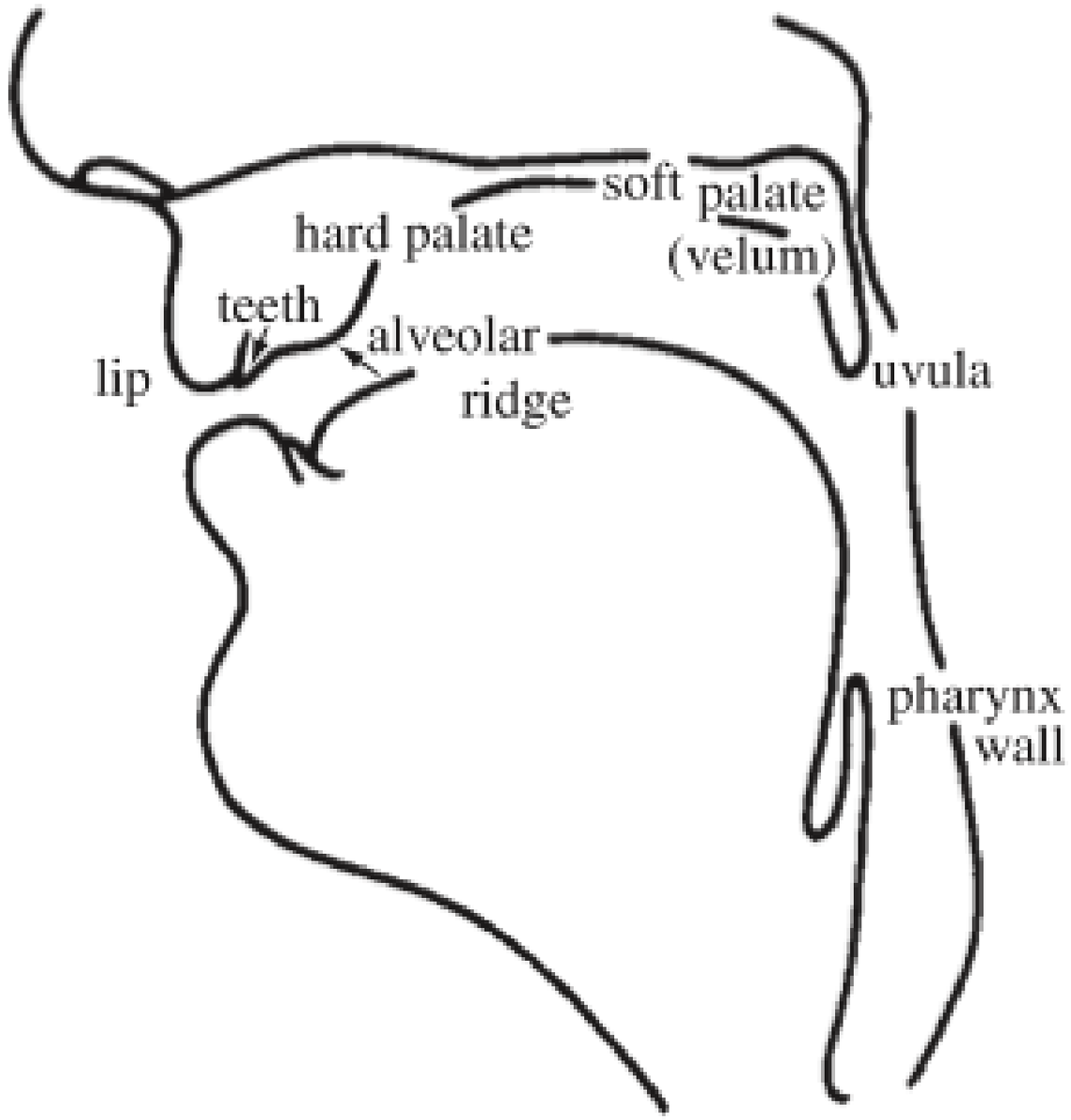
The vocal tract



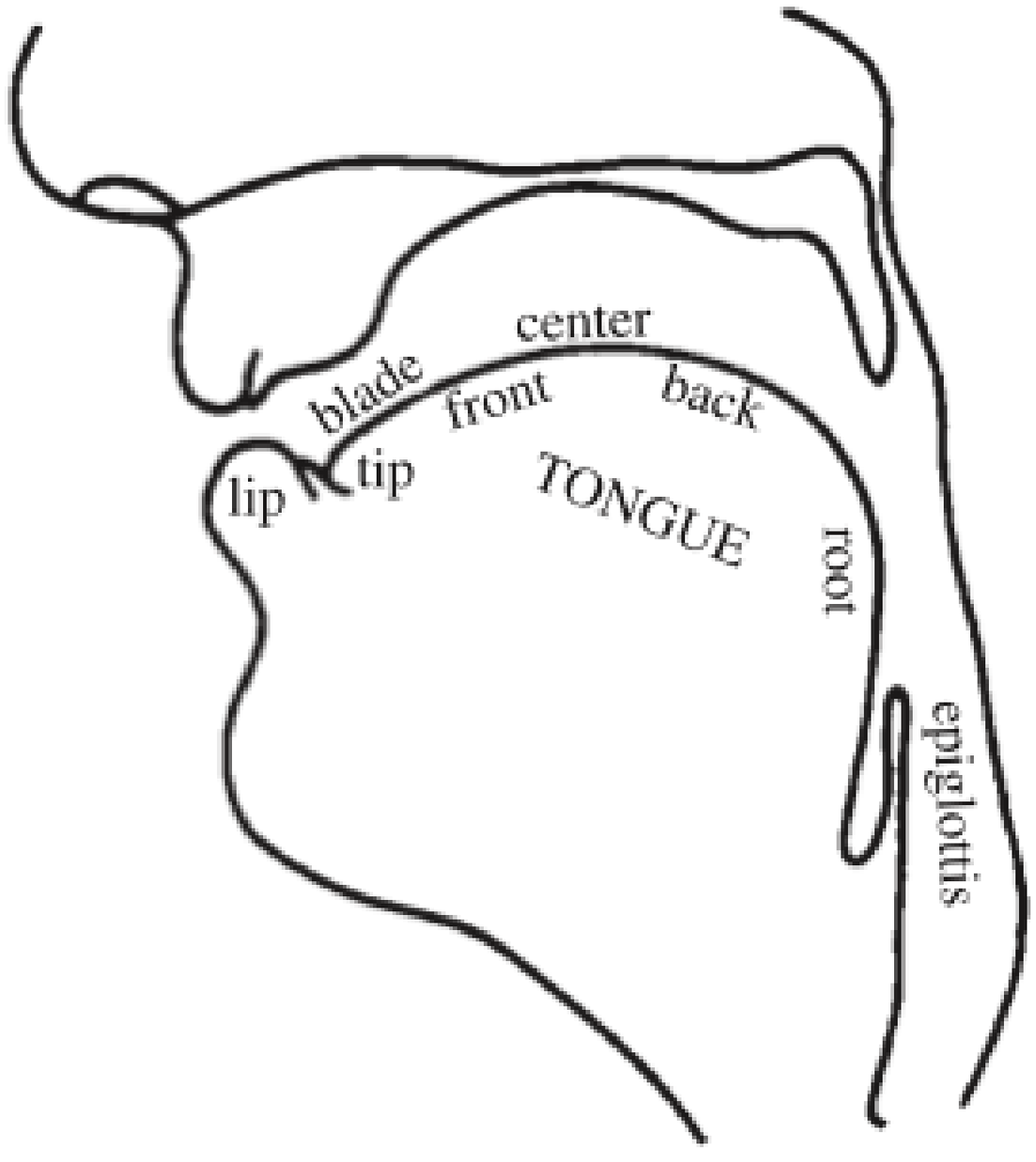
The vocal tract



Vocal tract (upper)



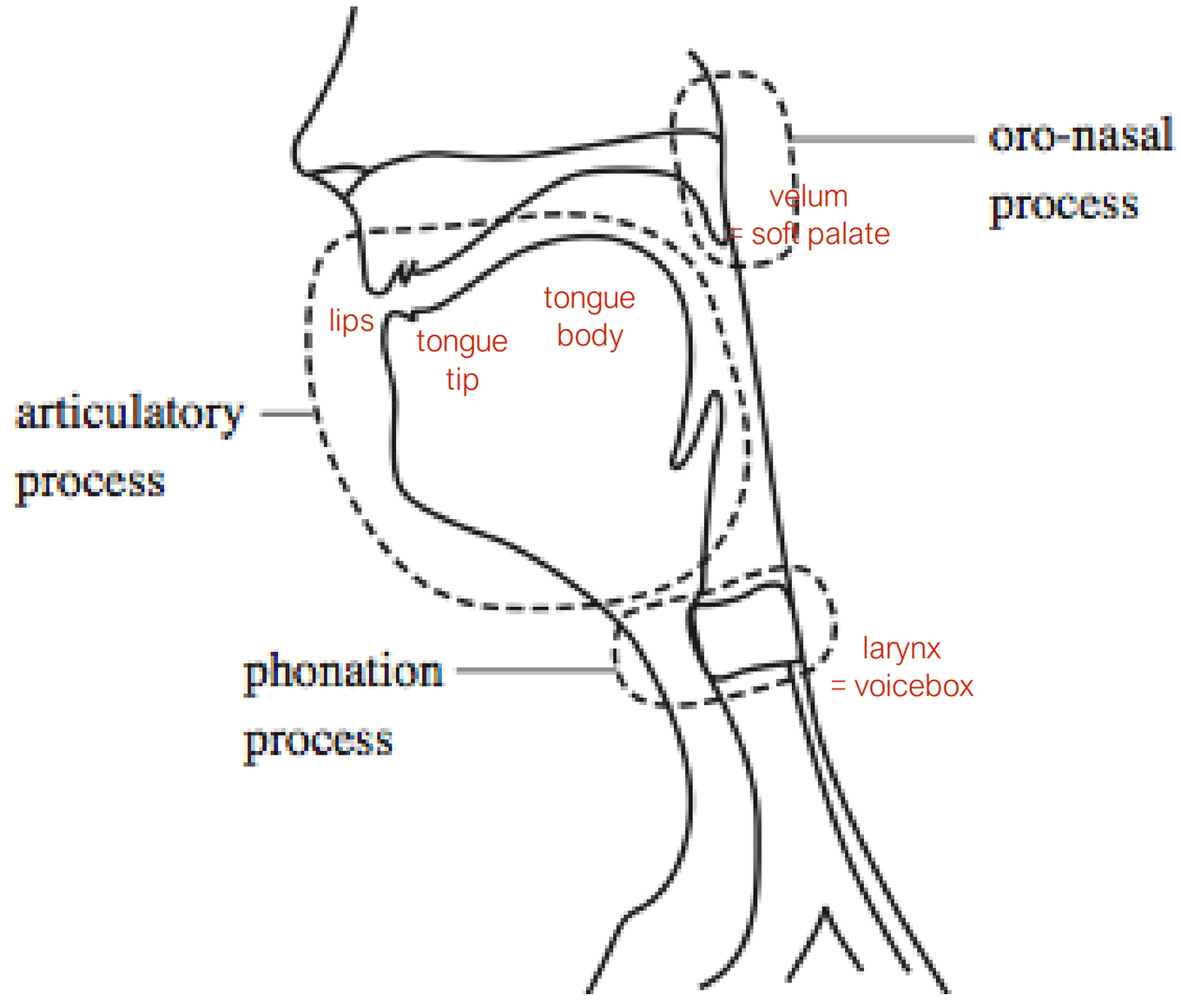
Vocal tract (lower)



Inside the vocal tract

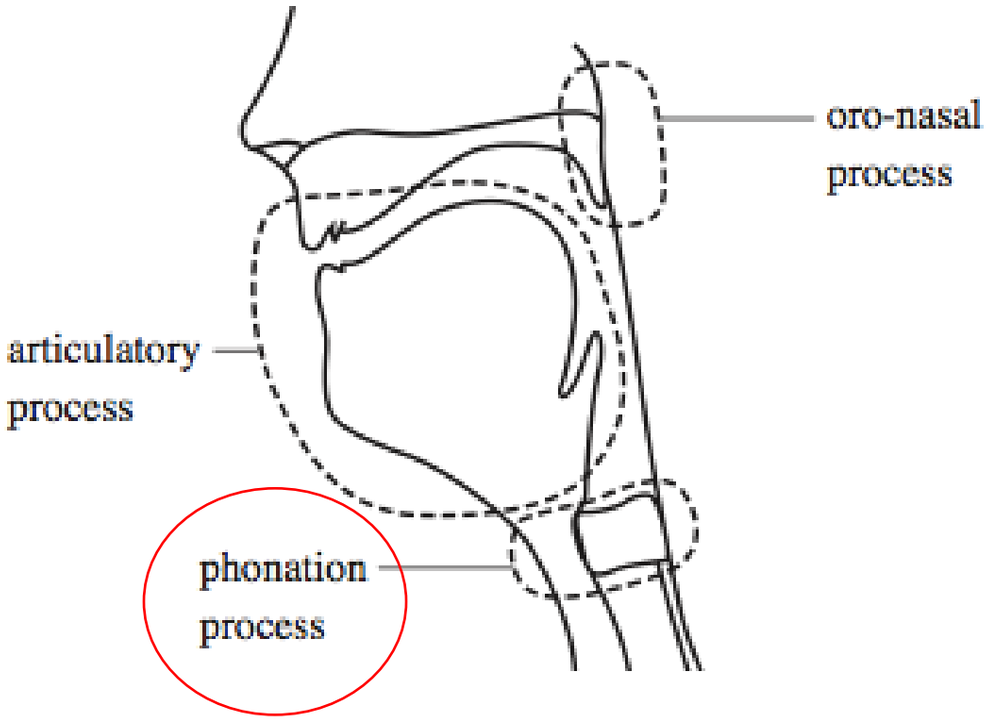
Articulatory phonetics
=speech production (dance of the tongue)

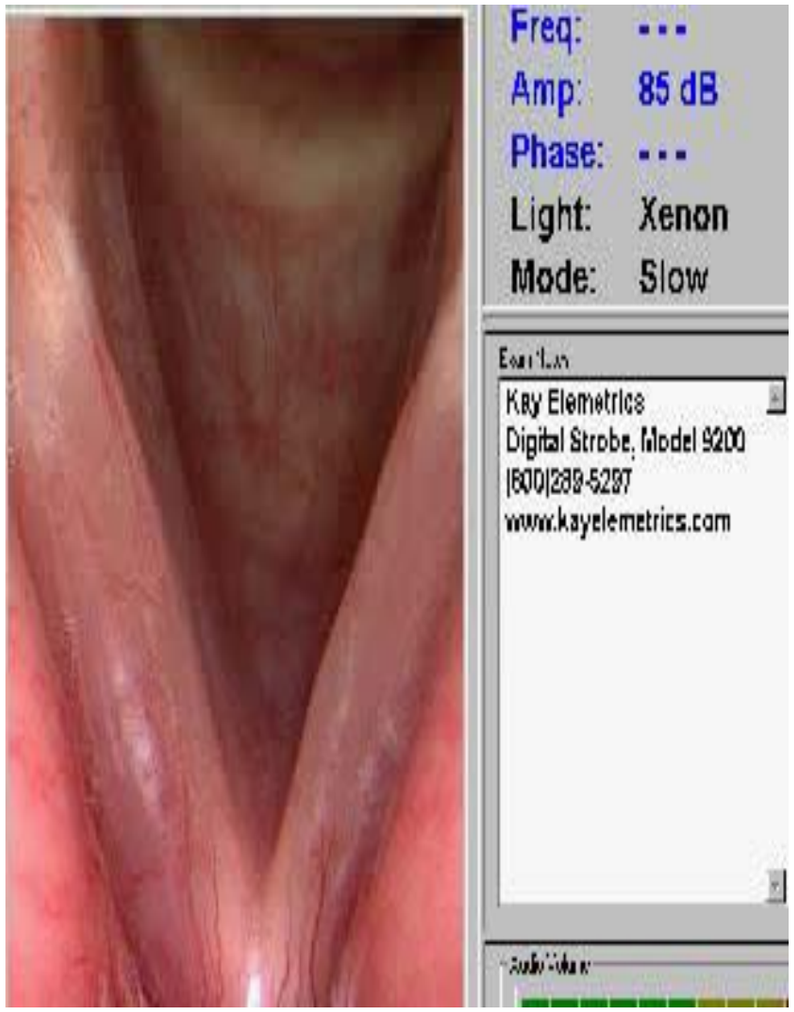
5 speech organs
=constrictors
=articulators



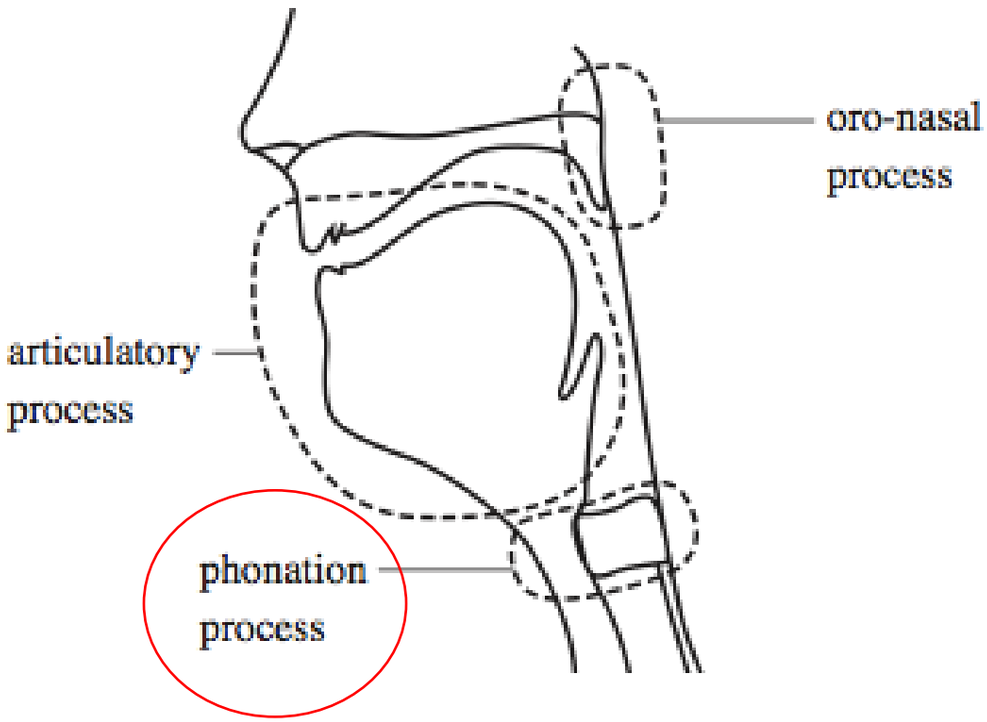
Phonation process in larynx

Vocal cords vibration

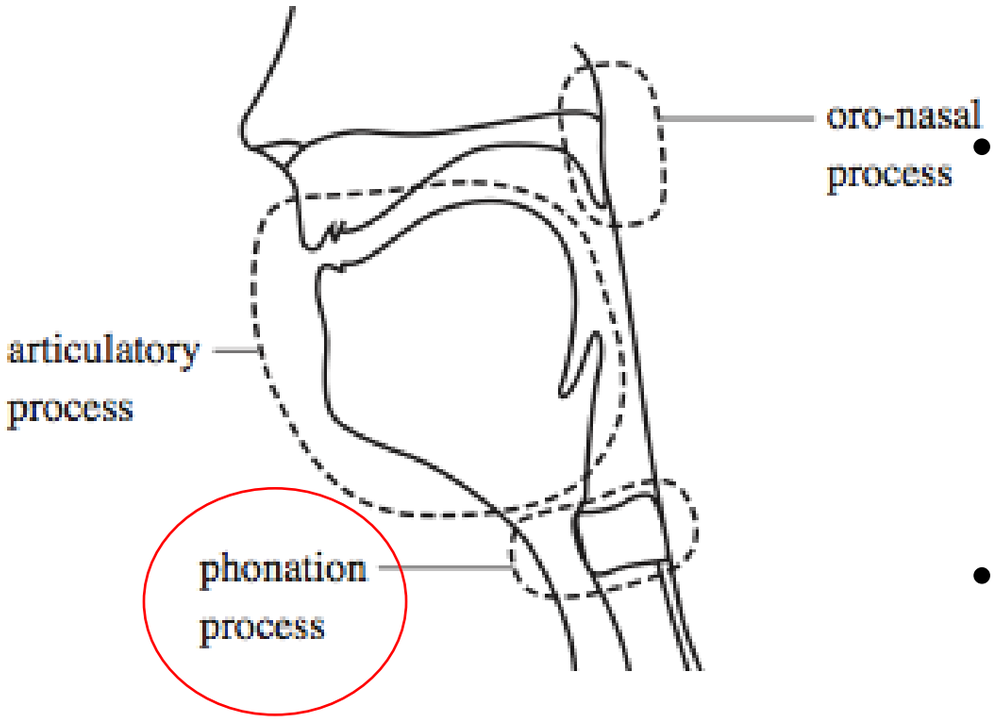




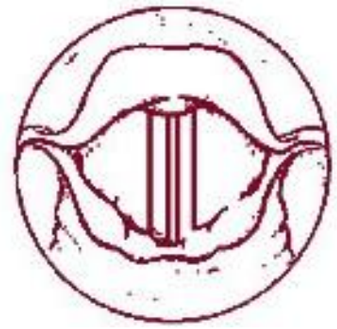
Slow motion
of vocal cords vibration



Larynx = voicebox

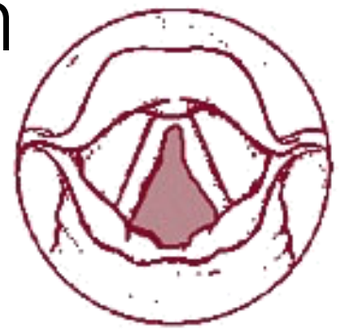


- voiced: can feel vibration



e.g. v, z, l, m, a, i, …

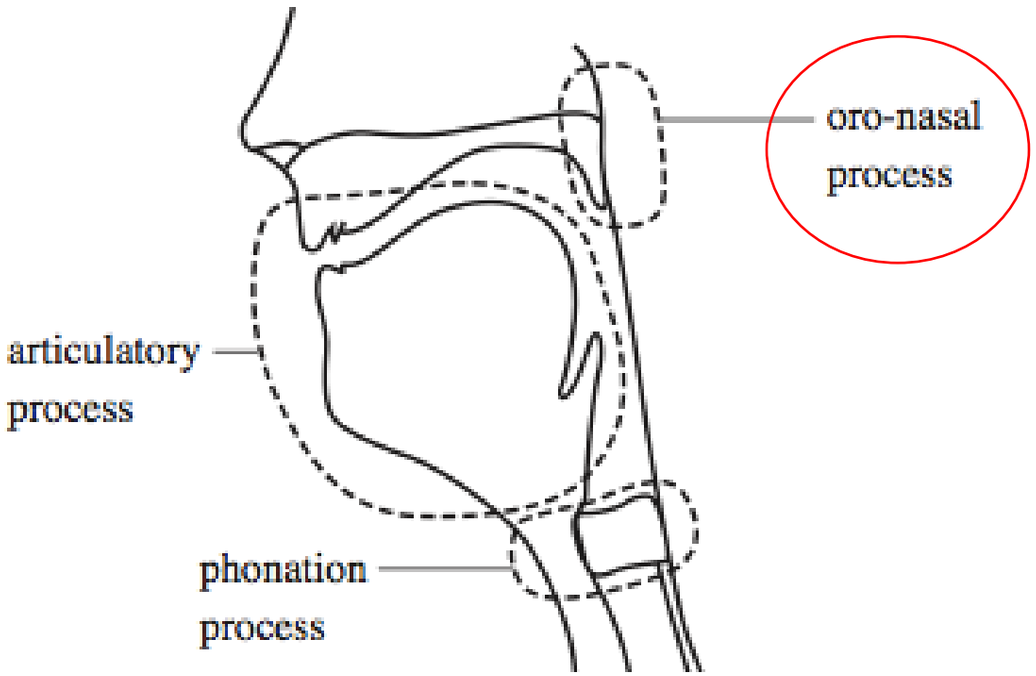
- voiceless: can't feel vibration

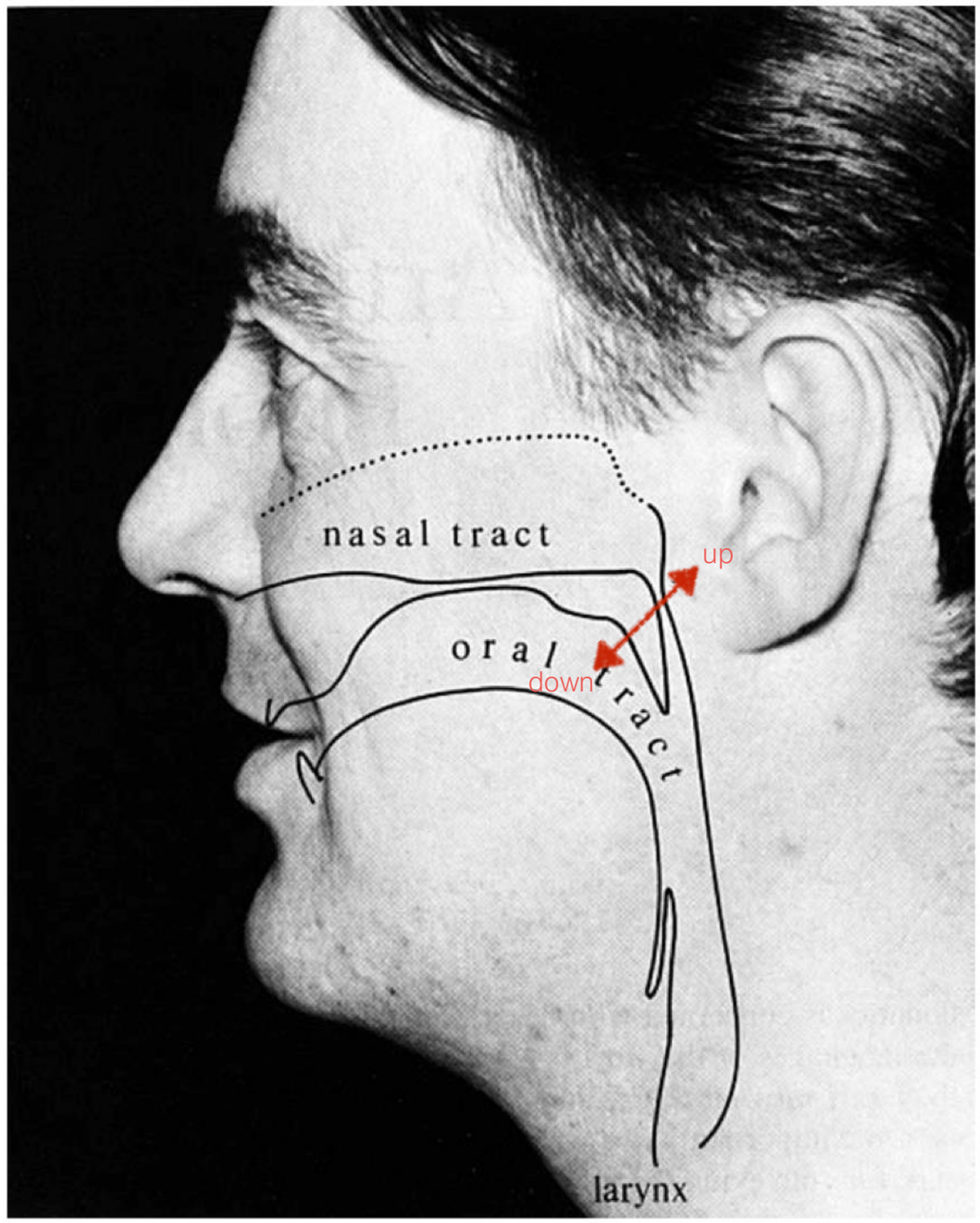


e.g. f, s, k, p, h, …

Oro-nasal process in velum

Whether velum lowered



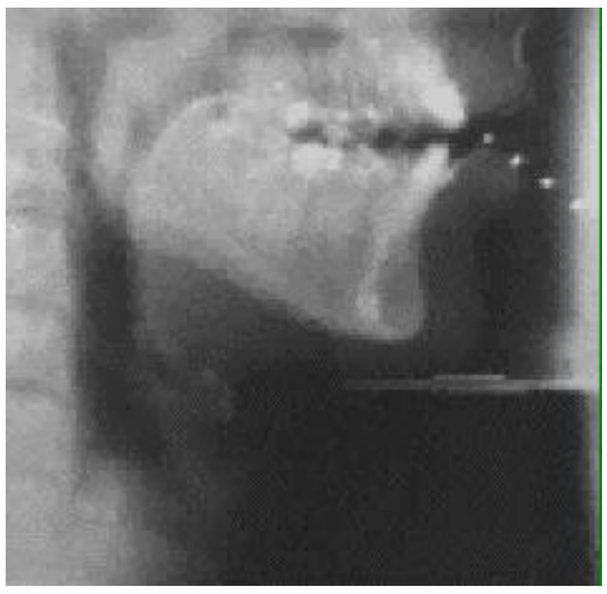


- nasals: m n ng …
- when breathing?

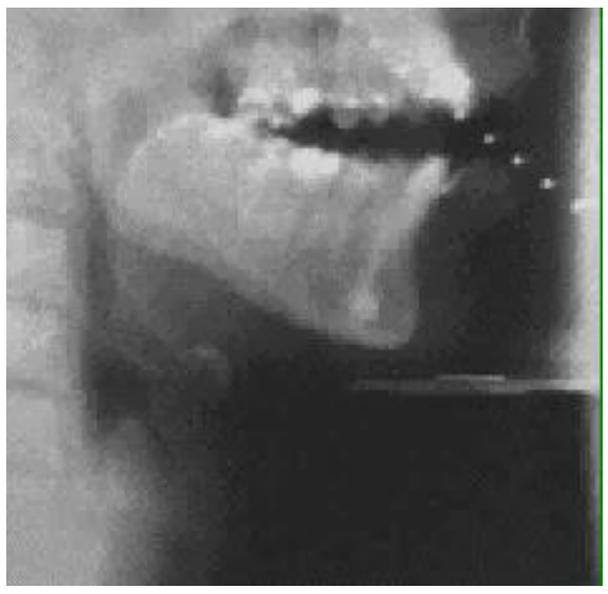
Articulatory process in lips / tongue tip / tongue body

constriction

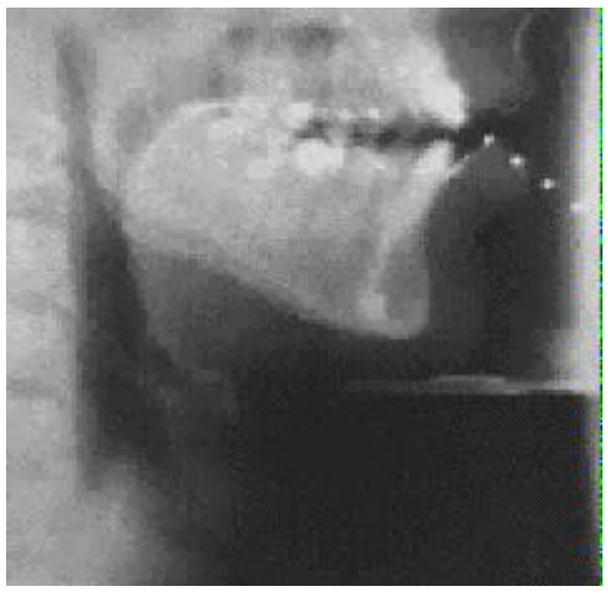
lips



tongue tip



tongue body



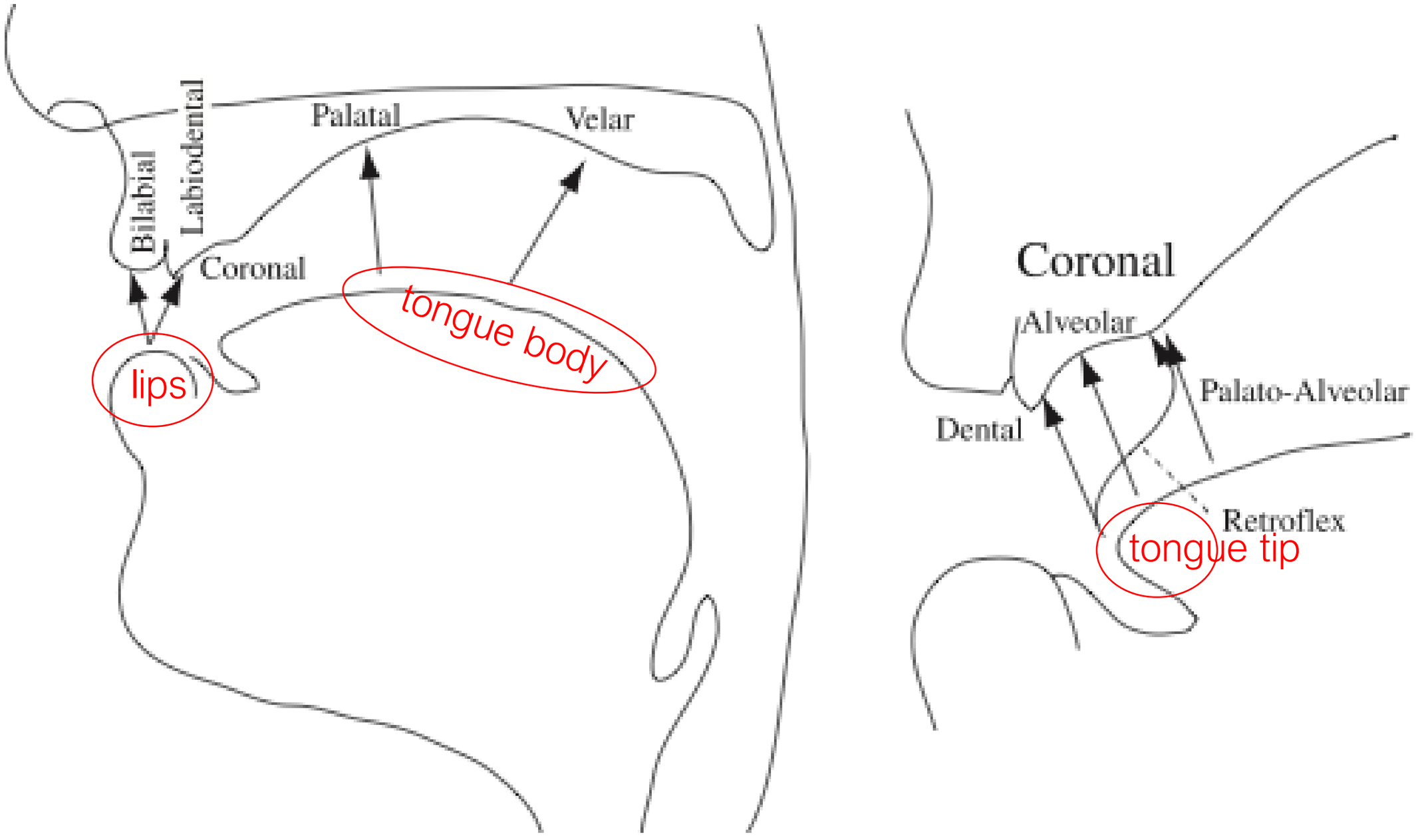
Control of constrictors(articulators)

By CL and CD

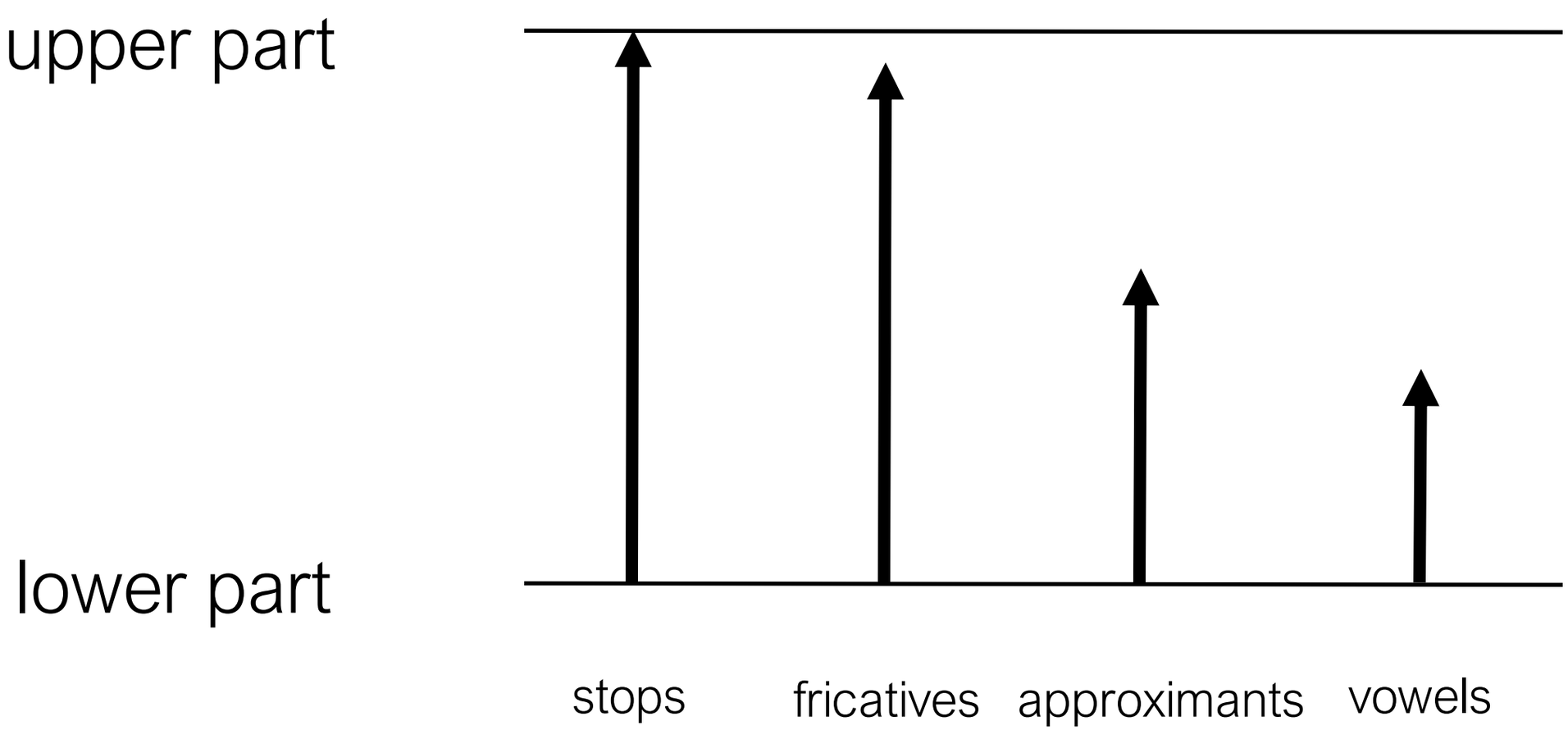
Each constrictor
needs to be more specific in geometry

- Constriction location (CL): where exactly?
- Constriction degree (CD): how much exactly?

Constriction location (CL)



Constriction degree (CD)



How to produce English consonants & vowels?

By specifying
constrictors, CD, and CL

e.g. which sound is produced?

- Constrictor: Lips
- CL: Bilabial
- CD: Closure

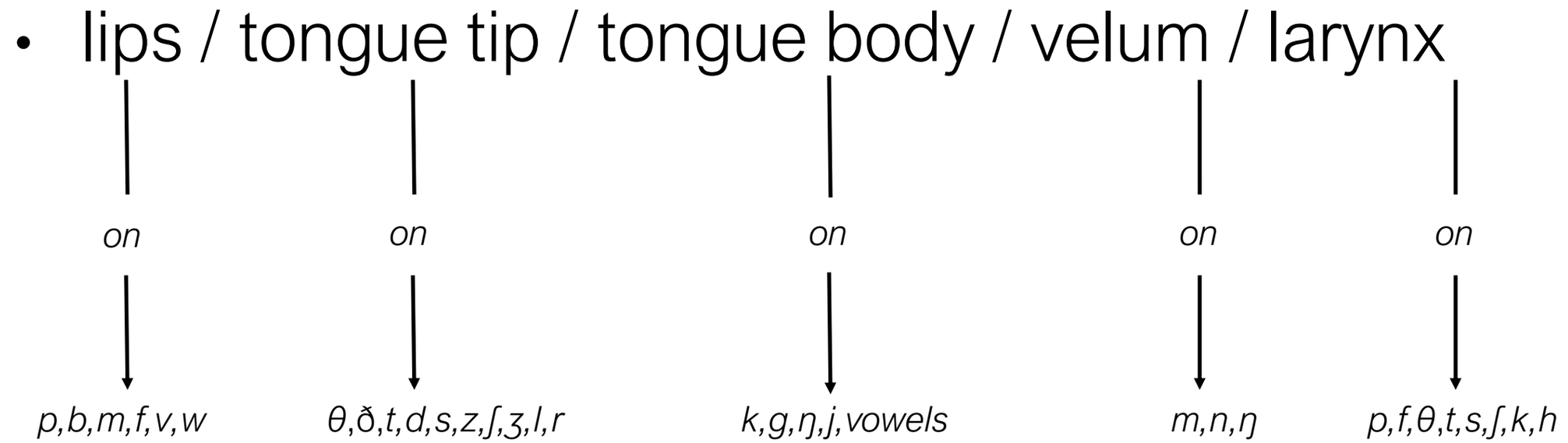
From now on
let’s call a sound
(a consonant or vowel)
a phoneme.

Phonemes

- Individual sounds that form words
  - e.g., ‘psycho’ /s aɪ k oʊ/

phonemes

- a combination of speech organs’ actions



N.B. on = active = effortful

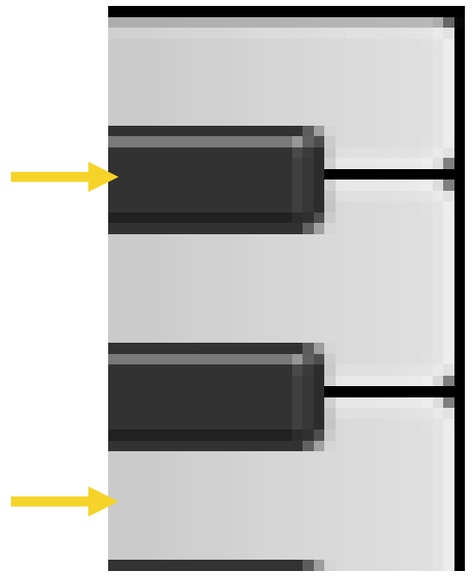
off = inactive = effortless =default

How to produce a phoneme?

Piano playing analogy

- e.g., /t/

speech organ	on/off
lips	off
tongue tip	on
tongue body	off
velum	off
larynx	on



- Five speech organs analogous to 5 keyboards on piano

- Producing a phoneme analogous to hitting a certain set of keys

Phonemes (practice)

- e.g., /t/

speech organ	on/off
lips	off
tongue tip	on
tongue body	off
velum	off
larynx	on

- e.g., /d/ ?

Phonemes (practice)

- e.g., /m/

speech organ	on/off
lips	on
tongue tip	off
tongue body	off
velum	on
larynx	off

- e.g., /n/ ?

Phonemes (practice)

- e.g., /g/

speech organ	on/off
lips	off
tongue tip	off
tongue body	on
velum	off
larynx	off

- e.g., /k/ ?

/t/ vs. /s/?

speech organ	on/off
lips	off
tongue tip	on
tongue body	off
velum	off
larynx	on

speech organ	on/off
lips	off
tongue tip	on
tongue body	off
velum	off
larynx	on

- not different! More specification needed.

Constriction Degree (CD)

- How much constriction?
  - stops: full constriction *(e.g. p, t, k, b, d, g, m, n, ŋ, …)*
  - fricatives: critical constriction - *(e.g. s, z, f, z, ʃ, ʒ,* θ, ð, *…)*
  - approximants: little constriction - *(e.g. r, l, w, y, …)*
  - vowels: no constriction - *(e.g. vowels)*

- For active (=on) speech organs only among lips, tongue tip, tongue body.

/t/ speech organ	on/off	CD
lips	off	-
tongue tip	on	stop
tongue body	off	-
velum	off	-
larynx	on	-

/t/ vs. /s/

speech organ	on/off	CD
lips	off	-
tongue tip	on	stop
tongue body	off	-
velum	off	-
larynx	on	-

speech organ	on/off	CD
lips	off	-
tongue tip	on	fric.
tongue body	off	-
velum	off	-
larynx	on	-

/t/ vs. /r/?

speech organ	on/off	CD
lips	off	-
tongue tip	on	stop
tongue body	off	-
velum	off	-
larynx	on	-

speech organ	on/off	CD
lips	off	-
tongue tip	on	approx.
tongue body	off	-
velum	off	-
larynx	off	-

/a/ vs. /g/?

speech organ	on/off	CD
lips	off	-
tongue tip	off	-
tongue body	on	vowel
velum	off	-
larynx	off	-

speech organ	on/off	CD
lips	off	-
tongue tip	off	-
tongue body	on	stop
velum	off	-
larynx	off	-

/s/ vs. /ʃ/?

speech organ	on/off	CD
lips	off	-
tongue tip	on	fric.
tongue body	off	-
velum	off	-
larynx	on	-

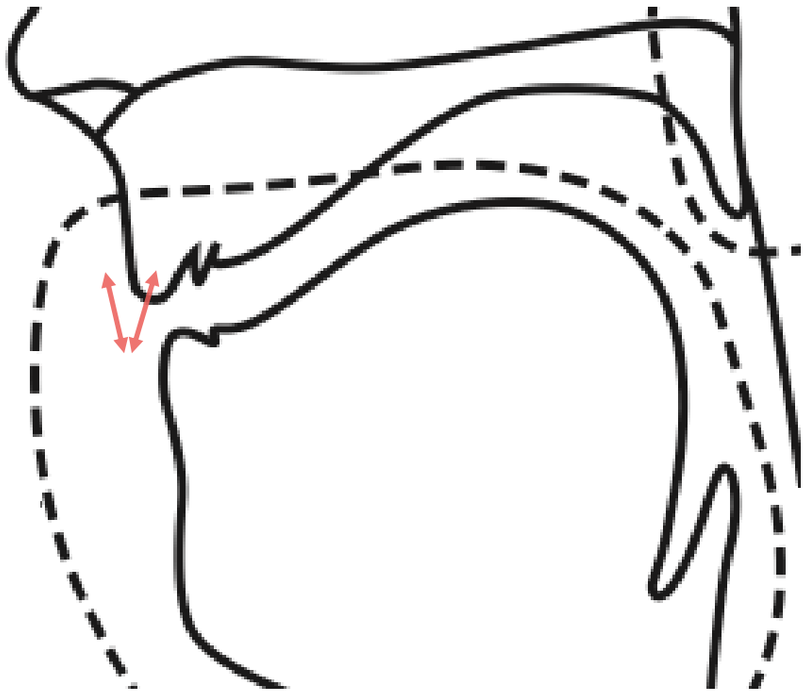
speech organ	on/off	CD
lips	off	-
tongue tip	on	fric.
tongue body	off	-
velum	off	-
larynx	on	-

- not different! More specification needed.

Constriction Location (CL)

- Where constriction occurs at a speech organ?

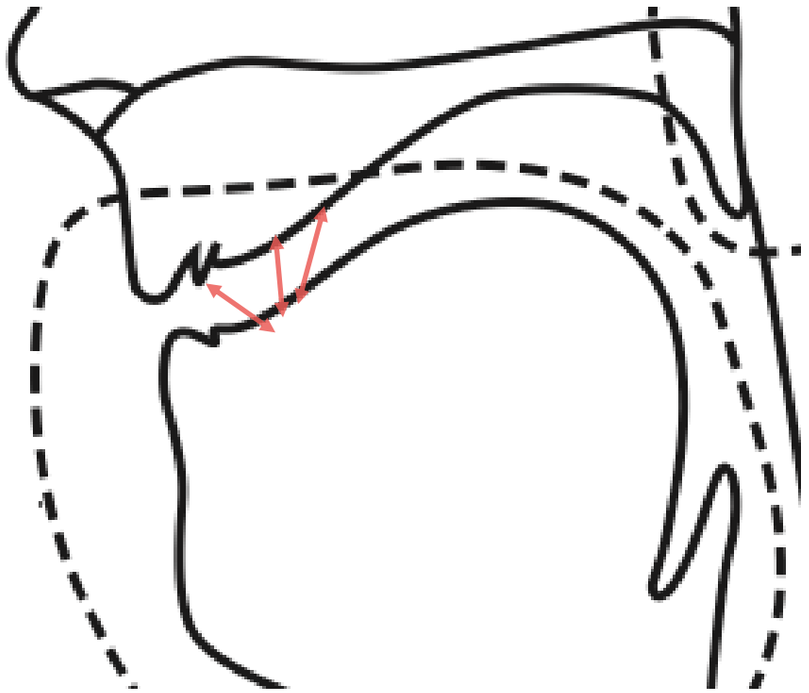
lips



labial / labiodental

p,b,m,w f,v

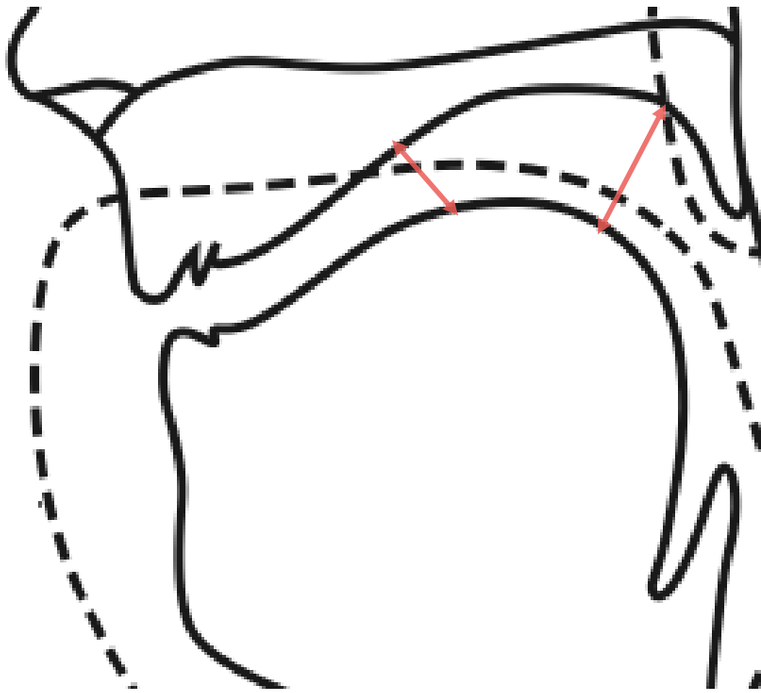
tongue tip



dental / alveolar / palato-alveolar

θ,ð t,d,s,z,l,r ʃ,ʒ

tongue body



palatal / velar

ɟ,ɲ,ʝ

c k,g,ng

/s/ vs. /ʃ/

speech organ	on/off	CD	CL
lips	off	-	-
tongue tip	on	fric.	alv.
tongue body	off	-	-
velum	off	-	-
larynx	on	-	-

speech organ	on/off	CD	CL
lips	off	-	-
tongue tip	on	fric.	pal-alv
tongue body	off	-	-
velum	off	-	-
larynx	on	-	-

Phonemes (practice)

/p/

speech organ	on/off	CD	CL
lips			
tongue tip			
tongue body			
velum			
larynx			

Phonemes (practice)

/h/

speech organ	on/off	CD	CL
lips			
tongue tip			
tongue body			
velum			
larynx			

Phonemes (practice)

/j/

speech organ	on/off	CD	CL
lips			
tongue tip			
tongue body			
velum			
larynx			

Phonemes (practice)

/n/

speech organ	on/off	CD	CL
lips			
tongue tip			
tongue body			
velum			
larynx			

Phonemes (practice)

/v/

speech organ	on/off	CD	CL
lips			
tongue tip			
tongue body			
velum			
larynx			

Phonemes (practice)

/ŋ/

speech organ	on/off	CD	CL
lips			
tongue tip			
tongue body			
velum			
larynx			

Phonemes (practice)

/w/

speech organ	on/off	CD	CL
lips			
tongue tip			
tongue body			
velum			
larynx			

Phonemes (practice)

/n/

speech organ	on/off	CD	CL
lips			
tongue tip			
tongue body			
velum			
larynx			

Phonemes (practice)

/z/

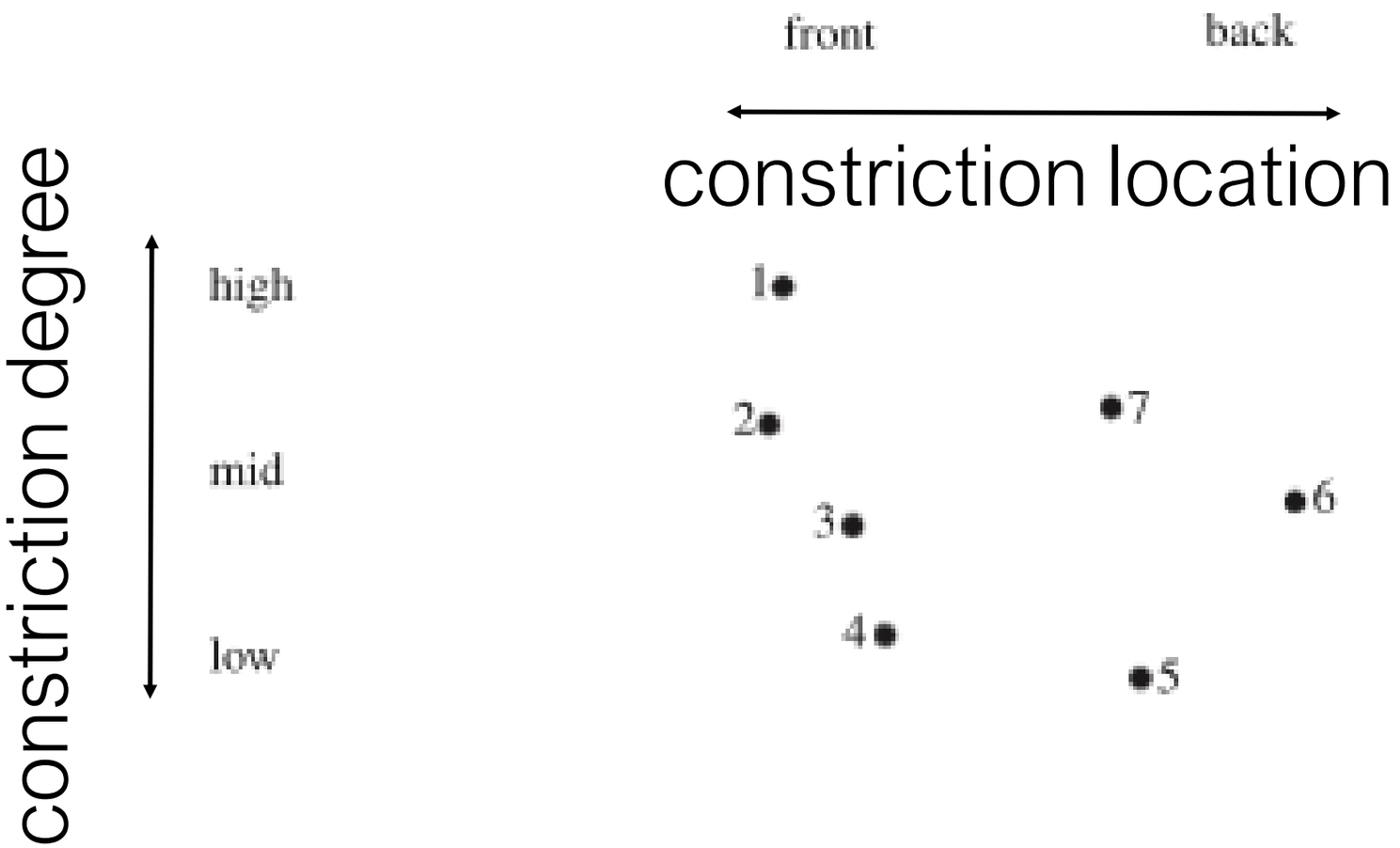
speech organ	on/off	CD	CL
lips			
tongue tip			
tongue body			
velum			
larynx			

Vowels
by tongue body only

Figure 1.12 The positions of the vocal organs for the vowels in the words 1 *heed*, 2 *hid*, 3 *head*, 4 *had*, 5 *father*, 6 *good*, 7 *food*. The lip positions for vowels 2, 3, and 4 are between those shown for 1 and 5. The lip position for vowel 6 is between those shown for 1 and 7.

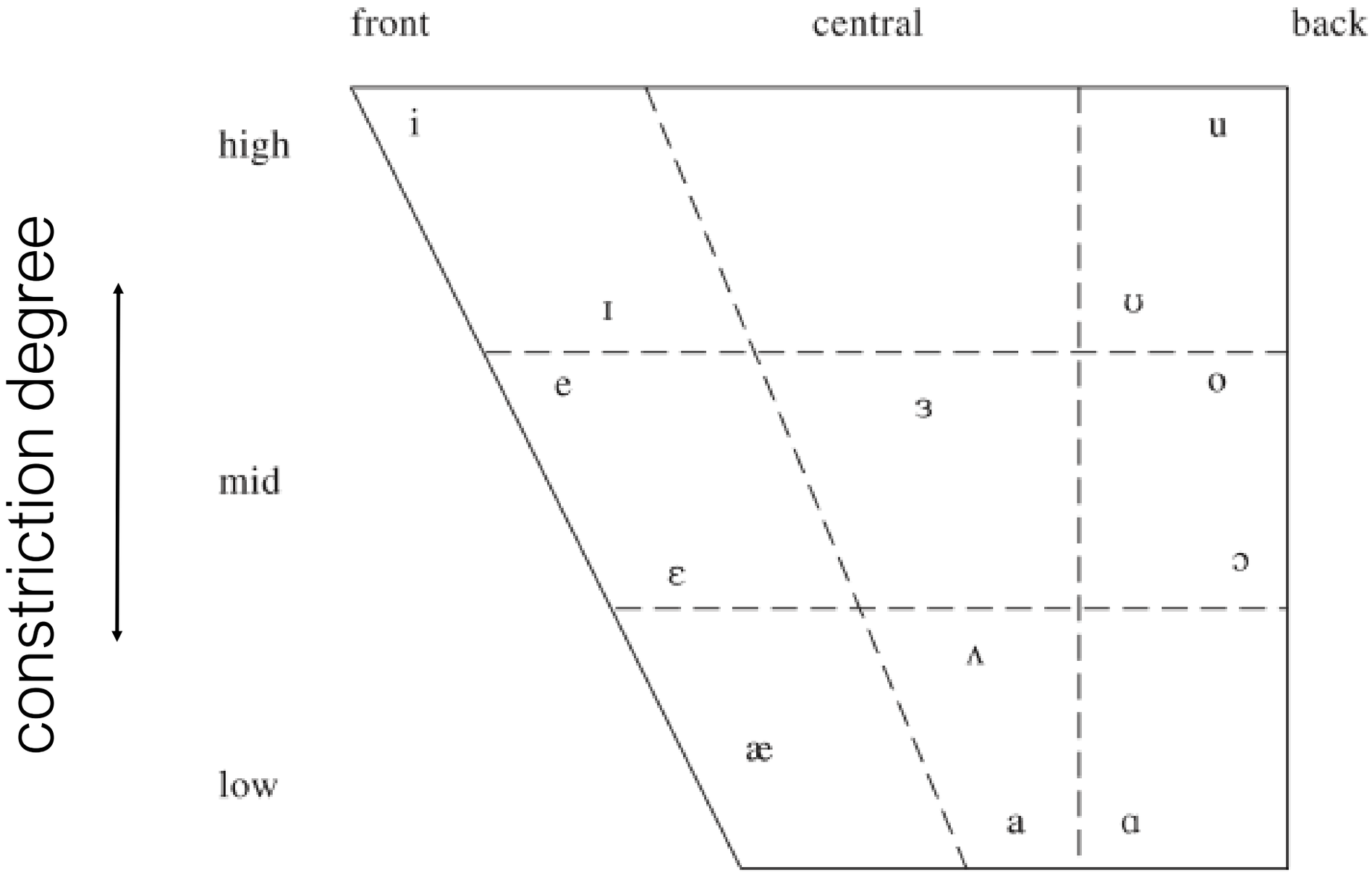
CL & CD specifications

Figure 1.13 The relative positions of the highest points of the tongue in the vowels in 1 *heed*, 2 *hid*, 3 *head*, 4 *had*, 5 *father*, 6 *good*, 7 *food*.



Vowel chart

constriction location



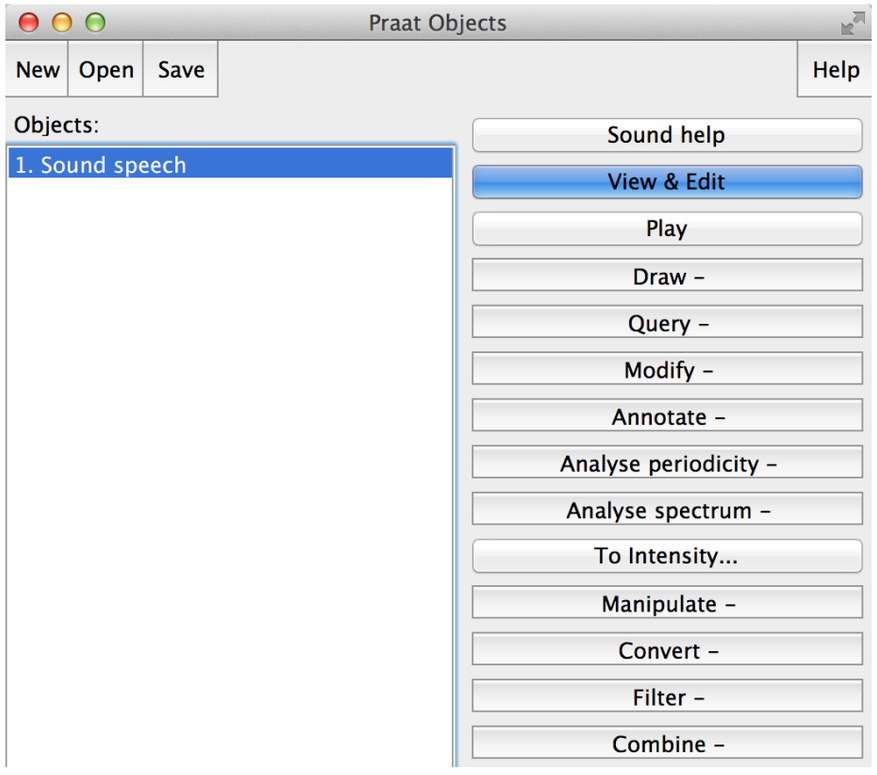
Geometry of tongue body!!

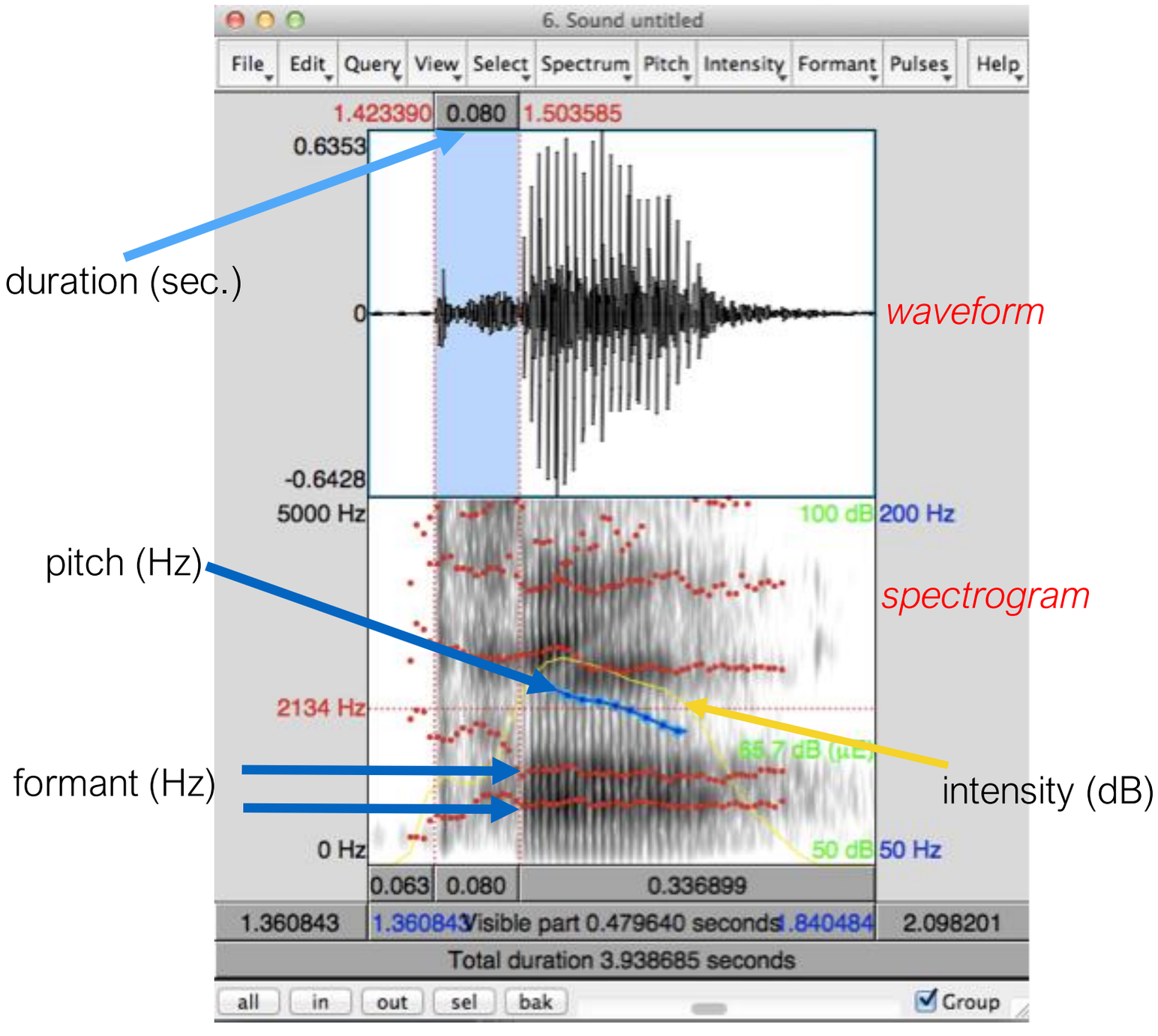
Acoustics

Sound generation

- How is sound generated within the vocal tract?
  - Vocal fold vibration (e.g. vowels)
  - Turbulence noise (e.g. fricatives)
  - Release of cavity under high pressure (e.g. stops)
- All these sound generation mechanisms require getting air to flow
  - English: airflow caused by a single mechanism (action of the lungs)
  - Other languages: alternate methods for getting air to flow

Acoustics in Praat





Praat

- Duration:
  - select (click and drag on waveform or spectrogram)
  - read a value (sec.) on the top
  - zoom in (if not visible)

Praat

- Intensity:
- show intensity
- click on green
- read a value (dB) on the right

Praat

- Pitch:
  - show pitch
  - pitch setting - pitch range
    - 65-200Hz for male speech
    - 145-275Hz for female speech
- click on blue
- read a value (Hz) on the right

Praat

- Formant:
- show formants
- place the cursor on one of the trajectories
- read a value (Hz) on the left

Autosegmentation

Consonant acoustics

Types of Stops
voiced vs. voiceless vs. aspirated

- English ‘bases’ Spanish ‘pesos’ English ‘paces’

Voice Onset Time: VOT

- The interval between the release of a closure and the start of the voicing

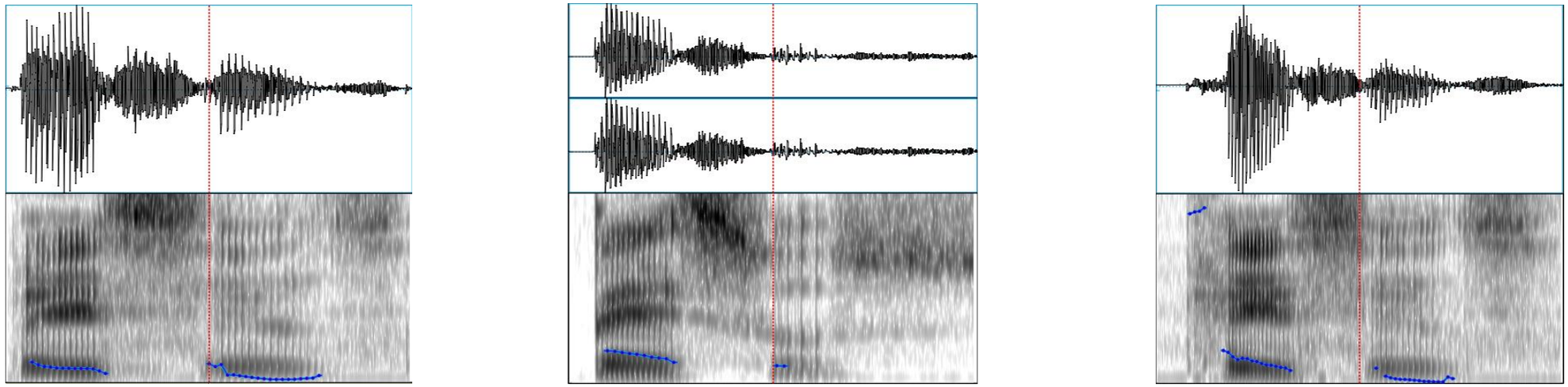
- The easiest way to visualize VOT is by reference to the waveform of a sound

- Compare VOT:

  - /ba/: English as L1 vs. English as L2

Voice Onset Time: VOT

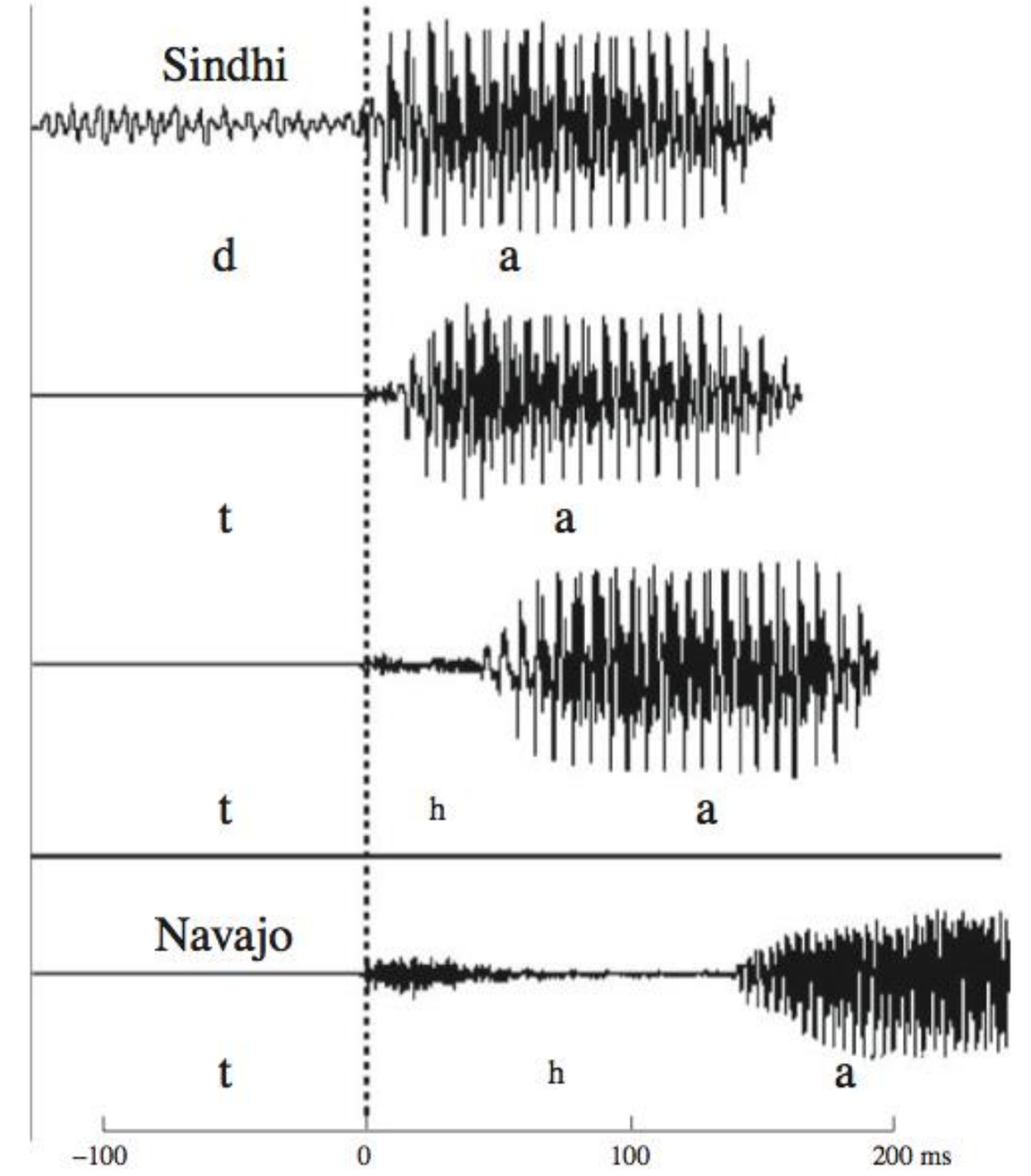
- English ‘bases’ Spanish ‘pesos’ English ‘paces’



- Measure VOTs
  - download VOT.Collection in “course materials”

Voice Onset Time

Figure 6.7 Waveforms showing stops with different degrees of voicing and aspiration.



Vowel acoustics

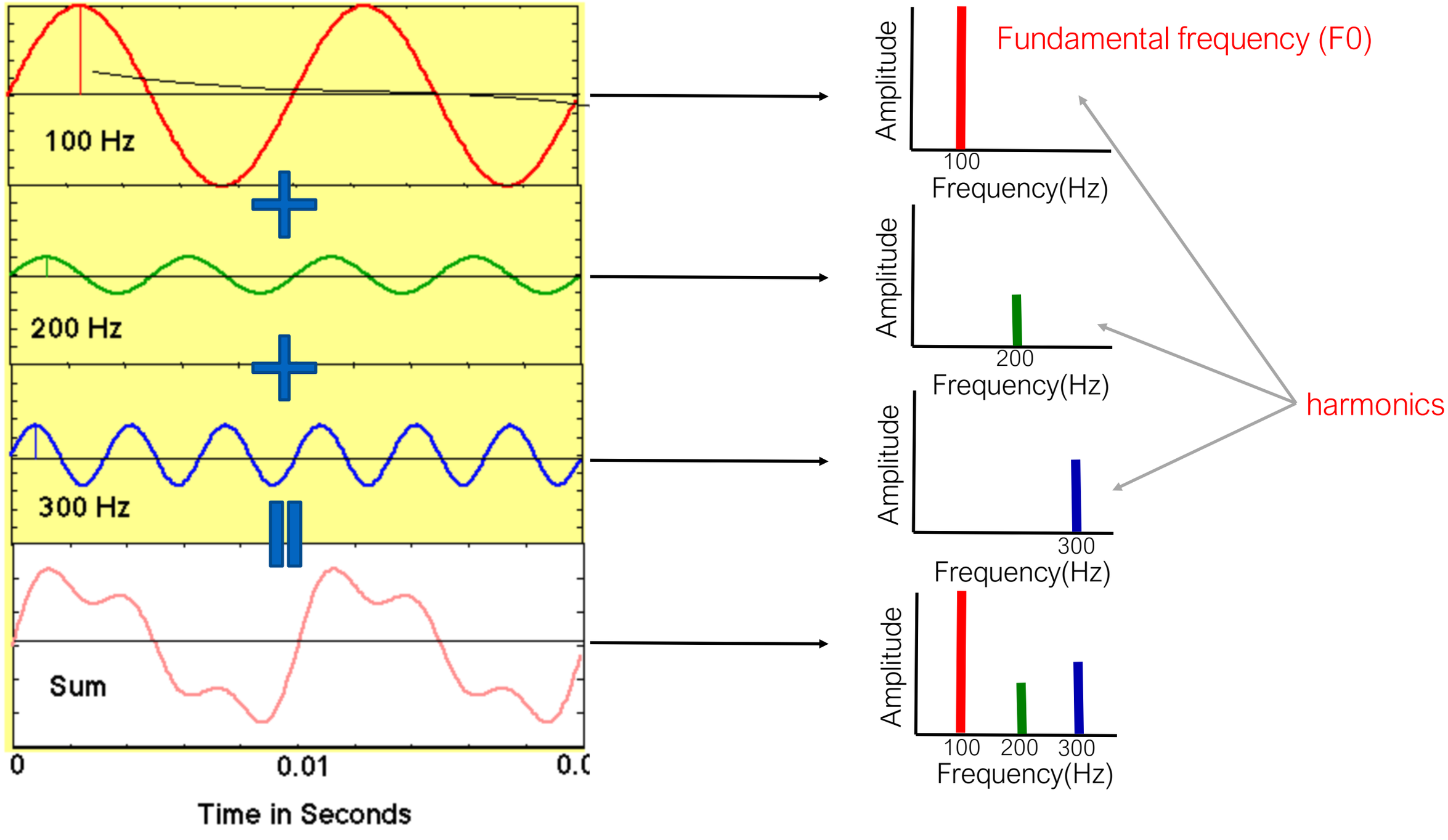
Vowel acoustics

- Record yourself with the vowel /a/
- Measure pitch using Praat
  - The number of occurrences of a repeating event per second (Hz)
    - repeating event = vibration of vocal folds
    - Repeating? This might remind you of a sine wave!
- Let's create a sine wave (=pure tone) with the same frequency using Praat
- Why different in sound quality?
  - Source & Filter

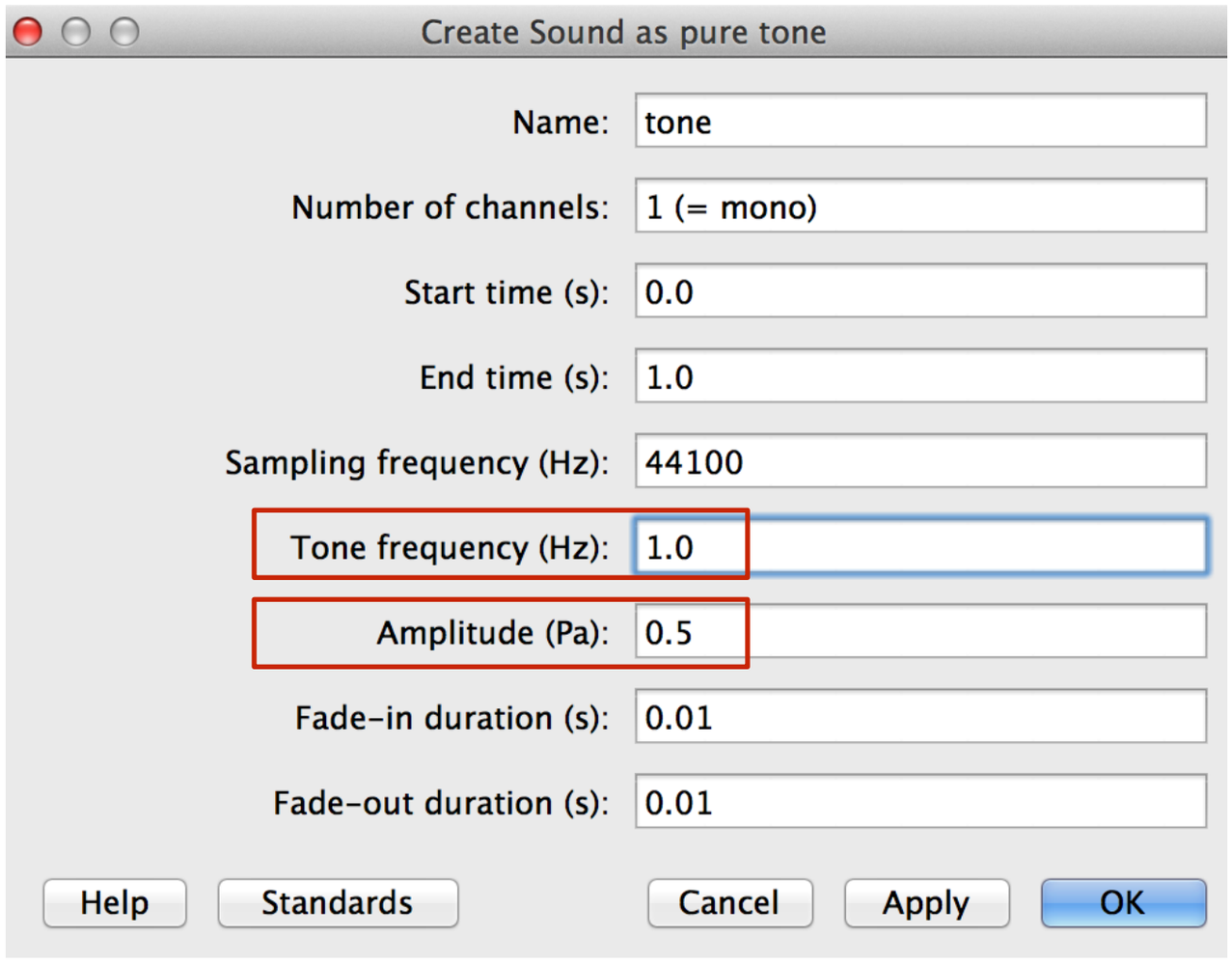
Source

- Sound at larynx
- Listen to a sentence from a SiSwati (Southern Bantu Language; Swaziland)
- Listen to the glottal wave recorded from subject's larynx while the sentence is being produced
- Doesn’t sound like a pure or simplex tone
- A complex tone

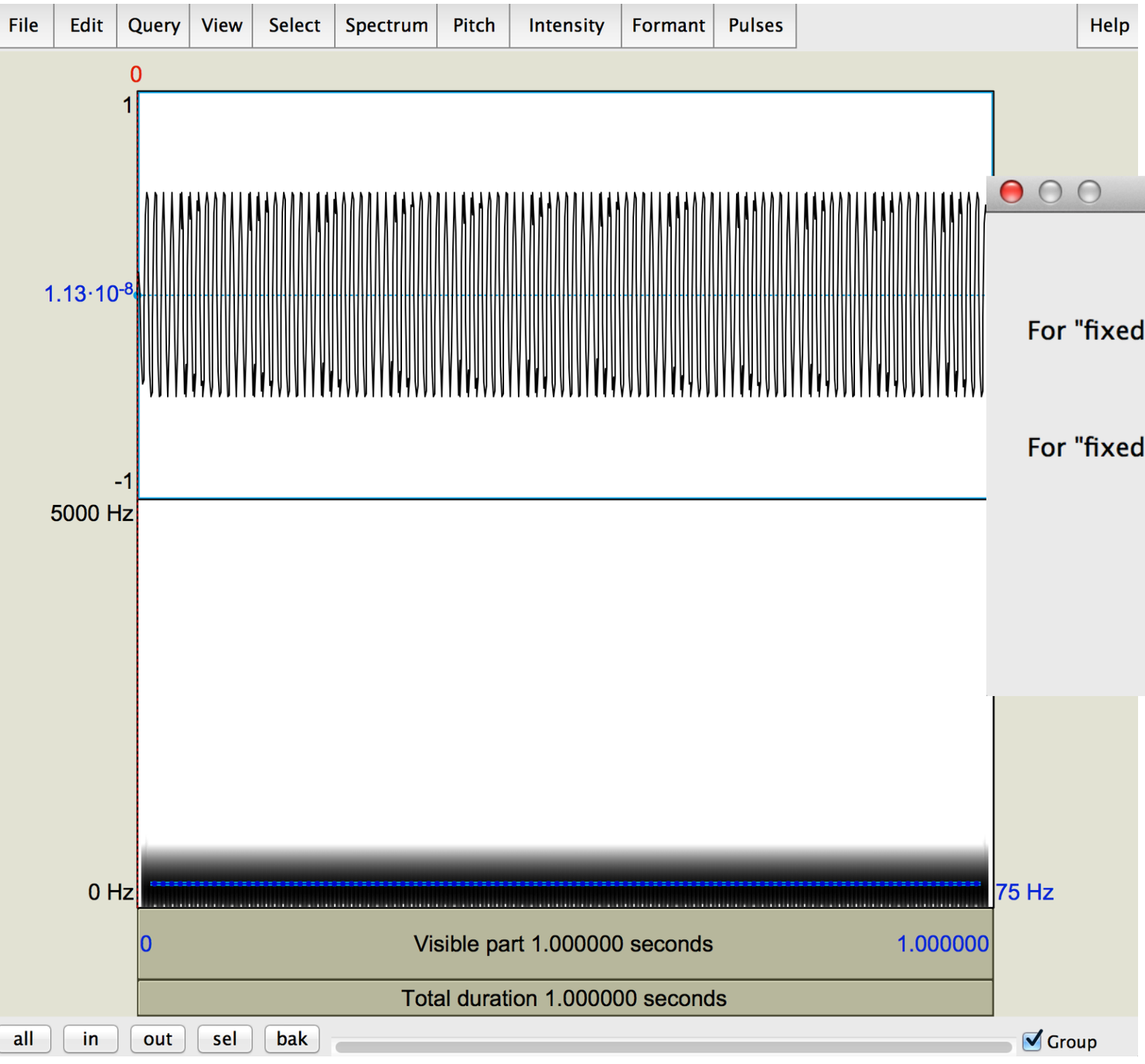
Complex tone in spectrum



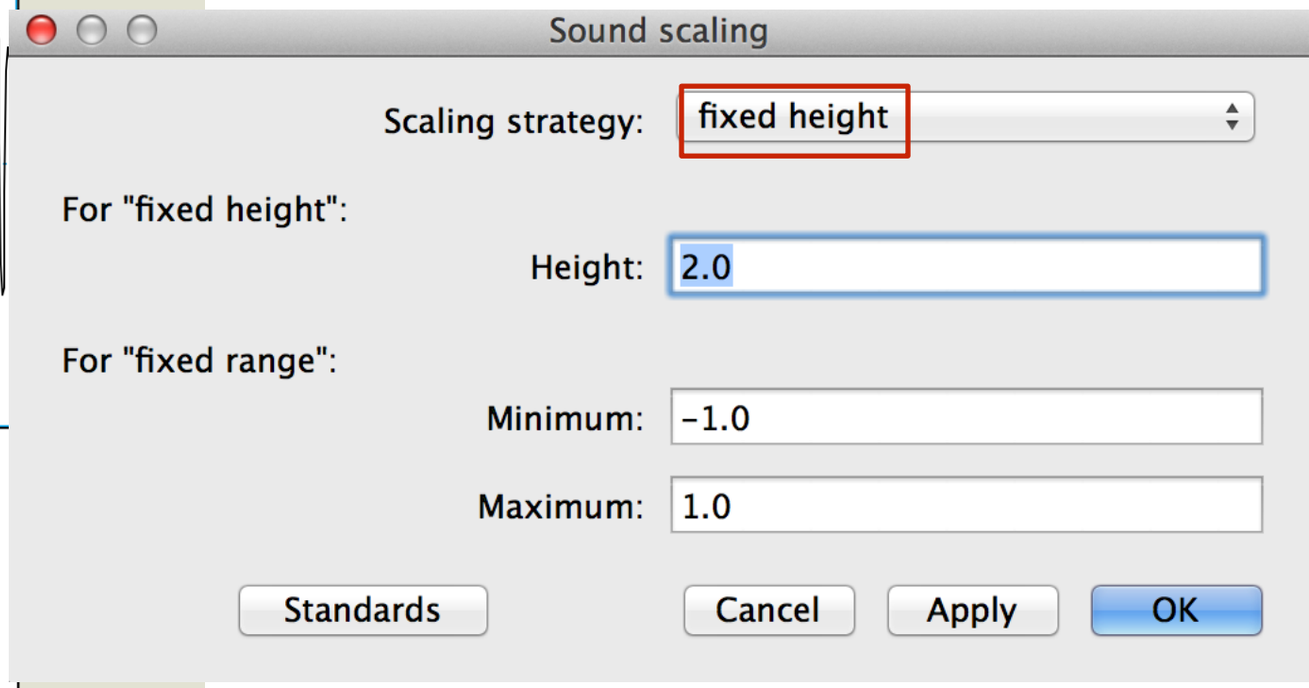
Practice with pure tone & spectrum



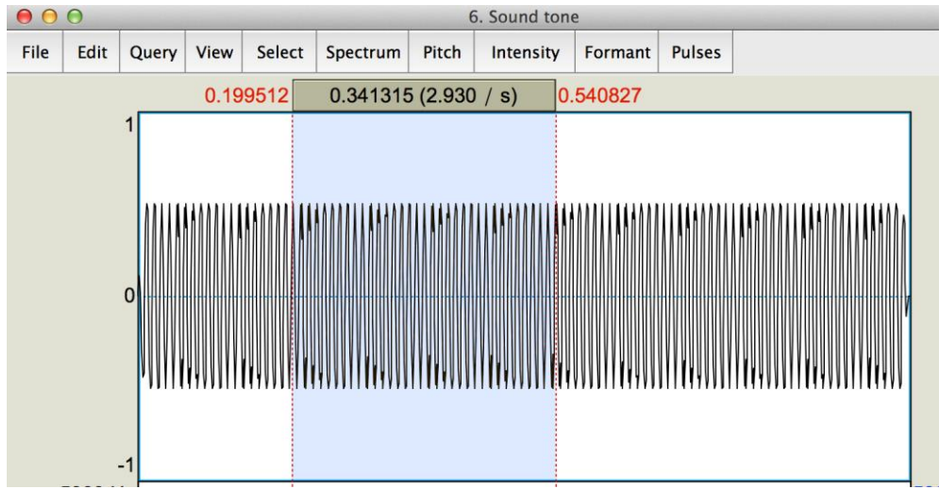
Practice with pure tone & spectrum



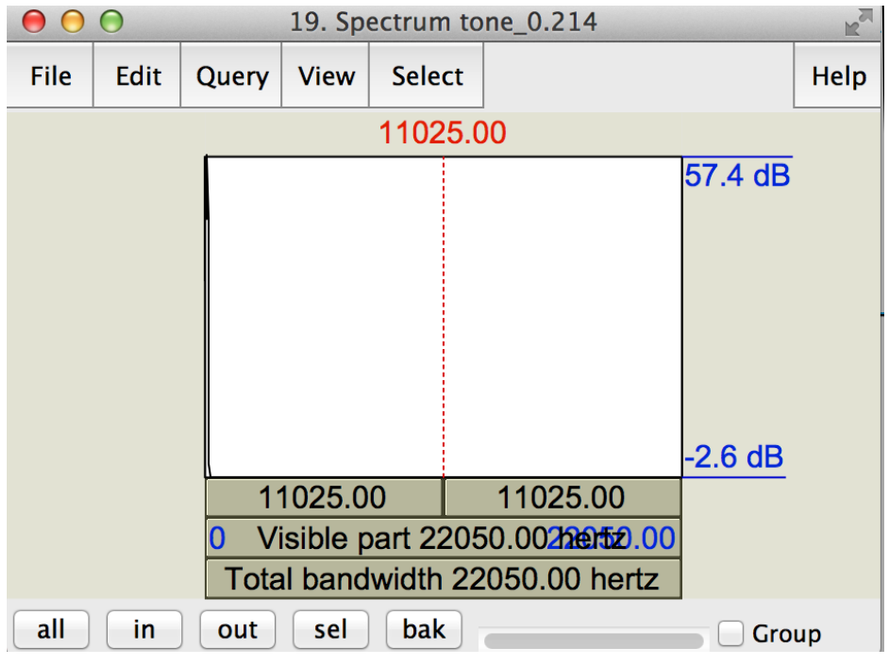
View:Sound scaling



Practice with pure tone & spectrum



Select at least several cycles
Spectrum:Spectral slice

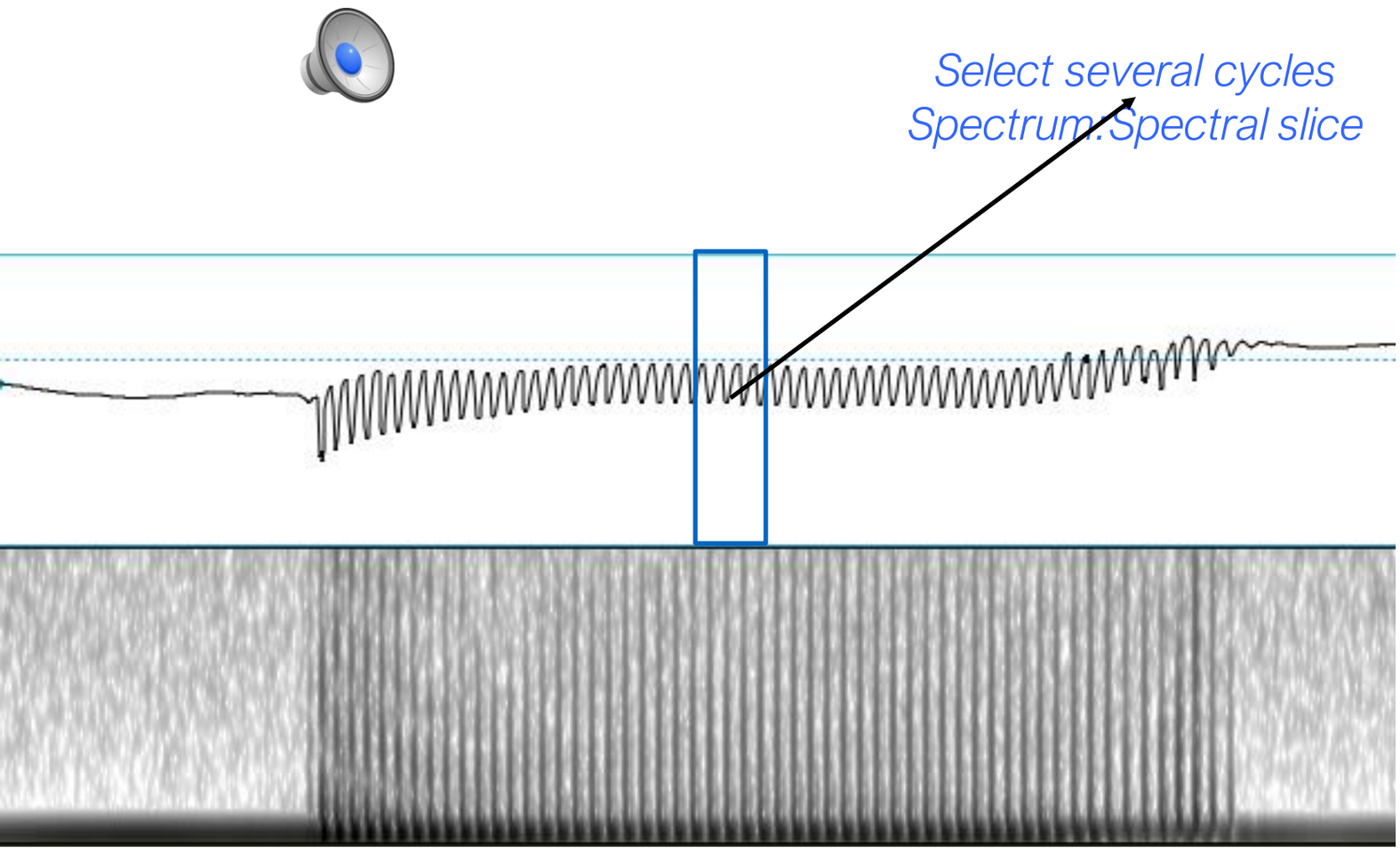


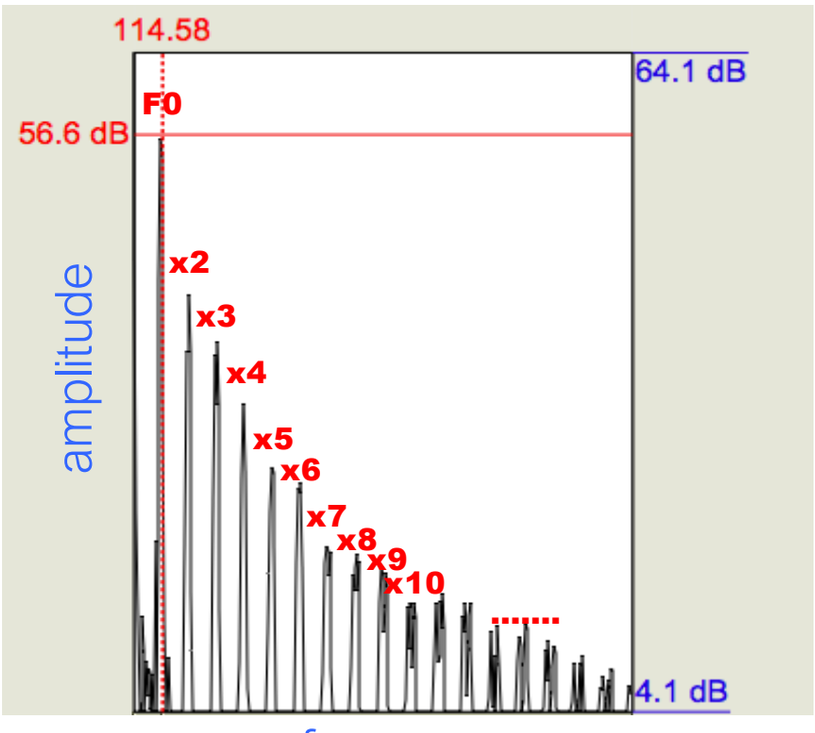
View:Zoom
From 0 to 3000

Human voice source

- Measured by ElectroGlottoGraph (EGG)
- Open vowelEGG.collection in Praat
- Compare spectrums between EGG and audio

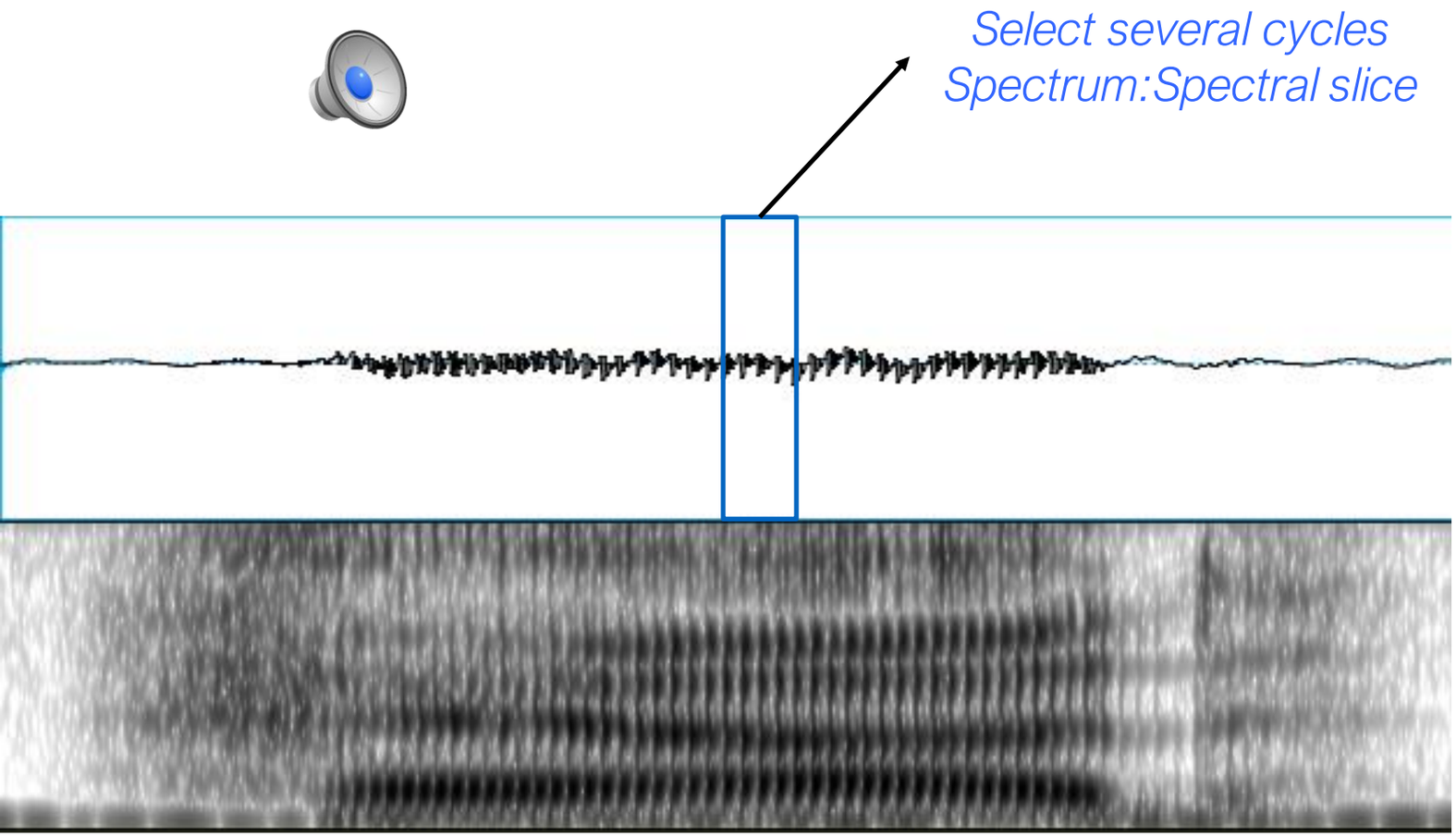
Human voice source

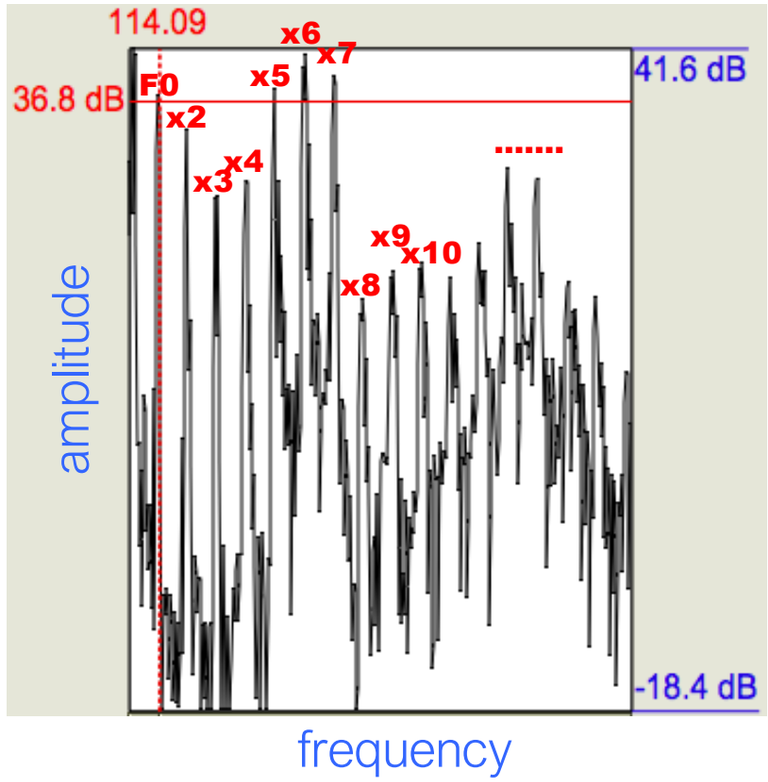




gradually decreasing

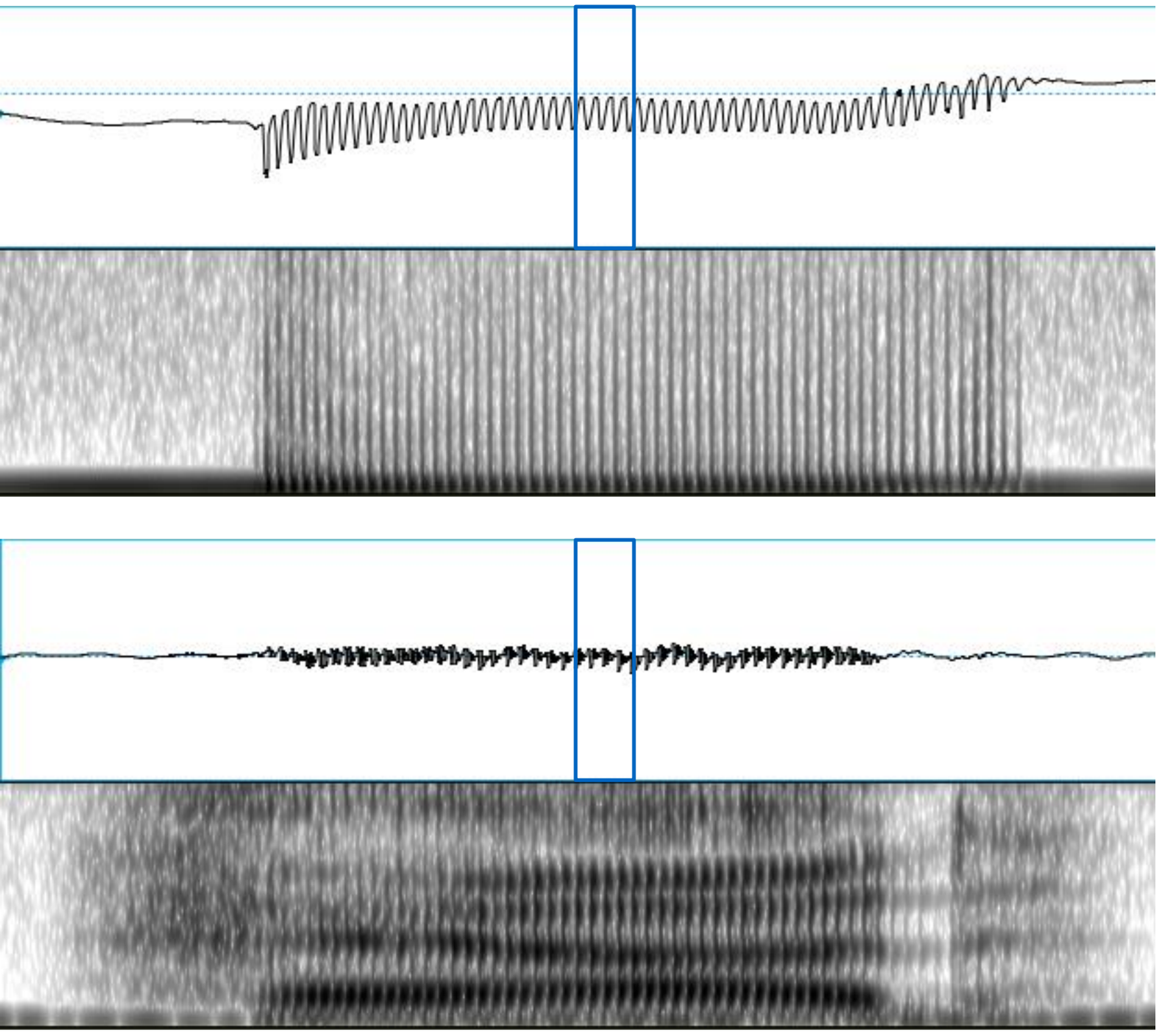
Filtered by vocal tract

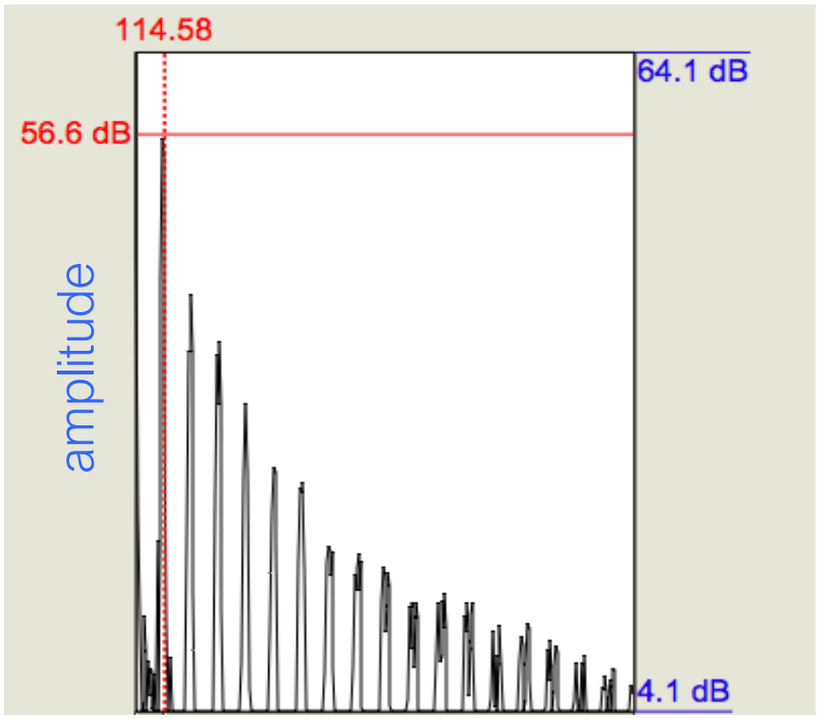


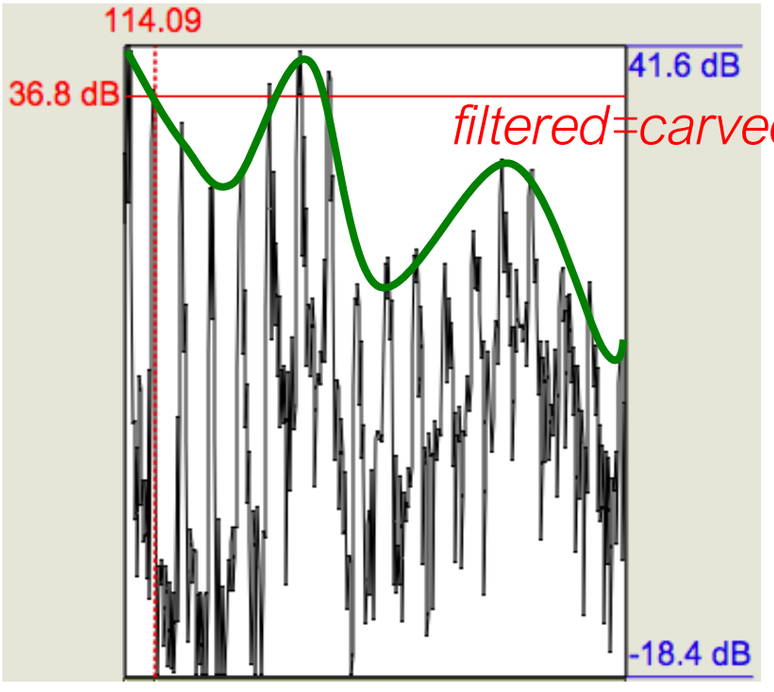


Jigjagging
with peaks and valleys

source & filter

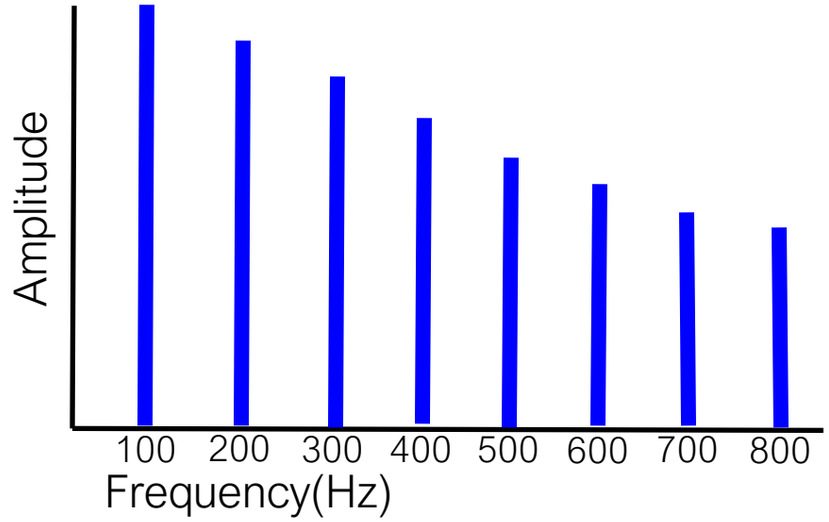






source

- Human voice source consists of *harmonics*
- A complex tone = sum of pure tones at integer multiples of the lowest pure tone
  - the lowest pure tone
    - *Fundamental frequency (F0)*
    - rate of vibration of the larynx
    - the number of opening-closing cycles of the larynx per second
  - Amplitude of pure tones gradually decreases



filter

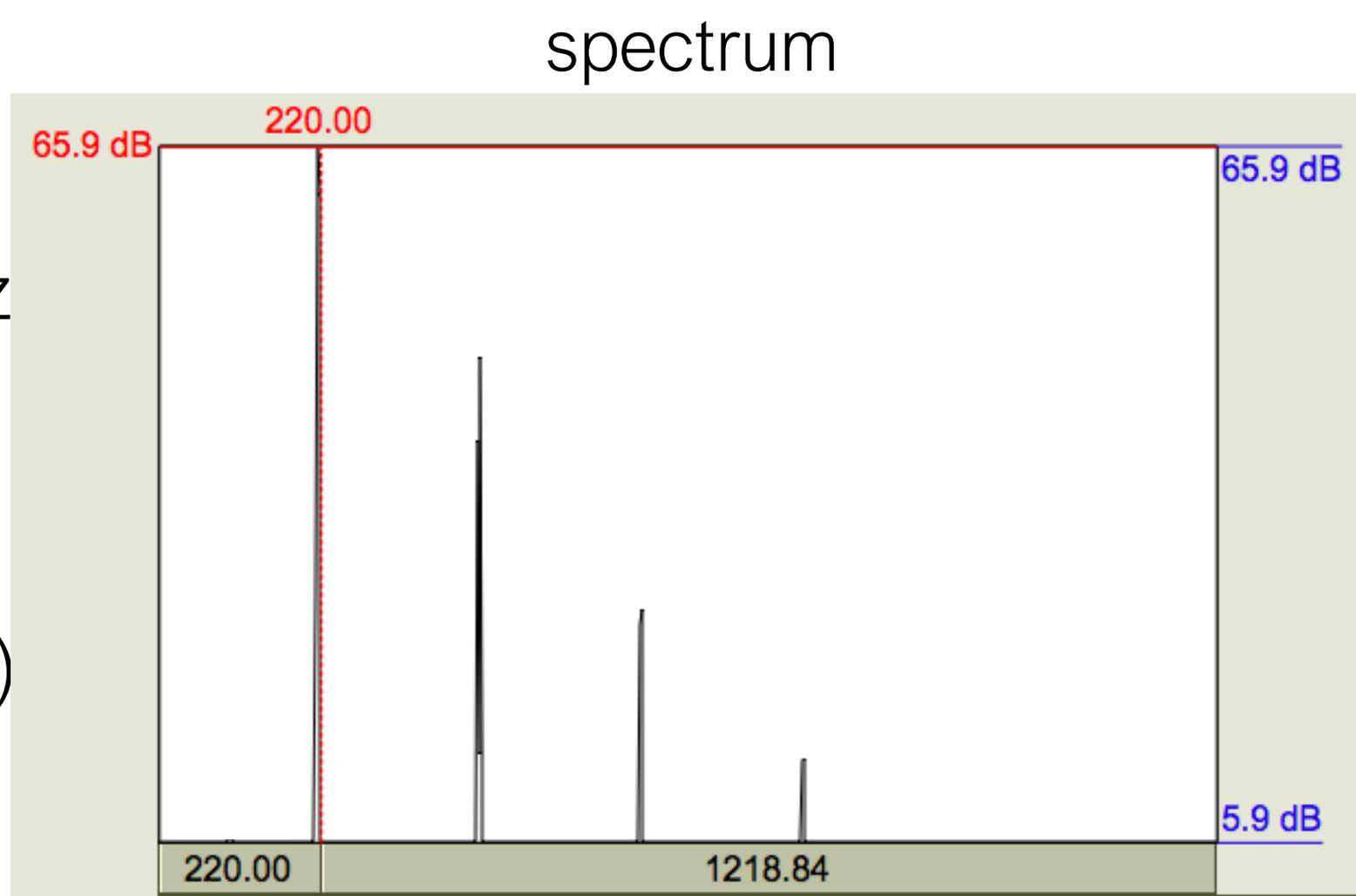
- Compare spectrums between audio and EGG
- EGG: gradual decreasing
- audio: peaks/mountains and valleys
- Because it is filtered by the vocal tract (VT)
  - peaks/mountains: frequencies VT likes = formants
  - valleys: frequencies VT does NOT like
- Matlab 3D view

Guitar plucking (similar to voice source)

- A complex tone consisting of pure tone components
- Decreasing amplitude

guitar note A (220Hz

pure tone A (220Hz)

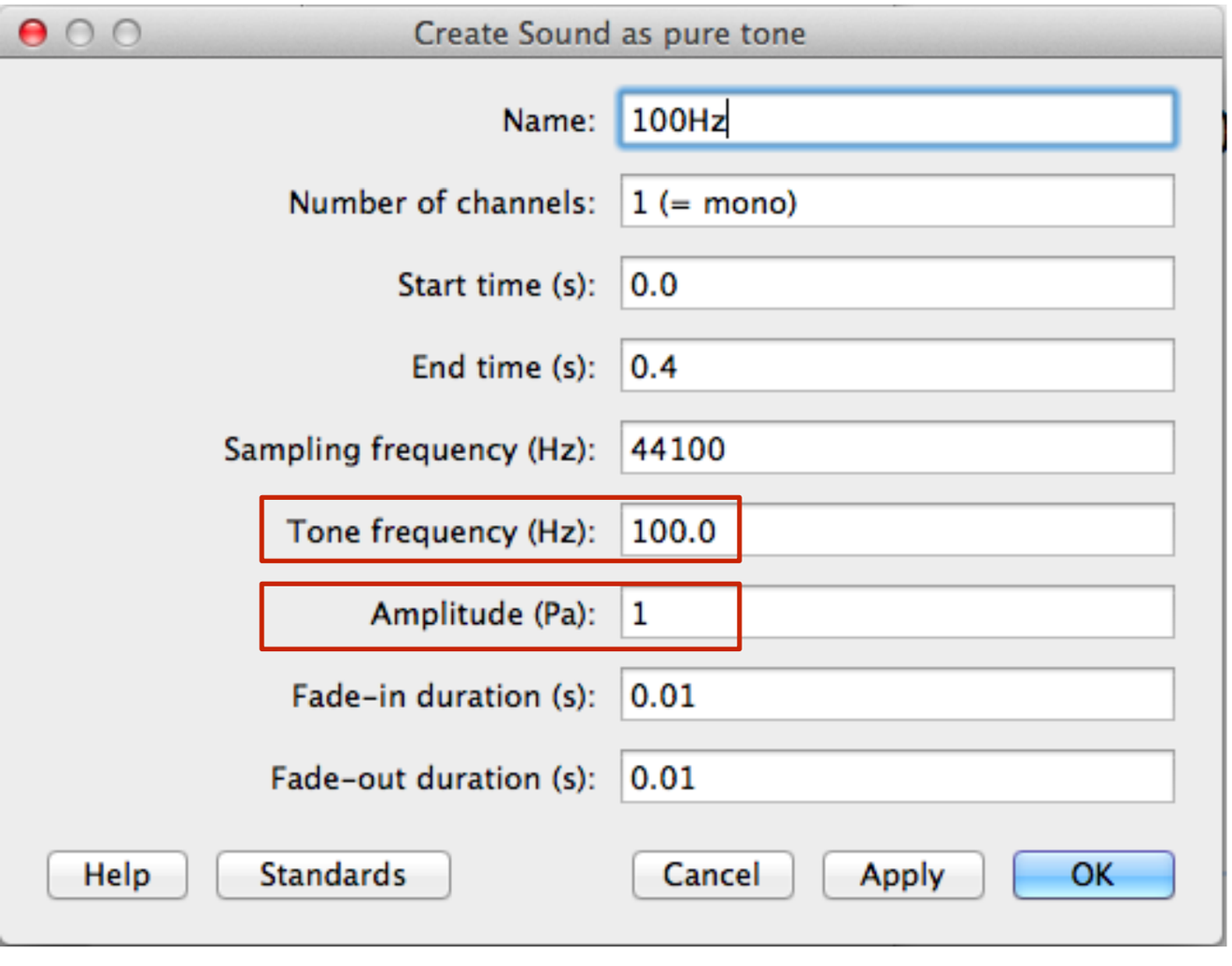


Synthesizing Source

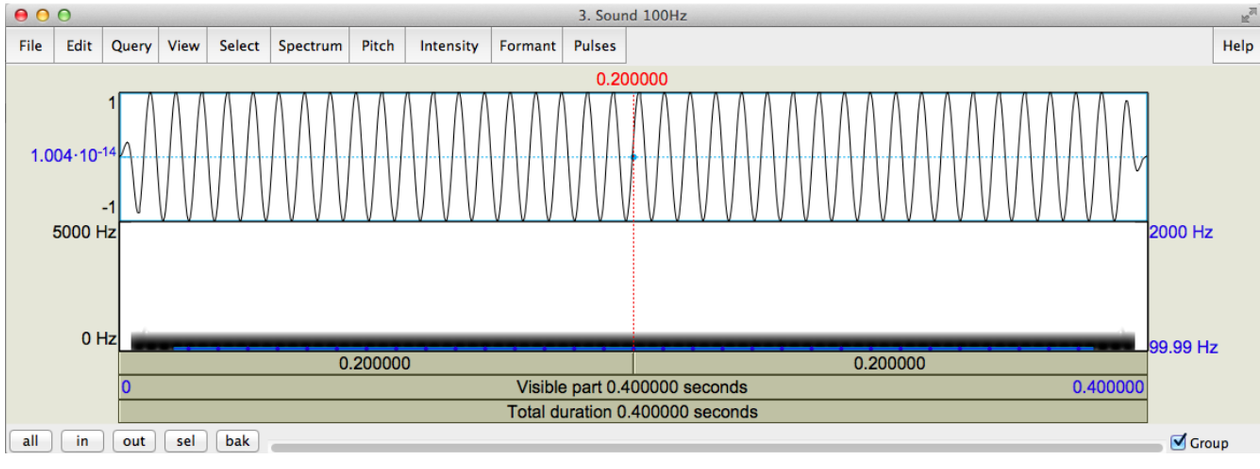
- Praat allows one to synthesize Source
- Create harmonics from 100Hz to 1000Hz
- with decreasing amplitude by .05

Synthesizing Source

- Create Sound as pure tone
- Frequency: 100Hz

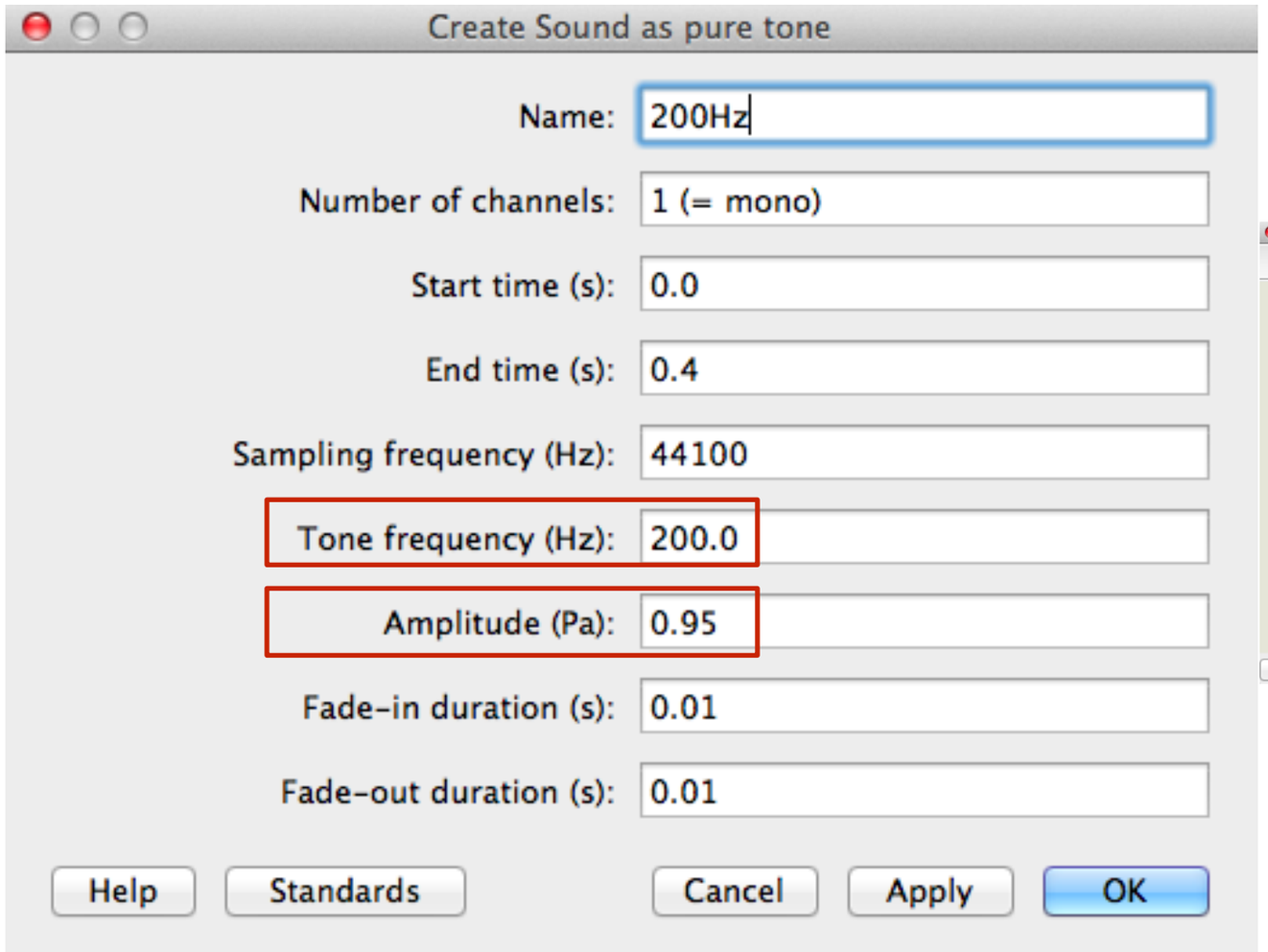


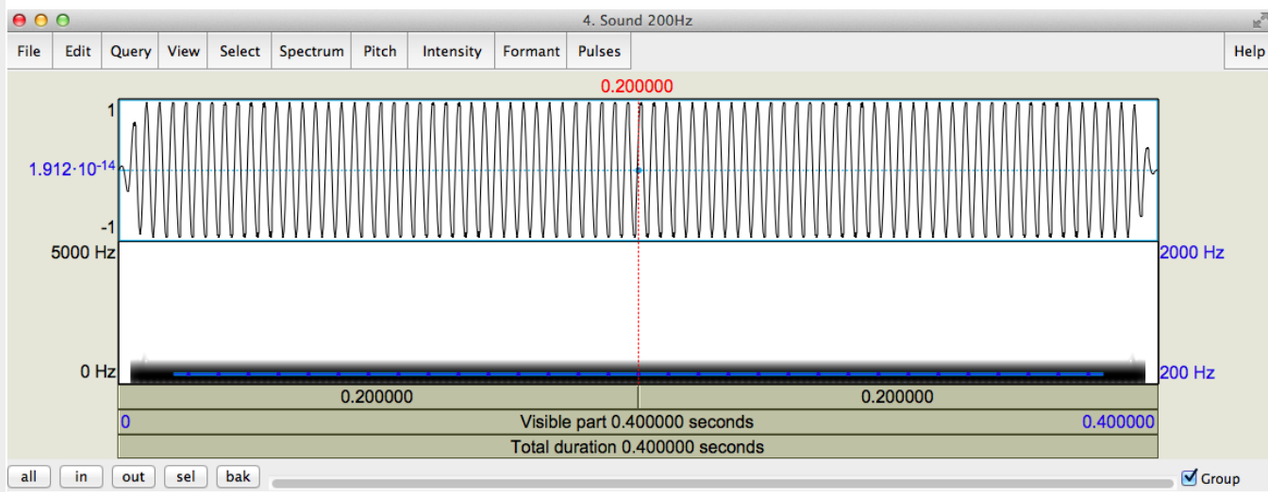
- Amplitude: 1



Synthesizing Source

- Create Sound as pure tone
- Frequency: 200Hz
- Amplitude: 0.95





Synthesizing Source

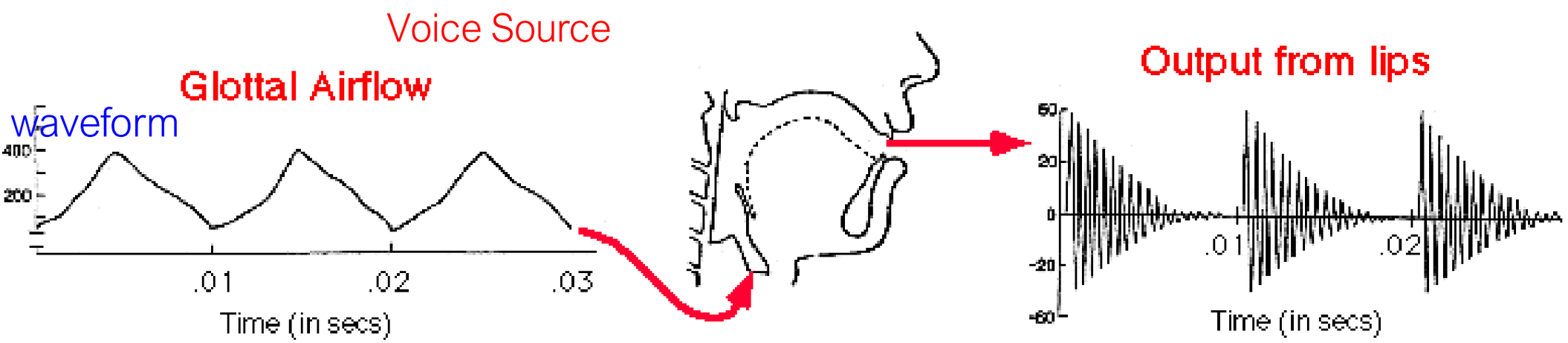
- Create pure tones
  - with different frequencies and amplitudes
- Combine to stereo
- Convert to mono

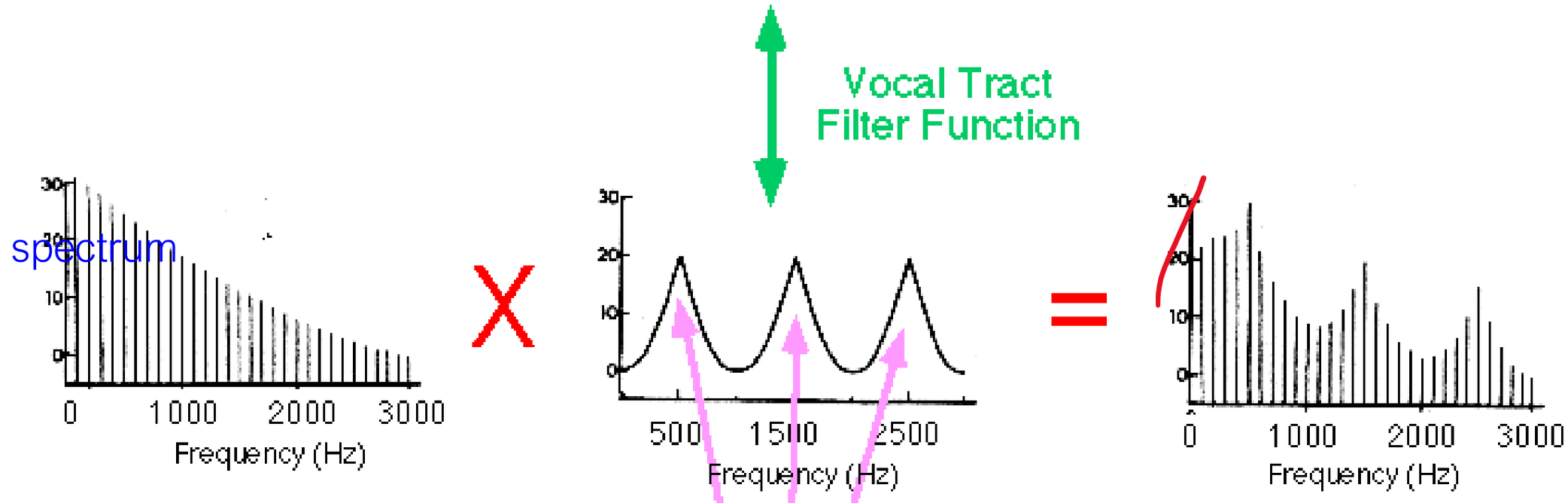
Synthesizing Source

- Open complex100to1000Hz.collection in Praat
- Plot a spectrum
- Measure pitch

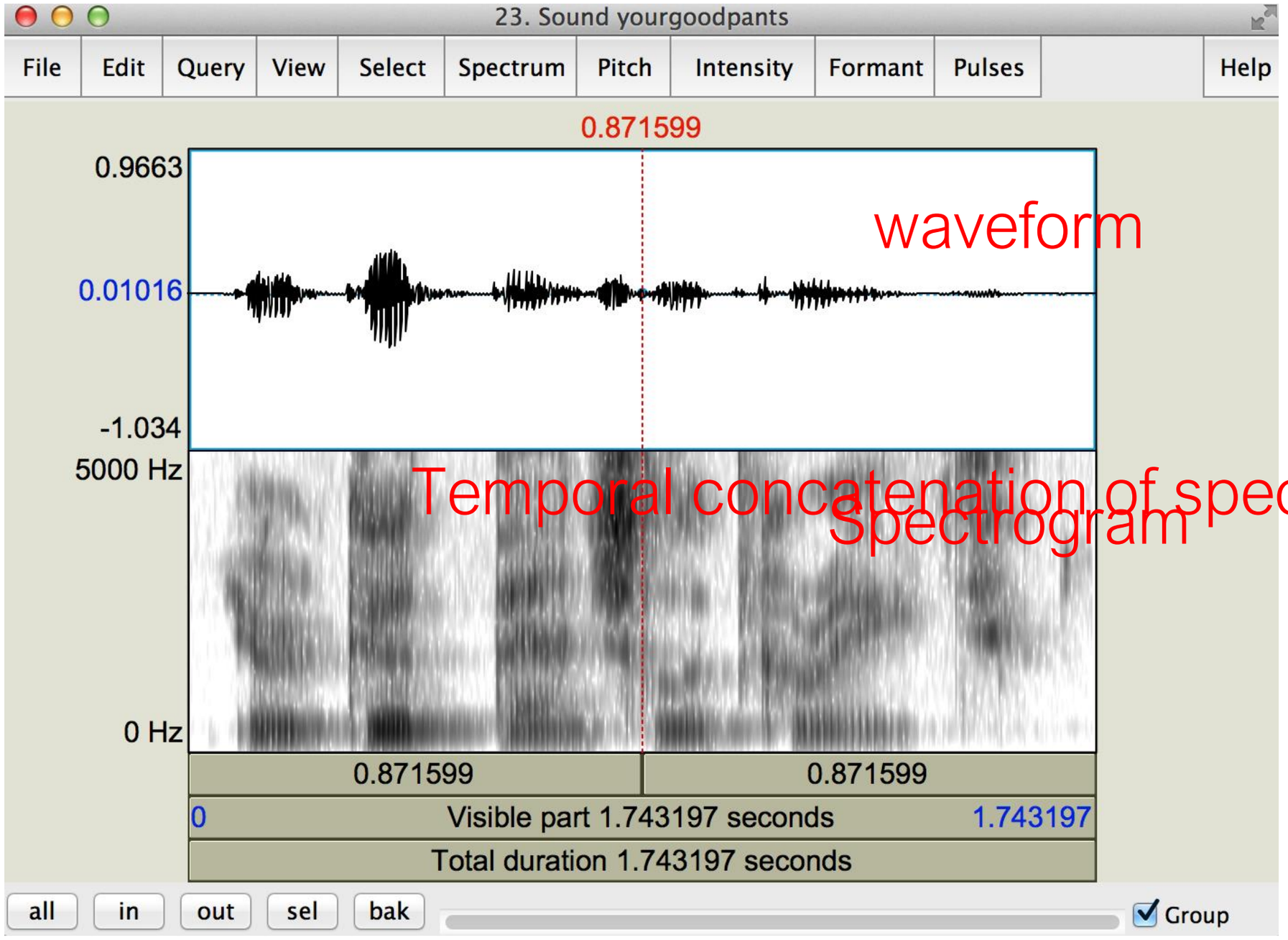
Source-filter theory

(from larynx) (by vocal tract)





Spectrogram



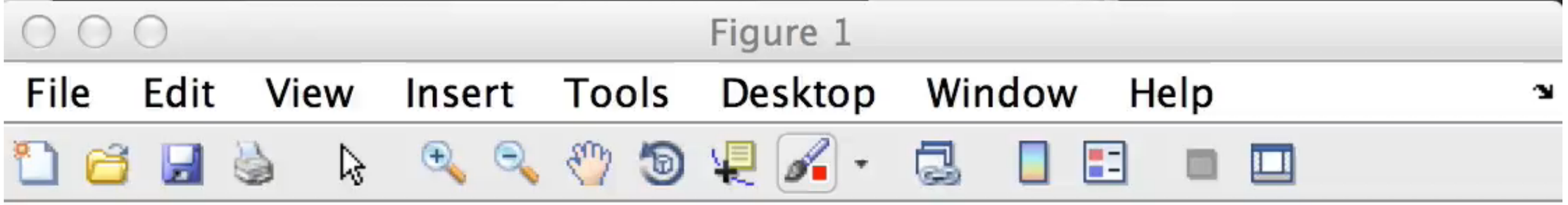
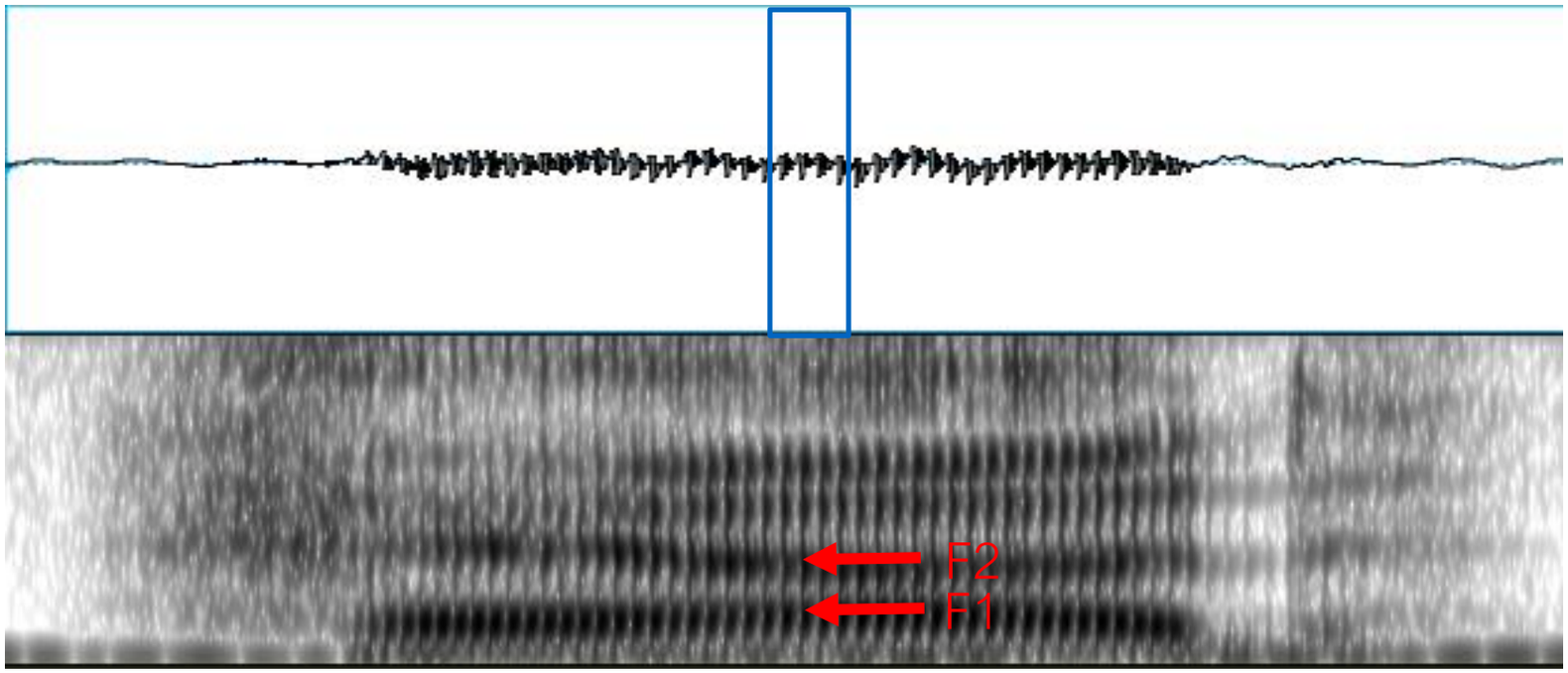
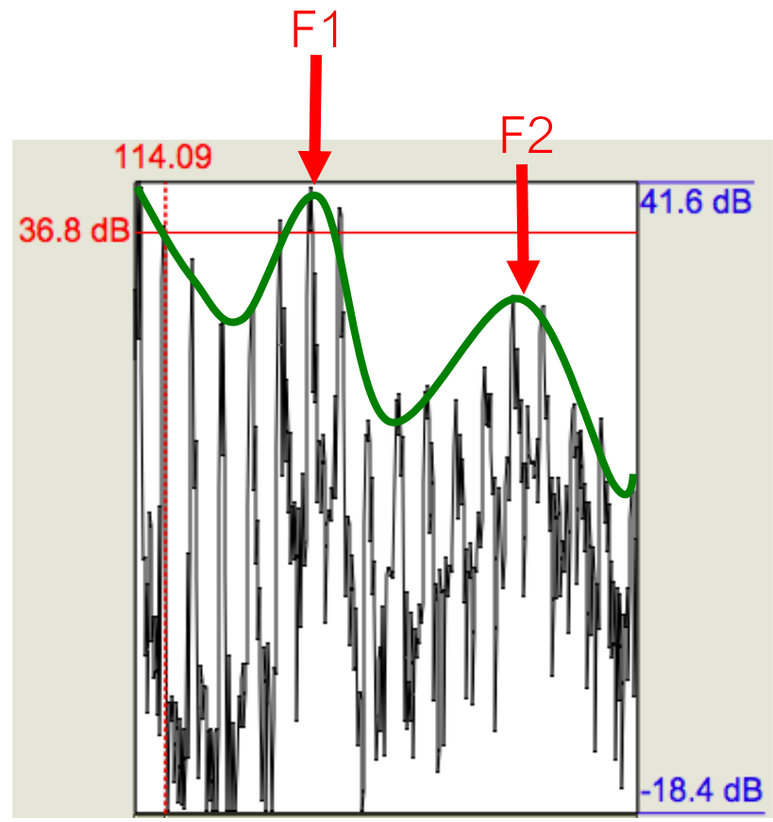


Figure 1
File
Edit
View
Insert
Tools
Desktop
Window
Help

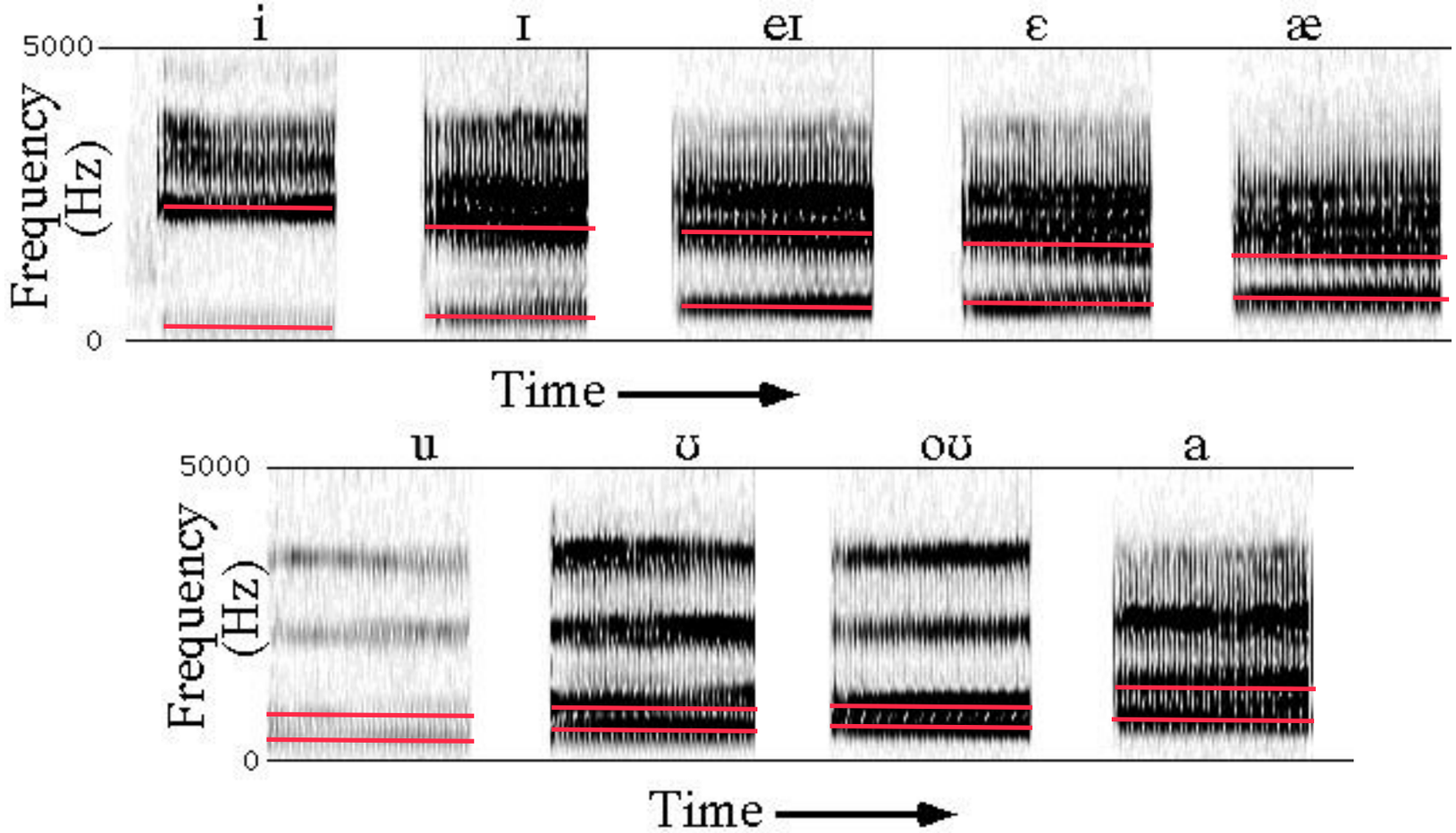
Spectrogram





- Airplane view of temporal concatenation of spectrum!
- Dark band: mountains = Formants

Formants



F1 and F2 are enough to disambiguate vowels.

Vowel space

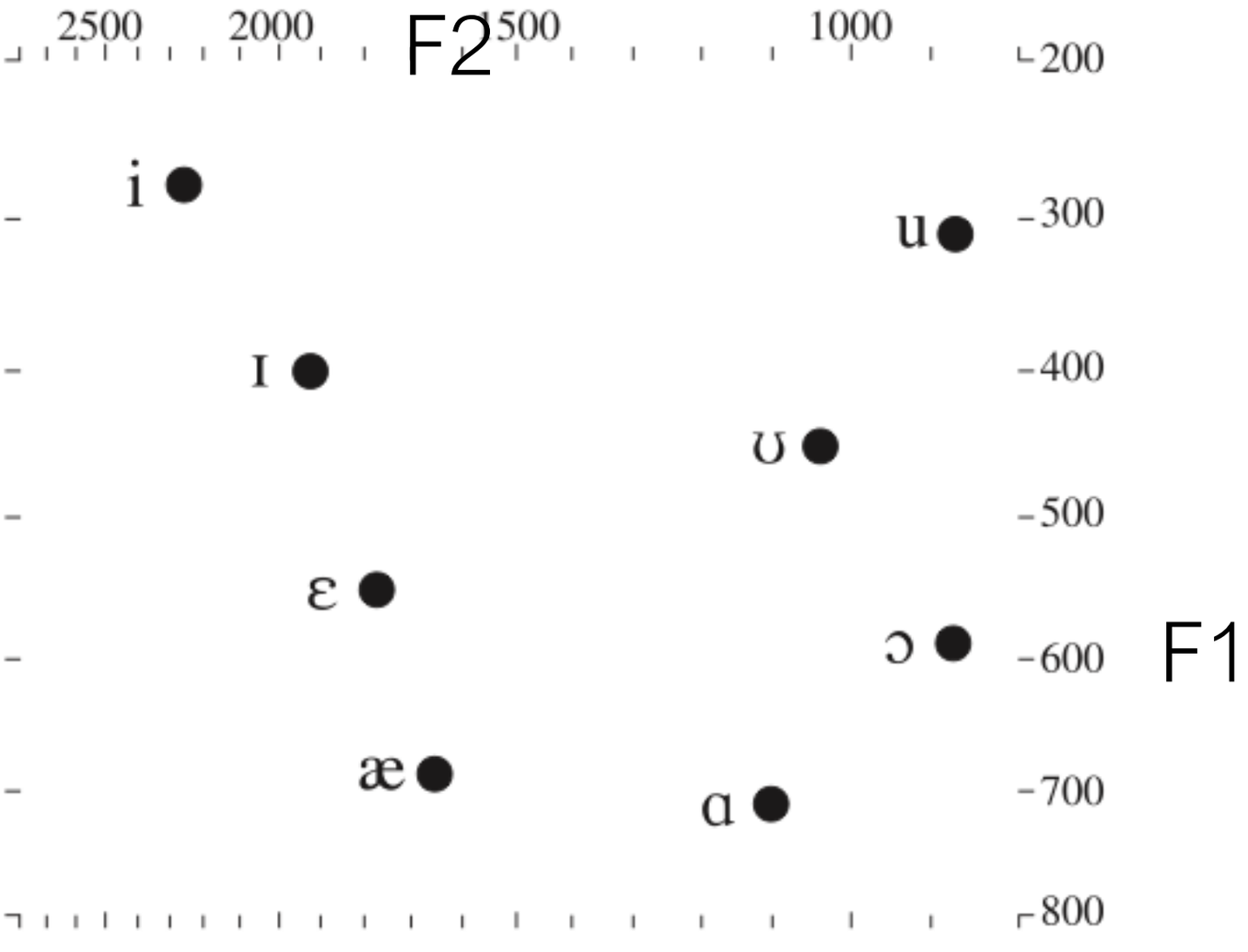


Figure 1.12 The positions of the vocal organs for the vowels in the words 1 *heed*, 2 *hid*, 3 *head*, 4 *had*, 5 *father*, 6 *good*, 7 *food*. The lip positions for vowels 2, 3, and 4 are between those shown for 1 and 5. The lip position for vowel 6 is between those shown for 1 and 7.

Practice with VowelEditor in Praat

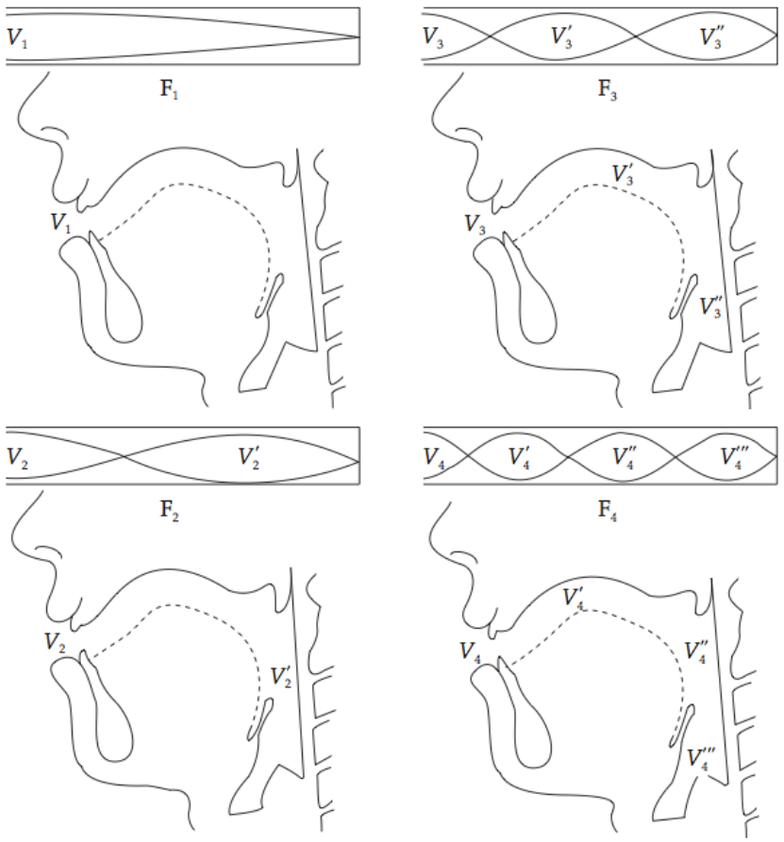
How formants determined

- What frequencies does the vocal tract tube prefer?
  = What frequencies resonate in VT?

- Need to understand standing waves

  - determining resonance frequencies of VT shape for a vowel.

Standing waves in VT



- The vowel, schwa is the most similar to this uniform tube
- The tube prefers the frequencies of these standing waves
- These frequencies resonate in VT
- The resonant frequencies are formants for schwa

What is a standing wave?

- http://www.acs.psu.edu/drussell/Demos/superposition/superposition.html

What is a standing wave?

- http://www.walter-fendt.de/html5/phen/standingwavereflection_en.htm

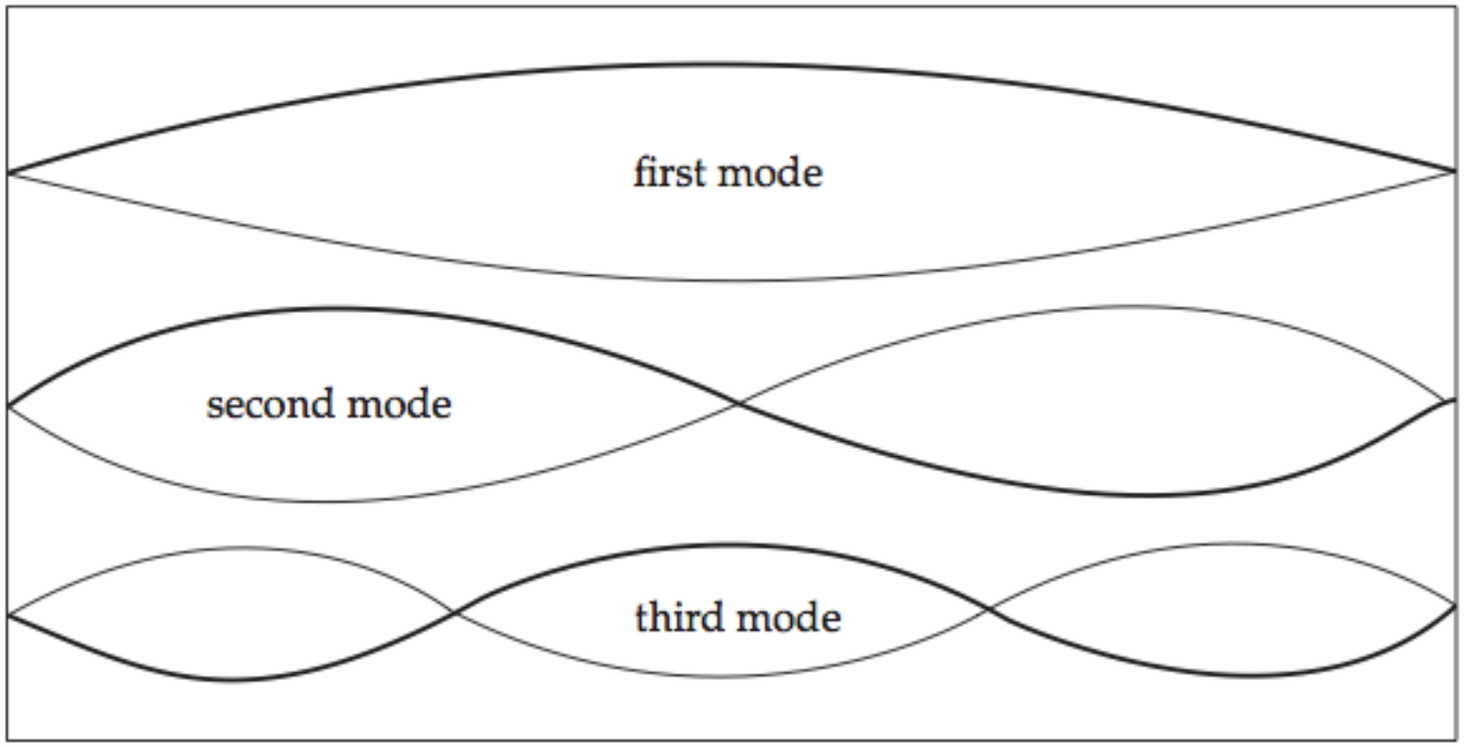
What is a standing wave?

What is a standing wave?

http://www.walter-fendt.de/html5/phen/standinglongitudinalwaves_en.htm

Standing waves

When both ends are all fixed/closed



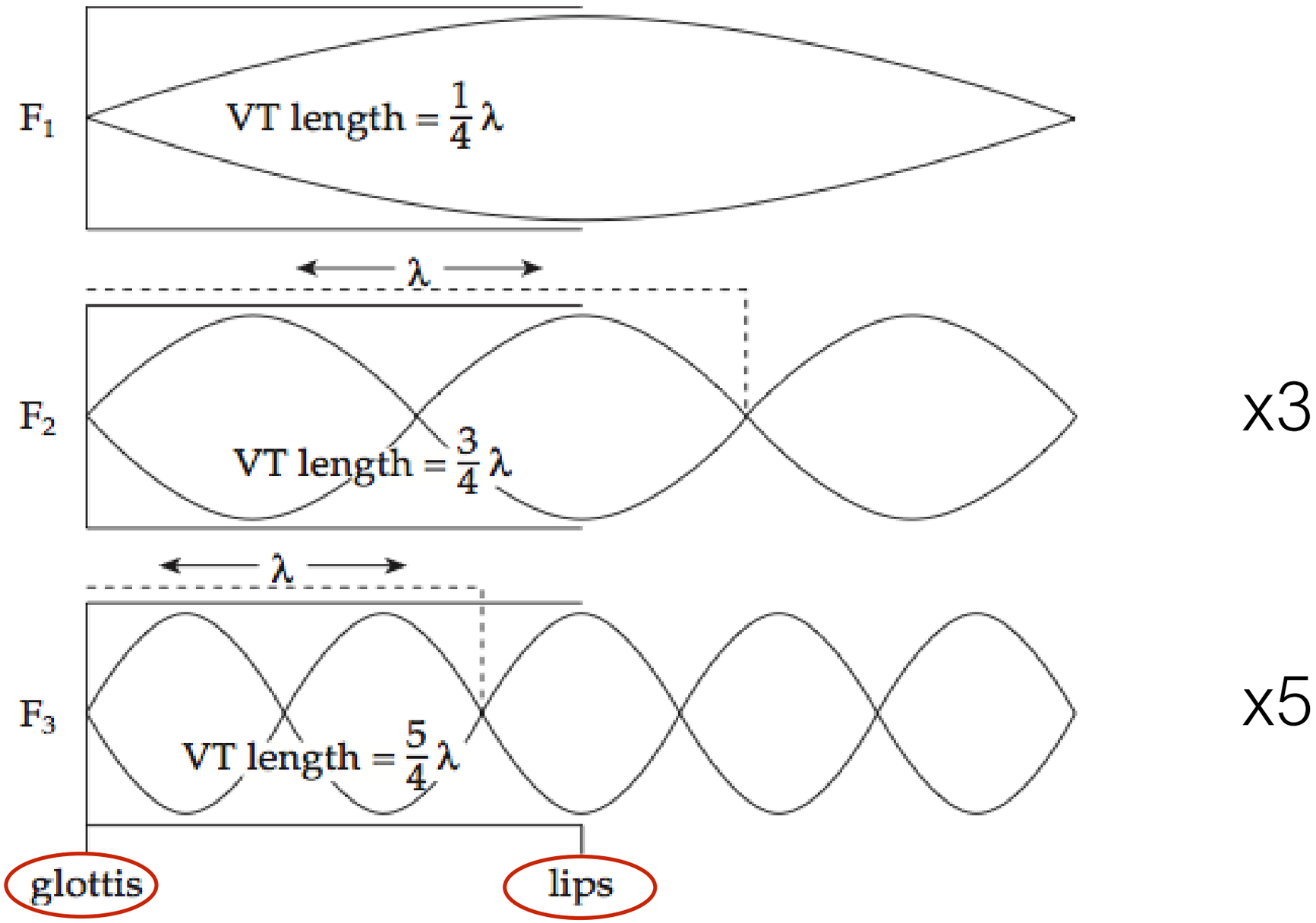
x2

x3

⋮

Standing waves in VT

When only one end is fixed/closed



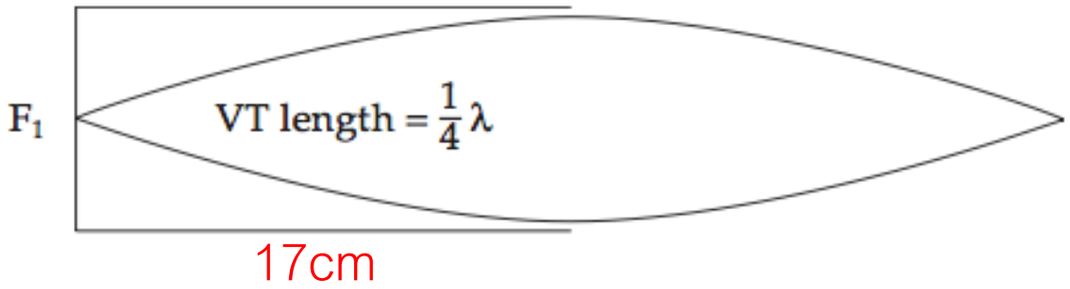
x3

x5

How to compute frequencies of standing waves

- Need to know the duration of a cycle to calculate frequency
- We know the distance instead and how fast it is (speed of sound = 340m/sec)

Length
of 1st standing wave



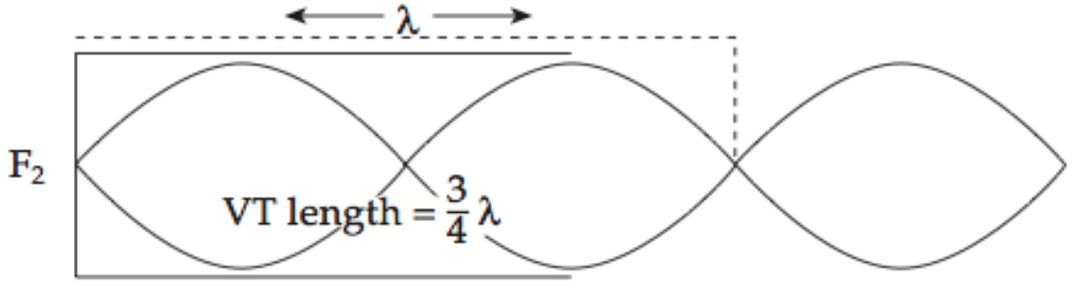
unfold

Compute frequency (F1) of 1st standing wave

- Length (λ: Lamda) of 1st standing wave:
  - 0.17 (VT length) x 4 = 0.68 m
- Duration: 0.68/340 sec. = 0.002 sec

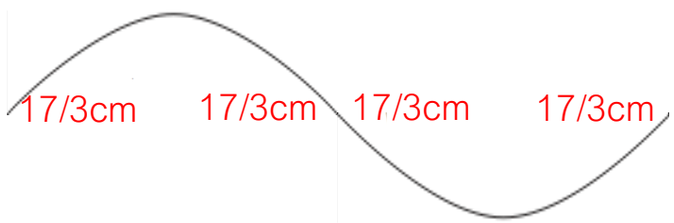
  ← speed of sound: 340 m/s
- Frequency (F1): 1/0.002 = 500 Hz

Length
of 2nd standing wave



17cm

unfold

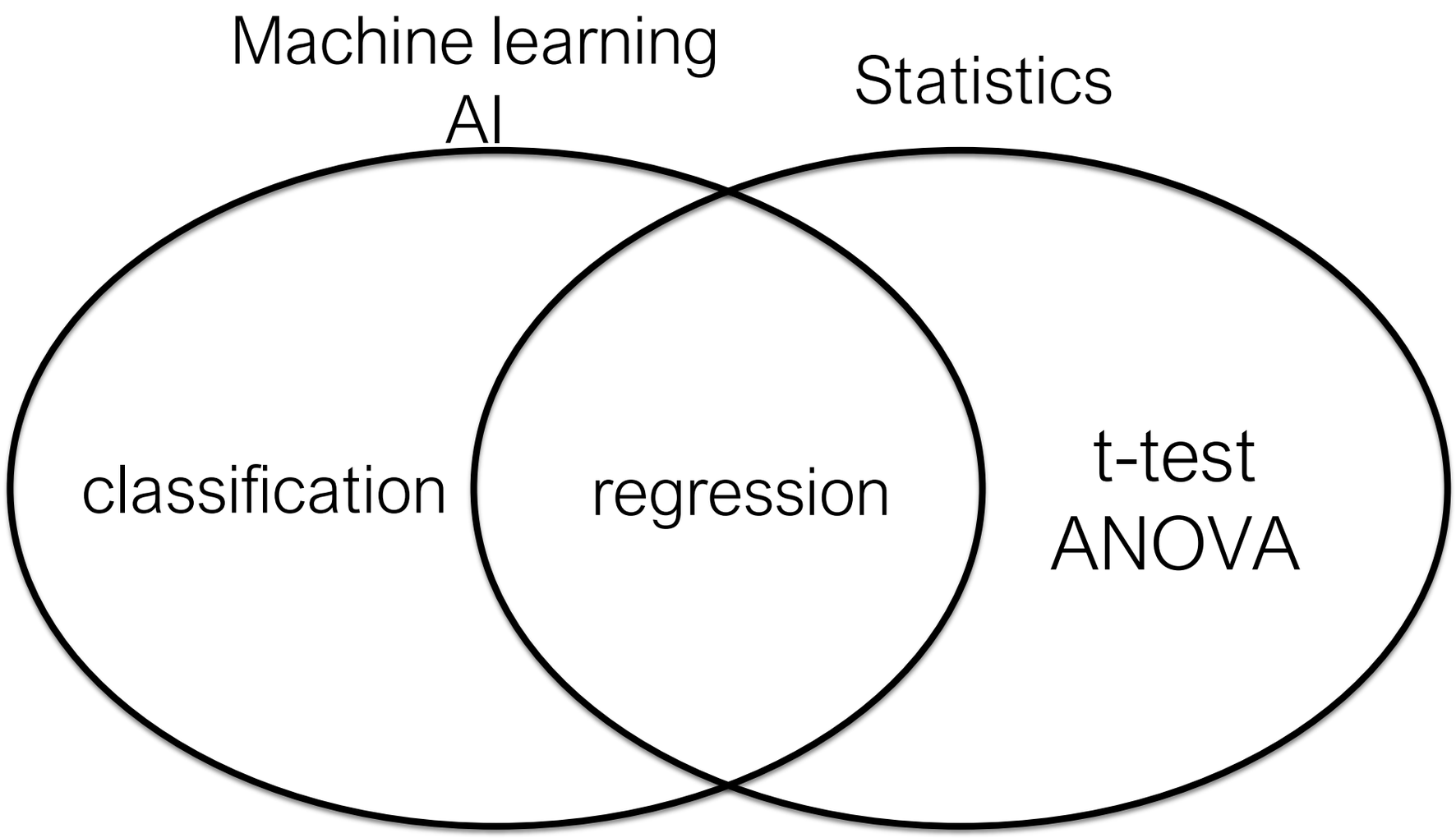


Compute frequency (F2) of 2nd standing wave

- Length (λ: Lamda) of 1st standing wave:
  - 0.17/3 (VT length) x 4 = 0.68/3 m
- Duration: 0.68/3/340 sec. = 0.002/3 sec

  ← speed of sound: 340 m/s
- Frequency (F2): 1/(0.002/3) = 1500 Hz

Machine learning vs. Statistics



인식(perception)
classification

- continous, physical, numerical, concrete

- discrete, cognitive, nominal, categorical, abstract

Variables vs. Constants

- Variables (input or output): may change
  - **Types**: Numerical vs. Nominal
  - e.g. temperature = {32.7° F, 90° F, 0° C}
    - e.g. country = {‘Portugal’, ‘Ireland’, ‘Italy’, ‘Greece’, Spain’}

- Constants: not change
  - e.g. π = 3.141592…

in-out mapping in a function

Classification: Num-Nom

Regression: Num-Num

t-test, ANOVA: Nom-Num

classification: Num-Nom

smoke양 – cancer유무

regression: Num-Num

hours of studying– math score

t-test, ANOVA: Nom-Num

gender – math score

statistics

input: independent variable (IV)

독립변수

output: dependent variable (DV)

종속변수

statistics

A study on the effect/impact/influence of IV on DV

A study on the effect of sleep, calory & workout on weight

regression: 학습량이 수학점수에 미치는 영향

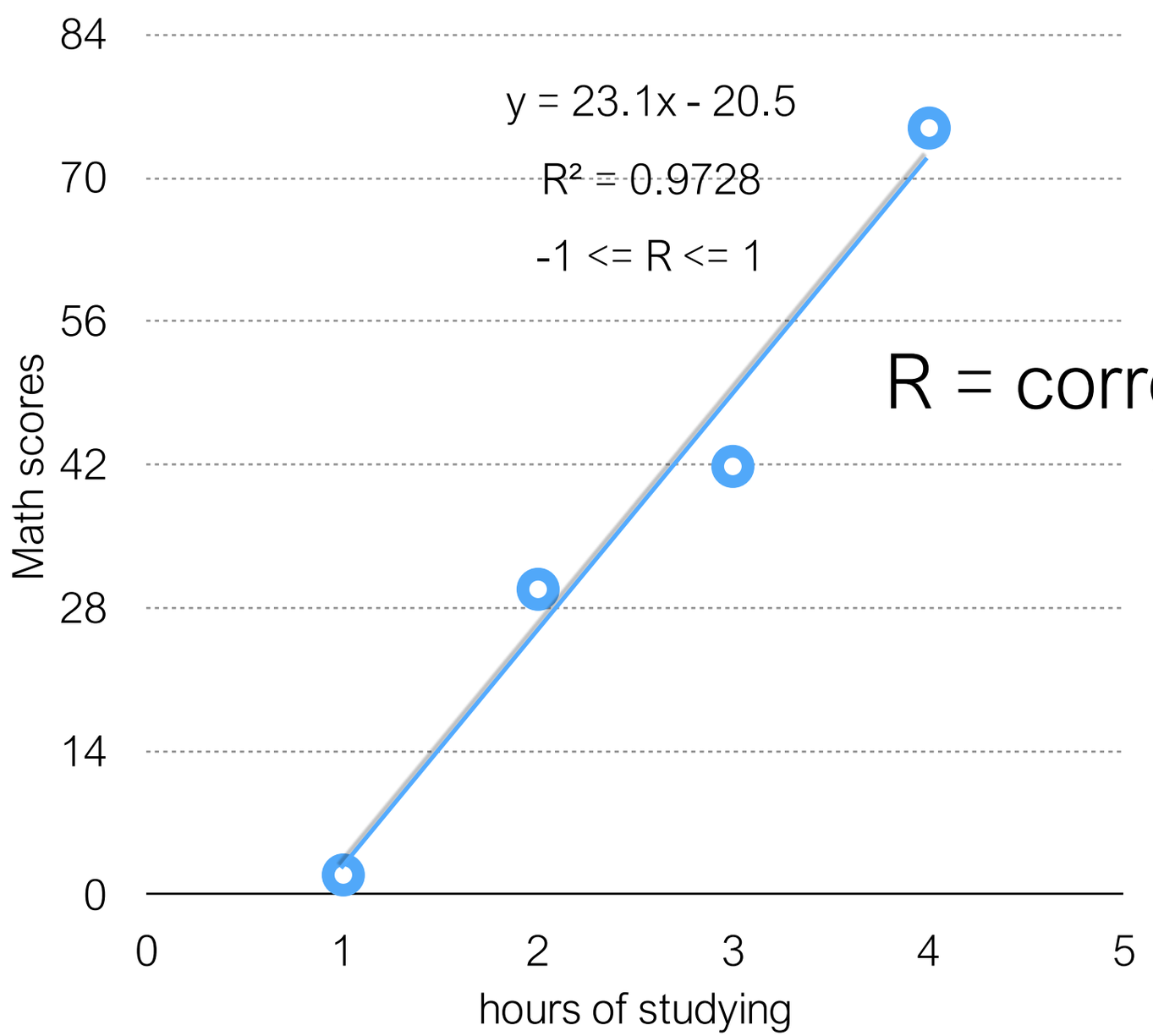
$r = 0.7, p = 0.012$

t-test (or ANOVA): 성별이 수학점수에 미치는 영향

$p = 0.09$

statistics

Regression: 학습량이 수학점수에 미치는 영향



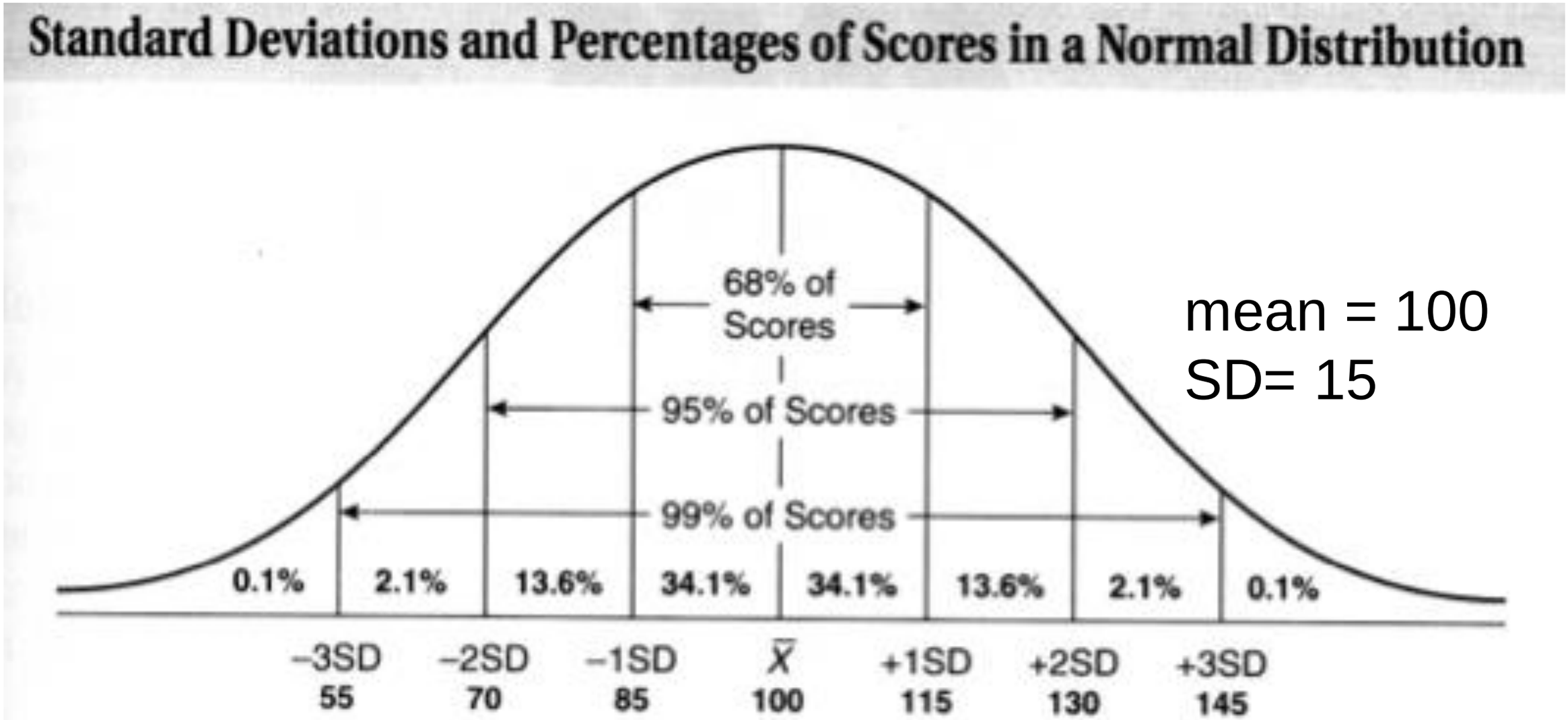
R = correlation coefficie
상관계수

statistics

t-test: 성별이 수학점수에 미치는 영향

Comparing two groups' normal distributions

Normal distribution



mean = 100
SD= 15

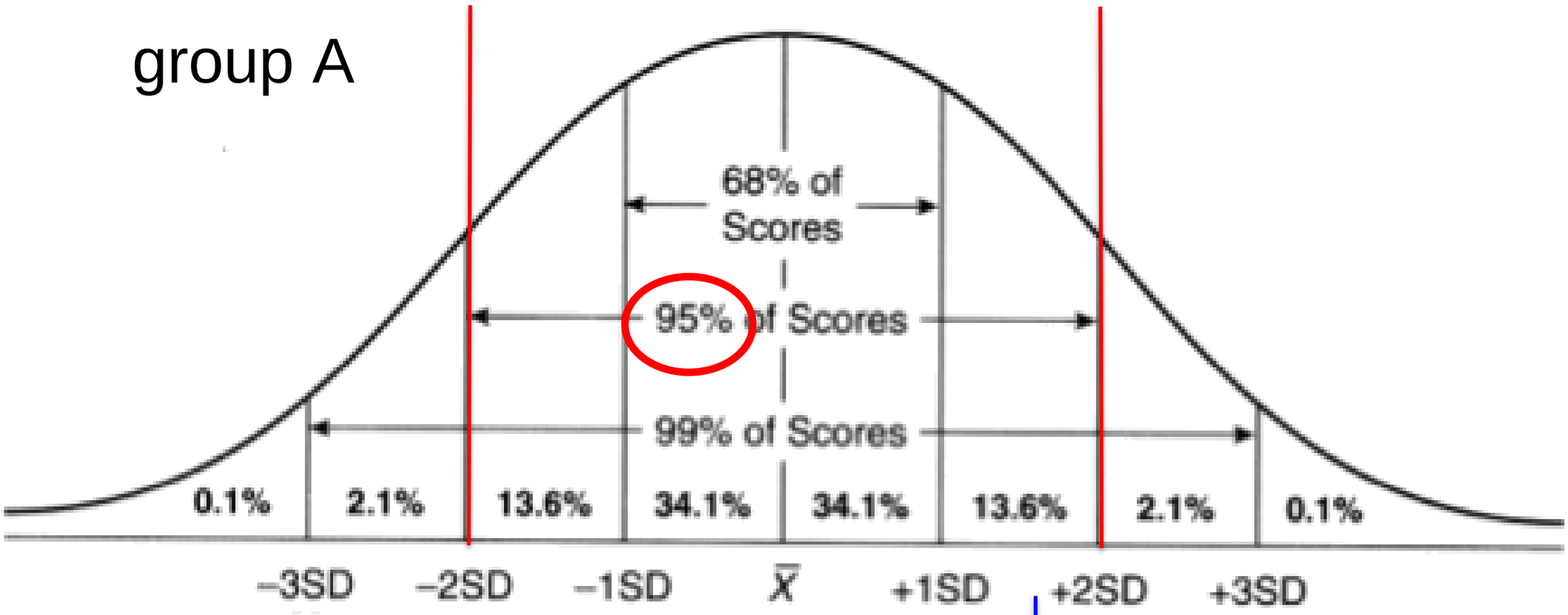
Normal distribution

- Normal distribution
  - salary
  - life span
  - height & weight

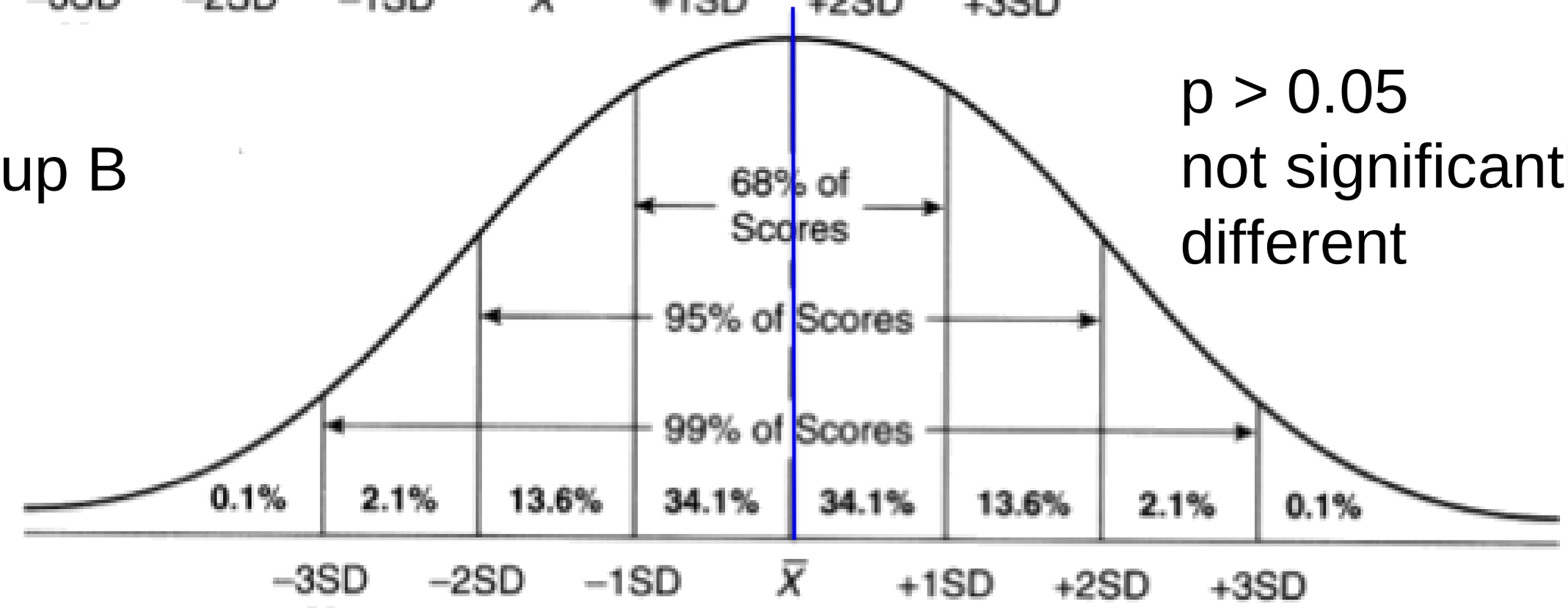
- non-normal distribution
  - throwing a dice/ flipping a coin
  - sunrise or sunset time
  - wealth in communist countries (?)

Comparing two groups' normal distributions

group A



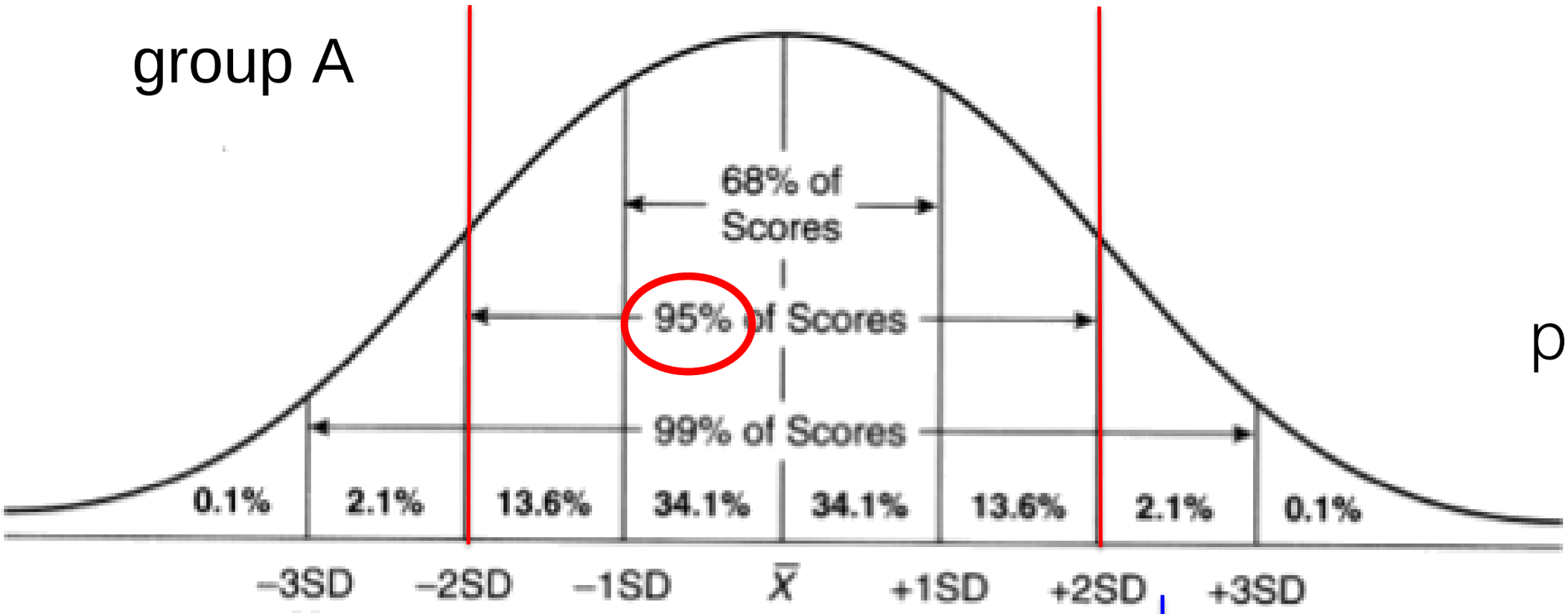
group B



$p > 0.05$
not significantly different

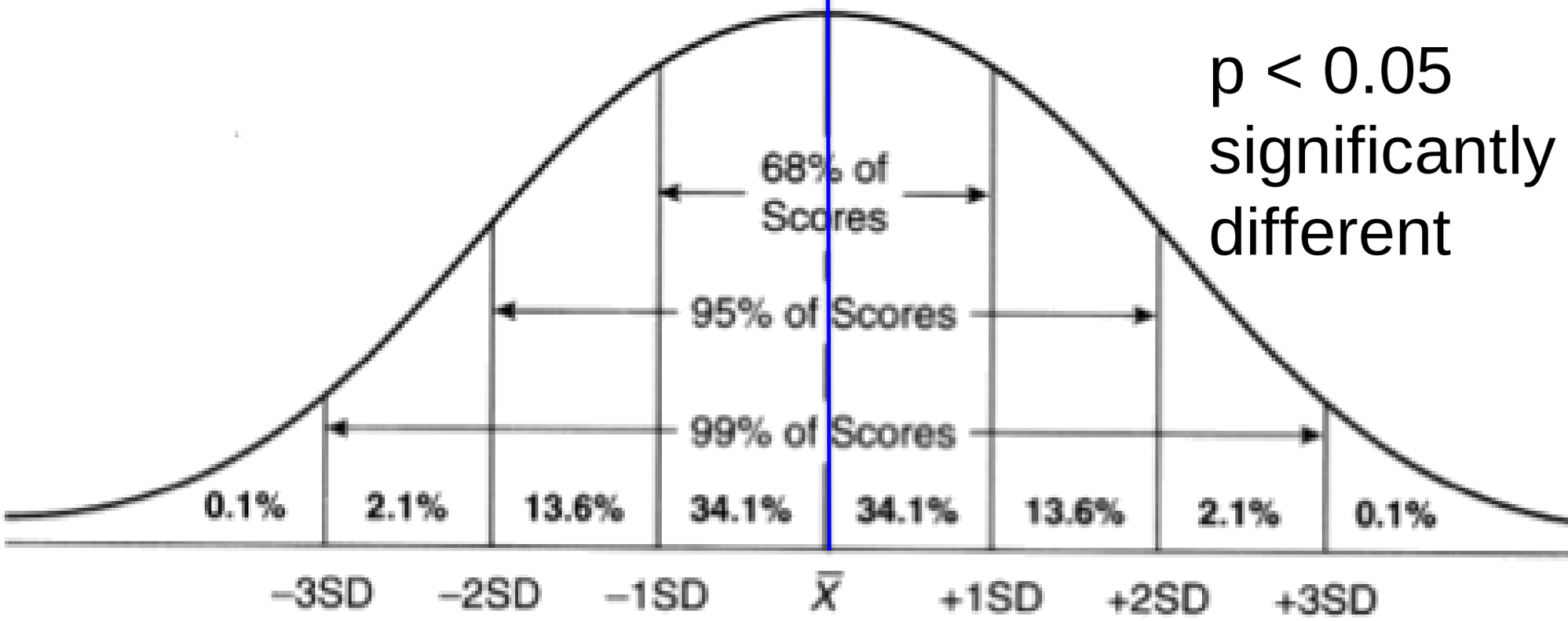
Comparing two groups' normal distributions

group A



p = 0.012

group B



p < 0.05 significantly different

T-test practices

- A = 30, B = 35, std = 2
- A = 60, B = 80, std = 10.5
- A와 B는 통계적으로 유의미하게 ($p < 0.05$) 차이 있나?

Linear algebra

Matrices

$$\begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix}$$

Vectors

$$\begin{bmatrix} a_1 \\ \vdots \\ a_m \end{bmatrix}$$

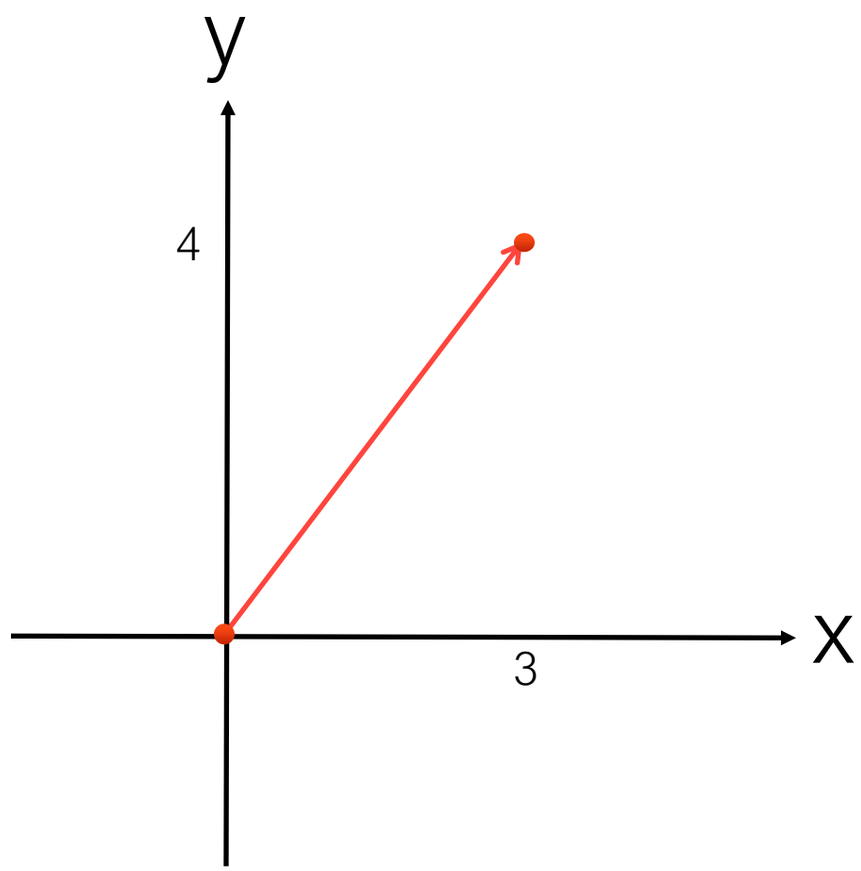
$$\begin{bmatrix} a_1 & \cdots & a_n \end{bmatrix}$$

Scalar

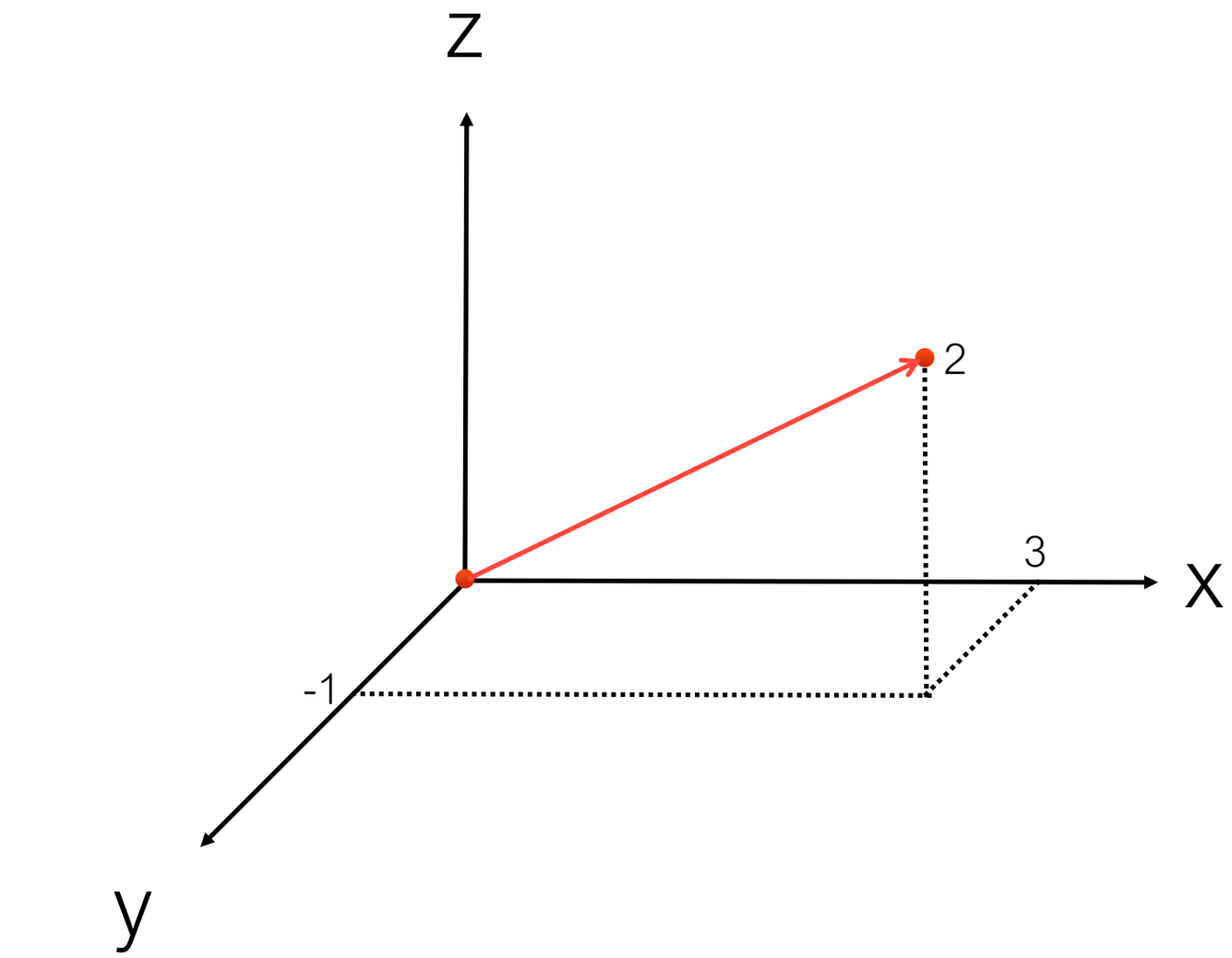
$$\begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$$

a sequence of numbers

$$\begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

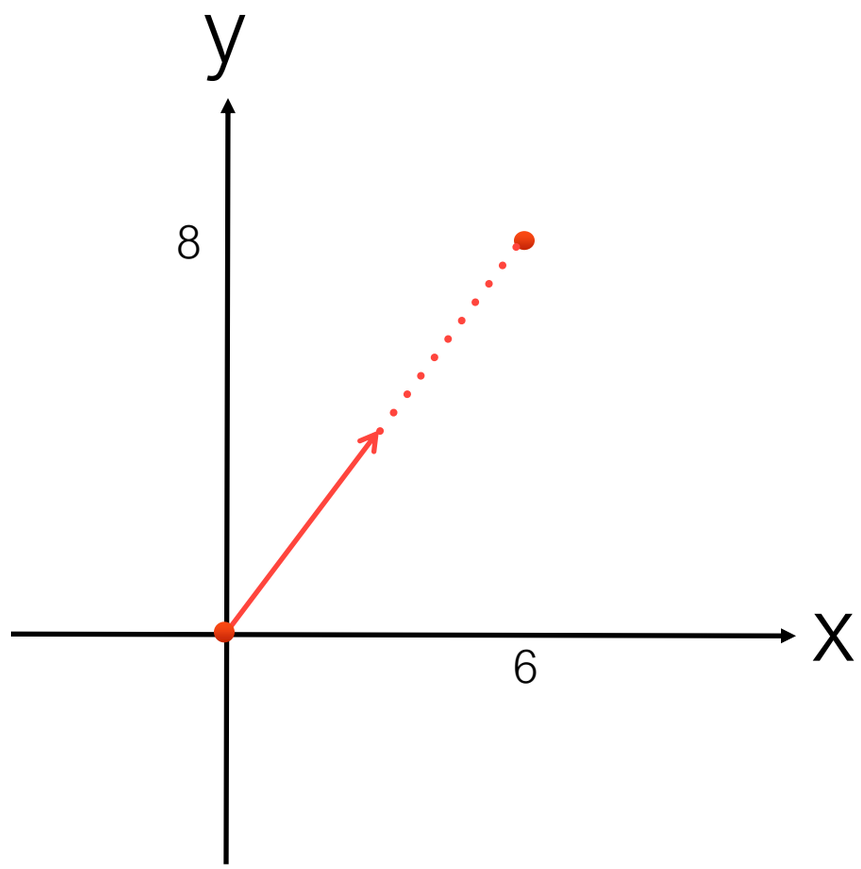


$$\begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$$

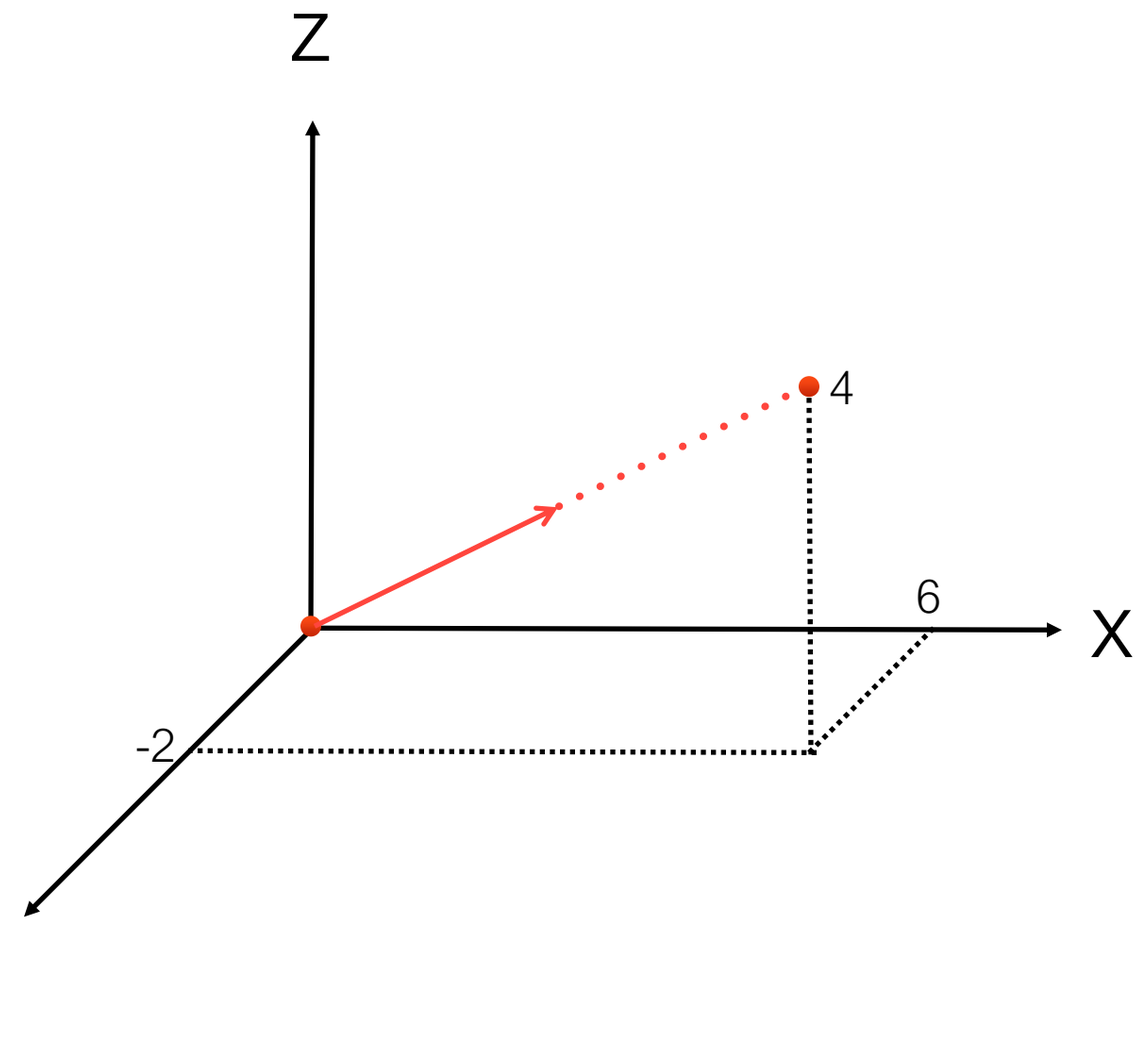


vector multiplication

$$0.5\begin{bmatrix} 6 \\ 8 \end{bmatrix}$$

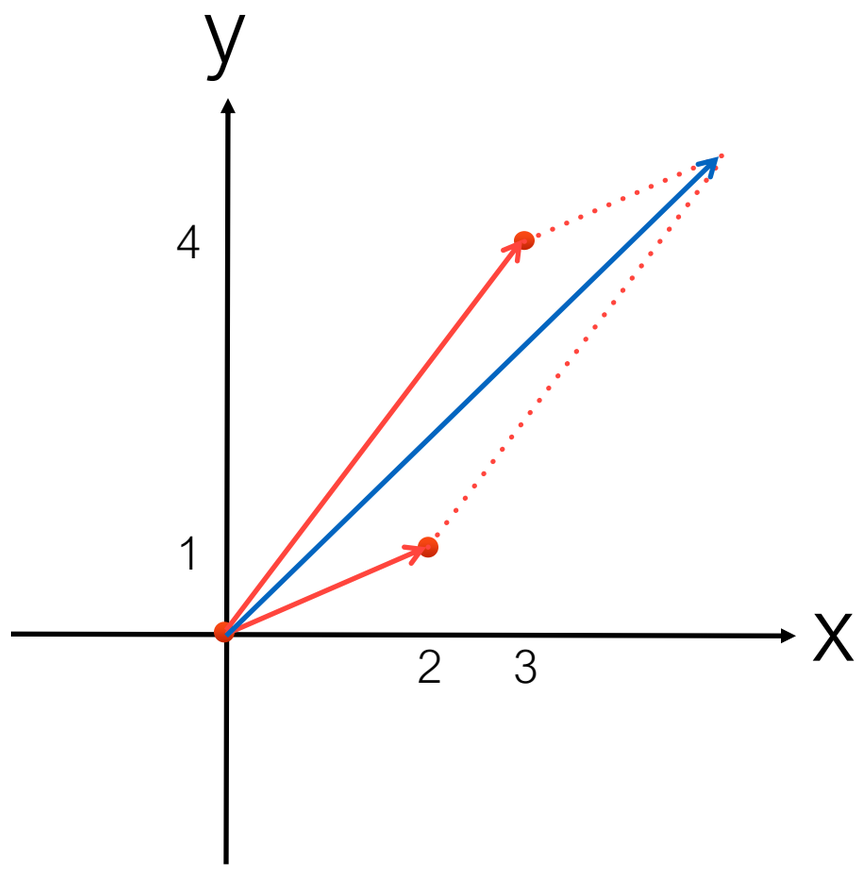


$$0.5\begin{bmatrix} 6 \\ -2 \\ 4 \end{bmatrix}$$

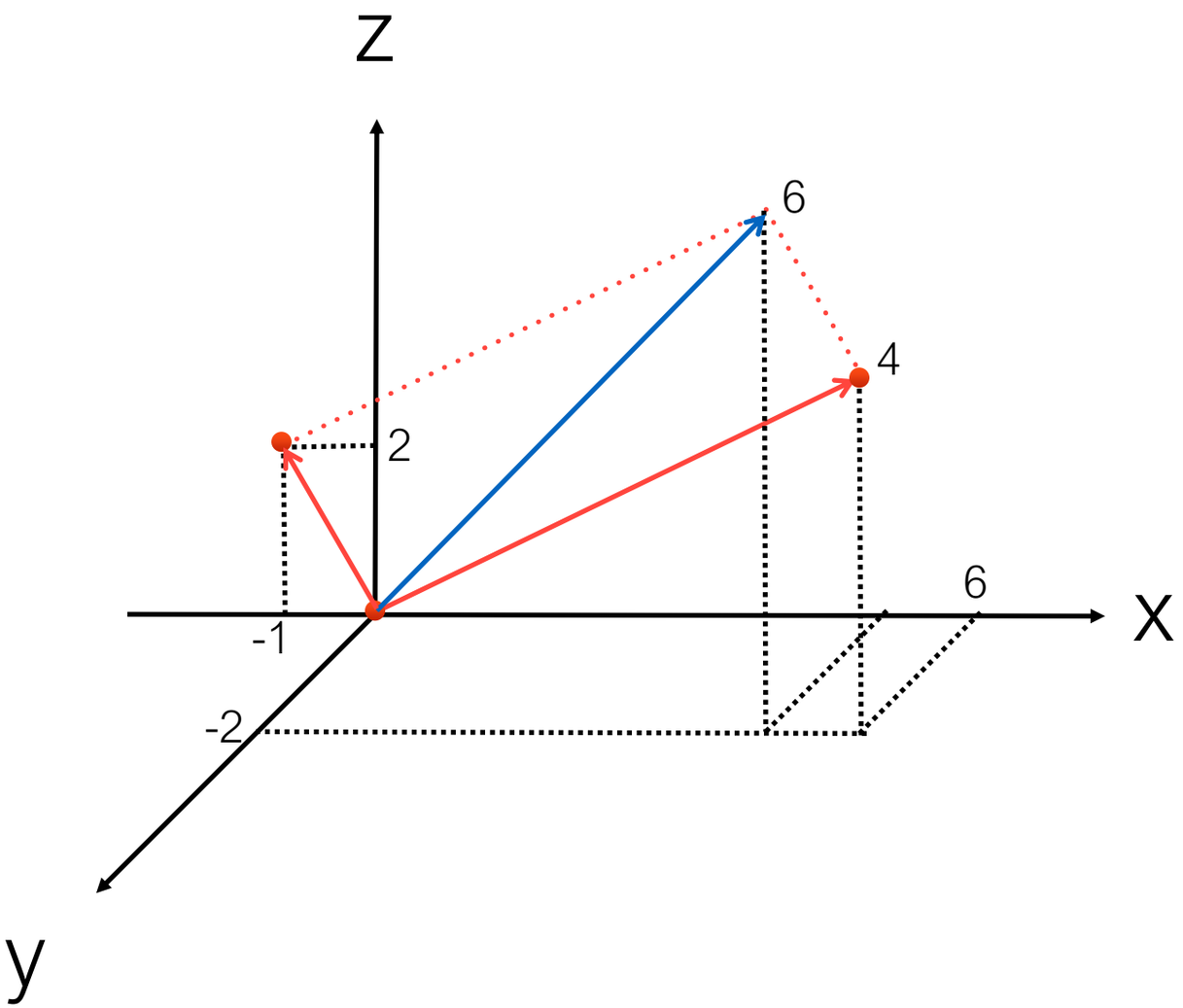


vector addition

$$\begin{bmatrix} 3 \\ 4 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$



$$\begin{bmatrix} 6 \\ -2 \\ 4 \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$$



vector product

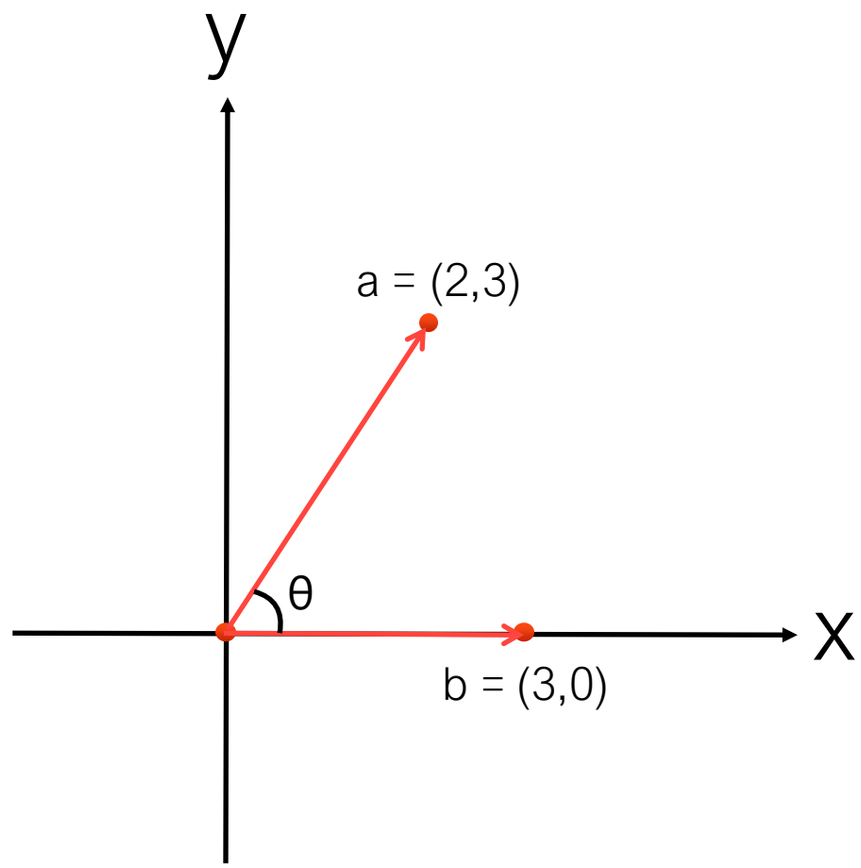
inner product

outer product

tensor product

inner product = dot product = scalar product

a = [2, 3]
b = [3, 0]



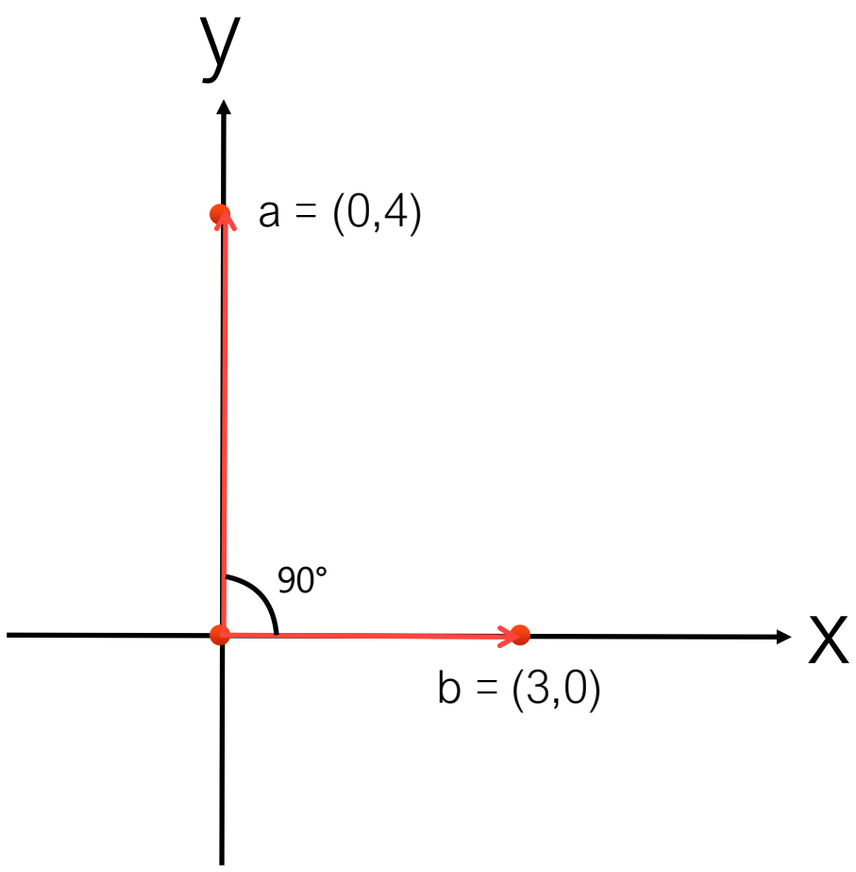
algebraic: a·b = 2x3 + 3x0

geometric: a·b = |a||b|cos(θ)

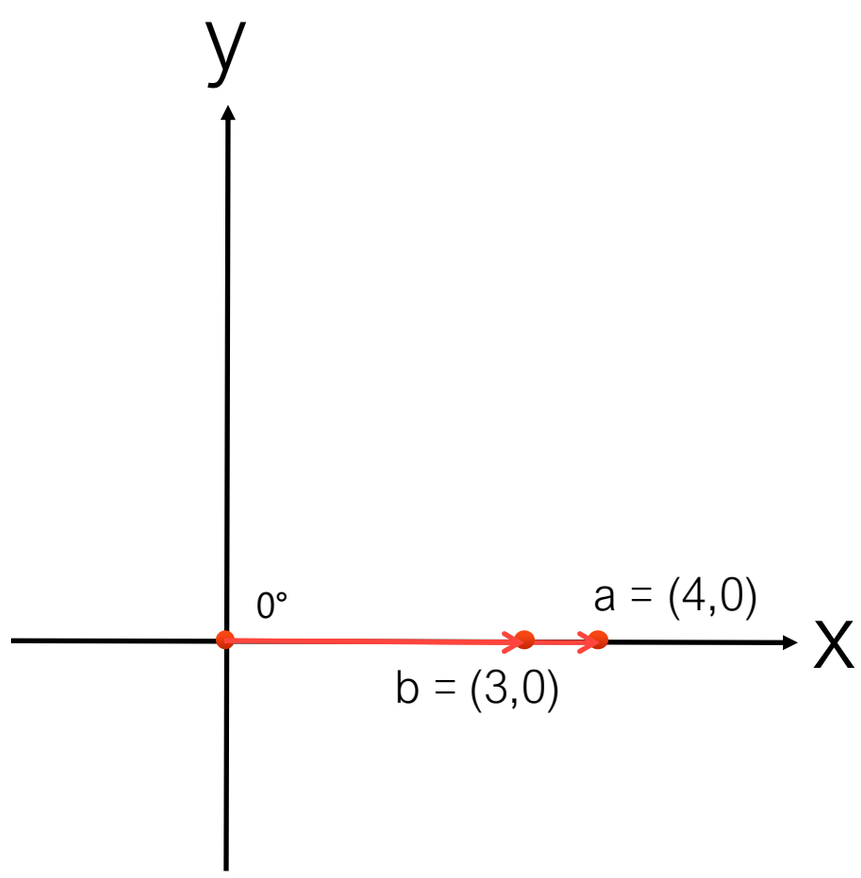
How to measure informational similarity between a and b?

- cosine(θ)
- a-b distance

inner product



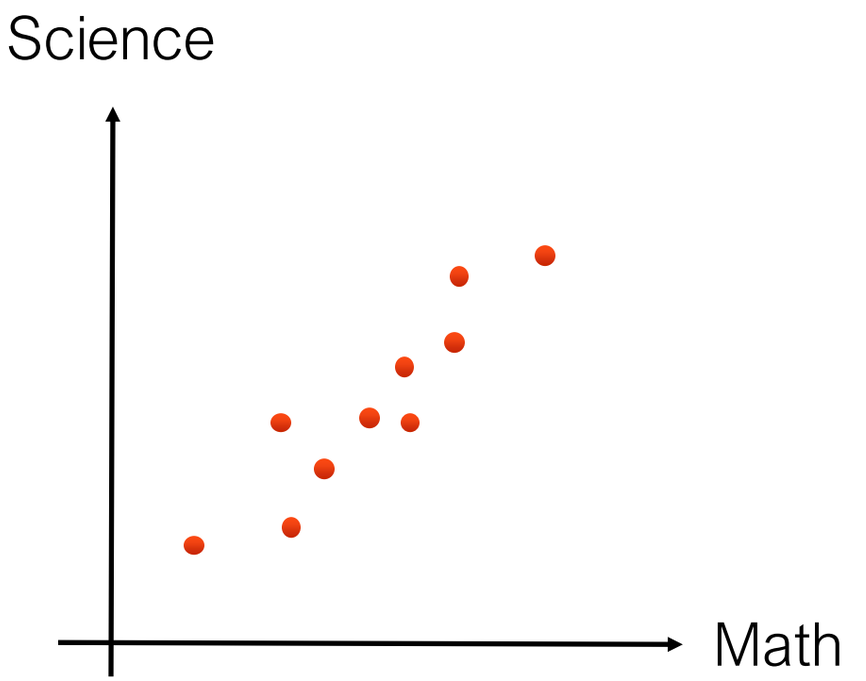
$a \cdot b = 4x3x\cos(\pi/2)$

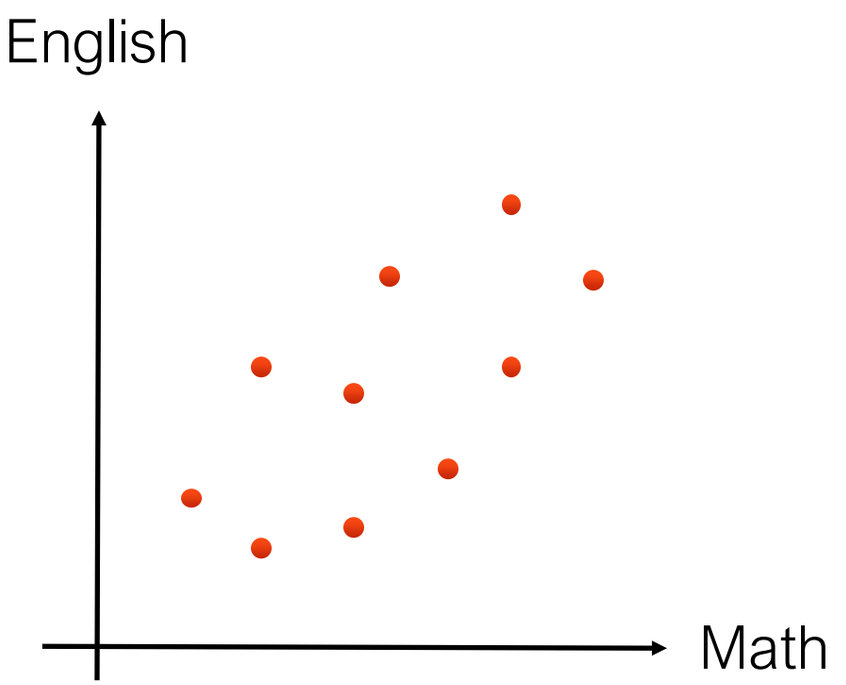


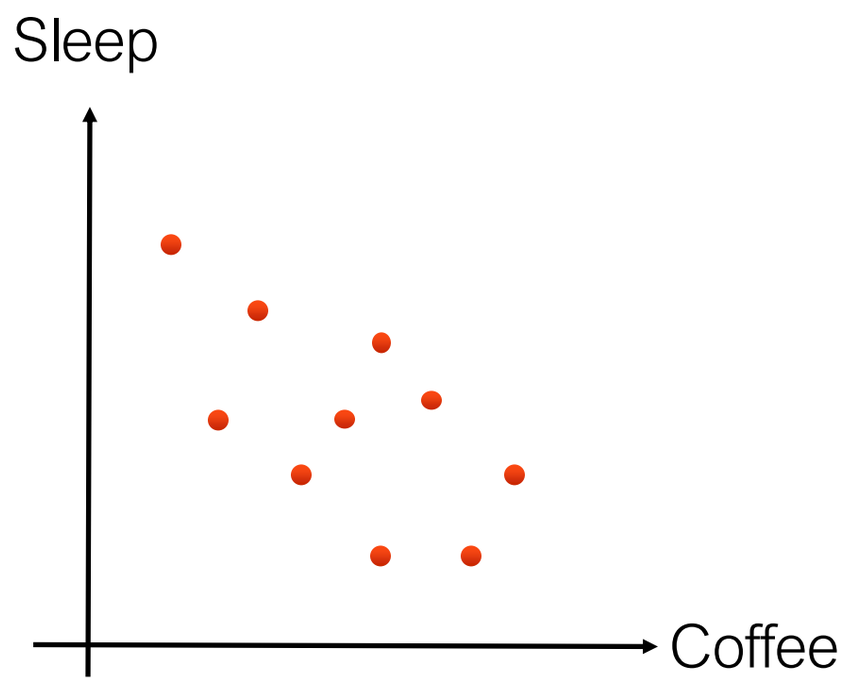
$a \cdot b = 4x3x\cos(0)$

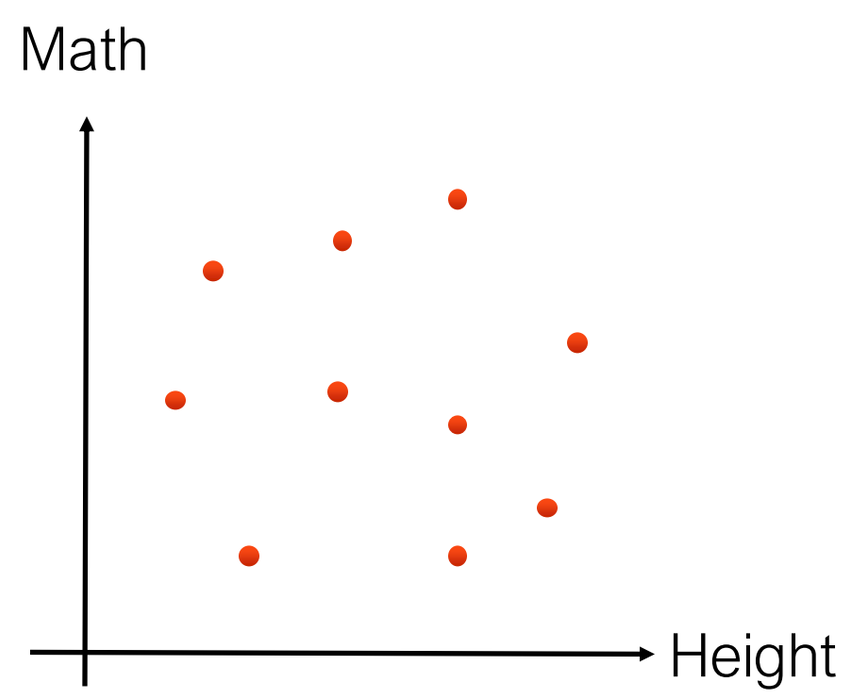
correlation

$-1 \leq r \leq 1$



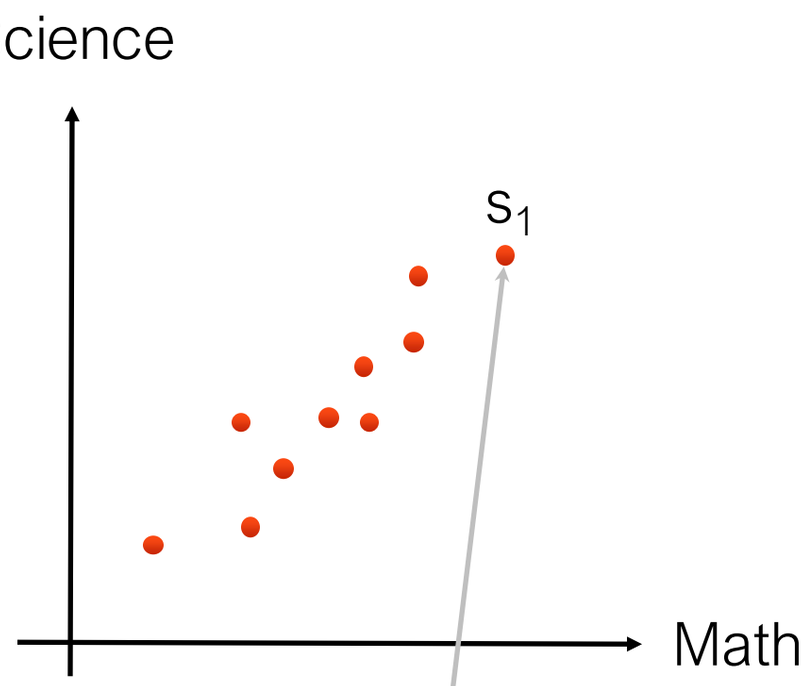


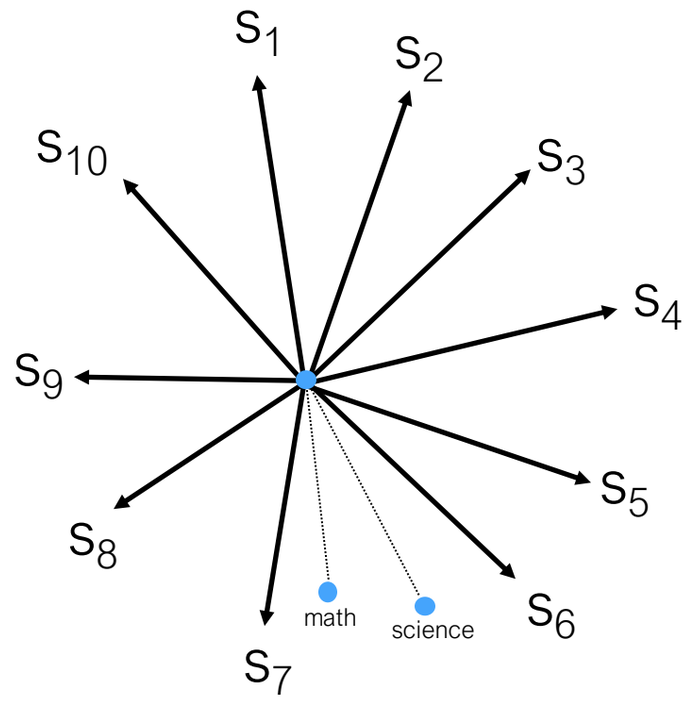




correlation = cosine similarity

variable space





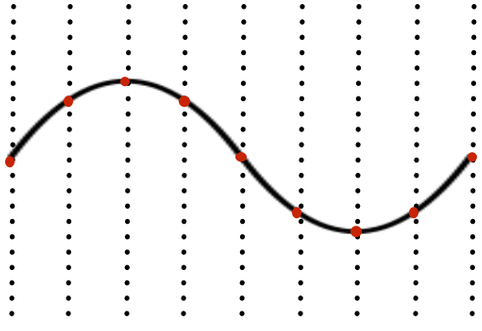
subject space

science = [90, 48, 60, 89, 25, 48, 55, 72, 36, 51]
math = [92, 54, 55, 92, 30, 41, 56, 74, 39, 48]

R=0.997

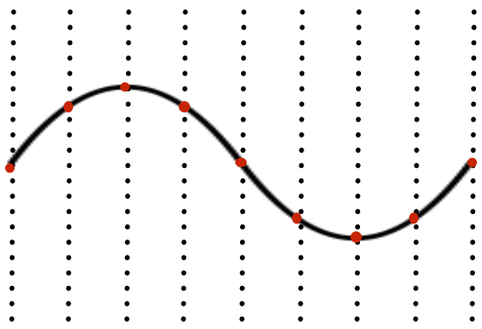
$$a \cdot b = |a||b|\cos(\theta)$$
$$\rightarrow \cos(\theta) = a \cdot b \,/\, |a||b|$$

inner product
signal vectors



a = [0, 0.85, 1, 0.85, 0, -0.85, -1, -0.85, 0]

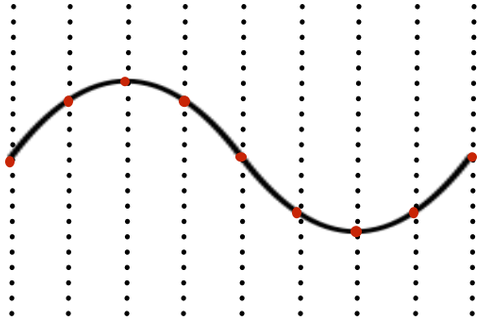
0°



b = [0, 0.85, 1, 0.85, 0, -0.85, -1, -0.85, 0]

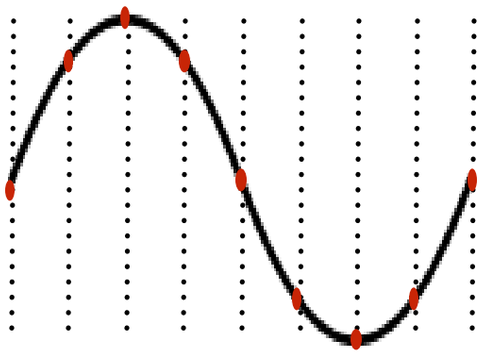
$\cos(\theta) = ?$

inner product
signal vectors



a = [0, 0.85, 1, 0.85, 0, -0.85, -1, -0.85, 0]

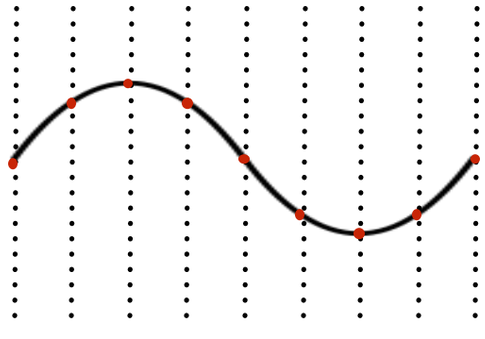
0°



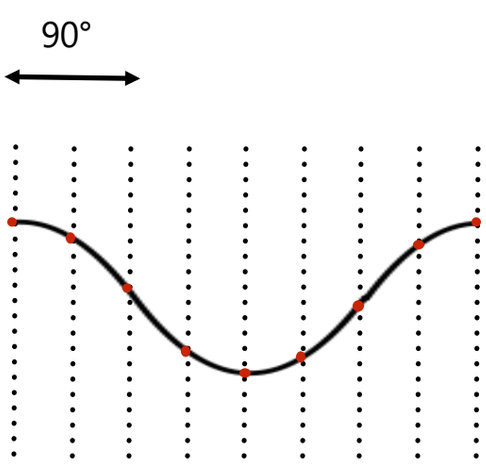
b = [0, 1.7, 2, 1.7, 0, -1.7, -2, -1.7, 0]

$\cos(\theta) = ?$

inner product
signal vectors



a = [0, 0.85, 1, 0.85, 0, -0.85, -1, -0.85, 0]



b = [1, 0.85, 0, -0.85, -1, -0.85, 0, 0.85, 1]

$\cos(\theta) = ?$