FABER-CASTELL

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ID5055: Assignment 2

Q1) $F(y, \alpha) = \frac{1}{\alpha^2} y e^{-y/\alpha}$

$\alpha \in (0, \infty) \quad y \in (0, \infty)$

Likelihood function

$$L(\alpha) = \prod_{i=1}^{n} F(y_i, \alpha) = \prod_{i=1}^{n} \frac{1}{\alpha^2} y_i e^{-y_i/\alpha}$$

$$= \alpha^{-2n} \times e^{-1/\alpha \sum_{i=1}^{n} y_i}$$

$$\ln(L(\alpha)) = -2n \ln(\alpha) + \sum_{i=1}^{n} \ln(y_i) - \frac{1}{\alpha} \sum_{i=1}^{n} y_i$$

$$= -2n \ln(\alpha) + \sum_{i=1}^{n} \ln(y_i) - \frac{n\bar{y}}{\alpha}$$

To find maximum likelihood estimator for parameter α we maximize the log-likelihood function. For this we differentiate function and equate to zero

$$\frac{d}{d\alpha}\left[\ln(L(\alpha))\right] = \frac{-2n}{\alpha} + \frac{n\bar{y}}{\alpha^2} = 0$$

As $\alpha \neq 0$

$$n\bar{y} = 2n\alpha$$

$$\alpha = \bar{y}/2$$

Now we check if it is maxima.

$$\frac{d}{d\alpha^2}(\ln(L(\alpha))) = \frac{2n}{\alpha^2} - \frac{2}{\alpha^3} \sum_{i=1}^{n} y_i = \frac{2n}{\alpha^2} - \frac{2 \times n\bar{y}}{\alpha^3}$$

Substitute $\bar{y} = 2\alpha$

$$\frac{d}{d\alpha^2} \ln(L(\alpha)) = \frac{2n}{\alpha^2} - \frac{2n \times 2\alpha}{\cancel{\alpha^3}\alpha^2}$$

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$$= \frac{-2n}{\alpha^2} < 0$$

Since second derivative < 0, the estimator $\alpha = \bar{y}/2$ maximies the likelihood function

$$\therefore \alpha = \bar{y}/2$$

3) Given $f(y;\theta) = \frac{3y^2}{\theta^3}$

The likelihood function is

$$L(\theta) = \left(\frac{3}{\theta^3}\right)^n \prod_{i=1}^{n} y_i^2$$

We need to find θ that maximises the likelihood function. So from the above function, θ should be minimum. But it is given that all y_i lie inside $[0, \theta]$. So θ should be greater than or equal to largest value in the data.

$\therefore$ Maximum likelihood estimator of θ is $\max\{y_i\}$