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Assignment 11

Q1) $F(y;\alpha) = \frac{1}{\alpha^2} y e^{-y/\alpha}$

$\alpha \in (0, \alpha)$, $y \in [0, \alpha)$

Likelihood Function,

$$L(\theta) = \prod_{i=1}^{n} F(y_i;\alpha)$$

$$= \prod_{i=1}^{n} \frac{1}{\alpha^2} y_i e^{-\frac{y_i}{\alpha}}$$

$$= \alpha^{-2n} \times \left(\prod_{i=1}^{n} y_i\right) \times e^{-1/\alpha \sum_{i=1}^{n} y_i}$$

$$\ln(L(\theta)) = -2n\ln(\alpha) + \sum_{i=1}^{n} \ln(y_i) - \frac{1}{\alpha}\sum_{i=1}^{n} y_i$$

we have to minimize log likelihood Function

$$\frac{\partial \ln(L(\theta))}{\partial \alpha} = \frac{-2n}{\alpha} + \frac{1}{\alpha^2}\sum_{i=1}^{n} y_i = 0$$

$$\alpha = \frac{1}{2}\frac{\sum_{i=1}^{n} y_i}{n} = \frac{\bar{y}}{2}$$

$$\frac{\partial^2 \ln(L(\theta))}{\partial \alpha^2} = \frac{2n}{\alpha^2} - \frac{2}{\alpha^3}\sum_{i=1}^{n} y_i$$

$$= \frac{2}{\alpha^2}\left(n - \frac{1}{\alpha}\sum_{i=1}^{n} y_i\right)$$

$$= \frac{2}{\alpha^2}(n - 2n)$$

$$= -\frac{2n}{\alpha^2} < 0 \quad \{n > 0 \;\; \alpha^2 > 0\}$$

$\therefore \alpha = \frac{\bar{y}}{2} \rightarrow$ sample mean