COMP 3711 – Design and Analysis of Algorithms
2022 Fall Semester – Written Assignment 4
Distributed: November 14, 2022
Due: November 30, 2022, 23:59

Your solution should contain
(i) your name, (ii) your student ID #, and (iii) your email address
at the top of its first page.

Some Notes:
• Please write clearly and briefly. In particular, your solutions should be written or printed on clean white paper with no watermarks, i.e., student society paper is not allowed.
• Please also follow the guidelines on doing your own work and avoiding plagiarism as described on the class home page. You must acknowledge individuals who assisted you, or sources where you found solutions. Failure to do so will be considered plagiarism.
• The term Documented Pseudocode means that your pseudocode must contain documentation, i.e., comments, inside the pseudocode, briefly explaining what each part does.
• Many questions ask you to explain things, e.g., what an algorithm is doing, why it is correct, etc. To receive full points, the explanation must also be understandable as well as correct.
• Please make a copy of your assignment before submitting it. If we can't find your submission, we will ask you to resubmit the copy.
• Submit a SOFTCOPY of your assignment to Canvas by the deadline. If your submission is a scan of a handwritten solution, make sure that it is of high enough resolution to be easily read. At least 300dpi and possibly denser.

Question 1 (20%): Cycle Detection

Let $G(V,E)$ be a undirected graph, not necessarily connected. The degree of a vertex is equal to the number of neighbors of that vertex in G. Assume that there are no nodes with degree 0.

a) (10%) Prove that if every vertex of G has an even degree, every vertex of G appears in some cycle.

   Hint: Explore the graph to remove one cycle at a time as well as the vertices on that cycle. Argue that this can be repeated until no vertex is left.

b) (10%) Based on part (a), design an $O(V+E)$ algorithm that returns TRUE if G contains a cycle, and FALSE otherwise. In case of TRUE, your algorithm should output **all** nodes that are part of some cycle (you are not required to output the individual cycles, if there are multiple). Your algorithm should be not be recursive, and should not be based on DFS.

   Hint: A vertex of degree 1 cannot appear on any cycle and hence can be deleted.

Question 2 (20%): All Pairs Shortest Paths

Assume an undirected connected graph $G(V,E)$, where all edges have the same weight w where $w > 0$. Give an $O(VE)$ algorithm for computing the shortest distance between all pairs of nodes.

Question 3 (30%): Currency Exchange

Let $x_1,..x_n$ a set of n currencies. You are given a nxn matrix M that contains the exchange rates for pairs of currencies (x_i,x_j), e.g., (HK$,US$) = 0.13 means that we can exchange HK$ 1 for US$ 0.13. Similarly, (US$, HK$) = 7.85 means that we can exchange US$ 1 for HK$ 7.85. The exchange rates for some pairs of currencies may be missing. By taking advantage of exchange rates, a trader can earn profit. For instance, in addition to the above assume the following exchange rates: (US$, EUR) = 1 and (EUR, $HK) = 7.9. One can start with HK$ 1,000, exchange to US$ 130, then exchange to EUR 130, and finally exchange back to HK$ 1,027 (130x7.9), for a profit of HK$ 27. Design an algorithm, which given M, it detects if there are any profit opportunities. The output should be TRUE or FALSE. TRUE implies that there is an opportunity, and FALSE that there is not. In case of TRUE, you do not have to output any of the profit opportunities.

Hint: Convert the problem to a problem of detecting whether there is a negative cycle in a directed graph.

Question 4 (30%): Maximum Flow

Let A be a $n \times m$ matrix of non-negative *real* numbers such that the sum of the entries in every row and in every column is an integer. Prove that there is an $n \times m$ matrix B of non-negative *integers* with the same sums as in A, in every row and every column.