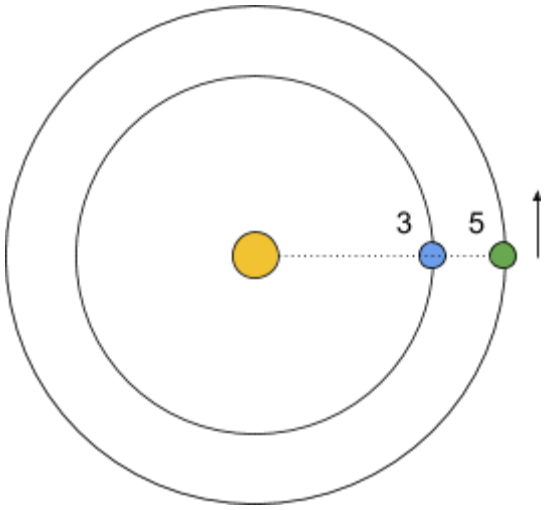
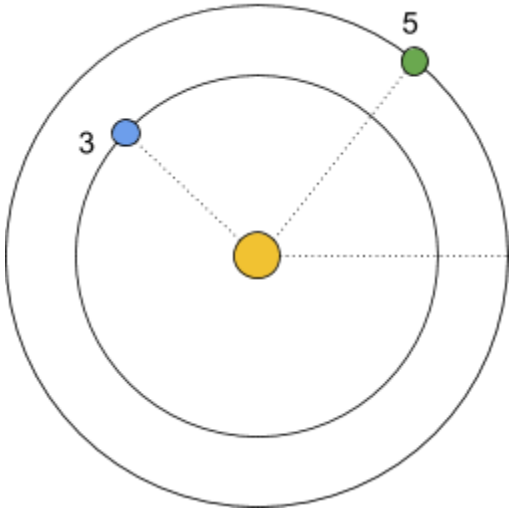
Asteroids

(posted Friday 20 November 2020)

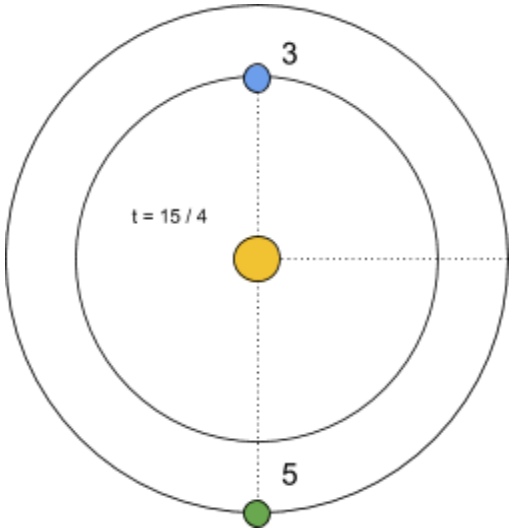
Asteroids with **distinct integer** periods orbit a sun in the same plane, along perfectly circular paths in the same direction. Initially, all asteroids are lined up on the same side of the sun. In this diagram, two asteroids with periods 3 and 5 are shown:



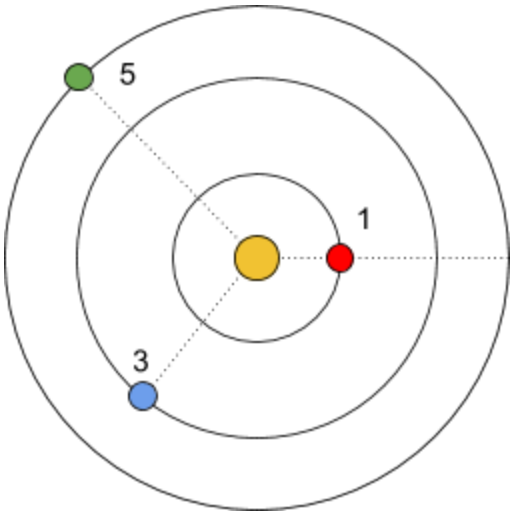
The asteroid labeled 3 takes 3 years to orbit the sun, while the asteroid labeled 5 takes 5 years. As for real asteroids, those with larger periods are farther out. As the asteroids rotate, they move apart due to differences in their orbital speeds, as shown in this diagram:



After 3.75 years, the asteroids will line up with the sun for the first time (after their initial line-up). In this case, the asteroids will be on opposite sides of the sun, as shown in this diagram:



The time it takes a configuration [a_1, a_2, …] of asteroids **to line up for the first time** is its alignment period, denoted by A[…]. We have that A[3, 5] = 3.75 years.
Other configurations of asteroids exist that also have an alignment period of 3.75 years. A[1, 3, 5] = 3.75, for example. The configuration [1, 3, 5] is shown below at the 2-year mark, before the three asteroids align:



There are 9 configurations of asteroids that have an alignment period of 3.75 years. These 9 configurations are [1, 3, 5], [1, 3, 5, 15], [1, 3, 15], [1, 5, 15], [3, 5], [3, 5, 15], [5, 15], [6, 30], and [7, 105] (we do not consider permutations of asteroids to be distinct configurations).

As additional examples, you are given that A[23, 29, 67] = 11172.25 years, that A[2, 3, 5, 6, 8, 11, 18] = 1980 years, and that there are 3747 configurations of asteroids with a period of 30 years.

How many configurations of asteroids have a period of 510510 years? Supply the last 12 digits of this number as your answer.

Goal to meet and beat: 3.5s (Python under PyPy)