UNITY strings

Strings are constructed using the letters of the word UNITY. Valid strings are 5 or more letters long and contain the substring “UNITY” exactly once. A valid string of length 8 is “IIUNITYN”.

We write Count(X) for the number of valid strings of length X. Trivially, Count(5) = 1.

Count(6) = 10. Among these 10 valid strings are “UUNITY” and “UNITYN”.

Count(10) = 18748. Among these is the string “UUUUUUNITY”, but not “UNITYUNITY”.

What is Count(1000)? (provide only the last 9 digits of this number as your answer)

Goal to meet and beat: 1.57s (interpreted Python)