PH107 Formulas

LM-2: Black Body Radiation

Rayleigh-Jean's Law

$$U(\nu)d\nu = \frac{8\pi\nu^2}{c^3}k_BT\,d\nu$$

Planck's Law

$$U(\nu)d\nu = \frac{8\pi\nu^2}{c^3}\cdot\frac{h\nu}{e^{h\nu/k_BT}-1}\,d\nu = \frac{8\pi\nu^2}{c^3}k_BT\cdot\frac{h\nu/k_BT}{e^{h\nu/k_BT}-1}\,d\nu$$

$$QC = \frac{h\nu/k_BT}{e^{h\nu/k_BT}-1}, \text{ and thus we obtain:}$$
$$U(\nu)d\nu = \frac{8\pi\nu^2}{c^3}k_BT\,d\nu\cdot QC(\text{quantum correction})$$

LM-3: Compton Effect

$$\lambda' - \lambda = \frac{h}{m_ec}(1-\cos\theta)$$

LM-4: Heat Capacity and Quantum Theory

Heat Capacity of Gases

$$c_v = \frac{f}{2}R$$

Where f is the number of degrees of freedom of the molecules of the gas

Energy available per molecule at room temperature is given by $E/molecule \approx 25meV$

Dulong-Petit's Law: Classical Theory fot r heat capacity of solids

$$\text{for a single atom moving in one direction } \langle E\rangle = k_BT$$
$$\text{for a single atom moving in all 3 directions } \langle E\rangle = 3k_BT$$

$$E = 3Nk_BT = 3RT$$

$$c_v = \frac{dE}{dT} = 3R$$

Einstein's Quantum Mechanical Theory for heat capacity of Solids

$$E_n = \left(n + \frac{1}{2}\right) \cdot h\nu_E$$

$$\text{for one direction:}$$
$$E = \frac{h\nu}{e^{h\nu/k_BT} - 1}$$
$$\text{and for that same atom oscillating in all 3 directions, we have:}$$
$$E = \frac{3h\nu}{e^{h\nu/k_BT} - 1}$$

$$E = N \cdot \frac{3h\nu}{e^{h\nu/k_BT} - 1} = 3Nk_BT \cdot \frac{h\nu/k_BT}{e^{h\nu/k_BT} - 1} = 3RT \cdot \frac{h\nu/k_BT}{e^{h\nu/k_BT} - 1}$$

We define $\theta_E = h\nu/k_B$ as the Einstein temperature of the solid and thus get

$$E = 3RT \cdot \frac{\theta_E/T}{e^{\theta_E/T} - 1}$$

In all the equations stated above, $\nu = \nu_E$, which is known as the Einstein frequency of the solid.

Debye Model

A few new assumptions made by Debye

$$\nu_{max} = \nu_D$$
$$\lambda_{min} = \lambda_D$$
$$\lambda_D = 2d \text{ where d is distance between atoms}$$

At low temperatures

$$c_v \propto T^3$$

LM-5: Wave Particle duality and de Broglie's hypothesis

de Broglie Hypothesis

$$\lambda_D = \frac{h}{p}$$

Bragg's Law

$$\text{(path difference)}\Delta\lambda = 2d\sin\theta$$

Where θ is the angle of the incident rays with the surface of the lattice.

Davisson-Germer Experiment

We generally look at only first order phenomena, ie $\Delta\lambda = \lambda$, and find that for the angle b/w the electron gun and detector being ϕ and applying Bragg's Law, we get

$$\lambda = 2d\sin\theta = 2d\cos\phi/2$$

LM-6: Wave Packets, Group Velocity and Phase Velocity

$$\begin{gathered}\text{Here are some basic and useful formulas to keep in mind from here on}\\ p = \hbar k = \frac{h}{\lambda}\\ \text{for particles: } E = \frac{p^2}{2m} = \frac{1}{2}mv^2\\ \text{for photons: } E = \hbar\omega = h\nu\end{gathered}$$

Group and Phase Velocity

$$v_p = \frac{\omega}{k} = \nu\lambda$$
$$v_g = \frac{d\omega}{dk}$$

For photons, we also define

$$v_p = \frac{\omega}{k} = \frac{E}{p}$$
$$v_g = \frac{d\omega}{dk} = \frac{dE}{dp}$$

Dispersive and non-dispersive mediums

Whenever $v_g \neq v_p$ we say that the medium is dispersive:

- if $v_p > v_g \implies \textit{normal dispersion}$
- if $v_g > v_p \implies \textit{anomalous dispersion}$

When we have the condition $v_p = v_g$ we say that medium is non dispersive.

LM-7: Fourier Transform and Heisenberg's Uncertainty Principle

Heisenberg's Uncertainty Principle

1. In terms of momentum and position

$$\Delta p_k \cdot \Delta k \geq \frac{\hbar}{2} \text{ where } k \in \{x, y, z\}$$

2. In terms of energy and time

$$\Delta E \cdot \Delta t \geq \frac{\hbar}{2}$$

LM-8: The Schrödinger Equation and its properties

Schrödinger Equation

- Time Dependent Schrödinger Equation(TDSE)

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} + U\Psi = i\hbar\frac{\partial \Psi}{\partial t}$$

- Time Independent Schrödinger Equation(TISE)

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} + U\Psi = E\Psi$$

Normalisation of a wave function

$$\int_{-\infty}^{\infty} |\Psi(x)|^2 dx = 1$$

Observables and Operators

Obesrvable	Symbol	Operator
Position	$\hat{x}$	x
Momentum	$\hat{p}$	$-i\hbar\frac{\partial}{\partial x}$
Potential Energy	$\hat{U}$	$U(x)$
Kinetic Energy	$\hat{K}$	$\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}$
Total Energy	$\hat{E}$	$i\hbar\frac{\partial}{\partial t}$

For a normalised wave function:

$$\langle o\rangle = \int_{-\infty}^{\infty} \Psi^* \hat{O}\Psi dx$$
$$\langle o^2\rangle = \int_{-\infty}^{\infty} \Psi^* \hat{O}^2\Psi dx$$

Eigen functions and values

$$\hat{O}\Psi = e\Psi$$

- Ψ is an Eigen function of the operator $\hat{O}$
- e is the Eigen value

Some more topics

Relativistic effects

When to consider it:

- v is close to c
- Energy/Kinetic Energy of electron is comparable to or larger than rest energy of electron($m_e c^2 = 511 keV$)

What are the effects of considering relativity:

- $E_{total} = \sqrt{m_0^2 c^4 + p^2 c^2}$, this includes the rest mass energy
- $KE = E_{total} - m_0 c^2$, where $E_{rest} = m_0 c^2$
- Now you may not use $KE = p^2/2m$ and should use only the above definition

Boltzmann distribution

It states that the probability of an atom to be in a state i, ie p_i, is proportional to the given expression, where E_i is the energy of the state i:

$$p_i \propto \exp \frac{-E_i}{k_B T}$$