Final Report:

Comparing different cooling methods for common hot beverages

Submitted to:
Professor Dr. Ramya Thinniyam
Department of Statistics

Report Prepared By:
Group 20
Aamir Ali, 1004733835
Arib Shaikh, 1005422348
Prashant Patel, 1003096437
Owen Chiu, 1005413571
Lanhui Qiu, 1003819291
Ankit Jhurani, 1003697065
Stefan Bozic, 1001716315
Khadijah Mosaheb, 1004622116

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August 23, 2021

Introduction

Newton's law of cooling states that the heat loss rate of an object is directly proportional to the difference in the temperatures between the object and its surroundings (Dill et al., 2010; Greiner et al., 2012). Meanwhile, increased air flow rate speeds up the evaporation process (Raimundo et al., 2014), which leads to a dropped temperature (He et al., 2012).

Using statistical modeling, this report compares the effectiveness of four common cooling methods (stirring, fanning, ice bath and natural evaporation) on three popular hot beverages in Canada - coffee, milk and tea (Garriguet et al., 2019). The purpose of this report is to share our findings on the most effective cooling method based on the type of the drink. The target population is all kinds of daily hot drinks. Our study is significant because people expect a drinkable warm beverage, especially in winter, but mostly they get a boiling hot one from a cafe. Additionally, this report intends to serve as a template for describing the experiment setup and statistical analysis methods in the evaluation of cooling techniques.

Experiment Setup

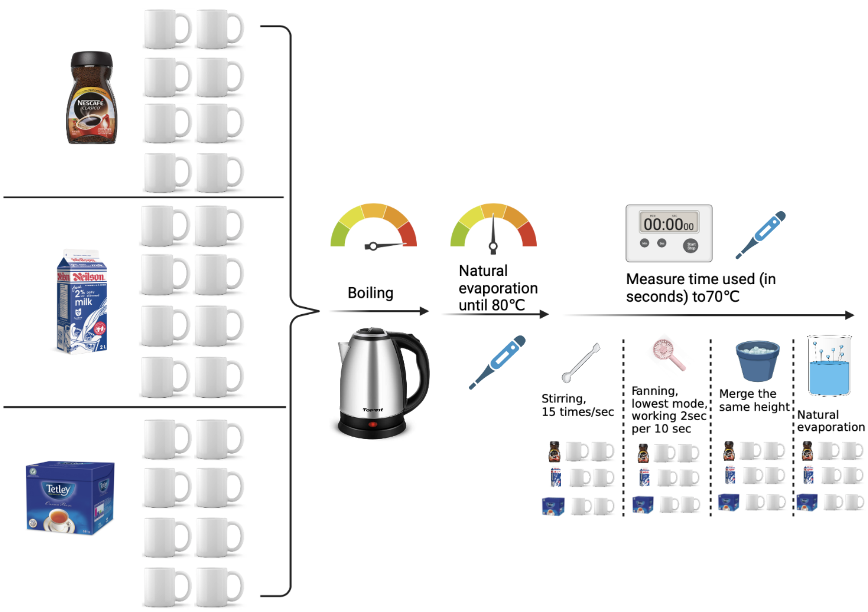

Figure 1. Diagram of experimental design

The experiment was conducted using the following procedure:

1. Heat 250mL of the beverage to its boiling point;
2. Start timing when the temperature drops to 80℃;
3. Measure the time taken for the beverage to reach 70℃.

The predictors are Beverage Type (**K**), and Cooling Method (**M**). The response variable is the time (**T**), in seconds, taken to cool the beverage from 80℃ to 70℃. 80℃ is the starting temperature as different types of drinks have different boiling points. Note that while volume changes with temperature (unlike mass), it is more commonly used to measure liquids.

The experiment is a between-subjects factorial Randomized Complete Design (RCD). This is because between-subject experiments are easier and faster to set up since the same drink does not need to be heated and cooled repeatedly (note, the K-M combination is the treatment), and they facilitate comparing groups that differ on key characteristics: K and M. The temperatures are measured by Cole-Parmer® Traceable Waterproof Food Thermometer with Calibration, which is ±0.4°C accuracy at tested points.

The control parameters are drink volume (250mL), mugs (same shape with 330mL volume), room temperature and humidity (26℃, 42%), stirring speed (stirring with a teaspoon 15 times per 10 seconds), fanning speed (electric fan set to lowest setting and blowing air across the liquid's surface for 2 seconds every 10 seconds to simulate a person blowing). Each replicate is randomly assigned one out of the total 12 treatments.

The sample size is determined by R (pwr2 package), with Type-one error α = 0.05, and Type-two error β = 0.2. The effective size of 0.8 is artificially chosen (a medium level) for both factors. The output n = 2 (Supplement 1) indicates the total number of experiments (N) should be 2*3*4 = 24.

Data Analysis

Raw data is obtained from Supplement 2. Figure 2 Shows the overview of all experiment results:

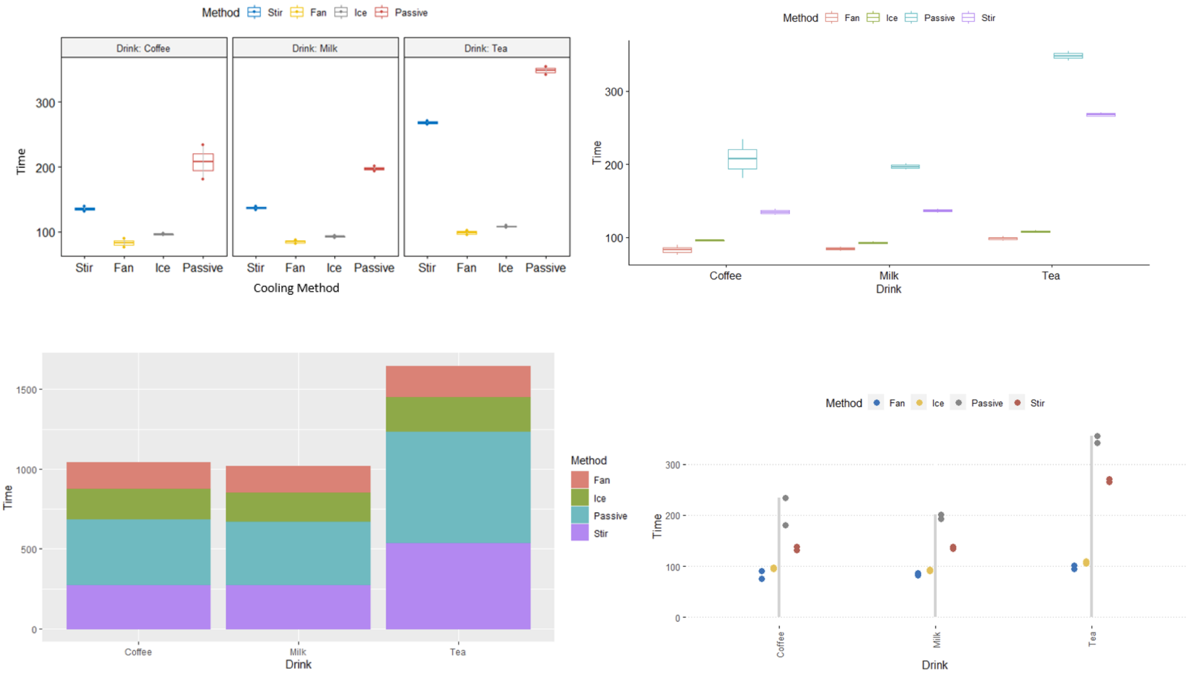

Figure 2. Overview experiment data. Clockwise from top left: box plot of T vs. M, box plot of T vs. K, subgroup bar plot of T based on K, the line plot of T vs. K.

Figure 2 and Table 1 relate to Model 1 which tests H_α:K and T are significantly related versus H_0:K and T are not related. Table 1 shows that while the regression model predicts the dependent variable well, the regression model does not statistically significantly predict the outcome variable since the *p value* (0.098) is greater than 0.05.

Table 1. Linear regression analysis for model 1

Coefficients: Estimate	Std.	Error	t value	Pr(>|t|)
(Intercept)	138.38	27.51	4.74	0.000111
Drink Milk	-2.75	38.9	-0.071	0.944311
Drink Tea	75.38	38.9	1.938	0.066235

Factor	df	SSM	Mean Sq	F value	Pr(>F)
Drink	2	31447	15723.3	2.5976	0.09817

Residuals	21	127115	6053.1

Tukey HSD test is used for model 1 (Table 2) to test the linear hypotheses. The first column shows the comparisons between each type of the hot beverages, and the last column (Pr(>|t|)) shows the adjusted p-values for each comparison. H_0: the two groups are equal vsH_a: the two groups are different.

Adjusted p-values are used to test whether two groups are significantly different. All of the comparisons had an error rate of 0.05, indicating the rejection of H_0. This result is further confirmed by Kruskal-Wallis Test (Figure 3).

Table 2. Tukey's HSD for Model 1.

Type of Drink	diff	lwr	upr	Adjusted p-value
Milk-Coffee	-2.8	-100.8	95.3	0.997
Tea-Coffee	75.4	-22.7	173.4	0.153
Tea-Milk	78.1	-19.9	176.2	0.135

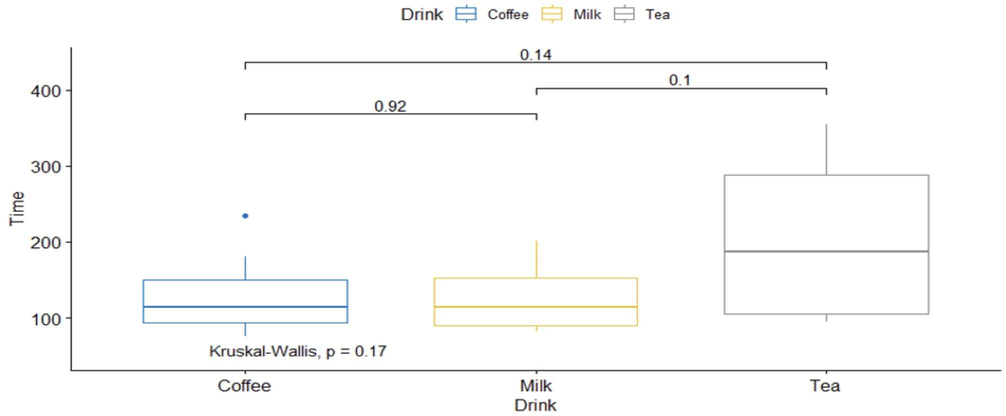

Figure 3: Box plot with Kruskal-Wallis Test.

Therefore, a Two-Way ANOVA test is performed as we are interested in finding out whether:

1. M is a significant predictor of T or whether K is a significant explanatory variable.
2. Whether T depends on K for different levels of M and vice-versa.

Lines are parallel in the interaction plot (Figure 4). Hence, there is no significant interaction between the two predictors, K and M, suggesting that T is not affected by combinations of K and M, but rather mainly by one of the two factors.

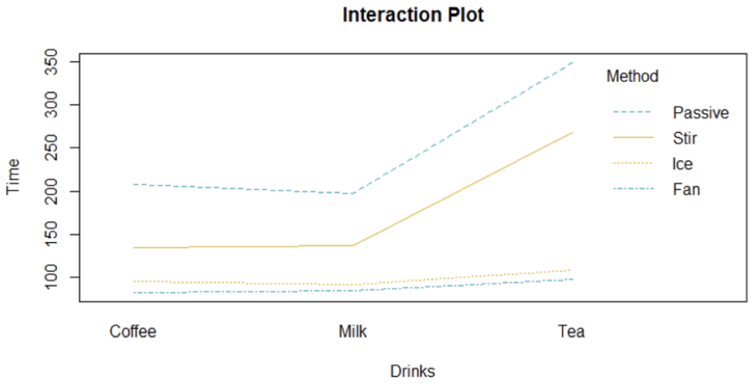

Figure 4. Interaction plot.

Next, an additive model is applied, interpreting the effects of K and M separately and additively. Model 2: $Y_{ijk}=\mu+\alpha_j+\beta_k+(\alpha\beta)_{jk}+\varepsilon_{ijk}$ is generated (Descriptions in Supplement 3). H_0 is K and M do not affect the response separately; H_α is K and M affect the response separately. The results are shown in Table 3.

Table 3. Linear model with two way Anova Model without interaction (main effects only)

	Estimate	Std. Error t	value	Pr(>|t|)	
(Intercept)	63.46	17.79	3.623	0.001946	***
DrinkMilk	-2.75	17.79	-0.155	0.878891	
DrinkTea	75.38	17.79	4.236	0.000497	***
MethodIce Bath	10.17	20.55	0.495	0.626702	
MethodNatural Cooling	162.33	20.55	7.901	2.92E-07	***
MethodStir	91.17	20.55	4.437	0.000318	***

ANOVA table (model 2)

	df	SSM	Mean Sq	F-value	P-Value	
Drink	2	31447	15723	12.416	0.000408	***
Method	3	104321	34774	27.460	6.187e-07	***
Residuals	18	22794	1266			

Model 2 predicts the dependent variable well. The above ANOVA table shows that $p < F$, suggesting that both K and M affect the response individually. Tukey HSD test (Supplement 4) shows that all comparisons have an error rate of 0.05.

The slope confidence intervals (Table 4) of milk and ice contain 0, suggesting the possibility that the regression coefficients to be 0 for these variables. Hence, the relationship between the response and both of these variables (Milk and Ice) respectively might be insignificant.

Table 4. Confidence interval.

	2.5%	97.5%
Intercept	27.07673	101.83994
Milk (K)	-40.13160	34.63160
Tea (K)	37.99340	112.75660
Ice Bath (M)	-32.99789	53.33122
Evaporation (M)	119.16878	205.49789
Stir (M)	48.00211	134.33122

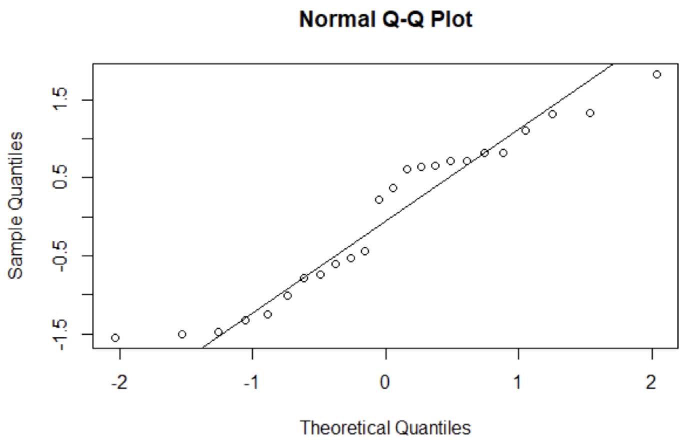

Figure 6. Normal QQ plot.

Normality is checked before adopting the F test. The QQ plot (Figure 6) shows that most of the points lie on the straight line, indicating the errors are normally distributed. By using R's powerTransform function, we find that transformation of variables is not needed (Supplement 5).

Discussion

For practical reasons, our experiment was affected by the following nuisance variables:

- Experiments were conducted in different rooms so ambient conditions could not be controlled
- Experimenters were unable to obtain identical equipment, resulting in unaccounted calibration errors
- Maintaining a consistent stirring rate for extended durations is difficult without practice

The limitations of the conclusions we draw from this experiment are:

- While tea, milk and coffee are popular, they cannot represent all hot beverages
- Specific brands do not represent all brands of a beverage
- A 'just right' 70℃ is subjective, individuals may prefer hotter or colder temperatures

To improve the experiments, future studies can target more types of beverages, different cooling methods (e.g. saucering and refrigerating) and ambient conditions (e.g. room temperature, humidity, altitude etc.), which are reported to affect cooling speed (Jódar et al., 2016)(He, Chao, et al., 2012). Additionally, future experiments should be conducted with identical equipment (e.g. ice bowl, fan etc.) as intended in the procedure.

Conclusion

In conclusion, the studied cooling methods - stirring, fanning, and ice bath - were statistically more effective than natural evaporation (control) based on the time taken to cool the three hot beverages (coffee, milk and tea) from 80℃ to 70℃. Meanwhile, the type of hot drink statistically affects the time taken to cool said drink. The overall interaction effect of the type of drink and cooling method is statistically insignificant in the time taken to cool.

We observed that fanning is the most effective method to cool all three types of hot beverage and coffee cooled the quickest while Tea cooled the slowest. If you find yourself in a rush, fanning or blowing on a mug of hot coffee will get you a drinkable beverage the quickest, but if you have time on your side, letting your beverage of choice cool passively will do the job while requiring the least effort.

References

Dill, Ken A., Sarina Bromberg, and Dirk Stigter. *Molecular driving forces: statistical thermodynamics in biology, chemistry, physics, and nanoscience*. Garland Science, 2010.

Garriguet, Didier. "Changes in beverage consumption in Canada." *Health reports* 30.7 (2019): 20-31.

Greiner, Walter, Ludwig Neise, and Horst Stöcker. *Thermodynamics and statistical mechanics*. Springer Science & Business Media, 2012.

Jódar, J., et al. "Correlation of the seasonal isotopic amplitude of precipitation with annual evaporation and altitude in alpine regions." *Science of The Total Environment* 550 (2016): 27-37.

He, Chao, et al. "The optimal evaporation temperature and working fluids for subcritical organic Rankine cycle." *Energy* 38.1 (2012): 136-143.

Raimundo, António M., et al. "Wind tunnel measurements and numerical simulations of water evaporation in forced convection airflow." *International journal of thermal sciences* 86 (2014): 28-40.

Supplementary data

Supplement 1. Sample size calculation by R. pwr2 package: Power and Sample Size Analysis for One-way and Two-way ANOVA Models.

```
Balanced two-way analysis of variance sample size adjustment
              a = 3
              b = 4
      sig.level = 0.05
          power = 0.8
              n = 2
NOTE: n is number in each group, total sample = 24
```

Supplement 2. Raw data from experiments.

Attempt #	Type of Hot Drink	Cooling Method Used	Time Taken To Cool (Seconds)	Attempt #	Type of Hot Drink	Cooling Method Used	Time Taken To Cool (Seconds)
1	Coffee	Stir	139	2	Coffee	Stir	131
1	Coffee	Fan	76	2	Coffee	Fan	90
1	Coffee	Ice Bath	95	2	Coffee	Ice Bath	97

1	Coffee	Natural Cooling	181	2	Coffee	Natural Cooling	234
1	Milk	Stir	134	2	Milk	Stir	139
1	Milk	Fan	82	2	Milk	Fan	87
1	Milk	Ice Bath	94	2	Milk	Ice Bath	91
1	Milk	Natural Cooling	201	2	Milk	Natural Cooling	193
1	Tea	Stir	271	2	Tea	Stir	265
1	Tea	Fan	95	2	Tea	Fan	102
1	Tea	Ice Bath	106	2	Tea	Ice Bath	110
1	Tea	Natural Cooling	342	2	Tea	Natural Cooling	355

Supplement 3. Model 2 descriptions.

- Y_{ijk} : ith observation from the jth level of the drink type factor (Factor K) and kth level of the cooling method factor (Factor M)
- μ: overall mean
- α_j =deviation of Factor K's jth level mean from overall mean
- β_k =deviation of Factor M's kth level mean from overall mean
- $(\alpha\beta)_{jk}$: deviation remaining after main effects are removed (interaction effect)
- ε_{ijk} : random error. Usually assume ε_{ijk} ~iid N(0,σ^2)

Supplement 4: Turkey HSD with two way Anova Model with interaction.

Drink:Method`	diff	lwr	upr	p adj
Milk:Fan-Coffee:Fan	2	-46	49	1.000
Tea:Fan-Coffee:Fan	16	-32	63	0.965
Coffee:Ice Bath-Coffee:Fan	13	-35	61	0.990
Milk:Ice Bath-Coffee:Fan	10	-38	57	0.999
Tea:Ice Bath-Coffee:Fan	25	-23	73	0.643
Coffee:Natural Cooling-Coffee:Fan	125	77	172	0.000
Milk:Natural Cooling-Coffee:Fan	114	66	162	0.000
Tea:Natural Cooling-Coffee:Fan	266	218	313	0.000
Coffee:Stir-Coffee:Fan	52	4	100	0.028
Milk:Stir-Coffee:Fan	54	6	101	0.023
Tea:Stir-Coffee:Fan	185	137	233	0.000
Tea:Fan-Milk:Fan	14	-34	62	0.983
Coffee:Ice Bath-Milk:Fan	12	-36	59	0.996

Milk:Ice Bath-Milk:Fan	8	-40	56	1.000
Tea:Ice Bath-Milk:Fan	24	-24	71	0.712
Coffee:Natural Cooling-Milk:Fan	123	75	171	0.000
Milk:Natural Cooling-Milk:Fan	113	65	160	0.000
Tea:Natural Cooling-Milk:Fan	264	216	312	0.000
Coffee:Stir-Milk:Fan	51	3	98	0.035
Milk:Stir-Milk:Fan	52	4	100	0.028
Tea:Stir-Milk:Fan	184	136	231	0.000
Coffee:Ice Bath-Tea:Fan	-3	-50	45	1.000
Milk:Ice Bath-Tea:Fan	-6	-54	42	1.000
Tea:Ice Bath-Tea:Fan	10	-38	57	0.999
Coffee:Natural Cooling-Tea:Fan	109	61	157	0.000
Milk:Natural Cooling-Tea:Fan	99	51	146	0.000
Tea:Natural Cooling-Tea:Fan	250	202	298	0.000
Coffee:Stir-Tea:Fan	37	-11	84	0.206
Milk:Stir-Tea:Fan	38	-10	86	0.172
Tea:Stir-Tea:Fan	170	122	217	0.000
Milk:Ice Bath-Coffee:Ice Bath	-4	-51	44	1.000
Tea:Ice Bath-Coffee:Ice Bath	12	-36	60	0.995
Coffee:Natural Cooling-Coffee:Ice Bath	112	64	159	0.000
Milk:Natural Cooling-Coffee:Ice Bath	101	53	149	0.000
Tea:Natural Cooling-Coffee:Ice Bath	253	205	300	0.000
Coffee:Stir-Coffee:Ice Bath	39	-9	87	0.153
Milk:Stir-Coffee:Ice Bath	41	-7	88	0.127
Tea:Stir-Coffee:Ice Bath	172	124	220	0.000
Tea:Ice Bath-Milk:Ice Bath	16	-32	63	0.965
Coffee:Natural Cooling-Milk:Ice Bath	115	67	163	0.000
Milk:Natural Cooling-Milk:Ice Bath	105	57	152	0.000
Tea:Natural Cooling-Milk:Ice Bath	256	208	304	0.000

Coffee:Stir-Milk:Ice Bath	43	-5	90	0.098
Milk:Stir-Milk:Ice Bath	44	-4	92	0.081
Tea:Stir-Milk:Ice Bath	176	128	223	0.000
Coffee:Natural Cooling-Tea:Ice Bath	100	52	147	0.000
Milk:Natural Cooling-Tea:Ice Bath	89	41	137	0.000
Tea:Natural Cooling-Tea:Ice Bath	241	193	288	0.000
Coffee:Stir-Tea:Ice Bath	27	-21	75	0.550
Milk:Stir-Tea:Ice Bath	29	-19	76	0.483
Tea:Stir-Tea:Ice Bath	160	112	208	0.000
Milk:Natural Cooling-Coffee:Natural Cooling	-11	-58	37	0.998
Tea:Natural Cooling-Coffee:Natural Cooling	141	93	189	0.000
Coffee:Stir-Coffee:Natural Cooling	-73	-120	-25	0.002
Milk:Stir-Coffee:Natural Cooling	-71	-119	-23	0.002
Tea:Stir-Coffee:Natural Cooling	61	13	108	0.009
Tea:Natural Cooling-Milk:Natural Cooling	152	104	199	0.000
Coffee:Stir-Milk:Natural Cooling	-62	-110	-14	0.008
Milk:Stir-Milk:Natural Cooling	-61	-108	-13	0.009
Tea:Stir-Milk:Natural Cooling	71	23	119	0.002
Coffee:Stir-Tea:Natural Cooling	-214	-261	-166	0.000
Milk:Stir-Tea:Natural Cooling	-212	-260	-164	0.000
Tea:Stir-Tea:Natural Cooling	-81	-128	-33	0.001
Milk:Stir-Coffee:Stir	2	-46	49	1.000
Tea:Stir-Coffee:Stir	133	85	181	0.000
Tea:Stir-Milk:Stir	132	84	179	0.000

Supplement 5: PowerTransform Table

	Est Power	Rounded Pwr	Lwr Bound	Upper Bound
Y1	-0.8442	0	-1.9212	0.2327
Y2	0.6573	1	-0.4891	1.8037
Y3	0.9299	1	-0.1354	1.9952

R-Code Appendix

1.
```
>#Sample size determination
>install.packages("pwr2")
>library(pwr2)
>ss.2way(a=3, b=4, alpha=0.05, beta=0.2, f.A=0.8, f.B=0.8, B=100)
```

2.
```
>#Creating graphs
>library("ggpubr")
>ggboxplot(my_data, x = "Drink", y = "Time", color = "Method")
>ggplot(my_data, aes(x = Drink, y = Time)) +geom_bar(aes(color =
Method, fill = Method), stat = "identity", position =
position_stack())
>ggdotchart(my_data, x = "Drink", y ="Time",color = "Method", palette
= "jco", size = 3,add = "segment", add.params = list(color =
"lightgray", size = 1.5),position = position_dodge(0.3),ggtheme =
theme_pubclean())
>my_comparisons <- list( c("Coffee", "Milk"), c("Milk", "Tea"),
c("Coffee", "Tea"))ggboxplot(my_data, x = "Drink", y = "Time",color =
"Drink", palette = "jco")+ stat_compare_means(comparisons =
my_comparisons)+ stat_compare_means(label.y = 50)
># Use only p.format as label. Remove method name.ggplot(my_data,
>aes(Method, Time)) + geom_boxplot(aes(color = Method))+
facet_wrap(~Drink) + scale_color_manual(values = c("#00AFBB",
"#E7B800")) + stat_compare_means(label = "p.format")
```

3.
```
>#Box plot facetted by "dose"
>p <- ggpaired(my_data, x = "Method", y = "Time",color = "Method",
palette = "jco", line.color = "gray", line.size = 0.4, facet.by =
"Drink", short.panel.labs = FALSE)
># Use only p.format as label. Remove method name.
>p + stat_compare_means(label = "p.format", paired = TRUE)
```

4.
```
>#One way Anova Model without interaction
>my_data <- read.csv(file.choose())
>View(my_data)
>attach(my_data)
>model1 <- lm(Time~ Drink, data=my_data)
>anova(model1)
>TukeyHSD(aov(model1))
```

5.
```
#Interaction Plot
>with(my_data,interaction.plot(Drink, Method, Time,col = c("#00AFBB",
"#E7B800"),main="Interaction Plot",xlab="Drinks",
ylab="Time"))
```

6.
```
>#Model with additive effect
>model2 <- lm(Time~Drink + Method)
>summary(model2)
>anova(model2)
>TukeyHSD(aov(model2))
>confint(model2)
```

7.
```
># two way Anova Model- Model with interaction
>model3 <- lm(Time~Drink*Method)
>summary(model3)
>anova(model3)
>TukeyHSD(aov(model3))
```

8.
```
># QQplot
>qqnorm(rstandard(model2))
```

9.
```
>#PowerTransform
>mult <- lm(cbind(my_data$Time, my_data$Drink, my_data$Method) ~ 1)
>bc <- powerTransform(mult)
>summary(bc)
```