

Time Geography Analysis of Toronto Public Transit

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1 Introduction

This project is based on Rafael Periera’s 2022 paper titled *Exploring the Time Geography of Public Transit Networks*. I have replicated his methods and analysis with data from Toronto and the Toronto Transit Commission (TTC) by adapting Rafael’s scripts found at https://github.com/ipeaGIT/gtfs2gps-time_geography. My adapted scripts can be found at https://github.com/jujujames/4MS3_JV

Original Paper:

R.H.M.Pereira et al. (2022). Exploring the Time Geography of Public Transit Networks. Journal of Geographical Systems (2023) 25:453–466 <https://doi.org/10.1007/s10109-022-00400-x>

2 0.0 Setup and Data Preparation

This section covers the initial setup, including loading libraries and preparing the geographic boundary and GTFS data specific to Toronto.

2.1 Load required packages

```
library(gtfs2gps)
```

```
## gtfs2gps version 2.1-2 is now loaded  
## NOTE: All filter functions from gtfs2gps were removed  
## Please replace them by similar functions from gtfstools
```

```
library(gtfstools)
```

```
##  
## Attaching package: 'gtfstools'  
  
## The following objects are masked from 'package:gtfs2gps':  
##  
##   read_gtfs, write_gtfs
```

```
library(data.table)  
library(sf)
```

```
## Linking to GEOS 3.12.2, GDAL 3.9.3, PROJ 9.4.1; sf_use_s2() is TRUE
```

```
library(dplyr)
```

```
##  
## Attaching package: 'dplyr'  
  
## The following objects are masked from 'package:data.table':  
##  
##   between, first, last
```

```
## The following objects are masked from 'package:stats':  
##  
##   filter, lag
```

```
## The following objects are masked from 'package:base':  
##  
##   intersect, setdiff, setequal, union
```

```
library(magrittr)  
library(ggplot2)  
library(ggmap)
```

```
## i Google's Terms of Service: <https://mapsplatform.google.com>  
##   Stadia Maps' Terms of Service: <https://stadiamaps.com/terms-of-service/>  
##   OpenStreetMap's Tile Usage Policy: <https://operations.osmfoundation.org/policies/tiles/>  
## i Please cite ggmap if you use it! Use 'citation("ggmap")' for details.
```

```

##
## Attaching package: 'ggmap'

## The following object is masked from 'package:magrittr':
##
##      inset

library(rayshader)
library(rayrender)

##
## Attaching package: 'rayrender'

## The following object is masked from 'package:rayshader':
##
##      run_documentation

## The following object is masked from 'package:ggplot2':
##
##      arrow

## The following object is masked from 'package:data.table':
##
##      cube

library(rayimage)

##
## Attaching package: 'rayimage'

## The following object is masked from 'package:rayrender':
##
##      run_documentation

## The following object is masked from 'package:rayshader':
##
##      run_documentation

library(cancensus)
library(viridis)

## Loading required package: viridisLite

library(sfheaders)
library(progressr)
library(pbapply)
library(patchwork)
library(stringr)
library(raster)

```

```
## Loading required package: sp

##
## Attaching package: 'raster'

## The following object is masked from 'package:dplyr':
##
##      select

library(terra)

## terra 1.8.21

##
## Attaching package: 'terra'

## The following object is masked from 'package:patchwork':
##
##      area

## The following object is masked from 'package:ggmap':
##
##      inset

## The following objects are masked from 'package:magrittr':
##
##      extract, inset

## The following object is masked from 'package:data.table':
##
##      shift

library(mapview)
library(tinytex)
```

2.2 Retrieve Toronto's Census Subdivision boundary

```
# Visit [https://censusmapper.ca/api](https://censusmapper.ca/api) to obtain an API key for the censensus
# options(cancensus.api_key = "api key")

toronto_bound <- cancensus::get_census(
  dataset = "CA21",           # Use the 2021 Canadian Census
  regions = list(CSD = "3520005"), # CSD code for Toronto
  level = "CSD",             # Specify the census subdivision level
  geo_format = "sf",         # Request output in sf format
  use_cache = TRUE           # Use cached data if available
)

## Reading geo data from local cache.
```

```
# Transform to WGS84 coordinate system
toronto_bound <- sf::st_transform(toronto_bound, 4326)

# Save the boundary data
readr::write_rds(toronto_bound, "data/toronto_bound_CSD.rds")
```

2.3 Prepare TTC GTFS data

```
# ttc_gtfs_url <- "https://ckan0.cf.opendata.inter.prod-toronto.ca/dataset/7795b45e-e65a-4465-81fc-c36b"
# download.file(url = ttc_gtfs_url, destfile = "data-raw/ttc_gtfs.zip")
ttc_gtfs_raw <- gtfstools::read_gtfs("data-raw/ttc_gtfs.zip")
```

2.4 Process TTC GTFS data

Add shape_id to stop_times

```
ttc_gtfs_raw$stop_times[ttc_gtfs_raw$trips,on = "trip_id",shape_id := i.shape_id]
```

Filter GTFS for sections 1.1, 1.2.1, & 1.2.2

```
# Filter by a specific shape_id (a single bus route variation) for use in scripts 1.2.1 & 1.2.2
ttc_gtfs <- gtfstools::filter_by_shape_id(ttc_gtfs_raw, "1048831")

# Save this filtered GTFS data
gtfs2gps::write_gtfs(ttc_gtfs,"data/gtfs_ttc_1048831.zip")
```

```
## Writing text files to C:/Users/julia/AppData/Local/Temp/Rtmpmos3M2/gtfsio22604b91492e
```

```
## - Writing agency.txt
```

```
## - Writing calendar.txt
```

```
## - Writing calendar_dates.txt
```

```
## - Writing routes.txt
```

```
## - Writing shapes.txt
```

```
## - Writing stops.txt
```

```
## - Writing stop_times.txt
```

```
## - Writing trips.txt
```

```
## GTFS object successfully zipped to data/gtfs_ttc_1048831.zip
```

```

# Filter specific intersecting trips for section 1.1 ... I did this by manually sifting through the GTFS
ttc_gtfs_tmp <- gtfs2tools::filter_by_shape_id(ttc_gtfs_raw, c("1048833", "1049212"))
ttc_gtfs_tmp <- gtfs2tools::filter_by_trip_id(ttc_gtfs_tmp, c("48337409", "48334845"))

# Convert these specific trips to GPS-like points
gps_tmp <- gtfs2gps::gtfs2gps(ttc_gtfs_tmp)

## Converting shapes to sf objects

## Using 27 CPU cores

## Processing the data

## Some 'speed' values are NA in the returned data.

# Adjust speed estimates between points
gps_tmp <- gtfs2gps::adjust_speed(gps_tmp)

# Save the processed GPS data for section 1.1
readr::write_rds(x = gps_tmp, "data/ttc_intersection_gps.rds")

```

3 1.1: Visualizing Trip Intersections

This section loads the pre-processed TTC intersection data created in the previous step and uses `rayshader` to create a 3D visualization. Time is represented on the vertical axis to illustrate the timing of the two selected intersecting trips.

```

rm(list=ls())
gc(reset = TRUE)

##           used (Mb) gc trigger (Mb) max used (Mb)
## Ncells 3831614 204.7   6356562 339.5  3831614 204.7
## Vcells 6060396  46.3   59747135 455.9  6060396  46.3

# Load the RDS file saved at the end of script 0.0
gps_dt <- readr::read_rds("data/ttc_intersection_gps.rds")

```

3.1 Adjust data for visualization & deal with midnight trips

```

# Prepare tmp_gps: calculate time differences, add segment end coordinates
tmp_gps <- data.table::copy(gps_dt) %>%
  .[!is.na(timestamp),] %>%
  .[,.SD[1],by = .(shape_pt_lon,shape_pt_lat)] %>% # Keep unique points
  .[,time := as.numeric(timestamp)] %>%
  .[,time1 := data.table::shift(time,1,NA,"lead"),by = trip_number] %>%
  .[,diff := time1 - time]

```

```

tmp_gps <- data.table::copy(tmp_gps) %>%
  .[!is.na(timestamp),] %>%
  .[,.SD[1],by = .(shape_pt_lon,shape_pt_lat)] %>%
  .[,time := as.numeric(timestamp)] %>%
  .[,shape_pt_lon_end := data.table::shift(shape_pt_lon,-1,NA), by = shape_id] %>%
  .[,shape_pt_lat_end := data.table::shift(shape_pt_lat,-1,NA), by = shape_id]

# Prepare tmp_stops: filter for valid stop records, create altitude based on time, rename coordinates
tmp_stops <- data.table::copy(tmp_gps) %>%
  .[,.SD[1],by = .(shape_pt_lon,shape_pt_lat)] %>%
  .[!is.na(cumtime) & !is.na(stop_id) & !is.na(timestamp),] %>%
  .[,time := as.numeric(timestamp)] %>%
  .[,altitude := 100 * time/max(time)] %>% # Simple scaling of time to altitude
  data.table::setnames(.,old = c("shape_pt_lon","shape_pt_lat")
    , new = c("X","Y"))

```

3.2 Create Spatial Objects & Bounding Box

```

# Create sf object for stops
# Transforming to UTM Zone 17N (EPSG:32617) for Toronto area
view_tmp_stops <- data.table::copy(tmp_stops) %>%
  sfheaders::sf_multipoint(.,x = "X"
    , y = "Y"
    , multipoint_id = "shape_id") %>%
  sf::st_set_crs(4326) %>% # WGS 1984
  sf::st_transform(32617) # UTM zone 17N for Toronto

# Convert stop points to sf LINESTRING object
tmp_line <- data.table::copy(tmp_gps) %>%
  .[,.SD[1],by = .(shape_pt_lon,shape_pt_lat)] %>%
  .[!is.na(cumtime) & !is.na(stop_id) & !is.na(timestamp),] %>%
  sfheaders::sf_linestring(obj = .
    , x = "shape_pt_lon"
    , y = "shape_pt_lat"
    , linestring_id = "shape_id"
    , keep = TRUE) %>%
  sf::st_set_crs(4326)

# Create a bounding box polygon around the lines
# Buffer is applied in UTM coordinates (meters) then transformed back to WGS84
tmp_gps_bbox <- tmp_line %>%
  sf::st_transform(4326) %>%
  sf::st_transform(32617) %>% # Use Toronto's UTM Zone 17N
  sf::st_buffer(x = .,dist = 8000) %>% # 8km buffer
  sf::st_transform(4326) %>%
  sf::st_bbox() %>%
  as.numeric() %>%
  data.frame("X" = c(.[1],.[1],.[3],.[3])
    , "Y" = c(.[2],.[4],.[4],.[2])) %>%
  sfheaders::sf_polygon(.,x = "X",y = "Y") %>%
  sf::st_set_crs(4326)

```


3.3 Download base map tile

```
# Extract bounding box coordinates for ggmap/get_stadiamap
osm_bbox = tmp_gps_bbox %>%
  raster::extent() %>%
  as.vector() %>%
  .[c(1,3,2,4)]

# Stamen map tiles previously used are now hosted by Stadia Maps.
# An API key is required.
# Visit [https://client.stadiamaps.com/signup/] (https://client.stadiamaps.com/signup/) to obtain an API
# Register api key
# ggmap::register_stadiamaps(key = "api key", write = TRUE)

# Check ggmap's file drawer for cached tiles
ggmap::file_drawer()
```

```
## [1] "C:\\Users\\julia\\AppData\\Local\\Temp\\Rtmpmos3M2\\ggmap"
```

```
dir(ggmap::file_drawer())
```

```
## character(0)
```

```
# Get the terrain basemap from Stadia Maps
base_map <- ggmap::get_stadiamap(bbox = c(left = osm_bbox[1],
                                           bottom = osm_bbox[2],
                                           right = osm_bbox[3],
                                           top = osm_bbox[4]),
                                maptype = "stamen_terrain",
                                crop = TRUE,
                                zoom = 12)
```

```
## i © Stadia Maps © Stamen Design © OpenMapTiles © OpenStreetMap contributors.
```

3.4 Prepare basemap for ray shader

```
# Create a transparent version of the base map
my_plot_trans <- matrix(adjustcolor(base_map,
                                     alpha.f = 0.01), # Mostly transparent
                        nrow = nrow(base_map))
attributes(my_plot_trans) <- attributes(base_map)

# Create ggmap objects for rayshader input
point_plot <- ggmap(base_map) +
  labs(x = NULL, y = NULL) +
  scale_x_continuous(expand = c(0, 0)) +
  scale_y_continuous(expand = c(0, 0)) +
  theme(axis.ticks.x = element_blank(),
```

```
axis.text.x = element_blank(),
axis.ticks.y = element_blank(),
axis.text.y = element_blank()+
theme(plot.margin=unit(-c(1,1,1,1), "mm"))
```

```
## Scale for x is already present.
## Adding another scale for x, which will replace the existing scale.
## Scale for y is already present.
## Adding another scale for y, which will replace the existing scale.
```

```
together_plot <- ggmap(my_plot_trans)+
  labs(x = NULL,y = NULL) +
  scale_x_continuous(expand = c(0, 0)) +
  scale_y_continuous(expand = c(0, 0)) +
  theme(axis.ticks.x = element_blank(),
        axis.text.x = element_blank(),
        axis.ticks.y = element_blank(),
        axis.text.y = element_blank()+
  theme(plot.margin=unit(-c(1,1,1,1), "mm"))
```

```
## Scale for x is already present.
## Adding another scale for x, which will replace the existing scale.
## Scale for y is already present.
## Adding another scale for y, which will replace the existing scale.
```

```
# Combine plots into a list
list_plot <- list(point_plot,together_plot)
```

3.5 Render 3D scene with Rayshader

Render basemap

```
# Clear the rgl device window
rgl::clear3d()

# Camera angles (fov, zoom, theta, phi) adjusted for specific bus routes
plot_gg(list_plot, height = nrow(base_map)/200
, width = ncol(base_map)/200, scale = 100
, raytrace = FALSE, windowsize = c(1200, 1200)
, fov = 160.50793457 , zoom = 0.06098146 , theta = -30.33646027, phi = 10.38603214
, max_error = 0.01, verbose = TRUE)
```

```
## 100.0% reduction: Number of triangles reduced from 4741296 to 2. Error: 0.007843
```

```
# Find angles interactively in the pop-up RGL window then run:
# rayshader::render_camera(theta = NULL,phi = NULL,zoom = NULL,fov = NULL)
```

Add paths for the two intersecting trips over time

```

# Define scaling factor for altitude
scale_altitude <- 5

# Calculate scaled altitude
tmp_gps[, new_scale_altitude := ( time - min(time) ) * scale_altitude]

# Define colors for paths
scale_color_shape_id <- viridis::viridis(n = 3)

# Get the specific TTC shape IDs being visualized
unique_shape_id <- unique(tmp_stops$shape_id)

# Loop through shapes to render paths
for(i in seq_along(unique_shape_id)){

  current_shape_id <- unique_shape_id[i]
  unique_trip_id <- unique(tmp_gps[shape_id == current_shape_id]$trip_number)

  for(j in unique_trip_id){

    trip_data <- tmp_gps[trip_number == j & shape_id == current_shape_id,]

    rayshader::render_path(extent = raster::extent(tmp_gps_bbox)
                          , lat = trip_data$shape_pt_lat
                          , long = trip_data$shape_pt_lon
                          , altitude = trip_data$new_scale_altitude
                          , zscale = 100
                          , linewidth = 5
                          , clear_previous = F
                          , color = scale_color_shape_id[i]
    )

  }
}

```

Add points for stops

```

# update id & shape_id values
tmp_stops_id <- data.table::copy(tmp_stops)[shape_id %in% unique(tmp_gps$shape_id)]
tmp_stops_id[, new_scale_altitude := ( time - min(tmp_gps$time) ) * scale_altitude]
tmp_stops_id[, N := .N, by = stop_id]

# Render regular stops (black points)
rayshader::render_points(extent = raster::extent(tmp_gps_bbox),
                        lat = tmp_stops_id[N==1]$Y, long = tmp_stops_id[N==1]$X,
                        altitude = tmp_stops_id[N==1]$new_scale_altitude,
                        size = 5, zscale = 100,
                        clear_previous = TRUE, color = "black") # Clear previous points first

# Render intersection stops (red points)
rayshader::render_points(extent = raster::extent(tmp_gps_bbox),
                        lat = tmp_stops_id[N>1]$Y, long = tmp_stops_id[N>1]$X,
                        altitude = tmp_stops_id[N>1]$new_scale_altitude,
                        size = 5, zscale = 100,

```

```

        clear_previous = FALSE, color = "red") # Add intersection points

# Render trip start point (large red points)
rayshader::render_points(extent = raster::extent(tmp_gps_bbox)
, lat = tmp_stops_id[id == 1 & shape_id == "1049212"]$Y # Adapted TTC Shape ID
, long = tmp_stops_id[id == 1 & shape_id == "1049212"]$X
, altitude = tmp_stops_id[id==1 & shape_id == "1049212"]$new_scale_altitude
, size = 10, zscale = 100
, clear_previous = FALSE, color = "red")

# Start point for the other shape
rayshader::render_points(extent = raster::extent(tmp_gps_bbox)
, lat = tmp_stops_id[id == 2 & shape_id == "1048833"]$Y # Adapted TTC Shape ID
, long = tmp_stops_id[id == 2 & shape_id == "1048833"]$X
, altitude = tmp_stops_id[id==2 & shape_id == "1048833"]$new_scale_altitude
, size = 10, zscale = 100
, clear_previous = FALSE, color = "red")

```

Add vertical lines at stops

```

# Calculate the minimum time from tmp_stops to use as base for altitude scaling
min_time_for_plot <- min(tmp_stops$time, na.rm = TRUE)

# Prepare stop data for drawing lines, scaling altitude relative to min_time_for_plot
tmp_stops_for_lines <- data.table::copy(tmp_stops) %>%
  .[, new_scale_altitude := (time - min_time_for_plot) * scale_altitude] %>%
  .[!is.na(new_scale_altitude) & new_scale_altitude >= 1]

# Loop through stops and render each vertical line using render_path
for (i in 1:nrow(tmp_stops_for_lines)) {
  stop_lat <- tmp_stops_for_lines$Y[i]
  stop_lon <- tmp_stops_for_lines$X[i]
  stop_alt <- tmp_stops_for_lines$new_scale_altitude[i]
  base_alt <- 1

  # Define the 2-point path for the vertical line
  line_lats <- c(stop_lat, stop_lat)
  line_lons <- c(stop_lon, stop_lon)
  line_alts <- c(base_alt, round(stop_alt))

  rayshader::render_path(
    extent = raster::extent(tmp_gps_bbox),
    lat = line_lats,
    long = line_lons,
    altitude = line_alts,
    zscale = 100,
    linewidth = 1,
    clear_previous = FALSE,
    color = "black")
}

```

Add paths at ground level

```

for(i in seq_along(unique_shape_id)){
  current_shape_id <- unique_shape_id[i]
  unique_trip_id <- unique(tmp_gps[shape_id == current_shape_id]$trip_number)

  for(j in unique_trip_id){
    trip_data <- tmp_gps[trip_number == j & shape_id == current_shape_id,]

    rayshader::render_path(extent = raster::extent(tmp_gps_bbox)
                          , lat = trip_data$shape_pt_lat
                          , long = trip_data$shape_pt_lon
                          , altitude = 150
                          , zscale = 100
                          , linewidth = 1
                          , clear_previous = F
                          , color = "black"
    )
  }
}

```

Prepare data for time labels

```

# Format timestamps into HH:MM text
tmp_stops1 <- data.table::copy(tmp_stops_id) %>%
  .[,timestamp := timestamp] %>%
  .[,text_hour := data.table::hour(as.ITime(timestamp))] %>%
  .[,text_min := data.table::minute(as.ITime(timestamp))] %>%
  .[,text_min := ifelse(nchar(text_min)==1,paste0("0",text_min),text_min)] %>%
  .[,text_plot := sprintf('%s:%s',text_hour,text_min)]

# get artificial heightmap to add @ render_label function
elev_matrix <- raster::raster(nrows=nrow(base_map), ncols=ncol(base_map))
values(elev_matrix) <- 0
raster::extent(elev_matrix) <- raster::extent(tmp_gps_bbox)

```

Add labels for start & end times

```

# Label for start of trip/shape 1 (1049212)
rayshader::render_label(heightmap = elev_matrix
  , lat = tmp_gps[shape_id == "1049212" & id == 1,]$shape_pt_lat+0.04
  , long = tmp_gps[shape_id == "1049212" & id == 1,]$shape_pt_lon-0.1
  , altitude = tmp_gps[shape_id == "1049212" & id == 1,]$new_scale_altitude + 3300
  , zscale = 100
  , textsize = 3
  , adjustvec = c(0,-0.5)
  , alpha = 0
  , extent = raster::extent(elev_matrix)
  , fonttype = "standard"
  , text = tmp_stops1[shape_id == "1049212"& id == 1,]$text_plot
  , clear_previous = T)

# Label for start of trip/shape 2 (1048833)
rayshader::render_label(heightmap = elev_matrix

```

```

, lat = tmp_gps[shape_id == "1048833" & id == 2,]$shape_pt_lat-0.0155
, long = tmp_gps[shape_id == "1048833" & id == 2,]$shape_pt_lon+0.015
, altitude = tmp_gps[shape_id == "1048833"& id == 2,]$new_scale_altitude -480
, zscale = 100
, textsize = 3
, linewidth = 0
, alpha = 0
, adjustvec = c(-.5,-2.5)
, extent = raster::extent(elev_matrix)
, fonttype = "standard"
, text = tmp_stops1[shape_id == "1048833"& id == 2,]$text_plot
, clear_previous = F)
# Label for end of trip/shape 1 (1049212)
rayshader::render_label(heightmap = elev_matrix
, lat = tmp_gps[shape_id == "1049212",.SD[.N]]$shape_pt_lat+0
, long = tmp_gps[shape_id == "1049212",.SD[.N]]$shape_pt_lon-0.01
, altitude = tmp_gps[shape_id == "1049212",.SD[.N]]$new_scale_altitude -6500
, zscale = 100
, textsize = 3
, linewidth = 0
, alpha = 0
, adjustvec = c(-0.25,-2)
, extent = raster::extent(elev_matrix)
, fonttype = "standard"
, text = tmp_stops1[shape_id == "1049212",.SD[.N]]$text_plot
, clear_previous = F)
# Label for end of trip/shape 2 (1048833)
rayshader::render_label(heightmap = elev_matrix
, lat = tmp_gps[shape_id == "1048833",.SD[.N]]$shape_pt_lat
, long = tmp_gps[shape_id == "1048833",.SD[.N]]$shape_pt_lon
, altitude = tmp_gps[shape_id == "1048833",.SD[.N]]$new_scale_altitude -4750
, zscale = 100
, textsize = 3
, linewidth = 0
, alpha = 0
, adjustvec = c(+1.25,-2)
, extent = raster::extent(elev_matrix)
, fonttype = "standard"
, text = tmp_stops1[shape_id == "1048833",.SD[.N]]$text_plot
, clear_previous = F)

```

3.6 Save 3D visualization

```

dir.create("figures", showWarnings = FALSE)

# Save the current rgl scene as a PNG file
rayshader::render_snapshot(filename = "figures/intersection_ttc_final.png"
, width = 500
, height = 500)

```

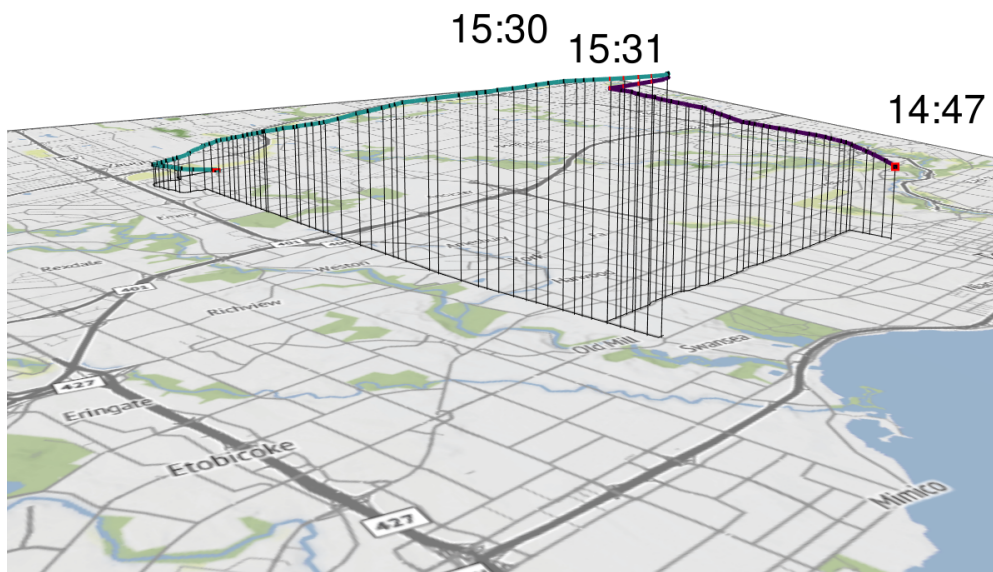


Figure 1: Trip intersection of 26 Dupont West and 35 Jane South.

4 1.2.1: Visualizing Multiple Trips Within a Time Window (Monday)

This section visualizes trips for a selected bus route (`shape_id 1048831`) that occur during a specific time window (Monday, 5 AM to 10 AM). It uses `rayshader` to plot these trips concurrently, with time represented in the z-axis as altitude.

4.1 Load Filtered TTC GTFS and Convert to GPS Format

```
rm(list=ls())
gc(reset = TRUE)
```

```
##          used  (Mb) gc trigger  (Mb) max used  (Mb)
## Ncells  4202563 224.5   6356562 339.5  4202563 224.5
## Vcells 13067309  99.7   47886732 365.4 13067309  99.7
```

```
# Read the GTFS file containing shape_id 1048831, saved in the first section
ttc_gtfs <- gtfstools::read_gtfs("data/gtfs_ttc_1048831.zip")
```

```

# Filter for Monday service
tmp_gtfs <- gtfstools::filter_by_weekday( ttc_gtfs,"monday")

# Convert the Monday GTFS data to GPS-like points
ttc_gps <- progressr::with_progress(
  gtfs2gps::gtfs2gps(tmp_gtfs)
)

```

```
## Converting shapes to sf objects
```

```
## Using 27 CPU cores
```

```
## Processing the data
```

```
## Some 'speed' values are NA in the returned data.
```

4.2 Filter GPS data by time window

```

# Define the start and end times for filtering
time_start = "05:00:00"
time_end = "10:00:00"

# Filter trips that start and end completely within the defined window
gps_dt <- rbind(ttc_gps[,day := "monday"]) %>%
  .[!is.na(timestamp)] %>%
  .[,timestamp := data.table::as.ITime(timestamp)] %>%
  .[,timestart := timestamp[1],by = .(trip_number,day)] %>%
  .[,timeend := timestamp[.N],by = .(trip_number,day)] %>%
  .[timestart >= as.ITime(time_start) & timeend <= as.ITime(time_end),] %>%
  .[timeend > timestart,]

# Check number of trips included (optional output)
gps_dt[,.N,by=day]

```

```

##      day      N
##    <char> <int>
## 1: monday  2700

```

4.3 Prepare data for visualization

```

# Check GPS connections
tmp_gps <- data.table::copy(gps_dt) %>%
  .[,time := as.numeric(timestamp)] %>%
  .[,shape_pt_lon_end := data.table::shift(shape_pt_lon,-1,NA), by = shape_id] %>%
  .[,shape_pt_lat_end := data.table::shift(shape_pt_lat,-1,NA), by = shape_id]

# create stops

```



```
tmp_stops <- data.table::copy(tmp_gps) %>%
  .[!is.na(cumtime) & !is.na(stop_id),] %>%
  .[,time := as.numeric(timestamp)] %>%
  .[,altitude := 100 * time/max(time)] %>% # Simple time scaling
  data.table::setnames(.,old = c("shape_pt_lon","shape_pt_lat")
    , new = c("X","Y"))
```

4.4 Create spatial objects and bounding box

```
# Create sf object for stops
view_tmp_stops <- data.table::copy(tmp_stops) %>%
  sfheaders::sf_multipoint(.,x = "X"
    , y = "Y"
    , multipoint_id = "shape_id") %>%
  sf::st_set_crs(4326) %>%
  sf::st_transform(32617) # Toronto UTM

# Convert stop points to sf LINESTRING object
tmp_line <- data.table::copy(tmp_gps) %>%
  .[!is.na(cumtime) & !is.na(stop_id) & !is.na(timestamp),] %>%
  sfheaders::sf_linestring(obj = .
    , x = "shape_pt_lon"
    , y = "shape_pt_lat"
    , linestring_id = "shape_id"
    , keep = TRUE) %>%
  sf::st_set_crs(4326)

# Create a bounding box polygon around the lines
tmp_gps_bbox <- tmp_line %>%
  sf::st_transform(4326) %>%
  sf::st_transform(32617) %>% # Toronto UTM
  sf::st_buffer(x = .,dist = 8000) %>%
  sf::st_transform(4326) %>%
  sf::st_bbox() %>%
  as.numeric() %>%
  data.frame("X" = c(.[1],.[1],.[3],.[3])
    ,"Y" = c(.[2],.[4],.[4],.[2])) %>%
  sfheaders::sf_polygon(.,x = "X",y = "Y") %>%
  sf::st_set_crs(4326)
```

4.5 Download basemap tile

```
# Extract bounding box coordinates for get_stadiamap
osm_bbox = tmp_gps_bbox %>%
  raster::extent() %>%
  as.vector() %>%
  .[c(1,3,2,4)]

# ggmap::register_stadiamaps(key = "api key", write = TRUE)
```

```
# Check ggmap's file drawer for cached tiles
ggmap::file_drawer()
```

```
## [1] "C:\\Users\\julia\\AppData\\Local\\Temp\\Rtmpmos3M2\\ggmap"
```

```
dir(ggmap::file_drawer())
```

```
## [1] "0f1bbb1a9e38b1cb0764b86cb971e5db.rds"
## [2] "3e6fa2fe72dd75085c82a287f29f0074.rds"
## [3] "42df73f1d408723a83b8e8229ee68172.rds"
## [4] "434b04134c1fd40674f45dfcac6baaa7.rds"
## [5] "55a66644be6289f4fa897e887ebd59d2.rds"
## [6] "5bb3e9f1a318dd90bd414f06aeffa7f4.rds"
## [7] "5fd63ced95fb314f5bd4bee4c1483270.rds"
## [8] "6478b303648e5153aa8e843ed569e5f3.rds"
## [9] "654615f52d0783eb38302f59bf00657a.rds"
## [10] "6e92ab24231d544d91c04705b4ada268.rds"
## [11] "716f9de1009c6b669541d73c0f5ba2e6.rds"
## [12] "75813349fb5b55a922dbde0500d30889.rds"
## [13] "78fb276ff60a9d9172a8d7ed5f9e6967.rds"
## [14] "8dc5ea052d24784e1a11e095d71a9149.rds"
## [15] "9e4f5feaf64ea75504fb78bc433eb530.rds"
## [16] "a250ea0871608499a582e8845b372f95.rds"
## [17] "a51c85dee8aade0d416bf24e584eb03f.rds"
## [18] "a6d7c73bad85229f4721c0d9bbc496de.rds"
## [19] "b5559d0e93d0ebcf904e0bf868b5f2cf.rds"
## [20] "bdf52df1ab4d6f54aa3adc3ef119e90c.rds"
## [21] "c3253d84b46bc6abee92db17054a520e.rds"
## [22] "c3de65e9f091b944c26b896de0877ab7.rds"
## [23] "c4683977c5a178859e7d6397e0001aa6.rds"
## [24] "dfc40465b4778646ac0a9d0cc050f08c.rds"
## [25] "e0186e779c357cd2d88bdb9b4dbb1c23.rds"
## [26] "e332cdbaeeb03eeb89205191ae488aff.rds"
## [27] "f0cb0720f583382d98d1a3bb718fce14.rds"
## [28] "f1da8b8396b5f11a4c45490e74885b5e.rds"
## [29] "f7be8b1404fa778a8eeb54724ef47183.rds"
## [30] "fd3181ffc3a2baf7ecb120ba3350dea8.rds"
## [31] "index.rds"
```

```
# Get the terrain basemap from Stadia Maps
base_map <- ggmap::get_stadiamap(bbox = c(left = osm_bbox[1],
                                           bottom = osm_bbox[2],
                                           right = osm_bbox[3],
                                           top = osm_bbox[4]),
                                maptype = "stamen_terrain",
                                crop = TRUE,
                                zoom = 12)
```

```
## i © Stadia Maps © Stamen Design © OpenMapTiles © OpenStreetMap contributors.
```

4.6 Prepare basemap for rayshader

```
# Create transparent version of base map
my_plot_trans <- matrix(adjustcolor(base_map,
                                   alpha.f = 0.01),
                       nrow = nrow(base_map))
attributes(my_plot_trans) <- attributes(base_map)

# Create ggmap objects
point_plot <- ggmap(base_map) +
  theme_nothing() +
  labs(x = NULL, y = NULL) +
  scale_x_continuous(expand = c(0, 0)) +
  scale_y_continuous(expand = c(0, 0)) +
  theme(axis.ticks.x = element_blank(), axis.text.x = element_blank(),
        axis.ticks.y = element_blank(), axis.text.y = element_blank()) +
  theme(plot.margin=unit(-c(1,1,1,1), "mm"))

## Scale for x is already present.
## Adding another scale for x, which will replace the existing scale.
## Scale for y is already present.
## Adding another scale for y, which will replace the existing scale.

together_plot <- ggmap(my_plot_trans) +
  theme_nothing() +
  labs(x = NULL, y = NULL) +
  scale_x_continuous(expand = c(0, 0)) +
  scale_y_continuous(expand = c(0, 0)) +
  theme(axis.ticks.x = element_blank(), axis.text.x = element_blank(),
        axis.ticks.y = element_blank(), axis.text.y = element_blank()) +
  theme(plot.margin=unit(-c(1,1,1,1), "mm"))

## Scale for x is already present.
## Adding another scale for x, which will replace the existing scale.
## Scale for y is already present.
## Adding another scale for y, which will replace the existing scale.

# Combine plots into a list
list_plot <- list(point_plot, together_plot)
```

4.7 Render 3D scene with rayshader

Render basemap

```
rgl::clear3d()

plot_gg(list_plot, height = nrow(base_map)/200
        , width = ncol(base_map)/200, scale = 100
        , raytrace = FALSE, windowsize = c(1200, 1200)
        , fov = 155.0611572, zoom = 0.1039339
        , theta = -1.1449540, phi = 12.2367451
        , max_error = 0.001, verbose = TRUE)
```

89.6% reduction: Number of triangles reduced from 2586906 to 268689. Error: 0.000000

Add trip paths

```
# Define scaling factor for altitude
scale_altitude <- 5
tmp_gps1 <- data.table::copy(tmp_gps)

# Calculate scaled altitude
tmp_gps1[, new_scale_altitude := ( time - min(time)) * scale_altitude]

# Define colors
scale_color_shape_id <- viridis::viridis(n = 3)
unique_shape_id <- unique(tmp_stops$shape_id)

# Loop through the shape's filtered trips
for(i in seq_along(unique_shape_id)){
  current_shape_id <- unique_shape_id[i]
  unique_trip_id <- unique(tmp_gps1[shape_id == current_shape_id]$trip_number)

  for(j in unique_trip_id){
    trip_data <- tmp_gps1[trip_number == j & shape_id == current_shape_id,]

    rayshader::render_path(extent = raster::extent(tmp_gps_bbox)
                           , lat = trip_data$shape_pt_lat
                           , long = trip_data$shape_pt_lon
                           , altitude = trip_data$new_scale_altitude
                           , zscale = 100, linewidth = 2
                           , clear_previous = F
                           , color = scale_color_shape_id[i]
                           )
  }
}
```

Add ground path

```
for(i in seq_along(unique_shape_id)){
  current_shape_id <- unique_shape_id[i]
  unique_trip_id <- unique(tmp_gps1[shape_id == current_shape_id]$trip_number)

  for(j in unique_trip_id){
    trip_data <- tmp_gps1[trip_number == j & shape_id == current_shape_id,]

    rayshader::render_path(extent = raster::extent(tmp_gps_bbox)
                           , lat = trip_data$shape_pt_lat
                           , long = trip_data$shape_pt_lon
                           , altitude = 150
                           , zscale = 100
                           , linewidth = 1
                           , clear_previous = F
                           , color = "black"
                           )
  }
}
```

```
}
}
```

Add stop points

```
tmp_stops_id <- data.table::copy(tmp_stops) %>%
  .[shape_id %in% unique(tmp_gps1$shape_id)] %>%
  .[, new_scale_altitude := (time - min(tmp_gps1$time)) * scale_altitude]

# Render all stops (red points)
rayshader::render_points(extent = raster::extent(tmp_gps_bbox),
  lat = tmp_stops_id$Y, long = tmp_stops_id$X,
  altitude = tmp_stops_id$new_scale_altitude,
  size = 2.5, zscale = 100,
  clear_previous = TRUE, color = "red")
```

Add vertical lines at stops

```
# Calculate min time in the window
min_time_in_window <- min(tmp_stops$time, na.rm = TRUE)

# Prepare stop data with altitude scaled relative to min time
tmp_stops_for_lines <- data.table::copy(tmp_stops) %>%
  .[, new_scale_altitude := (time - min_time_in_window) * scale_altitude]

# Select only the stops from the last trip in the time window (the one at the top of the visualization)
unique_trip_numbers_in_window <- unique(tmp_stops_for_lines$trip_number)
last_trip_number <- max(unique_trip_numbers_in_window, na.rm = TRUE)
last_trip_stops <- tmp_stops_for_lines[trip_number == last_trip_number, ]

# Loop through each stop and draw vertical lines using render_path
for (i in 1:nrow(last_trip_stops)) {
  stop_lat <- last_trip_stops$Y[i]
  stop_lon <- last_trip_stops$X[i]
  stop_alt <- last_trip_stops$new_scale_altitude[i]
  base_alt <- 1

  line_lats <- c(stop_lat, stop_lat)
  line_lons <- c(stop_lon, stop_lon)
  line_alts <- c(base_alt, round(stop_alt))

  rayshader::render_path(
    extent = raster::extent(tmp_gps_bbox),
    lat = line_lats, long = line_lons, altitude = line_alts,
    zscale = 100, linewidth = 1,
    clear_previous = FALSE, color = "black")
}
```

Add labels for start and end times of each trip

```
# Prepare data with HH:MM text labels and scaled altitude
tmp_stops1 <- data.table::copy(tmp_stops) %>%
```

```

[, new_scale_altitude := ( time - min(tmp_gps1$time) ) * scale_altitude] %>% # Use same min time
[,text_hour := as.ITime(timestamp) %>% data.table::hour()] %>%
[,text_min := as.ITime(timestamp) %>% data.table::minute()] %>%
[,text_min := ifelse(nchar(text_min)==1,paste0("0",text_min),text_min)] %>%
[,text_plot := sprintf('%s:%s',text_hour,text_min)]

elev_matrix <- raster::raster(nrows=nrow(base_map), ncols=ncol(base_map))
values(elev_matrix) <- 0
raster::extent(elev_matrix) <- raster::extent(tmp_gps_bbox)

# Prepare data for start labels (first stop of each trip)
label_data_start <- tmp_stops1[, .SD[1], by = trip_number]

# Loop to render start labels
for (i in 1:nrow(label_data_start)) {
  rayshader::render_label(
    heightmap = elev_matrix,
    lat = label_data_start$Y[i],
    long = label_data_start$X[i],
    altitude = label_data_start$new_scale_altitude[i],
    zscale = 100, textsize = 2.5, alpha = 0,
    adjustvec = c(2.5, 0),
    extent = raster::extent(elev_matrix),
    fonttype = "standard",
    text = label_data_start$text_plot[i],
    clear_previous = ifelse(i == 1, TRUE, FALSE)
  )
}

# Prepare data for end labels (last stop of each trip)
label_data_end <- tmp_stops1[, .SD[.N], by = trip_number]

# Loop to render end labels
for (i in 1:nrow(label_data_end)) {
  rayshader::render_label(
    heightmap = elev_matrix,
    lat = label_data_end$Y[i],
    long = label_data_end$X[i],
    altitude = label_data_end$new_scale_altitude[i],
    zscale = 100, textsize = 2.5, alpha = 0,
    adjustvec = -c(1.5, 0.35),
    extent = raster::extent(elev_matrix),
    fonttype = "standard",
    text = label_data_end$text_plot[i],
    clear_previous = FALSE
  )
}

```

Save 3D visualization

```

# Save the current rgl scene as a PNG file
rayshader::render_snapshot(filename = "figures/26_monday.png",
                           width = 1000,

```

```
height = 2000)
```

26 Dupont Eastbound on a Monday Morning.

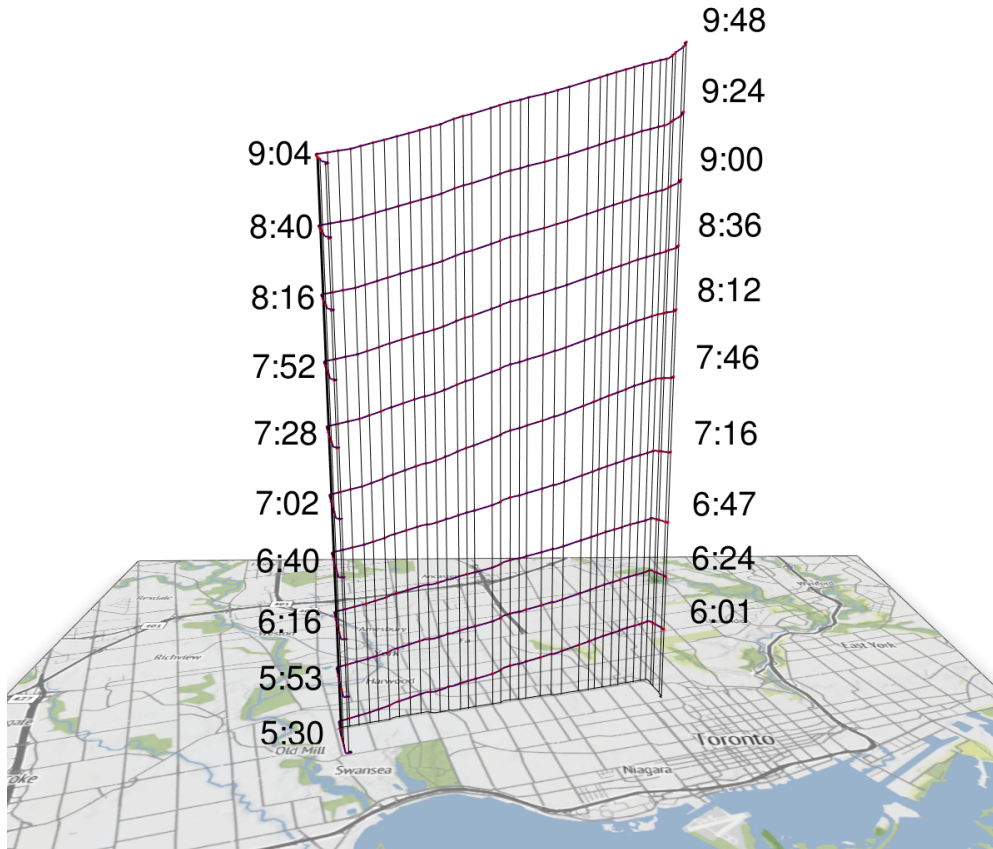


Figure 2: 26 Dupont Eastbound on a Monday Morning..

5 1.2.2: Visualizing Multiple Trips Within a Time Window (Sunday)

This section visualizes trips for a selected bus route (`shape_id` 1048831) that occur during a specific time window (Monday, 5 AM to 10 AM). It uses `rayshader` to plot these trips concurrently, with time represented in the z-axis as altitude.

5.1 Load Filtered TTC GTFS and Convert to GPS Format

```
rm(list=ls())
gc(reset = TRUE)
```

```
##          used (Mb) gc trigger (Mb) max used (Mb)
## Ncells  4224228 225.6   6356562 339.5  4224228 225.6
## Vcells 10007961  76.4   36828211 281.0 10007961  76.4
```

```

# Read the GTFS file containing shape_id 1048831, saved in the first section
ttc_gtfs <- gtfstools::read_gtfs("data/gtfs_ttc_1048831.zip")

# Filter for Monday service
tmp_gtfs <- gtfstools::filter_by_weekday( ttc_gtfs,"sunday")

# Convert the Monday GTFS data to GPS-like points
ttc_gps <- progressr::with_progress(
  gtfs2gps::gtfs2gps(tmp_gtfs)
)

```

```
## Converting shapes to sf objects
```

```
## Using 27 CPU cores
```

```
## Processing the data
```

```
## Some 'speed' values are NA in the returned data.
```

5.2 Filter GPS data by time window

```

# Define the start and end times for filtering
time_start = "05:00:00"
time_end = "10:00:00"

# Filter trips that start and end completely within the defined window
gps_dt <- rbind(ttc_gps[,day := "sunday"]) %>%
  .[!is.na(timestamp)] %>%
  .[,timestamp := data.table::as.ITime(timestamp)] %>%
  .[,timestart := timestamp[1],by = .(trip_number,day)] %>%
  .[,timeend := timestamp[.N],by = .(trip_number,day)] %>%
  .[timestart >= as.ITime(time_start) & timeend <= as.ITime(time_end),] %>%
  .[timeend > timestart,]

# Check number of trips included (optional output)
gps_dt[,.N,by=day]

```

```

##      day      N
##   <char> <int>
## 1:  sunday 1620

```

5.3 Prepare data for visualization

```

# Check GPS connections
tmp_gps <- data.table::copy(gps_dt) %>%
  .[,time := as.numeric(timestamp)] %>%
  .[,shape_pt_lon_end := data.table::shift(shape_pt_lon,-1,NA), by = shape_id] %>%
  .[,shape_pt_lat_end := data.table::shift(shape_pt_lat,-1,NA), by = shape_id]

```



```

# create stops
tmp_stops <- data.table::copy(tmp_gps) %>%
  .[!is.na(cumtime) & !is.na(stop_id),] %>%
  .[,time := as.numeric(timestamp)] %>%
  .[,altitude := 100 * time/max(time)] %>% # Simple time scaling
data.table::setnames(.,old = c("shape_pt_lon","shape_pt_lat")
  , new = c("X","Y"))

```

5.4 Create spatial objects and bounding box

```

# Create sf object for stops
view_tmp_stops <- data.table::copy(tmp_stops) %>%
  sfheaders::sf_multipoint(.,x = "X"
    , y = "Y"
    , multipoint_id = "shape_id") %>%
  sf::st_set_crs(4326) %>%
  sf::st_transform(32617) # Toronto UTM

# Convert stop points to sf LINESTRING object
tmp_line <- data.table::copy(tmp_gps) %>%
  .[!is.na(cumtime) & !is.na(stop_id) & !is.na(timestamp),] %>%
  sfheaders::sf_linestring(obj = .
    , x = "shape_pt_lon"
    , y = "shape_pt_lat"
    , linestring_id = "shape_id"
    , keep = TRUE) %>%
  sf::st_set_crs(4326)

# Create a bounding box polygon around the lines
tmp_gps_bbox <- tmp_line %>%
  sf::st_transform(4326) %>%
  sf::st_transform(32617) %>% # Toronto UTM
  sf::st_buffer(x = .,dist = 8000) %>%
  sf::st_transform(4326) %>%
  sf::st_bbox() %>%
  as.numeric() %>%
  data.frame("X" = c(.[1],.[1],.[3],.[3])
    , "Y" = c(.[2],.[4],.[4],.[2])) %>%
  sfheaders::sf_polygon(.,x = "X",y = "Y") %>%
  sf::st_set_crs(4326)

```

5.5 Download basemap tile

```

# Extract bounding box coordinates for get_stadiamap
osm_bbox = tmp_gps_bbox %>%
  raster::extent() %>%
  as.vector() %>%
  .[c(1,3,2,4)]

```

```

# ggmap::register_stadiamaps(key = "api key", write = TRUE)

# Check ggmap's file drawer for cached tiles
ggmap::file_drawer()

## [1] "C:\\Users\\julia\\AppData\\Local\\Temp\\Rtmpmos3M2\\ggmap"

dir(ggmap::file_drawer())

## [1] "0f1bbb1a9e38b1cb0764b86cb971e5db.rds"
## [2] "3e6fa2fe72dd75085c82a287f29f0074.rds"
## [3] "42df73f1d408723a83b8e8229ee68172.rds"
## [4] "434b04134c1fd40674f45dfcac6baaa7.rds"
## [5] "55a66644be6289f4fa897e887ebd59d2.rds"
## [6] "5bb3e9f1a318dd90bd414f06aeffa7f4.rds"
## [7] "5fd63ced95fb314f5bd4bee4c1483270.rds"
## [8] "6478b303648e5153aa8e843ed569e5f3.rds"
## [9] "654615f52d0783eb38302f59bf00657a.rds"
## [10] "6e92ab24231d544d91c04705b4ada268.rds"
## [11] "716f9de1009c6b669541d73c0f5ba2e6.rds"
## [12] "75813349fb5b55a922dbde0500d30889.rds"
## [13] "78fb276ff60a9d9172a8d7ed5f9e6967.rds"
## [14] "8dc5ea052d24784e1a11e095d71a9149.rds"
## [15] "9e4f5feaf64ea75504fb78bc433eb530.rds"
## [16] "a250ea0871608499a582e8845b372f95.rds"
## [17] "a51c85dee8aade0d416bf24e584eb03f.rds"
## [18] "a6d7c73bad85229f4721c0d9bbc496de.rds"
## [19] "b5559d0e93d0ebcf904e0bf868b5f2cf.rds"
## [20] "bdf52df1ab4d6f54aa3adc3ef119e90c.rds"
## [21] "c3253d84b46bc6abee92db17054a520e.rds"
## [22] "c3de65e9f091b944c26b896de0877ab7.rds"
## [23] "c4683977c5a178859e7d6397e0001aa6.rds"
## [24] "dfc40465b4778646ac0a9d0cc050f08c.rds"
## [25] "e0186e779c357cd2d88bdb9b4dbb1c23.rds"
## [26] "e332cdbaeeb03eeb89205191ae488aff.rds"
## [27] "f0cb0720f583382d98d1a3bb718fce14.rds"
## [28] "f1da8b8396b5f11a4c45490e74885b5e.rds"
## [29] "f7be8b1404fa778a8eeb54724ef47183.rds"
## [30] "fd3181ffc3a2baf7ecb120ba3350dea8.rds"
## [31] "index.rds"

# Get the terrain basemap from Stadia Maps
base_map <- ggmap::get_stadiamap(bbox = c(left = osm_bbox[1],
                                           bottom = osm_bbox[2],
                                           right = osm_bbox[3],
                                           top = osm_bbox[4]),
                                maptype = "stamen_terrain",
                                crop = TRUE,
                                zoom = 12)

```

```
## i © Stadia Maps © Stamen Design © OpenMapTiles © OpenStreetMap contributors.
```

5.6 Prepare basemap for rayshader

```
# Create transparent version of base map
my_plot_trans <- matrix(adjustcolor(base_map,
                                   alpha.f = 0.01),
                        nrow = nrow(base_map))
attributes(my_plot_trans) <- attributes(base_map)

# Create ggmap objects
point_plot <- ggmap(base_map) +
  theme_nothing() +
  labs(x = NULL, y = NULL) +
  scale_x_continuous(expand = c(0, 0)) +
  scale_y_continuous(expand = c(0, 0)) +
  theme(axis.ticks.x = element_blank(), axis.text.x = element_blank(),
        axis.ticks.y = element_blank(), axis.text.y = element_blank()) +
  theme(plot.margin=unit(-c(1,1,1,1), "mm"))

## Scale for x is already present.
## Adding another scale for x, which will replace the existing scale.
## Scale for y is already present.
## Adding another scale for y, which will replace the existing scale.

together_plot <- ggmap(my_plot_trans) +
  theme_nothing() +
  labs(x = NULL, y = NULL) +
  scale_x_continuous(expand = c(0, 0)) +
  scale_y_continuous(expand = c(0, 0)) +
  theme(axis.ticks.x = element_blank(), axis.text.x = element_blank(),
        axis.ticks.y = element_blank(), axis.text.y = element_blank()) +
  theme(plot.margin=unit(-c(1,1,1,1), "mm"))

## Scale for x is already present.
## Adding another scale for x, which will replace the existing scale.
## Scale for y is already present.
## Adding another scale for y, which will replace the existing scale.

# Combine plots into a list
list_plot <- list(point_plot, together_plot)
```

5.7 Render 3D scene with rayshader

Render basemap

```
rgl::clear3d()

plot_gg(list_plot, height = nrow(base_map)/200
        , width = ncol(base_map)/200, scale = 100
        , raytrace = FALSE, windowsize = c(1200, 1200)
        , fov = 155.0611572, zoom = 0.09427112
        , theta = 37.30995527, phi = 14.17542222
        , max_error = 0.001, verbose = TRUE)
```

89.6% reduction: Number of triangles reduced from 2586906 to 268689. Error: 0.000000

Add trip paths

```
# Define scaling factor for altitude
scale_altitude <- 5
tmp_gps1 <- data.table::copy(tmp_gps)

# Calculate scaled altitude
tmp_gps1[, new_scale_altitude := ( time - min(time)) * scale_altitude]

# Define colors
scale_color_shape_id <- viridis::viridis(n = 3)
unique_shape_id <- unique(tmp_stops$shape_id)

# Loop through the shape's filtered trips
for(i in seq_along(unique_shape_id)){
  current_shape_id <- unique_shape_id[i]
  unique_trip_id <- unique(tmp_gps1[shape_id == current_shape_id]$trip_number)

  for(j in unique_trip_id){
    trip_data <- tmp_gps1[trip_number == j & shape_id == current_shape_id,]

    rayshader::render_path(extent = raster::extent(tmp_gps_bbox)
                           , lat = trip_data$shape_pt_lat
                           , long = trip_data$shape_pt_lon
                           , altitude = trip_data$new_scale_altitude
                           , zscale = 100, linewidth = 2
                           , clear_previous = F
                           , color = scale_color_shape_id[i]
                           )
  }
}
```

Add ground path

```
for(i in seq_along(unique_shape_id)){
  current_shape_id <- unique_shape_id[i]
  unique_trip_id <- unique(tmp_gps1[shape_id == current_shape_id]$trip_number)

  for(j in unique_trip_id){
    trip_data <- tmp_gps1[trip_number == j & shape_id == current_shape_id,]

    rayshader::render_path(extent = raster::extent(tmp_gps_bbox)
                           , lat = trip_data$shape_pt_lat
                           , long = trip_data$shape_pt_lon
                           , altitude = 150
                           , zscale = 100
                           , linewidth = 1
                           , clear_previous = F
                           , color = "black"
                           )
  }
}
```

```
}
}
```

Add stop points

```
tmp_stops_id <- data.table::copy(tmp_stops) %>%
  .[shape_id %in% unique(tmp_gps1$shape_id)] %>%
  .[, new_scale_altitude := (time - min(tmp_gps1$time)) * scale_altitude]

# Render all stops (red points)
rayshader::render_points(extent = raster::extent(tmp_gps_bbox),
  lat = tmp_stops_id$Y, long = tmp_stops_id$X,
  altitude = tmp_stops_id$new_scale_altitude,
  size = 2.5, zscale = 100,
  clear_previous = TRUE, color = "red")
```

Add vertical lines at stops

```
# Calculate min time in the window
min_time_in_window <- min(tmp_stops$time, na.rm = TRUE)

# Prepare stop data with altitude scaled relative to min time
tmp_stops_for_lines <- data.table::copy(tmp_stops) %>%
  .[, new_scale_altitude := (time - min_time_in_window) * scale_altitude]

# Select only the stops from the last trip in the time window (the one at the top of the visualization)
unique_trip_numbers_in_window <- unique(tmp_stops_for_lines$trip_number)
last_trip_number <- max(unique_trip_numbers_in_window, na.rm = TRUE)
last_trip_stops <- tmp_stops_for_lines[trip_number == last_trip_number, ]

# Loop through each stop and draw vertical lines using render_path
for (i in 1:nrow(last_trip_stops)) {
  stop_lat <- last_trip_stops$Y[i]
  stop_lon <- last_trip_stops$X[i]
  stop_alt <- last_trip_stops$new_scale_altitude[i]
  base_alt <- 1

  line_lats <- c(stop_lat, stop_lat)
  line_lons <- c(stop_lon, stop_lon)
  line_alts <- c(base_alt, round(stop_alt))

  rayshader::render_path(
    extent = raster::extent(tmp_gps_bbox),
    lat = line_lats, long = line_lons, altitude = line_alts,
    zscale = 100, linewidth = 1,
    clear_previous = FALSE, color = "black")
}
```

Add labels for start and end times of each trip

```
# Prepare data with HH:MM text labels and scaled altitude
tmp_stops1 <- data.table::copy(tmp_stops) %>%
```

```

[, new_scale_altitude := ( time - min(tmp_gps1$time) ) * scale_altitude] %>% # Use same min time
[,text_hour := as.ITime(timestamp) %>% data.table::hour()] %>%
[,text_min := as.ITime(timestamp) %>% data.table::minute()] %>%
[,text_min := ifelse(nchar(text_min)==1,paste0("0",text_min),text_min)] %>%
[,text_plot := sprintf('%s:%s',text_hour,text_min)]

elev_matrix <- raster::raster(nrows=nrow(base_map), ncols=ncol(base_map))
values(elev_matrix) <- 0
raster::extent(elev_matrix) <- raster::extent(tmp_gps_bbox)

# Prepare data for start labels (first stop of each trip)
label_data_start <- tmp_stops1[, .SD[1], by = trip_number]

# Loop to render start labels
for (i in 1:nrow(label_data_start)) {
  rayshader::render_label(
    heightmap = elev_matrix,
    lat = label_data_start$Y[i],
    long = label_data_start$X[i],
    altitude = label_data_start$new_scale_altitude[i],
    zscale = 100, textsize = 2.5, alpha = 0,
    adjustvec = c(2.5, 0),
    extent = raster::extent(elev_matrix),
    fonttype = "standard",
    text = label_data_start$text_plot[i],
    clear_previous = ifelse(i == 1, TRUE, FALSE)
  )
}

# Prepare data for end labels (last stop of each trip)
label_data_end <- tmp_stops1[, .SD[.N], by = trip_number]

# Loop to render end labels
for (i in 1:nrow(label_data_end)) {
  rayshader::render_label(
    heightmap = elev_matrix,
    lat = label_data_end$Y[i],
    long = label_data_end$X[i],
    altitude = label_data_end$new_scale_altitude[i],
    zscale = 100, textsize = 2.5, alpha = 0,
    adjustvec = -c(1.5, 0.35),
    extent = raster::extent(elev_matrix),
    fonttype = "standard",
    text = label_data_end$text_plot[i],
    clear_previous = FALSE
  )
}

```

Save 3D visualization

```

# Save the current rgl scene as a PNG file
rayshader::render_snapshot(filename = "figures/26_Sunday.png",
                           width = 1000,

```

```
height = 2000)
```

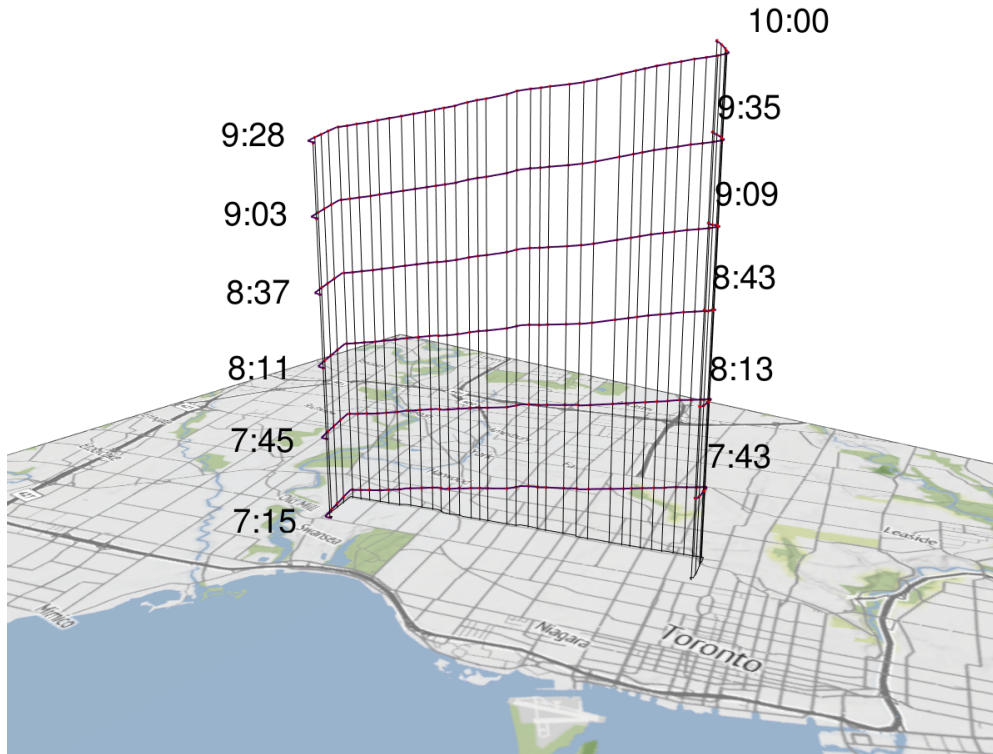


Figure 3: 26 Dupont Eastbound on a Sunday Morning.

6 1.3: Frequency Analysis & Canadian Census Data Integration

This section calculates the frequency of bus arrivals at stops within Toronto, aggregates these frequencies to a hexagonal grid, integrates census data (population & income) into the grid, and graphically visualizes the results.

6.1 Load Toronto Boundary and Raw TTC GTFS

```
rm(list=ls())
gc(reset = TRUE)
```

```
##          used (Mb) gc trigger (Mb) max used (Mb)
## Ncells  4225957 225.7   6356562 339.5  4225957 225.7
## Vcells 10040852  76.7   40915423 312.2 10040852  76.7
```

```

# Load Toronto boundary saved from the first script
toronto_bound <- readr::read_rds("data/toronto_bound_CSD.rds")

# Path to the raw TTC GTFS zip file
ttc_path <- "data-raw/ttc_gtfs.zip"
ttc_gtfs <- gtfstools::read_gtfs(path = ttc_path)

# Filter GTFS for Wednesday
ttc_gtfs <- gtfstools::filter_by_weekday(ttc_gtfs, "wednesday")

```

6.2 Convert GTFS to GPS

```

# Create directories to store intermediate GPS files
dir.create("data/gps/", showWarnings = FALSE) # Might exist from previous runs
dir.create("data/gps/ttc", showWarnings = FALSE)

# Convert GTFS to GPS format, saving each shape/trip to a separate file
# progressr::with_progress(
#   gtfs2gps::gtfs2gps(gtfs_data = ttc_gtfs,
#                       parallel = FALSE,
#                       filepath = "data/gps/ttc/")
# )

```

6.3 Read files and filter stops within Toronto Boundary

```

# List all generated GPS data files
ttc_files <- list.files("data/gps/ttc/", full.names = TRUE)

ttc_stops <- pbapply::pblapply(ttc_files, function(i){
  tmp <- data.table::fread(i, select = c('shape_id', 'trip_id', 'stop_id',
                                         'timestamp', 'dist',
                                         'shape_pt_lat', 'shape_pt_lon'))

  tmp <- tmp[!is.na(stop_id) & dist != 0]
  return(tmp)
}) %>% data.table::rbindlist()

all_stops <- ttc_stops
all_stops[, stop_id := as.character(stop_id)]

# Find bus stops inside Toronto
unique_stops_sf <- sfheaders::sf_multipoint(
  obj = all_stops[, .SD[1], by = .(stop_id)], # Get unique stops by id
  x = "shape_pt_lon", y = "shape_pt_lat",
  multipoint_id = "stop_id", keep = FALSE) %>%
  sf::st_set_crs(4326) # Set CRS to WGS84

# Spatial join to find stops within Toronto
tmp_id <- sf::st_within(x = unique_stops_sf, y = toronto_bound, sparse = FALSE)

```



```

# Get the IDs of stops inside Toronto
unique_stops_inside_toronto <- unique_stops_sf[which(tmp_id),]$stop_id
unique_stops_inside_toronto <- as.character(unique_stops_inside_toronto)

# Filter the main stops data table to keep only stops inside Toronto
all_stops_filtered <- all_stops[stop_id %in% unique_stops_inside_toronto,]

# Save the filtered stops data
readr::write_rds(all_stops_filtered, "data/all_stops_toronto_filtered.rds", compress = "gz")

```

6.4 Load Filtered Stops & Calculate Time Bins

```

rm(list=ls())
gc(reset = TRUE)

```

```

##           used (Mb) gc trigger (Mb) max used (Mb)
## Ncells  4228390 225.9   8438575 450.7  4228390 225.9
## Vcells 10112983  77.2   53275995 406.5 10112983  77.2

```

```

# Load the filtered stops data
all_stops <- readr::read_rds("data/all_stops_toronto_filtered.rds")

# Adjust time
all_stops[, time_to_sec := gtfstools::cpp_time_to_seconds(timestamp)]
all_stops[, minu_time := round(time_to_sec/60, 1)]
all_stops <- all_stops[minu_time < 1440]

# Add time classes
# 60 min intervals
tp2 <- sprintf("%02d", 0:23) # Format hours 00-23
label_60min <- paste0(tp2, ":00")
all_stops[, time_60min := cut(minu_time, breaks = seq(0, 1440, by=60), right = FALSE, labels = label_60min)]

# 30 min intervals
tp2 <- sprintf("%02d", rep(0:23, each=2))
label_30min <- paste0(tp2, c(":00", ":30"))
all_stops[, time_30min := cut(minu_time, breaks = seq(0, 1440, by=30), right = FALSE, labels = label_30min)]

# 15 min intervals
tp2 <- sprintf("%02d", rep(0:23, each=4))
label_15min <- paste0(tp2, c(":00", ":15", ":30", ":45"))
all_stops[, time_15min := cut(minu_time, breaks = seq(0, 1440, by=15), right = FALSE, labels = label_15min)]

# 10 min intervals
tp2 <- sprintf("%02d", rep(0:23, each=6))
label_10min <- paste0(tp2, c(":00", ":10", ":20", ":30", ":40", ":50"))
all_stops[, time_10min := cut(minu_time, breaks = seq(0, 1440, by=10), right = FALSE, labels = label_10min)]

# 5 min intervals

```

```

tp1 <- sprintf("%02d", seq(0,55,by = 5)) # Format minutes 00, 05, ..., 55
tp2 <- sprintf("%02d", rep(0:23, each=12))
label_05min <- paste0(tp2,":",tp1)
all_stops[, time_05min := cut(minu_time, breaks = seq(0,1440, by=5), right = FALSE, labels = label_05min)]

# Clean up temporary label variables
rm(list = c("label_10min","label_15min","label_30min","label_60min", "label_05min","tp1","tp2"))

```

6.5 Calculate Frequency per stop/bin

```

# Calculate frequency (N) for each stop within each time interval bin
all_stops[,N_60min := .N,by = .(stop_id,time_60min)]
all_stops[,N_30min := .N,by = .(stop_id,time_30min)]
all_stops[,N_15min := .N,by = .(stop_id,time_15min)]
all_stops[,N_10min := .N,by = .(stop_id,time_10min)]
all_stops[,N_05min := .N,by = .(stop_id,time_05min)]

all_stops <- data.table::melt.data.table(
  data = all_stops
  ,id.vars = c('stop_id','shape_pt_lat', 'shape_pt_lon')
  ,measure.vars = list(
    "time" = c('time_60min','time_30min','time_15min', 'time_10min', 'time_05min')
    ,"N" = c('N_60min','N_30min','N_15min', 'N_10min', 'N_05min')
  ))

# Add a column indicating the time interval length
all_stops[,time_interval := fcase(
  variable == 1,"60 min",
  variable == 2,"30 min",
  variable == 3,"15 min",
  variable == 4,"10 min",
  variable == 5,"05 min" )]

all_stops[1]

```

```

##      stop_id shape_pt_lat shape_pt_lon variable   time      N time_interval
##      <char>      <num>      <num>    <fctr> <char> <int>      <char>
## 1:    15182     43.77592    -79.34697       1 05:00      3        60 min

```

```
all_stops$time %>% unique() %>% sort()
```

```

##      [1] "00:00" "00:05" "00:10" "00:15" "00:20" "00:25" "00:30" "00:35" "00:40"
##     [10] "00:45" "00:50" "00:55" "01:00" "01:05" "01:10" "01:15" "01:20" "01:25"
##     [19] "01:30" "01:35" "01:40" "01:45" "01:50" "01:55" "02:00" "02:05" "02:10"
##     [28] "02:15" "02:20" "02:25" "02:30" "02:35" "02:40" "02:45" "02:50" "02:55"
##     [37] "03:00" "03:05" "03:10" "03:15" "03:20" "03:25" "03:30" "03:35" "03:40"
##     [46] "03:45" "03:50" "03:55" "04:00" "04:05" "04:10" "04:15" "04:20" "04:25"
##     [55] "04:30" "04:35" "04:40" "04:45" "04:50" "04:55" "05:00" "05:05" "05:10"
##     [64] "05:15" "05:20" "05:25" "05:30" "05:35" "05:40" "05:45" "05:50" "05:55"

```

```
## [73] "06:00" "06:05" "06:10" "06:15" "06:20" "06:25" "06:30" "06:35" "06:40"
## [82] "06:45" "06:50" "06:55" "07:00" "07:05" "07:10" "07:15" "07:20" "07:25"
## [91] "07:30" "07:35" "07:40" "07:45" "07:50" "07:55" "08:00" "08:05" "08:10"
## [100] "08:15" "08:20" "08:25" "08:30" "08:35" "08:40" "08:45" "08:50" "08:55"
## [109] "09:00" "09:05" "09:10" "09:15" "09:20" "09:25" "09:30" "09:35" "09:40"
## [118] "09:45" "09:50" "09:55" "10:00" "10:05" "10:10" "10:15" "10:20" "10:25"
## [127] "10:30" "10:35" "10:40" "10:45" "10:50" "10:55" "11:00" "11:05" "11:10"
## [136] "11:15" "11:20" "11:25" "11:30" "11:35" "11:40" "11:45" "11:50" "11:55"
## [145] "12:00" "12:05" "12:10" "12:15" "12:20" "12:25" "12:30" "12:35" "12:40"
## [154] "12:45" "12:50" "12:55" "13:00" "13:05" "13:10" "13:15" "13:20" "13:25"
## [163] "13:30" "13:35" "13:40" "13:45" "13:50" "13:55" "14:00" "14:05" "14:10"
## [172] "14:15" "14:20" "14:25" "14:30" "14:35" "14:40" "14:45" "14:50" "14:55"
## [181] "15:00" "15:05" "15:10" "15:15" "15:20" "15:25" "15:30" "15:35" "15:40"
## [190] "15:45" "15:50" "15:55" "16:00" "16:05" "16:10" "16:15" "16:20" "16:25"
## [199] "16:30" "16:35" "16:40" "16:45" "16:50" "16:55" "17:00" "17:05" "17:10"
## [208] "17:15" "17:20" "17:25" "17:30" "17:35" "17:40" "17:45" "17:50" "17:55"
## [217] "18:00" "18:05" "18:10" "18:15" "18:20" "18:25" "18:30" "18:35" "18:40"
## [226] "18:45" "18:50" "18:55" "19:00" "19:05" "19:10" "19:15" "19:20" "19:25"
## [235] "19:30" "19:35" "19:40" "19:45" "19:50" "19:55" "20:00" "20:05" "20:10"
## [244] "20:15" "20:20" "20:25" "20:30" "20:35" "20:40" "20:45" "20:50" "20:55"
## [253] "21:00" "21:05" "21:10" "21:15" "21:20" "21:25" "21:30" "21:35" "21:40"
## [262] "21:45" "21:50" "21:55" "22:00" "22:05" "22:10" "22:15" "22:20" "22:25"
## [271] "22:30" "22:35" "22:40" "22:45" "22:50" "22:55" "23:00" "23:05" "23:10"
## [280] "23:15" "23:20" "23:25" "23:30" "23:35" "23:40" "23:45" "23:50" "23:55"
```

```
# Stops to SF
stops_sf <- sfheaders::sf_point(obj = all_stops[,.SD[1],by = .(stop_id)]
                                ,x = "shape_pt_lon"
                                ,y = "shape_pt_lat"
                                ,keep = TRUE)
stops_sf <- sf::st_set_crs(stops_sf,4326)
```

6.6 Create Toronto hexagonal grid

```
# Project Toronto boundary and stops to UTM Zone 17N

toronto_bound <- readr::read_rds("data/toronto_bound_CSD.rds")
toronto_crs <- 26917
toronto_bound_proj <- sf::st_transform(toronto_bound, toronto_crs)
stops_sf_proj <- sf::st_transform(stops_sf, toronto_crs)

# Target Area ~ 0.11 km^2 (similar to H3 level 9) = 110,000 m^2
target_area_m2 <- 110000
target_side_length <- sqrt(2 * target_area_m2 / (3 * sqrt(3))) # approx 205m
target_cellsize <- sqrt(3) * target_side_length # approx 355m

# Use the bounding box of the projected boundary to create grid
grid_poly <- sf::st_make_grid(toronto_bound_proj,
                              cellsizes = target_cellsize,
                              what = "polygons",
                              square = FALSE)
```

```

# Convert to sf object
toronto_hex_grid_sf <- sf::st_sf(geometry = grid_poly)

# Add a unique hex_id column
toronto_hex_grid_sf$hex_id <- 1:nrow(toronto_hex_grid_sf)

# Assign CRS
toronto_hex_grid_sf <- sf::st_set_crs(toronto_hex_grid_sf, toronto_crs)

# Clip the grid to the Toronto boundary
toronto_hex_grid_proj <- sf::st_intersection(toronto_hex_grid_sf, toronto_bound_proj)

## Warning: attribute variables are assumed to be spatially constant throughout
## all geometries

# Select only the hex_id and geometry after intersection
toronto_hex_grid_proj <- dplyr::select(toronto_hex_grid_proj, hex_id, geometry)

```

6.7 Retrieve Toronto census data at aggregation level of Dissemination area

```

options(cancensus.cache_path = "data/cancensus_cache")
census_dataset <- "CA21"
toronto_da_data <- cancensus::get_census(
  dataset = census_dataset,
  regions = list(CSD = "3520005"), # Toronto CSD
  level = "DA",
  vectors = c("v_CA21_1", "v_CA21_906"), # Population and Median Total Household Income
  geo_format = "sf",
  use_cache = TRUE
)

## Reading vectors data from local cache.

## Reading geo data from local cache.

# Project DA data to the same CRS as hex grid
toronto_da_data_proj <- sf::st_transform(toronto_da_data, toronto_crs)

toronto_da_data_proj <- toronto_da_data_proj %>%
  dplyr::rename(
    Population2021 = `v_CA21_1: Population, 2021`,
    MedianTotalIncome = `v_CA21_906: Median total income of household in 2020 ($)`
  )

# Calculate DA area
toronto_da_data_proj$da_area <- sf::st_area(toronto_da_data_proj)

```

6.8 Apportion census data to hex grid

```

# Ensure geometries are valid
toronto_hex_grid_proj <- sf::st_make_valid(toronto_hex_grid_proj)
toronto_da_data_proj <- sf::st_make_valid(toronto_da_data_proj)

# Intersect hexagons and DAs
intersection_sf <- sf::st_intersection(toronto_hex_grid_proj, toronto_da_data_proj)

## Warning: attribute variables are assumed to be spatially constant throughout
## all geometries

# Calculate area of each intersection polygon
intersection_sf$intersection_area <- sf::st_area(intersection_sf)

# Convert sf to data.table
intersection_dt <- data.table::setDT(sf::st_drop_geometry(intersection_sf))

# Calculate apportioned values per hexagon
hex_apportioned_data <- intersection_dt[, .(
  # Area-weighted income: sum(Income * intersection_area) / sum(intersection_area)
  weighted_income = sum(MedianTotalIncome * intersection_area, na.rm = TRUE) / sum(intersection_area, na.rm = TRUE),
  # Apportioned population: sum(Population_Density * intersection_area)
  apportioned_pop = sum( (Population2021 / fifelse(da_area > units::set_units(0, "m^2"), units::drop_units(Population2021), 0)) * intersection_area, na.rm = TRUE),
  by = hex_id]

# Handle potential NaNs if sum(intersection_area) was 0 for any hex
hex_apportioned_data$weighted_income[is.nan(hex_apportioned_data$weighted_income)] <- 0 # Or NA
hex_apportioned_data$apportioned_pop <- round(hex_apportioned_data$apportioned_pop) # Round population

# Calculate Income Decile Breaks
breaks <- quantile(hex_apportioned_data$weighted_income, probs = seq(0, 1, 0.1), na.rm = TRUE)

# Assign income deciles
hex_apportioned_data[, income_decile := cut(weighted_income,
  breaks = breaks,
  labels = 1:10,
  include.lowest = TRUE,
  right = TRUE)]

# Convert factor to numeric
hex_apportioned_data$income_decile <- as.numeric(as.character(hex_apportioned_data$income_decile))
# Assign NA to hexagons with no income data
hex_apportioned_data$income_decile[is.na(hex_apportioned_data$weighted_income)] <- NA

# Join apportioned data back to hex grid
toronto_hex_grid_final_proj <- merge(toronto_hex_grid_proj, hex_apportioned_data, by = "hex_id", all.x = TRUE)

```

6.9 Join stops to hexagons and aggregate frequency

```

stops_with_hex <- sf::st_join(stops_sf_proj, toronto_hex_grid_final_proj, join = sf::st_within)

# Select relevant columns (stop_id, hex_id, income_decile, apportioned_pop)
stops_hex_lookup <- sf::st_drop_geometry(stops_with_hex) %>%
  dplyr::select(stop_id, hex_id, income_decile, apportioned_pop) %>%
  data.table::setDT()

# Allocate Hexagon Info to Stops Frequency Data
# Merge hex_id from the lookup table into the frequency data
all_stops[stops_hex_lookup, on = "stop_id", hex_id := i.hex_id]

# Sum frequency (N) per hexagon, time interval, and time bin
hex_freq_agg <- all_stops[!is.na(hex_id),
  list(N = sum(N, na.rm = TRUE)),
  by = .(hex_id, time, time_interval)]

```

6.10 Merge aggregated data and save output

```

hex_data_for_merge <- hex_apportioned_data[, .(hex_id, apportioned_pop, weighted_income, income_decile)]
setnames(hex_data_for_merge,
  old = c("apportioned_pop", "weighted_income", "income_decile"),
  new = c("total_pop", "avg_inc", "decil_ind"))

final_hex_freq_data <- merge(hex_freq_agg, hex_data_for_merge, by = "hex_id", all.x = TRUE)

# Merge final data with hex grid geometry
toronto_hex_freq_sf_proj <- merge(toronto_hex_grid_final_proj, final_hex_freq_data, by = "hex_id", all.x = TRUE)

# Convert back to WGS84
toronto_hex_freq_sf <- sf::st_transform(toronto_hex_freq_sf_proj, 4326)

# Save the final sf object
readr::write_rds(x = toronto_hex_freq_sf,
  file = "data/toronto_hex_freq_sf.rds",
  compress = "gz")

```

6.11 Summary Statistics

```

rm(list=ls())
gc(reset = TRUE)

```

```

##          used (Mb) gc trigger (Mb) max used (Mb)
## Ncells  4244381 226.7   21282294 1136.6  4244381 226.7
## Vcells 11976315  91.4   161626329 1233.2 11976315  91.4

```

```

# Load the processed data
toronto_hex_freq_sf <- readr::read_rds("data/toronto_hex_freq_sf.rds")
data.table::setDT(toronto_hex_freq_sf) # Convert sf to data.table for faster processing

```

```
# check hexagons in morning peak by income group
tmp <- toronto_hex_freq_sf[time_interval == "60 min",] %>%
  .[time %in% c("06:00","07:00","08:00"),peak := "Morning"] %>%
  .[!is.na(peak),] %>%
  .[,decil_class := fcase(decil_ind %in% 9:10,"20p_richest"
    , decil_ind %in% 1:5,"50p_poorest")] %>%
  .[!is.na(decil_class),] %>%
  .[,list("N_vehicles" = sum(N), "N_hex" = .N, "Minutes" = 180),by = .(peak,decil_class)]

tmp[,vehicles_by_hex_by_minute := N_vehicles / (N_hex * Minutes)]
tmp[,vehicles_by_hex := N_vehicles / (N_hex )]
tmp[,prop_vehicles_by_hex := round(100 * vehicles_by_hex / min(vehicles_by_hex))]
tmp[]
```

```
##      peak decil_class N_vehicles N_hex Minutes vehicles_by_hex_by_minute
##      <char>      <char>      <int> <int>   <num>              <num>
## 1: Morning 50p_poorest   1553958  4928    180              1.751847
## 2: Morning 20p_richest   229229  1225    180              1.039587
##      vehicles_by_hex prop_vehicles_by_hex
##      <num>              <num>
## 1:      315.3324              169
## 2:      187.1257              100
```

```
tmp[decil_class == '20p_richest']$vehicles_by_hex / tmp[decil_class == '50p_poorest']$vehicles_by_hex
```

```
## [1] 0.5934237
```

```
#> 4839.267 / 3225.112 = 1.500496
```

```
# check mean in morning peak by income group
tmp <- toronto_hex_freq_sf[time_interval == "05 min",] %>%
  .[time %in% c("06:00","07:00","08:00"),peak := "Morning"] %>%
  .[!is.na(peak),] %>%
  .[,hour := stringr::str_split(time,":",n = 2,simplify = TRUE)[1],by = .(hex_id,time)] %>%
  .[,minute := stringr::str_split(time,":",n = 2,simplify = TRUE)[2],by = .(hex_id,time)] %>%
  .[,time_minute := as.numeric(hour) * 60 + as.numeric(minute)] %>%
  .[,decil_class := fcase(decil_ind %in% 9:10,"20p_richest"
    , decil_ind %in% 1:5,"50p_poorest")] %>%
  .[!is.na(decil_class),]

tmp <- tmp[,weighted.mean(x = time_minute,w = N),by = .(decil_class)]

tmp[,hour := V1/%60]
tmp[,minute := (V1 - hour*60)]
tmp[,time := paste0(hour,":",round(minute,0))]
tmp[]
```

```
##      decil_class      V1 hour minute time
##      <char>      <num> <num>   <num> <char>
## 1: 50p_poorest 427.6068    7 7.606850    7:8
## 2: 20p_richest 429.9875    7 9.987547    7:10
```

```
tmp[decil_class == '20p_richest']$vehicles_by_hex / tmp[decil_class == '50p_poorest']$vehicles_by_hex
```

```
## numeric(0)
```

6.12 Producing Final Plots

```
rm(list=ls())
gc(reset = TRUE)
```

```
##          used (Mb) gc trigger (Mb) max used (Mb)
## Ncells  4246293 226.8   17025836 909.3  4246293 226.8
## Vcells 12121831  92.5   129301064 986.5 12121831  92.5
```

```
# Load data
toronto_hex_freq_sf <- readr::read_rds("data/toronto_hex_freq_sf.rds")
data.table::setDT(toronto_hex_freq_sf) # Convert to data.table

# Remove hexagons with NA or 0 values
toronto_hex_freq_sf <- toronto_hex_freq_sf[!is.na(total_pop) & total_pop > 0 & !is.na(decil_ind)]

# save tmp data
vec <- unique(toronto_hex_freq_sf$time_interval)

list_plots <- lapply(seq_along(vec),function(i){ # i = 4

  tmp <- toronto_hex_freq_sf[total_pop > 0 &
    time_interval == vec[i] &
    !is.na(time ),] %>%
    .[,weighted.mean(N,total_pop),by = .(time,decil_ind )]

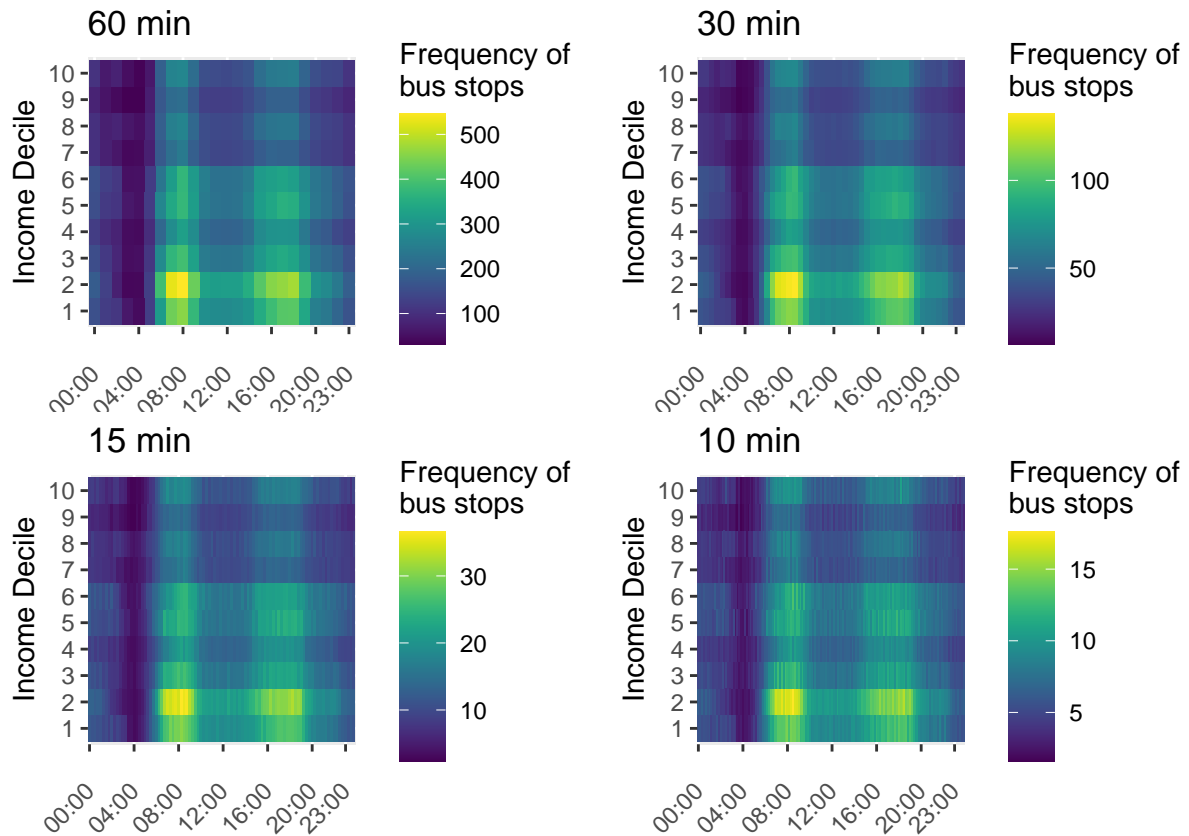
  fixed_time <- c("00:00","04:00","08:00","12:00","16:00","20:00","23:00")

  plot <- ggplot(tmp)+
    geom_tile(aes(x = time,y= as.factor(decil_ind),fill = V1))+
    scale_x_discrete(breaks = fixed_time,labels = fixed_time, drop = TRUE)+
    labs(title = vec[i]
      ,x = NULL
      ,y = "Income Decile"
      , fill = "Frequency of \nbus stops")+
    viridis::scale_fill_viridis()+
    theme(axis.text.x = element_text(angle = 45, vjust = 0.5, hjust=1))

  return(plot)
})

library(patchwork)

(list_plots[[1]] | list_plots[[2]])/
(list_plots[[3]] | list_plots[[4]])
```

```
# rayshader ----
future::plan("multisession", workers = 19)

tmp <- toronto_hex_freq_sf[total_pop > 0 &
  time_interval == "10 min" &
  !is.na(time ),] %>%
  .[,weighted.mean(N,total_pop),by = .(time,decil_ind )] # depois

fixed_time <- c("00:00","04:00","08:00","12:00","16:00","20:00","23:00")

plot <- ggplot(tmp)+
  geom_tile(aes(x = time,y = as.factor(decil_ind),fill = V1))+
  scale_x_discrete(breaks = fixed_time,labels = fixed_time, drop = TRUE)+
  coord_cartesian(expand = FALSE)+
  labs(title = NULL
    ,x = NULL
    ,y = "Income decile")+
  scale_fill_continuous(type = "viridis",direction = +1, name = "Mean frequency\nof vehicles at\npublic
  theme_minimal() +
  theme(
    plot.background = element_rect(fill = "white", colour = "white"),
    axis.text.x = element_text(angle = 0),
    legend.position = "right",
    legend.text.position = "left",
    legend.title.position = "bottom",
```

```

    legend.title.align = 0.5,
    legend.title = element_text(size=8),
    legend.key.width = unit(0.5, "cm")
  ) +
  guides(fill = guide_colourbar())

```

```

## Warning: The 'legend.title.align' argument of 'theme()' is deprecated as of ggplot2
## 3.5.0.
## i Please use theme(legend.title = element_text(hjust)) instead.
## This warning is displayed once every 8 hours.
## Call 'lifecycle::last_lifecycle_warnings()' to see where this warning was
## generated.

```

```

ggplot2::ggsave(plot
  ,filename = "figures/10min_freq_2d_WednesdayFix.png"
  ,width = 10
  ,height = 8
  ,dpi = 300
  ,scale = 0.65)

rayshader::plot_gg(ggobj = plot
  , multicore = TRUE
  , width = 5
  , height = 5
  , scale = 250
  , windowsize = c(1400,866)
  , zoom = 0.5391094
  , phi = 30.4472961
  , theta = -23.2254651 )

# find angle view
# rayshader::render_camera(theta = NULL,phi = NULL,zoom = NULL,fov = NULL)
rayshader::render_snapshot(filename = "figures/10min_freq_3d_rayshader_Wednesdayfix.png"
  ,width = 1000
  ,height = 1000
)

```

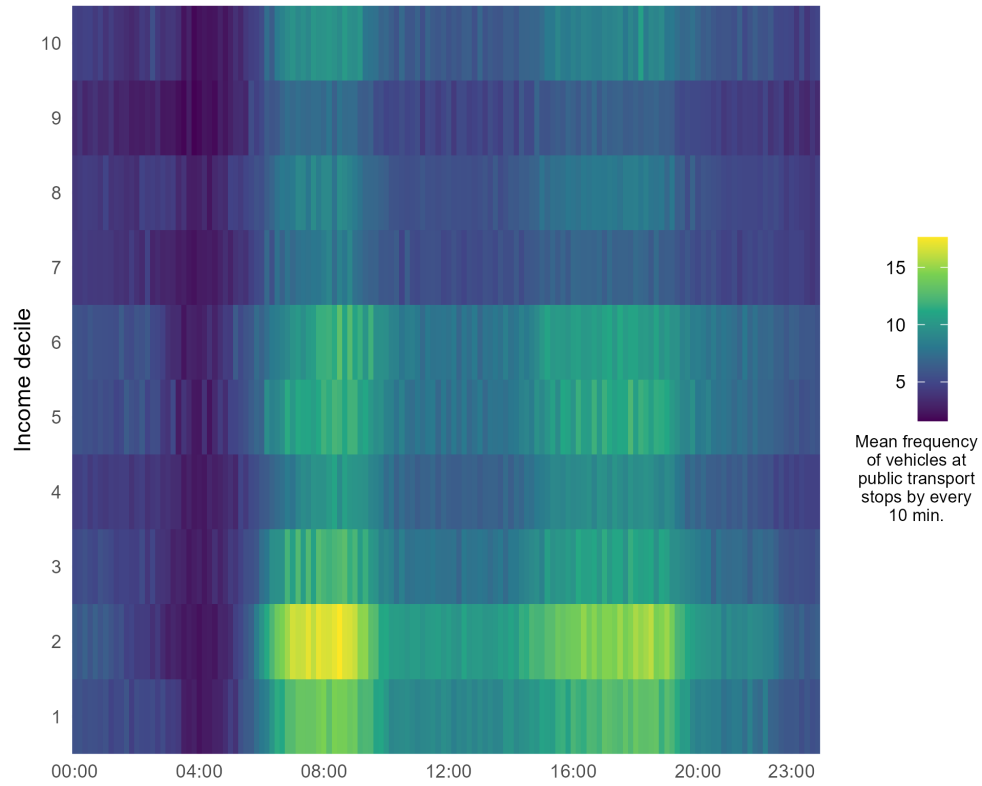


Figure 4: 10-Minute Frequency-Time-Income 2D Plot.

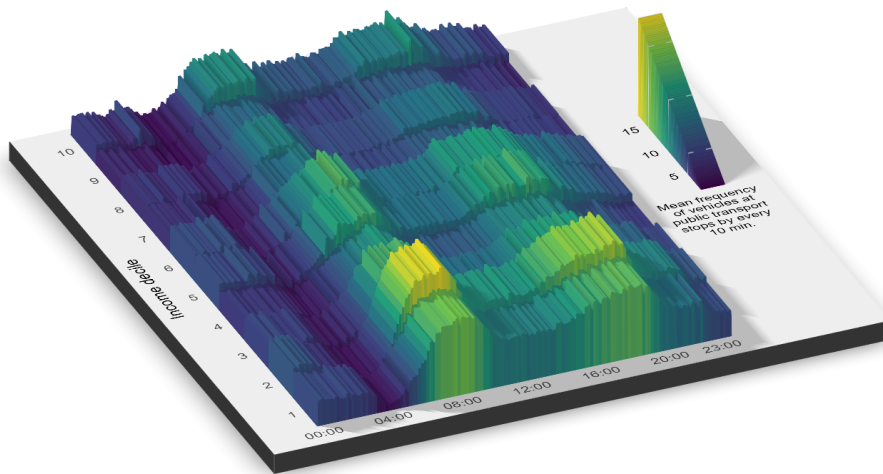


Figure 5: 10-Minute Frequency-Time-Income 3D Plot.