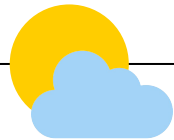


Discrete-Time Markov Chains

21-344 Final Project



Juliette Wong, jnwong

May 14th, 2021





Markov Chains Overview

- Stochastic Process: $\{X_t : t \in T\}$, state space S
- First-Order MC:

$$\underset{\text{future}}{P(X_{n+1} = j)} \mid \underset{\text{past}}{X_0 = i_0, X_1 = i_1, \dots, X_n = i_n} = \underset{\text{future}}{P(X_{n+1} = j)} \mid \underset{\text{current}}{X_n = i_n}$$

- Transition Matrix P
- $P_{ij} = P(X_{n+1} = j \mid X_n = i)$
- $P_{ij} \geq 0$
- $\sum_{j \in S} P_{ij} = 1$



Application: Pittsburgh Weather

- Weather Underground, May 2019–May 2021

- Mostly Sunny



- Partly Cloudy



- Mostly Cloudy



- Cloudy



- Foggy



- Scattered Showers



- Thunderstorms



















- Snow





Application: Pittsburgh Weather

$P =$

| |  |  |  |  |  |  |  |  |
|---|---|---|---|--|---|---|---|---|
|  | 0.3204 | 0.1553 | 0.1748 | 0.3301 | 0 | 0 | 0.0097 | 0.0097 |
|  | 0.1831 | 0.1408 | 0.2113 | 0.3521 | 0.0141 | 0.0986 | 0 | 0 |
|  | 0.1513 | 0.1513 | 0.2773 | 0.3277 | 0 | 0.0504 | 0.0252 | 0.0168 |
|  | 0.0714 | 0.0652 | 0.1273 | 0.5373 | 0.0155 | 0.1056 | 0 | 0.0776 |
|  | 0.25 | 0 | 0.25 | 0.375 | 0.125 | 0 | 0 | 0 |
|  | 0.1481 | 0.0741 | 0.0926 | 0.4444 | 0.0185 | 0.1481 | 0 | 0.0741 |
|  | 0.5 | 0 | 0.25 | 0.25 | 0 | 0 | 0 | 0 |
|  | 0.0816 | 0.0408 | 0.0816 | 0.449 | 0 | 0 | 0 | 0.3469 |



N-Step Transition Probabilities

















- $P_{ij}^n = P(X_{n+k} = j | X_k = i)$
- Chapman-Kolmogorov Equations: $P_{ij}^{n+m} = \sum_{k \in S} P_{kj}^m P_{ik}^n$
- Let $P^{(n)}$ be the matrix of P_{ij}^n ,

$$P^{(m+n)} = P^{(m)} \cdot P^{(n)}$$

$$P^{(n)} = P^n$$



Application: Pittsburgh Weather

| |  |  |  |  |  |  |  |  |
|---|---|---|---|---|---|---|---|---|
| $P^7 =$  | 0.1415 | 0.0974 | 0.1632 | 0.4393 | 0.011 | 0.0754 | 0.0055 | 0.0667 |
|  | 0.1414 | 0.0974 | 0.1631 | 0.4394 | 0.011 | 0.0754 | 0.0055 | 0.0668 |
|  | 0.1414 | 0.0974 | 0.1631 | 0.4394 | 0.011 | 0.0754 | 0.0055 | 0.0668 |
|  | 0.1411 | 0.0972 | 0.1629 | 0.4397 | 0.011 | 0.0754 | 0.0055 | 0.0673 |
|  | 0.1415 | 0.0974 | 0.1632 | 0.4393 | 0.011 | 0.0754 | 0.0055 | 0.0667 |
|  | 0.1412 | 0.0972 | 0.1629 | 0.4396 | 0.011 | 0.0754 | 0.0055 | 0.0672 |
|  | 0.1416 | 0.0975 | 0.1632 | 0.4392 | 0.0109 | 0.0754 | 0.0055 | .0667 |
|  | 0.1408 | 0.097 | 0.1627 | 0.4399 | 0.011 | 0.0754 | 0.0055 | 0.0677 |



Limiting Distribution

- Estimates the long-term behavior of the Markov Chain
- $\pi = (\pi_0 \quad \pi_1 \quad \cdots \quad \pi_s)$
 $\pi_j = \lim_{n \rightarrow \infty} P(X_n = j | X_0 = i)$
- Independent of initial state
- Not all MCs have a limiting distribution!
- To calculate:
 - See if there are identical rows in P^∞
 - Solve $\pi = \pi P$



Application: Pittsburgh Weather



$\pi = (0.14124 \quad 0.09728 \quad 0.16297 \quad 0.43957 \quad 0.01096 \quad 0.07539 \quad 0.00548 \quad 0.0671)$

Thank You!

