CPS変換メモ

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1 DS項の定義

ここでは、CPS 変換前の項、すなわち CPS 変換を行う対象の言語を示す。

1.1 構文

$$
\begin{array}{rcll}
\tau & ::= & \mathsf{Nat} \mid \mathsf{bool} \mid \tau_1 \to \tau_2 @\mathrm{cps}[\tau_3, \tau_4] & \text{型} \\
v & ::= & n \mid x \mid \lambda x.\, e \mid \mathcal{S} & \text{値} \\
e & ::= & v \mid e_1 @ e_2 \mid \langle e \rangle \mid \mathsf{let}\ x = e_1\ \mathsf{in}\ e_2 & \text{項}
\end{array}
$$

1.2 DS項の型付け規則

$$
\frac{\Gamma(x) = \tau_1}{\Gamma \vdash x : \tau_1 @\mathrm{cps}[\tau, \tau]}\ (\textsc{TVar}) \qquad \frac{}{\Gamma \vdash n : \mathsf{Nat} @\mathrm{cps}[\tau, \tau]}\ (\textsc{TNum})
$$

$$
\frac{\Gamma, x : \tau_2 \vdash e : \tau_1 @\mathrm{cps}[\tau_3, \tau_4]}{\Gamma \vdash \lambda x.\, e : (\tau_2 \to \tau_1 @\mathrm{cps}[\tau_3, \tau_4]) @\mathrm{cps}[\tau, \tau]}\ (\textsc{TFun})
$$

$$
\frac{}{\Gamma \vdash \mathcal{S} : (((\tau_3 \to \tau_4 @\mathrm{cps}[\tau, \tau]) \to \tau_1 @\mathrm{cps}[\tau_1, \tau_2]) \to \tau_3 @\mathrm{cps}[\tau_4, \tau_2]) @\mathrm{cps}[\tau, \tau]}\ (\textsc{TShift})
$$

$$
\frac{\Gamma \vdash e_1 : (\tau_2 \to \tau_1 @\mathrm{cps}[\tau_3, \tau_4]) @\mathrm{cps}[\tau_5, \tau_6] \quad \Gamma \vdash e_2 : \tau_2 @\mathrm{cps}[\tau_4, \tau_5]}{\Gamma \vdash e_1 @ e_2 : \tau_1 @\mathrm{cps}[\tau_3, \tau_6]}\ (\textsc{TApp})
$$

$$
\frac{\Gamma \vdash e : \tau_1 @\mathrm{cps}[\tau_1, \tau_2]}{\Gamma \vdash \langle e \rangle : \tau_2 @\mathrm{cps}[\tau_3, \tau_3]}\ (\textsc{TReset})
$$

$$
\frac{\Gamma \vdash e_1 : \tau_1 @\mathrm{cps}[\beta, \gamma] \quad \Gamma, x : \tau_1 \vdash e_2 : \tau_2 @\mathrm{cps}[\alpha, \beta]}{\Gamma \vdash (\mathsf{let}\ x = e_1\ \mathsf{in}\ e_2) : \tau_2 @\mathrm{cps}[\alpha, \gamma]}\ (\textsc{TLet})
$$

1.3 DS 項の代入規則

代入規則は、$e[v/x] = e'$ と表現することができ、「項 e の中に現れる変数 x を値 v に置き換える」と読む。

今回用いる代入規則は以下である。

$$\frac{}{x[v/x] = x}\ (\textsc{sVar=}) \qquad \frac{}{y[v/x] = y}\ (\textsc{sVar} \neq) \qquad \frac{}{n[v/x] = n}\ (\textsc{sNum})$$

$$\frac{\forall x.(e[v/y] = e')}{(\lambda x.\, e)[v/y] = \lambda x.\, e'}\ (\textsc{sFun}) \qquad \frac{e_1[v/x] = e_1' \quad e_2[v/x] = e_2'}{(e_1 \mathbin{@} e_2)[v/x] = (e_1' \mathbin{@} e_2')}\ (\textsc{sApp})$$

$$\frac{}{\mathcal{S}[v/x] = \mathcal{S}}\ (\textsc{sShift}) \qquad \frac{e_1[v/x] = e_1'}{\langle e_1 \rangle[v/x] = \langle e_1' \rangle}\ (\textsc{sReset})$$

$$\frac{e_1[v/y] = e_1' \quad \forall x.(e_2[v/y] = e_2')}{(\mathsf{let}\ x = e_1\ \mathsf{in}\ e_2)[v/y] = \mathsf{let}\ x = e_1'\ \mathsf{in}\ e_2'}\ (\textsc{sLet})$$

1.4 DS 項の簡約規則

次に、簡約規則について示す。ただし、簡約規則を示す前に、フレームを定義する必要がある。フレームを定義することによって、簡約の順序を決めることができる。

$$\begin{array}{lrcl} \text{フレーム} & F & = & [\ ] \mathbin{@} e_2 \mid v_1 \mathbin{@} [\ ] \mid \langle\, [\ ]\,\rangle \mid \mathsf{let}\ x = [\ ]\ \mathsf{in}\ e_2 \\ \text{評価文脈 (コンテキスト)} & E & = & [\ ] \mid F \circ E \end{array}$$

$$\frac{\Gamma \vdash e_2 : \tau_2\ @\mathrm{cps}[\tau_4, \tau_6]}{\Gamma \vdash ([\ ] \mathbin{@} e_2) : [\ (\tau_2 \to \tau_1 @\mathrm{cps}[\tau_3, \tau_4]) @\mathrm{cps}[\tau_5, \tau_6]\ ]_{\mathsf{f}}\ \tau_1 @\mathrm{cps}[\tau_3, \tau_6]}\ (\text{F-}\textsc{App}_1)$$

$$\frac{\Gamma \vdash v_1 : (\tau_2 \to \tau_1 @\mathrm{cps}[\tau_3, \tau_4])\ @\mathrm{cps}[\tau_5, \tau_5]}{\Gamma \vdash (v_1 \mathbin{@} [\ ]) : [\ \tau_2 @\mathrm{cps}[\tau_4, \tau_5]\ ]_{\mathsf{f}}\ \tau_1 @\mathrm{cps}[\tau_3, \tau_5]}\ (\text{F-}\textsc{App}_2)$$

$$\frac{}{\Gamma \vdash \langle [\ ] \rangle : [\ \tau_1 @\mathrm{cps}[\tau_1, \tau_2]\ ]_{\mathsf{f}}\ \tau_2 @\mathrm{cps}[\tau_3, \tau_3]}\ (\text{F-}\textsc{Reset})$$

$$\frac{\Gamma, x : \tau_1 \vdash e_2 : \tau_2\ @\mathrm{cps}[\alpha, \beta]}{\Gamma \vdash \mathsf{let}\ x = [\ ]\ \mathsf{in}\ e_2 : [\ \tau_1 @\mathrm{cps}[\beta, \gamma]\ ]_{\mathsf{f}}\ \tau_2 @\mathrm{cps}[\alpha, \gamma]}\ (\text{F-}\textsc{Let})$$

[ ] は、「次に計算を行う部分」である。「それ以外の部分」は、「文脈」と呼ぶ。評価文脈はフレームを何枚も重ね合わせたものであり、フレームの列として表すことができる。そのように表せるとすると、評価文脈 E に式 e を入れる関数 $plug_F$ を以下のように定義できる。

$$\begin{array}{rcl} plug_F([\ ] \mathbin{@} e_2, e_1) & = & e_1 \mathbin{@} e_2 \\ plug_F(v_1 \mathbin{@} [\ ], e_2) & = & v_1 \mathbin{@} e_2 \\ plug_F(\langle [\ ] \rangle, e_1) & = & \langle e_1 \rangle \\ plug_F(\mathsf{let}\ x = [\ ]\ \mathsf{in}\ e_2, e_1) & = & \mathsf{let}\ x = e_1\ \mathsf{in}\ e_2 \end{array}$$

次に、reset を含まないフレームである、ピュアフレームを定義する。

$$
\begin{array}{rcl}
\text{ピュアフレーム} \quad F_p & = & [\ ]\ @\ e_2 \mid v_1\ @\ [\ ] \mid \mathsf{let}\ x = [\ ]\ \mathsf{in}\ e_2 \\
\text{ピュアコンテキスト} \quad E_p & = & [\ ] \mid F_p \circ E_p
\end{array}
$$

$\boxed{\text{ピュアフレーム} \quad F_p}$

$$
\frac{\Gamma \vdash e_2 : \tau_2\ @\mathrm{cps}[\tau_4, \tau_5]}{\Gamma \vdash ([\ ]\ @\ e_2) : [\ (\tau_2 \to \tau_1 @\mathrm{cps}[\tau_3, \tau_4]) @\mathrm{cps}[\tau_5, \tau_6]\ ]_{\mathsf{pf}}\ \tau_1 @\mathrm{cps}[\tau_3, \tau_6]}\ (\mathrm{F}_p\text{-}\textsc{App}_1)
$$

$$
\frac{\Gamma \vdash v_1 : (\tau_2 \to \tau_1 @\mathrm{cps}[\tau_3, \tau_4])\ @\mathrm{cps}[\tau_5, \tau_5]}{\Gamma \vdash (v_1\ @\ [\ ]) : [\ \tau_2 @\mathrm{cps}[\tau_4, \tau_5]\ ]_{\mathsf{pf}}\ \tau_1 @\mathrm{cps}[\tau_3, \tau_5]}\ (\mathrm{F}_p\text{-}\textsc{App}_2)
$$

$$
\frac{\Gamma, x : \tau_1 \vdash e_2 : \tau_2\ @\mathrm{cps}[\alpha, \beta]}{\Gamma \vdash \mathsf{let}\ x = [\ ]\ \mathsf{in}\ e_2 : [\ \tau_1 @\mathrm{cps}[\beta, \gamma]\ ]_{\mathsf{pf}}\ \tau_2 @\mathrm{cps}[\alpha, \gamma]}\ [(\mathrm{F}_p\text{-}\textsc{Let})
$$

$\boxed{\text{ピュアコンテキスト} \quad E_p}$

$$
\frac{}{\Gamma \vdash [\ ] : [\ \tau_1 @\mathrm{cps}[\tau_2, \tau_3]\ ]_{\mathsf{pc}}\ \tau_1 @\mathrm{cps}[\tau_2, \tau_3]}\ (\mathrm{E}_p\text{-}\textsc{Hole})
$$

$$
\frac{\Gamma \vdash F_p : [\ \tau_4 @\mathrm{cps}[\tau_5, \tau_6]\ ]_{\mathsf{pf}}\ \tau_7 @\mathrm{cps}[\tau_8, \tau_9] \quad \Gamma \vdash E_p : [\ \tau_1 @\mathrm{cps}[\tau_2, \tau_3]\ ]_{\mathsf{pf}}\ \tau_4 @\mathrm{cps}[\tau_5, \tau_6]}{\Gamma \vdash F_p \circ E_p : [\ \tau_1 @\mathrm{cps}[\tau_2, \tau_3]\ ]_{\mathsf{pc}}\ \tau_7 @\mathrm{cps}[\tau_8, \tau_9]}\ (\mathrm{E}_p\text{-}\textsc{Frame})
$$

$\boxed{\text{ピュアフレーム同士の関係} \quad F_p \cong_{\mathsf{c}} F_p}$

$$
\frac{}{([\ ]\ @\ e_2) \cong_{\mathsf{f}} ([\ ]\ @\ e_2)}\ (\cong_{\mathsf{pf}}\ \text{-}\textsc{App}_1)
$$

同様に、関数 $plug_{F_p}$ を定義する。

$$
\begin{array}{rcl}
plug_{F_p}([\ ]\ @\ e_2, e_1) & = & e_1\ @\ e_2 \\
plug_{F_p}(v_1\ @\ [\ ], e_2) & = & v_1\ @\ e_2 \\
plug_{F_p}(\mathsf{let}\ x = [\ ]\ \mathsf{in}\ e_2, e_1) & = & \mathsf{let}\ x = e_1\ \mathsf{in}\ e_2
\end{array}
$$

関数 $plug_{E_p}$ は以下のように定義できる。

$$
\begin{array}{rcl}
plug_{E_p}([\ ], e_1) & = & e_1 \\
plug_{E_p}(F_p \circ E_p, e_2) & = & plug_{F_p}(F_p, plug_{E_p}(E_p, e_1))
\end{array}
$$

以上より、簡約規則は以下のように表せる。

$$
\frac{e_1[v_2/x] = e_1'}{(\lambda x.\, e_1)\ @\ v_2 \rightsquigarrow e_1'}\ (\mathrm{R}\textsc{Beta}) \qquad \frac{e_1 \rightsquigarrow e_2}{plug_F(F, e_1) \rightsquigarrow plug_F(F, e_2)}\ (\mathrm{R}\textsc{Frame})
$$

$$
\frac{E_{p_1} \cong_{\mathsf{c}} E_{p_2}}{\langle E_{p_1}[\ \mathcal{S}\ @\ v_2\ ] \rangle \rightsquigarrow \langle v_2\ @\ (\lambda y.\, \langle E_{p_2}[\ y\ ] \rangle) \rangle}\ (\mathrm{R}\textsc{Shift}) \qquad \frac{}{\langle v_1 \rangle \rightsquigarrow v_1}\ (\mathrm{R}\textsc{Reset})
$$

$$
\frac{e_2[v_1/x] = e_2'}{\mathsf{let}\ x = v_1\ \mathsf{in}\ e_2 \rightsquigarrow e_2'}\ (\mathrm{R}\textsc{Let})
$$

2　CPS 項の定義

CPS 変換後の項を示す。

ここで、$\overline{\lambda}.$ や $\overline{@}$ のように、上付きの線が書かれているものは、static な項。また、$\underline{\lambda}.$ や $\underline{@}$ のように、下付きの線が描かれているものは、dynamic な項と呼ぶ。

2.1　CPS 項の構文

$$
\begin{array}{rcll}
\tau & ::= & \mathsf{Nat} \mid \mathsf{bool} \mid \tau_1 \to \tau_2 & \text{型} \\
v & ::= & n \mid x \mid \underline{\lambda} x.\, \underline{\lambda} k.\, e \mid \mathcal{S} & \text{値} \\
\mathcal{S} & ::= & \underline{\lambda} w.\, \underline{\lambda} k.\, (w \,\underline{@}\, (\underline{\lambda} a.\, \underline{\lambda} k'.\, k' \,\underline{@}\, (k \,\underline{@}\, a))) \,\underline{@}\, (\underline{\lambda} m.\, m) & \texttt{shift} \\
e & ::= & v \mid e_1 \,\underline{@}\, e_2 \mid \underline{\mathsf{let}}\ x = e_1\ \underline{\mathsf{in}}\ e_2 & \text{項}
\end{array}
$$

2.2　CPS 項の型付け規則

$$
\frac{\Gamma(x) = \tau_1}{\Gamma \vdash x : \tau_1}\ (\text{TVar}_{\mathsf{C}})
\qquad
\frac{}{\Gamma \vdash n : \mathsf{Nat}}\ (\text{TNum}_{\mathsf{C}})
\qquad
\frac{\Gamma, x : \tau_2 \vdash e : \tau_1}{\Gamma \vdash \underline{\lambda} x.\, \underline{\lambda} k.\, e : \tau_2 \to \tau_1}\ (\text{TFun}_{\mathsf{C}})
$$

$$
\frac{}{\Gamma \vdash \mathcal{S} : ((\tau_1 \to (\tau_2 \to \tau_3) \to \tau_3) \to (\tau_4 \to \tau_4) \to \tau_5) \to (\tau_1 \to \tau_2) \to \tau_5}\ (\text{TShift}_{\mathsf{C}})
$$

$$
\frac{\Gamma \vdash e_1 : \tau_2 \to \tau_1 \quad \Gamma \vdash e_2 : \tau_2}{\Gamma \vdash e_1 \,\underline{@}\, e_2 : \tau_1}\ (\text{TApp}_{\mathsf{C}})
\qquad
\frac{\Gamma \vdash e_1 : \tau_1 \quad \Gamma, x : \tau_1 \vdash e_2 : \tau_2}{\Gamma \vdash (\mathsf{let}\ x = e_1\ \mathsf{in}\ e_2) : \tau_2}\ (\text{TLet}_{\mathsf{C}})
$$

2.3　CPS 項の代入規則

$$
\frac{}{x[v/x] = x}\ (\text{sVar}{=}_{\mathsf{C}})
\qquad
\frac{}{y[v/x] = y}\ (\text{sVar} \neq_{\mathsf{C}})
\qquad
\frac{}{n[v/x] = n}\ (\text{sNum}_{\mathsf{C}})
$$

$$
\frac{\forall x.(e[v/y] = e')}{(\lambda x.\, e)[v/y] = \lambda x.\, e'}\ (\text{sFun}_{\mathsf{C}})
\qquad
\frac{e_1[v/x] = e_1' \quad e_2[v/x] = e_2'}{(e_1 \,@\, e_2)[v/x] = (e_1' \,@\, e_2')}\ (\text{sApp}_{\mathsf{C}})
$$

$$
\frac{}{\mathcal{S}[v/x] = \mathcal{S}}\ (\text{sShift})
\qquad
\frac{e_1[v/y] = e_1' \quad \forall x.(e_2[v/y] = e_2')}{(\mathsf{let}\ x = e_1\ \mathsf{in}\ e_2)[v/y] = \mathsf{let}\ x = e_1'\ \mathsf{in}\ e_2'}\ (\text{sLet})
$$

2.4 CPS 項の簡約規則

$$\frac{e_1[v_2/x] = e_1'}{(\lambda x.\, e_1) @ v_2 \rightsquigarrow e_1'}\ (\textsc{eqBeta}_{\mathsf{C}}) \qquad \frac{e_2[v_1/x] = e_2'}{\mathsf{let}\ x = v_1\ \mathsf{in}\ e_2 \rightsquigarrow e_2'}\ (\textsc{eqLet}_{\mathsf{C}})$$

$$\frac{}{(\underline{\lambda} w.\, \underline{\lambda} k.\, (w \,\underline{@}\, (\underline{\lambda} a.\, \underline{\lambda} k'.\, k' \,\underline{@}\, (k \,\underline{@}\, a))) \,\underline{@}\, (\underline{\lambda} m.\, m) \,\underline{@}\, v_2) \,\underline{@}\, k \rightsquigarrow (v_2 \,\underline{@}\, (\underline{\lambda} a.\, \underline{\lambda} k'.\, k' \,\underline{@}\, (k \,\underline{@}\, a))) \,\underline{@}\, (\underline{\lambda} m.\, m)}\ (\textsc{eqShift}_{\mathsf{C}})$$

$$\frac{e_1 \rightsquigarrow e_1'}{e_1 \,\underline{@}\, e_2 \rightsquigarrow e_1' \,\underline{@}\, e_2}\ (\textsc{eqApp1}_{\mathsf{C}}) \qquad \frac{e_2 \rightsquigarrow e_2'}{e_1 \,\underline{@}\, e_2 \rightsquigarrow e_1 \,\underline{@}\, e_2'}\ (\textsc{eqApp2}_{\mathsf{C}})$$

$$\frac{e_1 \rightsquigarrow e_1'}{\underline{\mathsf{let}}\ x = e_1\ \underline{\mathsf{in}}\ e_2 \rightsquigarrow \underline{\mathsf{let}}\ x = e_1'\ \underline{\mathsf{in}}\ e_2}\ (\textsc{eqLet1}_{\mathsf{C}}) \qquad \frac{e_2 \rightsquigarrow e_2'}{\underline{\mathsf{let}}\ x = e_1\ \underline{\mathsf{in}}\ e_2 \rightsquigarrow \underline{\mathsf{let}}\ x = e_1\ \underline{\mathsf{in}}\ e_2'}\ (\textsc{eqLet2}_{\mathsf{C}})$$

$$\frac{}{e_1 \,\underline{@}\, e_2 \rightsquigarrow \underline{\mathsf{let}}\ x = e_1\ \underline{\mathsf{in}}\ x \,\underline{@}\, e_2}\ (\textsc{eqLetApp1}_{\mathsf{C}}) \qquad \frac{}{v_1 \,\underline{@}\, e_2 \rightsquigarrow \underline{\mathsf{let}}\ x = e_2\ \underline{\mathsf{in}}\ v_1 \,\underline{@}\, x}\ (\textsc{eqLetApp2}_{\mathsf{C}})$$

2.5 CPS 変換の定式化

η redex を作らない、one-pass の CPS 変換の定義を示す。

$$\begin{array}{rcl}
[\![n]\!]_v & = & n \\
[\![x]\!]_v & = & x \\
[\![\lambda x.\, e]\!]_v & = & \underline{\lambda} x.\, \underline{\lambda} k.\, [\![e]\!]' \,\overline{@}\, k \\
[\![\mathcal{S}]\!]_v & = & \underline{\lambda} wk.\, (w \,\underline{@}\, (\underline{\lambda} ak'.\, k' \,\underline{@}\, (k \,\underline{@}\, a))) \,\underline{@}\, (\underline{\lambda} m.\, m) \\
 & & \\
[\![v]\!] & = & \overline{\lambda} \kappa.\, \kappa \,\overline{@}\, [\![v]\!]_v \\
[\![e_1 @ e_2]\!] & = & \overline{\lambda} \kappa.\, [\![e_1]\!] \,\overline{@}\, (\overline{\lambda} m.\, [\![e_2]\!] \,\overline{@}\, (\overline{\lambda} n.\, (m \,\underline{@}\, n) \,\underline{@}\, (\underline{\lambda} a.\, \kappa \,\overline{@}\, a))) \\
[\![\langle e \rangle]\!] & = & \overline{\lambda} \kappa.\, \underline{\mathsf{let}}\ x = [\![e]\!] \,\overline{@}\, (\overline{\lambda} m.\, m)\ \underline{\mathsf{in}}\ \kappa \,\overline{@}\, x \\
[\![\mathsf{let}\ x = e_1\ \mathsf{in}\ e_2]\!] & = & \overline{\lambda} \kappa.\, [\![e_1]\!] \,\overline{@}\, (\overline{\lambda} m.\, \underline{\mathsf{let}}\ x = m\ \underline{\mathsf{in}}\ [\![e_2]\!] \,\overline{@}\, \kappa) \\
 & & \\
[\![v]\!]' & = & \overline{\lambda} k.\, k \,\underline{@}\, [\![v]\!]_v \\
[\![e_1 @ e_2]\!]' & = & \overline{\lambda} \kappa.\, [\![e_1]\!] \,\overline{@}\, (\overline{\lambda} m.\, [\![e_2]\!] \,\overline{@}\, (\overline{\lambda} n.\, (m \,\underline{@}\, n) \,\underline{@}\, k)) \\
[\![\langle e \rangle]\!]' & = & \overline{\lambda} k.\, \underline{\mathsf{let}}\ x = [\![e]\!] \,\overline{@}\, (\overline{\lambda} m.\, m)\ \underline{\mathsf{in}}\ k \,\underline{@}\, x \\
[\![\mathsf{let}\ x = e_1\ \mathsf{in}\ e_2]\!]' & = & \overline{\lambda} k.\, [\![e_1]\!] \,\overline{@}\, (\overline{\lambda} m.\, \underline{\mathsf{let}}\ x = m\ \underline{\mathsf{in}}\ [\![e_2]\!]' \,\overline{@}\, k)
\end{array}$$

3 補題の証明

CPS 変換の証明を行う前に、必要な補題をいくつか証明する。

3.1 CPS 項に関する代入補題の証明

補題 3.1.1: (eSubstV)

$v_1[v/x] = v_1'$ のとき、$[\![v_1]\!]_v[[\![v]\!]_v/x] = [\![v_1']\!]_v$

証明.

$v = x$ **のとき**

$$\begin{aligned} [\![x]\!]_v[[\![v]\!]_v/x] &= x[[\![v]\!]_v/x] \\ &= [\![v]\!]_v \qquad \texttt{(sVar=)} \end{aligned}$$

$v = y$ **のとき**

$$\begin{aligned} [\![y]\!]_v[[\![v]\!]_v/x] &= y[[\![v]\!]_v/x] \\ &= [\![y]\!]_v \qquad \texttt{(sVar} \neq\texttt{)} \end{aligned}$$

$v = \lambda x.\, e$ **のとき**

$$\begin{aligned} [\![\lambda x.\, e]\!]_v[[\![v]\!]_v/x] &\equiv (\underline{\lambda} x.\, \underline{\lambda} k.\, [\![e]\!]' \,\overline{@}\, k)[[\![v]\!]_v/x] \\ &= \underline{\lambda} x.\, \underline{\lambda} k.\, [\![e[v/x]]\!]' \,\overline{@}\, k \\ &= \underline{\lambda} x.\, \underline{\lambda} k.\, [\![e']\!]' \,\overline{@}\, k \qquad \text{(補題 3.1.3 ekSubst')} \\ &\equiv [\![\lambda x.\, e']\!]_v \end{aligned}$$

$v = \mathcal{S}$ **のとき**

$$\begin{aligned} [\![\mathcal{S}]\!]_v[[\![v]\!]_v/x] &= (\underline{\lambda} wk.\, (w \,\underline{@}\, (\underline{\lambda} ak'.\, k' \,\underline{@}\, (k \,\underline{@}\, a))) \,\underline{@}\, (\underline{\lambda} m.\, m))[[\![v]\!]_v/x] \\ &= [\![\mathcal{S}]\!]_v \end{aligned}$$

□

補題 3.1.2: (e κ Subst)

$e_1[v/x] = e_2$ かつ $\kappa_1[[\![v]\!]_v/x] = \kappa_2$ のとき、$([\![e_1]\!] \,\overline{@}\, \kappa_1)[[\![v]\!]_v/x] = [\![e_2]\!] \,\overline{@}\, \kappa_2$

証明.

$e_1 = v_1$ **(値) のとき**

$v_1[v/x] = v_2$ とすると、

$$
\begin{array}{rcll}
(\text{与式}) & \equiv & ([\![v_1]\!] \,\overline{@}\, \kappa_1)[[\![v]\!]_v/x] & \\
 & \equiv & (\kappa_1 \,\overline{@}\, [\![v_1]\!]_v)[[\![v]\!]_v/x] & \\
 & = & (\kappa_1[[\![v]\!]_v/x]) \,\overline{@}\, ([\![v_1]\!]_v[[\![v]\!]_v/x]) & \\
 & = & \kappa_2 \,\overline{@}\, [\![v_2]\!]_v & (\text{補題}\ 3.1.1\ \texttt{eSubstV}) \\
 & \equiv & [\![v_2]\!] \,\overline{@}\, \kappa_2 &
\end{array}
$$

e_1 **が App のとき**

$(e_1 \,@\, e_2)[v/x] = e_1' \,@\, e_2'$ とすると、

$$
\begin{array}{rcl}
(\text{与式}) & \equiv & ([\![e_1 \,@\, e_2]\!] \,\overline{@}\, \kappa_1)[[\![v]\!]_v/x] \\
 & \equiv & ([\![e_1]\!] \,\overline{@}\, (\overline{\lambda} m.\, [\![e_2]\!] \,\overline{@}\, (\overline{\lambda} n.\, (m \,\underline{@}\, n) \,\underline{@}\, (\underline{\lambda} a.\, \kappa_1 \,\overline{@}\, a))))[[\![v]\!]_v/x] \\
 & = & ([\![e_1]\!][[\![v]\!]_v/x]) \,\overline{@}\, ((\overline{\lambda} m.\, [\![e_2]\!] \,\overline{@}\, (\overline{\lambda} n.\, (m \,\underline{@}\, n) \,\underline{@}\, (\underline{\lambda} a.\, \kappa_1 \,\overline{@}\, a)))[[\![v]\!]_v/x]) \\
 & = & [\![e_1']\!] \,\overline{@}\, (\overline{\lambda} m.\, ([\![e_2]\!][[\![v]\!]_v/x]) \,\overline{@}\, ((\overline{\lambda} n.\, (m \,\underline{@}\, n) \,\underline{@}\, (\underline{\lambda} a.\, \kappa_1 \,\overline{@}\, a))[[\![v]\!]_v/x])) \\
 & = & [\![e_1']\!] \,\overline{@}\, (\overline{\lambda} m.\, [\![e_2']\!] \,\overline{@}\, (\overline{\lambda} n.\, (m \,\underline{@}\, n) \,\underline{@}\, (\underline{\lambda} a.\, \kappa_2 \,\overline{@}\, a))) \\
 & \equiv & ([\![e_1 \,@\, e_2]\!]) \,\overline{@}\, \kappa_2
\end{array}
$$

e_1 **が Reset のとき**

$\langle e \rangle[v/x] = \langle e' \rangle$ とすると、

$$
\begin{array}{rcll}
(\text{与式}) & \equiv & ([\![\langle e \rangle]\!] \,\overline{@}\, \kappa_1)[[\![v]\!]_v/x] & \\
 & \equiv & (\underline{\text{let}}\ \ c = [\![e]\!] \,\overline{@}\, (\overline{\lambda} m.\, m)\ \ \underline{\text{in}}\ \ \kappa_1 \,\overline{@}\, c)[[\![v]\!]_v/x] & \\
 & = & \underline{\text{let}}\ \ c = [\![e']\!] \,\overline{@}\, (\overline{\lambda} m.\, m)\ \ \underline{\text{in}}\ \ \kappa_2 \,\overline{@}\, c & (I.H.) \\
 & \equiv & [\![\langle e' \rangle]\!] \,\overline{@}\, \kappa_2 &
\end{array}
$$

e_1 **が Let のとき**

$(\text{let}\ x = e_1\ \text{in}\ e_2)[v/x] = \text{let}\ x = e_1'\ \text{in}\ e_2'$ とすると、

$$
\begin{array}{rcll}
(\text{与式}) & \equiv & \text{let}\ x = e_1\ \text{in}\ e_2 \,\overline{@}\, \kappa_1[[\![v]\!]_v/x] & \\
 & \equiv & ([\![e_1]\!] \,\overline{@}\, (\overline{\lambda} m.\, \underline{\text{let}}\ \ x = m\ \ \underline{\text{in}}\ \ [\![e_2]\!] \,\overline{@}\, \kappa_1))[[\![v]\!]_v/x] & \\
 & = & ([\![e_1]\!][[\![v]\!]_v/x]) \,\overline{@}\, (\overline{\lambda} m.\, \underline{\text{let}}\ \ x = m\ \ \underline{\text{in}}\ \ ([\![e_2]\!][[\![v]\!]_v/x]) \,\overline{@}\, (\kappa_1[[\![v]\!]_v/x])) & \\
 & = & [\![e_1[v/x]]\!] \,\overline{@}\, (\overline{\lambda} m.\, \underline{\text{let}}\ \ x = m\ \ \underline{\text{in}}\ \ ([\![e_2[v/x]]\!]) \,\overline{@}\, \kappa_2) & \\
 & = & ([\![e_1']\!] \,\overline{@}\, (\overline{\lambda} m.\, \underline{\text{let}}\ \ x = m\ \ \underline{\text{in}}\ \ [\![e_2']\!] \,\overline{@}\, \kappa_2))[[\![v]\!]_v/x] & (I.H.) \\
 & \equiv & [\![\text{let}\ x = e_1'\ \text{in}\ e_2']\!] \,\overline{@}\, \kappa_2 &
\end{array}
$$

□

補題 3.1.3: (ekSubst')

$e[v/x] = e'$ のとき、$(\llbracket e \rrbracket' \,\overline{@}\, k)[\llbracket v \rrbracket_v / x] = \llbracket e' \rrbracket' \,\overline{@}\, k$

証明.

$e = v$ (値) のとき

$v[v/x] = v'$ とすると、

$$
\begin{array}{rcll}
(\text{与式}) & \equiv & (\llbracket v \rrbracket' \,\overline{@}\, k)[\llbracket v_2 \rrbracket_v / x] & \\
& \equiv & (k \,\underline{@}\, \llbracket v \rrbracket_v)[\llbracket v_2 \rrbracket_v / x] & \\
& = & (k[\llbracket v_2 \rrbracket_v / x]) \,\underline{@}\, (\llbracket v \rrbracket_v [\llbracket v_2 \rrbracket_v / x]) & \\
& = & k \,\underline{@}\, \llbracket v[v_2/x] \rrbracket_v & (\texttt{sVar} \neq) \\
& = & k \,\underline{@}\, \llbracket v' \rrbracket_v & (\text{補題 } 1.1.1\ \ \texttt{eSubstV}) \\
& \equiv & \llbracket v' \rrbracket' \,\overline{@}\, k &
\end{array}
$$

e が App のとき

$(e_1 @ e_2)[v/x] = e'_1 @ e'_2$ とすると、

$$
\begin{array}{rcl}
(\text{与式}) & \equiv & (\llbracket e_1 @ e_2 \rrbracket' \,\overline{@}\, k)[\llbracket v \rrbracket_v / x] \\
& \equiv & (\llbracket e_1 \rrbracket \,\overline{@}\, (\overline{\lambda} m.\, \llbracket e_2 \rrbracket \,\overline{@}\, (\overline{\lambda} n.\, (m \,\underline{@}\, n) \,\underline{@}\, k)))[\llbracket v \rrbracket_v / x] \\
& = & (\llbracket e_1 \rrbracket [\llbracket v \rrbracket_v / x]) \,\overline{@}\, ((\overline{\lambda} m.\, \llbracket e_2 \rrbracket \,\overline{@}\, (\overline{\lambda} n.\, (m \,\underline{@}\, n) \,\underline{@}\, k))[\llbracket v \rrbracket_v / x]) \\
& = & \llbracket e_1[v/x] \rrbracket \,\overline{@}\, (\overline{\lambda} m.\, \llbracket e_2[v/x] \rrbracket \,\overline{@}\, ((\overline{\lambda} n.\, (m \,\underline{@}\, n) \,\underline{@}\, k)[\llbracket v \rrbracket_v / x])) \\
& = & \llbracket e'_1 \rrbracket \,\overline{@}\, (\overline{\lambda} m.\, \llbracket e'_2 \rrbracket \,\overline{@}\, (\overline{\lambda} n.\, (m \,\underline{@}\, n) \,\underline{@}\, k)) \\
& \equiv & \llbracket e'_1 @ e'_2 \rrbracket' \,\overline{@}\, k
\end{array}
$$

e が Reset のとき

$\langle e \rangle [\llbracket v \rrbracket_v / x] = \langle e' \rangle$ とすると、

$$
\begin{array}{rl}
(\text{与式}) & = (\llbracket \langle e \rangle \rrbracket' \,\overline{@}\, k)[\llbracket v \rrbracket_v / x] \\
& = (\underline{\mathsf{let}}\ \ c = \llbracket e \rrbracket \,\overline{@}\, (\overline{\lambda} m.\, m)\ \ \underline{\mathsf{in}}\ \ k \,\underline{@}\, c)[\llbracket v \rrbracket_v / x] \\
& = \underline{\mathsf{let}}\ \ c = \llbracket e' \rrbracket \,\overline{@}\, (\overline{\lambda} m.\, m)\ \ \underline{\mathsf{in}}\ \ k \,\underline{@}\, c \\
& = \llbracket \langle e' \rangle \rrbracket' \,\overline{@}\, k
\end{array}
$$

e が Let のとき

$(\mathsf{let}\ x = e_1\ \mathsf{in}\ e_2)[v/x] = \mathsf{let}\ x = e'_1\ \mathsf{in}\ e'_2$ とすると、

$$
\begin{array}{rcl}
(\text{与式}) & = & (\llbracket \mathsf{let}\ x = e_1\ \mathsf{in}\ e_2 \rrbracket' \,\overline{@}\, k)[\llbracket v \rrbracket_v / x] \\
& = & (\llbracket e_1 \rrbracket \,\overline{@}\, (\overline{\lambda} m.\, \underline{\mathsf{let}}\ \ x = m\ \ \underline{\mathsf{in}}\ \ \llbracket e_2 \rrbracket' \,\overline{@}\, k))[\llbracket v \rrbracket_v / x] \\
& = & (\llbracket e_1 \rrbracket [\llbracket v \rrbracket_v / x]) \,\overline{@}\, (\overline{\lambda} m.\, \underline{\mathsf{let}}\ \ x = m\ \ \underline{\mathsf{in}}\ \ (\llbracket e_2 \rrbracket' [\llbracket v \rrbracket_v / x]) \,\overline{@}\, k) \\
& = & \llbracket e_1[v/x] \rrbracket \,\overline{@}\, (\overline{\lambda} m.\, \underline{\mathsf{let}}\ \ x = m\ \ \underline{\mathsf{in}}\ \ \llbracket e_2[v/x] \rrbracket' \,\overline{@}\, k) \\
& = & \llbracket e'_1 \rrbracket \,\overline{@}\, (\overline{\lambda} m.\, \underline{\mathsf{let}}\ \ x = m\ \ \underline{\mathsf{in}}\ \ \llbracket e'_2 \rrbracket' \,\overline{@}\, k) \\
& = & \llbracket \mathsf{let}\ x = e'_1\ \mathsf{in}\ e'_2 \rrbracket' \,\overline{@}\, k
\end{array}
$$

□

補題 3.1.4: (κ Subst)

schematic な κ ($\kappa[v/k] = \kappa'$) について、$(\llbracket e \rrbracket \,\overline{@}\, \kappa)[v/k] = v \,\overline{@}\, \kappa'$ が成り立つ

証明.

$e = v_1$ **(値) のとき**

$$\begin{aligned}(\text{与式}) &= (\llbracket v_1 \rrbracket \,\overline{@}\, \kappa)[v/x] \\ &= \kappa \,\overline{@}\, \llbracket v_1 \rrbracket_v [v/k] \\ &= (\kappa[v/k]) \,\overline{@}\, (\llbracket v_1 \rrbracket_v [v/k]) \\ &= \kappa' \,\overline{@}\, \llbracket v \rrbracket_v \\ &= \llbracket v \rrbracket \,\overline{@}\, \kappa'\end{aligned}$$

$e = e_1 @ e_2$ **(App) のとき**

$$\begin{aligned}(\text{与式}) &= (\llbracket e_1 @ e_2 \rrbracket \,\overline{@}\, \kappa)[v/x] \\ &= (\llbracket e_1 \rrbracket \,\overline{@}\, (\overline{\lambda} m. \llbracket e_2 \rrbracket \,\overline{@}\, (\overline{\lambda} n. (m \,\underline{@}\, n) \,\underline{@}\, (\underline{\lambda} a. \kappa \,\overline{@}\, a))))[v/x] \\ &= (\llbracket e_1 \rrbracket[v/x]) \,\overline{@}\, ((\overline{\lambda} m. \llbracket e_2 \rrbracket \,\overline{@}\, (\overline{\lambda} n. (m \,\underline{@}\, n) \,\underline{@}\, (\underline{\lambda} a. \kappa \,\overline{@}\, a)))[v/x]) \\ &= \llbracket e_1 \rrbracket \,\overline{@}\, (\overline{\lambda} m. (\llbracket e_2 \rrbracket[v/x]) \,\overline{@}\, ((\overline{\lambda} n. (m \,\underline{@}\, n) \,\underline{@}\, (\underline{\lambda} a. \kappa \,\overline{@}\, a))[v/x])) \\ &= \llbracket e_1 \rrbracket \,\overline{@}\, (\overline{\lambda} m. \llbracket e_2 \rrbracket \,\overline{@}\, (\overline{\lambda} n. (m \,\underline{@}\, n) \,\underline{@}\, (\underline{\lambda} a. \kappa' \,\overline{@}\, a))) \\ &= (\llbracket e_1 @ e_2 \rrbracket) \,\overline{@}\, \kappa'\end{aligned}$$

$e = \langle e \rangle$ **(Reset) のとき**

$$\begin{aligned}(\text{与式}) &= (\llbracket \langle e \rangle \rrbracket \,\overline{@}\, \kappa)[v/x] \\ &= (\underline{\mathsf{let}}\ \ c = \llbracket e \rrbracket \,\overline{@}\, (\overline{\lambda} m. m)\ \ \underline{\mathsf{in}}\ \ \kappa \,\overline{@}\, c)[v/x] \\ &= \underline{\mathsf{let}}\ \ c = \llbracket e \rrbracket \,\overline{@}\, (\overline{\lambda} m. m)\ \ \underline{\mathsf{in}}\ \ \kappa' \,\overline{@}\, c \\ &= \llbracket \langle e \rangle \rrbracket \,\overline{@}\, \kappa'\end{aligned}$$

e **が Let のとき**

$$\begin{aligned}(\text{与式}) \quad &= \quad (\llbracket \mathsf{let}\ x = e_1\ \mathsf{in}\ e_2 \rrbracket \,\overline{@}\, \kappa)[v/x] \\ &= \quad (\llbracket e_1 \rrbracket \,\overline{@}\, (\overline{\lambda} m. \underline{\mathsf{let}}\ \ x = m\ \ \underline{\mathsf{in}}\ \ \llbracket e_2 \rrbracket \,\overline{@}\, \kappa))[v/x] \\ &= \quad \llbracket e_1 \rrbracket \,\overline{@}\, (\overline{\lambda} m. \underline{\mathsf{let}}\ \ x = m\ \ \underline{\mathsf{in}}\ \ \llbracket e_2 \rrbracket \,\overline{@}\, \kappa') \\ &= \quad \llbracket \mathsf{let}\ x = e_1\ \mathsf{in}\ e_2 \rrbracket \,\overline{@}\, \kappa'\end{aligned}$$

□

補題 3.1.5: (kSubst')

$(\llbracket e \rrbracket' \,\overline{@}\, k)[v/k] = \llbracket e \rrbracket' \,\overline{@}\, v$ が成り立つ

証明.

e **が値** $(e = v_1)$ **のとき**

$$\begin{aligned}(\text{与式}) &= \llbracket v_1 \rrbracket' \,\overline{@}\, k[v/x] \\ &= (k \,\underline{@}\, \llbracket v_1 \rrbracket_v)[v/k] \\ &= v \,\underline{@}\, \llbracket v_1 \rrbracket_v \\ &= \llbracket v_1 \rrbracket' \,\overline{@}\, v\end{aligned}$$

e **が App** $(e = e_1 \,@\, e_2)$ **のとき**

$$\begin{aligned}(\text{与式}) &= (\llbracket e_1 \,@\, e_2 \rrbracket' \,\overline{@}\, k)[v/x] \\ &= (\llbracket e_1 \rrbracket \,\overline{@}\, (\overline{\lambda} m.\, \llbracket e_2 \rrbracket \,\overline{@}\, (\overline{\lambda} n.\, (m \,\underline{@}\, n) \,\underline{@}\, k)))[v/x] \\ &= (\llbracket e_1 \rrbracket[v/x]) \,\overline{@}\, ((\overline{\lambda} m.\, \llbracket e_2 \rrbracket \,\overline{@}\, (\overline{\lambda} n.\, (m \,\underline{@}\, n) \,\underline{@}\, k))[v/x]) \\ &= \llbracket e_1 \rrbracket \,\overline{@}\, (\overline{\lambda} m.\, (\llbracket e_2 \rrbracket[v/x]) \,\overline{@}\, ((\overline{\lambda} n.\, (m \,\underline{@}\, n) \,\underline{@}\, k)[v/x])) \\ &= \llbracket e_1 \rrbracket \,\overline{@}\, (\overline{\lambda} m.\, \llbracket e_2 \rrbracket \,\overline{@}\, (\overline{\lambda} n.\, (m \,\underline{@}\, n) \,\underline{@}\, v)) \\ &= (\llbracket e_1 \,@\, e_2 \rrbracket') \,\overline{@}\, v\end{aligned}$$

e **が Reset** $(e = \langle e \rangle)$ **のとき**

$$\begin{aligned}(\text{与式}) &= (\llbracket \langle e \rangle \rrbracket' \,\overline{@}\, k)[v/x] \\ &= (\underline{\mathsf{let}}\ \ c = \llbracket e \rrbracket \,\overline{@}\, (\overline{\lambda} m.\, m)\ \ \underline{\mathsf{in}}\ \ k \,\underline{@}\, c)[v/x] \\ &= \underline{\mathsf{let}}\ \ c = \llbracket e \rrbracket \,\overline{@}\, (\overline{\lambda} m.\, m)\ \ \underline{\mathsf{in}}\ \ v \,\underline{@}\, c \\ &= \llbracket \langle e \rangle \rrbracket' \,\overline{@}\, v\end{aligned}$$

e **が Let** $(e = \textbf{let}\ c = e_1\ \textbf{in}\ e_2)$ **のとき**

$$\begin{aligned}(\text{与式}) \quad &= \quad (\llbracket \mathsf{let}\ c = e_1\ \mathsf{in}\ e_2 \rrbracket' \,\overline{@}\, k)[v/x] \\ &= \quad (\llbracket e_1 \rrbracket \,\overline{@}\, (\overline{\lambda} m.\, \underline{\mathsf{let}}\ \ c = m\ \ \underline{\mathsf{in}}\ \ \llbracket e_2 \rrbracket' \,\overline{@}\, k))[v/x] \\ &= \quad \llbracket e_1 \rrbracket \,\overline{@}\, (\overline{\lambda} m.\, \underline{\mathsf{let}}\ \ c = m\ \ \underline{\mathsf{in}}\ \ \llbracket e_2 \rrbracket' \,\overline{@}\, v) \\ &= \quad \llbracket \mathsf{let}\ x = e_1\ \mathsf{in}\ e_2 \rrbracket' \,\overline{@}\, v\end{aligned}$$

□

3.2 $[\![\cdot]\!]'$ と $[\![\cdot]\!]$ の関係性についての補題の証明

補題 3.2.1: (correctCont)

任意の項 e と schematic な 継続 κ_1, κ_2 について、$(\kappa_1 \,\overline{@}\, v) \sim (\kappa_2 \,\overline{@}\, v)$ が成り立つならば、$[\![e]\!] \,\overline{@}\, \kappa_1 \sim [\![e]\!] \,\overline{@}\, \kappa_2$ が成り立つ

証明.

e **が値** $(e = v_1)$ **のとき**

$$\begin{aligned} (\text{左式}) &\equiv [\![v]\!] \,\overline{@}\, \kappa_1 \\ &\equiv \kappa_1 \,\overline{@}\, [\![v]\!]_v \\ &\sim \kappa_2 \,\overline{@}\, [\![v]\!]_v \\ &\equiv [\![v]\!] \,\overline{@}\, \kappa_2 \end{aligned}$$

e **が App** $(e = e_1 @ e_2)$ **のとき**

$$\begin{aligned} (\text{左式}) &\equiv [\![e_1 @ e_2]\!] \,\overline{@}\, \kappa_1 \\ &\equiv [\![e_1]\!] \,\overline{@}\, (\overline{\lambda} m.\, [\![e_2]\!] \,\overline{@}\, (\overline{\lambda} n.\, (m \,\overline{@}\, n) \,\overline{@}\, (\overline{\lambda} a.\, \kappa_1 \,\overline{@}\, a))) \\ &\sim [\![e_1]\!] \,\overline{@}\, (\overline{\lambda} m.\, [\![e_2]\!] \,\overline{@}\, (\overline{\lambda} n.\, (m \,\overline{@}\, n) \,\overline{@}\, (\overline{\lambda} a.\, \kappa_2 \,\overline{@}\, a))) \\ &\equiv [\![e_1 @ e_2]\!] \,\overline{@}\, \kappa_2 \end{aligned}$$

e **が Reset** $(e = \langle e \rangle)$ **のとき**

$$\begin{aligned} (\text{左式}) &\equiv [\![\langle e \rangle]\!] \,\overline{@}\, \kappa_1 \\ &\equiv \underline{\mathsf{let}}\ x = [\![e]\!] \,\overline{@}\, (\overline{\lambda} m.\, m)\ \underline{\mathsf{in}}\ \kappa_1 \,\overline{@}\, x \\ &\sim \underline{\mathsf{let}}\ x = [\![e]\!] \,\overline{@}\, (\overline{\lambda} m.\, m)\ \underline{\mathsf{in}}\ \kappa_2 \,\overline{@}\, x \\ &\equiv [\![\langle e \rangle]\!] \,\overline{@}\, \kappa_2 \end{aligned}$$

e **が Let** $(e = \textbf{let}\ x = e_1\ \textbf{in}\ e_2)$ **のとき**

$$\begin{aligned} (\text{左式}) &\equiv [\![\mathsf{let}\ x = e_1\ \mathsf{in}\ e_2]\!] \,\overline{@}\, \kappa_1 \\ &\equiv [\![e_1]\!] \,\overline{@}\, (\overline{\lambda} m.\, \underline{\mathsf{let}}\ x = m\ \underline{\mathsf{in}}\ [\![e_2]\!] \,\overline{@}\, \kappa_1) \\ &\sim [\![e_1]\!] \,\overline{@}\, (\overline{\lambda} m.\, \underline{\mathsf{let}}\ x = m\ \underline{\mathsf{in}}\ [\![e_2]\!] \,\overline{@}\, \kappa_2) \\ &\equiv [\![\mathsf{let}\ x = e_1\ \mathsf{in}\ e_2]\!] \,\overline{@}\, \kappa_2 \end{aligned}$$

□

補題 3.2.2: (correctEtaEta')

$[\![e]\!]' \mathbin{\overline{@}} (\underline{\lambda} a.\, \kappa \mathbin{\overline{@}} a) \rightsquigarrow^{*} [\![e]\!] \mathbin{\overline{@}} \kappa$ が成り立つ

証明.

e **が値** $(e = v_1)$ **のとき**

$$\begin{array}{rcl}
(\text{左式}) & \equiv & [\![v]\!]' \mathbin{\overline{@}} (\underline{\lambda} a.\, \kappa \mathbin{\overline{@}} a) \\
& \equiv & (\overline{\lambda} k.\, k \mathbin{\underline{@}} [\![v]\!]_v) \mathbin{\overline{@}} (\underline{\lambda} a.\, \kappa \mathbin{\overline{@}} a) \\
& \equiv & (\underline{\lambda} a.\, \kappa \mathbin{\overline{@}} a) \mathbin{\underline{@}} [\![v]\!]_v \\
& \rightsquigarrow & \kappa \mathbin{\overline{@}} [\![v]\!]_v \\
& \equiv & [\![v]\!] \mathbin{\overline{@}} \kappa
\end{array}$$

e **が App** $(e = e_1 \mathbin{@} e_2)$ **のとき**

$$\begin{array}{rcl}
(\text{左式}) & \equiv & [\![e_1 \mathbin{@} e_2]\!]' \mathbin{\overline{@}} (\underline{\lambda} a.\, \kappa \mathbin{\overline{@}} a) \\
& \equiv & [\![e_1]\!] \mathbin{\overline{@}} (\overline{\lambda} m.\, [\![e_2]\!] \mathbin{\overline{@}} (\overline{\lambda} n.\, (m \mathbin{\underline{@}} n) \mathbin{\underline{@}} (\underline{\lambda} a.\, \kappa \mathbin{\overline{@}} a))) \\
& \equiv & [\![e_1 \mathbin{@} e_2]\!] \mathbin{\overline{@}} \kappa
\end{array}$$

e **が Reset** $(e = \langle e \rangle)$ **のとき**

$$\begin{array}{rcl}
(\text{左式}) & \equiv & [\![\langle e \rangle]\!]' \mathbin{\overline{@}} (\underline{\lambda} a.\, \kappa \mathbin{\overline{@}} a) \\
& \equiv & \underline{\mathsf{let}}\ \ c = [\![e]\!] \mathbin{\overline{@}} (\overline{\lambda} m.\, m)\ \ \underline{\mathsf{in}}\ \ (\underline{\lambda} a.\, \kappa \mathbin{\overline{@}} a) \mathbin{\underline{@}} c \\
& \rightsquigarrow & \underline{\mathsf{let}}\ \ c = [\![e]\!] \mathbin{\overline{@}} (\overline{\lambda} m.\, m)\ \ \underline{\mathsf{in}}\ \ \kappa \mathbin{\underline{@}} c \\
& \equiv & [\![\langle e \rangle]\!] \mathbin{\overline{@}} \kappa
\end{array}$$

e **が Let** $(e = \textbf{let}\ x = e_1\ \textbf{in}\ e_2)$ **のとき**

$$\begin{array}{rcl}
(\text{左式}) & \equiv & [\![\mathsf{let}\ x = e_1\ \mathsf{in}\ e_2]\!]' \mathbin{\overline{@}} (\underline{\lambda} a.\, \kappa \mathbin{\overline{@}} a) \\
& \equiv & [\![e_1]\!] \mathbin{\overline{@}} (\overline{\lambda} m.\, \underline{\mathsf{let}}\ \ x = m\ \ \underline{\mathsf{in}}\ \ [\![e_2]\!]' \mathbin{\overline{@}} (\underline{\lambda} a.\, \kappa \mathbin{\overline{@}} a)) \\
& \sim & [\![e_1]\!] \mathbin{\overline{@}} (\overline{\lambda} m.\, \underline{\mathsf{let}}\ \ x = m\ \ \underline{\mathsf{in}}\ \ [\![e_2]\!] \mathbin{\overline{@}} \kappa) \\
& \equiv & [\![\mathsf{let}\ x = e_1\ \mathsf{in}\ e_2]\!] \mathbin{\overline{@}} \kappa
\end{array}$$

$\square$

3.3 ピュアコンテキストに関する代入補題

補題 3.3.1: (subst-context)

任意のピュアコンテキスト con について、$E_{\mathsf{con}}[\ x\ ][v/x] = E_{\mathsf{con}}[\ v\ ]$ が成り立つ

証明.

con が Hole のとき

$$\begin{aligned}(\text{左式}) &\equiv x[v/x] \\ &= v\end{aligned}$$

con が Frame ($\mathbf{App}_1$ $\mathbf{e}_2$) のとき

$$\begin{aligned}(\text{左式}) &\equiv (x @ e_2)[v/x] \\ &= v @ e_2\end{aligned}$$

con が Frame ($\mathbf{App}_2$ $\mathbf{v}_1$) のとき

$$\begin{aligned}(\text{左式}) &\equiv (v_1 @ x)[v/x] \\ &= v_1 @ v\end{aligned}$$

con が Frame (Let $\mathbf{e}_2$) のとき

$$\begin{aligned}(\text{左式}) &\equiv (\mathsf{let}\ c = x\ \mathsf{in}\ e_2)[v/x] \\ &= \mathsf{let}\ c = v\ \mathsf{in}\ e_2\end{aligned}$$

□

3.4 Shift に関する補題

補題 3.4.1: (contextContE)

$\llbracket E_{p_1}[\ \mathcal{S} @ v\ ] \rrbracket \,\overline{@}\, \kappa \equiv \llbracket \mathcal{S} @ v \rrbracket' \,\overline{@}\, (\underline{\lambda} a.\, \llbracket E_{p_2}[\ a\ ] \rrbracket \,\overline{@}\, \kappa)$ が成り立つことを証明する

証明.

p_1, p_2 **が Hole のとき**

$$
\begin{aligned}
(\text{左式}) &\equiv \llbracket \mathcal{S} @ v \rrbracket \,\overline{@}\, \kappa \\
&\equiv \llbracket \mathcal{S} \rrbracket \,\overline{@}\, (\overline{\lambda} m.\, \llbracket v \rrbracket \,\overline{@}\, (\overline{\lambda} n.\, (m \,\overline{@}\, n) \,\overline{@}\, (\underline{\lambda} a.\, \kappa \,\overline{@}\, a))) \\
&\equiv \llbracket \mathcal{S} @ v \rrbracket' \,\overline{@}\, (\underline{\lambda} a.\, \kappa \,\overline{@}\, a) \\
&\equiv \llbracket \mathcal{S} @ v \rrbracket' \,\overline{@}\, \underline{\lambda} a.\, \llbracket a \rrbracket \,\overline{@}\, \kappa \\
&\equiv \llbracket \mathcal{S} @ v \rrbracket' \,\overline{@}\, \underline{\lambda} a.\, \llbracket E_{p_2}[\ a\ ] \rrbracket \,\overline{@}\, \kappa
\end{aligned}
$$

p_1, p_2 **が Frame (App$_1$ e$_2$) のとき**

$$
\begin{aligned}
(\text{左式}) &\equiv \llbracket E_{p'_1}[\ \mathcal{S} @ v\ ] @ e_2 \rrbracket \,\overline{@}\, \kappa \\
&\equiv \llbracket E_{p'_1}[\ \mathcal{S} @ v\ ] @ e_2 \rrbracket \,\overline{@}\, (\overline{\lambda} m.\, \llbracket e_2 \rrbracket \,\overline{@}\, (\overline{\lambda} n.\, (m \,\overline{@}\, n) \,\overline{@}\, (\underline{\lambda} a.\, \kappa \,\overline{@}\, a))) \\
&\equiv \llbracket \mathcal{S} @ v \rrbracket' \,\overline{@}\, \underline{\lambda} a.\, \llbracket E_{p'_2}[\ a\ ] \rrbracket \,\overline{@}\, (\overline{\lambda} m.\, \llbracket e_2 \rrbracket \,\overline{@}\, (\overline{\lambda} n.\, (m \,\overline{@}\, n) \,\overline{@}\, (\underline{\lambda} a.\, \kappa \,\overline{@}\, a))) \\
&\equiv \llbracket \mathcal{S} @ v \rrbracket' \,\overline{@}\, (\underline{\lambda} a.\, \llbracket E_{p'_2}[\ a\ ] @ e_2 \rrbracket \,\overline{@}\, \kappa) \\
&\equiv \llbracket \mathcal{S} @ v \rrbracket' \,\overline{@}\, (\underline{\lambda} a.\, \llbracket E_{p_2}[\ a\ ] \rrbracket \,\overline{@}\, \kappa)
\end{aligned}
$$

p_1, p_2 **が Frame (App$_2$ v$_1$) のとき**

$$
\begin{aligned}
(\text{左式}) &\equiv \llbracket v_1 @ E_{p'_1}[\ \mathcal{S} @ v\ ] \rrbracket \,\overline{@}\, \kappa \\
&\equiv \llbracket v_1 \rrbracket \,\overline{@}\, (\overline{\lambda} m.\, \llbracket E_{p'_1}[\ \mathcal{S} @ v\ ] \rrbracket \,\overline{@}\, (\overline{\lambda} n.\, (m \,\overline{@}\, n) \,\overline{@}\, (\underline{\lambda} a.\, \kappa \,\overline{@}\, a))) \\
&\equiv \llbracket E_{p'_1}[\ \mathcal{S} @ v\ ] \rrbracket \,\overline{@}\, (\overline{\lambda} n.\, (\llbracket v_1 \rrbracket_v \,\underline{@}\, n) \,\underline{@}\, (\underline{\lambda} a.\, \kappa \,\overline{@}\, a)) \\
&\equiv \llbracket \mathcal{S} @ v \rrbracket' \,\overline{@}\, (\underline{\lambda} a.\, \llbracket E_{p'_2}[\ a\ ] \rrbracket \,\overline{@}\, (\overline{\lambda} n.\, (\llbracket v_1 \rrbracket_v \,\underline{@}\, n) \,\underline{@}\, (\underline{\lambda} a.\, \kappa \,\overline{@}\, a))) \\
&\equiv \llbracket \mathcal{S} @ v \rrbracket' \,\overline{@}\, (\underline{\lambda} a.\, \llbracket v_1 @ E_{p'_2}[\ a\ ] \rrbracket \,\overline{@}\, \kappa) \\
&\equiv \llbracket \mathcal{S} @ v \rrbracket' \,\overline{@}\, (\underline{\lambda} a.\, \llbracket E_{p_2}[\ a\ ] \rrbracket \,\overline{@}\, \kappa)
\end{aligned}
$$

p_1, p_2 **が Frame (Let e$_2$) のとき**

$$
\begin{aligned}
(\text{左式}) &\equiv \llbracket \mathsf{let}\ x = E_{p'_1}[\ \mathcal{S} @ v\ ]\ \mathsf{in}\ e_2 \rrbracket \,\overline{@}\, \kappa \\
&\equiv \llbracket E_{p'_1}[\ \mathcal{S} @ v\ ] \rrbracket \,\overline{@}\, (\overline{\lambda} m.\, \underline{\mathsf{let}}\ \ x = m\ \ \underline{\mathsf{in}}\ \ e_2 \,\overline{@}\, \kappa) \\
&\equiv \llbracket \mathcal{S} @ v \rrbracket' \,\overline{@}\, (\underline{\lambda} a.\, \llbracket E_{p'_2}[\ a\ ] \rrbracket \,\overline{@}\, (\overline{\lambda} m.\, \underline{\mathsf{let}}\ \ x = m\ \ \underline{\mathsf{in}}\ \ e_2 \,\overline{@}\, \kappa)) \\
&\equiv \llbracket \mathcal{S} @ v \rrbracket' \,\overline{@}\, (\underline{\lambda} a.\, \llbracket \mathsf{let}\ x = E_{p'_2}[\ a\ ]\ \mathsf{in}\ e_2 \rrbracket \,\overline{@}\, \kappa) \\
&\equiv \llbracket \mathcal{S} @ v \rrbracket' \,\overline{@}\, (\underline{\lambda} a.\, \llbracket E_{p_2}[\ a\ ] \rrbracket \,\overline{@}\, \kappa)
\end{aligned}
$$

□

4 CPS 変換の正当性の証明

この節では、CPS 変換の正当性の証明として、CPS 変換が項の簡約関係を保存することを示す。

4.1 変換の証明

定理 4.1: (CPS 変換の正当性の証明)

任意の項 e, e' について $e \to e'$ が成り立つならば、任意の schematic な継続 κ について $\llbracket e \rrbracket \,\overline{@}\, \kappa \to *e' \,\overline{@}\, \kappa$

これは、以下のような図を意味する。

$$\begin{tikzcd}
e \arrow[rr, "\text{Reduce}"] \arrow[d, "\text{CPS}"'] & & e' \arrow[d, "\text{CPS}"] \\
\llbracket e \rrbracket \arrow[dr, "\text{Substitute } \kappa"'] & & \llbracket e' \rrbracket \arrow[dl, "\text{Substitute } \kappa"] \\
& \llbracket e \rrbracket \,\overline{@}\, \kappa =_\beta \llbracket e' \rrbracket \,\overline{@}\, \kappa &
\end{tikzcd}$$

この図にある Reduce の部分について、RBeta、RFrame (App_1)、RFrame (App_2)、RReset、RShift のケースについて場合分けをして帰納的に解く。

4.1.1 RBeta のケース

RBeta のケースでの証明を行う。

$$\begin{tikzcd}
(\lambda x.\, e_1) \,@\, v_2 \arrow[rr, "\text{RBeta}"] \arrow[d, "\text{CPS}"'] & & e_2 \arrow[d, "\text{CPS}"] \\
\llbracket (\lambda x.\, e_1) \,@\, v_2 \rrbracket \arrow[dr, "\text{Substitute } \kappa"'] & & \llbracket e_2 \rrbracket \arrow[dl, "\text{Substitute } \kappa"] \\
& \llbracket (\lambda x.\, e_1) \,@\, v_2 \rrbracket \,\overline{@}\, \kappa =_\beta \llbracket e_2 \rrbracket \,\overline{@}\, \kappa &
\end{tikzcd}$$

$$\begin{array}{lll}
 & \llbracket (\lambda x.\, e_1) \,@\, v_2 \rrbracket \,\overline{@}\, \kappa & \\
= & ((\underline{\lambda} x.\, \underline{\lambda} k.\, \llbracket e_1 \rrbracket' \,\overline{@}\, k) \,\underline{@}\, \llbracket v_2 \rrbracket_v) \,\underline{@}\, (\underline{\lambda} a.\, \kappa \,\overline{@}\, a) & \\
\to & (\underline{\lambda} k.\, \llbracket e_2 \rrbracket' \,\overline{@}\, k) \,\underline{@}\, (\underline{\lambda} a.\, \kappa \,\overline{@}\, a) & \text{(補題 1.1 ekSubst')} \\
\to & \llbracket e_2 \rrbracket' \,\underline{@}\, (\underline{\lambda} a.\, \kappa \,\overline{@}\, a) & \text{(補題 1.2 kSubst')} \\
\to^* & e_2 \,\overline{@}\, \kappa & \text{(補題 1.3 cpsEtaEta')}
\end{array}$$

4.1.2 RFrame($\mathbf{App}_1$) のケース

$$
\begin{tikzcd}
e_1 \mathbin{@} e_2 \arrow[rr, "\mathsf{RFrame(App_1\ e_2)}"] \arrow[d, "\mathsf{CPS}"'] & & e_1' \mathbin{@} e_2 \arrow[d, "\mathsf{CPS}"] \\
\llbracket e_1 \mathbin{@} e_2 \rrbracket \arrow[dr, "\mathsf{Substitute}\ \kappa"'] & & \llbracket e_1' \mathbin{@} e_2 \rrbracket \arrow[dl, "\mathsf{Substitute}\ \kappa"] \\
& \llbracket e_1 \mathbin{@} e_2 \rrbracket \mathbin{\overline{@}} \kappa =_\beta \llbracket e_1' \mathbin{@} e_2 \rrbracket \mathbin{\overline{@}} \kappa &
\end{tikzcd}
$$

$$
\begin{array}{ll}
 & \llbracket e_1 \mathbin{@} e_2 \rrbracket \mathbin{\overline{@}} \kappa \\
\equiv & \llbracket e_1 \rrbracket \mathbin{\overline{@}} (\overline{\lambda} m.\, \llbracket e_2 \rrbracket \mathbin{\overline{@}} (\overline{\lambda} n.\, (m \mathbin{\underline{@}} n) \mathbin{\underline{@}} (\underline{\lambda} a.\, \kappa \mathbin{\overline{@}} a))) \\
\leadsto & \llbracket e_1' \rrbracket \mathbin{\overline{@}} (\overline{\lambda} m.\, \llbracket e_2 \rrbracket \mathbin{\overline{@}} (\overline{\lambda} n.\, (m \mathbin{\underline{@}} n) \mathbin{\underline{@}} (\underline{\lambda} a.\, \kappa \mathbin{\overline{@}} a))) \\
\equiv & \llbracket e_1' \mathbin{@} e_2 \rrbracket \mathbin{\overline{@}} \kappa
\end{array}
$$

4.1.3 RFrame($\mathbf{App}_2$) のケース

$$
\begin{tikzcd}
v_1 \mathbin{@} e_2 \arrow[rr, "\mathsf{RFrame(App_2\ v_1)}"] \arrow[d, "\mathsf{CPS}"'] & & v_1 \mathbin{@} e_2' \arrow[d, "\mathsf{CPS}"] \\
\llbracket v_1 \mathbin{@} e_2 \rrbracket \arrow[dr, "\mathsf{Substitute}\ \kappa"'] & & \llbracket v_1 \mathbin{@} e_2' \rrbracket \arrow[dl, "\mathsf{Substitute}\ \kappa"] \\
& \llbracket v_1 \mathbin{@} e_2 \rrbracket \mathbin{\overline{@}} \kappa =_\beta \llbracket v_1 \mathbin{@} e_2' \rrbracket \mathbin{\overline{@}} \kappa &
\end{tikzcd}
$$

$$
\begin{array}{ll}
 & \llbracket v_1 \mathbin{@} e_2 \rrbracket \mathbin{\overline{@}} \kappa \\
\equiv & \llbracket v_1 \rrbracket \mathbin{\overline{@}} (\overline{\lambda} m.\, \llbracket e_2 \rrbracket \mathbin{\overline{@}} (\overline{\lambda} n.\, (m \mathbin{\underline{@}} n) \mathbin{\underline{@}} (\underline{\lambda} a.\, \kappa \mathbin{\overline{@}} a))) \\
\equiv & \llbracket e_2 \rrbracket \mathbin{\overline{@}} (\overline{\lambda} n.\, \llbracket v_1 \rrbracket_v \mathbin{\underline{@}} n \mathbin{\underline{@}} (\underline{\lambda} a.\, \kappa \mathbin{\overline{@}} a)) \\
\leadsto & \llbracket e_2' \rrbracket \mathbin{\overline{@}} (\overline{\lambda} n.\, \llbracket v_1 \rrbracket_v \mathbin{\underline{@}} n \mathbin{\underline{@}} (\underline{\lambda} a.\, \kappa \mathbin{\overline{@}} a)) \\
\equiv & \llbracket v_1 \mathbin{@} e_2' \rrbracket \mathbin{\overline{@}} \kappa
\end{array}
$$

4.1.4 RFrame(Let) のケース

$$
\begin{tikzcd}
\mathsf{let}\ x = e_1\ \mathsf{in}\ e_2 \arrow[rr, "\mathsf{RFrame(Let\ e_2)}"] \arrow[d, "\mathsf{CPS}"'] & & \mathsf{let}\ x = e_1'\ \mathsf{in}\ e_2 \arrow[d, "\mathsf{CPS}"] \\
[\![\mathsf{let}\ x = e_1\ \mathsf{in}\ e_2]\!] \arrow[dr, "\mathsf{Substitute}\ \kappa"'] & & [\![\mathsf{let}\ x = e_1'\ \mathsf{in}\ e_2]\!] \arrow[dl, "\mathsf{Substitute}\ \kappa"] \\
& [\![\mathsf{let}\ x = e_1\ \mathsf{in}\ e_2]\!]\,\overline{@}\,\kappa =_\beta [\![\mathsf{let}\ x = e_1'\ \mathsf{in}\ e_2]\!]\,\overline{@}\,\kappa &
\end{tikzcd}
$$

$$
\begin{array}{ll}
 & [\![\mathsf{let}\ x = e_1\ \mathsf{in}\ e_2]\!]\,\overline{@}\,\kappa \\
\equiv & [\![e_1]\!]\,\overline{@}\,(\overline{\lambda} m.\, \underline{\mathsf{let}}\ \ x = m\ \ \underline{\mathsf{in}}\ \ [\![e_2]\!]\,\overline{@}\,\kappa) \\
\rightsquigarrow & [\![e_1']\!]\,\overline{@}\,(\overline{\lambda} m.\, \underline{\mathsf{let}}\ \ x = m\ \ \underline{\mathsf{in}}\ \ [\![e_2]\!]\,\overline{@}\,\kappa) \\
\equiv & [\![\mathsf{let}\ x = e_1'\ \mathsf{in}\ e_2]\!]\,\overline{@}\,\kappa
\end{array}
$$

4.1.5 RFrame(Reset) のケース

$$
\begin{tikzcd}
\langle e \rangle \arrow[rr, "\mathsf{RFrame(Reset\ e)}"] \arrow[d, "\mathsf{CPS}"'] & & \langle e' \rangle \arrow[d, "\mathsf{CPS}"] \\
[\![\langle e \rangle]\!] \arrow[dr, "\mathsf{Substitute}\ \kappa"'] & & [\![\langle e' \rangle]\!] \arrow[dl, "\mathsf{Substitute}\ \kappa"] \\
& [\![\langle e \rangle]\!]\,\overline{@}\,\kappa =_\beta [\![\langle e' \rangle]\!]\,\overline{@}\,\kappa &
\end{tikzcd}
$$

$$
\begin{array}{ll}
 & [\![\langle e \rangle]\!]\,\overline{@}\,\kappa \\
\equiv & \underline{\mathsf{let}}\ \ x = [\![e]\!]\,\overline{@}\,(\overline{\lambda} m.\, m)\ \ \underline{\mathsf{in}}\ \ \kappa\,\overline{@}\,x \\
\rightsquigarrow & \underline{\mathsf{let}}\ \ x = [\![e']\!]\,\overline{@}\,(\overline{\lambda} m.\, m)\ \ \underline{\mathsf{in}}\ \ \kappa\,\overline{@}\,x \\
\equiv & [\![\langle e' \rangle]\!]\,\overline{@}\,\kappa
\end{array}
$$

4.1.6 RLet のケース

$$
\begin{tikzcd}
\mathsf{let}\ x = v_1\ \mathsf{in}\ e_2 \arrow[rr, "\mathsf{RLet}"] \arrow[d, "\mathsf{CPS}"'] & & e_2' \arrow[d, "\mathsf{CPS}"] \\
[\![\mathsf{let}\ x = v_1\ \mathsf{in}\ e_2]\!] \arrow[dr, "\mathsf{Substitute}\ \kappa"'] & & [\![e_2']\!] \arrow[dl, "\mathsf{Substitute}\ \kappa"] \\
& [\![\mathsf{let}\ x = v_1\ \mathsf{in}\ e_2]\!]\,\overline{@}\,\kappa =_\beta [\![e_2']\!]\,\overline{@}\,\kappa &
\end{tikzcd}
$$

$$
\begin{array}{ll}
 & [\![\mathsf{let}\ x = v_1\ \mathsf{in}\ e_2]\!]\,\overline{@}\,\kappa \\
\equiv & [\![v_1]\!]\,\overline{@}\,(\overline{\lambda} m.\, \underline{\mathsf{let}}\ \ x = m\ \ \underline{\mathsf{in}}\ \ [\![e_2]\!]\,\overline{@}\,\kappa) \\
\equiv & \underline{\mathsf{let}}\ \ x = [\![v_1]\!]_v\ \ \underline{\mathsf{in}}\ \ [\![e_2]\!]\,\overline{@}\,\kappa \\
\rightsquigarrow & [\![e_2']\!]\,\overline{@}\,\kappa
\end{array}
$$

4.1.7 RReset のケース

$$
\begin{tikzcd}
\langle v \rangle \arrow[rr, "\mathsf{RReset}"] \arrow[d, "\mathsf{CPS}"'] & & v \arrow[d, "\mathsf{CPS}"] \\
\llbracket \langle v \rangle \rrbracket \arrow[dr, "\mathsf{Substitute}\ \kappa"'] & & \llbracket v \rrbracket \arrow[dl, "\mathsf{Substitute}\ \kappa"] \\
& \llbracket \langle v \rangle \rrbracket \mathbin{\overline{@}} \kappa =_\beta \llbracket v \rrbracket \mathbin{\overline{@}} \kappa &
\end{tikzcd}
$$

$$
\begin{array}{ll}
& \llbracket \langle v \rangle \rrbracket \mathbin{\overline{@}} \kappa \\
\equiv & \underline{\mathsf{let}}\ \ x = \llbracket v \rrbracket \mathbin{\overline{@}} (\overline{\lambda} m.\, m)\ \ \underline{\mathsf{in}}\ \ \kappa \mathbin{\overline{@}} x \\
\equiv & \underline{\mathsf{let}}\ \ x = \llbracket v \rrbracket_v\ \ \underline{\mathsf{in}}\ \ \kappa \mathbin{\overline{@}} x \\
\rightsquigarrow & \kappa \mathbin{\overline{@}} \llbracket v \rrbracket_v \\
\equiv & \llbracket v \rrbracket \mathbin{\overline{@}} \kappa
\end{array}
$$

4.1.8 RShift のケース

$$
\begin{tikzcd}
\langle E_{p_1} [\ \mathcal{S} @ v\ ] \rangle \arrow[rr, "\mathsf{RShift}"] \arrow[d, "\mathsf{CPS}"'] & & \langle v @ (\lambda y.\, \langle E_{p_2} [\ y\ ] \rangle) \rangle \arrow[d, "\mathsf{CPS}"] \\
\llbracket \langle E_{p_1} [\ \mathcal{S} @ v\ ] \rangle \rrbracket \arrow[dr, "\mathsf{Substitute}\ \kappa"'] & & \llbracket \langle v @ (\lambda y.\, \langle E_{p_2} [\ y\ ] \rangle) \rangle \rrbracket \arrow[dl, "\mathsf{Substitute}\ \kappa"] \\
& \llbracket \langle E_{p_1} [\ \mathcal{S} @ v\ ] \rangle \rrbracket \mathbin{\overline{@}} \kappa =_\beta \llbracket \langle v @ (\lambda y.\, \langle E_{p_2} [\ y\ ] \rangle) \rangle \rrbracket \mathbin{\overline{@}} \kappa &
\end{tikzcd}
$$

$$
\begin{array}{ll}
& \llbracket \langle E_{p_1} [\ \mathcal{S} @ v\ ] \rangle \rrbracket \mathbin{\overline{@}} \kappa \\
\equiv & \underline{\mathsf{let}}\ \ x = \llbracket E_{p_1} [\ \mathcal{S} @ v\ ] \rrbracket \mathbin{\overline{@}} (\overline{\lambda} m.\, m)\ \ \underline{\mathsf{in}}\ \ \kappa \mathbin{\overline{@}} x \\
\equiv & \underline{\mathsf{let}}\ \ x = \llbracket \mathcal{S} @ v \rrbracket' \mathbin{\overline{@}} (\underline{\lambda} a.\, \llbracket E_{p_2} [\ a\ ] \rrbracket \mathbin{\overline{@}} (\overline{\lambda} m.\, m))\ \ \underline{\mathsf{in}}\ \ \kappa \mathbin{\overline{@}} x \\
\equiv & \underline{\mathsf{let}}\ \ x = (\llbracket \mathcal{S} \rrbracket_v \mathbin{\underline{@}} \llbracket v \rrbracket_v) \mathbin{\underline{@}} (\underline{\lambda} a.\, \llbracket E_{p_2} [\ a\ ] \rrbracket \mathbin{\overline{@}} (\overline{\lambda} m.\, m))\ \ \underline{\mathsf{in}}\ \ \kappa \mathbin{\overline{@}} x \\
\equiv & \underline{\mathsf{let}}\ \ x = ((\underline{\lambda} wk.\, (w \mathbin{\underline{@}} (\underline{\lambda} ak'.\, k' \mathbin{\underline{@}} (k \mathbin{\underline{@}} a))) \mathbin{\underline{@}} (\underline{\lambda} m.\, m)) \mathbin{\underline{@}} \llbracket v \rrbracket_v) \mathbin{\underline{@}} (\underline{\lambda} a.\, \llbracket E_{p_2} [\ a\ ] \rrbracket \mathbin{\overline{@}} (\overline{\lambda} m.\, m))\ \ \underline{\mathsf{in}}\ \ \kappa \mathbin{\overline{@}} x \\
\rightsquigarrow & \underline{\mathsf{let}}\ \ x = (\llbracket v \rrbracket_v \mathbin{\underline{@}} (\underline{\lambda} ak'.\, k' \mathbin{\underline{@}} ((\underline{\lambda} a.\, \llbracket E_{p_2} [\ a\ ] \rrbracket \mathbin{\overline{@}} (\overline{\lambda} m.\, m)) \mathbin{\underline{@}} a))) \mathbin{\underline{@}} (\underline{\lambda} m.\, m)\ \ \underline{\mathsf{in}}\ \ \kappa \mathbin{\overline{@}} x \\
\equiv & \underline{\mathsf{let}}\ \ x = (\llbracket v \rrbracket_v \mathbin{\underline{@}} (\underline{\lambda} ak'.\, k' \mathbin{\underline{@}} (\llbracket a \rrbracket' \mathbin{\overline{@}} (\underline{\lambda} a.\, \llbracket E_{p_2} [\ a\ ] \rrbracket \mathbin{\overline{@}} (\overline{\lambda} m.\, m))))) \mathbin{\underline{@}} (\underline{\lambda} m.\, m)\ \ \underline{\mathsf{in}}\ \ \kappa \mathbin{\overline{@}} x \\
\equiv & \underline{\mathsf{let}}\ \ x = (\llbracket v \rrbracket_v \mathbin{\underline{@}} (\underline{\lambda} ak'.\, k' \mathbin{\underline{@}} (\llbracket E_{p_2} [\ a\ ] \rrbracket \mathbin{\overline{@}} (\overline{\lambda} m.\, m)))) \mathbin{\underline{@}} (\underline{\lambda} m.\, m)\ \ \underline{\mathsf{in}}\ \ \kappa \mathbin{\overline{@}} x \\
\sim & \underline{\mathsf{let}}\ \ x = (\llbracket v \rrbracket_v \mathbin{\underline{@}} (\underline{\lambda} ak'.\, \llbracket \langle E_{p_2} [\ a\ ] \rangle \rrbracket' \mathbin{\overline{@}} k')) \mathbin{\underline{@}} (\underline{\lambda} m.\, m)\ \ \underline{\mathsf{in}}\ \ \kappa \mathbin{\overline{@}} x \\
\equiv & \underline{\mathsf{let}}\ \ x = (\llbracket v \rrbracket_v \mathbin{\underline{@}} \llbracket \lambda a.\, E_{p_2} [\ a\ ] \rrbracket_v) \mathbin{\underline{@}} (\underline{\lambda} m.\, m)\ \ \underline{\mathsf{in}}\ \ \kappa \mathbin{\overline{@}} x \\
\equiv & \llbracket \langle v @ (\lambda y.\, \langle E_{p_2} [\ y\ ] \rangle) \rangle \rrbracket \mathbin{\overline{@}} \kappa
\end{array}
$$