CCS 変換メモ

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2021 年 8 月 3 日

1 DS 項の定義

ここでは、CPS 変換前の項、すなわち CPS 変換を行う対象の言語を示す。

1.1 構文

$$
\begin{array}{lcll}
\tau & ::= & \mathsf{Nat} \mid \mathsf{bool} \mid \tau_1 \to \tau_2 @\mathrm{cps}[\tau_3, \tau_4] & \text{型} \\
v & ::= & n \mid x \mid \lambda x.\, e \mid \mathcal{S} & \text{値} \\
e & ::= & v \mid e_1 @ e_2 \mid \langle e \rangle \mid \mathsf{let}\ x = e_1\ \mathsf{in}\ e_2 & \text{項}
\end{array}
$$

1.2 DS 項の型付け規則

$$
\frac{\Gamma(x) = \tau_1}{\Gamma \vdash x : \tau_1 @\mathrm{cps}[\tau, \tau]} \ (\mathrm{TV{\scriptsize AR}}) \qquad \frac{}{\Gamma \vdash n : \mathsf{Nat} @\mathrm{cps}[\tau, \tau]} \ (\mathrm{TN{\scriptsize UM}})
$$

$$
\frac{\Gamma, x : \tau_2 \vdash e : \tau_1 @\mathrm{cps}[\tau_3, \tau_4]}{\Gamma \vdash \lambda x.\, e : (\tau_2 \to \tau_1 @\mathrm{cps}[\tau_3, \tau_4]) @\mathrm{cps}[\tau, \tau]} \ (\mathrm{TF{\scriptsize UN}})
$$

$$
\frac{}{\Gamma \vdash \mathcal{S} : (((\tau_3 \to \tau_4 @\mathrm{cps}[\tau, \tau]) \to \tau_1 @\mathrm{cps}[\tau_1, \tau_2]) \to \tau_3 @\mathrm{cps}[\tau_4, \tau_2]) @\mathrm{cps}[\tau, \tau]} \ (\mathrm{TS{\scriptsize HIFT}})
$$

$$
\frac{\Gamma \vdash e_1 : (\tau_2 \to \tau_1 @\mathrm{cps}[\tau_3, \tau_4]) @\mathrm{cps}[\tau_5, \tau_6] \quad \Gamma \vdash e_2 : \tau_2 @\mathrm{cps}[\tau_4, \tau_5]}{\Gamma \vdash e_1 @ e_2 : \tau_1 @\mathrm{cps}[\tau_3, \tau_6]} \ (\mathrm{TA{\scriptsize PP}})
$$

$$
\frac{\Gamma \vdash e : \tau_1 @\mathrm{cps}[\tau_1, \tau_2]}{\Gamma \vdash \langle e \rangle : \tau_2 @\mathrm{cps}[\tau_3, \tau_3]} \ (\mathrm{TR{\scriptsize ESET}})
$$

$$
\frac{\Gamma \vdash e_1 : \tau_1 @\mathrm{cps}[\beta, \gamma] \quad \Gamma, x : \tau_1 \vdash e_2 : \tau_2 @\mathrm{cps}[\alpha, \beta]}{\Gamma \vdash (\mathsf{let}\ x = e_1\ \mathsf{in}\ e_2) : \tau_2 @\mathrm{cps}[\alpha, \gamma]} \ (\mathrm{TL{\scriptsize ET}})
$$

1.3 DS 項の代入規則

代入規則は、$e[v/x] = e'$ と表現することができ、「項 e の中に現れる変数 x を値 v に置き換える」と読む。

今回用いる代入規則は以下である。

$$\frac{}{x[v/x] = x}\ (\text{sVAR=}) \qquad \frac{}{y[v/x] = y}\ (\text{sVAR} \neq) \qquad \frac{}{n[v/x] = n}\ (\text{sNUM})$$

$$\frac{\forall x.(e[v/y] = e')}{(\lambda x.\, e)[v/y] = \lambda x.\, e'}\ (\text{sFUN}) \qquad \frac{e_1[v/x] = e_1' \quad e_2[v/x] = e_2'}{(e_1 \mathbin{@} e_2)[v/x] = (e_1' \mathbin{@} e_2')}\ (\text{sAPP})$$

$$\frac{}{\mathcal{S}[v/x] = \mathcal{S}}\ (\text{sSHIFT}) \qquad \frac{e_1[v/x] = e_1'}{\langle e_1 \rangle [v/x] = \langle e_1' \rangle}\ (\text{sRESET})$$

$$\frac{e_1[v/y] = e_1' \quad \forall x.(e_2[v/y] = e_2')}{(\mathsf{let}\ x = e_1\ \mathsf{in}\ e_2)[v/y] = \mathsf{let}\ x = e_1'\ \mathsf{in}\ e_2'}\ (\text{sLET})$$

1.4 DS 項の簡約規則

次に、簡約規則について示す。ただし、簡約規則を示す前に、フレームを定義する必要がある。フレームを定義することによって、簡約の順序を決めることができる。

$$\begin{array}{rcl} \text{フレーム}\ \ F & = & [\ ] \mathbin{@} e_2 \mid v_1 \mathbin{@} [\ ] \mid \langle\, [\ ]\, \rangle \mid \mathsf{let}\ x = [\ ]\ \mathsf{in}\ e_2 \\ \text{評価文脈 (コンテキスト)}\ \ E & = & [\ ] \mid F \circ E \end{array}$$

$$\frac{\Gamma \vdash e_2 : \tau_2 \mathbin{@} \mathrm{cps}[\tau_4, \tau_6]}{\Gamma \vdash ([\ ] \mathbin{@} e_2) : [\ (\tau_2 \to \tau_1 @\mathrm{cps}[\tau_3, \tau_4]) @\mathrm{cps}[\tau_5, \tau_6]\ ]_{\mathsf{f}}\ \tau_1 @\mathrm{cps}[\tau_3, \tau_6]}\ (\text{F-APP}_1)$$

$$\frac{\Gamma \vdash v_1 : (\tau_2 \to \tau_1 @\mathrm{cps}[\tau_3, \tau_4]) \mathbin{@} \mathrm{cps}[\tau_5, \tau_5]}{\Gamma \vdash (v_1 \mathbin{@} [\ ]) : [\ \tau_2 @\mathrm{cps}[\tau_4, \tau_5]\ ]_{\mathsf{f}}\ \tau_1 @\mathrm{cps}[\tau_3, \tau_5]}\ (\text{F-APP}_2)$$

$$\frac{}{\Gamma \vdash \langle [\ ] \rangle : [\ \tau_1 @\mathrm{cps}[\tau_1, \tau_2]\ ]_{\mathsf{f}}\ \tau_2 @\mathrm{cps}[\tau_3, \tau_3]}\ (\text{F-RESET})$$

$$\frac{\Gamma, x : \tau_1 \vdash e_2 : \tau_2 \mathbin{@} \mathrm{cps}[\alpha, \beta]}{\Gamma \vdash \mathsf{let}\ x = [\ ]\ \mathsf{in}\ e_2 : [\ \tau_1 @\mathrm{cps}[\beta, \gamma]\ ]_{\mathsf{f}}\ \tau_2 @\mathrm{cps}[\alpha, \gamma]}\ (\text{F-LET})$$

[　] は、「次に計算を行う部分」である。「それ以外の部分」は、「文脈」と呼ぶ。評価文脈はフレームを何枚も重ね合わせたものであり、フレームの列として表すことができる。そのように表せるとすると、評価文脈 E に式 e を入れる関数 $plug_F$ を以下のように定義できる。

$$\begin{array}{rcl} plug_F([\ ] \mathbin{@} e_2, e_1) & = & e_1 \mathbin{@} e_2 \\ plug_F(v_1 \mathbin{@} [\ ], e_2) & = & v_1 \mathbin{@} e_2 \\ plug_F(\langle [\ ] \rangle, e_1) & = & \langle e_1 \rangle \\ plug_F(\mathsf{let}\ x = [\ ]\ \mathsf{in}\ e_2, e_1) & = & \mathsf{let}\ x = e_1\ \mathsf{in}\ e_2 \end{array}$$

次に、reset を含まないフレームである、ピュアフレームを定義する。

$$\begin{array}{rcl} \text{ピュアフレーム} \quad F_p & = & [\ ]\ @\ e_2 \mid v_1\ @\ [\ ] \mid \mathsf{let}\ x = [\ ]\ \mathsf{in}\ e_2 \\ \text{ピュアコンテキスト} \quad E_p & = & [\ ] \mid F_p \circ E_p \end{array}$$

$\boxed{\text{ピュアフレーム} \quad F_p}$

$$\frac{\Gamma \vdash e_2 : \tau_2\ @\mathrm{cps}[\tau_4, \tau_5]}{\Gamma \vdash ([\ ]\ @\ e_2) : [\ (\tau_2 \to \tau_1 @\mathrm{cps}[\tau_3, \tau_4]) @\mathrm{cps}[\tau_5, \tau_6]\ ]_{\mathsf{pf}}\ \tau_1 @\mathrm{cps}[\tau_3, \tau_6]}\ (\mathrm{F}_p\text{-}\mathrm{App}_1)$$

$$\frac{\Gamma \vdash v_1 : (\tau_2 \to \tau_1 @\mathrm{cps}[\tau_3, \tau_4])\ @\mathrm{cps}[\tau_5, \tau_5]}{\Gamma \vdash (v_1\ @\ [\ ]) : [\ \tau_2 @\mathrm{cps}[\tau_4, \tau_5]\ ]_{\mathsf{pf}}\ \tau_1 @\mathrm{cps}[\tau_3, \tau_5]}\ (\mathrm{F}_p\text{-}\mathrm{App}_2)$$

$$\frac{\Gamma, x : \tau_1 \vdash e_2 : \tau_2\ @\mathrm{cps}[\alpha, \beta]}{\Gamma \vdash \mathsf{let}\ x = [\ ]\ \mathsf{in}\ e_2 : [\ \tau_1 @\mathrm{cps}[\beta, \gamma]\ ]_{\mathsf{pf}}\ \tau_2 @\mathrm{cps}[\alpha, \gamma]}\ [(\mathrm{F}_p\text{-}\mathrm{Let})$$

$\boxed{\text{ピュアコンテキスト} \quad E_p}$

$$\frac{}{\Gamma \vdash [\ ] : [\ \tau_1 @\mathrm{cps}[\tau_2, \tau_3]\ ]_{\mathsf{pc}}\ \tau_1 @\mathrm{cps}[\tau_2, \tau_3]}\ (\mathrm{E}_p\text{-}\mathrm{Hole})$$

$$\frac{\Gamma \vdash F_p : [\ \tau_4 @\mathrm{cps}[\tau_5, \tau_6]\ ]_{\mathsf{pf}}\ \tau_7 @\mathrm{cps}[\tau_8, \tau_9] \quad \Gamma \vdash E_p : [\ \tau_1 @\mathrm{cps}[\tau_2, \tau_3]\ ]_{\mathsf{pc}}\ \tau_4 @\mathrm{cps}[\tau_5, \tau_6]}{\Gamma \vdash F_p \circ E_p : [\ \tau_1 @\mathrm{cps}[\tau_2, \tau_3]\ ]_{\mathsf{pc}}\ \tau_7 @\mathrm{cps}[\tau_8, \tau_9]}\ (\mathrm{E}_p\text{-}\mathrm{Frame})$$

$\boxed{\text{ピュアフレーム同士の関係} \quad F_p \cong_{\mathsf{c}} F_p}$

$$\frac{}{([\ ]\ @\ e_2) \cong_{\mathsf{f}} ([\ ]\ @\ e_2)}\ (\cong_{\mathsf{pf}}\text{-}\mathrm{App}_1)$$

同様に、関数 $plug_{F_p}$ を定義する。

$$\begin{array}{rcl} plug_{F_p}([\ ]\ @\ e_2, e_1) & = & e_1\ @\ e_2 \\ plug_{F_p}(v_1\ @\ [\ ], e_2) & = & v_1\ @\ e_2 \\ plug_{F_p}(\mathsf{let}\ x = [\ ]\ \mathsf{in}\ e_2, e_1) & = & \mathsf{let}\ x = e_1\ \mathsf{in}\ e_2 \end{array}$$

関数 $plug_{E_p}$ は以下のように定義できる。

$$\begin{array}{rcl} plug_{E_p}([\ ], e) & = & e \\ plug_{E_p}(F_p \circ E_p, e) & = & plug_{F_p}(F_p, plug_{E_p}(E_p, e)) \end{array}$$

以上より、簡約規則は以下のように表せる。

$$\frac{e_1[v_2/x] = e_1'}{(\lambda x.\, e_1)\ @\ v_2 \rightsquigarrow e_1'}\ (\mathrm{RBeta}) \qquad \frac{e_1 \rightsquigarrow e_2}{plug_F(F, e_1) \rightsquigarrow plug_F(F, e_2)}\ (\mathrm{RFrame})$$

$$\frac{E_{p_1} \cong_{\mathsf{c}} E_{p_2}}{\langle E_{p_1}[\ \mathcal{S}\ @\ v_2\ ] \rangle \rightsquigarrow \langle v_2\ @\ (\lambda y.\, \langle E_{p_2}[\ y\ ] \rangle) \rangle}\ (\mathrm{RShift}) \qquad \frac{}{\langle v_1 \rangle \rightsquigarrow v_1}\ (\mathrm{RReset})$$

$$\frac{e_2[v_1/x] = e_2'}{\mathsf{let}\ x = v_1\ \mathsf{in}\ e_2 \rightsquigarrow e_2'}\ (\mathrm{RLet})$$

2　CPS 項の定義

2.1　CPS 項の構文

$$
\begin{array}{rcll}
\tau & ::= & \mathsf{Nat} \mid \mathsf{bool} \mid \tau_2 \to (\tau_1 \to \tau_3) \to \tau_4 \mid \tau_1 \to \tau_2 & \text{型} \\
v & ::= & n \mid x \mid \lambda x.\, \lambda k.\, e \mid \mathcal{S} & \text{値} \\
\mathcal{S} & ::= & \lambda w.\, \lambda k.\, (w\ (\lambda a.\, \lambda k'.\, k'\ (k\ a)))\ (\lambda m.\, m) & \texttt{shift} \\
e & ::= & c\ v \mid v\ w\ c \mid c\ e & \text{項} \\
c & ::= & k \mid \lambda x.\, x \mid \lambda x.\, e & \text{継続}
\end{array}
$$

2.2　CPS 項の型付け規則

$$
\frac{\Gamma(x) = \tau_1}{\Gamma \vdash x : \tau_1}\ (\textsc{TVar}_{\mathsf{C}}) \qquad \frac{}{\Gamma \vdash n : \mathsf{Nat}}\ (\textsc{TNum}_{\mathsf{C}})
$$

$$
\frac{\Gamma, x : \tau_2,\ k : \tau_1 \to \tau_3 \vdash e_k : (\tau_3 \to \tau_3)\ \tau_4}{\Gamma \vdash \lambda x.\, \lambda k.\, e : \tau_2 \to (\tau_1 \to \tau_3) \to \tau_4}\ (\textsc{TFun}_{\mathsf{C}})
$$

$$
\frac{}{\Gamma \vdash \mathcal{S} : ((\tau_1 \to (\tau_2 \to \tau_3) \to \tau_3) \to (\tau_4 \to \tau_4) \to \tau_5) \to (\tau_1 \to \tau_2) \to \tau_5}\ (\textsc{TShift}_{\mathsf{C}})
$$

$$
\frac{\Gamma \vdash c_\Delta : (\tau_3 \to \tau_3)\ (\tau_1 \to \tau_2) \quad \Gamma \vdash v : \tau_1}{\Gamma \vdash c\ v : (\tau_3 \to \tau_3)\ \tau_2}\ (\textsc{TRet}_{\mathsf{C}})
$$

$$
\frac{\Gamma \vdash v_1 : \tau_2 \to (\tau_1 \to \tau_3) \to \tau_4 \quad \Gamma \vdash v_2 : \tau_2 \quad \Gamma \vdash c_\Delta : (\tau_5 \to \tau_5)\ (\tau_1 \to \tau_3)}{\Gamma \vdash v\ w\ c : (\tau_5 \to \tau_5)\ \tau_1}\ (\textsc{TApp}_{\mathsf{C}})
$$

$$
\frac{\Gamma \vdash c_\Delta : (\tau_0 \to \tau_0)\ (\tau_1 \to \tau_2) \quad \Gamma \vdash e_\bullet : (\tau_3 \to \tau_3)\ \tau_2}{\Gamma \vdash c\ e : (\tau_3 \to \tau_3)\ \tau_2}\ (\textsc{TRetE}_{\mathsf{C}})
$$

$$
\frac{\Delta(k) = \tau_1 \to \tau_2}{\Gamma \vdash k : (\tau_2 \to \tau_2)\ (\tau_1 \to \tau_2)}\ (\textsc{TCVar}_{\mathsf{C}}) \qquad \frac{}{(\Delta = \bullet) \vdash \lambda x.\, x : (\tau_1 \to \tau_1)\ (\tau_1 \to \tau_1)}\ (\textsc{TCId}_{\mathsf{C}})
$$

$$
\frac{\Gamma, x : \tau_1 \vdash e_\Delta : (\tau_4 \to \tau_4)\ \tau_2}{\Delta \vdash \lambda x.\, e_\Delta : (\tau_4 \to \tau_4)\ (\tau_1 \to \tau_2)}\ (\textsc{TCCont}_{\mathsf{C}})
$$

2.3　CPS 項の代入規則

2.4　CPS 項の簡約規則

2.5 CPS 変換の定式化

Biernacki の `shift/reset` を含む computational な $\lambda_{c\mathcal{S}}$ 計算からの CCS 変換を示す。

$$
\begin{array}{lcl}
x^\dagger & = & x \\
\lambda x.\, e^\dagger & = & \lambda xk.\,(e : k) \\
\mathcal{S}^\dagger & = & \lambda wj.\,(w\ (\lambda yk.\, k\ (j\ y)))\ (\lambda m.\, m) \\
\\
v : K & = & K\ v^\dagger \\
(e_1 \mathbin{@} e_2) : K & = & e_1 : (\lambda m.\,(e_1 : (\lambda n.\, m\ n\ K))) \\
(e_1 \mathbin{@} v_2) : K & = & e_1 : (\lambda m.\, m\ {v_2}^\dagger\ K) \\
(v_1 \mathbin{@} e_2) : K & = & e_2 : (\lambda n.\, {v_1}^\dagger\ n\ K) \\
(v_1 \mathbin{@} v_2) : K & = & {v_1}^\dagger\ {v_2}^\dagger\ K \\
(\mathsf{let}\ x = e_1\ \mathsf{in}\ e_2) : K & = & e_1 : (\lambda m.\,(e_2 : K)) \\
\langle e \rangle : K & = & K\ (e : (\lambda m.\, m))
\end{array}
$$

3 DS 項 (kernel) の定義

3.1 DS 項 (kernel) の構文

$$
\begin{array}{lcll}
\tau & ::= & \mathsf{Nat} \mid \mathsf{bool} \mid \tau_1 \to \tau_2 @\mathrm{cps}[\tau_3, \tau_4] & \text{型} \\
v & ::= & n \mid x \mid \lambda x.\, \mathcal{S}\ (\lambda k.\, e) \mid \mathcal{S} & \text{値} \\
p & ::= & v \mid e_1 \mathbin{@} e_2 \mid \langle e \rangle \mid \mathsf{let}\ x = e_1\ \mathsf{in}\ e_2 & \text{値以外} \\
e & ::= & K[v] \mid K[p] & \text{項}
\end{array}
$$

3.2 DS 変換の定義

$$
\begin{array}{lcl}
x^\natural & = & x \\
(\lambda xk.\, e)^\natural & = & \lambda x.\, \mathcal{S} \mathbin{@} (\lambda k.\, e^\sharp_k) \\
(\lambda wj.\,(w\ (\lambda yk.\, k\ (j\ y)))\ (\lambda m.\, m))^\natural & = & \mathcal{S} \\
\\
k^\flat & = & k\ \texttt{[ ]} \\
(\lambda x.\, x)^\flat & = & \texttt{[ ]} \\
(\lambda x.\, e_\Delta)^\flat & = & \mathsf{let}\ x = \texttt{[ ]}\ \mathsf{in}\ e^\sharp_\Delta \\
\\
(K_\Delta\ V)^\sharp & = & K^\flat_\Delta[V^\natural] \\
(V\ W\ K_\Delta)^\sharp & = & K^\flat_\Delta[V^\natural\ W^\natural] \\
(K_\Delta\ e_\bullet)^\sharp & = & K^\flat_\Delta[\langle {e_\bullet}^\sharp \rangle]
\end{array}
$$

3.3 Isomorphism : DS_k to CPS

$$
\begin{array}{lcl}
x^\dagger & = & x \\
(\lambda x.\, \mathcal{S}\ (\lambda k.\, e))^\dagger & = & \lambda x k.\, e^\circ \\
\mathcal{S}^\dagger & = & \lambda w j.\, (w\ (\lambda y k.\, k\ (j\ y)))\ (\lambda m.\, m) \\
\\
(k\ \texttt{[ ]})^\ddagger & = & k \\
\texttt{[ ]}^\ddagger & = & \lambda x.\, x \\
(\mathsf{let}\ x = \texttt{[ ]}\ \mathsf{in}\ e)^\ddagger & = & \lambda x.\, e^\circ \\
\\
(K[V])^\circ & = & K^\ddagger\ V^\dagger \\
(K[V\ W])^\circ & = & V^\dagger\ W^\dagger\ K^\ddagger \\
(K[\langle e\rangle])^\circ & = & K^\ddagger\ e^\circ
\end{array}
$$

3.4 Inclusion : DS of DS_k

$$
\begin{array}{lcl}
x^\dagger & = & x \\
(\lambda x.\, e)^\dagger & = & \lambda x.\, \mathcal{S}\ (\lambda k.\, (e : k\ \texttt{[ ]})) \\
\mathcal{S}^\dagger & = & \lambda w j.\, (w\ (\lambda y k.\, k\ (j\ y)))\ (\lambda m.\, m) \\
\\
V : K & = & K[V^\dagger] \\
(PQ) : K & = & P : (\mathsf{let}\ m = \texttt{[ ]}\ \mathsf{in}\ (Q : (\mathsf{let}\ n = \texttt{[ ]}\ \mathsf{in}\ K[m\ n]))) \\
(PW) : K & = & P : (\mathsf{let}\ m = \texttt{[ ]}\ \mathsf{in}\ K[m\ W^\dagger]) \\
(VQ) : K & = & Q : (\mathsf{let}\ n = \texttt{[ ]}\ \mathsf{in}\ K[V^\dagger\ n]) \\
(\mathsf{let}\ x = e_1\ \mathsf{in}\ e_2) : K & = & e_1 : (\mathsf{let}\ m = \texttt{[ ]}\ \mathsf{in}\ (e_2 : K)) \\
\langle e\rangle : K & = & K[\langle e : \texttt{[ ]}\rangle]
\end{array}
$$