

CPS 変換メモ

Juko Yamamoto

2021 年 1 月 3 日

1 DS 項の定義

ここでは、CPS 変換前の項、すなわち CPS 変換を行う対象の言語を示す。

1.1 構文

$$\begin{aligned}\tau &::= \text{Nat} \mid \text{bool} \mid \tau_1 \rightarrow \tau_2 @ \text{cps}[\tau_3, \tau_4] && \text{型} \\ v &::= n \mid x \mid \lambda x. e \mid \mathcal{S} && \text{値} \\ e &::= v \mid e_1 @ e_2 \mid \langle e \rangle \mid \text{let } x = e_1 \text{ in } e_2 && \text{項}\end{aligned}$$

1.2 DS 項の型付け規則

$$\begin{aligned}\frac{\Gamma(x) = \tau_1}{\Gamma \vdash x : \tau_1 @ \text{cps}[\tau, \tau]} \text{ (TVAR)} \quad & \frac{}{\Gamma \vdash n : \text{Nat} @ \text{cps}[\tau, \tau]} \text{ (TNUM)} \\ \frac{\Gamma, x : \tau_2 \vdash e : \tau_1 @ \text{cps}[\tau_3, \tau_4]}{\Gamma \vdash \lambda x. e : (\tau_2 \rightarrow \tau_1 @ \text{cps}[\tau_3, \tau_4]) @ \text{cps}[\tau, \tau]} \text{ (TFUN)} \\ \frac{}{\Gamma \vdash \mathcal{S} : (((\tau_3 \rightarrow \tau_4 @ \text{cps}[\tau, \tau]) \rightarrow \tau_1 @ \text{cps}[\tau_1, \tau_2]) \rightarrow \tau_3 @ \text{cps}[\tau_4, \tau_2]) @ \text{cps}[\tau, \tau]} \text{ (TSHIFT)} \\ \frac{\Gamma \vdash e_1 : (\tau_2 \rightarrow \tau_1 @ \text{cps}[\tau_3, \tau_4]) @ \text{cps}[\tau_5, \tau_6] \quad \Gamma \vdash e_2 : \tau_2 @ \text{cps}[\tau_4, \tau_5]}{\Gamma \vdash e_1 @ e_2 : \tau_1 @ \text{cps}[\tau_3, \tau_6]} \text{ (TAPP)} \\ \frac{\Gamma \vdash e : \tau_1 @ \text{cps}[\tau_1, \tau_2]}{\Gamma \vdash \langle e \rangle : \tau_2 @ \text{cps}[\tau_3, \tau_3]} \text{ (TRESET)} \\ \frac{\Gamma \vdash e_1 : \tau_1 @ \text{cps}[\beta, \gamma] \quad \Gamma, x : \tau_1 \vdash e_2 : \tau_2 @ \text{cps}[\alpha, \beta]}{\Gamma \vdash (\text{let } x = e_1 \text{ in } e_2) : \tau_2 @ \text{cps}[\alpha, \gamma]} \text{ (TLET)}\end{aligned}$$

1.3 DS 項の代入規則

代入規則は、 $e[v/x] = e'$ と表現することができ、「項 e の中に現れる変数 x を値 v に置き換える」と読む。

今回用いる代入規則は以下である。

$$\begin{array}{c}
\frac{}{x[v/x] = x} \text{ (SVAR=)} \quad \frac{}{y[v/x] = y} \text{ (SVAR } \neq \text{)} \quad \frac{}{n[v/x] = n} \text{ (SNUM)} \\
\\
\frac{\forall x.(e[v/y] = e')}{(\lambda x.e)[v/y] = \lambda x.e'} \text{ (SFUN)} \quad \frac{e_1[v/x] = e'_1 \quad e_2[v/x] = e'_2}{(e_1 @ e_2)[v/x] = (e'_1 @ e'_2)} \text{ (SAPP)} \\
\\
\frac{}{\mathcal{S}[v/x] = \mathcal{S}} \text{ (SSHIFT)} \quad \frac{e_1[v/x] = e'_1}{\langle e_1 \rangle[v/x] = \langle e'_1 \rangle} \text{ (SRESET)} \\
\\
\frac{e_1[v/y] = e'_1 \quad \forall x.(e_2[v/y] = e'_2)}{(\text{let } x = e_1 \text{ in } e_2)[v/y] = \text{let } x = e'_1 \text{ in } e'_2} \text{ (SLET)}
\end{array}$$

1.4 DS 項の簡約規則

次に、簡約規則について示す。ただし、簡約規則を示す前に、フレームを定義する必要がある。フレームを定義することによって、簡約の順序を決めることができる。

$$\begin{array}{lcl}
\text{フレーム } F & = & [] @ e_2 \mid v_1 @ [] \mid \langle [] \rangle \mid \text{let } x = [] \text{ in } e_2 \\
\text{評価文脈 (コンテキスト) } E & = & [] \mid F \circ E
\end{array}$$

$$\begin{array}{c}
\frac{\Gamma \vdash e_2 : \tau_2 @ \text{cps}[\tau_4, \tau_6]}{\Gamma \vdash ([] @ e_2) : [(\tau_2 \rightarrow \tau_1 @ \text{cps}[\tau_3, \tau_4]) @ \text{cps}[\tau_5, \tau_6]]_f \tau_1 @ \text{cps}[\tau_3, \tau_6]} \text{ (F-APP}_1\text{)} \\
\\
\frac{\Gamma \vdash v_1 : (\tau_2 \rightarrow \tau_1 @ \text{cps}[\tau_3, \tau_4]) @ \text{cps}[\tau_5, \tau_5]}{\Gamma \vdash (v_1 @ []) : [\tau_2 @ \text{cps}[\tau_4, \tau_5]]_f \tau_1 @ \text{cps}[\tau_3, \tau_5]} \text{ (F-APP}_2\text{)} \\
\\
\frac{}{\Gamma \vdash \langle [] \rangle : [\tau_1 @ \text{cps}[\tau_1, \tau_2]]_f \tau_2 @ \text{cps}[\tau_3, \tau_3]} \text{ (F-RESET)} \\
\\
\frac{\Gamma, x : \tau_1 \vdash e_2 : \tau_2 @ \text{cps}[\alpha, \beta]}{\Gamma \vdash \text{let } x = [] \text{ in } e_2 : [\tau_1 @ \text{cps}[\beta, \gamma]]_f \tau_2 @ \text{cps}[\alpha, \gamma]} \text{ (F-LET)}
\end{array}$$

$[]$ は、「次に計算を行う部分」である。「それ以外の部分」は、「文脈」と呼ぶ。評価文脈はフレームを何枚も重ね合わせたものであり、フレームの列として表すことができる。そのように表せるとすると、評価文脈 E に式 e を入れる関数 $plug_F$ を以下のように定義できる。

$$\begin{array}{lcl}
plug_F([] @ e_2, e_1) & = & e_1 @ e_2 \\
plug_F(v_1 @ [], e_2) & = & v_1 @ e_2 \\
plug_F(\langle [] \rangle, e_1) & = & \langle e_1 \rangle \\
plug_F(\text{let } x = [] \text{ in } e_2, e_1) & = & \text{let } x = e_1 \text{ in } e_2
\end{array}$$

次に、reset を含まないフレームである、ピュアフレームを定義する。

$$\begin{aligned} \text{ピュアフレーム } F_p &= [\] @ e_2 \mid v_1 @ [\] \mid \text{let } x = [\] \text{ in } e_2 \\ \text{ピュアコンテキスト } E_p &= [\] \mid F_p \circ E_p \end{aligned}$$

$$\boxed{\text{ピュアフレーム } F_p}$$

$$\frac{\Gamma \vdash e_2 : \tau_2 @ \text{cps}[\tau_4, \tau_5]}{\Gamma \vdash ([\] @ e_2) : [\ (\tau_2 \rightarrow \tau_1 @ \text{cps}[\tau_3, \tau_4]) @ \text{cps}[\tau_5, \tau_6] \]_{\text{pf}} \tau_1 @ \text{cps}[\tau_3, \tau_6]} \text{ (F}_p\text{-APP}_1\text{)}$$

$$\frac{\Gamma \vdash v_1 : (\tau_2 \rightarrow \tau_1 @ \text{cps}[\tau_3, \tau_4]) @ \text{cps}[\tau_5, \tau_5]}{\Gamma \vdash (v_1 @ [\]) : [\ \tau_2 @ \text{cps}[\tau_4, \tau_5] \]_{\text{pf}} \tau_1 @ \text{cps}[\tau_3, \tau_5]} \text{ (F}_p\text{-APP}_2\text{)}$$

$$\frac{\Gamma, x : \tau_1 \vdash e_2 : \tau_2 @ \text{cps}[\alpha, \beta]}{\Gamma \vdash \text{let } x = [\] \text{ in } e_2 : [\ \tau_1 @ \text{cps}[\beta, \gamma] \]_{\text{pf}} \tau_2 @ \text{cps}[\alpha, \gamma]} \text{ [(F}_p\text{-LET)}$$

$$\boxed{\text{ピュアコンテキスト } E_p}$$

$$\overline{\Gamma \vdash [\] : [\ \tau_1 @ \text{cps}[\tau_2, \tau_3] \]_{\text{pc}} \tau_1 @ \text{cps}[\tau_2, \tau_3]} \text{ (E}_p\text{-HOLE)}$$

$$\frac{\Gamma \vdash F_p : [\ \tau_4 @ \text{cps}[\tau_5, \tau_6] \]_{\text{pf}} \tau_7 @ \text{cps}[\tau_8, \tau_9] \quad \Gamma \vdash E_p : [\ \tau_1 @ \text{cps}[\tau_2, \tau_3] \]_{\text{pf}} \tau_4 @ \text{cps}[\tau_5, \tau_6]}{\Gamma \vdash F_p \circ E_p : [\ \tau_1 @ \text{cps}[\tau_2, \tau_3] \]_{\text{pc}} \tau_7 @ \text{cps}[\tau_8, \tau_9]} \text{ (E}_p\text{-FRAME)}$$

$$\boxed{\text{ピュアフレーム同士の関係 } F_p \cong_c F_p}$$

$$\overline{([\] @ e_2) \cong_f ([\] @ e_2)} \text{ (}\cong_{\text{pf}}\text{-APP}_1\text{)}$$

同様に、関数 plug_{F_p} を定義する。

$$\begin{aligned} \text{plug}_{F_p}([\] @ e_2, e_1) &= e_1 @ e_2 \\ \text{plug}_{F_p}(v_1 @ [\], e_2) &= v_1 @ e_2 \\ \text{plug}_{F_p}(\text{let } x = [\] \text{ in } e_2, e_1) &= \text{let } x = e_1 \text{ in } e_2 \end{aligned}$$

関数 plug_{E_p} は以下のように定義できる。

$$\begin{aligned} \text{plug}_{E_p}([\], e_1) &= e_1 \\ \text{plug}_{E_p}(F_p \circ E_p, e_2) &= \text{plug}_{F_p}(F_p, \text{plug}_{E_p}(E_p, e_1)) \end{aligned}$$

以上より、簡約規則は以下のように表せる。

$$\begin{aligned} \frac{e_1[v_2/x] = e'_1}{(\lambda x. e_1) @ v_2 \rightsquigarrow e'_1} \text{ (RBETA)} \quad & \frac{e_1 \rightsquigarrow e_2}{\text{plug}_F(F, e_1) \rightsquigarrow \text{plug}_F(F, e_2)} \text{ (RFRAME)} \\ \frac{E_{p_1} \cong_c E_{p_2}}{\langle E_{p_1} [\ \mathcal{S} @ v_2 \] \rangle \rightsquigarrow \langle v_2 @ (\lambda y. \langle E_{p_2} [\ y \] \rangle) \rangle} \text{ (RSHIFT)} \quad & \overline{\langle v_1 \rangle \rightsquigarrow v_1} \text{ (RRESET)} \\ & \frac{e_2[v_1/x] = e'_2}{\text{let } x = v_1 \text{ in } e_2 \rightsquigarrow e'_2} \text{ (RLET)} \end{aligned}$$

2 CPS 項の定義

CPS 変換後の項を示す。

ここで、 $\bar{\lambda}$. や $\bar{@}$ のように、上付きの線が書かれているものは、static な項。また、 $\underline{\lambda}$. や $\underline{@}$ のように、下付きの線が描かれているものは、dynamic な項と呼ぶ。

2.1 CPS 項の構文

$$\begin{array}{ll}
 \tau ::= \text{Nat} \mid \text{bool} \mid \tau_1 \rightarrow \tau_2 & \text{型} \\
 v ::= n \mid x \mid \underline{\lambda}x. \underline{\lambda}k. e \mid \mathcal{S} & \text{値} \\
 \mathcal{S} ::= \underline{\lambda}w. \underline{\lambda}k. (w \underline{@} (\underline{\lambda}a. \underline{\lambda}k'. k' \underline{@} (k \underline{@} a))) \underline{@} (\underline{\lambda}m. m) & \text{shift 項} \\
 e ::= v \mid e_1 \underline{@} e_2 \mid \underline{\text{let}} \ x = e_1 \ \underline{\text{in}} \ e_2 & \text{項}
 \end{array}$$

2.2 CPS 項の型付け規則

$$\begin{array}{c}
 \frac{\Gamma(x) = \tau_1}{\Gamma \vdash x : \tau_1} \text{(TVAR}_C\text{)} \quad \frac{}{\Gamma \vdash n : \text{Nat}} \text{(TNUM}_C\text{)} \quad \frac{\Gamma, x : \tau_2 \vdash e : \tau_1}{\Gamma \vdash \underline{\lambda}x. \underline{\lambda}k. e : \tau_2 \rightarrow \tau_1} \text{(TFUNC}_C\text{)} \\
 \frac{}{\Gamma \vdash \mathcal{S} : ((\tau_1 \rightarrow (\tau_2 \rightarrow \tau_3) \rightarrow \tau_3) \rightarrow (\tau_4 \rightarrow \tau_4) \rightarrow \tau_5) \rightarrow (\tau_1 \rightarrow \tau_2) \rightarrow \tau_5} \text{(TSHIFT}_C\text{)} \\
 \frac{\Gamma \vdash e_1 : \tau_2 \rightarrow \tau_1 \quad \Gamma \vdash e_2 : \tau_2}{\Gamma \vdash e_1 \underline{@} e_2 : \tau_1} \text{(TAPPC}_C\text{)} \quad \frac{\Gamma \vdash e_1 : \tau_1 \quad \Gamma, x : \tau_1 \vdash e_2 : \tau_2}{\Gamma \vdash (\underline{\text{let}} \ x = e_1 \ \underline{\text{in}} \ e_2) : \tau_2} \text{(TLET}_C\text{)}
 \end{array}$$

2.3 CPS 項の代入規則

$$\begin{array}{c}
 \frac{}{x[v/x] = x} \text{(SVAR}=_C\text{)} \quad \frac{}{y[v/x] = y} \text{(SVAR} \neq_C\text{)} \quad \frac{}{n[v/x] = n} \text{(SNUM}_C\text{)} \\
 \frac{\forall x. (e[v/y] = e')}{(\underline{\lambda}x. e)[v/y] = \underline{\lambda}x. e'} \text{(SFUNC}_C\text{)} \quad \frac{e_1[v/x] = e'_1 \quad e_2[v/x] = e'_2}{(e_1 \underline{@} e_2)[v/x] = (e'_1 \underline{@} e'_2)} \text{(SAPPC}_C\text{)} \\
 \frac{}{\mathcal{S}[v/x] = \mathcal{S}} \text{(SSHIFT}_C\text{)} \quad \frac{e_1[v/y] = e'_1 \quad \forall x. (e_2[v/y] = e'_2)}{(\underline{\text{let}} \ x = e_1 \ \underline{\text{in}} \ e_2)[v/y] = \underline{\text{let}} \ x = e'_1 \ \underline{\text{in}} \ e'_2} \text{(SLET}_C\text{)}
 \end{array}$$

2.4 CPS 項の簡約規則

$$\begin{array}{c}
\frac{e_1[v_2/x] = e'_1}{(\lambda x. e_1) @ v_2 \rightsquigarrow e'_1} \text{ (EQBETA}_C\text{)} \qquad \frac{e_2[v_1/x] = e'_2}{\text{let } x = v_1 \text{ in } e_2 \rightsquigarrow e'_2} \text{ (EQLET}_C\text{)} \\
\\
\frac{}{(\lambda w. \lambda k. (w @ (\lambda a. \lambda k'. k' @ (k @ a))) @ (\lambda m. m) @ v_2) @ k \rightsquigarrow (v_2 @ (\lambda a. \lambda k'. k' @ (k @ a))) @ (\lambda m. m)} \text{ (EQSHIFT}_C\text{)} \\
\\
\frac{e_1 \rightsquigarrow e'_1}{e_1 @ e_2 \rightsquigarrow e'_1 @ e_2} \text{ (EQAPP1}_C\text{)} \qquad \frac{e_2 \rightsquigarrow e'_2}{e_1 @ e_2 \rightsquigarrow e_1 @ e'_2} \text{ (EQAPP2}_C\text{)} \\
\\
\frac{e_1 \rightsquigarrow e'_1}{\text{let } x = e_1 \text{ in } e_2 \rightsquigarrow \text{let } x = e'_1 \text{ in } e_2} \text{ (EQLET1}_C\text{)} \qquad \frac{e_2 \rightsquigarrow e'_2}{\text{let } x = e_1 \text{ in } e_2 \rightsquigarrow \text{let } x = e_1 \text{ in } e'_2} \text{ (EQLET2}_C\text{)} \\
\\
\frac{}{e_1 @ e_2 \rightsquigarrow \text{let } x = e_1 \text{ in } x @ e_2} \text{ (EQLETAAPP1}_C\text{)} \qquad \frac{}{v_1 @ e_2 \rightsquigarrow \text{let } x = e_2 \text{ in } v_1 @ x} \text{ (EQLETAAPP2}_C\text{)}
\end{array}$$

2.5 CPS 変換の定式化

η redex を作らない、one-pass の CPS 変換の定義を示す。

$$\begin{array}{lcl}
\llbracket n \rrbracket_v & = & n \\
\llbracket x \rrbracket_v & = & x \\
\llbracket \lambda x. e \rrbracket_v & = & \lambda x. \lambda k. \llbracket e \rrbracket' @ k \\
\llbracket S \rrbracket_v & = & \lambda w k. (w @ (\lambda a k'. k' @ (k @ a))) @ (\lambda m. m) \\
\\
\llbracket v \rrbracket & = & \bar{\lambda} \kappa. \kappa @ \llbracket v \rrbracket_v \\
\llbracket e_1 @ e_2 \rrbracket & = & \bar{\lambda} \kappa. \llbracket e_1 \rrbracket @ (\bar{\lambda} m. \llbracket e_2 \rrbracket @ (\bar{\lambda} n. (m @ n) @ (\lambda a. \kappa @ a))) \\
\llbracket \langle e \rangle \rrbracket & = & \bar{\lambda} \kappa. \text{let } x = \llbracket e \rrbracket @ (\bar{\lambda} m. m) \text{ in } \kappa @ x \\
\llbracket \text{let } x = e_1 \text{ in } e_2 \rrbracket & = & \bar{\lambda} \kappa. \llbracket e_1 \rrbracket @ (\bar{\lambda} m. \text{let } x = m \text{ in } \llbracket e_2 \rrbracket @ \kappa) \\
\\
\llbracket v \rrbracket' & = & \bar{\lambda} k. k @ \llbracket v \rrbracket_v \\
\llbracket e_1 @ e_2 \rrbracket' & = & \bar{\lambda} \kappa. \llbracket e_1 \rrbracket @ (\bar{\lambda} m. \llbracket e_2 \rrbracket @ (\bar{\lambda} n. (m @ n) @ k)) \\
\llbracket \langle e \rangle \rrbracket' & = & \bar{\lambda} k. \text{let } x = \llbracket e \rrbracket @ (\bar{\lambda} m. m) \text{ in } k @ x \\
\llbracket \text{let } x = e_1 \text{ in } e_2 \rrbracket' & = & \bar{\lambda} k. \llbracket e_1 \rrbracket @ (\bar{\lambda} m. \text{let } x = m \text{ in } \llbracket e_2 \rrbracket' @ k)
\end{array}$$

3 補題の証明

CPS 変換の証明を行う前に、必要な補題をいくつか証明する。

3.1 CPS 項に関する代入補題の証明

補題 3.1.1: (eSubstV)

$$v_1[v/x] = v'_1 \text{ のとき、 } \llbracket v_1 \rrbracket_v \llbracket [v]_v / x \rrbracket = \llbracket v'_1 \rrbracket_v$$

証明.

$v = x$ のとき

$$\begin{aligned} \llbracket x \rrbracket_v \llbracket [v]_v / x \rrbracket &= x \llbracket [v]_v / x \rrbracket \\ &= \llbracket v \rrbracket_v \quad (\text{sVar} =) \end{aligned}$$

$v = y$ のとき

$$\begin{aligned} \llbracket y \rrbracket_v \llbracket [v]_v / x \rrbracket &= y \llbracket [v]_v / x \rrbracket \\ &= \llbracket y \rrbracket_v \quad (\text{sVar} \neq) \end{aligned}$$

$v = \lambda x. e$ のとき

$$\begin{aligned} \llbracket \lambda x. e \rrbracket_v \llbracket [v]_v / x \rrbracket &\equiv (\lambda x. \lambda k. \llbracket e \rrbracket' \bar{\text{@}} k) \llbracket [v]_v / x \rrbracket \\ &= \lambda x. \lambda k. \llbracket e[v/x] \rrbracket' \bar{\text{@}} k \\ &= \lambda x. \lambda k. \llbracket e' \rrbracket' \bar{\text{@}} k \quad (\text{補題 3.1.3 ekSubst'}) \\ &\equiv \llbracket \lambda x. e' \rrbracket_v \end{aligned}$$

$v = \mathcal{S}$ のとき

$$\begin{aligned} \llbracket \mathcal{S} \rrbracket_v \llbracket [v]_v / x \rrbracket &= (\lambda w k. (w \text{ @ } (\lambda a k'. k' \text{ @ } (k \text{ @ } a))) \text{ @ } (\lambda m. m)) \llbracket [v]_v / x \rrbracket \\ &= \llbracket \mathcal{S} \rrbracket_v \end{aligned}$$

□

補題 3.1.2: (e κ Subst)

$e_1[v/x] = e_2$ かつ $\kappa_1[[v]_v/x] = \kappa_2$ のとき、 $([e_1] \bar{\otimes} \kappa_1)[[v]_v/x] = [e_2] \bar{\otimes} \kappa_2$

証明.

$e_1 = v_1$ (値) のとき

$v_1[v/x] = v_2$ とすると、

$$\begin{aligned}
 (\text{与式}) &\equiv ([v_1] \bar{\otimes} \kappa_1)[[v]_v/x] \\
 &\equiv (\kappa_1 \bar{\otimes} [v_1]_v)[[v]_v/x] \\
 &= (\kappa_1[[v]_v/x]) \bar{\otimes} ([v_1]_v[[v]_v/x]) \\
 &= \kappa_2 \bar{\otimes} [v_2]_v && (\text{補題 3.1.1 eSubstV}) \\
 &\equiv [v_2] \bar{\otimes} \kappa_2
 \end{aligned}$$

e_1 が **App** のとき

$(e_1 @ e_2)[v/x] = e'_1 @ e'_2$ とすると、

$$\begin{aligned}
 (\text{与式}) &\equiv ([e_1 @ e_2] \bar{\otimes} \kappa_1)[[v]_v/x] \\
 &\equiv ([e_1] \bar{\otimes} (\bar{\lambda} m. [e_2] \bar{\otimes} (\bar{\lambda} n. (m @ n) @ (\bar{\lambda} a. \kappa_1 \bar{\otimes} a)))) [[v]_v/x] \\
 &= ([e_1][[v]_v/x]) \bar{\otimes} ((\bar{\lambda} m. [e_2] \bar{\otimes} (\bar{\lambda} n. (m @ n) @ (\bar{\lambda} a. \kappa_1 \bar{\otimes} a)))) [[v]_v/x] \\
 &= [e'_1] \bar{\otimes} (\bar{\lambda} m. ([e_2][[v]_v/x]) \bar{\otimes} ((\bar{\lambda} n. (m @ n) @ (\bar{\lambda} a. \kappa_1 \bar{\otimes} a)) [[v]_v/x])) \\
 &= [e'_1] \bar{\otimes} (\bar{\lambda} m. [e'_2] \bar{\otimes} (\bar{\lambda} n. (m @ n) @ (\bar{\lambda} a. \kappa_2 \bar{\otimes} a))) \\
 &\equiv ([e_1 @ e_2]) \bar{\otimes} \kappa_2
 \end{aligned}$$

e_1 が **Reset** のとき

$\langle e \rangle[v/x] = \langle e' \rangle$ とすると、

$$\begin{aligned}
 (\text{与式}) &\equiv ([\langle e \rangle] \bar{\otimes} \kappa_1)[[v]_v/x] \\
 &\equiv (\text{let } c = [e] \bar{\otimes} (\bar{\lambda} m. m) \text{ in } \kappa_1 \bar{\otimes} c)[[v]_v/x] \\
 &= \text{let } c = [e'] \bar{\otimes} (\bar{\lambda} m. m) \text{ in } \kappa_2 \bar{\otimes} c && (I.H.) \\
 &\equiv [\langle e' \rangle] \bar{\otimes} \kappa_2
 \end{aligned}$$

e_1 が **Let** のとき

$(\text{let } x = e_1 \text{ in } e_2)[v/x] = \text{let } x = e'_1 \text{ in } e'_2$ とすると、

$$\begin{aligned}
 (\text{与式}) &\equiv \text{let } x = e_1 \text{ in } e_2 \bar{\otimes} \kappa_1[[v]_v/x] \\
 &\equiv ([e_1] \bar{\otimes} (\bar{\lambda} m. \text{let } x = m \text{ in } [e_2] \bar{\otimes} \kappa_1)) [[v]_v/x] \\
 &= ([e_1][[v]_v/x]) \bar{\otimes} (\bar{\lambda} m. \text{let } x = m \text{ in } ([e_2][[v]_v/x]) \bar{\otimes} (\kappa_1[[v]_v/x])) \\
 &= [e_1[v/x]] \bar{\otimes} (\bar{\lambda} m. \text{let } x = m \text{ in } ([e_2[v/x]]) \bar{\otimes} \kappa_2) \\
 &= ([e'_1] \bar{\otimes} (\bar{\lambda} m. \text{let } x = m \text{ in } [e'_2] \bar{\otimes} \kappa_2)) [[v]_v/x] && (I.H.) \\
 &\equiv [\text{let } x = e'_1 \text{ in } e'_2] \bar{\otimes} \kappa_2
 \end{aligned}$$

□

補題 3.1.3: (ekSubst')

$e[v/x] = e'$ のとき、 $([e]'\overline{\text{@}}k)[[v]_v/x] = [e']'\overline{\text{@}}k$

証明.

$e = v$ (値) のとき

$v[v/x] = v'$ とすると、

$$\begin{aligned}
 (\text{与式}) &\equiv ([v]'\overline{\text{@}}k)[[v_2]_v/x] \\
 &\equiv (k\text{@}[v]_v)[[v_2]_v/x] \\
 &= (k[[v_2]_v/x])\text{@}([v]_v[[v_2]_v/x]) \\
 &= k\text{@}[v[v_2/x]]_v && (\text{sVar} \neq) \\
 &= k\text{@}[v']_v && (\text{補題 1.1.1 eSubstV}) \\
 &\equiv [v']'\overline{\text{@}}k
 \end{aligned}$$

e が **App** のとき

$(e_1 \text{@} e_2)[v/x] = e'_1 \text{@} e'_2$ とすると、

$$\begin{aligned}
 (\text{与式}) &\equiv ([e_1 \text{@} e_2]'\overline{\text{@}}k)[[v]_v/x] \\
 &\equiv ([e_1]\overline{\text{@}}(\overline{\lambda}m. [e_2]\overline{\text{@}}(\overline{\lambda}n. (m\text{@}n)\text{@}k)))[[v]_v/x] \\
 &= ([e_1][[v]_v/x])\overline{\text{@}}(\overline{\lambda}m. [e_2]\overline{\text{@}}(\overline{\lambda}n. (m\text{@}n)\text{@}k))[[v]_v/x] \\
 &= [e_1[v/x]]\overline{\text{@}}(\overline{\lambda}m. [e_2[v/x]]\overline{\text{@}}(\overline{\lambda}n. (m\text{@}n)\text{@}k))[[v]_v/x] \\
 &= [e'_1]\overline{\text{@}}(\overline{\lambda}m. [e'_2]\overline{\text{@}}(\overline{\lambda}n. (m\text{@}n)\text{@}k)) \\
 &\equiv [e'_1 \text{@} e'_2]'\overline{\text{@}}k
 \end{aligned}$$

e が **Reset** のとき

$\langle e \rangle[[v]_v/x] = \langle e' \rangle$ とすると、

$$\begin{aligned}
 (\text{与式}) &= ([\langle e \rangle]'\overline{\text{@}}k)[[v]_v/x] \\
 &= (\text{let } c = [e]\overline{\text{@}}(\overline{\lambda}m. m) \text{ in } k\text{@}c)[[v]_v/x] \\
 &= \text{let } c = [e']\overline{\text{@}}(\overline{\lambda}m. m) \text{ in } k\text{@}c \\
 &= [\langle e' \rangle]'\overline{\text{@}}k
 \end{aligned}$$

e が **Let** のとき

$(\text{let } x = e_1 \text{ in } e_2)[v/x] = \text{let } x = e'_1 \text{ in } e'_2$ とすると、

$$\begin{aligned}
 (\text{与式}) &= ([\text{let } x = e_1 \text{ in } e_2]'\overline{\text{@}}k)[[v]_v/x] \\
 &= ([e_1]\overline{\text{@}}(\overline{\lambda}m. \text{let } x = m \text{ in } [e_2]'\overline{\text{@}}k))[[v]_v/x] \\
 &= ([e_1][[v]_v/x])\overline{\text{@}}(\overline{\lambda}m. \text{let } x = m \text{ in } ([e_2]'\overline{\text{@}}k))[[v]_v/x] \\
 &= [e_1[v/x]]\overline{\text{@}}(\overline{\lambda}m. \text{let } x = m \text{ in } [e_2[v/x]]'\overline{\text{@}}k) \\
 &= [e'_1]\overline{\text{@}}(\overline{\lambda}m. \text{let } x = m \text{ in } [e'_2]'\overline{\text{@}}k) \\
 &= [\text{let } x = e'_1 \text{ in } e'_2]'\overline{\text{@}}k
 \end{aligned}$$

□

補題 3.1.4: (κ Subst)

schematic な κ ($\kappa[v/k] = \kappa'$) について、($\llbracket e \rrbracket \bar{\otimes} \kappa$)[v/k] = $v \bar{\otimes} \kappa'$ が成り立つ

証明.

$e = v_1$ (値) のとき

$$\begin{aligned}
 (\text{与式}) &= (\llbracket v_1 \rrbracket \bar{\otimes} \kappa)[v/x] \\
 &= \kappa \bar{\otimes} \llbracket v_1 \rrbracket_v[v/k] \\
 &= (\kappa[v/k]) \bar{\otimes} (\llbracket v_1 \rrbracket_v[v/k]) \\
 &= \kappa' \bar{\otimes} \llbracket v \rrbracket_v \\
 &= \llbracket v \rrbracket \bar{\otimes} \kappa'
 \end{aligned}$$

$e = e_1 @ e_2$ (App) のとき

$$\begin{aligned}
 (\text{与式}) &= (\llbracket e_1 @ e_2 \rrbracket \bar{\otimes} \kappa)[v/x] \\
 &= (\llbracket e_1 \rrbracket \bar{\otimes} (\bar{\lambda} m. \llbracket e_2 \rrbracket \bar{\otimes} (\bar{\lambda} n. (m @ n) @ (\bar{\lambda} a. \kappa \bar{\otimes} a))))[v/x] \\
 &= (\llbracket e_1 \rrbracket[v/x]) \bar{\otimes} ((\bar{\lambda} m. \llbracket e_2 \rrbracket \bar{\otimes} (\bar{\lambda} n. (m @ n) @ (\bar{\lambda} a. \kappa \bar{\otimes} a))))[v/x] \\
 &= \llbracket e_1 \rrbracket \bar{\otimes} (\bar{\lambda} m. (\llbracket e_2 \rrbracket[v/x]) \bar{\otimes} ((\bar{\lambda} n. (m @ n) @ (\bar{\lambda} a. \kappa \bar{\otimes} a))[v/x])) \\
 &= \llbracket e_1 \rrbracket \bar{\otimes} (\bar{\lambda} m. \llbracket e_2 \rrbracket \bar{\otimes} (\bar{\lambda} n. (m @ n) @ (\bar{\lambda} a. \kappa' \bar{\otimes} a))) \\
 &= (\llbracket e_1 @ e_2 \rrbracket) \bar{\otimes} \kappa'
 \end{aligned}$$

$e = \langle e \rangle$ (Reset) のとき

$$\begin{aligned}
 (\text{与式}) &= (\llbracket \langle e \rangle \rrbracket \bar{\otimes} \kappa)[v/x] \\
 &= (\underline{\text{let}} \ c = \llbracket e \rrbracket \bar{\otimes} (\bar{\lambda} m. m) \ \underline{\text{in}} \ \kappa \bar{\otimes} c)[v/x] \\
 &= \underline{\text{let}} \ c = \llbracket e \rrbracket \bar{\otimes} (\bar{\lambda} m. m) \ \underline{\text{in}} \ \kappa' \bar{\otimes} c \\
 &= \llbracket \langle e \rangle \rrbracket \bar{\otimes} \kappa'
 \end{aligned}$$

e が Let のとき

$$\begin{aligned}
 (\text{与式}) &= (\llbracket \text{let } x = e_1 \text{ in } e_2 \rrbracket \bar{\otimes} \kappa)[v/x] \\
 &= (\llbracket e_1 \rrbracket \bar{\otimes} (\bar{\lambda} m. \underline{\text{let}} \ x = m \ \underline{\text{in}} \ \llbracket e_2 \rrbracket \bar{\otimes} \kappa))[v/x] \\
 &= \llbracket e_1 \rrbracket \bar{\otimes} (\bar{\lambda} m. \underline{\text{let}} \ x = m \ \underline{\text{in}} \ \llbracket e_2 \rrbracket \bar{\otimes} \kappa') \\
 &= \llbracket \text{let } x = e_1 \text{ in } e_2 \rrbracket \bar{\otimes} \kappa'
 \end{aligned}$$

□

補題 3.1.5: (kSubst')

$([e]'\overline{\text{@}}k)[v/k] = [e]'\overline{\text{@}}v$ が成り立つ

証明.

e が値 ($e = v_1$) のとき

$$\begin{aligned}(\text{与式}) &= [v_1]'\overline{\text{@}}k[v/x] \\ &= (k \text{ @ } [v_1]_v)[v/k] \\ &= v \text{ @ } [v_1]_v \\ &= [v_1]'\overline{\text{@}}v\end{aligned}$$

e が **App** ($e = e_1 \text{ @ } e_2$) のとき

$$\begin{aligned}(\text{与式}) &= ([e_1 \text{ @ } e_2]'\overline{\text{@}}k)[v/x] \\ &= ([e_1] \overline{\text{@}} (\overline{\lambda}m. [e_2] \overline{\text{@}} (\overline{\lambda}n. (m \text{ @ } n) \text{ @ } k)))[v/x] \\ &= ([e_1][v/x]) \overline{\text{@}} ((\overline{\lambda}m. [e_2] \overline{\text{@}} (\overline{\lambda}n. (m \text{ @ } n) \text{ @ } k))[v/x]) \\ &= [e_1] \overline{\text{@}} (\overline{\lambda}m. ([e_2][v/x]) \overline{\text{@}} ((\overline{\lambda}n. (m \text{ @ } n) \text{ @ } k)[v/x])) \\ &= [e_1] \overline{\text{@}} (\overline{\lambda}m. [e_2] \overline{\text{@}} (\overline{\lambda}n. (m \text{ @ } n) \text{ @ } v)) \\ &= ([e_1 \text{ @ } e_2]') \overline{\text{@}} v\end{aligned}$$

e が **Reset** ($e = \langle e \rangle$) のとき

$$\begin{aligned}(\text{与式}) &= ([\langle e \rangle]'\overline{\text{@}}k)[v/x] \\ &= (\text{let } c = [e] \overline{\text{@}} (\overline{\lambda}m. m) \text{ in } k \text{ @ } c)[v/x] \\ &= \text{let } c = [e] \overline{\text{@}} (\overline{\lambda}m. m) \text{ in } v \text{ @ } c \\ &= [\langle e \rangle]'\overline{\text{@}}v\end{aligned}$$

e が **Let** ($e = \text{let } c = e_1 \text{ in } e_2$) のとき

$$\begin{aligned}(\text{与式}) &= ([\text{let } c = e_1 \text{ in } e_2]'\overline{\text{@}}k)[v/x] \\ &= ([e_1] \overline{\text{@}} (\overline{\lambda}m. \text{let } c = m \text{ in } [e_2]'\overline{\text{@}}k))[v/x] \\ &= [e_1] \overline{\text{@}} (\overline{\lambda}m. \text{let } c = m \text{ in } [e_2]'\overline{\text{@}}v) \\ &= [\text{let } x = e_1 \text{ in } e_2]'\overline{\text{@}}v\end{aligned}$$

□

3.2 $\llbracket \cdot \rrbracket'$ と $\llbracket \cdot \rrbracket$ の関係性についての補題の証明

補題 3.2.1: (correctCont)

任意の項 e と schematic な 継続 κ_1, κ_2 について、 $(\kappa_1 \bar{\text{@}} v) \sim (\kappa_2 \bar{\text{@}} v)$ が成り立つならば、 $\llbracket e \rrbracket \bar{\text{@}} \kappa_1 \sim \llbracket e \rrbracket \bar{\text{@}} \kappa_2$ が成り立つ

証明.

e が値 ($e = v_1$) のとき

$$\begin{aligned} (\text{左式}) &\equiv \llbracket v \rrbracket \bar{\text{@}} \kappa_1 \\ &\equiv \kappa_1 \bar{\text{@}} \llbracket v \rrbracket_v \\ &\sim \kappa_2 \bar{\text{@}} \llbracket v \rrbracket_v \\ &\equiv \llbracket v \rrbracket \bar{\text{@}} \kappa_2 \end{aligned}$$

e が **App** ($e = e_1 @ e_2$) のとき

$$\begin{aligned} (\text{左式}) &\equiv \llbracket e_1 @ e_2 \rrbracket \bar{\text{@}} \kappa_1 \\ &\equiv \llbracket e_1 \rrbracket \bar{\text{@}} (\bar{\lambda} m. \llbracket e_2 \rrbracket \bar{\text{@}} (\bar{\lambda} n. (m \bar{\text{@}} n) \bar{\text{@}} (\bar{\lambda} a. \kappa_1 \bar{\text{@}} a))) \\ &\sim \llbracket e_1 \rrbracket \bar{\text{@}} (\bar{\lambda} m. \llbracket e_2 \rrbracket \bar{\text{@}} (\bar{\lambda} n. (m \bar{\text{@}} n) \bar{\text{@}} (\bar{\lambda} a. \kappa_2 \bar{\text{@}} a))) \\ &\equiv \llbracket e_1 @ e_2 \rrbracket \bar{\text{@}} \kappa_2 \end{aligned}$$

e が **Reset** ($e = \langle e \rangle$) のとき

$$\begin{aligned} (\text{左式}) &\equiv \llbracket \langle e \rangle \rrbracket \bar{\text{@}} \kappa_1 \\ &\equiv \text{let } x = \llbracket e \rrbracket \bar{\text{@}} (\bar{\lambda} m. m) \text{ in } \kappa_1 \bar{\text{@}} x \\ &\sim \text{let } x = \llbracket e \rrbracket \bar{\text{@}} (\bar{\lambda} m. m) \text{ in } \kappa_2 \bar{\text{@}} x \\ &\equiv \llbracket \langle e \rangle \rrbracket \bar{\text{@}} \kappa_2 \end{aligned}$$

e が **Let** ($e = \text{let } x = e_1 \text{ in } e_2$) のとき

$$\begin{aligned} (\text{左式}) &\equiv \llbracket \text{let } x = e_1 \text{ in } e_2 \rrbracket \bar{\text{@}} \kappa_1 \\ &\equiv \llbracket e_1 \rrbracket \bar{\text{@}} (\bar{\lambda} m. \text{let } x = m \text{ in } \llbracket e_2 \rrbracket \bar{\text{@}} \kappa_1) \\ &\sim \llbracket e_1 \rrbracket \bar{\text{@}} (\bar{\lambda} m. \text{let } x = m \text{ in } \llbracket e_2 \rrbracket \bar{\text{@}} \kappa_2) \\ &\equiv \llbracket \text{let } x = e_1 \text{ in } e_2 \rrbracket \bar{\text{@}} \kappa_2 \end{aligned}$$

□

補題 3.2.2: (correctEtaEta')

$\llbracket e \rrbracket' \bar{\otimes} (\lambda a. \kappa \bar{\otimes} a) \rightsquigarrow^* \llbracket e \rrbracket \bar{\otimes} \kappa$ が成り立つ

証明.

e が値 ($e = v_1$) のとき

$$\begin{aligned}
 (\text{左式}) &\equiv \llbracket v \rrbracket' \bar{\otimes} (\lambda a. \kappa \bar{\otimes} a) \\
 &\equiv (\bar{\lambda} k. k \underline{\otimes} \llbracket v \rrbracket_v) \bar{\otimes} (\lambda a. \kappa \bar{\otimes} a) \\
 &\equiv (\lambda a. \kappa \bar{\otimes} a) \underline{\otimes} \llbracket v \rrbracket_v \\
 &\rightsquigarrow \kappa \bar{\otimes} \llbracket v \rrbracket_v \\
 &\equiv \llbracket v \rrbracket \bar{\otimes} \kappa
 \end{aligned}$$

e が **App** ($e = e_1 \underline{\otimes} e_2$) のとき

$$\begin{aligned}
 (\text{左式}) &\equiv \llbracket e_1 \underline{\otimes} e_2 \rrbracket' \bar{\otimes} (\lambda a. \kappa \bar{\otimes} a) \\
 &\equiv \llbracket e_1 \rrbracket \bar{\otimes} (\bar{\lambda} m. \llbracket e_2 \rrbracket \bar{\otimes} (\bar{\lambda} n. (m \underline{\otimes} n) \underline{\otimes} (\lambda a. \kappa \bar{\otimes} a))) \\
 &\equiv \llbracket e_1 \underline{\otimes} e_2 \rrbracket \bar{\otimes} \kappa
 \end{aligned}$$

e が **Reset** ($e = \langle e \rangle$) のとき

$$\begin{aligned}
 (\text{左式}) &\equiv \llbracket \langle e \rangle \rrbracket' \bar{\otimes} (\lambda a. \kappa \bar{\otimes} a) \\
 &\equiv \underline{\text{let}} \ c = \llbracket e \rrbracket \bar{\otimes} (\bar{\lambda} m. m) \ \underline{\text{in}} \ (\lambda a. \kappa \bar{\otimes} a) \underline{\otimes} c \\
 &\rightsquigarrow \underline{\text{let}} \ c = \llbracket e \rrbracket \bar{\otimes} (\bar{\lambda} m. m) \ \underline{\text{in}} \ \kappa \underline{\otimes} c \\
 &\equiv \llbracket \langle e \rangle \rrbracket \bar{\otimes} \kappa
 \end{aligned}$$

e が **Let** ($e = \text{let } x = e_1 \text{ in } e_2$) のとき

$$\begin{aligned}
 (\text{左式}) &\equiv \llbracket \text{let } x = e_1 \text{ in } e_2 \rrbracket' \bar{\otimes} (\lambda a. \kappa \bar{\otimes} a) \\
 &\equiv \llbracket e_1 \rrbracket \bar{\otimes} (\bar{\lambda} m. \underline{\text{let}} \ x = m \ \underline{\text{in}} \ \llbracket e_2 \rrbracket' \bar{\otimes} (\lambda a. \kappa \bar{\otimes} a)) \\
 &\sim \llbracket e_1 \rrbracket \bar{\otimes} (\bar{\lambda} m. \underline{\text{let}} \ x = m \ \underline{\text{in}} \ \llbracket e_2 \rrbracket \bar{\otimes} \kappa) \\
 &\equiv \llbracket \text{let } x = e_1 \text{ in } e_2 \rrbracket \bar{\otimes} \kappa
 \end{aligned}$$

□

3.3 ピュアコンテキストに関する代入補題

補題 3.3.1: (subst-context)

任意のピュアコンテキスト con について、 $E_{\text{con}}[x][v/x] = E_{\text{con}}[v]$ が成り立つ

証明.

con が Hole のとき

$$\begin{aligned} (\text{左式}) &\equiv x[v/x] \\ &= v \end{aligned}$$

con が Frame ($\text{App}_1 e_2$) のとき

$$\begin{aligned} (\text{左式}) &\equiv (x @ e_2)[v/x] \\ &= v @ e_2 \end{aligned}$$

con が Frame ($\text{App}_2 v_1$) のとき

$$\begin{aligned} (\text{左式}) &\equiv (v_1 @ x)[v/x] \\ &= v_1 @ v \end{aligned}$$

con が Frame (Let e_2) のとき

$$\begin{aligned} (\text{左式}) &\equiv (\text{let } c = x \text{ in } e_2)[v/x] \\ &= \text{let } c = v \text{ in } e_2 \end{aligned}$$

□

3.4 Shift に関する補題

補題 3.4.1: (contextContE)

$\llbracket E_{p_1} [S @ v] \rrbracket \bar{\otimes} \kappa \equiv \llbracket S @ v \rrbracket' \bar{\otimes} (\lambda a. \llbracket E_{p_2} [a] \rrbracket \bar{\otimes} \kappa)$ が成り立つことを証明する

証明.

p_1, p_2 が Hole のとき

$$\begin{aligned}
 (\text{左式}) &\equiv \llbracket S @ v \rrbracket \bar{\otimes} \kappa \\
 &\equiv \llbracket S \rrbracket \bar{\otimes} (\bar{\lambda} m. \llbracket v \rrbracket \bar{\otimes} (\bar{\lambda} n. (m \bar{\otimes} n) \bar{\otimes} (\lambda a. \kappa \bar{\otimes} a))) \\
 &\equiv \llbracket S @ v \rrbracket' \bar{\otimes} (\lambda a. \kappa \bar{\otimes} a) \\
 &\equiv \llbracket S @ v \rrbracket' \bar{\otimes} (\lambda a. \llbracket a \rrbracket \bar{\otimes} \kappa) \\
 &\equiv \llbracket S @ v \rrbracket' \bar{\otimes} (\lambda a. \llbracket E_{p_2} [a] \rrbracket \bar{\otimes} \kappa)
 \end{aligned}$$

p_1, p_2 が Frame (App₁ e₂) のとき

$$\begin{aligned}
 (\text{左式}) &\equiv \llbracket E_{p'_1} [S @ v] @ e_2 \rrbracket \bar{\otimes} \kappa \\
 &\equiv \llbracket E_{p'_1} [S @ v] @ e_2 \rrbracket \bar{\otimes} (\bar{\lambda} m. \llbracket e_2 \rrbracket \bar{\otimes} (\bar{\lambda} n. (m \bar{\otimes} n) \bar{\otimes} (\lambda a. \kappa \bar{\otimes} a))) \\
 &\equiv \llbracket S @ v \rrbracket' \bar{\otimes} (\lambda a. \llbracket E_{p'_2} [a] \rrbracket \bar{\otimes} (\bar{\lambda} m. \llbracket e_2 \rrbracket \bar{\otimes} (\bar{\lambda} n. (m \bar{\otimes} n) \bar{\otimes} (\lambda a. \kappa \bar{\otimes} a)))) \\
 &\equiv \llbracket S @ v \rrbracket' \bar{\otimes} (\lambda a. \llbracket E_{p'_2} [a] @ e_2 \rrbracket \bar{\otimes} \kappa) \\
 &\equiv \llbracket S @ v \rrbracket' \bar{\otimes} (\lambda a. \llbracket E_{p_2} [a] \rrbracket \bar{\otimes} \kappa)
 \end{aligned}$$

p_1, p_2 が Frame (App₂ v₁) のとき

$$\begin{aligned}
 (\text{左式}) &\equiv \llbracket v_1 @ E_{p'_1} [S @ v] \rrbracket \bar{\otimes} \kappa \\
 &\equiv \llbracket v_1 \rrbracket \bar{\otimes} (\bar{\lambda} m. \llbracket E_{p'_1} [S @ v] \rrbracket \bar{\otimes} (\bar{\lambda} n. (m \bar{\otimes} n) \bar{\otimes} (\lambda a. \kappa \bar{\otimes} a))) \\
 &\equiv \llbracket E_{p'_1} [S @ v] \rrbracket \bar{\otimes} (\bar{\lambda} n. (\llbracket v_1 \rrbracket_v @ n) @ (\lambda a. \kappa \bar{\otimes} a)) \\
 &\equiv \llbracket S @ v \rrbracket' \bar{\otimes} (\lambda a. \llbracket E_{p'_2} [a] \rrbracket \bar{\otimes} (\bar{\lambda} n. (\llbracket v_1 \rrbracket_v @ n) @ (\lambda a. \kappa \bar{\otimes} a))) \\
 &\equiv \llbracket S @ v \rrbracket' \bar{\otimes} (\lambda a. \llbracket v_1 @ E_{p'_2} [a] \rrbracket \bar{\otimes} \kappa) \\
 &\equiv \llbracket S @ v \rrbracket' \bar{\otimes} (\lambda a. \llbracket E_{p_2} [a] \rrbracket \bar{\otimes} \kappa)
 \end{aligned}$$

p_1, p_2 が Frame (Let e₂) のとき

$$\begin{aligned}
 (\text{左式}) &\equiv \llbracket \text{let } x = E_{p'_1} [S @ v] \text{ in } e_2 \rrbracket \bar{\otimes} \kappa \\
 &\equiv \llbracket E_{p'_1} [S @ v] \rrbracket \bar{\otimes} (\bar{\lambda} m. \underline{\text{let}} \ x = m \ \underline{\text{in}} \ e_2 \bar{\otimes} \kappa) \\
 &\equiv \llbracket S @ v \rrbracket' \bar{\otimes} (\lambda a. \llbracket E_{p'_2} [a] \rrbracket \bar{\otimes} (\bar{\lambda} m. \underline{\text{let}} \ x = m \ \underline{\text{in}} \ e_2 \bar{\otimes} \kappa)) \\
 &\equiv \llbracket S @ v \rrbracket' \bar{\otimes} (\lambda a. \llbracket \text{let } x = E_{p'_2} [a] \text{ in } e_2 \rrbracket \bar{\otimes} \kappa) \\
 &\equiv \llbracket S @ v \rrbracket' \bar{\otimes} (\lambda a. \llbracket E_{p_2} [a] \rrbracket \bar{\otimes} \kappa)
 \end{aligned}$$

□

4 CPS 変換の正当性の証明

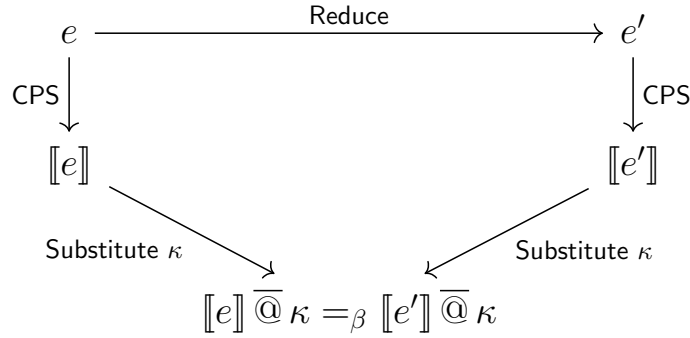
この節では、CPS 変換の正当性の証明として、CPS 変換が項の簡約関係を保存することを示す。

4.1 変換の証明

定理 4.1: (CPS 変換の正当性の証明)

任意の項 e, e' について $e \rightarrow e'$ が成り立つならば、任意の schematic な継続 κ について $\llbracket e \rrbracket \bar{\kappa} \rightarrow^* \llbracket e' \rrbracket \bar{\kappa}$

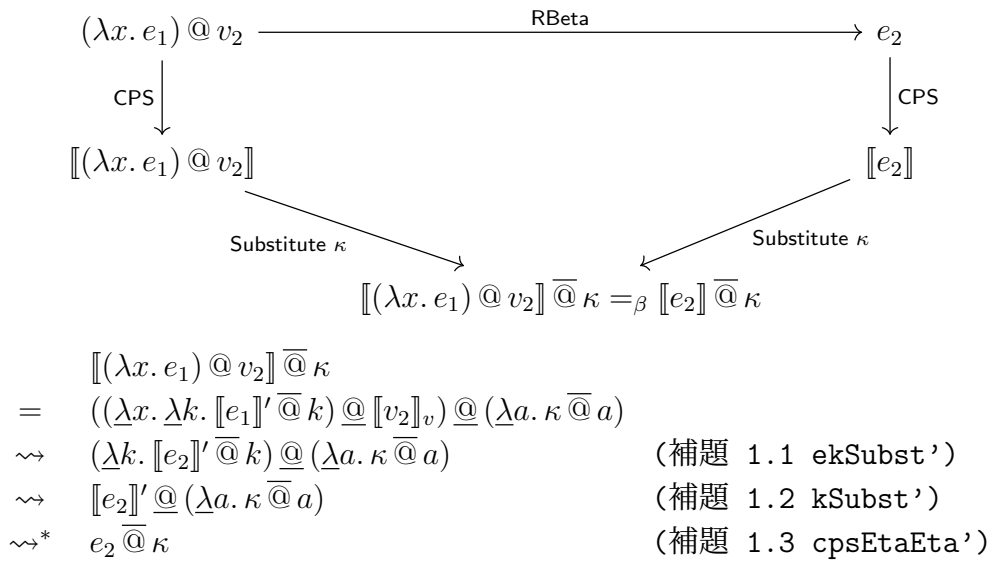
これは、以下のような図を意味する。



この図にある Reduce の部分について、RBeta、RFrame (App₁)、RFrame (App₂)、RReset、RShift のケースについて場合分けをして帰納的に解く。

4.1.1 RBeta のケース

RBeta のケースでの証明を行う。



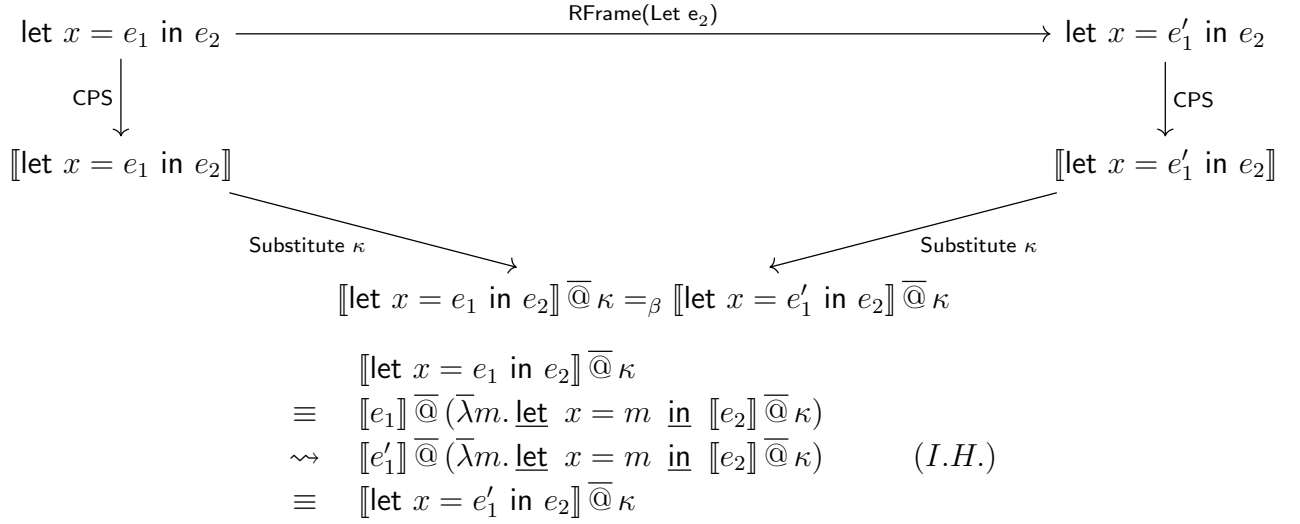
4.1.2 RFrame(App₁) のケース

$$\begin{array}{ccc}
e_1 @ e_2 & \xrightarrow{\text{RFrame}(\text{App}_1 e_2)} & e'_1 @ e_2 \\
\text{CPS} \downarrow & & \downarrow \text{CPS} \\
\llbracket e_1 @ e_2 \rrbracket & & \llbracket e'_1 @ e_2 \rrbracket \\
\searrow \text{Substitute } \kappa & & \swarrow \text{Substitute } \kappa \\
& \llbracket e_1 @ e_2 \rrbracket \bar{\alpha} \kappa =_\beta \llbracket e'_1 @ e_2 \rrbracket \bar{\alpha} \kappa & \\
\equiv & \llbracket e_1 @ e_2 \rrbracket \bar{\alpha} (\bar{\lambda} m. \llbracket e_2 \rrbracket \bar{\alpha} (\bar{\lambda} n. (m @ n) @ (\underline{\lambda} a. \kappa \bar{\alpha} a))) & \\
\rightsquigarrow & \llbracket e'_1 @ e_2 \rrbracket \bar{\alpha} (\bar{\lambda} m. \llbracket e_2 \rrbracket \bar{\alpha} (\bar{\lambda} n. (m @ n) @ (\underline{\lambda} a. \kappa \bar{\alpha} a))) & (I.H.) \\
\equiv & \llbracket e'_1 @ e_2 \rrbracket \bar{\alpha} \kappa &
\end{array}$$

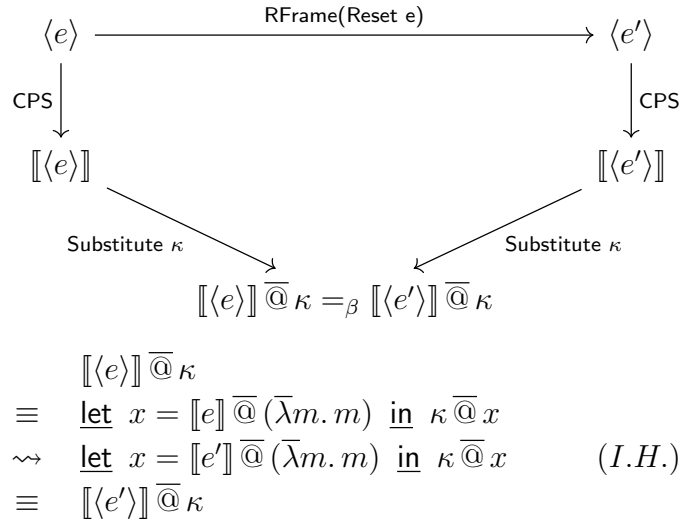
4.1.3 RFrame(App₂) のケース

$$\begin{array}{ccc}
v_1 @ e_2 & \xrightarrow{\text{RFrame}(\text{App}_2 v_1)} & v_1 @ e'_2 \\
\text{CPS} \downarrow & & \downarrow \text{CPS} \\
\llbracket v_1 @ e_2 \rrbracket & & \llbracket v_1 @ e'_2 \rrbracket \\
\searrow \text{Substitute } \kappa & & \swarrow \text{Substitute } \kappa \\
& \llbracket v_1 @ e_2 \rrbracket \bar{\alpha} \kappa =_\beta \llbracket v_1 @ e'_2 \rrbracket \bar{\alpha} \kappa & \\
\equiv & \llbracket v_1 @ e_2 \rrbracket \bar{\alpha} (\bar{\lambda} m. \llbracket e_2 \rrbracket \bar{\alpha} (\bar{\lambda} n. (m @ n) @ (\underline{\lambda} a. \kappa \bar{\alpha} a))) & \\
\equiv & \llbracket e_2 \rrbracket \bar{\alpha} (\bar{\lambda} n. \llbracket v_1 \rrbracket_v @ n @ (\underline{\lambda} a. \kappa \bar{\alpha} a)) & \\
\rightsquigarrow & \llbracket e'_2 \rrbracket \bar{\alpha} (\bar{\lambda} n. \llbracket v_1 \rrbracket_v @ n @ (\underline{\lambda} a. \kappa \bar{\alpha} a)) & (I.H.) \\
\equiv & \llbracket v_1 @ e'_2 \rrbracket \bar{\alpha} \kappa &
\end{array}$$

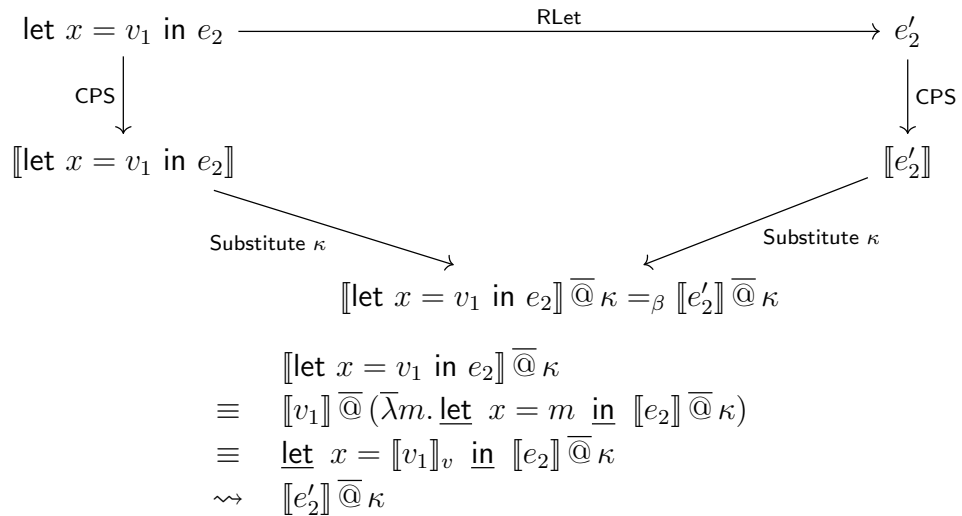
4.1.4 RFrame(Let) のケース



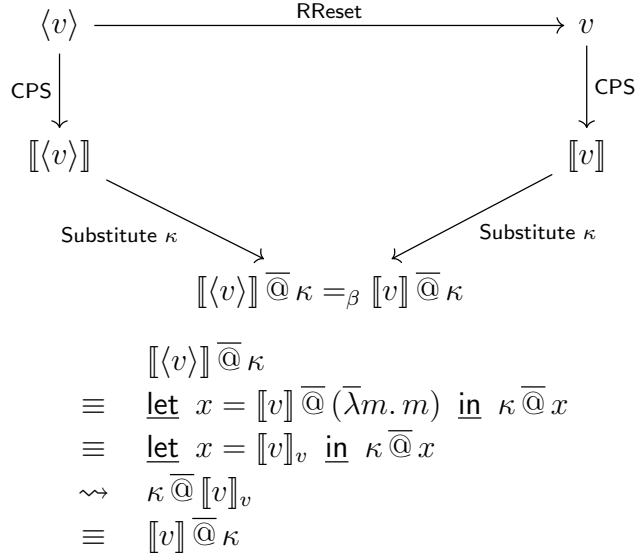
4.1.5 RFrame(Reset) のケース



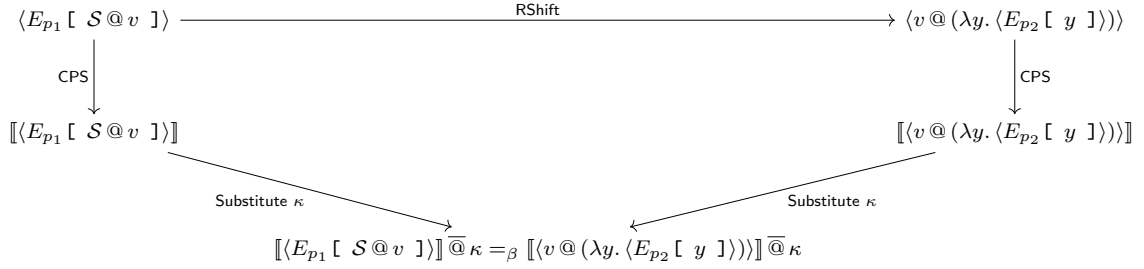
4.1.6 RLet のケース



4.1.7 RReset のケース



4.1.8 RShift のケース



$$\begin{aligned}
 & \llbracket \langle E_{p_1} [\mathcal{S} @ v] \rangle \rrbracket \bar{\text{@}} \kappa \\
 \equiv & \text{let } x = \llbracket E_{p_1} [\mathcal{S} @ v] \rrbracket \bar{\text{@}} (\bar{\lambda} m. m) \text{ in } \kappa \bar{\text{@}} x \\
 \equiv & \text{let } x = \llbracket \mathcal{S} @ v \rrbracket' \bar{\text{@}} (\lambda a. \llbracket E_{p_2} [a] \rrbracket \bar{\text{@}} (\bar{\lambda} m. m)) \text{ in } \kappa \bar{\text{@}} x & (\text{補題 3.4.1 contextContE}) \\
 \equiv & \text{let } x = (\llbracket \mathcal{S} \rrbracket_v \bar{\text{@}} \llbracket v \rrbracket_v) \bar{\text{@}} (\lambda a. \llbracket E_{p_2} [a] \rrbracket \bar{\text{@}} (\bar{\lambda} m. m)) \text{ in } \kappa \bar{\text{@}} x \\
 \equiv & \text{let } x = ((\lambda w k. (w \bar{\text{@}} (\lambda a k'. k' \bar{\text{@}} (k \bar{\text{@}} a))) \bar{\text{@}} (\lambda m. m)) \bar{\text{@}} \llbracket v \rrbracket_v) \bar{\text{@}} (\lambda a. \llbracket E_{p_2} [a] \rrbracket \bar{\text{@}} (\bar{\lambda} m. m)) \text{ in } \kappa \bar{\text{@}} x \\
 \rightsquigarrow & \text{let } x = (\llbracket v \rrbracket_v \bar{\text{@}} (\lambda a k'. k' \bar{\text{@}} ((\lambda a. \llbracket E_{p_2} [a] \rrbracket \bar{\text{@}} (\bar{\lambda} m. m)) \bar{\text{@}} a))) \bar{\text{@}} (\lambda m. m) \text{ in } \kappa \bar{\text{@}} x & (\text{eqShiftc}) \\
 \equiv & \text{let } x = (\llbracket v \rrbracket_v \bar{\text{@}} (\lambda a k'. k' \bar{\text{@}} (\llbracket a \rrbracket' \bar{\text{@}} (\lambda a. \llbracket E_{p_2} [a] \rrbracket \bar{\text{@}} (\bar{\lambda} m. m))))) \bar{\text{@}} (\lambda m. m) \text{ in } \kappa \bar{\text{@}} x \\
 \equiv & \text{let } x = (\llbracket v \rrbracket_v \bar{\text{@}} (\lambda a k'. k' \bar{\text{@}} (\llbracket E_{p_2} [a] \rrbracket \bar{\text{@}} (\bar{\lambda} m. m))))) \bar{\text{@}} (\lambda m. m) \text{ in } \kappa \bar{\text{@}} x \\
 \sim & \text{let } x = (\llbracket v \rrbracket_v \bar{\text{@}} (\lambda a k'. \text{let } x = \llbracket E_{p_2} [a] \rrbracket \bar{\text{@}} (\bar{\lambda} m. m) \text{ in } k' \bar{\text{@}} x)) \bar{\text{@}} (\lambda m. m) \text{ in } \kappa \bar{\text{@}} x & (\text{eqLetApp2}) \\
 \equiv & \text{let } x = (\llbracket v \rrbracket_v \bar{\text{@}} (\lambda a k'. \llbracket \langle E_{p_2} [a] \rangle \rrbracket' \bar{\text{@}} k')) \bar{\text{@}} (\lambda m. m) \text{ in } \kappa \bar{\text{@}} x \\
 \equiv & \text{let } x = (\llbracket v \rrbracket_v \bar{\text{@}} \llbracket \lambda a. E_{p_2} [a] \rrbracket_v) \bar{\text{@}} (\lambda m. m) \text{ in } \kappa \bar{\text{@}} x \\
 \equiv & \llbracket \langle v @ (\lambda y. \langle E_{p_2} [y] \rangle) \rangle \rrbracket \bar{\text{@}} \kappa
 \end{aligned}$$