CPS変換メモ

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1 DS項の定義

ここでは、CPS 変換前の項、すなわち CPS 変換を行う対象の言語を示す。

1.1 構文

$$
\begin{array}{rcll}
\tau & ::= & \mathsf{Nat} \mid \mathsf{bool} \mid \tau_1 \to \tau_2 @\mathrm{cps}[\tau_3, \tau_4] & \text{型} \\
v & ::= & n \mid x \mid \lambda x.\, e \mid \mathcal{S} & \text{値} \\
e & ::= & v \mid e_1 @ e_2 \mid \langle e \rangle \mid \mathsf{let}\ x = e_1\ \mathsf{in}\ e_2 & \text{項}
\end{array}
$$

1.2 DS項の型付け規則

$$
\frac{\Gamma(x) = \tau_1}{\Gamma \vdash x : \tau_1 @\mathrm{cps}[\tau, \tau]}\ (\textsc{TVar}) \qquad \frac{}{\Gamma \vdash n : \mathsf{Nat} @\mathrm{cps}[\tau, \tau]}\ (\textsc{TNum})
$$

$$
\frac{\Gamma, x : \tau_2 \vdash e : \tau_1 @\mathrm{cps}[\tau_3, \tau_4]}{\Gamma \vdash \lambda x.\, e : (\tau_2 \to \tau_1 @\mathrm{cps}[\tau_3, \tau_4]) @\mathrm{cps}[\tau, \tau]}\ (\textsc{TFun})
$$

$$
\frac{}{\Gamma \vdash \mathcal{S} : (((\tau_3 \to \tau_4 @\mathrm{cps}[\tau, \tau]) \to \tau_1 @\mathrm{cps}[\tau_1, \tau_2]) \to \tau_3 @\mathrm{cps}[\tau_4, \tau_2]) @\mathrm{cps}[\tau, \tau]}\ (\textsc{TShift})
$$

$$
\frac{\Gamma \vdash e_1 : (\tau_2 \to \tau_1 @\mathrm{cps}[\tau_3, \tau_4]) @\mathrm{cps}[\tau_5, \tau_6] \quad \Gamma \vdash e_2 : \tau_2 @\mathrm{cps}[\tau_4, \tau_5]}{\Gamma \vdash e_1 @ e_2 : \tau_1 @\mathrm{cps}[\tau_3, \tau_6]}\ (\textsc{TApp})
$$

$$
\frac{\Gamma \vdash e : \tau_1 @\mathrm{cps}[\tau_1, \tau_2]}{\Gamma \vdash \langle e \rangle : \tau_2 @\mathrm{cps}[\tau_3, \tau_3]}\ (\textsc{TReset})
$$

$$
\frac{\Gamma \vdash e_1 : \tau_1 @\mathrm{cps}[\beta, \gamma] \quad \Gamma, x : \tau_1 \vdash e_2 : \tau_2 @\mathrm{cps}[\alpha, \beta]}{\Gamma \vdash (\mathsf{let}\ x = e_1\ \mathsf{in}\ e_2) : \tau_2 @\mathrm{cps}[\alpha, \gamma]}\ (\textsc{TLet})
$$

1.3 DS 項の代入規則

代入規則は、$e[v/x] = e'$ と表現することができ、「項 e の中に現れる変数 x を値 v に置き換える」と読む。

今回用いる代入規則は以下である。

$$\frac{}{x[v/x] = x}\ (\textsc{sVar=}) \qquad \frac{}{y[v/x] = y}\ (\textsc{sVar}\neq) \qquad \frac{}{n[v/x] = n}\ (\textsc{sNum})$$

$$\frac{\forall x.(e[v/y] = e')}{(\lambda x.\, e)[v/y] = \lambda x.\, e'}\ (\textsc{sFun}) \qquad \frac{e_1[v/x] = e_1' \quad e_2[v/x] = e_2'}{(e_1 \mathbin{@} e_2)[v/x] = (e_1' \mathbin{@} e_2')}\ (\textsc{sApp})$$

$$\frac{}{\mathcal{S}[v/x] = \mathcal{S}}\ (\textsc{sShift}) \qquad \frac{e_1[v/x] = e_1'}{\langle e_1\rangle[v/x] = \langle e_1'\rangle}\ (\textsc{sReset})$$

$$\frac{e_1[v/y] = e_1' \quad \forall x.(e_2[v/y] = e_2')}{(\mathsf{let}\ x = e_1\ \mathsf{in}\ e_2)[v/y] = \mathsf{let}\ x = e_1'\ \mathsf{in}\ e_2'}\ (\textsc{sLet})$$

1.4 DS 項の簡約規則

次に、簡約規則について示す。ただし、簡約規則を示す前に、フレームを定義する必要がある。フレームを定義することによって、簡約の順序を決めることができる。

$$\begin{array}{rcl} \text{フレーム}\ \ F & = & [\ ] \mathbin{@} e_2 \mid v_1 \mathbin{@} [\ ] \mid \langle\, [\ ]\, \rangle \mid \mathsf{let}\ x = [\ ]\ \mathsf{in}\ e_2 \\ \text{評価文脈 (コンテキスト)}\ \ E & = & [\ ] \mid F \circ E \end{array}$$

$$\frac{\Gamma \vdash e_2 : \tau_2 \mathbin{@}\mathrm{cps}[\tau_4, \tau_6]}{\Gamma \vdash ([\ ] \mathbin{@} e_2) : [\ (\tau_2 \to \tau_1 @\mathrm{cps}[\tau_3, \tau_4]) @\mathrm{cps}[\tau_5, \tau_6]\ ]_{\mathsf{f}}\ \tau_1 @\mathrm{cps}[\tau_3, \tau_6]}\ (\text{F-}\textsc{App}_1)$$

$$\frac{\Gamma \vdash v_1 : (\tau_2 \to \tau_1 @\mathrm{cps}[\tau_3, \tau_4]) \mathbin{@}\mathrm{cps}[\tau_5, \tau_5]}{\Gamma \vdash (v_1 \mathbin{@} [\ ]) : [\ \tau_2 @\mathrm{cps}[\tau_4, \tau_5]\ ]_{\mathsf{f}}\ \tau_1 @\mathrm{cps}[\tau_3, \tau_5]}\ (\text{F-}\textsc{App}_2)$$

$$\frac{}{\Gamma \vdash \langle [\ ] \rangle : [\ \tau_1 @\mathrm{cps}[\tau_1, \tau_2]\ ]_{\mathsf{f}}\ \tau_2 @\mathrm{cps}[\tau_3, \tau_3]}\ (\text{F-}\textsc{Reset})$$

$$\frac{\Gamma, x : \tau_1 \vdash e_2 : \tau_2 \mathbin{@}\mathrm{cps}[\alpha, \beta]}{\Gamma \vdash \mathsf{let}\ x = [\ ]\ \mathsf{in}\ e_2 : [\ \tau_1 @\mathrm{cps}[\beta, \gamma]\ ]_{\mathsf{f}}\ \tau_2 @\mathrm{cps}[\alpha, \gamma]}\ (\text{F-}\textsc{Let})$$

[ ] は、「次に計算を行う部分」である。「それ以外の部分」は、「文脈」と呼ぶ。評価文脈はフレームを何枚も重ね合わせたものであり、フレームの列として表すことができる。そのように表せるとすると、評価文脈 E に式 e を入れる関数 $plug_F$ を以下のように定義できる。

$$\begin{array}{rcl} plug_F([\ ] \mathbin{@} e_2, e_1) & = & e_1 \mathbin{@} e_2 \\ plug_F(v_1 \mathbin{@} [\ ], e_2) & = & v_1 \mathbin{@} e_2 \\ plug_F(\langle [\ ] \rangle, e_1) & = & \langle e_1 \rangle \\ plug_F(\mathsf{let}\ x = [\ ]\ \mathsf{in}\ e_2, e_1) & = & \mathsf{let}\ x = e_1\ \mathsf{in}\ e_2 \end{array}$$

次に、reset を含まないフレームである、ピュアフレームを定義する。

$$\begin{array}{rcl} \text{ピュアフレーム} \quad F_p & = & [\;]\,@\,e_2 \mid v_1\,@\,[\;] \mid \mathsf{let}\ x = [\;]\ \mathsf{in}\ e_2 \\ \text{ピュアコンテキスト} \quad E_p & = & [\;] \mid F_p \circ E_p \end{array}$$

$\boxed{\text{ピュアフレーム} \quad F_p}$

$$\frac{\Gamma \vdash e_2 : \tau_2\,@\mathrm{cps}[\tau_4,\tau_5]}{\Gamma \vdash ([\;]\,@\,e_2) : [\;(\tau_2 \to \tau_1 @\mathrm{cps}[\tau_3,\tau_4]) @\mathrm{cps}[\tau_5,\tau_6]\;]_{\mathsf{pf}}\ \tau_1 @\mathrm{cps}[\tau_3,\tau_6]}\ (\mathrm{F}_p\text{-}\mathrm{A{\scriptsize PP}}_1)$$

$$\frac{\Gamma \vdash v_1 : (\tau_2 \to \tau_1 @\mathrm{cps}[\tau_3,\tau_4])\,@\mathrm{cps}[\tau_5,\tau_5]}{\Gamma \vdash (v_1\,@\,[\;]) : [\;\tau_2 @\mathrm{cps}[\tau_4,\tau_5]\;]_{\mathsf{pf}}\ \tau_1 @\mathrm{cps}[\tau_3,\tau_5]}\ (\mathrm{F}_p\text{-}\mathrm{A{\scriptsize PP}}_2)$$

$$\frac{\Gamma, x : \tau_1 \vdash e_2 : \tau_2\,@\mathrm{cps}[\alpha,\beta]}{\Gamma \vdash \mathsf{let}\ x = [\;]\ \mathsf{in}\ e_2 : [\;\tau_1 @\mathrm{cps}[\beta,\gamma]\;]_{\mathsf{pf}}\ \tau_2 @\mathrm{cps}[\alpha,\gamma]}\ [(\mathrm{F}_p\text{-}\mathrm{L{\scriptsize ET}})$$

$\boxed{\text{ピュアコンテキスト} \quad E_p}$

$$\frac{}{\Gamma \vdash [\;] : [\;\tau_1 @\mathrm{cps}[\tau_2,\tau_3]\;]_{\mathsf{pc}}\ \tau_1 @\mathrm{cps}[\tau_2,\tau_3]}\ (\mathrm{E}_p\text{-}\mathrm{H{\scriptsize OLE}})$$

$$\frac{\Gamma \vdash F_p : [\;\tau_4 @\mathrm{cps}[\tau_5,\tau_6]\;]_{\mathsf{pf}}\ \tau_7 @\mathrm{cps}[\tau_8,\tau_9] \quad \Gamma \vdash E_p : [\;\tau_1 @\mathrm{cps}[\tau_2,\tau_3]\;]_{\mathsf{pf}}\ \tau_4 @\mathrm{cps}[\tau_5,\tau_6]}{\Gamma \vdash F_p \circ E_p : [\;\tau_1 @\mathrm{cps}[\tau_2,\tau_3]\;]_{\mathsf{pc}}\ \tau_7 @\mathrm{cps}[\tau_8,\tau_9]}\ (\mathrm{E}_p\text{-}\mathrm{F{\scriptsize RAME}})$$

$\boxed{\text{ピュアフレーム同士の関係} \quad F_p \cong_{\mathsf{c}} F_p}$

$$\frac{}{([\;]\,@\,e_2) \cong_{\mathsf{f}} ([\;]\,@\,e_2)}\ (\cong_{\mathsf{pf}}\text{-}\mathrm{A{\scriptsize PP}}_1)$$

同様に、関数 $plug_{F_p}$ を定義する。

$$\begin{array}{rcl} plug_{F_p}([\;]\,@\,e_2, e_1) & = & e_1\,@\,e_2 \\ plug_{F_p}(v_1\,@\,[\;], e_2) & = & v_1\,@\,e_2 \\ plug_{F_p}(\mathsf{let}\ x = [\;]\ \mathsf{in}\ e_2, e_1) & = & \mathsf{let}\ x = e_1\ \mathsf{in}\ e_2 \end{array}$$

関数 $plug_{E_p}$ は以下のように定義できる。

$$\begin{array}{rcl} plug_{E_p}([\;], e_1) & = & e_1 \\ plug_{E_p}(F_p \circ E_p, e_2) & = & plug_{F_p}(F_p, plug_{E_p}(E_p, e_1)) \end{array}$$

以上より、簡約規則は以下のように表せる。

$$\frac{e_1[v_2/x] = e_1'}{(\lambda x.\,e_1)\,@\,v_2 \rightsquigarrow e_1'}\ (\mathrm{RB{\scriptsize ETA}}) \qquad \frac{e_1 \rightsquigarrow e_2}{plug_F(F, e_1) \rightsquigarrow plug_F(F, e_2)}\ (\mathrm{RF{\scriptsize RAME}})$$

$$\frac{E_{p_1} \cong_{\mathsf{c}} E_{p_2}}{\langle E_{p_1}[\;\mathcal{S}\,@\,v_2\;]\rangle \rightsquigarrow \langle v_2\,@\,(\lambda y.\,\langle E_{p_2}[\;y\;]\rangle)\rangle}\ (\mathrm{RS{\scriptsize HIFT}}) \qquad \frac{}{\langle v_1 \rangle \rightsquigarrow v_1}\ (\mathrm{RR{\scriptsize ESET}})$$

$$\frac{e_2[v_1/x] = e_2'}{\mathsf{let}\ x = v_1\ \mathsf{in}\ e_2 \rightsquigarrow e_2'}\ (\mathrm{RL{\scriptsize ET}})$$

2 CPS 項の定義

2.1 CPS 項の構文

$$
\begin{array}{rcll}
\tau & ::= & \mathsf{Nat} \mid \mathsf{bool} \mid \tau_1 \to \tau_2 & \text{型} \\
v & ::= & n \mid x \mid \underline{\lambda} x.\, \underline{\lambda} k.\, e \mid \mathcal{S} & \text{値} \\
\mathcal{S} & ::= & \underline{\lambda} w.\, \underline{\lambda} k.\, (w \mathbin{\underline{@}} (\underline{\lambda} a.\, \underline{\lambda} k'.\, k' \mathbin{\underline{@}} (k \mathbin{\underline{@}} a))) \mathbin{\underline{@}} (\underline{\lambda} m.\, m) & \texttt{shift} \\
e & ::= & v \mid e_1 \mathbin{\underline{@}} e_2 \mid \underline{\mathsf{let}}\ x = e_1\ \underline{\mathsf{in}}\ e_2 & \text{項}
\end{array}
$$

2.2 CPS 項の型付け規則

$$
\frac{\Gamma(x) = \tau_1}{\Gamma \vdash x : \tau_1}\ (\text{TVar}_\mathsf{C}) \qquad \frac{}{\Gamma \vdash n : \mathsf{Nat}}\ (\text{TNum}_\mathsf{C}) \qquad \frac{\Gamma, x : \tau_2 \vdash e : \tau_1}{\Gamma \vdash \underline{\lambda} x.\, \underline{\lambda} k.\, e : \tau_2 \to \tau_1}\ (\text{TFun}_\mathsf{C})
$$

$$
\frac{}{\Gamma \vdash \mathcal{S} : ((\tau_1 \to (\tau_2 \to \tau_3) \to \tau_3) \to (\tau_4 \to \tau_4) \to \tau_5) \to (\tau_1 \to \tau_2) \to \tau_5}\ (\text{TShift}_\mathsf{C})
$$

$$
\frac{\Gamma \vdash e_1 : \tau_2 \to \tau_1 \quad \Gamma \vdash e_2 : \tau_2}{\Gamma \vdash e_1 \mathbin{\underline{@}} e_2 : \tau_1}\ (\text{TApp}_\mathsf{C}) \qquad \frac{\Gamma \vdash e_1 : \tau_1 \quad \Gamma, x : \tau_1 \vdash e_2 : \tau_2}{\Gamma \vdash (\mathsf{let}\ x = e_1\ \mathsf{in}\ e_2) : \tau_2}\ (\text{TLet}_\mathsf{C})
$$

2.3 CPS 項の代入規則

$$
\frac{}{x[v/x] = x}\ (\text{sVar=}_\mathsf{C}) \qquad \frac{}{y[v/x] = y}\ (\text{sVar} \neq_\mathsf{C}) \qquad \frac{}{n[v/x] = n}\ (\text{sNum}_\mathsf{C})
$$

$$
\frac{\forall x.(e[v/y] = e')}{(\lambda x.\, e)[v/y] = \lambda x.\, e'}\ (\text{sFun}_\mathsf{C}) \qquad \frac{e_1[v/x] = e_1' \quad e_2[v/x] = e_2'}{(e_1 @ e_2)[v/x] = (e_1' @ e_2')}\ (\text{sApp}_\mathsf{C})
$$

$$
\frac{}{\mathcal{S}[v/x] = \mathcal{S}}\ (\text{sShift}) \qquad \frac{e_1[v/y] = e_1' \quad \forall x.(e_2[v/y] = e_2')}{(\mathsf{let}\ x = e_1\ \mathsf{in}\ e_2)[v/y] = \mathsf{let}\ x = e_1'\ \mathsf{in}\ e_2'}\ (\text{sLet})
$$

2.4 CPS 項の簡約規則

$$
\frac{e_1[v_2/x] = e_1'}{(\lambda x.\, e_1) @ v_2 \rightsquigarrow e_1'}\ (\text{eqBeta}_\mathsf{C}) \qquad \frac{e_2[v_1/x] = e_2'}{\mathsf{let}\ x = v_1\ \mathsf{in}\ e_2 \rightsquigarrow e_2'}\ (\text{eqLet}_\mathsf{C})
$$

$$
\frac{}{(\underline{\lambda} w.\, \underline{\lambda} k.\, (w \mathbin{\underline{@}} (\underline{\lambda} a.\, \underline{\lambda} k'.\, k' \mathbin{\underline{@}} (k \mathbin{\underline{@}} a))) \mathbin{\underline{@}} (\underline{\lambda} m.\, m) \mathbin{\underline{@}} v_2) \mathbin{\underline{@}} k \rightsquigarrow (v_2 \mathbin{\underline{@}} (\underline{\lambda} a.\, \underline{\lambda} k'.\, k' \mathbin{\underline{@}} (k \mathbin{\underline{@}} a))) \mathbin{\underline{@}} (\underline{\lambda} m.\, m)}\ (\text{eqShift}_\mathsf{C})
$$

$$
\frac{e_1 \rightsquigarrow e_1'}{e_1 \mathbin{\underline{@}} e_2 \rightsquigarrow e_1' \mathbin{\underline{@}} e_2}\ (\text{eqApp1}_\mathsf{C}) \qquad \frac{e_2 \rightsquigarrow e_2'}{e_1 \mathbin{\underline{@}} e_2 \rightsquigarrow e_1 \mathbin{\underline{@}} e_2'}\ (\text{eqApp2}_\mathsf{C})
$$

$$
\frac{e_1 \rightsquigarrow e_1'}{\underline{\mathsf{let}}\ x = e_1\ \underline{\mathsf{in}}\ e_2 \rightsquigarrow \underline{\mathsf{let}}\ x = e_1'\ \underline{\mathsf{in}}\ e_2}\ (\text{eqLet1}_\mathsf{C}) \qquad \frac{}{v_1 \mathbin{\underline{@}} e_2 \rightsquigarrow \underline{\mathsf{let}}\ x = e_2\ \underline{\mathsf{in}}\ v_1 \mathbin{\underline{@}} x}\ (\text{eqLetApp2}_\mathsf{C})
$$

2.5 CPS 変換の定式化

η redex を作らない、one-pass の CPS 変換の定義を示す。

$$
\begin{array}{rcl}
[\![n]\!]_v &=& n \\
[\![x]\!]_v &=& x \\
[\![\lambda x.\, e]\!]_v &=& \underline{\lambda} x.\, \underline{\lambda} k.\, [\![e]\!]' \,\overline{@}\, k \\
[\![\mathcal{S}]\!]_v &=& \underline{\lambda} wk.\, (w \,\underline{@}\, (\underline{\lambda} ak'.\, k' \,\underline{@}\, (k \,\underline{@}\, a))) \,\underline{@}\, (\underline{\lambda} m.\, m) \\
\\
[\![v]\!] &=& \overline{\lambda} \kappa.\, \kappa \,\overline{@}\, [\![v]\!]_v \\
[\![e_1 \,@\, e_2]\!] &=& \overline{\lambda} \kappa.\, [\![e_1]\!] \,\overline{@}\, (\overline{\lambda} m.\, [\![e_2]\!] \,\overline{@}\, (\overline{\lambda} n.\, (m \,\underline{@}\, n) \,\underline{@}\, (\underline{\lambda} a.\, \kappa \,\overline{@}\, a))) \\
[\![\langle e \rangle]\!] &=& \overline{\lambda} \kappa.\, \underline{\mathsf{let}}\ \ x = [\![e]\!] \,\overline{@}\, (\overline{\lambda} m.\, m)\ \ \underline{\mathsf{in}}\ \ \kappa \,\overline{@}\, x \\
[\![\mathsf{let}\ x = e_1\ \mathsf{in}\ e_2]\!] &=& \overline{\lambda} \kappa.\, [\![e_1]\!] \,\overline{@}\, (\overline{\lambda} m.\, \underline{\mathsf{let}}\ \ x = m\ \ \underline{\mathsf{in}}\ \ [\![e_2]\!] \,\overline{@}\, \kappa) \\
\\
[\![v]\!]' &=& \overline{\lambda} k.\, k \,\underline{@}\, [\![v]\!]_v \\
[\![e_1 \,@\, e_2]\!]' &=& \overline{\lambda} \kappa.\, [\![e_1]\!] \,\overline{@}\, (\overline{\lambda} m.\, [\![e_2]\!] \,\overline{@}\, (\overline{\lambda} n.\, (m \,\underline{@}\, n) \,\underline{@}\, k)) \\
[\![\langle e \rangle]\!]' &=& \overline{\lambda} k.\, \underline{\mathsf{let}}\ \ x = [\![e]\!] \,\overline{@}\, (\overline{\lambda} m.\, m)\ \ \underline{\mathsf{in}}\ \ k \,\underline{@}\, x \\
[\![\mathsf{let}\ x = e_1\ \mathsf{in}\ e_2]\!]' &=& \overline{\lambda} k.\, [\![e_1]\!] \,\overline{@}\, (\overline{\lambda} m.\, \underline{\mathsf{let}}\ \ x = m\ \ \underline{\mathsf{in}}\ \ [\![e_2]\!]' \,\overline{@}\, k)
\end{array}
$$

3 補題の証明

CPS 変換の証明を行う前に、必要な補題をいくつか証明する。

3.1 CPS 項に関する代入補題の証明

補題 3.1.1: (eSubstV)

$v_1[v/x] = v_1'$ のとき、$[\![v_1]\!]_v[[\![v]\!]_v/x] = [\![v_1']\!]_v$

証明.

$v = x$ のとき

$$\begin{array}{ll} [\![x]\!]_v[[\![v]\!]_v/x] & = x[[\![v]\!]_v/x] \\ & = [\![v]\!]_v \qquad \texttt{(sVar=)} \end{array}$$

$v = y$ のとき

$$\begin{array}{lcl} [\![y]\!]_v[[\![v]\!]_v/x] & = & y[[\![v]\!]_v/x] \\ & = & [\![y]\!]_v \qquad \texttt{(sVar} \neq \texttt{)} \end{array}$$

$v = \lambda x.\, e$ のとき

$$\begin{array}{lcl} [\![\lambda x.\, e]\!]_v[[\![v]\!]_v/x] & = & (\underline{\lambda} x.\, \underline{\lambda} k.\, [\![e]\!]' \,\overline{@}\, k)[[\![v]\!]_v/x] \\ & = & [\![y]\!]_v \qquad \texttt{(sVar} \neq \texttt{)} \end{array}$$

$v = \mathcal{S}$ のとき

$$\begin{array}{lcl} [\![\mathcal{S}]\!]_v[[\![v]\!]_v/x] & = & (\underline{\lambda} wk.\, (w \,\underline{@}\, (\underline{\lambda} ak'.\, k' \,\underline{@}\, (k \,\underline{@}\, a))) \,\underline{@}\, (\underline{\lambda} m.\, m))[[\![v]\!]_v/x] \\ & = & [\![\mathcal{S}]\!]_v \end{array}$$

□

補題 3.1.2: (e κ Subst)

$e_1[v/x] = e_2$ かつ $\kappa_1[[\![v]\!]_v/x] = \kappa_2$ のとき、$([\![e_1]\!] \,\overline{@}\, \kappa_1)[[\![v]\!]_v/x] = [\![e_2]\!] \,\overline{@}\, \kappa_2$

証明.

$e_1 = v_1$ (値) のとき

$v_1[v/x] = v_2$ とすると、

$$
\begin{aligned}
(\text{与式}) &= (\llbracket v_1 \rrbracket \mathbin{\overline{@}} \kappa_1)[\llbracket v \rrbracket_v / x] \\
&= ((\overline{\lambda}\kappa.\, \kappa \mathbin{\overline{@}} \llbracket v_1 \rrbracket_v) \mathbin{\overline{@}} \kappa_1)[\llbracket v \rrbracket_v / x] \\
&= (\kappa_1 \mathbin{\overline{@}} \llbracket v_1 \rrbracket_v)[\llbracket v \rrbracket_v / x] \\
&= (\kappa_1[\llbracket v \rrbracket_v / x]) \mathbin{\overline{@}} (\llbracket v_1 \rrbracket_v [\llbracket v \rrbracket_v / x]) \\
&= \kappa_2 \mathbin{\overline{@}} \llbracket v_2 \rrbracket_v & (\text{補題 } 1.1.1\ \texttt{eSubstV}) \\
&= \llbracket v_2 \rrbracket \mathbin{\overline{@}} \kappa_2
\end{aligned}
$$

e_1 **が App のとき**

$(e_1 \mathbin{@} e_2)[v/x] = e_1' \mathbin{@} e_2'$ とすると、

$$
\begin{aligned}
(\text{与式}) &= (\llbracket e_1 \mathbin{@} e_2 \rrbracket \mathbin{\overline{@}} \kappa_1)[\llbracket v \rrbracket_v / x] \\
&= ((\overline{\lambda}\kappa.\, \llbracket e_1 \rrbracket \mathbin{\overline{@}} (\overline{\lambda}m.\, \llbracket e_2 \rrbracket \mathbin{\overline{@}} (\overline{\lambda}n.\, (m \mathbin{\underline{@}} n) \mathbin{\underline{@}} (\underline{\lambda}a.\, \kappa_1 \mathbin{\overline{@}} a)))) \mathbin{\overline{@}} \kappa)[\llbracket v \rrbracket_v / x] \\
&= (\llbracket e_1 \rrbracket \mathbin{\overline{@}} (\overline{\lambda}m.\, \llbracket e_2 \rrbracket \mathbin{\overline{@}} (\overline{\lambda}n.\, (m \mathbin{\underline{@}} n) \mathbin{\underline{@}} (\underline{\lambda}a.\, \kappa_1 \mathbin{\overline{@}} a))))[\llbracket v \rrbracket_v / x] \\
&= (\llbracket e_1 \rrbracket [\llbracket v \rrbracket_v / x]) \mathbin{\overline{@}} ((\overline{\lambda}m.\, \llbracket e_2 \rrbracket \mathbin{\overline{@}} (\overline{\lambda}n.\, (m \mathbin{\underline{@}} n) \mathbin{\underline{@}} (\underline{\lambda}a.\, \kappa_1 \mathbin{\overline{@}} a)))[\llbracket v \rrbracket_v / x]) \\
&= \llbracket e_1' \rrbracket \mathbin{\overline{@}} (\overline{\lambda}m.\, (\llbracket e_2 \rrbracket [\llbracket v \rrbracket_v / x]) \mathbin{\overline{@}} ((\overline{\lambda}n.\, (m \mathbin{\underline{@}} n) \mathbin{\underline{@}} (\underline{\lambda}a.\, \kappa_1 \mathbin{\overline{@}} a))[\llbracket v \rrbracket_v / x])) \\
&= \llbracket e_1' \rrbracket \mathbin{\overline{@}} (\overline{\lambda}m.\, \llbracket e_2' \rrbracket \mathbin{\overline{@}} (\overline{\lambda}n.\, (m \mathbin{\underline{@}} n) \mathbin{\underline{@}} (\underline{\lambda}a.\, \kappa_2 \mathbin{\overline{@}} a))) \\
&= (\llbracket e_1 \mathbin{@} e_2 \rrbracket) \mathbin{\overline{@}} \kappa_2
\end{aligned}
$$

e_1 **が Reset のとき**

$\langle e \rangle [v/x] = \langle e' \rangle$ とすると、

$$
\begin{aligned}
(\text{与式}) &= (\llbracket \langle e \rangle \rrbracket \mathbin{\overline{@}} \kappa_1)[\llbracket v \rrbracket_v / x] \\
&= (\underline{\mathsf{let}}\ \ c = \llbracket e \rrbracket \mathbin{\overline{@}} (\overline{\lambda}m.\, m)\ \ \underline{\mathsf{in}}\ \ \kappa_1 \mathbin{\overline{@}} c)[\llbracket v \rrbracket_v / x] \\
&= \underline{\mathsf{let}}\ \ c = \llbracket e' \rrbracket \mathbin{\overline{@}} (\overline{\lambda}m.\, m)\ \ \underline{\mathsf{in}}\ \ \kappa_2 \mathbin{\overline{@}} c \\
&= \llbracket \langle e' \rangle \rrbracket \mathbin{\overline{@}} \kappa_2
\end{aligned}
$$

e_1 **が Let のとき**

$(\mathsf{let}\ x = e_1\ \mathsf{in}\ e_2)[v/x] = \mathsf{let}\ x = e_1'\ \mathsf{in}\ e_2'$ とすると、

$$
\begin{aligned}
(\text{与式}) &= \mathsf{let}\ x = e_1\ \mathsf{in}\ e_2 \mathbin{\overline{@}} \kappa_1[\llbracket v \rrbracket_v / x] \\
&= (\llbracket e_1 \rrbracket \mathbin{\overline{@}} (\overline{\lambda}m.\, \underline{\mathsf{let}}\ \ x = m\ \ \underline{\mathsf{in}}\ \ \llbracket e_2 \rrbracket \mathbin{\overline{@}} \kappa_1))[\llbracket v \rrbracket_v / x] \\
&= (\llbracket e_1 \rrbracket [\llbracket v \rrbracket_v / x]) \mathbin{\overline{@}} (\overline{\lambda}m.\, \underline{\mathsf{let}}\ \ x = m\ \ \underline{\mathsf{in}}\ \ (\llbracket e_2 \rrbracket [\llbracket v \rrbracket_v / x]) \mathbin{\overline{@}} (\kappa_1 [\llbracket v \rrbracket_v / x])) \\
&= \llbracket e_1[v/x] \rrbracket \mathbin{\overline{@}} (\overline{\lambda}m.\, \underline{\mathsf{let}}\ \ x = m\ \ \underline{\mathsf{in}}\ \ (\llbracket e_2[v/x] \rrbracket) \mathbin{\overline{@}} \kappa_2) \\
&= \llbracket \mathsf{let}\ x = e_1'\ \mathsf{in}\ e_2' \rrbracket \mathbin{\overline{@}} \kappa_2
\end{aligned}
$$

□

補題 3.1.3: (ekSubst')

$e[v/x] = e'$ のとき、$(\llbracket e \rrbracket' \mathbin{\overline{@}} k)[\llbracket v \rrbracket_v / x] = \llbracket e' \rrbracket' \mathbin{\overline{@}} k$

証明.

$e = v$ (値) のとき

$v[\llbracket v \rrbracket_v / x] = v'$ とすると、

$$\begin{aligned}
(\text{与式}) &= \llbracket v \rrbracket' \,\overline{@}\, k[\llbracket v_2 \rrbracket_v / x] \\
&= ((\overline{\lambda} k.\, k \,\underline{@}\, \llbracket v \rrbracket_v) \,\overline{@}\, k)[\llbracket v_2 \rrbracket_v / x] \\
&= (k \,\underline{@}\, \llbracket v \rrbracket_v)[\llbracket v_2 \rrbracket_v / x] \\
&= (k[\llbracket v_2 \rrbracket_v / x]) \,\underline{@}\, (\llbracket v \rrbracket_v [\llbracket v_2 \rrbracket_v / x]) \\
&= k \,\underline{@}\, (\llbracket v \rrbracket_v [\llbracket v_2 \rrbracket_v / x]) && (\texttt{sVar} \neq) \\
&= k \,\underline{@}\, \llbracket v' \rrbracket_v && (\text{補題 } 1.1.1\ \ \texttt{eSubstV}) \\
&= \llbracket v' \rrbracket' \,\overline{@}\, k
\end{aligned}$$

e が App のとき

$(e_1 \,@\, e_2)[\llbracket v \rrbracket_v / x] = e'_1 \,@\, e'_2$ とすると、

$$\begin{aligned}
(\text{与式}) &= (\llbracket e_1 \,@\, e_2 \rrbracket' \,\overline{@}\, k)[\llbracket v \rrbracket_v / x] \\
&= ((\overline{\lambda} k.\, \llbracket e_1 \rrbracket \,\overline{@}\, (\overline{\lambda} m.\, \llbracket e_2 \rrbracket \,\overline{@}\, (\overline{\lambda} n.\, (m \,\underline{@}\, n) \,\underline{@}\, k))) \,\overline{@}\, k)[\llbracket v \rrbracket_v / x] \\
&= (\llbracket e_1 \rrbracket \,\overline{@}\, (\overline{\lambda} m.\, \llbracket e_2 \rrbracket \,\overline{@}\, (\overline{\lambda} n.\, (m \,\underline{@}\, n) \,\underline{@}\, k)))[\llbracket v \rrbracket_v / x] \\
&= (\llbracket e_1 \rrbracket [\llbracket v \rrbracket_v / x]) \,\overline{@}\, ((\overline{\lambda} m.\, \llbracket e_2 \rrbracket \,\overline{@}\, (\overline{\lambda} n.\, (m \,\underline{@}\, n) \,\underline{@}\, k))[\llbracket v \rrbracket_v / x]) \\
&= \llbracket e'_1 \rrbracket \,\overline{@}\, (\overline{\lambda} m.\, (\llbracket e_2 \rrbracket [\llbracket v \rrbracket_v / x]) \,\overline{@}\, ((\overline{\lambda} n.\, (m \,\underline{@}\, n) \,\underline{@}\, k)[\llbracket v \rrbracket_v / x])) \\
&= \llbracket e'_1 \rrbracket \,\overline{@}\, (\overline{\lambda} m.\, \llbracket e'_2 \rrbracket \,\overline{@}\, (\overline{\lambda} n.\, (m \,\underline{@}\, n) \,\underline{@}\, k)) \\
&= (\llbracket e_1 \,@\, e_2 \rrbracket') \,\overline{@}\, k
\end{aligned}$$

e が Reset のとき

$\langle e \rangle [\llbracket v \rrbracket_v / x] = \langle e' \rangle$ とすると、

$$\begin{aligned}
(\text{与式}) &= (\llbracket \langle e \rangle \rrbracket' \,\overline{@}\, k)[\llbracket v \rrbracket_v / x] \\
&= (\underline{\mathsf{let}}\ \ c = \llbracket e \rrbracket \,\overline{@}\, (\overline{\lambda} m.\, m)\ \ \underline{\mathsf{in}}\ \ k \,\underline{@}\, c)[\llbracket v \rrbracket_v / x] \\
&= \underline{\mathsf{let}}\ \ c = \llbracket e' \rrbracket \,\overline{@}\, (\overline{\lambda} m.\, m)\ \ \underline{\mathsf{in}}\ \ k \,\underline{@}\, c \\
&= \llbracket \langle e' \rangle \rrbracket' \,\overline{@}\, k
\end{aligned}$$

e が Let のとき

$(\mathsf{let}\ x = e_1\ \mathsf{in}\ e_2)[v/x] = \mathsf{let}\ x = e'_1\ \mathsf{in}\ e'_2$ とすると、

$$\begin{aligned}
(\text{与式}) &= (\llbracket \mathsf{let}\ x = e_1\ \mathsf{in}\ e_2 \rrbracket' \,\overline{@}\, k)[\llbracket v \rrbracket_v / x] \\
&= (\llbracket e_1 \rrbracket \,\overline{@}\, (\overline{\lambda} m.\, \underline{\mathsf{let}}\ \ x = m\ \ \underline{\mathsf{in}}\ \ \llbracket e_2 \rrbracket' \,\overline{@}\, k))[\llbracket v \rrbracket_v / x] \\
&= (\llbracket e_1 \rrbracket [\llbracket v \rrbracket_v / x]) \,\overline{@}\, (\overline{\lambda} m.\, \underline{\mathsf{let}}\ \ x = m\ \ \underline{\mathsf{in}}\ \ (\llbracket e_2 \rrbracket' [\llbracket v \rrbracket_v / x]) \,\overline{@}\, k) \\
&= \llbracket e_1 [v/x] \rrbracket \,\overline{@}\, (\overline{\lambda} m.\, \underline{\mathsf{let}}\ \ x = m\ \ \underline{\mathsf{in}}\ \ \llbracket e_2 [v/x] \rrbracket' \,\overline{@}\, k) \\
&= \llbracket e'_1 \rrbracket \,\overline{@}\, (\overline{\lambda} m.\, \underline{\mathsf{let}}\ \ x = m\ \ \underline{\mathsf{in}}\ \ \llbracket e'_2 \rrbracket' \,\overline{@}\, k) \\
&= \llbracket \mathsf{let}\ x = e'_1\ \mathsf{in}\ e'_2 \rrbracket' \,\overline{@}\, k
\end{aligned}$$

□

補題 3.1.4: (κ Subst)

schematic な κ ($\kappa[v/k] = \kappa'$) について、$([\![e]\!] \overline{@} \kappa)[v/k] = v \overline{@} \kappa'$ が成り立つ

証明.

$e = v_1$ **(値) のとき**

$$
\begin{aligned}
(\text{与式}) &= ([\![v_1]\!] \overline{@} \kappa)[v/x] \\
&= \kappa \overline{@} [\![v_1]\!]_v[v/k] \\
&= (\kappa[v/k]) \overline{@} ([\![v_1]\!]_v[v/k]) \\
&= \kappa' \overline{@} [\![v]\!]_v \\
&= [\![v]\!] \overline{@} \kappa'
\end{aligned}
$$

$e = e_1 @ e_2$ **(App) のとき**

$$
\begin{aligned}
(\text{与式}) &= ([\![e_1 @ e_2]\!] \overline{@} \kappa)[v/x] \\
&= ([\![e_1]\!] \overline{@} (\overline{\lambda} m.\, [\![e_2]\!] \overline{@} (\overline{\lambda} n.\, (m \underline{@} n) \underline{@} (\underline{\lambda} a.\, \kappa \overline{@} a))))[v/x] \\
&= ([\![e_1]\!][v/x]) \overline{@} ((\overline{\lambda} m.\, [\![e_2]\!] \overline{@} (\overline{\lambda} n.\, (m \underline{@} n) \underline{@} (\underline{\lambda} a.\, \kappa \overline{@} a)))[v/x]) \\
&= [\![e_1]\!] \overline{@} (\overline{\lambda} m.\, ([\![e_2]\!][v/x]) \overline{@} ((\overline{\lambda} n.\, (m \underline{@} n) \underline{@} (\underline{\lambda} a.\, \kappa \overline{@} a))[v/x])) \\
&= [\![e_1]\!] \overline{@} (\overline{\lambda} m.\, [\![e_2]\!] \overline{@} (\overline{\lambda} n.\, (m \underline{@} n) \underline{@} (\underline{\lambda} a.\, \kappa' \overline{@} a))) \\
&= ([\![e_1 @ e_2]\!]) \overline{@} \kappa'
\end{aligned}
$$

$e = \langle e \rangle$ **(Reset) のとき**

$$
\begin{aligned}
(\text{与式}) &= ([\![\langle e \rangle]\!] \overline{@} \kappa)[v/x] \\
&= (\underline{\mathsf{let}}\ c = [\![e]\!] \overline{@} (\overline{\lambda} m.\, m)\ \underline{\mathsf{in}}\ \kappa \overline{@} c)[v/x] \\
&= \underline{\mathsf{let}}\ c = [\![e]\!] \overline{@} (\overline{\lambda} m.\, m)\ \underline{\mathsf{in}}\ \kappa' \overline{@} c \\
&= [\![\langle e \rangle]\!] \overline{@} \kappa'
\end{aligned}
$$

e **が Let のとき**

$$
\begin{aligned}
(\text{与式}) &= ([\![\mathsf{let}\ x = e_1\ \mathsf{in}\ e_2]\!] \overline{@} \kappa)[v/x] \\
&= ([\![e_1]\!] \overline{@} (\overline{\lambda} m.\, \underline{\mathsf{let}}\ x = m\ \underline{\mathsf{in}}\ [\![e_2]\!] \overline{@} \kappa))[v/x] \\
&= [\![e_1]\!] \overline{@} (\overline{\lambda} m.\, \underline{\mathsf{let}}\ x = m\ \underline{\mathsf{in}}\ [\![e_2]\!] \overline{@} \kappa') \\
&= [\![\mathsf{let}\ x = e_1\ \mathsf{in}\ e_2]\!] \overline{@} \kappa'
\end{aligned}
$$

□

補題 3.1.5: (kSubst')

$([\![e]\!]' \,\overline{@}\, k)[v/k] = [\![e]\!]' \,\overline{@}\, v$ が成り立つ

証明.

e **が値** $(e = v_1)$ **のとき**

$$\begin{aligned}(\text{与式}) &= [\![v_1]\!]' \,\overline{@}\, k[v/x] \\ &= (k \,\underline{@}\, [\![v_1]\!]_v)[v/k] \\ &= v \,\underline{@}\, [\![v_1]\!]_v \\ &= [\![v_1]\!]' \,\overline{@}\, v\end{aligned}$$

e **が App** $(e = e_1 \,@\, e_2)$ **のとき**

$$\begin{aligned}(\text{与式}) &= ([\![e_1 \,@\, e_2]\!]' \,\overline{@}\, k)[v/x] \\ &= ([\![e_1]\!] \,\overline{@}\, (\overline{\lambda} m.\, [\![e_2]\!] \,\overline{@}\, (\overline{\lambda} n.\, (m \,\underline{@}\, n) \,\underline{@}\, k)))[v/x] \\ &= ([\![e_1]\!][v/x]) \,\overline{@}\, ((\overline{\lambda} m.\, [\![e_2]\!] \,\overline{@}\, (\overline{\lambda} n.\, (m \,\underline{@}\, n) \,\underline{@}\, k))[v/x]) \\ &= [\![e_1]\!] \,\overline{@}\, (\overline{\lambda} m.\, ([\![e_2]\!][v/x]) \,\overline{@}\, ((\overline{\lambda} n.\, (m \,\underline{@}\, n) \,\underline{@}\, k)[v/x])) \\ &= [\![e_1]\!] \,\overline{@}\, (\overline{\lambda} m.\, [\![e_2]\!] \,\overline{@}\, (\overline{\lambda} n.\, (m \,\underline{@}\, n) \,\underline{@}\, v)) \\ &= ([\![e_1 \,@\, e_2]\!]') \,\overline{@}\, v\end{aligned}$$

e **が Reset** $(e = \langle e \rangle)$ **のとき**

$$\begin{aligned}(\text{与式}) &= ([\![\langle e \rangle]\!]' \,\overline{@}\, k)[v/x] \\ &= (\underline{\mathsf{let}}\ \ c = [\![e]\!] \,\overline{@}\, (\overline{\lambda} m.\, m)\ \ \underline{\mathsf{in}}\ \ k \,\underline{@}\, c)[v/x] \\ &= \underline{\mathsf{let}}\ \ c = [\![e]\!] \,\overline{@}\, (\overline{\lambda} m.\, m)\ \ \underline{\mathsf{in}}\ \ v \,\underline{@}\, c \\ &= [\![\langle e \rangle]\!]' \,\overline{@}\, v\end{aligned}$$

e **が Let** $(e = \textbf{let}\ c = e_1\ \textbf{in}\ e_2)$ **のとき**

$$\begin{aligned}(\text{与式}) &= ([\![\mathsf{let}\ c = e_1\ \mathsf{in}\ e_2]\!]' \,\overline{@}\, k)[v/x] \\ &= ([\![e_1]\!] \,\overline{@}\, (\overline{\lambda} m.\, \underline{\mathsf{let}}\ \ c = m\ \ \underline{\mathsf{in}}\ \ [\![e_2]\!]' \,\overline{@}\, k))[v/x] \\ &= [\![e_1]\!] \,\overline{@}\, (\overline{\lambda} m.\, \underline{\mathsf{let}}\ \ c = m\ \ \underline{\mathsf{in}}\ \ [\![e_2]\!]' \,\overline{@}\, v) \\ &= [\![\mathsf{let}\ x = e_1\ \mathsf{in}\ e_2]\!]' \,\overline{@}\, v\end{aligned}$$

□

3.2 $[\![\cdot]\!]'$ と $[\![\cdot]\!]$ の関係性についての補題の証明

補題 3.2.1: (correctCont)

任意の項 e と schematic な 継続 κ_1, κ_2 について、$(\kappa_1 \overline{@}\, v) \sim (\kappa_2 \overline{@}\, v)$ が成り立つならば、$[\![e]\!] \overline{@}\, \kappa_1 \sim [\![e]\!] \overline{@}\, \kappa_2$ が成り立つ

証明.

e **が値** $(e = v_1)$ **のとき**

$$\begin{array}{rcl}(\text{左式}) & \equiv & [\![v]\!] \overline{@}\, \kappa_1 \\ & \equiv & \kappa_1 \overline{@}\, [\![v]\!]_v \\ & \sim & \kappa_2 \overline{@}\, [\![v]\!]_v \\ & \equiv & [\![v]\!] \overline{@}\, \kappa_2\end{array}$$

e **が App** $(e = e_1 @ e_2)$ **のとき**

$$\begin{array}{rcl}(\text{左式}) & \equiv & [\![e_1 @ e_2]\!] \overline{@}\, \kappa_1 \\ & \equiv & [\![e_1]\!] \overline{@}\, (\overline{\lambda} m. [\![e_2]\!] \overline{@}\, (\overline{\lambda} n. (m \overline{@}\, n) \overline{@}\, (\overline{\lambda} a. \kappa_1 \overline{@}\, a))) \\ & \sim & [\![e_1]\!] \overline{@}\, (\overline{\lambda} m. [\![e_2]\!] \overline{@}\, (\overline{\lambda} n. (m \overline{@}\, n) \overline{@}\, (\overline{\lambda} a. \kappa_2 \overline{@}\, a))) \\ & \equiv & [\![e_1 @ e_2]\!] \overline{@}\, \kappa_2\end{array}$$

e **が Reset** $(e = \langle e \rangle)$ **のとき**

$$\begin{array}{rcl}(\text{左式}) & \equiv & [\![\langle e \rangle]\!] \overline{@}\, \kappa_1 \\ & \equiv & \underline{\mathsf{let}}\ \ x = [\![e]\!] \overline{@}\, (\overline{\lambda} m. m)\ \ \underline{\mathsf{in}}\ \ \kappa_1 \overline{@}\, x \\ & \sim & \underline{\mathsf{let}}\ \ x = [\![e]\!] \overline{@}\, (\overline{\lambda} m. m)\ \ \underline{\mathsf{in}}\ \ \kappa_2 \overline{@}\, x \\ & \equiv & [\![\langle e \rangle]\!] \overline{@}\, \kappa_2\end{array}$$

e **が Let** $(e = \textbf{let}\ x = e_1\ \textbf{in}\ e_2)$ **のとき**

$$\begin{array}{rcl}(\text{左式}) & \equiv & [\![\mathsf{let}\ x = e_1\ \mathsf{in}\ e_2]\!] \overline{@}\, \kappa_1 \\ & \equiv & [\![e_1]\!] \overline{@}\, (\overline{\lambda} m. \underline{\mathsf{let}}\ \ x = m\ \ \underline{\mathsf{in}}\ \ [\![e_2]\!] \overline{@}\, \kappa_1) \\ & \sim & [\![e_1]\!] \overline{@}\, (\overline{\lambda} m. \underline{\mathsf{let}}\ \ x = m\ \ \underline{\mathsf{in}}\ \ [\![e_2]\!] \overline{@}\, \kappa_2) \\ & \equiv & [\![\mathsf{let}\ x = e_1\ \mathsf{in}\ e_2]\!] \overline{@}\, \kappa_2\end{array}$$

$\square$

補題 3.2.2: (correctEtaEta')

$[\![e]\!]' \,\overline{@}\, (\underline{\lambda} a.\, \kappa \,\overline{@}\, a) \rightsquigarrow^* [\![e]\!] \,\overline{@}\, \kappa$ が成り立つ

証明.

e **が値** $(e = v_1)$ **のとき**

$$\begin{array}{rcl} (\text{左式}) & \equiv & [\![v]\!]' \,\overline{@}\, (\underline{\lambda} a.\, \kappa \,\overline{@}\, a) \\ & \equiv & (\overline{\lambda} k.\, k \,\underline{@}\, [\![v]\!]_v) \,\overline{@}\, (\underline{\lambda} a.\, \kappa \,\overline{@}\, a) \\ & \equiv & (\underline{\lambda} a.\, \kappa \,\overline{@}\, a) \,\underline{@}\, [\![v]\!]_v \\ & \rightsquigarrow & \kappa \,\overline{@}\, [\![v]\!]_v \\ & \equiv & [\![v]\!] \,\overline{@}\, \kappa \end{array}$$

e **が App** $(e = e_1 \,@\, e_2)$ **のとき**

$$\begin{array}{rcl} (\text{左式}) & \equiv & [\![e_1 \,@\, e_2]\!]' \,\overline{@}\, (\underline{\lambda} a.\, \kappa \,\overline{@}\, a) \\ & \equiv & [\![e_1]\!] \,\overline{@}\, (\overline{\lambda} m.\, [\![e_2]\!] \,\overline{@}\, (\overline{\lambda} n.\, (m \,\underline{@}\, n) \,\underline{@}\, (\underline{\lambda} a.\, \kappa \,\overline{@}\, a))) \\ & \equiv & [\![e_1 \,@\, e_2]\!] \,\overline{@}\, \kappa \end{array}$$

e **が Reset** $(e = \langle e \rangle)$ **のとき**

$$\begin{array}{rcl} (\text{左式}) & \equiv & [\![\langle e \rangle]\!]' \,\overline{@}\, (\underline{\lambda} a.\, \kappa \,\overline{@}\, a) \\ & \equiv & \underline{\mathsf{let}}\ \ c = [\![e]\!] \,\overline{@}\, (\overline{\lambda} m.\, m)\ \ \underline{\mathsf{in}}\ \ (\underline{\lambda} a.\, \kappa \,\overline{@}\, a) \,\underline{@}\, c \\ & \rightsquigarrow & \underline{\mathsf{let}}\ \ c = [\![e]\!] \,\overline{@}\, (\overline{\lambda} m.\, m)\ \ \underline{\mathsf{in}}\ \ \kappa \,\underline{@}\, c \\ & \equiv & [\![\langle e \rangle]\!] \,\overline{@}\, \kappa \end{array}$$

e **が Let** $(e = \textbf{let}\ x = e_1\ \textbf{in}\ e_2)$ **のとき**

$$\begin{array}{rcl} (\text{左式}) & \equiv & [\![\mathsf{let}\ x = e_1\ \mathsf{in}\ e_2]\!]' \,\overline{@}\, (\underline{\lambda} a.\, \kappa \,\overline{@}\, a) \\ & \equiv & [\![e_1]\!] \,\overline{@}\, (\overline{\lambda} m.\, \underline{\mathsf{let}}\ \ x = m\ \ \underline{\mathsf{in}}\ \ [\![e_2]\!]' \,\overline{@}\, (\underline{\lambda} a.\, \kappa \,\overline{@}\, a)) \\ & \sim & [\![e_1]\!] \,\overline{@}\, (\overline{\lambda} m.\, \underline{\mathsf{let}}\ \ x = m\ \ \underline{\mathsf{in}}\ \ [\![e_2]\!] \,\overline{@}\, \kappa) \\ & \equiv & [\![\mathsf{let}\ x = e_1\ \mathsf{in}\ e_2]\!] \,\overline{@}\, \kappa \end{array}$$

□

3.3 ピュアコンテキストに関する代入補題

補題 3.3.1: (subst-context)

任意のピュアコンテキスト con について、$E_{\mathsf{con}}[\ x\ ][v/x] = E_{\mathsf{con}}[\ v\ ]$ が成り立つ

証明.

con が Hole のとき

$$\begin{array}{rcl}(\text{左式}) & \equiv & x[v/x] \\ & = & v\end{array}$$

con が Frame (App_1 e_2) のとき

$$\begin{array}{rcl}(\text{左式}) & \equiv & (x \mathbin{@} e_2)[v/x] \\ & = & v \mathbin{@} e_2\end{array}$$

con が Frame (App_2 v_1) のとき

$$\begin{array}{rcl}(\text{左式}) & \equiv & (v_1 \mathbin{@} x)[v/x] \\ & = & v_1 \mathbin{@} v\end{array}$$

con が Frame (Let e_2) のとき

$$\begin{array}{rcl}(\text{左式}) & \equiv & (\mathsf{let}\ c = x\ \mathsf{in}\ e_2)[v/x] \\ & = & \mathsf{let}\ c = v\ \mathsf{in}\ e_2\end{array}$$

□

3.4 Shift に関する補題

補題 3.4.1: (contextContE)

$\llbracket E_{p_1}[\ \mathcal{S} @ v\ ]\rrbracket \mathbin{\overline{@}} \kappa \equiv \llbracket \mathcal{S} @ v\rrbracket' \mathbin{\overline{@}} (\underline{\lambda} a.\, \llbracket E_{p_2}[\ a\ ]\rrbracket \mathbin{\overline{@}} \kappa)$ が成り立つことを証明する

証明.

p_1, p_2 **が Hole のとき**

$$
\begin{aligned}
(\text{左式}) &\equiv \llbracket \mathcal{S} @ v\rrbracket \mathbin{\overline{@}} \kappa \\
&\equiv \llbracket \mathcal{S}\rrbracket \mathbin{\overline{@}} (\overline{\lambda} m.\, \llbracket v\rrbracket \mathbin{\overline{@}} (\overline{\lambda} n.\, (m \mathbin{\overline{@}} n) \mathbin{\overline{@}} (\underline{\lambda} a.\, \kappa \mathbin{\overline{@}} a))) \\
&\equiv \llbracket \mathcal{S} @ v\rrbracket' \mathbin{\overline{@}} (\underline{\lambda} a.\, \kappa \mathbin{\overline{@}} a) \\
&\equiv \llbracket \mathcal{S} @ v\rrbracket' \mathbin{\overline{@}} \underline{\lambda} a.\, \llbracket a\rrbracket \mathbin{\overline{@}} \kappa \\
&\equiv \llbracket \mathcal{S} @ v\rrbracket' \mathbin{\overline{@}} \underline{\lambda} a.\, \llbracket E_{p_2}[\ a\ ]\rrbracket \mathbin{\overline{@}} \kappa
\end{aligned}
$$

p_1, p_2 **が Frame (App_1 e_2) のとき**

$$
\begin{aligned}
(\text{左式}) &\equiv \llbracket E_{p_1'}[\ \mathcal{S} @ v\ ] @ e_2\rrbracket \mathbin{\overline{@}} \kappa \\
&\equiv \llbracket E_{p_1'}[\ \mathcal{S} @ v\ ] @ e_2\rrbracket \mathbin{\overline{@}} (\overline{\lambda} m.\, \llbracket e_2\rrbracket \mathbin{\overline{@}} (\overline{\lambda} n.\, (m \mathbin{\overline{@}} n) \mathbin{\overline{@}} (\underline{\lambda} a.\, \kappa \mathbin{\overline{@}} a))) \\
&\equiv \llbracket \mathcal{S} @ v\rrbracket' \mathbin{\overline{@}} \underline{\lambda} a.\, \llbracket E_{p_2'}[\ a\ ]\rrbracket \mathbin{\overline{@}} (\overline{\lambda} m.\, \llbracket e_2\rrbracket \mathbin{\overline{@}} (\overline{\lambda} n.\, (m \mathbin{\overline{@}} n) \mathbin{\overline{@}} (\underline{\lambda} a.\, \kappa \mathbin{\overline{@}} a))) \\
&\equiv \llbracket \mathcal{S} @ v\rrbracket' \mathbin{\overline{@}} (\underline{\lambda} a.\, \llbracket E_{p_2'}[\ a\ ] @ e_2\rrbracket \mathbin{\overline{@}} \kappa) \\
&\equiv \llbracket \mathcal{S} @ v\rrbracket' \mathbin{\overline{@}} (\underline{\lambda} a.\, \llbracket E_{p_2}[\ a\ ]\rrbracket \mathbin{\overline{@}} \kappa)
\end{aligned}
$$

p_1, p_2 **が Frame (App_2 v_1) のとき**

$$
\begin{aligned}
(\text{左式}) &\equiv \llbracket v_1 @ E_{p_1'}[\ \mathcal{S} @ v\ ]\rrbracket \mathbin{\overline{@}} \kappa \\
&\equiv \llbracket v_1\rrbracket \mathbin{\overline{@}} (\overline{\lambda} m.\, \llbracket E_{p_1'}[\ \mathcal{S} @ v\ ]\rrbracket \mathbin{\overline{@}} (\overline{\lambda} n.\, (m \mathbin{\overline{@}} n) \mathbin{\overline{@}} (\underline{\lambda} a.\, \kappa \mathbin{\overline{@}} a))) \\
&\equiv \llbracket E_{p_1'}[\ \mathcal{S} @ v\ ]\rrbracket \mathbin{\overline{@}} (\overline{\lambda} n.\, (\llbracket v_1\rrbracket_v \mathbin{\underline{@}} n) \mathbin{\underline{@}} (\underline{\lambda} a.\, \kappa \mathbin{\overline{@}} a)) \\
&\equiv \llbracket \mathcal{S} @ v\rrbracket' \mathbin{\overline{@}} (\underline{\lambda} a.\, \llbracket E_{p_2'}[\ a\ ]\rrbracket \mathbin{\overline{@}} (\overline{\lambda} n.\, (\llbracket v_1\rrbracket_v \mathbin{\underline{@}} n) \mathbin{\underline{@}} (\underline{\lambda} a.\, \kappa \mathbin{\overline{@}} a))) \\
&\equiv \llbracket \mathcal{S} @ v\rrbracket' \mathbin{\overline{@}} (\underline{\lambda} a.\, \llbracket v_1 @ E_{p_2'}[\ a\ ]\rrbracket \mathbin{\overline{@}} \kappa) \\
&\equiv \llbracket \mathcal{S} @ v\rrbracket' \mathbin{\overline{@}} (\underline{\lambda} a.\, \llbracket E_{p_2}[\ a\ ]\rrbracket \mathbin{\overline{@}} \kappa)
\end{aligned}
$$

p_1, p_2 **が Frame (Let e_2) のとき**

$$
\begin{aligned}
(\text{左式}) &\equiv \llbracket \mathsf{let}\ x = E_{p_1'}[\ \mathcal{S} @ v\ ]\ \mathsf{in}\ e_2\rrbracket \mathbin{\overline{@}} \kappa \\
&\equiv \llbracket E_{p_1'}[\ \mathcal{S} @ v\ ]\rrbracket \mathbin{\overline{@}} (\overline{\lambda} m.\, \underline{\mathsf{let}}\ \ x = m\ \ \underline{\mathsf{in}}\ \ e_2 \mathbin{\overline{@}} \kappa) \\
&\equiv \llbracket \mathcal{S} @ v\rrbracket' \mathbin{\overline{@}} (\underline{\lambda} a.\, \llbracket E_{p_2'}[\ a\ ]\rrbracket \mathbin{\overline{@}} (\overline{\lambda} m.\, \underline{\mathsf{let}}\ \ x = m\ \ \underline{\mathsf{in}}\ \ e_2 \mathbin{\overline{@}} \kappa)) \\
&\equiv \llbracket \mathcal{S} @ v\rrbracket' \mathbin{\overline{@}} (\underline{\lambda} a.\, \llbracket \mathsf{let}\ x = E_{p_2'}[\ a\ ]\ \mathsf{in}\ e_2\rrbracket \mathbin{\overline{@}} \kappa) \\
&\equiv \llbracket \mathcal{S} @ v\rrbracket' \mathbin{\overline{@}} (\underline{\lambda} a.\, \llbracket E_{p_2}[\ a\ ]\rrbracket \mathbin{\overline{@}} \kappa)
\end{aligned}
$$

□

4 CPS 変換の正当性の証明

この節では、CPS 変換の正当性の証明として、CPS 変換が項の簡約関係を保存することを示す。

4.1 変換の証明

定理 4.1: (CPS 変換の正当性の証明)

任意の項 e, e' について $e \to e'$ が成り立つならば、任意の schematic な継続 κ について $\llbracket e \rrbracket \,\overline{@}\, \kappa \to * e' \,\overline{@}\, \kappa$

これは、以下のような図を意味する。

$$
\begin{tikzcd}
e \arrow[rr, "\mathsf{Reduce}"] \arrow[d, "\mathsf{CPS}"'] & & e' \arrow[d, "\mathsf{CPS}"] \\
\llbracket e \rrbracket \arrow[dr, "\mathsf{Substitute}\ \kappa"'] & & \llbracket e' \rrbracket \arrow[dl, "\mathsf{Substitute}\ \kappa"] \\
& \llbracket e \rrbracket \,\overline{@}\, \kappa =_\beta \llbracket e' \rrbracket \,\overline{@}\, \kappa &
\end{tikzcd}
$$

この図にある Reduce の部分について、RBeta、RFrame (App_1)、RFrame (App_2)、RReset、RShift のケースについて場合分けをして帰納的に解く。

4.1.1 RBeta のケース

RBeta のケースでの証明を行う。

$$
\begin{tikzcd}
(\lambda x.\, e_1) \mathbin{@} v_2 \arrow[rr, "\mathsf{RBeta}"] \arrow[d, "\mathsf{CPS}"'] & & e_2 \arrow[d, "\mathsf{CPS}"] \\
\llbracket (\lambda x.\, e_1) \mathbin{@} v_2 \rrbracket \arrow[dr, "\mathsf{Substitute}\ \kappa"'] & & \llbracket e_2 \rrbracket \arrow[dl, "\mathsf{Substitute}\ \kappa"] \\
& \llbracket (\lambda x.\, e_1) \mathbin{@} v_2 \rrbracket \,\overline{@}\, \kappa =_\beta \llbracket e_2 \rrbracket \,\overline{@}\, \kappa &
\end{tikzcd}
$$

$$
\begin{array}{lll}
 & \llbracket (\lambda x.\, e_1) \mathbin{@} v_2 \rrbracket \,\overline{@}\, \kappa & \\
= & ((\underline{\lambda} x.\, \underline{\lambda} k.\, \llbracket e_1 \rrbracket' \,\overline{@}\, k) \,\underline{@}\, \llbracket v_2 \rrbracket_v) \,\underline{@}\, (\underline{\lambda} a.\, \kappa \,\overline{@}\, a) & \\
\to & (\underline{\lambda} k.\, \llbracket e_2 \rrbracket' \,\overline{@}\, k) \,\underline{@}\, (\underline{\lambda} a.\, \kappa \,\overline{@}\, a) & \text{(補題 1.1 ekSubst')} \\
\to & \llbracket e_2 \rrbracket' \,\underline{@}\, (\underline{\lambda} a.\, \kappa \,\overline{@}\, a) & \text{(補題 1.2 kSubst')} \\
\to^* & e_2 \,\overline{@}\, \kappa & \text{(補題 1.3 cpsEtaEta')}
\end{array}
$$

4.1.2 RFrame(App_1) のケース

$$
\begin{tikzcd}
e_1 \mathbin{@} e_2 \arrow[rr, "\mathsf{RFrame(App_1\ e_2)}"] \arrow[d, "\mathsf{CPS}"'] & & e'_1 \mathbin{@} e_2 \arrow[d, "\mathsf{CPS}"] \\
\llbracket e_1 \mathbin{@} e_2 \rrbracket \arrow[dr, "\mathsf{Substitute}\ \kappa"'] & & \llbracket e'_1 \mathbin{@} e_2 \rrbracket \arrow[dl, "\mathsf{Substitute}\ \kappa"] \\
& \llbracket e_1 \mathbin{@} e_2 \rrbracket \mathbin{\overline{@}} \kappa =_\beta \llbracket e'_1 \mathbin{@} e_2 \rrbracket \mathbin{\overline{@}} \kappa &
\end{tikzcd}
$$

$$
\begin{aligned}
& \llbracket e_1 \mathbin{@} e_2 \rrbracket \mathbin{\overline{@}} \kappa \\
\equiv\quad & \llbracket e_1 \rrbracket \mathbin{\overline{@}} (\overline{\lambda} m.\, \llbracket e_2 \rrbracket \mathbin{\overline{@}} (\overline{\lambda} n.\, (m \mathbin{\underline{@}} n) \mathbin{\underline{@}} (\underline{\lambda} a.\, \kappa \mathbin{\overline{@}} a))) \\
\rightsquigarrow\quad & \llbracket e'_1 \rrbracket \mathbin{\overline{@}} (\overline{\lambda} m.\, \llbracket e_2 \rrbracket \mathbin{\overline{@}} (\overline{\lambda} n.\, (m \mathbin{\underline{@}} n) \mathbin{\underline{@}} (\underline{\lambda} a.\, \kappa \mathbin{\overline{@}} a))) \\
\equiv\quad & \llbracket e'_1 \mathbin{@} e_2 \rrbracket \mathbin{\overline{@}} \kappa
\end{aligned}
$$

4.1.3 RFrame(App_2) のケース

$$
\begin{tikzcd}
v_1 \mathbin{@} e_2 \arrow[rr, "\mathsf{RFrame(App_2\ v_1)}"] \arrow[d, "\mathsf{CPS}"'] & & v_1 \mathbin{@} e'_2 \arrow[d, "\mathsf{CPS}"] \\
\llbracket v_1 \mathbin{@} e_2 \rrbracket \arrow[dr, "\mathsf{Substitute}\ \kappa"'] & & \llbracket v_1 \mathbin{@} e'_2 \rrbracket \arrow[dl, "\mathsf{Substitute}\ \kappa"] \\
& \llbracket v_1 \mathbin{@} e_2 \rrbracket \mathbin{\overline{@}} \kappa =_\beta \llbracket v_1 \mathbin{@} e'_2 \rrbracket \mathbin{\overline{@}} \kappa &
\end{tikzcd}
$$

$$
\begin{aligned}
& \llbracket v_1 \mathbin{@} e_2 \rrbracket \mathbin{\overline{@}} \kappa \\
\equiv\quad & \llbracket v_1 \rrbracket \mathbin{\overline{@}} (\overline{\lambda} m.\, \llbracket e_2 \rrbracket \mathbin{\overline{@}} (\overline{\lambda} n.\, (m \mathbin{\underline{@}} n) \mathbin{\underline{@}} (\underline{\lambda} a.\, \kappa \mathbin{\overline{@}} a))) \\
\equiv\quad & \llbracket e_2 \rrbracket \mathbin{\overline{@}} (\overline{\lambda} n.\, \llbracket v_1 \rrbracket_v \mathbin{\underline{@}} n \mathbin{\underline{@}} (\underline{\lambda} a.\, \kappa \mathbin{\overline{@}} a)) \\
\rightsquigarrow\quad & \llbracket e'_2 \rrbracket \mathbin{\overline{@}} (\overline{\lambda} n.\, \llbracket v_1 \rrbracket_v \mathbin{\underline{@}} n \mathbin{\underline{@}} (\underline{\lambda} a.\, \kappa \mathbin{\overline{@}} a)) \\
\equiv\quad & \llbracket v_1 \mathbin{@} e'_2 \rrbracket \mathbin{\overline{@}} \kappa
\end{aligned}
$$

4.1.4 RFrame(Let) のケース

$$
\begin{tikzcd}
\mathsf{let}\ x = e_1\ \mathsf{in}\ e_2 \arrow[rr, "\mathsf{RFrame(Let\ e_2)}"] \arrow[d, "\mathsf{CPS}"'] & & \mathsf{let}\ x = e'_1\ \mathsf{in}\ e_2 \arrow[d, "\mathsf{CPS}"] \\
\llbracket \mathsf{let}\ x = e_1\ \mathsf{in}\ e_2 \rrbracket \arrow[dr, "\mathsf{Substitute}\ \kappa"'] & & \llbracket \mathsf{let}\ x = e'_1\ \mathsf{in}\ e_2 \rrbracket \arrow[dl, "\mathsf{Substitute}\ \kappa"] \\
& \llbracket \mathsf{let}\ x = e_1\ \mathsf{in}\ e_2 \rrbracket \mathbin{\overline{@}} \kappa =_\beta \llbracket \mathsf{let}\ x = e'_1\ \mathsf{in}\ e_2 \rrbracket \mathbin{\overline{@}} \kappa &
\end{tikzcd}
$$

$$
\begin{array}{ll}
 & \llbracket \mathsf{let}\ x = e_1\ \mathsf{in}\ e_2 \rrbracket \mathbin{\overline{@}} \kappa \\
\equiv & \llbracket e_1 \rrbracket \mathbin{\overline{@}} (\overline{\lambda} m.\, \underline{\mathsf{let}}\ \ x = m\ \ \underline{\mathsf{in}}\ \ \llbracket e_2 \rrbracket \mathbin{\overline{@}} \kappa) \\
\rightsquigarrow & \llbracket e_1' \rrbracket \mathbin{\overline{@}} (\overline{\lambda} m.\, \underline{\mathsf{let}}\ \ x = m\ \ \underline{\mathsf{in}}\ \ \llbracket e_2 \rrbracket \mathbin{\overline{@}} \kappa) \\
\equiv & \llbracket \mathsf{let}\ x = e_1'\ \mathsf{in}\ e_2 \rrbracket \mathbin{\overline{@}} \kappa
\end{array}
$$

4.1.5 RFrame(Reset) のケース

$$
\begin{tikzcd}
\langle e \rangle \arrow[rr, "\mathsf{RFrame(Reset\ e)}"] \arrow[d, "\mathsf{CPS}"'] & & \langle e' \rangle \arrow[d, "\mathsf{CPS}"] \\
\llbracket \langle e \rangle \rrbracket \arrow[dr, "\mathsf{Substitute}\ \kappa"'] & & \llbracket \langle e' \rangle \rrbracket \arrow[dl, "\mathsf{Substitute}\ \kappa"] \\
& \llbracket \langle e \rangle \rrbracket \mathbin{\overline{@}} \kappa =_\beta \llbracket \langle e' \rangle \rrbracket \mathbin{\overline{@}} \kappa &
\end{tikzcd}
$$

$$
\begin{array}{ll}
 & \llbracket \langle e \rangle \rrbracket \mathbin{\overline{@}} \kappa \\
\equiv & \underline{\mathsf{let}}\ \ x = \llbracket e \rrbracket \mathbin{\overline{@}} (\overline{\lambda} m.\, m)\ \ \underline{\mathsf{in}}\ \ \kappa \mathbin{\overline{@}} x \\
\rightsquigarrow & \underline{\mathsf{let}}\ \ x = \llbracket e' \rrbracket \mathbin{\overline{@}} (\overline{\lambda} m.\, m)\ \ \underline{\mathsf{in}}\ \ \kappa \mathbin{\overline{@}} x \\
\equiv & \llbracket \langle e' \rangle \rrbracket \mathbin{\overline{@}} \kappa
\end{array}
$$

4.1.6 RLet のケース

$$
\begin{tikzcd}
\mathsf{let}\ x = v_1\ \mathsf{in}\ e_2 \arrow[rr, "\mathsf{RLet}"] \arrow[d, "\mathsf{CPS}"'] & & e_2' \arrow[d, "\mathsf{CPS}"] \\
\llbracket \mathsf{let}\ x = v_1\ \mathsf{in}\ e_2 \rrbracket \arrow[dr, "\mathsf{Substitute}\ \kappa"'] & & \llbracket e_2' \rrbracket \arrow[dl, "\mathsf{Substitute}\ \kappa"] \\
& \llbracket \mathsf{let}\ x = v_1\ \mathsf{in}\ e_2 \rrbracket \mathbin{\overline{@}} \kappa =_\beta \llbracket e_2' \rrbracket \mathbin{\overline{@}} \kappa &
\end{tikzcd}
$$

$$
\begin{array}{ll}
 & \llbracket \mathsf{let}\ x = v_1\ \mathsf{in}\ e_2 \rrbracket \mathbin{\overline{@}} \kappa \\
\equiv & \llbracket v_1 \rrbracket \mathbin{\overline{@}} (\overline{\lambda} m.\, \underline{\mathsf{let}}\ \ x = m\ \ \underline{\mathsf{in}}\ \ \llbracket e_2 \rrbracket \mathbin{\overline{@}} \kappa) \\
\equiv & \underline{\mathsf{let}}\ \ x = \llbracket v_1 \rrbracket_v\ \ \underline{\mathsf{in}}\ \ \llbracket e_2 \rrbracket \mathbin{\overline{@}} \kappa \\
\rightsquigarrow & \llbracket e_2' \rrbracket \mathbin{\overline{@}} \kappa
\end{array}
$$

4.1.7 RReset のケース

$$
\begin{tikzcd}
\langle v\rangle \arrow[rr, "\mathsf{RReset}"] \arrow[d, "\mathsf{CPS}"'] & & v \arrow[d, "\mathsf{CPS}"] \\
\llbracket\langle v\rangle\rrbracket \arrow[dr, "\mathsf{Substitute}\ \kappa"'] & & \llbracket v\rrbracket \arrow[dl, "\mathsf{Substitute}\ \kappa"] \\
& \llbracket\langle v\rangle\rrbracket\,\overline{@}\,\kappa =_\beta \llbracket v\rrbracket\,\overline{@}\,\kappa &
\end{tikzcd}
$$

$$
\begin{array}{ll}
 & \llbracket\langle v\rangle\rrbracket\,\overline{@}\,\kappa \\
\equiv & \underline{\mathsf{let}}\ \ x = \llbracket v\rrbracket\,\overline{@}\,(\overline{\lambda} m.\, m)\ \ \underline{\mathsf{in}}\ \ \kappa\,\overline{@}\,x \\
\equiv & \underline{\mathsf{let}}\ \ x = \llbracket v\rrbracket_v\ \ \underline{\mathsf{in}}\ \ \kappa\,\overline{@}\,x \\
\rightsquigarrow & \kappa\,\overline{@}\,\llbracket v\rrbracket_v \\
\equiv & \llbracket v\rrbracket\,\overline{@}\,\kappa
\end{array}
$$

4.1.8 RShift のケース

$$
\begin{tikzcd}
\langle E_{p_1}[\ \mathcal{S}\,@\,v\ ]\rangle \arrow[rr, "\mathsf{RShift}"] \arrow[d, "\mathsf{CPS}"'] & & \langle v\,@\,(\lambda y.\,\langle E_{p_2}[\ y\ ]\rangle)\rangle \arrow[d, "\mathsf{CPS}"] \\
\llbracket\langle E_{p_1}[\ \mathcal{S}\,@\,v\ ]\rangle\rrbracket \arrow[dr, "\mathsf{Substitute}\ \kappa"'] & & \llbracket\langle v\,@\,(\lambda y.\,\langle E_{p_2}[\ y\ ]\rangle)\rangle\rrbracket \arrow[dl, "\mathsf{Substitute}\ \kappa"] \\
& \llbracket\langle E_{p_1}[\ \mathcal{S}\,@\,v\ ]\rangle\rrbracket\,\overline{@}\,\kappa =_\beta \llbracket\langle v\,@\,(\lambda y.\,\langle E_{p_2}[\ y\ ]\rangle)\rangle\rrbracket\,\overline{@}\,\kappa &
\end{tikzcd}
$$

$$
\begin{array}{ll}
 & \llbracket\langle E_{p_1}[\ \mathcal{S}\,@\,v\ ]\rangle\rrbracket\,\overline{@}\,\kappa \\
\equiv & \underline{\mathsf{let}}\ \ x = \llbracket E_{p_1}[\ \mathcal{S}\,@\,v\ ]\rrbracket\,\overline{@}\,(\overline{\lambda} m.\, m)\ \ \underline{\mathsf{in}}\ \ \kappa\,\overline{@}\,x \\
\equiv & \underline{\mathsf{let}}\ \ x = \llbracket \mathcal{S}\,@\,v\rrbracket'\,\overline{@}\,(\underline{\lambda} a.\,\llbracket E_{p_2}[\ a\ ]\rrbracket\,\overline{@}\,(\overline{\lambda} m.\, m))\ \ \underline{\mathsf{in}}\ \ \kappa\,\overline{@}\,x \\
\equiv & \underline{\mathsf{let}}\ \ x = (\llbracket\mathcal{S}\rrbracket_v\,\underline{@}\,\llbracket v\rrbracket_v)\,\underline{@}\,(\underline{\lambda} a.\,\llbracket E_{p_2}[\ a\ ]\rrbracket\,\overline{@}\,(\overline{\lambda} m.\, m))\ \ \underline{\mathsf{in}}\ \ \kappa\,\overline{@}\,x \\
\equiv & \underline{\mathsf{let}}\ \ x = ((\underline{\lambda} wk.\,(w\,\underline{@}\,(\underline{\lambda} ak'.\,k'\,\underline{@}\,(k\,\underline{@}\,a)))\,\underline{@}\,(\underline{\lambda} m.\,m))\,\underline{@}\,\llbracket v\rrbracket_v)\,\underline{@}\,(\underline{\lambda} a.\,\llbracket E_{p_2}[\ a\ ]\rrbracket\,\overline{@}\,(\overline{\lambda} m.\, m))\ \ \underline{\mathsf{in}}\ \ \kappa\,\overline{@}\,x \\
\rightsquigarrow & \underline{\mathsf{let}}\ \ x = (\llbracket v\rrbracket_v\,\underline{@}\,(\underline{\lambda} ak'.\,k'\,\underline{@}\,((\underline{\lambda} a.\,\llbracket E_{p_2}[\ a\ ]\rrbracket\,\overline{@}\,(\overline{\lambda} m.\, m))\,\underline{@}\,a)))\,\underline{@}\,(\underline{\lambda} m.\,m)\ \ \underline{\mathsf{in}}\ \ \kappa\,\overline{@}\,x \\
\equiv & \underline{\mathsf{let}}\ \ x = (\llbracket v\rrbracket_v\,\underline{@}\,(\underline{\lambda} ak'.\,k'\,\underline{@}\,(\llbracket a\rrbracket'\,\overline{@}\,(\underline{\lambda} a.\,\llbracket E_{p_2}[\ a\ ]\rrbracket\,\overline{@}\,(\overline{\lambda} m.\, m)))))\,\underline{@}\,(\underline{\lambda} m.\,m)\ \ \underline{\mathsf{in}}\ \ \kappa\,\overline{@}\,x \\
\equiv & \underline{\mathsf{let}}\ \ x = (\llbracket v\rrbracket_v\,\underline{@}\,(\underline{\lambda} ak'.\,k'\,\underline{@}\,(\llbracket E_{p_2}[\ a\ ]\rrbracket\,\overline{@}\,(\overline{\lambda} m.\, m))))\,\underline{@}\,(\underline{\lambda} m.\,m)\ \ \underline{\mathsf{in}}\ \ \kappa\,\overline{@}\,x \\
\sim & \underline{\mathsf{let}}\ \ x = (\llbracket v\rrbracket_v\,\underline{@}\,(\underline{\lambda} ak'.\,\llbracket\langle E_{p_2}[\ a\ ]\rangle\rrbracket'\,\overline{@}\,k'))\,\underline{@}\,(\underline{\lambda} m.\,m)\ \ \underline{\mathsf{in}}\ \ \kappa\,\overline{@}\,x \\
\equiv & \underline{\mathsf{let}}\ \ x = (\llbracket v\rrbracket_v\,\underline{@}\,\llbracket\lambda a.\,E_{p_2}[\ a\ ]\rrbracket_v)\,\underline{@}\,(\underline{\lambda} m.\,m)\ \ \underline{\mathsf{in}}\ \ \kappa\,\overline{@}\,x \\
\equiv & \llbracket\langle v\,@\,(\lambda y.\,\langle E_{p_2}[\ y\ ]\rangle)\rangle\rrbracket\,\overline{@}\,\kappa
\end{array}
$$