CS 678 Machine Learning
Programming Assignment #4

Neural Networks for Handwritten Character Recognition

Specification

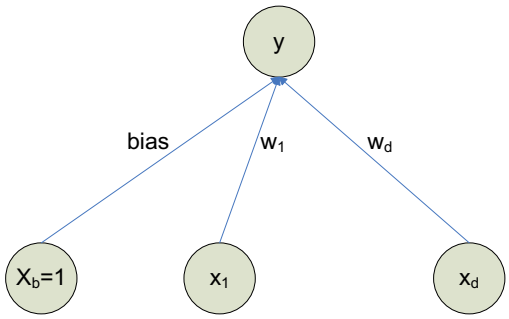
The goal is to create a neural network (i.e. a multilayered perceptron) that is capable of recognizing handwritten digits. A collection of Training data is provided to train the network to recognize different people's handwriting. The network should then be able to generalize its behavior such that it can correctly classify handwritten characters that it has not seen before, i.e. classify the provided Test set.

Perceptron

The basic processing element of a neural network is the perceptron. It is composed of inputs: $x_b=1$, $x_1..x_d$; connection weights: bias, $w_1..w_d$; and the Sum-of-Products function σ:

$$\sigma = \sum_{j=1}^{d} w_j x_j + bias$$

A multiple-input perceptron is represented as in the figure below:



The perceptron can be used as a regression or classification operator by using it to implement a linear discriminant (or threshold) function, as in:

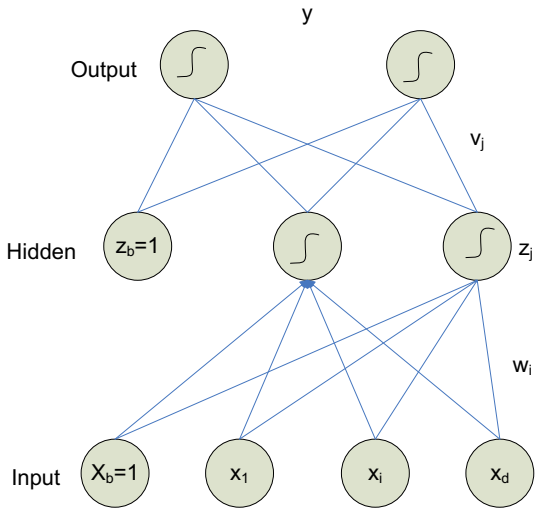
$$y = \begin{cases} 1, if \sigma > 0 \\ 0, otherwise \end{cases}$$

This assumes that the classes are linearly separable, representing linear functions. In cases where a non-linear function is required, one whose outputs are a smoothly differentiable function of its inputs, the *sigmoid* (squashing) function can be used:

$$y = sigmoid(\sigma) = \frac{1}{1+e^{-\sigma}}$$

Multi-layer Perceptrons

The multi-layer feedforward network includes an additional intermediate or *hidden* layer between the input and output layers (see figure below). It is used in conjunction with the sigmoid function to allow the network to approximate non-linear functions. There may be multiple hidden layers, but often there is simply one hidden layer with multiple perceptrons (the number of which may have to be experimentally determined). Generally, each perceptron is connected to every perceptron in the "next" layer. Note that the network may also include a *bias* input at the hidden layer.



Training the Network

Whether a single perceptron, or a multi-layer network, the training process consists of presenting examples to the system (as in all forms of Supervised learning) and adjusting the weights until the desired outcome is obtained. A process called gradient descent is used to implement the learning, converging iteratively via the form:

$$\text{Update} = \text{LearningFactor} \cdot (\text{DesiredOutput} - \text{ActualOutput}) \cdot \text{Input}$$

The notion of *training error* (distance from the target value *t*) is used to determine the direction of steepest descent. Training example error is commonly defined as:

$$E \equiv \tfrac{1}{2} \sum_{d \in D} (t_d - y_d)^2$$

The error *E* is computed after all input values have been fed forward. Then each weight is altered proportionally for each edge in order to minimize the error, as in:

$$\Delta w_i = -\eta \frac{\partial E}{\partial w_i}$$

where η is the learning rate (a convergence factor).

Error Backpropagation

Since there is no target value for perceptrons in the hidden layer, error is propagated backwards from the output layer and used for training:

For each training example do:

1. **Feedforward**: Propagate the input forward through the network
    a. Input the instance and calculate the output of each unit
2. **Backpropagate**: Compute and propagate error backwards through the network
    a. For each output unit *j*, calculate the error

    $$E_j = (t_j - y_j) y_j (1 - y_j) \quad \textit{// note: derivative of sigmoid func}$$

    b. For each hidden unit *i*, calculate the error

    $$E_i = h_i (1 - h_i) \sum_k w_{ik} E_k$$

3. **Learn**: Update each network weight proportionally

    $$w_j = w_j + \eta E_j z_j \quad \textit{// connecting hidden to output}$$

    $$w_i = w_i + \eta E_i x_i \quad \textit{// connecting input to hidden}$$

Datasets

Two datasets are posted on the course info page:

1. Fishing: a classic; used in the naïve-Bayes and decision tree tutorials.
    - Training instances: 14
    - Test sample: 1 (included in dataset)
    - Categorical data w/4 attributes:
        - Wind: Strong, Weak
        - Water temp: Warm, Moderate, Cold
        - Air temp: Warm, Cool
        - Forecast: Sunny, Cloudy, Rainy
    - Target: Yes/No
2. Digits: a collection of bitmaps of handwritten digits.
    - https://archive.ics.uci.edu/ml/datasets/optical+recognition+of+handwritten+digits
    - Training set instances: 3823
    - Test set instances: 1797
    - Integer data:
        - 64 values representing an 8x8 bitmap of a character
        - Each value ranges from 0..16 and represents an "intensity"
    - Targets: the digits 0..9

You may also find/use any dataset of your own choosing.
The data can be preprocessed, modified and normalized as desired.

Requirements:

Write modular code that implements the concept of perceptrons and their connections in a network, feeds forward the inputs, calculates the output function, calculates error, backpropagates error, and adjusts weights appropriately. Train your neural network using the provided training data, and test it using the provided test data.

As usual, your solution must be written in Pyton. Submit a written analysis of your project (pdf of ipython notebook with commentary/discussion or separate PDF of results and code uploaded directly). Be prepared to present your solution to the class

- □ Describe any interesting experiments, configuration details, problems, etc.
- □ Demonstrate the effectiveness/accuracy of your neural network.
- □ Include a discussion/analysis of your results.