Advanced Topics in Computer Graphics I - Sheet R02

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Assignment 2

Ray-Sphere intersection

Given a ray $r(t)$ with origin o and direction d:

$$r(t) = o + t \cdot d$$

and a sphere with center c and radius r defined by:

$$\|x - c\|^2 = r^2$$

Substitute ray equation into sphere equation:

$$\begin{aligned}
\|o + t \cdot d - c\|^2 &= r^2 \\
\|o + t \cdot d - c\|^2 - r^2 &= 0 && \mid \text{Substitute } o - c = s \\
(s + t \cdot d) \cdot (s + t \cdot d) - r^2 &= 0 \\
s \cdot s + t \cdot d \cdot s + t \cdot d \cdot s + t \cdot d \cdot t \cdot d - r^2 &= 0 \\
d \cdot d \cdot t^2 + 2 \cdot d \cdot s \cdot t + s \cdot s - r^2 &= 0 && \mid \text{Substitute back } o - c = s \\
d \cdot d \cdot t^2 + 2 \cdot d \cdot (o - c) \cdot t + (o - c) \cdot (o - c) - r^2 &= 0
\end{aligned}$$

This results in a quadratic equation of the form $at^2 + bt + c = 0$ with

$$\begin{aligned}
a &= d \cdot d \\
b &= 2 \cdot d \cdot (o - c) \\
c &= (o - c) \cdot (o - c) - r^2
\end{aligned}$$

The solutions of this are:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Assignment 3

Ray-Triangle intersection

Given a ray $r(t)$ with origin o and direction d:

$$r(t) = o + t \cdot d$$

and a triangle with vertices a, b, c in global coordinates.

Barycentric Coordinates

Precompute normal n of triangle plane:

$$n = (b - a) \times (c - a)$$

and value $p = -n \cdot a$ for all triangles.

1. **if** $n \cdot d = 0$: **return false** (3 mult, 2 add)
2. Compute parameter t: (3 mult, 3 add, 1 div)

$$t = -\frac{p + (n \cdot o)}{n \cdot d}$$

3. **if** $t \leq 0$: **return false**
4. Compute intersection point q: (3 mult, 3 add)

$$q = o + t \cdot d$$

5. Compute barycentric coordinates α, β, γ of intersection point q with respect to the triangle vertices: (27 mult, 17 add/sub, 2 div)

$$\alpha = \frac{((b - c) \times (q - c)) \cdot n}{((b - a) \times (c - a)) \cdot n}$$

$$\beta = \frac{((c - a) \times (q - a)) \cdot n}{((b - a) \times (c - a)) \cdot n}$$

$$\gamma = 1 - \alpha - \beta$$

6. **if** $\alpha \geq 0, \beta \geq 0, \gamma \geq 0$: **return true**

Total Operations with n and p precomputed:
Multiplications: 36

Additions/Subtractions: 25
Divisions: 3

Möller-Trumbore

1. Compute Edges $e1$, $e2$: (6 sub)
$$e_1 = b - a$$
$$e_2 = c - a$$
2. Compute Cross Product h: (6 mult, 3 sub)
$$h = d \times e_2$$
3. Compute the determinant: (3 mult, 2 add)
$$\det = e_1 \cdot h$$
4. **if** $|\det| < \epsilon$: **return false** (ray is parallel to triangle plane)
5. Compute inverse determinant: (1 div)
$$\text{det\_inv} = \frac{1}{\det}$$
6. Compute vector s from vertex a to ray origin: (3 sub)
$$s = o - a$$
7. Compute first barycentric coordinate u: (4 mult, 2 add)
$$u = (s \cdot h) \times \text{det\_inv}$$
8. **if** $u < 0$ **or** $u > 1$: **return false**
9. Compute cross product q: (6 mult, 3 sub)
$$q = s \times e_1$$
10. Compute second barycentric coordinate v: (4 mult, 2 add)
$$v = (d \cdot q) \times \text{det\_inv}$$
11. **if** $v < 0$ **or** $u + v > 1$: **return false**

12. Compute parameter t and intersection point q (7 mult, 5 add):

$$t = (e_2 \cdot q) \times \text{det\_inv}$$

$$q = o + t \cdot d$$

Total Operations:
Multiplications: 30
Additions/Subtractions: 26
Divisions: 1

In the case of a successful intersection, the Möller–Trumbore algorithm requires six fewer multiplications, 2 fewer (costly) divisions and one additional addition, assuming sequential execution without parallelization.
Fewer divisions and cross product calculations also lead to improved numerical stability compared to the approach based on barycentric coordinates.