Advanced Topics in Computer Graphics I

Universität Bonn
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Sheet R10 - Volumetric Rendering

Hand in your solutions via eCampus by Tue, 01.07.2025, **12:00 p.m.**. Compile your solution to the theoretical part into a single printable PDF file. For the practical part, hand in a single ZIP file containing only the exercise* folder within the src/ directory. Please refrain from sending the entire framework.

Assignment 1) Volumetric Pathtracing *(6 Pts)*

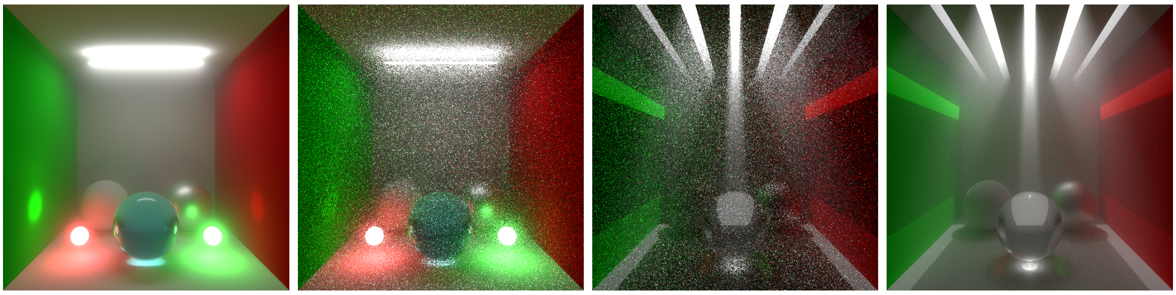

Figure 1: Cornell Box scene with volumetric scattering effects due to participating media. On the left the Cornell Box with area lights is filled with a scattering medium, and the glass sphere absorbs red and green light more than blue light. On the right, a point light source is situated above the Cornell Box and illuminates the scene through some gaps in the ceiling. The inner images have been rendered with 10 samples per pixel, and the outer images with 100k samples per pixel. To obtain these renderings, run `./bin/exercise10_VolumetricPathtracing -s data/exercise10_VolumetricPathtracing/cornell_box_spheres_area_lights_fog.xml` and `./bin/exercise10_VolumetricPathtracing -s data/exercise10_VolumetricPathtracing/cornell_box_spheres_god_rays.xml`, respectively.

In this exercise, the pathtracing framework is extended by volumetric scattering effects. The volume that fills the space between the surface in the scene now can absorb the light with a certain probability, and also scatter the light into a different direction. To make this work, we added two new component types: A `Medium` component is responsible for absorbing light between surfaces and sampling medium events in the space between the surfaces. A `PhaseFunction` component is responsible for scattering light at medium events, similarly to `BSDF`s on surfaces.

a) Implement a homogeneous medium with spatially constant scattering coefficients by completing the functions in `homogeneousmedium.cu`:
   - `...evalTransmittance()` simply evaluates the transmittance when traveling a given distance through the medium.
   - `...sampleMediumEvent()` samples a distance in the medium at which the next medium scattering event should occur.
   - `...evalMediumEventSamplingPdfAfter()` evaluates the probability the sampling routine did produce a scattering event after the given distance, which is used when a surface is intersected before a sampled medium event.

b) Implement the methods for evaluating and sampling the Henyey-Greenstein phase function in `phasefunctions.cu`. The Henyey-Greenstein phase function has the convenient property that it is a properly normalized probability density function.

$$p_{\mathrm{HG}}(\cos\theta \mid g) = \frac{1-g^2}{2} \cdot \left(1+g^2-2g\cos\theta\right)^{-1.5} \tag{1}$$

Given a uniformly distributed random variable $\xi \in [0,1]$, the inverse CDF is then

$$\cos\theta = P_{\mathrm{HG}}^{-1}(\xi \mid g) = \frac{g}{2} \cdot \left[1+g^2-\left(\frac{1-g^2}{1-g+2g\xi}\right)^2\right]. \tag{2}$$

Note that the scattered direction is isotropically distributed around the incoming "forward" ray direction, i.e. the azimuth angle $\phi \in [0, 2\pi]$ is chosen uniformly at random with $p_{\mathrm{HG}}(\phi \mid g) = \frac{1}{2\pi}$.

$$p_{\mathrm{HG}}(\omega_i \mid g) = p_{\mathrm{HG}}(\cos\theta \mid g) \cdot p_{\mathrm{HG}}(\phi \mid g) \tag{3}$$

Theoretical Assignments

Assignment 2) Henyey Greenstein Phase Function *(4Pts)*

a) Given the Henyey-Greenstein phase function $p_{\mathrm{HG}}(\mu \mid g)$ defined in Eq. 1. Let $\mu \in [-1,1]$ be a random variable distributed according to p_{HG}. Proof that the parameter g equals the expected value of μ, i.e.

$$\mathbb{E}[\mu] = g. \tag{4}$$

b) Given an incoming ray direction ω_i that is scattered according to $p_{\mathrm{HG}}(\mu \mid g)$ into direction ω_o. How is the distribution on ω_o shaped for $g \in \{-1, 0, 1\}$?

Good luck!