BRDF Models

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1 Introduction

In the lecture, the definitions of some formulas are given with angles between the normal or tangent/bitangent. This is rather inefficient in practice. Therefore, the formulas below are using the following identities so we only need to compute dot products in the shader.

$$\begin{aligned}
\cos(\cdot) &= \langle \cdot, \cdot \rangle \\
\sin^2(\cdot) &= 1 - \langle \cdot, \cdot \rangle^2 \\
\tan^2(\cdot) &= \frac{\sin^2(\cdot)}{\cos^2(\cdot)} = \frac{1 - \langle \cdot, \cdot \rangle^2}{\langle \cdot, \cdot \rangle^2}
\end{aligned}$$

2 Micro Facet BRDFs

The general definition of a microfacet BRDF is given as

$$f_{\text{BRDF}} = \frac{F \cdot D \cdot G}{4 \langle \boldsymbol{n}, \boldsymbol{l} \rangle \langle \boldsymbol{n}, \boldsymbol{v} \rangle} \tag{1}$$

where F is the fresnel reflectance, G is the geometry term (shadowing and masking) and D is the normal distribution function. In practice, this model sometimes also written as

$$f_{\text{BRDF}} = F \cdot D \cdot V \tag{2}$$

where $V = \frac{G}{4\langle \boldsymbol{n}, \boldsymbol{l} \rangle \langle \boldsymbol{n}, \boldsymbol{v} \rangle}$, which often simplifies the implementation.

3 Beckman normal distribution

The isotropic Beckmann-Spizzichino normal distribution is defined as:

$$D_{\text{Beckmann}} = \frac{1}{\langle \boldsymbol{n}, \boldsymbol{h} \rangle^4 \alpha^2 \pi} \cdot \exp\left(\frac{\langle \boldsymbol{n}, \boldsymbol{h} \rangle^2 - 1}{\langle \boldsymbol{n}, \boldsymbol{h} \rangle^2 \cdot \alpha^2} \right)$$

The anisotropic Beckmann-Spizzichino normal distribution is defined as:

$$D_{\text{AnBeckmann}} = \frac{1}{\langle \boldsymbol{n}, \boldsymbol{h} \rangle^4 \alpha_T \cdot \alpha_B \pi} \cdot \exp\left(-\frac{1}{\langle \boldsymbol{n}, \boldsymbol{h} \rangle^2} \left(\frac{\langle \boldsymbol{t}, \boldsymbol{h} \rangle^2}{\alpha_T^2} + \frac{\langle \boldsymbol{b}, \boldsymbol{h} \rangle^2}{\alpha_B^2} \right) \right)$$

4 GGX normal distribution

$$D_{\text{GGX}} = \frac{\alpha^2}{\pi \left(\left(\langle \boldsymbol{n}, \boldsymbol{h} \rangle \cdot \alpha^2 - \langle \boldsymbol{n}, \boldsymbol{h} \rangle \right) \cdot \langle \boldsymbol{n}, \boldsymbol{h} \rangle + 1 \right)^2}$$

$$D_{\text{AnGGX}} = \frac{1}{\alpha_T \cdot \alpha_B \pi \left(\frac{\langle \boldsymbol{t}, \boldsymbol{h} \rangle^2}{\alpha_T \cdot \alpha_T} + \frac{\langle \boldsymbol{b}, \boldsymbol{h} \rangle^2}{\alpha_B \cdot \alpha_B} + \langle \boldsymbol{n}, \boldsymbol{h} \rangle^2 \right)^2}$$

5 Shadowing and Masking

The isotropic Smith Shadowing function is defined as:

$$\Lambda_L = \frac{1}{2} \left(-1 + \sqrt{1 + \alpha^2 \left(\frac{1 - \langle \boldsymbol{n}, \boldsymbol{l} \rangle^2}{\langle \boldsymbol{n}, \boldsymbol{l} \rangle^2} \right)} \right)$$

$$\Lambda_V = \frac{1}{2} \left(-1 + \sqrt{1 + \alpha^2 \left(\frac{1 - \langle \boldsymbol{n}, \boldsymbol{v} \rangle^2}{\langle \boldsymbol{n}, \boldsymbol{v} \rangle^2} \right)} \right)$$

$$G_{\text{SmithJointGGX}} = \frac{1}{1 + \Lambda_L + \Lambda_V}$$

The anisotropic Smith Shadowing function is defined as:

$$\Lambda_L = \frac{1}{2} \left(-1 + \sqrt{1 + \frac{\alpha_T^2 \cdot \langle \boldsymbol{t}, \boldsymbol{l} \rangle^2 + \alpha_B^2 \cdot \langle \boldsymbol{b}, \boldsymbol{l} \rangle^2)}{\langle \boldsymbol{n}, \boldsymbol{l} \rangle^2}} \right)$$

$$\Lambda_V = \frac{1}{2} \left(-1 + \sqrt{1 + \frac{\alpha_T^2 \cdot \langle \boldsymbol{t}, \boldsymbol{v} \rangle^2 + \alpha_B^2 \cdot \langle \boldsymbol{b}, \boldsymbol{v} \rangle^2)}{\langle \boldsymbol{n}, \boldsymbol{v} \rangle^2}} \right)$$

$$G_{\text{AnSmithJointGGX}} = \frac{1}{1 + \Lambda_L + \Lambda_V}$$

By using $V = \frac{G}{4 \langle \boldsymbol{n}, \boldsymbol{l} \rangle \langle \boldsymbol{n}, \boldsymbol{v} \rangle}$, we can also define the quantities below and use Equation 2.

$$\Lambda_L = \langle \boldsymbol{n}, \boldsymbol{v} \rangle \cdot \sqrt{\langle \boldsymbol{n}, \boldsymbol{l} \rangle^2 \cdot (1 - \alpha^2) + \alpha^2}$$

$$\Lambda_V = \langle \boldsymbol{n}, \boldsymbol{l} \rangle \cdot \sqrt{\langle \boldsymbol{n}, \boldsymbol{v} \rangle^2 \cdot (1 - \alpha^2) + \alpha^2}$$

$$V_{\text{SmithJointGGX}} = \frac{1}{2 \left(\Lambda_L + \Lambda_V \right)}$$

$$
\begin{aligned}
\Lambda_L &= \langle \boldsymbol{n}, \boldsymbol{v} \rangle \cdot \sqrt{\langle \boldsymbol{t}, \boldsymbol{l} \rangle^2 \cdot \alpha_T^2 + \langle \boldsymbol{b}, \boldsymbol{l} \rangle^2 \alpha_B^2 + \langle \boldsymbol{n}, \boldsymbol{l} \rangle^2} \\
\Lambda_V &= \langle \boldsymbol{n}, \boldsymbol{l} \rangle \cdot \sqrt{\langle \boldsymbol{t}, \boldsymbol{v} \rangle^2 \cdot \alpha_T^2 + \langle \boldsymbol{b}, \boldsymbol{v} \rangle^2 \alpha_B^2 + \langle \boldsymbol{n}, \boldsymbol{v} \rangle^2} \\
V_{\text{AnSmithJointGGX}} &= \frac{1}{2\left(\Lambda_L + \Lambda_V\right)}
\end{aligned}
$$

6 Ward BRDF

The Ward BRDF is defined as

$$
f_{\text{Ward}} = \frac{F \cdot D_{\text{Beckmann}}}{4 \langle \boldsymbol{n}, \boldsymbol{l} \rangle \langle \boldsymbol{n}, \boldsymbol{v} \rangle}, \tag{3}
$$

i.e., a microfacet model without shadowing and masking. The Geisler-Moroder method shows how to compute this BRDF more efficiently:

$$
f_{\text{Ward}} = \frac{\langle \boldsymbol{l} + \boldsymbol{v}, \boldsymbol{l} + \boldsymbol{v} \rangle}{\pi \alpha_T \alpha_B \langle \boldsymbol{l} + \boldsymbol{v}, \boldsymbol{n} \rangle^4} \exp\left(-\frac{1}{\langle \boldsymbol{l} + \boldsymbol{v}, n \rangle^2} \cdot \left(\frac{\langle \boldsymbol{l} + \boldsymbol{v}, \boldsymbol{t} \rangle^2}{\alpha_T^2} + \frac{\langle \boldsymbol{l} + \boldsymbol{v}, \boldsymbol{b} \rangle^2}{\alpha_B^2}\right)\right)
$$