Advanced Topics in Computer Graphics I

Universität Bonn
Institut für Informatik II
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Sheet R07 - Multiple Importance Sampling

Hand in your solutions via eCampus by Tue, 03.06.2025, **12:00 p.m.**. Compile your solution to the theoretical part into a single printable PDF file. For the practical part, hand in a single ZIP file containing only the exercise* folder within the src/ directory. Please refrain from sending the entire framework.

Assignment 1) Pathtracer with Area Lights *(6Pts)*

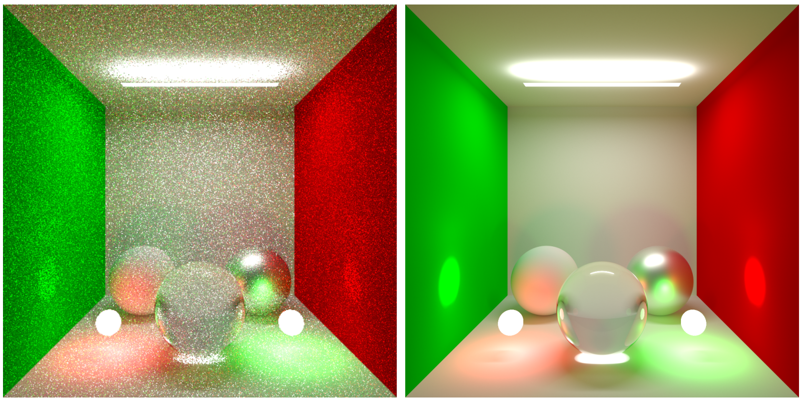


Figure 1: Cornell Box scene containing an elongated rectangular area light at the top and two colored sphere lights rendered with path tracing using 10 and 100 000 samples per pixel, respectively. To obtain these renderings, run `./bin/exercise07_LightsourceSampling -s data/exercise07_LightsourceSampling/cornell_box_spheres_area_lights.xml`.

In the previous exercise you have implemented BSDF importance sampling in a pathtracing framework. The light sources that we used in the previous exercises were infinitesimally small, so the direction towards to light source was always unique. This is unrealistic, but makes the computation of the illumination much easier, since the integral over the light source can be computed using a single sample. In this exercise you will implement importance sampling of area light sources and combine the BSDF importance sampling with the light source importance sampling via multiple importance sampling.

Your task is to extend the pathtracer from the last exercise with an implementation for spherical area lights and mesh-based area lights. The surface of the light sources is supposed to emit light diffusely into all directions. As before, the code for the light sources can be found in `lightsources.cu`. For this exercise the `__direct_callable__*_sampleLight()` and `__direct_callable__*_evalLightSamplingPDF()` functions are of interest. The former creates a ray direction from a given surface interaction that is the shading point towards the light source. The later is responsible for evaluating the probability of sampling the direction corresponding to a given shading point surface interaction and a surface interaction on the light source, which is required for multiple-importance sampling.

a) Implement the light source sampling for spherical area lights. When viewed from a surface element $\mathbf{x}$, a spherical area light (e.g. the sun) looks like a simple disc with constant radiance received by

$\mathbf{x}$. Therefore, we do not have to change variables to the area light surface, but can simply sample the light direction ω_i from a spherical cap[1]. The spherical cap is parameterized by the height h of the cap (as well as the direction towards the center of the light source). A direction is sampled uniformly from the spherical cap by by choosing $\cos\theta \in [1-h, 1]$ and $\phi \in [0, 2\pi]$ uniformly at random.

b) Implement the light source sampling for mesh-based area lights. Sample a light direction implicitly by sampling a position uniformly distributed from the surface of the light source. This effectively changes the integration domain from the light directions ω_i to the surface elements $\mathbf{x}_L$ of the light source with $\left|\frac{\partial \omega_i}{\partial \mathbf{x}_L}\right| = \frac{\cos\theta_L}{r^2}$.

$$L_o(\mathbf{x}, \omega_o) = \int_{\Omega_i} f(\mathbf{x}, \omega_i, \omega_o) L_i(\mathbf{x}, \omega_i) \cos\theta_i \, \mathrm{d}\omega_i = \int_A f(\mathbf{x}, \omega_i, \omega_o) L_o(\mathbf{x}_L, -\omega_i) \frac{\cos\theta_i \cos\theta_L}{r^2} \, \mathrm{d}\mathbf{x}_L \tag{1}$$

Since the integration domain of the Monte-Carlo estimator are still the incoming directions Ω_i, we have to use the sampling probability density

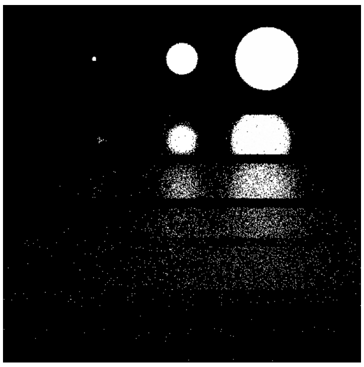
$$\mathrm{pdf}(\omega_i) = \left|\frac{\partial \omega_i}{\partial \mathbf{x}_L}\right|^{-1} \cdot \mathrm{pdf}(\mathbf{x}_L) = \frac{r^2}{\cos\theta_L} \cdot \mathrm{pdf}(\mathbf{x}_L). \tag{2}$$

c) With the light sources being represented as geometry in the scene, there are now two paths in the code that can lead to a light contribution from a given point on an area light source.

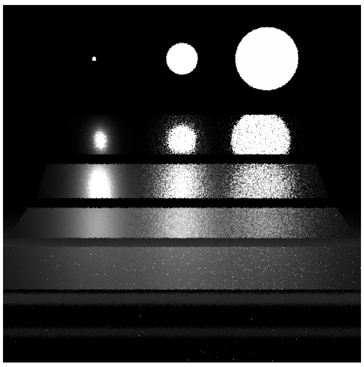
(i) A ray-scene intersection results in a surface interaction with an emitter surface.

(ii) The emitter was sampled successfully in the direct illumination computation.

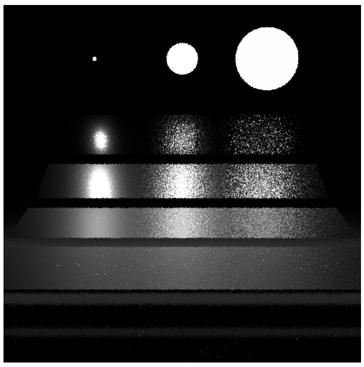
If we would simply add up both contributions the image would be too bright! Implement multiple importance sampling of the light source and the BSDF using the balance heuristic in the `__raygen__main()` in `pathtracingraygenerator.cu`. Note that you first need to complete the `__direct_callable__*_evalLightSamplingPDF()` functions for this to work.



(a) BRDF sampling



(b) Multiple importance sampling



(c) Light source sampling

Figure 2: MIS test scene containing three spherical lights of different size and four specular planes of varying roughness. Images are rendered with 1 sample per pixel and light source. To obtain these renderings, run `./bin/exercise07_LightsourceSampling -s data/exercise07_LightsourceSampling/mis_spheres.xml`. `data/exercise07_LightsourceSampling/mis_rects.xml` contains an equivalent scene with rectangular light sources.

[1] `https://en.wikipedia.org/wiki/Spherical_cap`

Theoretical Assignments

Assignment 2) Importance sampling of products *(4Pts)*

In this exercise, you should analytically calculate variances for the Monte-Carlo integration of a product function when using multiple importance sampling.

We want to compute the integral

$$\int_0^1 f_1(x) f_2(x) dx$$

where

$$\begin{aligned} f_1(x) &= 20x(x-0.5)(x-1)+1 \\ f_2(x) &= -20x(x-0.5)(x-1)+1 \end{aligned}$$

via Monte-Carlo integration. Your task is now to compute analytically the variance you obtain when using different PDFs for the importance sampling. Compute the variance for the following importance sampling strategies:

1) Use either f_1 or f_2 for the importance sampling.
2) Use the balance heuristic described in the lecture slides (Monte-Carlo Integration, Sampling Strategies) with an equal number of samples for both functions.
3) Use the function $f_1 \cdot f_2$ for the importance sampling.

Hints:

- To get the PDFs for sampling, you have to normalize the functions accordingly.
- You can use Maple or Wolfram Alpha to solve the resulting integrals (or any computer algebra system of your choice).

Good luck!