Advanced Topics in Computer Graphics I

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Sheet R05 - BRDF Models

Hand in your solutions via eCampus by Tue, 20.05.2025, **12:00 p.m.**. Compile your solution to the theoretical part into a single printable PDF file. For the practical part, hand in a single ZIP file containing only the exercise* folder within the src/ directory. Please refrain from sending the entire framework.

Assignment 1) BRDF Models *(7 Pts)*

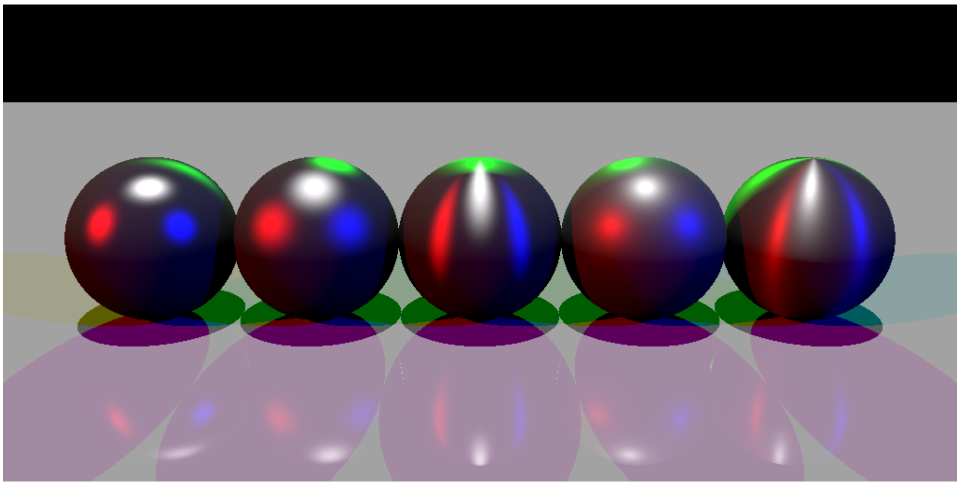

Figure 1: Renderings of the BRDF models that you will implement, illuminated by four directional light sources. From left to right: Phong-BRDF, isotropic Ward-BRDF, anisotropic Ward-BRDF, isotropic GGX-BRDF, anisotropic GGX-BRDF. To obtain this rendering, run `bin/exercise05_BRDFModels -s data/exercise05_BRDFModels/brdf_models.xml`.

In this exercise we extend the Whitted raytracing framework by by a few BRDF models. The diffuse Lambert BRDF is already in use by the opaque material. Your task is to complete the implementation of the `evalBSDF` callbacks in `bsdfmodels.cu` for some of the BRDF models discussed in the lecture (and additionally the Ward BRDF model).

- **Phong- [4] / Cosine-Lobe-BRDF [2]:** In the lecture, the Phong-BRDF was introduced as a simple phenomenological model. When accounting for energy conservation, it is usually called the Cosine-Lobe-BRDF. Fill in the corresponding equation of the Cosine-Lobe-BRDF from the slides.
- **Ward-BRDF [6, 1]:** The Ward-BRDF is again a “hybrid” model (i.e. not physically founded, but still similar to micro-facet BRDFs). With a modification it also becomes reciprocal and energy conserving [1]. It is similar to a micro-facet model which uses a Beckmann-Spizzichino normal distribution function. However, it does not contain a geometric shadowing-masking term. The most efficient implementation is based on the unnormalized halfway vector $\vec{l} + \vec{v}$, see slide 22 (Evaluation of the New BRDF) in the slides[1] for [1]. Complete the implementation using the

[1] `https://www.radiance-online.org/community/workshops/2010-freiburg/PDF/geisler-moroder_duer_thetmeyer_RW2010.pdf`

anisotropic version of the normal distribution function and the Geisler-Moroder-variant (partial points will be given for an implementation of the isotropic normal distribution function). Include Schlick's approximation for the Fresnel reflectance term of a micro-facet in your implementation.

- **GGX- [5] / Trowbridge-Reitz-BRDF [3]:** Nowadays, the GGX-BRDF with the Smith shadowing term is the most widely used micro-facet BRDF, both in the research context and in the industry. Implement the anisotropic version of the GGX normal distribution function, together with Smith's geometric shadowing-masking term and Schlick's approximation for the Fresnel reflectance term. Start with the high-level definition of the Cook-Torrance-BRDF model and plug in the definitions for the three terms. Note that for $\omega \in \{\omega_i, \omega_o\}$ in the anisotropic case of the Smith shadowing term we have
$$\alpha^2 \tan^2\theta = \alpha^2 \frac{1 - \langle \mathbf{n} \mid \omega \rangle^2}{\langle \mathbf{n} \mid \omega \rangle^2} = \frac{\alpha_t^2 \langle \mathbf{t} \mid \omega \rangle^2 + \alpha_b^2 \langle \mathbf{b} \mid \omega \rangle^2}{\langle \mathbf{n} \mid \omega \rangle^2},$$
where $\mathbf{n}$ is the surface normal, $\mathbf{t}$ is the tangent vector and $\mathbf{b}$ is the bitangent vector. α_t and α_b are the surface roughnesses in tangent and bitangent directions, respectively.

If you want to extend your knowledge of these models beyond what is presented in the lecture slides, you should read through the respective papers. They can be downloaded from the web page. If you finished this task successfully, your rendering should look like Figure 1.

Hints: You should be able to find all relevant formulae in the slides and on this exercise sheet but you can also find some additional material on eCampus. For the implementation of the anisotropic BRDF models, you do not only need the surface normal, but also the tangent and bitangent vector, pointing "forward" and "sideways" on the surface, respectively. The tangent and bitangent vector are provided in the `SurfaceInteraction` structure.

Theoretical Assignments

Assignment 2) Refraction in Slab *(2Pts)*

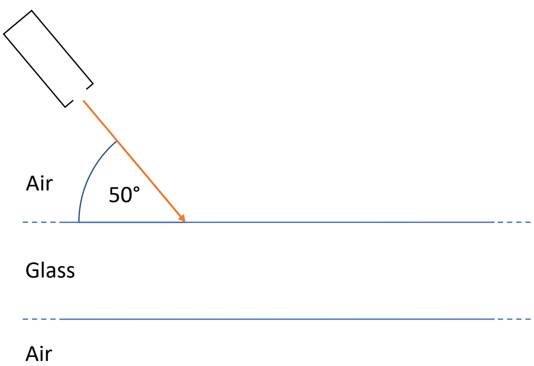

Figure 2: Slab illuminated by a laser.

Consider the scene depicted in Figure 2 in which a laser is shooting unpolarized light rays onto a glass slab (refractive index $n_g = 1.5$) surrounded by air (refractive index $n_a = 1$) at an angle of 50 degrees measured to the surface. Assume that the glass slab has parallel interfaces at the top and bottom, and extends infinitely to both sides in Figure 2.

a) Draw the light paths (using correct angles) of the laser light in the scene. Since there is an infinite number of paths, it is enough to only draw paths up to 6 reflections/refractions.

b) Calculate how much light is emitted to the top and how much light is emitted to the bottom of the slab as a fraction of total light emitted by the laser.

Hints:

- Use the properties of a geometric series.
- The light polarization does not change between the top and bottom interface of the slab.

Assignment 3) Properties of a Heitz Surface *(2Pts)*

In the lecture, the Heitz surface model was introduced. According to this model, the surface can be described as a height field

$$h(x, y) = \sqrt{\frac{2}{N}} \sum_{i=1}^{N} \cos\left(\Phi_i + x f_i^x + y f_i^y\right)$$

with slopes

$$\left(\frac{\partial h}{\partial x}, \frac{\partial h}{\partial y}\right) = \sqrt{\frac{2}{N}} \sum_{i=1}^{N} (-f_i^x \sin\left(\Phi_i + x f_i^x + y f_i^y\right), -f_i^y \sin\left(\Phi_i + x f_i^x + y f_i^y\right)).$$

a) Show that the height field distribution converges towards a Gaussian distribution with expected value 0 and variance 1.

b) Show that the slope distributions converge towards a Gaussian distribution with expected value 0 and variance of $\frac{\alpha^2_{\{x,y\}}}{2}$ in each direction respectively.

References

[1] David Geisler-Moroder and Arne Dür. A new ward brdf model with bounded albedo. In *Computer Graphics Forum*, volume 29, pages 1391–1398. Wiley Online Library, 2010.

[2] Robert R Lewis. Making shaders more physically plausible. In *Computer Graphics Forum*, volume 13, pages 109–120. Wiley Online Library, 1994.

[3] TS Trowbridge and Karl P Reitz. Average irregularity representation of a rough surface for ray reflection. *JOSA*, 65(5):531–536, 1975.

[4] Bui Tuong-Phong. Illumination for computer-generated images. *Technical Report 129, UTEC-CSC-73, Computer Science*, 1973.

[5] Bruce Walter, Stephen R Marschner, Hongsong Li, and Kenneth E Torrance. Microfacet models for refraction through rough surfaces. *Rendering techniques*, 2007:18th, 2007.

[6] Gregory J Ward. Measuring and modeling anisotropic reflection. *ACM SIGGRAPH Computer Graphics*, 26(2):265–272, 1992.

Good luck!