Advanced Topics in Computer Graphics I - Sheet R02

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Assignment 2

Ray-Sphere intersection

Given a ray $r(t)$ with origin o and direction d:

$$r(t) = o + t \cdot d$$

and a sphere with center c and radius r defined by:

$$\|x - c\|^2 = r^2$$

Substitute ray equation into sphere equation:

$$\begin{aligned}
\|o + t \cdot d - c\|^2 &= r^2 \\
\|o + t \cdot d - c\|^2 - r^2 &= 0 && \text{| Substitute } o - c = s \\
(s + t \cdot d) \cdot (s + t \cdot d) - r^2 &= 0 \\
s \cdot s + t \cdot d \cdot s + t \cdot d \cdot s + t \cdot d \cdot t \cdot d - r^2 &= 0 \\
d \cdot d \cdot t^2 + 2 \cdot d \cdot s \cdot t + s \cdot s - r^2 &= 0 && \text{| Substitute back } o - c = s \\
d \cdot d \cdot t^2 + 2 \cdot d \cdot (o - c) \cdot t + (o - c) \cdot (o - c) - r^2 &= 0
\end{aligned}$$

This results in a quadratic equation of the form $at^2 + bt + c = 0$ with

$$\begin{aligned}
a &= d \cdot d \\
b &= 2 \cdot d \cdot (o - c) \\
c &= (o - c) \cdot (o - c) - r^2
\end{aligned}$$

The solutions of this are:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Assignment 3

Ray-Triangle intersection