Advanced Topics in Computer Graphics I

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Sheet R06 - Monte-Carlo Integration

Hand in your solutions via eCampus by Tue, 27.05.2025, **12:00 p.m.**. Compile your solution to the theoretical part into a single printable PDF file. For the practical part, hand in a single ZIP file containing only the exercise* folder within the src/ directory. Please refrain from sending the entire framework.

Assignment 1) Pathtracer *(6Pts)*

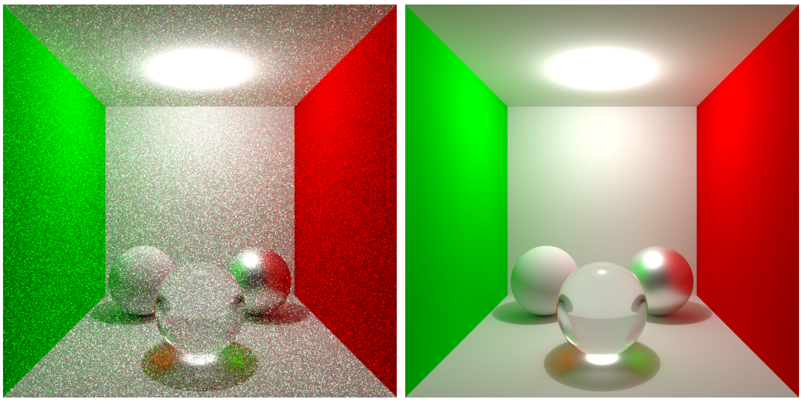

Figure 1: Cornell Box scene rendered with path tracing using 10 and 100 000 samples per pixel, respectively. To obtain these renderings, run `./bin/exercise06_Pathtracing -s data/exercise06_Pathtracing/cornell_box_spheres.xml`.

In this exercise, a we will implement a pathtracer that shoots (many) rays through each pixel. At each hit with the scene, the direct illumination from the light sources is computed via next-event estimation. For the indirect illumination, i.e. reflections of other objects in the scene, a ray is stochastically sampled, such that the sum over all rays through a pixel converges to the true solution of the rendering integral. To reduce the variance in the output, the goal is to choose the probability distribution function of the rays to be as proportional as possible to $\rho(\omega_i, \omega_o) \cdot \langle \mathbf{n}, \omega_i \rangle$.

As before, the exercise contains a ray generator, BSDFs and point light sources. In contrast to the Whitted raytracing framework, there is always (at most) a single recursive ray generated. This simplifies the ray generator since we do not have to manage an increasing number of reflected and transmitted rays that have to be traced. Instead, there is always a single next ray scattered from a surface (at most).

Your task is to implement the `__direct_callable__*_sampleBSDF()` functions in `bsdfmodels.cu`.

a) Implement the ray sampling for the diffuse component of the GGX BSDF by sampling ω_i with probability

$$p_{\omega_i}(\omega_i) = \max\left\{0, \frac{\langle \mathbf{n}, \omega_i \rangle}{\pi}\right\}, \tag{1}$$

given the surface normal $\mathbf{n}$.

b) Implement the ray sampling for the specular GGX microfacet component of the GGX BSDF. Here you first have to sample the halfway vector $\mathbf{h}$ using the inverse CDF $P^{-1}_{\langle \mathbf{n},\mathbf{h} \rangle}$

$$\langle \mathbf{n}, \mathbf{h} \rangle = P^{-1}_{\langle \mathbf{n},\mathbf{h} \rangle}(u) = \sqrt{\frac{1-u}{1+(\alpha^2-1)u}}, \tag{2}$$

where α is the surface roughness. Given the halfway vector $\mathbf{h}$ and view direction ω_o, perform a change of variables to ω_i.

$$\left| \frac{d\omega_i}{d\mathbf{h}} \right| = 4\langle \mathbf{h}, \omega_i \rangle \tag{3}$$

c) Implement the reflection and transmission on glass materials, where the transmission is refracted according to Snell's law. Since you can only generate a single ray at each scene interaction, you have to choose probabilistically whether you generate a transmission or reflection ray. You can use Schlick's approximation to determine the probability of light transmission vs. reflection.

If you finished all tasks successfully your results should look like the ones in Figure 1.

Theoretical Assignments

Assignment 2) Monte Carlo Integration *(6Pts)*

In this exercise you should manually determine the following integral using various methods:

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{-x^2} dx \tag{4}$$

When random numbers are needed, use the following 20 numbers sampled uniformly from the interval $[0, \ldots, 1]$:

0.4387	0.7655	0.1869	0.4456	0.7094	0.2760	0.6551	0.1190	0.9597	0.5853
0.3816	0.7952	0.4898	0.6463	0.7547	0.6797	0.1626	0.4984	0.3404	0.2238

a) Calculate the integral numerically with a tool of your choice (maple, wolfram alpha, ...).

b) Calculate the integral using naive Monte Carlo Integration $\langle I \rangle_A$ (samples drawn from a constant PDF). Specify PDF, CDF, inverse CDF, and the final result.

c) Calculate the integral using Monte Carlo Integration $\langle I \rangle_B$ and Importance Sampling. Draw the necessary samples from a probability distribution proportional to $g(x) = \cos(x)$. As in the previous task, specify PDF, CDF, inverse CDF, and the final result.

d) Calculate the variance of the estimators in b) and c). What do you notice about the variance of $\langle I \rangle_B$? How can this be explained?

e) Why is the Rendering Equation often solved using Monte Carlo Integration?

Hints:

- It might be a good idea to use a spreadsheet for the final calculations.
- Also try to use more samples and see what changes.

Good luck!