Advanced Topics in Computer Graphics I - Sheet R02

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Assignment 2

Ray-Sphere intersection

Given a ray $r(t)$ with origin o and direction d:

$$r(t) = o + t \cdot d$$

and a sphere with center c and radius r defined by:

$$\|x - c\|^2 = r^2$$

Substitute ray equation into sphere equation:

$$\begin{aligned}
\|o + t \cdot d - c\|^2 &= r^2 \\
\|o + t \cdot d - c\|^2 - r^2 &= 0 && \mid \text{Substitute } o - c = s \\
(s + t \cdot d) \cdot (s + t \cdot d) - r^2 &= 0 \\
s \cdot s + t \cdot d \cdot s + t \cdot d \cdot s + t \cdot d \cdot t \cdot d - r^2 &= 0 \\
d \cdot d \cdot t^2 + 2 \cdot d \cdot s \cdot t + s \cdot s - r^2 &= 0 && \mid \text{Substitute back } o - c = s \\
d \cdot d \cdot t^2 + 2 \cdot d \cdot (o - c) \cdot t + (o - c) \cdot (o - c) - r^2 &= 0
\end{aligned}$$

This results in a quadratic equation of the form $at^2 + bt + c = 0$ with

$$\begin{aligned}
a &= d \cdot d \\
b &= 2 \cdot d \cdot (o - c) \\
c &= (o - c) \cdot (o - c) - r^2
\end{aligned}$$

The solutions of this are:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Assignment 3

Ray-Triangle intersection

Given a ray $r(t)$ with origin o and direction d:

$$r(t) = o + t \cdot d$$

and a triangle with vertices a, b, c in global coordinates.

Barycentric Coordinates

Precompute normal n of triangle plane:

$$n = (b - a) \times (c - a)$$

and value $p = -n \cdot a$ for all Triangles.

1. **if** $n \cdot d = 0$: **return false** (3 mult, 2 add)
2. Compute parameter t: (3 mult, 3 add, 1 div)

$$t = -\frac{p + (n \cdot o)}{n \cdot d}$$

3. **if** $t \leq 0$: **return false**
4. Compute intersection point q: (3 mult, 3 add)

$$q = o + t \cdot d$$

5. Compute barycentric coordinates α, β, γ of intersection point q with respect to the triangle vertices: (27 mult, 17 add/sub)

$$\alpha = \frac{((b - c) \times (q - c)) \cdot n}{((b - a) \times (c - a)) \cdot n}$$

$$\beta = \frac{((c - a) \times (q - a)) \cdot n}{((b - a) \times (c - a)) \cdot n}$$

$$\gamma = 1 - \alpha - \beta$$

6. **if** $\alpha \geq 0, \beta \geq 0, \gamma \geq 0$: **return true**

Total Operations with n and p precomputed:
Multiplications: 36

Additions/Subtractions: 25
Divisions: 1

Möller-Trumbore

1. Compute Edges $e1$, $e2$: (6 sub)
$$e_1 = b - a$$
$$e_2 = c - a$$
2. Compute Cross Product h: (6 mult, 3 sub)
$$h = d \times e_2$$
3. Compute the determinant: (3 mult, 2 add)
$$\det = e_1 \cdot h$$
4. **if** $|\det| < \epsilon$: **return false** (ray is parallel to triangle plane)
5. Compute inverse determinant: (1 div)
$$\text{det\_inv} = \frac{1}{\det}$$
6. Compute vector s from vertex a to ray origin: (3 sub)
$$s = o - a$$
7. Compute first barycentric coordinate u: (4 mult, 2 add)
$$u = (s \cdot h) \times \text{det\_inv}$$
8. **if** $u < 0$ **or** $u > 1$: **return false**
9. Compute cross product q: (6 mult, 3 sub)
$$q = s \times e_1$$
10. Compute second barycentric coordinate v: (4 mult, 2 add)
$$v = (d \cdot q) \times \text{det\_inv}$$
11. **if** $v < 0$ **or** $u + v > 1$: **return false**

12. Compute parameter t and intersection point q (7 mult, 5 add):

$$t = (e_2 \cdot q) \times \text{det\_inv}$$

$$q = o + t \cdot d$$

Total Operations:
Multiplications: 30
Additions/Subtractions: 26
Divisions: 1

In the case of a successful intersection, the Möller–Trumbore algorithm requires six fewer multiplications and one additional addition or subtraction, assuming no parallelization.