Project 1: Martingale

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***Abstract*—**This first report seeks to measure the performance of the Martingale strategy through two experiments: the first with no money limit, and the second with a limit of \$256. The results include probabilities of winning \$80, expected values after 1000 spins, and standard deviation behavior. It highlight the strengths and weaknesses of the Martingale strategy under different conditions. The results seek to show the limitations of certain strategies.

1 INTRODUCTION

The Martingale strategy is a gambling approach where every time the player loses, he doubles his bet to recover his previous losses and achieve a net gain. While theoretically effective with an unlimited bankroll, real-world constraints often limit its feasibility.

We are going to carry out two experiments:

- Experiment 1: Unlimited bankroll.
- Experiment 2: Bankroll limited to \$256.

We analyze the probability of winning \$80, expected winnings after 1000 spins, and the stabilization and convergence of standard deviation lines.

2 EXPERIMENT 1: UNLIMITED BANKROLL

2.1 Objective

In this first experiment there will be no constraints, we can assume that our player has an infinite amount of money to evaluate the Martingale strategy's performance with no financial constraints.

2.2 Results and Figures

Question Set 1:
Probability of Winning \$80: The chance of reaching \$80 with 1000 spins :

$$P(\text{Winning } \$80) = \frac{\text{Number of episodes where winnings} \geqslant \$80}{\text{Total episodes}}$$

Calculated Probability: **1.0000** (100%).

We have 1,000 spins, which gives us some margin to win back what we've lost with this strategy. And on average you don't need 1000 spins to achieve this gain, just 200 spins are enough.

Question 2:
Expected Value of winnings after 1000 spins :

$$\text{Expected Value} = \frac{\sum_{i=1}^{N} W_{i,1000}}{N}$$

Calculated Expected Value: **$80.00**.

Where N is the number of episodes.

The expected value of winnings after 1000 spins is $80, which is normal because we have set ourselves a target of $80 as soon as we reach this amount we consider that we have won. What's more, in this first experiment we found that we had a 100% chance of winning.

Question Set 3:
Standard Deviation: The standard deviation of winnings at the final spin is calculated using:

$$\text{Standard Deviation} = \sqrt{\frac{\sum_{i=1}^{N} (W_{i,1000} - \bar{W})^2}{N}}$$

Final Standard Deviation: **0.00**.

Since the bankroll is unlimited, the Martingale strategy guarantees that every episode reaches the target winnings of $80. This ensures that there is no variation in the final outcomes.

As all the episodes are winners, there will be no standard deviation in the final result, the standard deviation across episodes becomes 0. Thus, the mean ± stdev bounds stabilize at the same value ($80).

Figures:

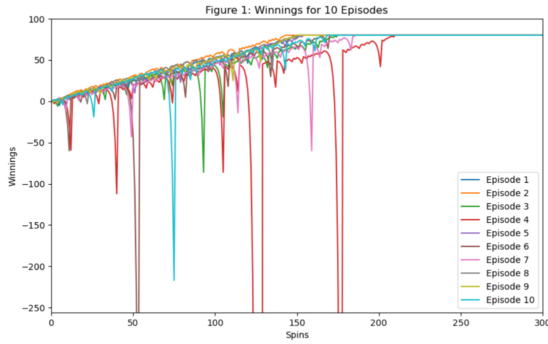

Figure 1—Winnings for 10 Episodes (Unlimited Bankroll). Each line represents one episode.

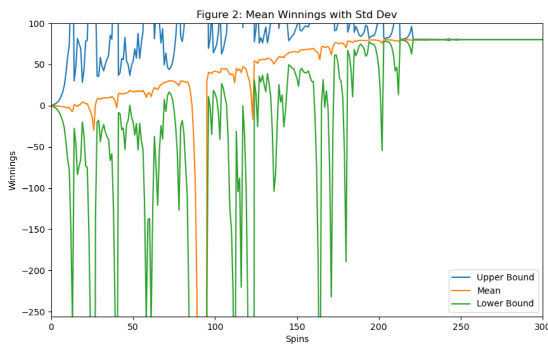

Figure 2—Mean Winnings with Standard Deviation (Unlimited Bankroll). The standard deviation lines stabilize at $80.

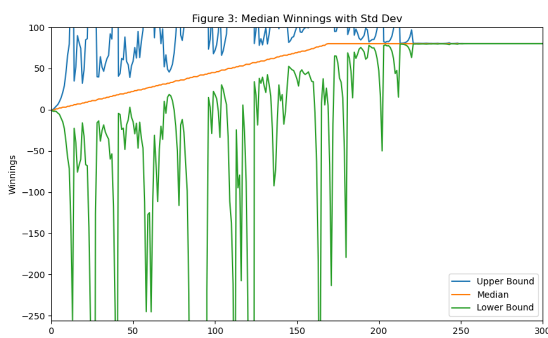

Figure 3—Median Winnings with Standard Deviation (Unlimited Bankroll). The standard deviation lines converge at $80.

2.3 Analysis

The Martingale strategy guarantees success ($80) when the bankroll is unlimited. The probability of winning is 100%, and there is no variance in results, as reflected by a standard deviation of 0.00.

3 EXPERIMENT 2: LIMITED BANKROLL

3.1 Objective

To evaluate the strategy with a $256 bankroll, simulating real-world constraints.

3.2 Results and Figures

Question 4:
Probability of Winning $80: The probability of reaching $80 is calculated as:

$$P(\text{Winning } \$80) = \frac{\text{Number of episodes where winnings} \geqslant \$80}{\text{Total episodes}}$$

Calculated Probability: **0.6180** (61.8%).

Some players have run out of money and can no longer play, so they have lost and can no longer win.

Question 5:
Expected Value of winnings after 1000 spins :

$$\text{Expected Value} = \frac{\sum_{i=1}^{N} W_{i,1000}}{N}$$

Calculated Expected Value: **$-47.06**.

The average player loses money

Question Set 6:
Standard Deviation: The standard deviation of winnings at the final spin is:

$$\text{Standard Deviation} = \sqrt{\frac{\sum_{i=1}^{N} (W_{i,1000} - \bar{W})^2}{N}}$$

Final Standard Deviation: **162.69**.

To calculate stabilisation, you need to take the standard deviation over the last few spins and see if it continues to evolve. The standard deviation stabilizes, which can be explained as follows: after a certain number of spins, the number of winners and losers is already fixed. Some players will have won the $80, while others will no longer have enough money to play.

For convergence to occur, the variance must be equal tends towards 0. All players' winnings should therefore tend towards the same level. And since at a certain point nobody wants to play or can play. So there is a disparity in their winnings depending on the player, which is why there is no convergence.

Figures:

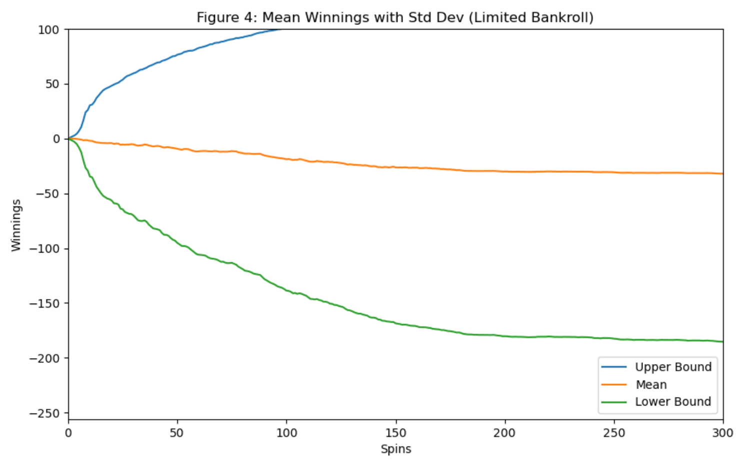

Figure 4—Mean Winnings with Standard Deviation (Limited Bankroll). Variability increases due to the bankroll limit.

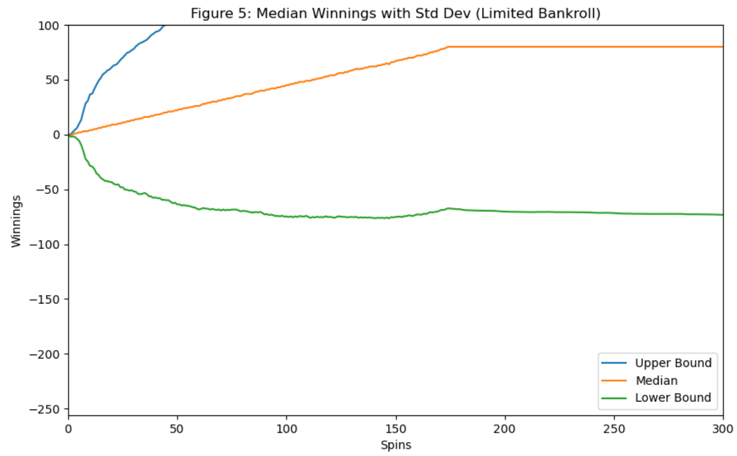

Figure 5—Median Winnings with Standard Deviation (Limited Bankroll). The wide range of outcomes prevents convergence.

3.3 Analysis

The bankroll limit significantly reduces the success probability (61.8%) and results in a negative expected value ($-47.40). Standard deviation does stabilize but does not converge, reflecting the wide range of outcomes caused by premature termination of episodes due to bankruptcy.

4 DISCUSSION

4.1 Expected Values vs. Single Episodes

Question 7:
Using expected values provides the following benefits:

- Reduces the impact of outliers.
- Reflects the average performance across many trials
- Single episodes may misrepresent the strategy due to high variance or extreme results.

5 CONCLUSION

This study highlights the theoretical effectiveness of the Martingale strategy under ideal conditions (unlimited bankroll) and its significant risks under realistic constraints (limited bankroll). While the strategy guarantees success when constraints are removed, it becomes highly volatile with a fixed bankroll.

6 REFERENCES