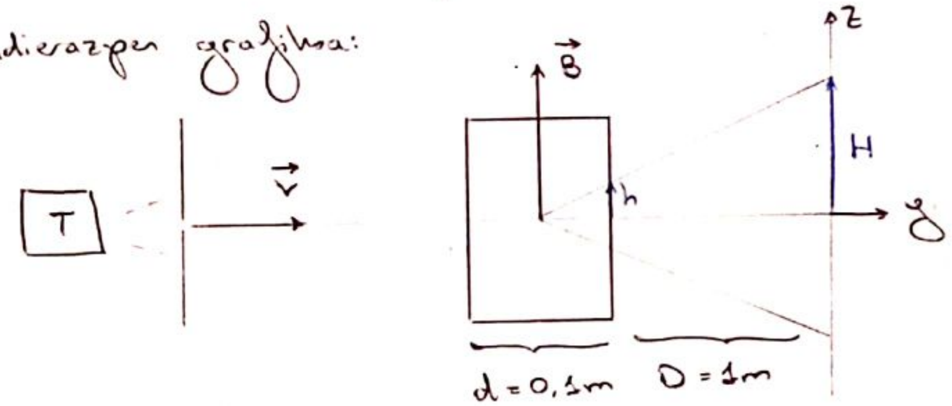
3. ORRIA

① Stern-Gerlach-en dispositiboaz baliatuz 1000K-eko labe batetik ateratako zilarrezko atomoak erabiltzen direla emanik eta eremu magnetikoaren z-rekiko deribatua $100\frac{T}{m}$-koa dela jakinik, lor bedi eremu magnetikoaren eraginpetik atera ondoren sorturiko bi atomo-sortek pantaila jo egiten duteneko posizioen arteko distantzia. Eremu magnetikoaren eraginpean atomoek deskribatzen duten ibilbidearen x ardatzarekiko proiekzioa 10cm-koa da eta eremu magnetikoaren eraginpetik kanpo atomoek deskribatzen duten ibilbidearen x ardatzarekiko proiekzioa 1m-koa da.

Adierazpen grafikoa:



Labeko energia termikoa energia zinetiko bihurtzen denez:

$$\frac{3}{2}K_B T = \frac{1}{2}mv_0^2 \Rightarrow v_0^2 = \frac{3K_BT}{m} \Rightarrow v_0 = \sqrt{\frac{3K_BT}{m}}$$

Dispositiboan sartzean, $\vec{B}$ eremu magnetikoak indar bat eragingo dio sortari, honen abiadura eta norabidea aldatuz:

$$U = -\vec{m}\cdot\vec{B} = -m_z\cdot B = -g_s\cdot \mu_B\cdot S\cdot B_z$$

$$F_z = -\frac{\partial}{\partial z}(U) = g\mu_B S\cdot 100$$

$$\left.\begin{aligned} V_y &= V_0 \quad (\text{kte}) \\ V_z &= V_{0t} + \frac{F_z}{m} t = \frac{100 g \mu_0 S}{m} t \end{aligned}\right\} \Rightarrow$$

$$d = V_{0y} \cdot t \longrightarrow t_h = \frac{d}{V_0} = d\sqrt{\frac{m}{3K_0T}}$$

$$z = z_0 + V_{0z}t + \frac{1}{2} \cdot \frac{F_z}{m} t^2$$

$$h = \frac{1}{2} \cdot \frac{g \cdot \mu_0 \cdot S \cdot 100}{m} (t_h)^2 = 50 \cdot \frac{g \mu_0 S}{m} \cdot d^2 \cdot \frac{m}{3K_0T}$$

Ondorioz: $h = 50 \cdot \frac{g \mu_0 S}{3K_0T} d^2$

$$V_{zh} = \frac{100 g \mu_0 S}{m} d \sqrt{\frac{m}{3K_0T}} \quad , \quad V_{yh} = V_0 = \sqrt{\frac{3K_0T}{m}}$$

$$D = 1m = V_{0y} \cdot t \longrightarrow t = \frac{D}{V_0} = D\sqrt{\frac{m}{3K_0T}}$$

$$z = V_z t = \frac{100 g \mu_0 S}{m} d \cdot D \cdot \frac{m}{3K_0T}$$

Beraz: $H = h + z = \frac{100 g \mu_0 S}{3K_0T} \left(\frac{d^2}{2} + d \cdot D \right)$

Distantzia totala: $2H$

Emaitza: $\boxed{2H = \frac{100 g \mu_0 S}{3K_0T} d \left(\frac{d}{2} + D \right)}$

(2)

a) Lor bitez, $s = 1/2$ kasuan, Su eragilearen autobektoreak eta autobalioak.

Badakigu: $\hat{S_u} = \vec{S} \cdot \hat{u} = (\hat{S_x} \cdot \hat{i} + \hat{S_y} \cdot \hat{j} + \hat{S_z} \cdot \hat{k}) \vec{u}$

non $\hat{u} = \sin\theta \cos\varphi \, \hat{i} + \sin\theta \sin\varphi \, \hat{j} + \cos\theta \cdot \hat{k}$

Oinarria $\{|+\rangle, |-\rangle\}$ bada:

$$\hat{S_x} = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \hat{S_y} = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \hat{S_z} = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \Rightarrow$$

$$\hat{S_u} = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}(\hat{i} \cdot \hat{u_x}) + \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}(\hat{j} \cdot \hat{u_y}) + \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}(\hat{k} \cdot \hat{u_z})$$

$$\hat{S_u} = \frac{\hbar}{2}\begin{pmatrix} \cos\theta & \sin\theta\cos\varphi - i\sin\theta\sin\varphi \\ \sin\theta\cos\varphi + i\sin\theta\sin\varphi & -\cos\theta \end{pmatrix} \Rightarrow$$

$$\hat{S_u} = \frac{\hbar}{2}\begin{pmatrix} \cos\theta & \sin\theta e^{-i\varphi} \\ \sin\theta e^{i\varphi} & -\cos\theta \end{pmatrix}$$ izango da. Autobalioak:

$$\begin{vmatrix} \frac{\hbar}{2}\cos\theta - \lambda & \frac{\hbar}{2}\sin\theta e^{-i\varphi} \\ \frac{\hbar}{2}\sin\theta e^{i\varphi} & -\frac{\hbar}{2}\cos\theta - \lambda \end{vmatrix} = 0 \Rightarrow \boxed{\lambda = \pm\frac{\hbar}{2}} \quad \text{(Mathematica)}$$

Jarraian, autobektoreak kalkulatuz:

$$\underline{\lambda = \hbar/2}: \begin{pmatrix} \cos\theta - 1 & \sin\theta e^{-i\varphi} \\ \sin\theta e^{i\varphi} & -\cos\theta - 1 \end{pmatrix}\begin{pmatrix} C_+ \\ C_- \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \rightarrow C_- = \frac{\sin\theta e^{i\varphi}}{\cos\theta + 1} C_+$$

Normalizatuz: $|C_-|^2 + |C_+|^2 = 1$

$$|C_+|^2\left(1 + \frac{\sin^2\theta}{(1+\cos\theta)^2}\right) = 1 = |C_+|^2\left(\frac{1 + 2\cos\theta + \sin^2\theta + \cos^2\theta}{(1+\cos\theta)^2}\right) =$$

$$= |C_+|^2 \cdot 2 \cdot \frac{1+\cos\theta}{(1+\cos\theta)^2} = \frac{2|C_+|^2}{1+\cos\theta} = 1 \Rightarrow C_+ \in \mathbb{R} \Rightarrow$$

$$C_+^2 = \frac{1+\cos\theta}{2} = \cos^2\left(\frac{\theta}{2}\right) \Rightarrow C_+ = \cos\frac{\theta}{2}$$

Ondorioz: $C_- = \frac{\sin\theta e^{i\varphi}}{1+\cos\theta} \cos\frac{\theta}{2} \Rightarrow C_- = \sin\frac{\theta}{2} e^{i\varphi}$

$|+\rangle_u = \cos\frac{\theta}{2}|+\rangle + e^{i\varphi}\sin\frac{\theta}{2}|-\rangle = e^{i\frac{\varphi}{2}}\left(\cos\frac{\theta}{2}e^{-i\frac{\varphi}{2}}|+\rangle + \sin\frac{\theta}{2}e^{i\frac{\varphi}{2}}|-\rangle\right)$

$e^{i\varphi/2}$ gaia beti aukeratu dezakegunez:

$$\boxed{|+\rangle_u = \cos\frac{\theta}{2}e^{-i\frac{\varphi}{2}}|+\rangle + \sin\frac{\theta}{2}e^{i\frac{\varphi}{2}}|-\rangle}$$

$\underline{\lambda = -\hbar/2}$: $\begin{pmatrix} \cos\theta + 1 & \sin\theta e^{-i\varphi} \\ \sin\theta e^{i\varphi} & 1-\cos\theta \end{pmatrix}\begin{pmatrix} C_+ \\ C_- \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \rightarrow C_+ = -\frac{\sin\theta e^{-i\varphi}}{1+\cos\theta} C_-$

Normalizazioa: $|C_+|^2 + |C_-|^2 = 1 \Rightarrow$

$$1 = |C_-|^2\left(1 + \frac{\sin^2\theta}{(1+\cos\theta)^2}\right) = \frac{1+2\cos\theta+\cos^2\theta+\sin^2\theta}{(1+\cos\theta)^2}|C_-|^2 =$$

$$= 2|C_-|^2\frac{1+\cos\theta}{(1+\cos\theta)^2} = |C_-|^2\sec^2\frac{\theta}{2} = 1 \Rightarrow C_- = \cos\frac{\theta}{2}.$$

Beraz: $C_+ = \frac{-\sin\theta}{1+\cos\theta}\cdot\cos\frac{\theta}{2}e^{-i\varphi} = -e^{-i\varphi}\sin\frac{\theta}{2} \rightarrow C_+ = -e^{-i\varphi}\sin\frac{\theta}{2}$

Ondorioz: $|-\rangle_u = -e^{-i\varphi}\sin\frac{\theta}{2}|+\rangle + \cos\frac{\theta}{2}|-\rangle$

Emaitza: $\boxed{|-\rangle_u = e^{i\frac{\varphi}{2}}\cos\frac{\theta}{2}|-\rangle - e^{-i\frac{\varphi}{2}}\sin\frac{\theta}{2}}$

② jarraipena

b) Froga bedi $S=1/2$ kasuan edozein spin-egoera honako era honetara adieraz daitekeela: $|\chi\rangle = e^{i\xi}|+\rangle_u$.

Lor bitez $\hat{u}$ bektore unitarioaren θ eta ϕ angeluak $|\chi\rangle$ spin-egoeraren C_+ eta C_- osagaien funtzioan.

Definizioz: $|\chi\rangle = C_+|+\rangle + C_-|-\rangle = e^{i\xi}|+\rangle$

Badakigu: $C_+|+\rangle + C_-|-\rangle = e^{i\xi}\left(\cos\frac{\theta}{2}|+\rangle + \sin\frac{\theta}{2}e^{-i\varphi}|-\rangle\right)$

Beraz: $C_+ = e^{i\xi}\cos\frac{\theta}{2}$ eta $C_- = e^{i(\xi-\varphi)}\sin\frac{\theta}{2}$ dira

Zatituz: $\frac{C_-}{C_+} = e^{-i\varphi}\tan\frac{\theta}{2} \Rightarrow \boxed{\theta = 2\arctan\left|\frac{C_-}{C_+}\right|}$

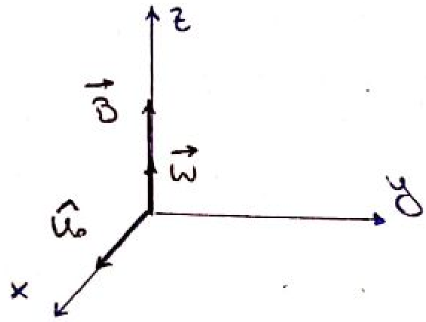
③ Kontsidera bedi $\vec{B} = B\cdot\hat{u}$ eremu magnetiko estatikoaren eragipean kokaturiko elektroia, beronen hasierako aldiuneko spin-egoera $|\Psi(0)\rangle = |+\rangle_x$ izanik.

a) Lor bitez $\hat{S}_x$ eta $\hat{S}_y$ behagarrien itxarondako balioak, denboraren funtzioan:

Hasieran: $|\Psi(0)\rangle = |+\rangle_x = \frac{1}{\sqrt{2}}\left(|+\rangle + |-\rangle\right)$

Adierazpen orokorra: $|+\rangle_u = e^{-i\frac{\varphi}{2}}\cos\frac{\theta}{2}|+\rangle + e^{i\frac{\varphi}{2}}\sin\frac{\theta}{2}|-\rangle$

Hasieran $\varphi=0$ da, baina $\vec{B}$ eremuak ω maiztasun batekin biratzea eragingo dio $\hat{u}_0$-ri $\vec{B}\parallel\vec{\omega}$ izanik.



Beraz: $\varphi(t) = \cancel{\varphi_0}^{\,0} + \omega t = \omega t$

$|\Psi(t)\rangle = \frac{1}{\sqrt{2}}\left(e^{-i\frac{\omega t}{2}}|+\rangle + e^{i\frac{\omega t}{2}}|-\rangle\right)$

θ angeluai ez dio eragingo eremuak, perpendikularki higitzen baita.

Egoera: $|\psi\rangle = \begin{pmatrix} e^{-i\frac{\omega t}{2}}/\sqrt{2} \\ e^{i\frac{\omega t}{2}}/\sqrt{2} \end{pmatrix} \rightarrow$ Itxarondako balioak:

$$\langle \hat{S_x} \rangle = \frac{1}{2}\begin{pmatrix} e^{\frac{\omega t}{2}i} & e^{-i\frac{\omega t}{2}} \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\frac{\hbar}{2}\begin{pmatrix} e^{-i\frac{\omega t}{2}} \\ e^{i\frac{\omega t}{2}} \end{pmatrix} \Rightarrow$$

$$\langle \hat{S_x} \rangle = \frac{\hbar}{4}\begin{pmatrix} e^{-i\frac{\omega t}{2}} & e^{i\frac{\omega t}{2}} \end{pmatrix}\begin{pmatrix} e^{-i\frac{\omega t}{2}} \\ e^{i\frac{\omega t}{2}} \end{pmatrix} \Rightarrow \boxed{\langle \hat{S_x} \rangle = \frac{\hbar}{2}\cos(\omega t)}$$

$$\langle \hat{S_y} \rangle = \frac{1}{2}\cdot\frac{\hbar}{2}\begin{pmatrix} e^{i\frac{\omega t}{2}} & e^{-i\frac{\omega t}{2}} \end{pmatrix}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} e^{-i\frac{\omega t}{2}} \\ e^{i\frac{\omega t}{2}} \end{pmatrix} \Rightarrow$$

$$\langle \hat{S_y} \rangle = \frac{\hbar}{4}i\begin{pmatrix} e^{-i\frac{\omega t}{2}} & -e^{i\frac{\omega t}{2}} \end{pmatrix}\begin{pmatrix} e^{-i\omega t/2} \\ e^{i\omega t/2} \end{pmatrix} \Rightarrow \boxed{\langle \hat{S_y} \rangle = \frac{\hbar}{2}\sin(\omega t)}$$

b) Aldiune jakin batean (t) elektroiak OX ardatzaren norabideko SG dispositiboa zeharkatzen duela jakinik, zein izango da elektroi hori $|+\rangle_x$ egoeran aurkitzeko probabilitatea?

Denboraren garapena: $|\psi\rangle = \frac{1}{\sqrt{2}}\left(e^{-i\frac{\omega t}{2}}|+\rangle + e^{i\frac{\omega t}{2}}|-\rangle\right)$

Orain, egoera X-en menpe idatziz:

$$\left.\begin{aligned} |+\rangle_x &= \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle) \\ |-\rangle_x &= \frac{1}{\sqrt{2}}(|+\rangle - |-\rangle) \end{aligned}\right\} \Rightarrow \left\{\begin{aligned} |+\rangle &= \frac{1}{\sqrt{2}}(|+\rangle_x + |-\rangle_x) \\ |-\rangle &= \frac{1}{\sqrt{2}}(|+\rangle_x - |-\rangle_x) \end{aligned}\right.$$

Beraz, egoera:

$$|\psi(t)\rangle = \frac{1}{2}\left(e^{-i\frac{\omega t}{2}}(|+\rangle_x + |-\rangle_x) + e^{i\frac{\omega t}{2}}(|+\rangle_x - |-\rangle_x)\right)$$

③ jarraipena

b) jarraipena

$$|\psi(t)\rangle = \frac{1}{2}\left(|+\rangle_x\left(e^{-i\frac{\omega t}{2}} + e^{i\frac{\omega t}{2}}\right) - |-\rangle_x\left(e^{i\frac{\omega t}{2}} - e^{-i\frac{\omega t}{2}}\right)\right) \Rightarrow$$

$$|\psi(t)\rangle = \cos\left(\frac{\omega t}{2}\right)|+\rangle_x + i|-\rangle_x \sin\left(\frac{\omega t}{2}\right)$$

Probabilitatea: $\boxed{P(\hbar/2) = \cos^2(\omega t/2)}$

c) Zein aldiunetan izango da ziurra (b) atalako neurketaren emaitza?

Ziurra izateko: $P = 1 \longrightarrow \cos\left(\frac{\omega t}{2}\right) = \pm 1 \Longrightarrow$

$\frac{\omega t}{2} = \pi n$. Aldiunea: $\boxed{t = \frac{2\pi n}{\omega}}$

④ Izan bedi $\vec{B} = \frac{B_0}{\sqrt{2}}(\hat{i} + \hat{k})$ eremu magnetiko uniforme eta estatikoaren eraginpeko elektroia

a) Lor bedi elektroiaren hamiltondarraren $\{|+\rangle, |-\rangle\}$ oinarriarekiko adierazpen matriziala.

Definizioz: $\hat{H} = \hat{H}_{\vec{r}} + H_s \begin{cases} \hat{H}_{\vec{r}} \to \text{Espaziala} \\ \hat{H}_s \to \text{Spina} \end{cases}$

Hamiltondarrerako $\hat{H_s}$ soilik hartuko dugu kontutan, ez baitaude akoplatuta. Elektroi batek, eremu magnetikoan egoteagatik duen energia potentziala:

$$U = -\vec{\mu}\cdot\vec{B} = -\gamma\vec{S}\cdot\vec{B} = -\gamma\vec{S}\cdot\frac{B_0}{\sqrt{2}}(\hat{i}+\hat{k}) = -\frac{\gamma B_0}{\sqrt{2}}(\hat{S_x} + \hat{S_z})$$

$\omega_0 = -\gamma B_0$ hartuz: $U = \frac{\omega_0}{\sqrt{2}}(\hat{S_x} + \hat{S_z})$

Hamiltondarra: $\hat{H} = \frac{w_0}{\sqrt{2}} (\hat{S_x} + \hat{S_z})$

$$(\hat{H}) = \frac{w_0}{\sqrt{2}} \left\{ \frac{\hbar}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right\} = \frac{w_0}{\sqrt{2}} \cdot \frac{\hbar}{2} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Adierazpen matriziala: $\boxed{(\hat{H}) = \frac{w_0}{\sqrt{2}} \cdot \frac{\hbar}{2} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}}$

b) Lor bitez hamiltondarraren autobalioak eta autobektoreak

Hamiltondarra: $\hat{H} = \frac{w_0 \hbar}{2\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \alpha \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \Rightarrow$

$$\begin{vmatrix} \alpha - \lambda & \alpha \\ \alpha & -\alpha - \lambda \end{vmatrix} = 0 = (\lambda + \alpha)(\lambda - \alpha) - \alpha^2 = \lambda^2 - 2\alpha^2 \Rightarrow$$

Autobalioak: $\boxed{\lambda = \pm \sqrt{2}\alpha}$ $\boxed{\lambda = \pm \frac{w_0 \hbar}{2}}$

Jarraian, autobektoreak kalkulatuz:

$\underline{\lambda = \alpha\sqrt{2}}$: $\begin{pmatrix} 1 - \sqrt{2} & 1 \\ 1 & -1 - \sqrt{2} \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \rightarrow a = (1 + \sqrt{2}) b$

Normalizazioa: $|a|^2 + |b|^2 = 1 \Rightarrow a, b \in \mathbb{R} \Rightarrow$

$$1 = b^2 \left(1 + (1 + \sqrt{2})^2\right) = b^2 (4 + 2\sqrt{2}) \Rightarrow b = \frac{1}{\sqrt{2(2+\sqrt{2})}}$$

Beraz: $\boxed{|+\rangle_1 = \frac{1}{\sqrt{2(2+\sqrt{2})}} \left((1+\sqrt{2}) \cdot |+\rangle + |-\rangle \right)}$

$\underline{\lambda = -\alpha\sqrt{2}}$: $\begin{pmatrix} 1 + \sqrt{2} & 1 \\ 1 & -1 + \sqrt{2} \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \rightarrow -b = (1 + \sqrt{2}) a$

Normalizazioa: $|a|^2 + |b|^2 = 1 \Rightarrow$

④ jarraipena

$$1 = a^2(4+2\sqrt{2}) \Longrightarrow a = \frac{1}{\sqrt{2(2+\sqrt{2})}}$$

Beraz: $$\boxed{|-\rangle_1 = \frac{1}{\sqrt{2(2+\sqrt{2})}}\left(|+\rangle - (1+\sqrt{2})|-\rangle\right)}$$

Beste modu bat:

$$\hat{H} = -\gamma\vec{S}\cdot\vec{B} = -\gamma B_0\cdot\vec{S}\cdot\frac{\hat{\imath}+\hat{k}}{\cancel{\sqrt{2}}} = -\gamma B_0\,\hat{S}_u \qquad \text{non} \qquad \hat{u} = \frac{\hat{\imath}+\hat{k}}{\cancel{\sqrt{2}}}$$

Beraz, autofuntzioak $\hat{S}_u$-renak izango dira:

$$\left.\begin{aligned} |+\rangle_u &= e^{-i\frac{\varphi}{2}}\cos\frac{\theta}{2}|+\rangle + e^{i\frac{\varphi}{2}}\sin\frac{\theta}{2}|-\rangle \\ |-\rangle_u &= -e^{-i\frac{\varphi}{2}}\sin\frac{\theta}{2}|+\rangle + e^{i\frac{\varphi}{2}}\cos\frac{\theta}{2}|-\rangle \end{aligned}\right\}$$

$\hat{u} = \hat{\imath}+\hat{k}$ denez: $\varphi = 0$, $\theta = \pi/4$. Beraz:

$$\left.\begin{aligned} |+\rangle_u &= \cos\frac{\pi}{8}|+\rangle + \sin\frac{\pi}{8}|-\rangle \\ |-\rangle_u &= -\sin\frac{\pi}{8}|+\rangle + \cos\frac{\pi}{8}|-\rangle \end{aligned}\right\} \quad \text{non} \quad \left\{\begin{aligned} \sin\frac{\pi}{8} &= \frac{1}{\sqrt{4+2\sqrt{2}}} \\ \cos\frac{\pi}{8} &= \frac{1+\sqrt{2}}{\sqrt{4+2\sqrt{2}}} \end{aligned}\right.$$

c) Spin-egoera $|\psi\rangle = |-\rangle$ denean zeintzuk dira energiaren neurketaren emaitzak? Lor bitez energiaren balio posible horiek aurkitzeko dauden probabilitateak.

Hasteko $|\psi\rangle$ egoera $|+\rangle_u$ eta $|-\rangle_u$-ren funtzioan idatzi behar dugu:

$$|-\rangle_u = a|+\rangle_u + b|-\rangle_u \Longrightarrow \text{Lehendik:}$$

$$a = \sin\frac{\pi}{8}, \quad b = \cos\frac{\pi}{8}$$

Beraz: $|-\rangle = \sin\frac{\pi}{8}|+\rangle_u + \cos\frac{\pi}{8}|-\rangle_u$

- $E_+ = \frac{w_0\hbar}{2} \Longrightarrow \boxed{P(E_+) = \sin^2\frac{\pi}{8}}$

- $E_- = -\frac{w_0\hbar}{2} \Longrightarrow \boxed{P(E_-) = \cos^2\frac{\pi}{8}}$

d) Lor bitez hasierako aldiuneko spin-egoera $|\psi\rangle = |-\rangle$ dela emanik, spin-egoera eta $\hat{S}x$ behagarriaren itxarotako balioa denboraren funtzioan. Eman bedi emaitza haren interpretazio geometrikoa

Hasteko, $|-\rangle$ $\{|\pm\rangle_u\}$-ren funtzioan garatu behar dugu denboraren funtzioan adierazteko. Goiko ataletik:

$|\psi(0)\rangle = |-\rangle = \sin\frac{\pi}{8}|+\rangle_u + \cos\frac{\pi}{8}|-\rangle_u$. Zuzenean:

$$|\psi(t)\rangle = \sin\frac{\pi}{8}|+\rangle_u \cdot e^{-i\frac{w_0t}{2}} + \cos\frac{\pi}{8}|-\rangle_u e^{i\frac{w_0t}{2}}$$

Jarraian, $\{|\pm\rangle\}$ oinarrira pasatuko dugu berriz, $\hat{S}x$-en adierazpena oinarri horretan lortu baitugu:

$$\text{Badakigu: } \begin{cases} |+\rangle_u = \cos\frac{\pi}{8}|+\rangle + \sin\frac{\pi}{8}|-\rangle \\ |-\rangle_u = -\sin\frac{\pi}{8}|+\rangle + \cos\frac{\pi}{8}|-\rangle \end{cases} \Longrightarrow$$

$$|\psi(t)\rangle = \sin\frac{\pi}{8}\left(\cos\frac{\pi}{8}|+\rangle + \sin\frac{\pi}{8}|-\rangle\right)e^{-i\frac{w_0t}{2}} +$$
$$\cos\frac{\pi}{8}\left(-\sin\frac{\pi}{8}|+\rangle + \cos\frac{\pi}{8}|-\rangle\right)e^{i\frac{w_0t}{2}} \Longrightarrow$$

④ jarraipena

d) jarraipena

$$|\psi(t)\rangle = |+\rangle \sin\frac{\pi}{8}\cos\frac{\pi}{8}\left(e^{-i\frac{\omega_0 t}{2}} - e^{i\frac{\omega_0 t}{2}}\right) +$$
$$|-\rangle\left(\sin^2\frac{\pi}{8}e^{-i\frac{\omega_0 t}{2}} + \cos^2\frac{\pi}{8}e^{i\frac{\omega_0 t}{2}}\right)$$

$$|\psi(t)\rangle = |+\rangle \sin\frac{\pi}{4}\sin\left(\frac{\omega_0 t}{2}\right)(-i) + |-\rangle\left(\sin^2\frac{\pi}{8}e^{-i\frac{\omega_0 t}{2}} + \cos^2\frac{\pi}{8}e^{i\frac{\omega_0 t}{2}}\right)$$

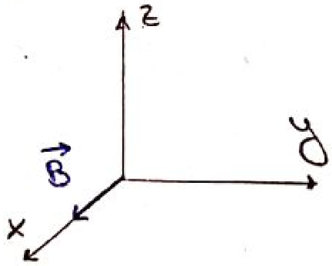
Beraz: $\langle \hat{S}_x\rangle = \langle\psi|\hat{S}_x|\psi\rangle = (\psi)^* \cdot \frac{\hbar}{2}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}(\psi)$

(Mathematica) $\rightarrow$ $\boxed{\langle\hat{S}_x\rangle = -\frac{\hbar}{2}\sin^2\left(\frac{\omega t}{2}\right)}$

⑤ Demagun $\hat{u}$ bektore unitarioa OXZ planoan kokatuta dagoela. $\hat{u}$ norabideko SG dispositibo batez baliatuz $|+\rangle_u$ spin-egoerako elektroi-sorta lortzen dugu.

a) Lor bedi elektroien spin-egoeraren denborarekiko menpekotasuna, elektroi horiek OX ardatzaren norabideko eremu magnetiko uniforme eta estatiko baten eraginpean higitzen ari direlarik

Hasierako egoera:



$$|\psi(0)\rangle = |+\rangle_u = \cos\frac{\theta}{2}|+\rangle + \sin\frac{\theta}{2}|-\rangle$$

Hamiltondarraren autofuntzioetan garatzen badugu denboraren garapena egiteko:

$$\hat{H} = -\gamma\vec{B}\cdot\vec{S} = -\gamma B\hat{i}\cdot\vec{S} = -\gamma B_0\cdot\hat{S}_x = \omega_0\hat{S}_x$$

$\hat{H} = \frac{\omega_0\hbar}{2}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$ $\longrightarrow$ $\hat{S}_x$-en autofuntzioak:

$$|+\rangle_x = \frac{1}{\sqrt{2}}\left(|+\rangle + |-\rangle\right) \quad \text{eta} \quad |-\rangle_x = \frac{1}{\sqrt{2}}\left(|+\rangle - |-\rangle\right)$$

Gure egoera: $|\psi\rangle = \alpha|+\rangle_x + \beta|-\rangle_x$

$\alpha = {}_x\langle + | \psi(0)\rangle = \left(|+\rangle + |-\rangle, \cos\frac{\theta}{2}|+\rangle + \sin\frac{\theta}{2}|-\rangle\right)\cdot\frac{1}{\sqrt{2}}$

$\alpha = \frac{1}{\sqrt{2}}\left(\cos\frac{\theta}{2} + \sin\frac{\theta}{2}\right)$

$\beta = {}_x\langle - | \psi(0)\rangle = \left(-|+\rangle + |-\rangle, \cos\frac{\theta}{2}|+\rangle + \sin\frac{\theta}{2}|-\rangle\right)\cdot\frac{1}{\sqrt{2}}$

$\beta = \frac{1}{\sqrt{2}}\left(-\cos\frac{\theta}{2} + \sin\frac{\theta}{2}\right)$. Beraz:

$|\psi(0)\rangle = \frac{1}{\sqrt{2}}\left\{\left(\cos\frac{\theta}{2} + \sin\frac{\theta}{2}\right)|+\rangle_x - \left(\cos\frac{\theta}{2} - \sin\frac{\theta}{2}\right)|-\rangle_x\right\}$

Denboraren garapena:

$$\boxed{|\psi(t)\rangle = \frac{1}{\sqrt{2}}\left\{\left(\cos\frac{\theta}{2} + \sin\frac{\theta}{2}\right)e^{i\frac{\omega_0 t}{2}}|+\rangle_x - \left(\cos\frac{\theta}{2} - \sin\frac{\theta}{2}\right)e^{-i\frac{\omega_0 t}{2}}|-\rangle_x\right\}}$$

b) Aldiune jakin batean (t) elektroi horien spin momentu angeluararen z osagaia neurtzen dugu OZ ardatzaren norabideko SG dispositibo batez baliaturik. Zeintzuk izango dira neurketaren emaitza posibleak eta emaitza posible horiek lortzeko probabilitateak?

Denboraren garapena $|\pm\rangle$-en mende idatziko dugu:

Badakigu: $|+\rangle_x = \frac{1}{\sqrt{2}}\left(|+\rangle + |-\rangle\right)$, $|-\rangle_x = \frac{-1}{\sqrt{2}}\left(|+\rangle - |-\rangle\right)$

Gure egoera: $|\psi(t)\rangle = \alpha|+\rangle + \beta|-\rangle$

α eta β kalkulatu behar dira:

Escaneado con CamScanner

⑤. jarraipena

b) jarraipena

$|+\rangle$ egoeran:

$$\alpha = \langle +|\Psi(t)\rangle = \Big(|+\rangle, \left(\cos\frac{\theta}{2} + \sin\frac{\theta}{2}\right) e^{i\frac{\omega_0 t}{2}} \left(|+\rangle + |-\rangle\right) +$$

$$+ \left(\cos\frac{\theta}{2} - \sin\frac{\theta}{2}\right) e^{-i\frac{\omega_0 t}{2}} \left(|+\rangle - |-\rangle\right)\Big) \cdot \frac{1}{2}$$

$$\alpha = \frac{1}{2}\left(\left(\cos\frac{\theta}{2} + \sin\frac{\theta}{2}\right) e^{i\frac{\omega_0 t}{2}} + \left(\cos\frac{\theta}{2} - \sin\frac{\theta}{2}\right) e^{-i\frac{\omega_0 t}{2}}\right)$$

$$\alpha = \frac{1}{2}\left(\cos\frac{\theta}{2}\left(e^{i\frac{\omega_0 t}{2}} + e^{-i\frac{\omega_0 t}{2}}\right) + \sin\frac{\theta}{2}\left(e^{i\frac{\omega_0 t}{2}} - e^{-i\frac{\omega_0 t}{2}}\right)\right)$$

$$\alpha = \frac{1}{2}\left(2\cos\frac{\theta}{2}\cos\left(\frac{\omega_0 t}{2}\right) + 2i\sin\frac{\theta}{2}\sin\left(\frac{\omega_0 t}{2}\right)\right) \longrightarrow P(|+\rangle) = |\alpha|^2$$

Beraz:
$$\boxed{P(|+\rangle) = \cos^2\frac{\theta}{2}\cos^2\left(\frac{\omega_0 t}{2}\right) + \sin^2\frac{\theta}{2}\sin^2\left(\frac{\omega_0 t}{2}\right)}$$

$|-\rangle$ egoeran:

$$\beta = \langle -|\Psi(t)\rangle = \Big(|-\rangle, \left(\cos\frac{\theta}{2} + \sin\frac{\theta}{2}\right) e^{i\frac{\omega_0 t}{2}} \left(|+\rangle + |-\rangle\right) +$$

$$\left(\cos\frac{\theta}{2} - \sin\frac{\theta}{2}\right) e^{-i\frac{\omega_0 t}{2}} \left(|+\rangle - |-\rangle\right)\Big) \cdot \frac{1}{2}$$

$$\beta = \frac{1}{2}\left(\left(\cos\frac{\theta}{2} + \sin\frac{\theta}{2}\right) e^{i\frac{\omega_0 t}{2}} - \left(\cos\frac{\theta}{2} - \sin\frac{\theta}{2}\right) e^{-i\frac{\omega_0 t}{2}}\right)$$

$$\beta = \frac{1}{2}\left(\cos\frac{\theta}{2}\left(e^{i\frac{\omega_0 t}{2}} - e^{-i\frac{\omega_0 t}{2}}\right) + \sin\frac{\theta}{2}\left(e^{i\frac{\omega_0 t}{2}} + e^{-i\frac{\omega_0 t}{2}}\right)\right)$$

$$\beta = i\cos\frac{\theta}{2}\sin\left(\frac{\omega_0 t}{2}\right) + \sin\frac{\theta}{2}\cos\left(\frac{\omega_0 t}{2}\right) \longrightarrow P(|-\rangle) = |\beta|^2$$

Beraz:
$$\boxed{P(|-\rangle) = \cos^2\frac{\theta}{2}\sin^2\left(\frac{\omega_0 t}{2}\right) + \sin^2\frac{\theta}{2}\cos^2\left(\frac{\omega_0 t}{2}\right)}$$