# Final project

***Monte-Carlo Simulations for Finance - Matlab***

## Structured Product, Autocallables
## Phoenix Option Pricing

M2 104 - Research in Finance
Université Paris-Dauphine - PSL University

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# Table of contents

# Introduction

Structured products, such as Phoenix options, offer attractive features to enhance yield by combining customized risk profiles with contingent payments. Phoenix options, a variation of the autocallable family, stand out due to their periodic coupon payments and distinctive autocall trigger and coupon barriers, enabling flexible cash flow dynamics for investors. The Phoenix payoff structure introduces key characteristics, including the potential for coupons to 'revive' through a memory effect and exposure to a down-and-in put, providing a combination of defensive and yield-generating elements. These features make Phoenix options particularly appealing in diverse market environments.

This project focuses on implementing pricing, sensitivity analysis, and value-at-risk (VaR) calculations for a Phoenix option using Monte Carlo simulations. By modeling key parameters such as coupon barriers, autocall triggers, and maturity, our goal is to explore the product's behavior under various market scenarios and understand its risk-return profile. In doing so, we also highlight how Monte Carlo methods can effectively capture the complex interactions inherent in structured products like the Phoenix option.

# 1 Preliminary : Down-and-In Put Barrier Option (DIP)

## 1.1 Pricing of the DIP Option

Phoenix products include a down-and-in put barrier option, which activates when the underlying asset price falls below a specified barrier level. As a preliminary work, it can be useful to price this put option to understand its behavior and impact on the overall Phoenix option. The DIP option behaves similarly to a standard put option but has to be activated. To be activated, the underlying asset price must go below the barrier level at least once during the option's lifetime. If this condition is reached, the option becomes a standard European put option. Otherwise, it expires worthless. The payoff of a DIP option can be expressed as :

$$\Phi(S_T) = \begin{cases} (K - S_T)^+ & \text{if } S_t \leq B \text{ for some } t \in [0, T] \\ 0 & \text{otherwise} \end{cases}$$

where $S_T$ is the underlying asset price at maturity, $K$ is the strike price, and $B$ is the barrier level.

At first sight, it seems logical that a DIP option should be priced lower than a standard European put option as it gives the same payoff but with an additional condition.

As for the rest of the project, we will use Monte Carlo simulations to simulate the underlying asset price paths and calculate the payoff of the products. We will simulate $N_{MC}$ paths with $N$ time steps each.

To simulate our paths, we will use the Black-Scholes model for the underlying asset price dynamics :

$$dS_t = S_t(rdt + \sigma dW_t)$$

where $r$ is the risk-free rate, $\sigma$ is the volatility, and $W_t$ is a standard Brownian motion. moreover, we know :

$$S_{i+1} = S_i \exp((r - \frac{\sigma^2}{2})\Delta t + \sigma\sqrt{\Delta t} Z_i)$$

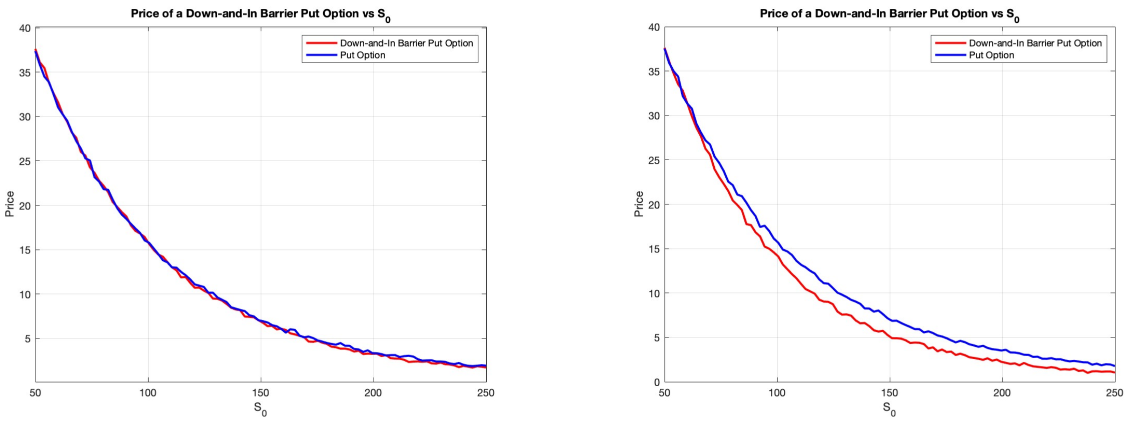
where $Z_i$ is a standard normal random variable.

To price the DIP option, for each monte carlo path, we will check if the barrier condition is met at least once. To do so we just check if the minimum of the path is below the barrier level. If it is, we will calculate the payoff of the put option at maturity. Otherwise, the payoff will be zero. Then we will calculate the average payoff of all the paths and discount it to get the price of the option. As a result for the following parameters, we get the following price for the DIP option :

- Initial stock price : $S_0 = 100$
- Strike price : $K = 100$
- Barrier level : $B = 80$
- Maturity : $T = 5$
- Risk-free rate : $r = 0.04$
- Volatility : $\sigma = 0.3$
- Number of Monte Carlo paths : $N_{MC} = 1000000$
- Number of time steps : $N = 100$

We get a price of 15.653 for the DIP option.

The payoff of the DIP option is represented in the figure ??. We compare it to the payoff of a standard European put option with the same parameters.




(a) Barrier at 80 (b) Barrier at 60

FIGURE 1 – Comparison of DIP and European Put Option Payoffs

Looking at the payoffs, for the barrier at 80, the payoffs of the DIP and European put options are almost identical. This is because the barrier level is too high, and the DIP option is almost always activated. However, for the barrier at 60, the DIP option's payoff is significantly lower than the European put option, as the barrier is less likely to be reached. Analysing the 60 barrier case, we see that for small values of the underlying, payoffs of both options are very close, the payoffs are close and high, close because the barrier is reached (even at time 0 when $S_0 \leq 60$) and high because the underlying is low. For very high values of the underlying, the payoffs are also close and low, close because the barrier is not reached and low because the underlying is high, making the European Put option worthless.

## 1.2 Greeks

To compute the greeks of the DIP option, we will use finite differences. We will focus on the Delta, the sensitivity of the option price to the underlying asset price.

$$\Delta = \frac{\partial V}{\partial S} = \frac{V(S+h) - V(S-h)}{2h}$$

To implement this, we will simulate the underlying asset price paths twice, once with $S+h$ and once with $S-h$ and calculate the option price for each path. Then we will calculate the average payoff for each path and use the formula above to get the Delta. We will have to take care that the random numbers used for the two simulations are the same to ensure that the only difference from the comes from the $S+h$ and $S-h$ values.

For the previous parameters, with h = 0.1 we get a Delta of $-0.264$ for the DIP option.

By computing the delta for differents values of $S_0$, we can plot the Delta as a function of the underlying asset price. This is represented in the figure 2.

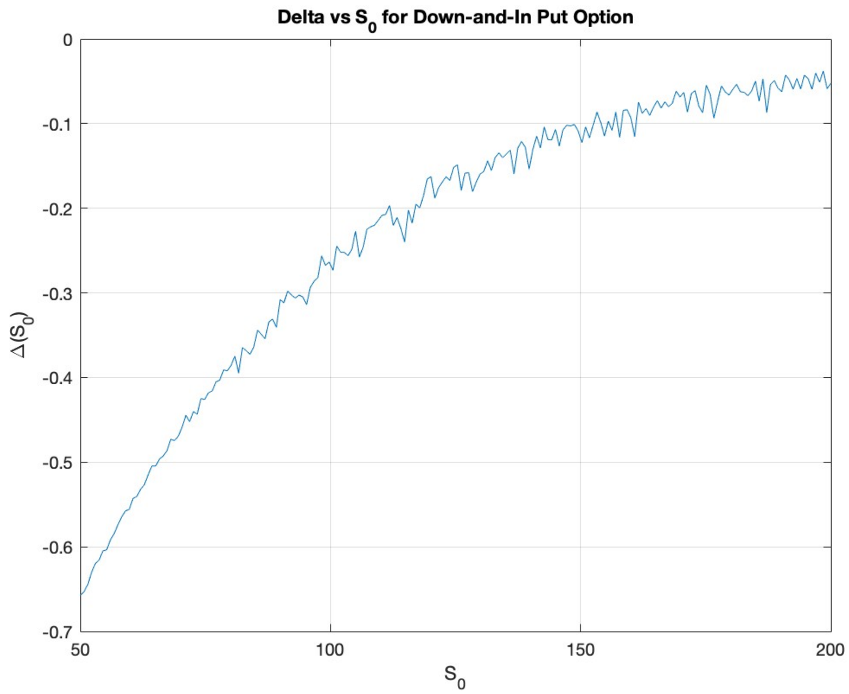



FIGURE 2 – Delta of the DIP Option as a Function of the Underlying Asset Price

We can see that the Delta is negative and decreases as the underlying asset price increases. This is consistent with the behavior of a put option, as the option price decreases when the underlying asset price rises.

# 2 Pricing of the Phoenix Option

## 2.1 Payoff of the Phoenix Option

The pricing for the Pheonix option is as follows :

$$\Phi(S_1, ..., S_N) = \begin{cases} \sum_{i=1}^{N-1} e^{-r(t_i - t_0)}(\Pi_0 + C_{\text{Ph}}) \cdot \mathbb{1}_{S_i > B_{\text{Ph}}} \cdot \mathbb{1}_{\max_{k\in\{1,\dots,i-1\}} S_k < B_{\text{Ph}}}, & \text{(A)} \\ + \sum_{i=1}^{N-1} e^{-r(t_i - t_0)} C_Y \cdot \mathbb{1}_{\max_{k\in\{1,\dots,i-1\}} S_k < B_{\text{Ph}}} \cdot \mathbb{1}_{S_i > B_Y}, & \text{(B)} \\ + e^{-r(t_N - t_0)}(\Pi_0 + C_{\text{Ph}}) \cdot \mathbb{1}_{\max_{k\in\{1,\dots,N-1\}} S_k < B_{\text{Ph}}} \cdot \mathbb{1}_{S_N > B_{\text{Ph}}}, & \text{(C)} \\ + e^{-r(t_N - t_0)}(\Pi_0 + C_Y) \cdot \mathbb{1}_{\max_{k\in\{1,\dots,N-1\}} S_k < B_{\text{Ph}}} \cdot \mathbb{1}_{B_Y < S_N < B_{\text{Ph}}}, & \text{(D)} \\ + e^{-r(t_N - t_0)}\Pi_0 \cdot \mathbb{1}_{\max_{k\in\{1,\dots,N-1\}} S_k < B_{\text{Ph}}} \cdot \mathbb{1}_{B_{\text{Put}} < S_N < B_Y}, & \text{(E)} \\ + e^{-r(t_N - t_0)} \max\left(\frac{K}{S_0} - \frac{S_N}{S_0}, 0\right) \cdot \mathbb{1}_{\max_{k\in\{1,\dots,N-1\}} S_k < B_{\text{Ph}}} \cdot \mathbb{1}_{S_N < B_{\text{Put}}}. & \text{(F)} \end{cases}$$

From an financial point of view, each of the terms of the payoff can be explain as follows :

— (A) : The autocall trigger is activated and the coupon is paid at each autocall date if the barrier is not reached.

— (B) : The autocall trigger is not activated but the coupon is paid at each autocall date if the barrier is not reached.

— (C) : The autocall trigger is activated and the coupon is paid at maturity if the barrier is not reached.

— (D) : The autocall trigger is not activated but the coupon is paid at maturity if the barrier is not reached.

- (E) : The autocall trigger is not activated and the barrier is not reached, the put option is activated.
- (F) : The autocall trigger is not activated and the barrier is not reached, the put option is not activated, the performance is calculated.

Dans le cas $S_N < B_{\text{Put}}$ (F), there exists another version that we will call performance

$$e^{-r(t_N-t_0)} S_N/S_0 \cdot \mathbb{1}_{\max_{k\in\{1,\ldots,N-1\}} S_k < B_{\text{Ph}}} \cdot \mathbb{1}_{S_N < B_{\text{Put}}}$$

**Algorithm 1** Payoff Calculation for a Phoenix Option

**Require:** $S$, $K$, $\Pi_0$, $C_{Ph}$, $C_Y$, $T$, $r$, $B_{Ph}$, $B_Y$, $B_{Put}$, $\Delta t$, $put\_or\_perf$
**Ensure:** $Payoff$

$timesteps \leftarrow \text{length}(S) - 1$
$t \leftarrow \Delta t \cdot (0 : timesteps)/timesteps \cdot T$
$PV \leftarrow 0$ ▷ Present Value
$autocall \leftarrow$ **false**
$observation\_date \leftarrow \text{round}(\text{linspace}(1, \text{length}(S), T/\Delta t + 1))$
**for** $i \in observation\_date$ **do**
    **if** $S[i] > B_{Ph}$ **then** ▷ Autocall trigger activated
        $PV \leftarrow (\Pi_0 + C_{Ph}) \cdot e^{-r\cdot t[i]}$
        $autocall \leftarrow$ **true**
        **break**
    **else if** $B_Y \leq S[i] \leq B_{Ph}$ **then** ▷ Coupon paid without autocall
        $PV \leftarrow PV + C_Y \cdot e^{-r\cdot t[i]}$
    **end if**
**end for**
**if** not $autocall$ **then**
    $S_N \leftarrow S[\text{end}]$
    **if** $S_N > B_{Ph}$ **then** ▷ Autocall trigger activated at maturity
        $PV \leftarrow PV + (\Pi_0 + C_{Ph}) \cdot e^{-r\cdot T}$
    **else if** $B_Y \leq S_N \leq B_{Ph}$ **then** ▷ Coupon paid at maturity without autocall
        $PV \leftarrow PV + (\Pi_0 + C_Y) \cdot e^{-r\cdot T}$
    **else if** $B_{Put} < S_N < B_Y$ **then** ▷ Put option activated
        $PV \leftarrow PV + \Pi_0 \cdot e^{-r\cdot T}$
    **else if** $S_N \leq B_{Put}$ **then** ▷ Performance calculated
        **if** $put\_or\_perf = 1$ **then**
            $Payoff \leftarrow \max\left(\frac{K-S_N}{S[1]}, 0\right)$ ▷ Put option
        **else**
            $Payoff \leftarrow \frac{S_N}{S[1]}$ ▷ Performance
        **end if**
        $PV \leftarrow PV + Payoff \cdot e^{-r\cdot T}$
    **end if**
**end if**
$Payoff \leftarrow PV$ ▷ Return the final present value

## 2.2 Pricing for a single value of $S_0$

First, we will compute the price of the Phoenix option for a single value $S0$, then we will plot the price of the Phoenix option as a function of $S0$. In this second step, we will study two cases, one with fixed barrier levels : the Barrier will be the same whatever the value of $S0$, and one with proportionnal barrier levels : the barrier will be a percentage of $S0$. For each case, we will study the impact of the payoff type for $S_N < B_{Put}$, we will compare the price of the Phoenix option with the put option and the performance. Even if we look at the underlying only $\Delta t$ times, it does not mean that the size of each path is $\Delta t$. Indeed, we chose to have $N$ time steps, for each of the $N_{MC}$ paths. This means that the size of the paths is $N \geq \Delta t$. One of the challenge of the Payoff function is to find the right time steps to observe the underlying. With the following parameters, we get the following price for the Phoenix option :

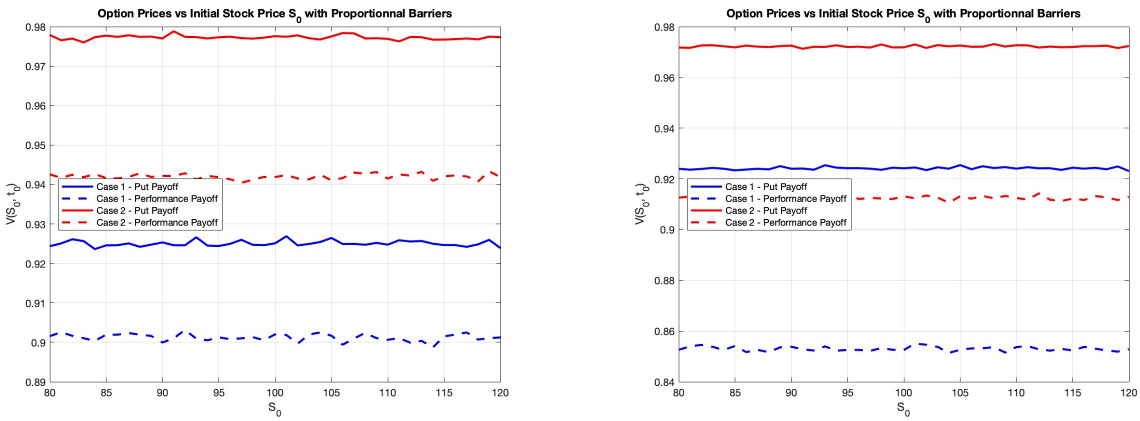
| **Parameter** | **Case 1.1** | **Case 1.2** | **Case 2.1** | **Case 2.2** |
|---|---|---|---|---|
| Initial Stock Price ($S_0$) | 100 | 100 | 100 | 100 |
| Strike Price ($K$) | 100 | 100 | 100 | 100 |
| Frequency of observatin ($\Delta t$) | 1 | 1 | 1 | 1 |
| Initial Product Value ($\Pi_0$) | 1 | 1 | 1 | 1 |
| Phoenix Coupon ($C_{Ph}$) | 0.1 | 0.1 | 0.08 | 0.08 |
| Yeti Coupon ($C_Y$) | 0.05 | 0.05 | 0.05 | 0.05 |
| Maturity ($T$) | 5 | 10 | 5 | 10 |
| Risk-free Rate ($r$) | 0.02 | 0.02 | 0.02 | 0.02 |
| Volatility ($\sigma$) | 0.3 | 0.3 | 0.3 | 0.3 |
| Number of Monte Carlo Paths ($N_{MC}$) | 10000 | 10000 | 10000 | 10000 |
| Number of Time Steps ($N$) | 100 | 100 | 100 | 100 |
| Barrier Levels ($B_{Ph}$, $B_Y$, $B_{Put}$) | (120, 80, 70) | (120, 80, 70) | (100, 70, 60) | (100, 70, 60) |
| **Price (Put Payoff)** | 0.927 | 0.922 | 0.976 | 0.972 |
| **Price (Performance Payoff)** | 0.904 | 0.856 | 0.938 | 0.911 |

TABLE 1 – Phoenix Option Prices for Different Parameters

We observe that the prices with the put payoff are higher than the prices with the performance payoff. This is because the put payoff is more valuable than the performance payoff when the underlying asset price is below the put barrier level ($B_{Put}$). Moreover, we observe that for a given maturity, the price of Phoenix option in the second case is higher than the price in the first case (second case correponds to lower barriers). This is because the autocall trigger is more likely to be activated with lower barriers, leading to more frequent coupon payments. Finally, we see that prices increases with the maturity, this is because the autocall trigger is more likely to be activated with a longer maturity.

## 2.3 Pricing for multiple values of $S_0$, plotting the price as a function of $S_0$

In this section, we will study 2 cases, one with fixed barrier levels and one with proportionnal barrier levels. We will plot the price of the Phoenix option as a function of $S_0$ for each case. We will also study the impact of the payoff type for $S_N < B_{Put}$, we will compare the price of the Phoenix option with the put option and the performance.

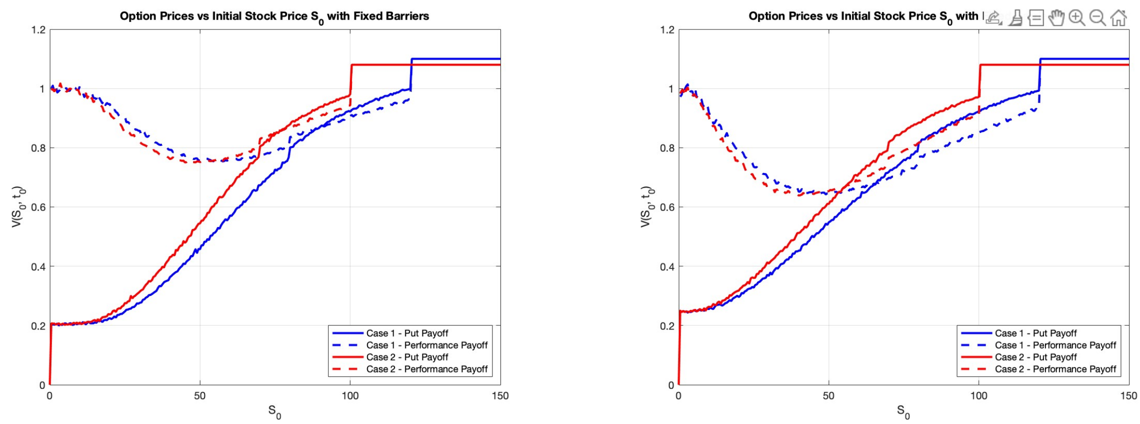



(a) $T = 5$ (b) $T = 10$

FIGURE 3 – Phoenix Option Price as a Function of $S_0$, with Proportional Barrier Levels : Case 1 : $B_{Ph} = 120\% S_0$, $B_Y = 80\% S_0$, $B_{Put} = 70\% S_0$, Case 2 : $B_{Ph} = 100\% S_0$, $B_Y = 70\% S_0$, $B_{Put} = 60\% S_0$

We can cleary notice that the price of the option remains constant regarding $S_0$, and for each case of the barrier levels. It confirms what we noticed on the previous section : the price with the performance payoff is lower than the price with the put payoff. Moreover, we can see that the price of the Phoenix option increases with the maturity. It is not surprising that the price remains constant with respect to $S_0$ since the barrier levels are proportionnal to $S_0$.

The second case is dedicated to fixed barrier levels. We will plot the price of the Phoenix option as a function of $S0$ for each case.




(a) $T = 5$ (b) $T = 10$

FIGURE 4 – Phoenix Option Price as a Function of $S_0$, with Fixed Barrier Levels : Case 1 : $B_{Ph} = 120$, $B_Y = 80$, $B_{Put} = 70$, Case 2 : $B_{Ph} = 100$, $B_Y = 70$, $B_{Put} = 60$

The graphs illustrate the evolution of Phoenix option prices as a function of $S_0$, the initial underlying price, for fixed barriers and two maturities ($T = 5$ and $T = 10$). Generally, prices increase with $S_0$ since a higher $S_0$ reduces the likelihood of hitting unfavorable barriers ($B_{Put}$) and increases the chances of reaching $B_{Ph}$ (autocall), making the option more attractive.

One of the first thing we notice is the curvature aspect of the performance payoff and the linear aspect of the put payoff. For low values of $S_0$, the performance payoff is higher than the put payoff. Indeed, with low values of $S_0$, the put barrier is more likely to be reached, and the Performance payoff is higher.

Regarding maturity, we do not notice particular differences between the two maturities. Comparing Case 1 (higher barriers : $B_{Ph} = 120$, $B_Y = 80$, $B_{Put} = 70$) and Case 2 (lower barriers : $B_{Ph} = 100$, $B_Y = 70$, $B_{Put} = 60$), we see that higher barriers make the option less likely to reach favorable levels but also less vulnerable to breaching $B_{Put}$. Lower barriers make the option more sensitive to moderate variations in $S_0$, with more reactive curves.

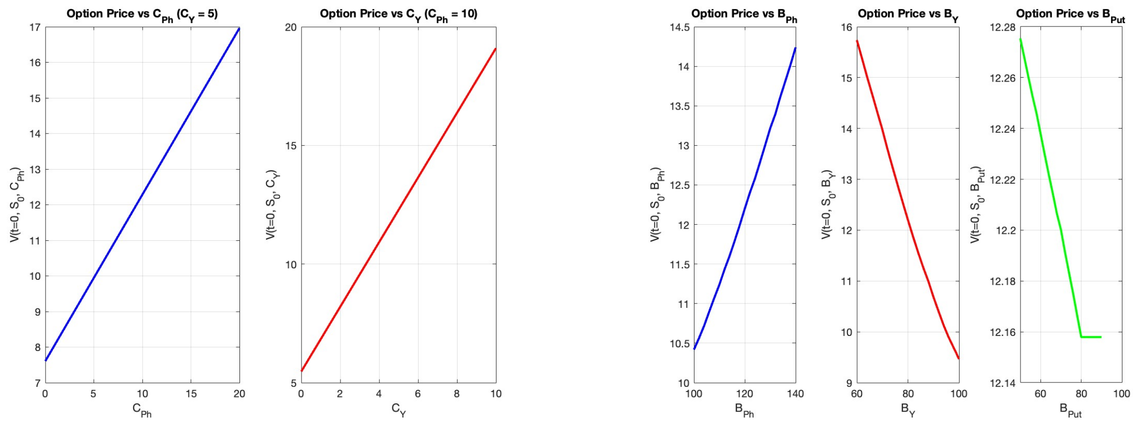
# 3 Sensitivity Analysis

## 3.1 Option Parameters

Two scripts were used to analyze the sensitivity of the Phoenix option price to its parameters :

— `Sensitivity_coupons.m` : Examines the impact of changing the coupons ($C_{Ph}$, $C_Y$) on the option price.

— `Sensitivity_barriers.m` : Investigates the effects of varying the barrier levels ($B_{Ph}, B_Y, B_{Put}$).




(a) Sensitivity to Coupons

(b) Sensitivity to Barriers

FIGURE 5 – Option Price Sensitivity to Coupons and Barriers

The graphs analyzing the sensitivity to coupons $C_{Ph}$ (Phoenix coupon) and $C_Y$ (Yeti coupon) show that the option price increases linearly with both parameters. This behavior is intuitive since higher coupon values directly enhance the payoffs, thereby increasing the option's overall present value. Importantly, the impact of $C_{Ph}$ is more significant than $C_Y$ because $C_{Ph}$ is linked to the autocall trigger, which occurs under favorable market conditions, while $C_Y$ represents a more frequent, albeit smaller, payout.

The sensitivity to barrier levels ($B_{Ph}, B_Y, B_{Put}$) reveals a convex relationship between the option price and the barrier values. For $B_{Ph}$ (autocall barrier), the option price increases as $B_{Ph}$ rises, indicating that the autocall condition becomes harder to trigger, prolonging the option's life and potentially enhancing its value. Conversely, the option price decreases with lower $B_Y$ (coupon barrier) and $B_{Put}$ (put barrier), as these barriers increase the likelihood of intermediate payouts or defensive mechanisms, limiting the option's upside potential. Notably, the graph for Option Price vs $B_{Put}$ resembles a put payoff structure, reflecting the down-and-in put activation, though the price does not converge to zero due to the remaining value of the option under default conditions.

## 3.2 Market Parameters

The script `Sensitivity_sigma_r_T.m` analyzes the sensitivity of the option price to :

— Volatility ($\sigma$)

— Maturity ($T$)

— Risk-free rate ($r$)

These results provide insights into how market conditions influence the option's value.

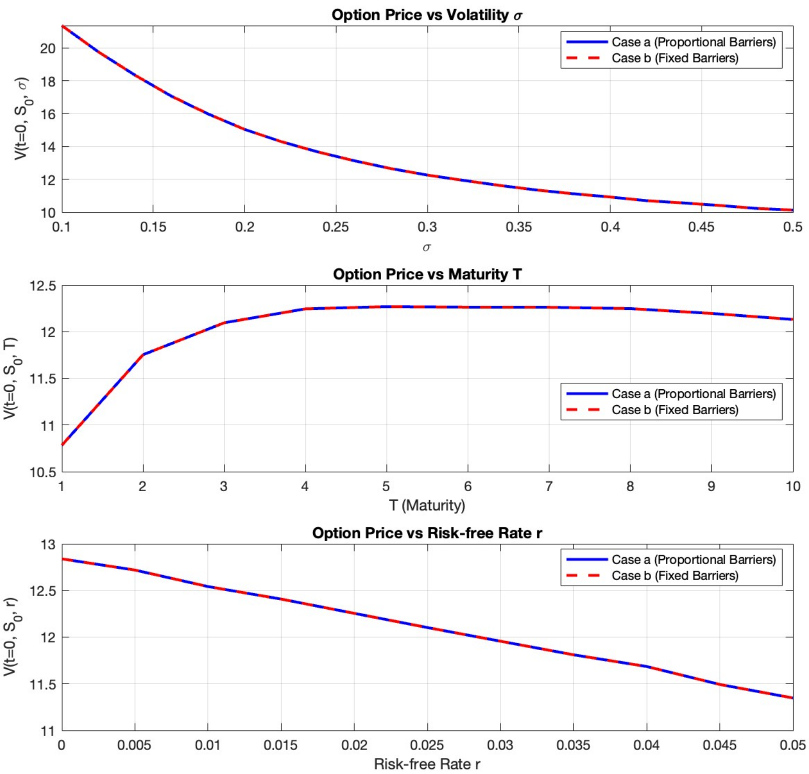



FIGURE 6 – Option Price Sensitivity to Market Parameters $(\sigma, T, r)$

The Option Price vs Volatility $\sigma$ graph shows a decreasing relationship between volatility and the price of the Phoenix option for both barrier configurations (proportional and fixed). This relationship can be explained by the fact that an increase in $\sigma$ amplifies the uncertainty surrounding the future trajectories of the underlying asset, making it less likely to hit the autocall barriers. The higher risk of underperformance reduces the option's value due to increased potential losses for the investor. Both configurations yield nearly identical results because volatility affects the fixed and proportional barriers proportionally.

The Option Price vs Maturity $T$ graph exhibits a nonlinear relationship : the price increases rapidly for short maturities and then tends to stabilize for longer maturities. This behavior arises because, over a short period, the coupons and autocall barriers play a crucial role in pricing, as reaching the thresholds triggers the early redemption of the option. For longer maturities, the likelihood of the underlying asset reaching the barriers decreases, leading to a stabilization of the product's value. Once again, the curves for fixed and proportional barriers remain very close, indicating a similar impact of maturity in both cases.

The Option Price vs Risk-free Rate $r$ graph reveals a decreasing relationship between the risk-free rate and the option price. An increase in $r$ reduces the present value of future cash flows, which mechanically lowers the option's value. Since the coupons and barriers are discounted less significantly at higher rates, the impact is particularly noticeable for longer maturities. Additionally, the proportional and fixed barrier configurations produce almost identical results, confirming that discounting applies uniformly to both structures.

These sensitivities to market parameters confirm that the price of the Phoenix option is highly dependent on volatility, the product's lifespan, and interest rate conditions. The similar shape of the curves for both barrier configurations indicates that, although the barrier structures differ, their impact remains consistent in response to changes in macroeconomic parameters.

## 3.3 Greeks

The script `Sensitivity_greeks.m` computes the following Greeks using finite differences :

- **Delta ($\Delta$)** : Sensitivity to changes in the underlying asset price.
- **Gamma ($\Gamma$)** : Sensitivity of Delta to changes in the underlying asset price.
- **Vega** : Sensitivity to changes in volatility ($\sigma$).

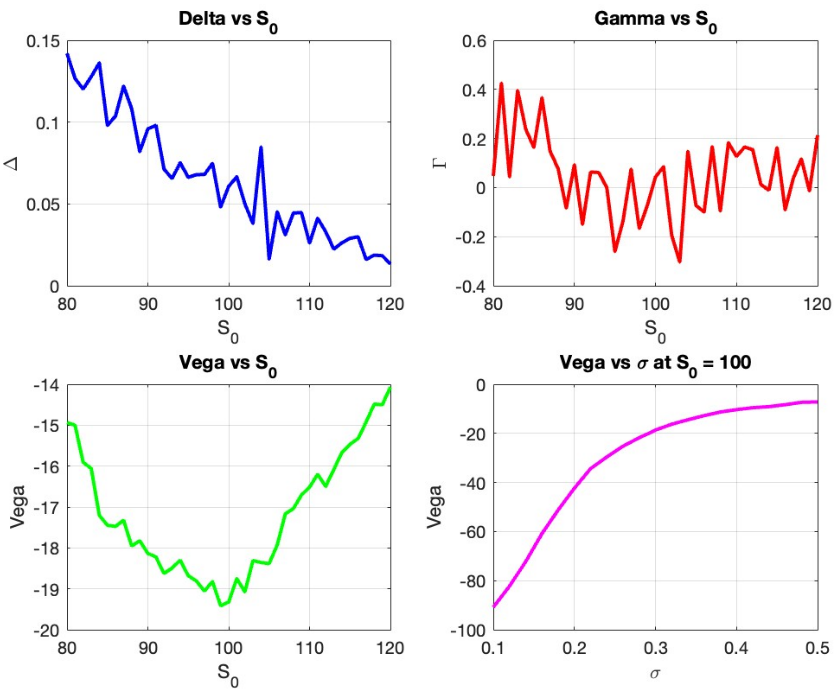



FIGURE 7 – Computed Greeks : Delta, Gamma, and Vega

The Delta graph shows a decreasing relationship between Delta and the initial stock price $S_0$ . Delta, which measures the sensitivity of the option's price to changes in the underlying asset price, is positive but decreases as $S_0$ rises. This behavior can be explained by the nature of the Phoenix option payoff : when $S_0$ increases significantly, the probability of triggering the autocallable barrier $B_{Ph}$ grows, reducing the additional gains from further increases in $S_0$ . As a result, the sensitivity to $S_0$ diminishes. The slight fluctuations observed near $S_0 \approx 100$ reflect the complex structure of the option, where coupon and barrier effects interact dynamically.

The Gamma graph exhibits erratic behavior, oscillating between positive and negative values as $S_0$ increases. Gamma, which represents the rate of change of Delta with respect to $S_0$ , captures the curvature of the option price with respect to changes in the underlying price. The irregular shape stems from the discrete nature of the barriers and the payoff structure.

The Vega graph demonstrates a U-shaped relationship with $S_0$ , where Vega is minimized near $S_0 \approx 100$ and increases for lower and higher values of $S_0$ . Vega measures the sensitivity of the option price to changes in volatility $\sigma$. At $S_0 \approx 100$ , the option price is less sensitive to volatility because the probabilities of barrier activation (autocallable or put barriers) are relatively balanced. However, for very high or low $S_0$ , the probability of hitting specific barriers becomes more sensitive to volatility, leading to a higher Vega.

The Vega vs Volatility graph shows that Vega decreases as $\sigma$ increases. Initially, Vega is high for low volatility, as small changes in $\sigma$ significantly impact the option price due to the uncertainty surrounding barrier activation. However, as volatility increases, the price impact of further increases in $\sigma$ diminishes, resulting in a concave shape. This behavior highlights the diminishing marginal effect of volatility changes on the option price at higher volatility levels.

In conclusion, the Greeks analysis reveals the Phoenix option's sensitivities to changes in $S_0$ and $\sigma$ . Delta decreases with rising $S_0$ , while Gamma shows volatility around critical ranges, reflecting the barrier structure's complexity. Vega exhibits a U-shaped relationship with $S_0$ and decreases with increasing $\sigma$ . These findings underscore the importance of monitoring the underlying asset price and volatility for effective hedging and risk management strategies.

# 4 Value at Risk (VaR)

The Value at Risk (VaR) analysis quantifies potential losses with a given confidence level $(1 - \alpha)$. The script `VaR.m` performs the following tasks :

— Simulate the distribution of $V_T - V_0$ for both the Put and Performance payoff scenarios.

— Calculate the quantile-based VaR for $\alpha = 10\%$ and $\alpha = 1\%$.

The empirical cumulative distribution functions (CDFs) below highlight the VaR levels for each confidence level.

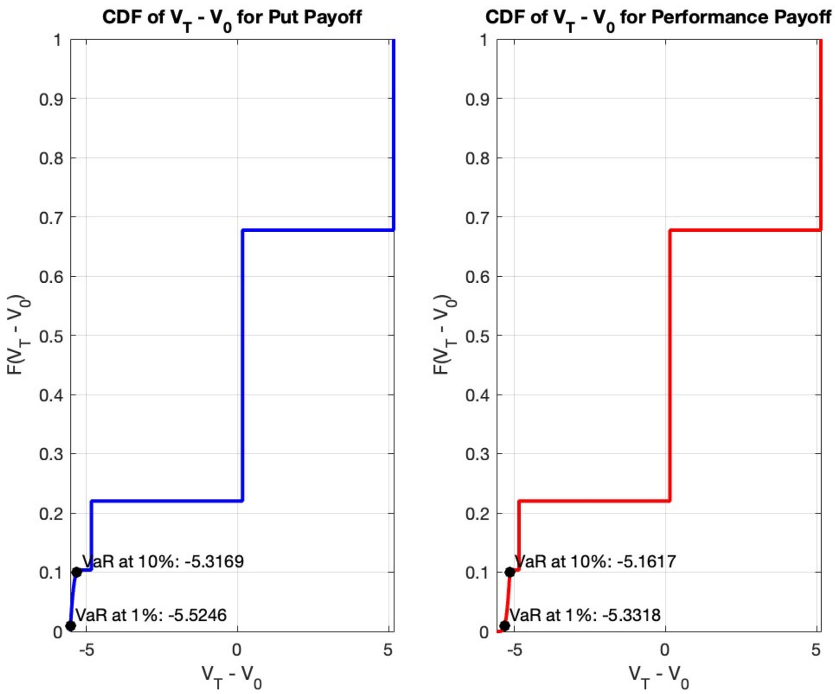



FIGURE 8 – Cumulative Distribution Functions (CDF) for Put and Performance Payoffs

The CDF of $V_T - V_0$ for the Put Payoff (left graph) demonstrates the tail risk, where the Value at Risk for $\alpha = 10\%$ and $\alpha = 1\%$ are approximately -5.3169 and -5.5246, respectively. This indicates that there is a 10% probability of losing more than 5.3169 units and a 1% probability of losing more than 5.5246 units. The Put Payoff reflects the protection feature of the option, which becomes active when the underlying price breaches the lower barrier.

For the Performance Payoff (right graph), the VaR values are slightly less negative : -5.1617 for $\alpha = 10\%$ and -5.3318 for $\alpha = 1\%$ . This suggests that the Performance Payoff structure reduces the tail risk compared to the Put Payoff. This behavior can be attributed to the contingent coupon payments that provide additional cashflows when the underlying asset remains above certain barriers, thereby mitigating the downside exposure.

The similarity in the shape of both CDFs highlights that the Phoenix option retains some risk symmetry across payoff structures, but the Performance Payoff structure offers slightly better protection against extreme losses. This demonstrates the trade-off between defensive features and yield enhancement within the Phoenix product. While the Put Payoff exposes investors to slightly higher downside risk, the Performance Payoff mitigates losses through periodic cashflows.

# Conclusion

The Phoenix option pricing project has provided a comprehensive framework for understanding and evaluating a structured autocallable product using Monte Carlo simulations. By incorporating pricing, sensitivity analysis, and Value at Risk (VaR) estimation, we have illustrated the versatility and complexity of Phoenix options.

## Key Insights

- **Pricing Analysis :**
  The comparison between fixed and proportional barriers highlighted distinct behaviors in the option's price evolution. Proportional barriers exhibit a linear relationship with the initial stock price, while fixed barriers result in a concave price curve. This underlines the importance of barrier structure in determining the option's valuation.
- **Sensitivity Analysis :**
  - *Option Parameters :* Variations in coupons and barrier levels showed that option prices are convex in both parameters. The influence of $B_{Put}$ closely mirrors a put payoff structure, emphasizing the significance of this barrier.
  - *Market Parameters :* The option price demonstrated strong sensitivity to volatility, maturity, and interest rates. Volatility increases led to price reductions due to higher uncertainty, while maturity induced a stabilizing effect after an initial rise. The impact of interest rates revealed a clear discounting mechanism, reducing the value of future cash flows.

  The consistent behavior observed between fixed and proportional barriers confirms that market conditions act uniformly across both configurations.
- **Risk Assessment with VaR :**
  The Value at Risk analysis quantified the potential losses for the Phoenix option under both put and performance payoffs. The empirical CDFs showcased the tail risks at confidence levels of 90% and 99%, providing critical insights into the product's risk profile. While the performance payoff demonstrated slightly lower tail risks compared to the put payoff, both highlighted the inherent exposure of the option to adverse market movements.

## Final Remarks

The Phoenix option, with its combination of autocall triggers, coupon payments, and memory effects, remains a compelling structured product for yield enhancement in volatile markets. However, its valuation is highly sensitive to market parameters and structural choices, requiring meticulous modeling and risk management. Monte Carlo simulations proved to be an effective tool for capturing these dynamics, providing a robust methodology for pricing, sensitivity analysis, and risk assessment.

Future research could explore more advanced pricing techniques, such as variance reduction methods or alternative stochastic models, to improve computational efficiency and accuracy. Additionally, analyzing the Phoenix option under real-world constraints, such as transaction costs or liquidity risks, could further enhance its practical applicability in portfolio management.

# Bibliography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