Test

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CURRENT ISSUES:

- Replacement issues
- ISSUES WITH NEGATIVES!!!!

QUESTION — should outside brackets have to be

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????

$$\partial_{\mu} A \Big(Y \partial_{\nu} h^{\mu\nu} + X \partial^{\mu} h^{\nu}_{\nu} \Big) \tag{1}$$

$$\partial_{\mu} \Big(Y A \partial_{\nu} h^{\mu\nu} + X A \partial^{\mu} h^{\nu}_{\nu} \Big) \tag{2}$$

Contract:

$$\partial_{\mu} A \Big(Y \partial_{\nu} h^{\mu\nu} + X \partial^{\mu} h^{\nu}_{\nu} \Big) \tag{3}$$

1 Representation

Reading in and understanding equations (display initial equation)

Reading in signs between brackets

$$(a) + (b) \rightarrow (a) + (b)$$
 (4)

$$(a) - (b) \rightarrow (a) - (b)$$
 (5)

basic reading in signs inside brackets

$$b_{\gamma} - g_j \to \left(b_{\gamma} - g_j\right) \tag{6}$$

inside and outside brackets

$$A^{\mu} + \left(A^{\gamma} - B^{\zeta}\right) \to A^{\mu} + \left(A^{\gamma} - B^{\zeta}\right) \tag{7}$$

$$A^{\mu} - \left(A^{\gamma} - B^{\zeta}\right) \to A^{\mu} - \left(A^{\gamma} - B^{\zeta}\right) \tag{8}$$

with partial derivatives

$$\partial_{\alpha} \Big(a + \Big(b_{\gamma} - g \Big) \Big) \to \partial_{\alpha} \Big(a + \Big(b_{\gamma} - g \Big) \Big)$$
 (9)

multiplied by a tensor

$$A^{\mu} + C^{\epsilon} \Big(A^{\gamma} - B^{\zeta} \Big) \to A^{\mu} + C^{\epsilon} \Big(A^{\gamma} - B^{\zeta} \Big)$$
 (10)

complicated bracket nesting

$$\left(\left(a\right) - \left(b - \left(c\right)\right)\right) + \left(\left(a - d\right) - b\right) \to \left(\left(a\right) - \left(b - \left(c\right)\right)\right) + \left(\left(a - d\right) - b\right) \tag{11}$$

complicated bracket nesting and tensor indices

$$\left(\left(a\right) - \left(b - \left(c\right)\right)\right) + \left(\left(a^{\gamma} - d^{\gamma}\right) - b\right) \to \left(\left(a\right) - \left(b - \left(c\right)\right)\right) + \left(\left(a^{\gamma} - d^{\gamma}\right) - b\right) \tag{12}$$

complicated bracket nesting and tensor indices, multiplication, and partials

$$\partial_{\zeta} \left(G^{\gamma} \right) \partial^{\zeta} \Box \left(\left(A \right) - \left(B_{\kappa}^{\kappa} - \left(C \right) \right) \right) + \left(\left(A^{\gamma} - D^{\gamma} \right) - B_{\alpha}^{\alpha \gamma} \right) \\ \rightarrow \partial_{\zeta} \left(G^{\gamma} \right) \partial^{\zeta} \Box \left(\left(A \right) - \left(B_{\kappa}^{\kappa} - \left(C \right) \right) \right) + \left(\left(A^{\gamma} - D^{\gamma} \right) - B_{\alpha}^{\alpha \gamma} \right)$$
(13)

with equals sign

$$G^{\mu} = A^{\mu} + C^{\mu\epsilon} \left(A_{\epsilon} - B_{\epsilon} \right) \to G^{\mu} = A^{\mu} + C^{\mu\epsilon} \left(A_{\epsilon} - B_{\epsilon} \right) \tag{14}$$

with begin equation command

```
\begin{equation}
G^{\mu} = A^{\mu} + C^{\mu} \exp(A_{\epsilon}) - B_{\epsilon}
\end{equation}
\begin{equation}
G^{\mathbb{T}} = A^{\mathbb{T}} + C^{\mathbb{T}} + C^{\mathbb{T}} 
\end{equation}
   with begin multline command (but short so it changes to equation)
\begin{multline}
G^{\mu} = A^{\mu} + C^{\mu} + C^{\mu} + C^{\mu}
\end{multline}
->
\begin{equation}
G^{\mu} = A^{\mu} + C^{\mu} \cdot A_{\phi} - B_{\phi} \cdot A_{\phi}
\end{equation}
   with begin multline command and long enough to stay multline
\begin{multline}
G^{\mathbb{T}} = A^{\mathbb{T}} + C^{\mathbb{T}} (A_{\epsilon}) - B_{\epsilon} + A^{\mathbb{T}} + A^{\mathbb{T}}
+ C^{\mu u \epsilon}(A_{\epsilon}) - B_{\epsilon}(mu) + A^{\mu \epsilon}(mu)
+ C^{\mu \epsilon}\(A_{\epsilon} - B_{\epsilon}\) + A^{\mu}
+ C^{\mu u \epsilon}(A_{\epsilon}) - B_{\epsilon}(n) + A^{\mu}
+\\ C^{\mu u \epsilon}(A_{\epsilon}) - B_{\epsilon}(n) + A^{\mu}
+ C^{\mu u \epsilon}(A_{\epsilon}) - B_{\epsilon}(mu) + A^{\mu \epsilon}
+ C^{\mu \nu \epsilon}(A_{\epsilon}) - B_{\epsilon}(mu) + A^{\mu \epsilon}(mu)
+ C^{\mu u \epsilon}(A_{\epsilon}) - B_{\epsilon}(mu) + A^{\mu \epsilon}
+\ C^{\mu u \epsilon}(A_{\epsilon}) - B_{\epsilon}(n) + A^{\mu}
+ C^{\mu u \epsilon}(A_{\epsilon}) - B_{\epsilon}(mu) + A^{\mu \epsilon}
+ C^{\mu \epsilon}\(A_{\epsilon} - B_{\epsilon}\)
\end{multline}
->
\begin{multline}
G^{\mathbb{T}} = A^{\mathbb{T}} + C^{\mathbb{T}} + C^{\mathbb{T}} + C^{\mathbb{T}} + C^{\mathbb{T}} + C^{\mathbb{T}} + C^{\mathbb{T}}
+ C^{\mu u \epsilon} \ (A_{\epsilon}) - B_{\epsilon} \ )+ A^{\mu \epsilon}
+ C^{\mu \epsilon} \( A_{\epsilon} - B_{\epsilon} \)+ A^{\mu} \\
+ C^{\mu \epsilon} \( A_{\epsilon} - B_{\epsilon} \)+ A^{\mu}
+ C^{\mu u \epsilon} \ (A_{\epsilon} - B_{\epsilon}) + A^{\mu \epsilon} 
+ C^{\mu u \epsilon} \ (A_{\epsilon} - B_{\epsilon} ) + A^{\mu \epsilon} \ (A_{\epsilon} \ C^{\mu u \epsilon} )
+ C^{\mu u \epsilon} \ (A_{\epsilon}) - B_{\epsilon} \ )+ A^{\mu \epsilon}
+ C^{\mu \epsilon} \( A_{\epsilon} - B_{\epsilon} \)+ A^{\mu}
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+ $C^{\mu u \epsilon} \ (A_{\epsilon} - B_{\epsilon}) + A^{\mu \epsilon} \ (A_{\epsilon} \ C^{\mu u \epsilon})$

+ C^{\mu \epsilon} \(A_{\epsilon} - B_{\epsilon} \) + A^{\mu}
+ C^{\mu \epsilon} \(A_{\epsilon} - B_{\epsilon} \)
\end{\multline}

also print above to test what it looks like visually: output is

$$G^{\mu} = A^{\mu} + C^{\mu\epsilon} \left(A_{\epsilon} - B_{\epsilon} \right) + A^{\mu} + C^{\mu\epsilon} \left(A_{\epsilon} - B_{\epsilon} \right) + A^{\mu} + C^{\mu\epsilon} \left(A_{\epsilon} - B_{\epsilon} \right) + A^{\mu}$$

$$+ C^{\mu\epsilon} \left(A_{\epsilon} - B_{\epsilon} \right) + A^{\mu} + C^{\mu\epsilon} \left(A_{\epsilon} - B_{\epsilon} \right) + A^{\mu} + C^{\mu\epsilon} \left(A_{\epsilon} - B_{\epsilon} \right) + A^{\mu}$$

$$+ C^{\mu\epsilon} \left(A_{\epsilon} - B_{\epsilon} \right) + A^{\mu} + C^{\mu\epsilon} \left(A_{\epsilon} - B_{\epsilon} \right) + A^{\mu} + C^{\mu\epsilon} \left(A_{\epsilon} - B_{\epsilon} \right) + A^{\mu}$$

$$+ C^{\mu\epsilon} \left(A_{\epsilon} - B_{\epsilon} \right) + A^{\mu} + C^{\mu\epsilon} \left(A_{\epsilon} - B_{\epsilon} \right)$$

$$+ C^{\mu\epsilon} \left(A_{\epsilon} - B_{\epsilon} \right) + A^{\mu} + C^{\mu\epsilon} \left(A_{\epsilon} - B_{\epsilon} \right)$$

$$+ C^{\mu\epsilon} \left(A_{\epsilon} - B_{\epsilon} \right) + A^{\mu} + C^{\mu\epsilon} \left(A_{\epsilon} - B_{\epsilon} \right)$$

$$+ C^{\mu\epsilon} \left(A_{\epsilon} - B_{\epsilon} \right) + A^{\mu} + C^{\mu\epsilon} \left(A_{\epsilon} - B_{\epsilon} \right)$$

$$+ C^{\mu\epsilon} \left(A_{\epsilon} - B_{\epsilon} \right) + A^{\mu} + C^{\mu\epsilon} \left(A_{\epsilon} - B_{\epsilon} \right)$$

$$+ C^{\mu\epsilon} \left(A_{\epsilon} - B_{\epsilon} \right) + A^{\mu} + C^{\mu\epsilon} \left(A_{\epsilon} - B_{\epsilon} \right)$$

$$+ C^{\mu\epsilon} \left(A_{\epsilon} - B_{\epsilon} \right) + A^{\mu} + C^{\mu\epsilon} \left(A_{\epsilon} - B_{\epsilon} \right)$$

$$+ C^{\mu\epsilon} \left(A_{\epsilon} - B_{\epsilon} \right) + A^{\mu} + C^{\mu\epsilon} \left(A_{\epsilon} - B_{\epsilon} \right)$$

$$+ C^{\mu\epsilon} \left(A_{\epsilon} - B_{\epsilon} \right) + A^{\mu} + C^{\mu\epsilon} \left(A_{\epsilon} - B_{\epsilon} \right)$$

$$+ C^{\mu\epsilon} \left(A_{\epsilon} - B_{\epsilon} \right) + A^{\mu} + C^{\mu\epsilon} \left(A_{\epsilon} - B_{\epsilon} \right)$$

$$+ C^{\mu\epsilon} \left(A_{\epsilon} - B_{\epsilon} \right) + A^{\mu} + C^{\mu\epsilon} \left(A_{\epsilon} - B_{\epsilon} \right)$$

$$+ C^{\mu\epsilon} \left(A_{\epsilon} - B_{\epsilon} \right) + A^{\mu} + C^{\mu\epsilon} \left(A_{\epsilon} - B_{\epsilon} \right)$$

$$+ C^{\mu\epsilon} \left(A_{\epsilon} - B_{\epsilon} \right) + A^{\mu} + C^{\mu\epsilon} \left(A_{\epsilon} - B_{\epsilon} \right)$$

$$+ C^{\mu\epsilon} \left(A_{\epsilon} - B_{\epsilon} \right) + A^{\mu} + C^{\mu\epsilon} \left(A_{\epsilon} - B_{\epsilon} \right)$$

$$+ C^{\mu\epsilon} \left(A_{\epsilon} - B_{\epsilon} \right) + A^{\mu} + C^{\mu\epsilon} \left(A_{\epsilon} - B_{\epsilon} \right)$$

$$+ C^{\mu\epsilon} \left(A_{\epsilon} - B_{\epsilon} \right) + A^{\mu} + C^{\mu\epsilon} \left(A_{\epsilon} - B_{\epsilon} \right)$$

$$+ C^{\mu\epsilon} \left(A_{\epsilon} - B_{\epsilon} \right) + A^{\mu} + C^{\mu\epsilon} \left(A_{\epsilon} - B_{\epsilon} \right)$$

each line could be longer before wrapping but I'm pretty happy with how it looks for now

with etas and deltas and fractions

$$7X\eta_{\mu\nu}\delta^{\gamma}_{\nu}\Box A^{\mu} + \frac{1}{3}\delta^{\gamma}_{\mu}C^{\mu\epsilon}\left(A_{\epsilon} - B_{\epsilon}\right) \to 7X\delta^{\gamma}_{\nu}\eta_{\mu\nu}\Box A^{\mu} + \frac{1}{3}\delta^{\gamma}_{\mu}C^{\mu\epsilon}\left(A_{\epsilon} - B_{\epsilon}\right) \tag{16}$$

multiple coefficients

$$\left(XYZ\partial_{\nu}h^{\mu\nu} + 6bX\partial^{\mu}h^{\nu}_{\nu}\right) \to \left(XYZ\partial_{\nu}h^{\mu\nu} + 6bX\partial^{\mu}h^{\nu}_{\nu}\right) \tag{17}$$

multiple coefficients in unusual order

$$\left(XY\partial_{\nu}h^{\mu\nu}Z + b6X\partial^{\mu}h^{\nu}_{\nu}\right) \to \left(XYZ\partial_{\nu}h^{\mu\nu} + 6bX\partial^{\mu}h^{\nu}_{\nu}\right) \tag{18}$$

breaks (is this good or no??)

$$A^{\alpha} \frac{1}{3} B_{\gamma} \to \text{ Equation: pop from empty list}$$
 (19)

this one seems to work...

$$A^{\alpha}3B_{\gamma} \to 3A^{\alpha}B_{\gamma}$$
 (20)

QUESTION: does multiplication always need brackets? If no, when does it need them? Why does the whole number work but the fraction breaks....

Please email jbrucero@uwo.ca for if you think this is a bug

****things that shouldn't work
*** uneven brackets

 $\Big(A+B \to WORKING (though the message could be greatly improved)" Equation: Please email jbrucero@uwo.ca for if you think the context of the provided in the provided$

*** uneven index brackets

 $A^{\{}\gamma \rightarrow Working in that its hould error but again the issue could probably be caught earlier and abetter message given)$ "Equation: string (22)

$$A^{\gamma} + B^{\{} \rightarrow thisonereads the equation fine - is this anissue???(A^{\gamma} + B)$$
 (23)

2 Multiply

2.1 FOIL out terms, distributing derivatives when necessary (recommended)

Basic case:

$$(A)\partial_{\mu}(Y\partial_{\nu}h^{\mu\nu} + X\partial^{\mu}h^{\nu}_{\nu}) \to (YA\partial_{\nu}\partial_{\mu}h^{\mu\nu} + XA\partial^{\mu}\partial_{\mu}h^{\nu}_{\nu})$$
(24)

Also need to test adding summations, with and without partials, and subtracting summations, with and without partials

2.2 distribute partial derivatives

Basic case:

$$\partial_{\mu} \left(Y \partial_{\nu} h^{\mu\nu} + X \partial^{\mu} h^{\nu}_{\nu} \right) \to \left(Y \partial_{\nu} \partial_{\mu} h^{\mu\nu} + X \partial^{\mu} \partial_{\mu} h^{\nu}_{\nu} \right) \tag{25}$$

** number coefficients

$$\partial_{\mu} \left(5\partial_{\nu} h^{\mu\nu} + 3\partial^{\mu} h^{\nu}_{\nu} \right) \to \left(5\partial_{\nu} \partial_{\mu} h^{\mu\nu} + 3\partial^{\mu} \partial_{\mu} h^{\nu}_{\nu} \right) \tag{26}$$

** mixed coefficients

$$\partial_{\mu} \left(5\partial_{\nu} h^{\mu\nu} + V \partial^{\mu} h^{\nu}_{\nu} \right) \to \left(5\partial_{\nu} \partial_{\mu} h^{\mu\nu} + V \partial^{\mu} \partial_{\mu} h^{\nu}_{\nu} \right) \tag{27}$$

** product rule

$$\partial_{\mu} \left(A^{\gamma} B^{\zeta} \right) \to \left(\partial_{\mu} A^{\gamma} B^{\zeta} + \partial_{\mu} B^{\zeta} A^{\gamma} \right)$$
 (28)

** three term product rule

$$\partial_{\mu} \left(A^{\gamma} B^{\zeta} G^{\alpha} \right) \to \left(\partial_{\mu} A^{\gamma} B^{\zeta} G^{\alpha} + \partial_{\mu} B^{\zeta} G^{\alpha} A^{\gamma} + \partial_{\mu} G^{\alpha} B^{\zeta} A^{\gamma} \right) \tag{29}$$

*** Distribute through multiple terms

$$\partial_{\mu} \left(A^{\gamma} B^{\zeta} \right) \partial_{\alpha} \left(A_{\gamma} \right) + \partial_{\alpha} \left(G_{\mu \zeta} \right) \to \left(\partial_{\mu} A^{\gamma} B^{\zeta} + \partial_{\mu} B^{\zeta} A^{\gamma} \right) \left(\partial_{\alpha} A_{\gamma} \right) + \left(\partial_{\alpha} G_{\mu \zeta} \right) \tag{30}$$

** multiple partials product rule

$$\partial_{\alpha}\partial_{\gamma}\Big(A^{\alpha}B^{\gamma}\Big) \to \Big(\partial_{\alpha}\partial_{\gamma}A^{\alpha}B^{\gamma} + \partial_{\gamma}B^{\gamma}\partial_{\alpha}A^{\alpha} + \partial_{\alpha}\partial_{\gamma}B^{\gamma}A^{\alpha} + \partial_{\gamma}A^{\alpha}\partial_{\alpha}B^{\gamma}\Big)$$
(31)

** product rule with squares

$$\Box \left(A^{\alpha} B_{\alpha} \right) \to \left(\Box A^{\alpha} B_{\alpha} + \partial^{\tau} B_{\alpha} \partial_{\tau} A^{\alpha} + \Box B_{\alpha} A^{\alpha} + \partial^{\tau} A^{\alpha} \partial_{\tau} B_{\alpha} \right) \tag{32}$$

****** also check with equals signssssss

It appears that numbers are working but symbolic coefficients are not

- ** non-tensor terms
- ** only partial terms

ALSO test product rule with multiple tensors!!!!

QUESTION: why num co, symco, AND tensorCos?? What happens with multiple coefficients???

2.3 FOIL out terms without distributing derivatives

Basic case:

$$(A)(Y\partial_{\nu}h^{\mu\nu} + X\partial^{\mu}h^{\nu}_{\nu}) \to (YA\partial_{\nu}h^{\mu\nu} + XA\partial^{\mu}h^{\nu}_{\nu})$$
(33)

Alternate cases:

*** constants that can move through the derivatives

$$\partial_{\mu} A \Big(Y \partial_{\nu} h^{\mu\nu} + X \partial^{\mu} h^{\nu}_{\nu} \Big) \to \partial_{\mu} \Big(Y A \partial_{\nu} h^{\mu\nu} + X A \partial^{\mu} h^{\nu}_{\nu} \Big)$$
(34)

$$A\partial_{\mu} \Big(Y \partial_{\nu} h^{\mu\nu} + X \partial^{\mu} h^{\nu}_{\nu} \Big) \to \partial_{\mu} \Big(Y A \partial_{\nu} h^{\mu\nu} + X A \partial^{\mu} h^{\nu}_{\nu} \Big)$$
 (35)

$$4\partial_{\mu} \left(Y \partial_{\nu} h^{\mu\nu} + X \partial^{\mu} h^{\nu}_{\nu} \right) \to \partial_{\mu} \left(4Y \partial_{\nu} h^{\mu\nu} + 4X \partial^{\mu} h^{\nu}_{\nu} \right) \tag{36}$$

*** multiple terms

$$\left(A + 5B + CD \right) \partial_{\mu} \left(Y \partial_{\nu} h^{\mu\nu} + X \partial^{\mu} h^{\nu}_{\nu} \right) \rightarrow \partial_{\mu} \left(Y A \partial_{\nu} h^{\mu\nu} + 5Y B \partial_{\nu} h^{\mu\nu} + Y C D \partial_{\nu} h^{\mu\nu} + X A \partial^{\mu} h^{\nu}_{\nu} + 5X B \partial^{\mu} h^{\nu}_{\nu} + X C D \partial^{\mu} h^{\nu}_{\nu} \right)$$

$$(37)$$

*** etas and deltas

$$\delta_{\zeta}^{\gamma} \eta^{\nu \alpha} \partial_{\mu} \left(Y \partial_{\nu} h^{\mu \zeta} + X \partial^{\mu} h_{\nu}^{\zeta} \right) \rightarrow \partial_{\mu} \left(Y \delta_{\zeta}^{\gamma} \eta^{\nu \alpha} \partial_{\nu} h^{\mu \zeta} + X \delta_{\zeta}^{\gamma} \eta^{\nu \alpha} \partial^{\mu} h_{\nu}^{\zeta} \right) \tag{38}$$

*** sum of mixed constants

$$\left(A\eta^{\gamma\epsilon} + A5B + 56\delta_{\xi}^{\phi}\right)\partial_{\mu}\left(Y\partial_{\nu}h^{\mu\nu} + X\partial^{\mu}h_{\nu}^{\nu}\right)
\rightarrow \partial_{\mu}\left(YA\eta^{\gamma\epsilon}\partial_{\nu}h^{\mu\nu} + 5YAB\partial_{\nu}h^{\mu\nu} + 56Y\delta_{\xi}^{\phi}\partial_{\nu}h^{\mu\nu} + XA\eta^{\gamma\epsilon}\partial^{\mu}h_{\nu}^{\nu} + 5XAB\partial^{\mu}h_{\nu}^{\nu}
+ 56X\delta_{\xi}^{\phi}\partial^{\mu}h_{\nu}^{\nu}\right) (39)$$

TODO: decide on how partials behave then implement test cases for them

Question: do all nodes have summation objects or do some have multgroup objects?? – they appear to all be summation objects

*** derivative is attached to a multgroup and doesn't need to be distributed

$$\partial_{\gamma} A^{\gamma} \Big(Y \partial_{\mu} \partial_{\nu} h^{\mu\nu} + X \partial_{\mu} \partial^{\mu} h^{\nu}_{\nu} \Big) \to \Big(Y \partial_{\gamma} A^{\gamma} \partial_{\mu} \partial_{\nu} h^{\mu\nu} + X \partial_{\gamma} A^{\gamma} \partial_{\mu} \partial^{\mu} h^{\nu}_{\nu} \Big) \tag{40}$$

** with added term at the end

$$\partial_{\mu}A^{\epsilon} \Big(Y \partial_{\nu} h^{\mu\nu} + X \partial^{\mu} h^{\nu}_{\nu} \Big) + B^{\epsilon} \to \Big(Y \partial_{\mu}A^{\epsilon} \partial_{\nu} h^{\mu\nu} + X \partial_{\mu}A^{\epsilon} \partial^{\mu} h^{\nu}_{\nu} + B^{\epsilon} \Big)$$

$$\tag{41}$$

***with subtracted term at the end

$$\partial_{\mu}A^{\epsilon} \Big(Y \partial_{\nu} h^{\mu\nu} + X \partial^{\mu} h^{\nu}_{\nu} \Big) - B^{\epsilon} \to \Big(Y \partial_{\mu}A^{\epsilon} \partial_{\nu} h^{\mu\nu} + X \partial_{\mu}A^{\epsilon} \partial^{\mu} h^{\nu}_{\nu} - B^{\epsilon} \Big)$$

$$\tag{42}$$

*** test FOIL ability ** 2 terms

$$\left(A^{\epsilon} + B^{\gamma}\right) \left(\partial_{\mu} X^{\mu} + M_{\gamma}^{\nu\zeta}\right) \to \left(A^{\epsilon} \partial_{\mu} X^{\mu} + A^{\epsilon} M_{\gamma}^{\nu\zeta} + B^{\gamma} \partial_{\mu} X^{\mu} + B^{\gamma} M_{\gamma}^{\nu\zeta}\right) \tag{43}$$

** 3 terms

$$H\left(A^{\epsilon}+B^{\gamma}\right)\left(\partial_{\mu}X^{\mu}+M_{\gamma}^{\nu\zeta}\right)\left(X+Y\right)\rightarrow\left(HXA^{\epsilon}\partial_{\mu}X^{\mu}+HYA^{\epsilon}\partial_{\mu}X^{\mu}+HXA^{\epsilon}M_{\gamma}^{\nu\zeta}+HYA^{\epsilon}M_{\gamma}^{\nu\zeta}+HXB^{\gamma}\partial_{\mu}X^{\mu}+HYB^{\gamma}\partial_{\mu}X^{\mu}+HXA^{\epsilon}M_{\gamma}^{\nu\zeta}+HYA^{\epsilon}M_{\gamma}^{\nu\zeta}+HXB^{\gamma}\partial_{\mu}X^{\mu}+HYB^{\gamma}\partial_{\mu}X^{\mu}+HXA^{\epsilon}M_{\gamma}^{\nu\zeta}+HXB^{\gamma}\partial_{\mu}X^{\mu}+HYB^{\gamma}\partial_{\mu}X^{\mu}+HXB^{\gamma}\partial_{\mu}X^{\mu}+HYB^{\gamma}\partial_{\mu}X$$

TODO figure out the part with switching and write test cases for it

ALSO etas and deltas

** multgroup also has constant factor

$$B\partial_{\gamma}A^{\gamma}\left(Y\partial_{\mu}\partial_{\nu}h^{\mu\nu} + X\partial_{\mu}\partial^{\mu}h^{\nu}_{\nu}\right) \to \left(BY\partial_{\gamma}A^{\gamma}\partial_{\mu}\partial_{\nu}h^{\mu\nu} + BX\partial_{\gamma}A^{\gamma}\partial_{\mu}\partial^{\mu}h^{\nu}_{\nu}\right) \tag{45}$$

**** Shouldn't be distributed

$$A^{\zeta} \partial_{\mu} \Big(Y \partial_{\nu} h^{\mu\nu} + X \partial^{\mu} h^{\nu}_{\nu} \Big) \to A^{\zeta} \partial_{\mu} \Big(Y \partial_{\nu} h^{\mu\nu} + X \partial^{\mu} h^{\nu}_{\nu} \Big) \tag{46}$$

$$\partial_{\mu} \Big(Y \partial_{\nu} h^{\mu\nu} + X \partial^{\mu} h^{\nu}_{\nu} \Big) + T^{\gamma} \Big(A_{\gamma} + B_{\gamma} \Big) \rightarrow \partial_{\mu} \Big(Y \partial_{\nu} h^{\mu\nu} + X \partial^{\mu} h^{\nu}_{\nu} \Big) + \Big(T^{\gamma} A_{\gamma} + T^{\gamma} B_{\gamma} \Big) \tag{47}$$

*** with added term at the end

$$A^{\epsilon}\partial_{\mu}\Big(Y\partial_{\nu}h^{\mu\nu} + X\partial^{\mu}h^{\nu}_{\nu}\Big) + B^{\epsilon} \to A^{\epsilon}\partial_{\mu}\Big(Y\partial_{\nu}h^{\mu\nu} + X\partial^{\mu}h^{\nu}_{\nu}\Big) + B^{\epsilon} \tag{48}$$

*** with subtracted term at the end

$$A^{\epsilon}\partial_{\mu}\left(Y\partial_{\nu}h^{\mu\nu} + X\partial^{\mu}h^{\nu}_{\nu}\right) - B^{\epsilon} \to A^{\epsilon}\partial_{\mu}\left(Y\partial_{\nu}h^{\mu\nu} + X\partial^{\mu}h^{\nu}_{\nu}\right) - B^{\epsilon} \tag{49}$$

Is this sensical or non-sensical???

$$\left(\partial_{alpha}\right)\left(A^{\gamma}B_{\gamma}\right) - > LEADSTO\partial_{alpha}\left(A^{\gamma}B_{\gamma}\right) \tag{50}$$

What about...

$$\left(\partial_{\alpha}\right)\partial^{\alpha}\left(A^{\gamma}B_{\gamma}\right) - > LEADSTO\partial_{\alpha}\partial^{\alpha}\left(A^{\gamma}B_{\gamma}\right) \tag{51}$$

Also need to test adding summations, with and without partials, and subtracting summations, with and without partials

3 Contract

3.1 contract etas and deltas

$$\delta^{\gamma}_{\beta} \eta^{\nu\alpha} \partial_{\mu} h^{\mu}_{\alpha} \partial_{\nu} h^{\beta}_{\gamma} \to \partial_{\mu} h^{\mu\nu} \partial_{\nu} h^{\gamma}_{\gamma} \tag{52}$$

3.2 contract only deltas

Basic case:

$$\delta^{\nu}_{\alpha}\partial_{\mu}h^{\mu}_{\alpha}\partial_{\nu}h^{\gamma}_{\gamma} \to \partial_{\mu}h^{\mu}_{\alpha}\partial_{\alpha}h^{\gamma}_{\gamma} \tag{53}$$

3.3 contract only etas

Basic case:

$$\eta^{\nu\alpha}\partial_{\mu}h^{\mu}_{\alpha}\partial_{\nu}h^{\gamma}_{\gamma} \to \partial_{\mu}h^{\mu\nu}\partial_{\nu}h^{\gamma}_{\gamma} \tag{54}$$

Alternative cases:

4 Factor

4.1 factor out GCF

Basic case:

$$\left(3Y\partial_{\nu}h^{\mu\nu} + Y\partial^{\mu}h^{\nu}_{\nu}\right) \to Y\left(3\partial_{\nu}h^{\mu\nu} + \partial^{\mu}h^{\nu}_{\nu}\right) \tag{55}$$

Alternative cases:

$$\left(X\partial_{\nu}h^{\mu\nu} + Y\partial_{\nu}h^{\mu\nu}\right) \to \partial_{\nu}h^{\mu\nu}\left(X + Y\right) \tag{56}$$

*** sum

$$\left(XA_{\alpha}^{\alpha} + YA_{\alpha}^{\alpha}\right) \to A_{\alpha}^{\alpha}\left(X + Y\right) \tag{57}$$

*** no outer brackets...

$$XA^{\alpha}_{\alpha} + YA^{\alpha}_{\alpha} \to A^{\alpha}_{\alpha} \Big(X + Y \Big)$$
 (58)

$$\left(X\partial_{\gamma}h^{\mu\gamma} + Y\partial_{\nu}h^{\mu\nu}\right) \to \partial_{\gamma}h^{\mu\gamma}\left(X + Y\right) \tag{59}$$

$$\left(X\partial_{\gamma}h^{\mu\gamma}A^{\alpha}_{\alpha} + Y\partial_{\nu}h^{\mu\nu}\right) \to \partial_{\gamma}h^{\mu\gamma}\left(XA^{\alpha}_{\alpha} + Y\right) \tag{60}$$

$$\left(X\partial_{\gamma}h^{\mu\gamma} + Yh^{\gamma\nu}\right) \to \left(X\partial_{\gamma}h^{\mu\gamma} + Yh^{\gamma\nu}\right) \tag{61}$$

$$\left(\frac{1}{2}\partial_{\gamma}h^{\mu\gamma} + \frac{1}{4}h^{\gamma\nu}\right) \to \frac{1}{4}\left(2\partial_{\gamma}h^{\mu\gamma} + h^{\gamma\nu}\right) \tag{62}$$

*** factor of 1 (implied)

$$\left(X\partial_{\nu}h^{\mu\nu} + \partial_{\nu}h^{\mu\nu}\right) \to \partial_{\nu}h^{\mu\nu}\left(X+1\right) \tag{63}$$

4.2 factor out user specified term

*** smaller numerical

$$\left(\frac{1}{2}\partial_{\gamma}h^{\mu\gamma} + \frac{1}{4}h^{\gamma\nu}\right), \frac{1}{8} \to \frac{1}{8}\left(4\partial_{\gamma}h^{\mu\gamma} + 2h^{\gamma\nu}\right) \tag{64}$$

*** bigger numerical

$$\left(\frac{1}{2}\partial_{\gamma}h^{\mu\gamma} + \frac{1}{4}h^{\gamma\nu}\right), 8 \to 8\left(\frac{1}{16}\partial_{\gamma}h^{\mu\gamma} + \frac{1}{32}h^{\gamma\nu}\right) \tag{65}$$

*** smaller tensor

$$(X\partial_{\gamma}h^{\mu\gamma}A^{\gamma} + Y\partial_{\nu}h^{\mu\nu}A^{\gamma}), A^{\gamma} \to A^{\gamma}(X\partial_{\gamma}h^{\mu\gamma} + Y\partial_{\nu}h^{\mu\nu})$$
(66)

*** not included tensor

$$X\partial_{\mu}\partial_{\nu}h^{\mu\nu} + Y\partial_{\nu}h^{\mu\nu}, A^{alpha} \to \left(X\partial_{\mu}\partial_{\nu}h^{\mu\nu} + Y\partial_{\nu}h^{\mu\nu}\right)$$
 (67)

*** different index

(***decide if this should be the expected behaviour (tensor logic question))

$$\left(X\partial_{\gamma}h^{\mu\gamma}A^{\gamma} + Y\partial_{\nu}h^{\mu\nu}A^{\gamma}\right), A^{\alpha} \to \left(X\partial_{\gamma}h^{\mu\gamma}A^{\gamma} + Y\partial_{\nu}h^{\mu\nu}A^{\gamma}\right) \tag{68}$$

*** make sure it's recognizing not to factor out tensors under partials

$$\left(X\partial_{\mu}\partial_{\nu}h^{\mu\nu} + Y\partial_{\nu}h^{\mu\nu}A^{\gamma}\right), \partial_{\nu}h^{\mu\nu} \to \partial_{\nu}h^{\mu\nu}\left(YA^{\gamma}\right) + \left(X\partial_{\mu}\partial_{\nu}h^{\mu\nu}\right) \tag{69}$$

*** what about if there's a different index with the same sum pattern, and what about coefficient of 1

$$\left(XA_{\alpha}^{\alpha} + A_{\alpha}^{\alpha}\right), A_{\beta}^{\beta} \to A_{\beta}^{\beta}\left(X + 1\right) \tag{70}$$

— variation, no brackets...

$$XA^{\alpha}_{\alpha} + A^{\alpha}_{\alpha}, A^{\beta}_{\beta} \to A^{\beta}_{\beta} \left(X + 1 \right) \tag{71}$$

5 Replace

5.1 replace indices

Basic case:

*** single replacement

$$4X\partial_{\mu}\partial_{\nu}h^{\mu\nu} + Y\partial_{\nu}h^{\mu\nu}A^{\gamma}, \mu \to \alpha \to \left(4X\partial_{\alpha}\partial_{\nu}h^{\alpha\nu} + Y\partial_{\nu}h^{\alpha\nu}A^{\gamma}\right)$$

$$\tag{72}$$

*** list replacement

$$4X\partial_{\mu}\partial_{\nu}h^{\mu\nu} + Y\partial_{\nu}h^{\mu\nu}A^{\gamma}, \mu, \nu, \gamma \to \alpha, \beta, \xi \to \left(4X\partial_{\alpha}\partial_{\beta}h^{\alpha\beta} + Y\partial_{\beta}h^{\alpha\beta}A^{\xi}\right)$$

$$(73)$$

*** replace index that doesn't exist

$$4X\partial_{\mu}\partial_{\nu}h^{\mu\nu} + Y\partial_{\nu}h^{\mu\nu}A^{\gamma}, \mu, \nu, \xi \to \alpha, \beta, \chi \to \left(4X\partial_{\alpha}\partial_{\beta}h^{\alpha\beta} + Y\partial_{\beta}h^{\alpha\beta}A^{\gamma}\right)$$

$$(74)$$

*** what if it's not there but it's one of the replacement indices

$$4X\partial_{\mu}\partial_{\nu}h^{\mu\nu} + Y\partial_{\nu}h^{\mu\nu}A^{\gamma}, \mu, \nu, \alpha \to \alpha, \beta, \xi \to \left(4X\partial_{\alpha}\partial_{\beta}h^{\alpha\beta} + Y\partial_{\beta}h^{\alpha\beta}A^{\gamma}\right)$$

$$(75)$$

*** long number of indices (mismatched list lengths)

5.2 replace terms

Basic case:

*** direct replacement

$$4X\partial_{\mu}\partial_{\nu}h^{\mu\nu} + Y\partial_{\nu}h^{\mu\nu}A^{\gamma}, A^{\gamma} \to B^{\gamma} \to \left(4X\partial_{\mu}\partial_{\nu}h^{\mu\nu} + Y\partial_{\nu}h^{\mu\nu}B^{\gamma}\right)$$

$$\tag{76}$$

*** more complicated replacement term

 $4X\partial_{\mu}\partial_{\nu}h^{\mu\nu} + Y\partial_{\nu}h^{\mu\nu}A^{\gamma}, \partial_{\nu}h^{\mu\nu} \rightarrow \partial_{\gamma}V^{\mu\gamma}ISSUE : "Equation: indicestoreplaced on ot matchin length! Please email jbrucero@uwo (77)$

 $4X\partial_{\mu}\partial_{\nu}h^{\mu\nu} + Y\partial_{\nu}h^{\mu\nu}A^{\gamma}, \partial_{\nu}h^{\mu\nu} \to V^{\mu} \to ISSUE : "Equation: indicestore placed on ot matchin length! Please email jbrucero@uwo (78)$

— ***probably one of the most complicated algorithms, need to look over in detail then finish cases***

6 Sort

6.1 combine like terms differing only by a numerical factor

Basic case:

$$3A^{\gamma} + \frac{5}{7}A^{\gamma} \to \left(\frac{26}{7}A^{\gamma}\right) \tag{79}$$

*** fraction simplification

$$3A^{\gamma} + \frac{4}{2}A^{\gamma} \to \left(5A^{\gamma}\right) \tag{80}$$

*** coefficient of 1 (implied)

$$A^{\gamma} + A^{\gamma} \to \left(2A^{\gamma}\right) \tag{81}$$

*** more complicated term

$$3\partial_{\gamma}A^{\gamma} + \frac{5}{7}\partial_{\beta}A^{\beta} \to \left(\frac{26}{7}\partial_{\gamma}A^{\gamma}\right) \tag{82}$$

$$3\partial_{\gamma}\partial^{\mu}A^{\gamma}_{\nu\mu} + 7\partial_{\beta}\partial^{\zeta}A^{\beta}_{\nu\zeta} \to \left(10\partial_{\gamma}\partial^{\mu}A^{\gamma}_{\nu\mu}\right) \tag{83}$$

*** not all terms combine

$$3\partial_{\gamma}A^{\gamma} + 4A^{\chi} + \frac{5}{7}\partial_{\beta}A^{\beta} \to \left(\frac{26}{7}\partial_{\gamma}A^{\gamma} + 4A^{\chi}\right) \tag{84}$$

*** things that shouldn't combine

*** different tensors

$$3A^{\gamma} + \frac{5}{7}B^{\gamma} \to \left(3A^{\gamma} + \frac{5}{7}B^{\gamma}\right) \tag{85}$$

*** not differing only by numerical factor

$$3XA^{\gamma} + 7A^{\gamma} \to \left(3XA^{\gamma} + 7A^{\gamma}\right) \tag{86}$$

***different free indices

$$A^{\beta} + A^{\alpha} \to \left(A^{\beta} + A^{\alpha}\right) \tag{87}$$

***different number of partials

$$3\partial_{\gamma}A^{\gamma}_{\nu} + 7\partial_{\beta}\partial^{\zeta}A^{\beta}_{\nu\zeta} \to \left(3\partial_{\gamma}A^{\gamma}_{\nu} + 7\partial_{\beta}\partial^{\zeta}A^{\beta}_{\nu\zeta}\right) \tag{88}$$

*** different position of free index

$$3\partial_{\gamma}\partial^{\mu}A^{\gamma}_{\nu\mu} + 7\partial_{\beta}\partial^{\zeta}A^{\beta}_{\zeta\nu} \to \left(3\partial_{\gamma}\partial^{\mu}A^{\gamma}_{\nu\mu} + 7\partial_{\beta}\partial^{\zeta}A^{\beta}_{\zeta\nu}\right) \tag{89}$$

***(Unless A is a symmetric tensor, in which case):

$$3\partial_{\gamma}\partial^{\mu}A^{\gamma}_{\nu\mu} + 7\partial_{\beta}\partial^{\zeta}A^{\beta}_{\zeta\nu} \to \left(10\partial_{\gamma}\partial^{\mu}A^{\gamma}_{\nu\mu}\right) \tag{90}$$

6.2 combine like terms differing by any (numerical or symbolic) coefficient

*** basic case

$$3XA^{\gamma} + 7A^{\gamma} \to \left(\left(3X + 7 \right) A^{\gamma} \right) \tag{91}$$

*** more complicated term

$$3B\partial_{\gamma}A^{\gamma} + \frac{5}{7}V\partial_{\beta}A^{\beta} \to \left(\left(3B + \frac{5}{7}V\right)\partial_{\gamma}A^{\gamma}\right) \tag{92}$$

$$3B\partial_{\gamma}A^{\gamma} + \frac{5}{7}B\partial_{\beta}A^{\beta} \to \left(\frac{26}{7}B\partial_{\gamma}A^{\gamma}\right) \tag{93}$$

$$3M\partial_{\gamma}\partial^{\mu}A^{\gamma}_{\nu\mu} + 7X\partial_{\beta}\partial^{\zeta}A^{\beta}_{\nu\zeta} \to \left(\left(3M + 7X \right) \partial_{\gamma}\partial^{\mu}A^{\gamma}_{\nu\mu} \right) \tag{94}$$

*** not all terms combine

$$3V\partial_{\gamma}A^{\gamma} + 4ZA^{\chi} + \frac{5}{7}N\partial_{\beta}A^{\beta} \to \left(\left(3V + \frac{5}{7}N\right)\partial_{\gamma}A^{\gamma} + 4ZA^{\chi}\right) \tag{95}$$

***shouldn't combine

 $3B\partial_{\gamma}A^{\gamma} + \frac{5}{7}V_{\alpha}^{\alpha}\partial_{\beta}A^{\beta} \rightarrow (IthinkthisiswhatIwantbutshoulddoublecheck/seeifanimprovementcombinethesetoo \rightarrow wouldthisbeheelddoublecheck/seeifanimprovementcombinethesetoo \rightarrow (96)$

*** different tensors

$$3CA^{\gamma} + \frac{5}{7}DB^{\gamma} \to \left(3CA^{\gamma} + \frac{5}{7}DB^{\gamma}\right) \tag{97}$$

***different free indices

$$XA^{\beta} + A^{\alpha} \to \left(XA^{\beta} + A^{\alpha}\right)$$
 (98)

***different number of partials

$$3\partial_{\gamma}A^{\gamma}_{\nu} + 7M\partial_{\beta}\partial^{\zeta}A^{\beta}_{\nu\zeta} \to \left(3\partial_{\gamma}A^{\gamma}_{\nu} + 7M\partial_{\beta}\partial^{\zeta}A^{\beta}_{\nu\zeta}\right) \tag{99}$$

*** different position of free index

$$3V\partial_{\gamma}\partial^{\mu}A^{\gamma}_{\nu\mu} + 7L\partial_{\beta}\partial^{\zeta}A^{\beta}_{\zeta\nu} \to \left(3V\partial_{\gamma}\partial^{\mu}A^{\gamma}_{\nu\mu} + 7L\partial_{\beta}\partial^{\zeta}A^{\beta}_{\zeta\nu}\right) \tag{100}$$

***(Unless A is a symmetric tensor, in which case):

$$3V\partial_{\gamma}\partial^{\mu}A^{\gamma}_{\nu\mu} + 7L\partial_{\beta}\partial^{\zeta}A^{\beta}_{\zeta\nu} \to \left(\left(3V + 7L\right)\partial_{\gamma}\partial^{\mu}A^{\gamma}_{\nu\mu}\right) \tag{101}$$

6.3 sort the tensors in each term by number of derivatives (least to greatest)

$$\partial_{\gamma} \Box G^{\nu\gamma} \partial_{\beta} \partial^{\xi} M_{\xi}^{\beta} \Box \Box \partial_{\chi} X_{\nu}^{\kappa\zeta} \partial_{\mu} T^{\mu} \rightarrow \partial_{\mu} T^{\mu} \partial_{\beta} \partial^{\xi} M_{\xi}^{\beta} \partial_{\gamma} \Box G^{\nu\gamma} \partial_{\chi} \Box \Box X_{\nu}^{\kappa\zeta}$$

$$\tag{102}$$

***What about with coefficients

$$4A\partial_{\gamma}\Box G^{\nu\gamma}\partial_{\beta}\partial^{\xi}M_{\xi}^{\beta}\Box\Box\partial_{\chi}X_{\nu}^{\kappa\zeta}\partial_{\mu}T^{\mu} \to WORKING4A\partial_{\mu}T^{\mu}\partial_{\beta}\partial^{\xi}M_{\xi}^{\beta}\partial_{\gamma}\Box G^{\nu\gamma}\partial_{\chi}\Box\Box X_{\nu}^{\kappa\zeta} \tag{103}$$

***What about multiple terms and coefficients

$$4A\partial_{\gamma}\Box G^{\nu\gamma}\partial_{\beta}\partial^{\xi}M_{\xi}^{\beta} + \frac{7}{8}C\Box\Box\partial_{\chi}X_{\nu}^{\kappa\zeta}\partial_{\mu}T^{\mu} \to \left(4A\partial_{\beta}\partial^{\xi}M_{\xi}^{\beta}\partial_{\gamma}\Box G^{\nu\gamma} + \frac{7}{8}C\partial_{\mu}T^{\mu}\partial_{\chi}\Box\Box X_{\nu}^{\kappa\zeta}\right) \tag{104}$$

***And with brackets

$$\left(4A\partial_{\gamma}\Box G^{\nu\gamma}\partial_{\beta}\partial^{\xi}M_{\xi}^{\beta}\right)\left(\frac{7}{8}C\Box\Box\partial_{\chi}X_{\nu}^{\kappa\zeta}\partial_{\mu}T^{\mu}\right) \to \left(4A\partial_{\beta}\partial^{\xi}M_{\xi}^{\beta}\partial_{\gamma}\Box G^{\nu\gamma}\right)\left(\frac{7}{8}C\partial_{\mu}T^{\mu}\partial_{\chi}\Box\Box X_{\nu}^{\kappa\zeta}\right) \tag{105}$$

6.4 sort terms by number of derivatives (least to greatest)

$$\partial_{\gamma}\partial_{\kappa}A^{\gamma} + \partial^{\chi}B_{\chi} + C \to \left(C + \partial^{\chi}B_{\chi} + \partial_{\gamma}\partial_{\kappa}A^{\gamma}\right) \tag{106}$$

***with brackets

$$\left(\partial_{\gamma}\partial_{\kappa}A^{\gamma} + \partial^{\chi}B_{\chi} + C\right) \to \left(C + \partial^{\chi}B_{\chi} + \partial_{\gamma}\partial_{\kappa}A^{\gamma}\right) \tag{107}$$

***with coefficients

$$\left(9\partial^{\chi}B_{\chi} + \frac{1}{2}T\partial_{\gamma}\partial_{\kappa}A^{\gamma} + MC\right) \to \left(MC + 9\partial^{\chi}B_{\chi} + \frac{1}{2}T\partial_{\gamma}\partial_{\kappa}A^{\gamma}\right) \tag{108}$$

*** with multiplied terms

$$\left(\partial_{\gamma}\partial_{\kappa}A^{\gamma} + \partial_{\omega}G\right)\left(\partial^{\chi}B_{\chi} + C\right) \to \left(\partial_{\omega}G + \partial_{\gamma}\partial_{\kappa}A^{\gamma}\right)\left(C + \partial^{\chi}B_{\chi}\right) \tag{109}$$

** with multiple tensors per term (I think it goes by either least or greatest...)

$$\left(\partial_{\gamma}\partial_{\kappa}\Box A^{\gamma}\partial_{\omega}G + \Box\partial^{\chi}B_{\chi} + \Box X_{\xi} + \Box VC\right) \to \left(\Box X_{\xi} + \Box VC + \partial^{\chi}\Box B_{\chi} + \partial_{\gamma}\partial_{\kappa}\Box A^{\gamma}\partial_{\omega}G\right) \tag{110}$$

(must sort by total)

$$\partial_{\xi} H \partial_{\omega} G \partial^{\xi} M + \Box A \to \left(\Box A + \partial_{\xi} H \partial_{\omega} G \partial^{\xi} M \right)$$
 (111)

*** interesting thing with brackets - should look into at some point -;. At the very least it should be a warning and not make it look like it worked...

*** same idea but with summation -; INTERESTING BRACKET STUFF - is it important??

$$(XA^{\alpha}_{\alpha} + YA^{\alpha}_{\alpha}) \to NOTWORKING(X+Y)A^{\alpha}_{\alpha}A^{\alpha}_{\alpha}$$
 (112)

— variations on above:

$$(X\partial_{\mu}A^{\alpha}_{\alpha} + Y\partial_{\mu}A^{\alpha}_{\alpha}) \to sameissue(X+Y)\partial_{\mu}A^{\alpha}_{\alpha}\partial_{\mu}A^{\alpha}_{\alpha} \tag{113}$$

$$(X\partial_{\alpha}A^{\alpha} + Y\partial_{\alpha}A^{\alpha}) \to ISSUE(X+Y)\partial_{\alpha}A^{\alpha}\partial_{\alpha}A^{\alpha} \tag{114}$$

$$(X\partial_{\alpha}h^{\alpha} + Y\partial_{\alpha}h^{\alpha}) \to (X+Y)\partial_{\alpha}h^{\alpha}\partial_{\alpha}h^{\alpha} \tag{115}$$

$$(X\partial_{\alpha}h^{\alpha\beta} + Y\partial_{\alpha}h^{\alpha\beta}) \to (X+Y)\partial_{\alpha}h^{\alpha\beta}\partial_{\alpha}h^{\alpha\beta} \tag{116}$$