

Test

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CURRENT ISSUES:

- Replacement issues
- ISSUES WITH NEGATIVES!!!!

QUESTION — should outside brackets have to be

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???

$$\partial_\mu A \left(Y \partial_\nu h^{\mu\nu} + X \partial^\mu h^\nu_\nu \right) \quad (1)$$

$$\partial_\mu \left(Y A \partial_\nu h^{\mu\nu} + X A \partial^\mu h^\nu_\nu \right) \quad (2)$$

Contract:

$$\partial_\mu A \left(Y \partial_\nu h^{\mu\nu} + X \partial^\mu h^\nu_\nu \right) \quad (3)$$

1 Representation

Reading in and understanding equations (display initial equation)

Reading in signs between brackets

$$(a) + (b) \rightarrow (a) + (b) \quad (4)$$

$$(a) - (b) \rightarrow (a) - (b) \quad (5)$$

basic reading in signs inside brackets

$$b_\gamma - g_j \rightarrow (b_\gamma - g_j) \quad (6)$$

inside and outside brackets

$$A^\mu + (A^\gamma - B^\zeta) \rightarrow A^\mu + (A^\gamma - B^\zeta) \quad (7)$$

$$A^\mu - (A^\gamma - B^\zeta) \rightarrow A^\mu - (A^\gamma - B^\zeta) \quad (8)$$

with partial derivatives

$$\partial_\alpha (a + (b_\gamma - g)) \rightarrow \partial_\alpha (a + (b_\gamma - g)) \quad (9)$$

multiplied by a tensor

$$A^\mu + C^\epsilon (A^\gamma - B^\zeta) \rightarrow A^\mu + C^\epsilon (A^\gamma - B^\zeta) \quad (10)$$

complicated bracket nesting

$$\left((a) - (b - (c)) \right) + ((a - d) - b) \rightarrow \left((a) - (b - (c)) \right) + ((a - d) - b) \quad (11)$$

complicated bracket nesting and tensor indices

$$\left((a) - (b - (c)) \right) + \left((a^\gamma - d^\gamma) - b \right) \rightarrow \left((a) - (b - (c)) \right) + \left((a^\gamma - d^\gamma) - b \right) \quad (12)$$

complicated bracket nesting and tensor indices, multiplication, and partials

$$\begin{aligned} \partial_\zeta \left(G^\gamma \right) \partial^\zeta \square \left((A) - (B_\kappa^\kappa - (C)) \right) + \left((A^\gamma - D^\gamma) - B_\alpha^{\alpha\gamma} \right) \\ \rightarrow \partial_\zeta \left(G^\gamma \right) \partial^\zeta \square \left((A) - (B_\kappa^\kappa - (C)) \right) + \left((A^\gamma - D^\gamma) - B_\alpha^{\alpha\gamma} \right) \end{aligned} \quad (13)$$

with equals sign

$$G^\mu = A^\mu + C^{\mu\epsilon} (A_\epsilon - B_\epsilon) \rightarrow G^\mu = A^\mu + C^{\mu\epsilon} (A_\epsilon - B_\epsilon) \quad (14)$$

with begin equation command

```
\begin{equation}
G^{\mu} = A^{\mu} + C^{\mu \epsilon} (A_{\epsilon} - B_{\epsilon})
\end{equation}
->
\begin{equation}
G^{\mu} = A^{\mu} + C^{\mu \epsilon} \ (A_{\epsilon} - B_{\epsilon}) \ )
\end{equation}
```

with begin multiline command (but short so it changes to equation)

```
\begin{multiline}
G^{\mu} = A^{\mu} + C^{\mu \epsilon} (A_{\epsilon} - B_{\epsilon})
\end{multiline}
->
\begin{equation}
G^{\mu} = A^{\mu} + C^{\mu \epsilon} \ (A_{\epsilon} - B_{\epsilon}) \ )
\end{equation}
```

with begin multiline command and long enough to stay multiline

```
\begin{multiline}
G^{\mu} = A^{\mu} + C^{\mu \epsilon} (A_{\epsilon} - B_{\epsilon}) + A^{\mu}
+ C^{\mu \epsilon} (A_{\epsilon} - B_{\epsilon}) + A^{\mu}
+ C^{\mu \epsilon} (A_{\epsilon} - B_{\epsilon}) + A^{\mu}
+ C^{\mu \epsilon} (A_{\epsilon} - B_{\epsilon}) + A^{\mu}
+ \ C^{\mu \epsilon} (A_{\epsilon} - B_{\epsilon}) + A^{\mu}
+ C^{\mu \epsilon} (A_{\epsilon} - B_{\epsilon}) + A^{\mu}
+ C^{\mu \epsilon} (A_{\epsilon} - B_{\epsilon}) + A^{\mu}
+ C^{\mu \epsilon} (A_{\epsilon} - B_{\epsilon}) + A^{\mu}
+ \ C^{\mu \epsilon} (A_{\epsilon} - B_{\epsilon}) + A^{\mu}
+ C^{\mu \epsilon} (A_{\epsilon} - B_{\epsilon}) + A^{\mu}
+ C^{\mu \epsilon} (A_{\epsilon} - B_{\epsilon})
\end{multiline}
->
\begin{multiline}
G^{\mu} = A^{\mu} + C^{\mu \epsilon} \ (A_{\epsilon} - B_{\epsilon}) \ ) + A^{\mu}
+ C^{\mu \epsilon} \ (A_{\epsilon} - B_{\epsilon}) \ ) + A^{\mu}
+ C^{\mu \epsilon} \ (A_{\epsilon} - B_{\epsilon}) \ ) + A^{\mu} \ \
+ C^{\mu \epsilon} \ (A_{\epsilon} - B_{\epsilon}) \ ) + A^{\mu}
+ C^{\mu \epsilon} \ (A_{\epsilon} - B_{\epsilon}) \ ) + A^{\mu} \ \
+ C^{\mu \epsilon} \ (A_{\epsilon} - B_{\epsilon}) \ ) + A^{\mu}
+ C^{\mu \epsilon} \ (A_{\epsilon} - B_{\epsilon}) \ ) + A^{\mu}
+ C^{\mu \epsilon} \ (A_{\epsilon} - B_{\epsilon}) \ ) + A^{\mu} \ \
+ C^{\mu \epsilon} \ (A_{\epsilon} - B_{\epsilon}) \ ) + A^{\mu} \ \
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+ C^{\mu \epsilon} \ ( \ A_{\epsilon} - B_{\epsilon} \ ) + A^{\mu}
+ C^{\mu \epsilon} \ ( \ A_{\epsilon} - B_{\epsilon} \ )
\end{multline}
```

also print above to test what it looks like visually: output is

$$\begin{aligned}
G^{\mu} = & A^{\mu} + C^{\mu\epsilon} (A_{\epsilon} - B_{\epsilon}) + A^{\mu} + C^{\mu\epsilon} (A_{\epsilon} - B_{\epsilon}) + A^{\mu} + C^{\mu\epsilon} (A_{\epsilon} - B_{\epsilon}) + A^{\mu} \\
& + C^{\mu\epsilon} (A_{\epsilon} - B_{\epsilon}) + A^{\mu} + C^{\mu\epsilon} (A_{\epsilon} - B_{\epsilon}) + A^{\mu} + C^{\mu\epsilon} (A_{\epsilon} - B_{\epsilon}) + A^{\mu} \\
& + C^{\mu\epsilon} (A_{\epsilon} - B_{\epsilon}) + A^{\mu} + C^{\mu\epsilon} (A_{\epsilon} - B_{\epsilon}) + A^{\mu} + C^{\mu\epsilon} (A_{\epsilon} - B_{\epsilon}) + A^{\mu} \\
& + C^{\mu\epsilon} (A_{\epsilon} - B_{\epsilon}) + A^{\mu} + C^{\mu\epsilon} (A_{\epsilon} - B_{\epsilon}) \quad (15)
\end{aligned}$$

each line could be longer before wrapping but I'm pretty happy with how it looks for now

with etas and deltas and fractions

$$7X\eta_{\mu\nu}\delta_{\nu}^{\gamma}\square A^{\mu} + \frac{1}{3}\delta_{\mu}^{\gamma}C^{\mu\epsilon}(A_{\epsilon} - B_{\epsilon}) \rightarrow 7X\delta_{\nu}^{\gamma}\eta_{\mu\nu}\square A^{\mu} + \frac{1}{3}\delta_{\mu}^{\gamma}C^{\mu\epsilon}(A_{\epsilon} - B_{\epsilon}) \quad (16)$$

multiple coefficients

$$(XYZ\partial_{\nu}h^{\mu\nu} + 6bX\partial^{\mu}h_{\nu}^{\nu}) \rightarrow (XYZ\partial_{\nu}h^{\mu\nu} + 6bX\partial^{\mu}h_{\nu}^{\nu}) \quad (17)$$

multiple coefficients in unusual order

$$(XY\partial_{\nu}h^{\mu\nu}Z + 6bX\partial^{\mu}h_{\nu}^{\nu}) \rightarrow (XYZ\partial_{\nu}h^{\mu\nu} + 6bX\partial^{\mu}h_{\nu}^{\nu}) \quad (18)$$

breaks (is this good or no??)

$$A^{\alpha}\frac{1}{3}B_{\gamma} \rightarrow \text{Equation: pop from empty list} \quad (19)$$

this one seems to work...

$$A^{\alpha}3B_{\gamma} \rightarrow 3A^{\alpha}B_{\gamma} \quad (20)$$

QUESTION: does multiplication always need brackets? If no, when does it need them? Why does the whole number work but the fraction breaks...

Please email jbrucero@uwo.ca for if you think this is a bug

*****things that shouldn't work

*** uneven brackets

$$(A+B \rightarrow \text{WORKING}(\text{though the message could be greatly improved})) \text{ Equation : Please email jbrucero@uwo.ca for if you think this is a bug} \quad (21)$$

*** uneven index brackets

$$A^{\{\gamma} \rightarrow \text{Working in that it should error but again the issue could probably be caught earlier and a better message given}) \text{ Equation : string} \quad (22)$$

$$A^{\gamma} + B^{\{\gamma} \rightarrow \text{this one reads the equation fine - is this an issue???} (A^{\gamma} + B) \quad (23)$$

2 Multiply

2.1 FOIL out terms, distributing derivatives when necessary (recommended)

Basic case:

$$(A)\partial_{\mu}(Y\partial_{\nu}h^{\mu\nu} + X\partial^{\mu}h_{\nu}^{\nu}) \rightarrow (YA\partial_{\nu}\partial_{\mu}h^{\mu\nu} + XA\partial^{\mu}\partial_{\mu}h_{\nu}^{\nu}) \quad (24)$$

Also need to test adding summations, with and without partials, and subtracting summations, with and without partials

2.2 distribute partial derivatives

Basic case:

$$\partial_\mu \left(Y \partial_\nu h^{\mu\nu} + X \partial^\mu h^\nu_\nu \right) \rightarrow \left(Y \partial_\nu \partial_\mu h^{\mu\nu} + X \partial^\mu \partial_\mu h^\nu_\nu \right) \quad (25)$$

** number coefficients

$$\partial_\mu \left(5 \partial_\nu h^{\mu\nu} + 3 \partial^\mu h^\nu_\nu \right) \rightarrow \left(5 \partial_\nu \partial_\mu h^{\mu\nu} + 3 \partial^\mu \partial_\mu h^\nu_\nu \right) \quad (26)$$

** mixed coefficients

$$\partial_\mu \left(5 \partial_\nu h^{\mu\nu} + V \partial^\mu h^\nu_\nu \right) \rightarrow \left(5 \partial_\nu \partial_\mu h^{\mu\nu} + V \partial^\mu \partial_\mu h^\nu_\nu \right) \quad (27)$$

** product rule

$$\partial_\mu \left(A^\gamma B^\zeta \right) \rightarrow \left(\partial_\mu A^\gamma B^\zeta + \partial_\mu B^\zeta A^\gamma \right) \quad (28)$$

** three term product rule

$$\partial_\mu \left(A^\gamma B^\zeta G^\alpha \right) \rightarrow \left(\partial_\mu A^\gamma B^\zeta G^\alpha + \partial_\mu B^\zeta G^\alpha A^\gamma + \partial_\mu G^\alpha B^\zeta A^\gamma \right) \quad (29)$$

*** Distribute through multiple terms

$$\partial_\mu \left(A^\gamma B^\zeta \right) \partial_\alpha \left(A_\gamma \right) + \partial_\alpha \left(G_{\mu\zeta} \right) \rightarrow \left(\partial_\mu A^\gamma B^\zeta + \partial_\mu B^\zeta A^\gamma \right) \left(\partial_\alpha A_\gamma \right) + \left(\partial_\alpha G_{\mu\zeta} \right) \quad (30)$$

** multiple partials product rule

$$\partial_\alpha \partial_\gamma \left(A^\alpha B^\gamma \right) \rightarrow \left(\partial_\alpha \partial_\gamma A^\alpha B^\gamma + \partial_\gamma B^\gamma \partial_\alpha A^\alpha + \partial_\alpha \partial_\gamma B^\gamma A^\alpha + \partial_\gamma A^\alpha \partial_\alpha B^\gamma \right) \quad (31)$$

** product rule with squares

$$\square \left(A^\alpha B_\alpha \right) \rightarrow \left(\square A^\alpha B_\alpha + \partial^\tau B_\alpha \partial_\tau A^\alpha + \square B_\alpha A^\alpha + \partial^\tau A^\alpha \partial_\tau B_\alpha \right) \quad (32)$$

***** also check with equals signssssss

It appears that numbers are working but symbolic coefficients are not

** non-tensor terms

** only partial terms

ALSO test product rule with multiple tensors!!!!

QUESTION: why num co, symco, AND tensorCos?? What happens with multiple coefficients???

2.3 FOIL out terms without distributing derivatives

Basic case:

$$\left(A \right) \left(Y \partial_\nu h^{\mu\nu} + X \partial^\mu h^\nu_\nu \right) \rightarrow \left(Y A \partial_\nu h^{\mu\nu} + X A \partial^\mu h^\nu_\nu \right) \quad (33)$$

Alternate cases:

*** constants that can move through the derivatives

$$\partial_\mu A \left(Y \partial_\nu h^{\mu\nu} + X \partial^\mu h^\nu_\nu \right) \rightarrow \partial_\mu \left(Y A \partial_\nu h^{\mu\nu} + X A \partial^\mu h^\nu_\nu \right) \quad (34)$$

$$A \partial_\mu \left(Y \partial_\nu h^{\mu\nu} + X \partial^\mu h^\nu_\nu \right) \rightarrow \partial_\mu \left(Y A \partial_\nu h^{\mu\nu} + X A \partial^\mu h^\nu_\nu \right) \quad (35)$$

$$4 \partial_\mu \left(Y \partial_\nu h^{\mu\nu} + X \partial^\mu h^\nu_\nu \right) \rightarrow \partial_\mu \left(4 Y \partial_\nu h^{\mu\nu} + 4 X \partial^\mu h^\nu_\nu \right) \quad (36)$$

*** multiple terms

$$\left(A + 5B + CD \right) \partial_\mu \left(Y \partial_\nu h^{\mu\nu} + X \partial^\mu h^\nu_\nu \right) \rightarrow \partial_\mu \left(Y A \partial_\nu h^{\mu\nu} + 5Y B \partial_\nu h^{\mu\nu} + Y C D \partial_\nu h^{\mu\nu} + X A \partial^\mu h^\nu_\nu + 5X B \partial^\mu h^\nu_\nu + X C D \partial^\mu h^\nu_\nu \right) \quad (37)$$

*** etas and deltas

$$\delta_\xi^\gamma \eta^{\nu\alpha} \partial_\mu \left(Y \partial_\nu h^{\mu\zeta} + X \partial^\mu h_\nu^\zeta \right) \rightarrow \partial_\mu \left(Y \delta_\xi^\gamma \eta^{\nu\alpha} \partial_\nu h^{\mu\zeta} + X \delta_\xi^\gamma \eta^{\nu\alpha} \partial^\mu h_\nu^\zeta \right) \quad (38)$$

*** sum of mixed constants

$$\begin{aligned} & \left(A \eta^{\gamma\epsilon} + A5B + 56\delta_\xi^\phi \right) \partial_\mu \left(Y \partial_\nu h^{\mu\nu} + X \partial^\mu h_\nu^\nu \right) \\ & \rightarrow \partial_\mu \left(Y A \eta^{\gamma\epsilon} \partial_\nu h^{\mu\nu} + 5YAB \partial_\nu h^{\mu\nu} + 56Y \delta_\xi^\phi \partial_\nu h^{\mu\nu} + X A \eta^{\gamma\epsilon} \partial^\mu h_\nu^\nu + 5XAB \partial^\mu h_\nu^\nu \right. \\ & \quad \left. + 56X \delta_\xi^\phi \partial^\mu h_\nu^\nu \right) \end{aligned} \quad (39)$$

TODO: decide on how partials behave then implement test cases for them

Question: do all nodes have summation objects or do some have multigroup objects?? – they appear to all be summation objects

*** derivative is attached to a multigroup and doesn't need to be distributed

$$\partial_\gamma A^\gamma \left(Y \partial_\mu \partial_\nu h^{\mu\nu} + X \partial_\mu \partial^\mu h_\nu^\nu \right) \rightarrow \left(Y \partial_\gamma A^\gamma \partial_\mu \partial_\nu h^{\mu\nu} + X \partial_\gamma A^\gamma \partial_\mu \partial^\mu h_\nu^\nu \right) \quad (40)$$

** with added term at the end

$$\partial_\mu A^\epsilon \left(Y \partial_\nu h^{\mu\nu} + X \partial^\mu h_\nu^\nu \right) + B^\epsilon \rightarrow \left(Y \partial_\mu A^\epsilon \partial_\nu h^{\mu\nu} + X \partial_\mu A^\epsilon \partial^\mu h_\nu^\nu + B^\epsilon \right) \quad (41)$$

***with subtracted term at the end

$$\partial_\mu A^\epsilon \left(Y \partial_\nu h^{\mu\nu} + X \partial^\mu h_\nu^\nu \right) - B^\epsilon \rightarrow \left(Y \partial_\mu A^\epsilon \partial_\nu h^{\mu\nu} + X \partial_\mu A^\epsilon \partial^\mu h_\nu^\nu - B^\epsilon \right) \quad (42)$$

*** test FOIL ability ** 2 terms

$$\left(A^\epsilon + B^\gamma \right) \left(\partial_\mu X^\mu + M_\gamma^{\nu\zeta} \right) \rightarrow \left(A^\epsilon \partial_\mu X^\mu + A^\epsilon M_\gamma^{\nu\zeta} + B^\gamma \partial_\mu X^\mu + B^\gamma M_\gamma^{\nu\zeta} \right) \quad (43)$$

** 3 terms

$$H \left(A^\epsilon + B^\gamma \right) \left(\partial_\mu X^\mu + M_\gamma^{\nu\zeta} \right) \left(X + Y \right) \rightarrow \left(H X A^\epsilon \partial_\mu X^\mu + H Y A^\epsilon \partial_\mu X^\mu + H X A^\epsilon M_\gamma^{\nu\zeta} + H Y A^\epsilon M_\gamma^{\nu\zeta} + H X B^\gamma \partial_\mu X^\mu + H Y B^\gamma \partial_\mu X^\mu + H X \right) \quad (44)$$

TODO figure out the part with switching and write test cases for it

ALSO etas and deltas

** multigroup also has constant factor

$$B \partial_\gamma A^\gamma \left(Y \partial_\mu \partial_\nu h^{\mu\nu} + X \partial_\mu \partial^\mu h_\nu^\nu \right) \rightarrow \left(B Y \partial_\gamma A^\gamma \partial_\mu \partial_\nu h^{\mu\nu} + B X \partial_\gamma A^\gamma \partial_\mu \partial^\mu h_\nu^\nu \right) \quad (45)$$

**** Shouldn't be distributed

$$A^\zeta \partial_\mu \left(Y \partial_\nu h^{\mu\nu} + X \partial^\mu h_\nu^\nu \right) \rightarrow A^\zeta \partial_\mu \left(Y \partial_\nu h^{\mu\nu} + X \partial^\mu h_\nu^\nu \right) \quad (46)$$

$$\partial_\mu \left(Y \partial_\nu h^{\mu\nu} + X \partial^\mu h_\nu^\nu \right) + T^\gamma \left(A_\gamma + B_\gamma \right) \rightarrow \partial_\mu \left(Y \partial_\nu h^{\mu\nu} + X \partial^\mu h_\nu^\nu \right) + \left(T^\gamma A_\gamma + T^\gamma B_\gamma \right) \quad (47)$$

*** with added term at the end

$$A^\epsilon \partial_\mu \left(Y \partial_\nu h^{\mu\nu} + X \partial^\mu h_\nu^\nu \right) + B^\epsilon \rightarrow A^\epsilon \partial_\mu \left(Y \partial_\nu h^{\mu\nu} + X \partial^\mu h_\nu^\nu \right) + B^\epsilon \quad (48)$$

*** with subtracted term at the end

$$A^\epsilon \partial_\mu \left(Y \partial_\nu h^{\mu\nu} + X \partial^\mu h_\nu^\nu \right) - B^\epsilon \rightarrow A^\epsilon \partial_\mu \left(Y \partial_\nu h^{\mu\nu} + X \partial^\mu h_\nu^\nu \right) - B^\epsilon \quad (49)$$

Is this sensical or non-sensical???

$$\left(\partial_{alpha} \right) \left(A^\gamma B_\gamma \right) - > LEADSTO \partial_{alpha} \left(A^\gamma B_\gamma \right) \quad (50)$$

What about...

$$\left(\partial_\alpha \right) \partial^\alpha \left(A^\gamma B_\gamma \right) - > LEADSTO \partial_\alpha \partial^\alpha \left(A^\gamma B_\gamma \right) \quad (51)$$

Also need to test adding summations, with and without partials, and subtracting summations, with and without partials

3 Contract

3.1 contract etas and deltas

$$\delta_{\beta}^{\gamma} \eta^{\nu\alpha} \partial_{\mu} h_{\alpha}^{\mu} \partial_{\nu} h_{\gamma}^{\beta} \rightarrow \partial_{\mu} h^{\mu\nu} \partial_{\nu} h_{\gamma}^{\gamma} \quad (52)$$

3.2 contract only deltas

Basic case:

$$\delta_{\alpha}^{\nu} \partial_{\mu} h_{\alpha}^{\mu} \partial_{\nu} h_{\gamma}^{\gamma} \rightarrow \partial_{\mu} h_{\alpha}^{\mu} \partial_{\alpha} h_{\gamma}^{\gamma} \quad (53)$$

3.3 contract only etas

Basic case:

$$\eta^{\nu\alpha} \partial_{\mu} h_{\alpha}^{\mu} \partial_{\nu} h_{\gamma}^{\gamma} \rightarrow \partial_{\mu} h^{\mu\nu} \partial_{\nu} h_{\gamma}^{\gamma} \quad (54)$$

Alternative cases:

4 Factor

4.1 factor out GCF

Basic case:

$$\left(3Y \partial_{\nu} h^{\mu\nu} + Y \partial^{\mu} h_{\nu}^{\nu} \right) \rightarrow Y \left(3\partial_{\nu} h^{\mu\nu} + \partial^{\mu} h_{\nu}^{\nu} \right) \quad (55)$$

Alternative cases:

$$\left(X \partial_{\nu} h^{\mu\nu} + Y \partial_{\nu} h^{\mu\nu} \right) \rightarrow \partial_{\nu} h^{\mu\nu} (X + Y) \quad (56)$$

*** sum

$$\left(X A_{\alpha}^{\alpha} + Y A_{\alpha}^{\alpha} \right) \rightarrow A_{\alpha}^{\alpha} (X + Y) \quad (57)$$

*** no outer brackets...

$$X A_{\alpha}^{\alpha} + Y A_{\alpha}^{\alpha} \rightarrow A_{\alpha}^{\alpha} (X + Y) \quad (58)$$

$$\left(X \partial_{\gamma} h^{\mu\gamma} + Y \partial_{\nu} h^{\mu\nu} \right) \rightarrow \partial_{\gamma} h^{\mu\gamma} (X + Y) \quad (59)$$

$$\left(X \partial_{\gamma} h^{\mu\gamma} A_{\alpha}^{\alpha} + Y \partial_{\nu} h^{\mu\nu} \right) \rightarrow \partial_{\gamma} h^{\mu\gamma} (X A_{\alpha}^{\alpha} + Y) \quad (60)$$

$$\left(X \partial_{\gamma} h^{\mu\gamma} + Y h^{\gamma\nu} \right) \rightarrow \left(X \partial_{\gamma} h^{\mu\gamma} + Y h^{\gamma\nu} \right) \quad (61)$$

$$\left(\frac{1}{2} \partial_{\gamma} h^{\mu\gamma} + \frac{1}{4} h^{\gamma\nu} \right) \rightarrow \frac{1}{4} \left(2 \partial_{\gamma} h^{\mu\gamma} + h^{\gamma\nu} \right) \quad (62)$$

*** factor of 1 (implied)

$$\left(X \partial_{\nu} h^{\mu\nu} + \partial_{\nu} h^{\mu\nu} \right) \rightarrow \partial_{\nu} h^{\mu\nu} (X + 1) \quad (63)$$

4.2 factor out user specified term

*** smaller numerical

$$\left(\frac{1}{2} \partial_{\gamma} h^{\mu\gamma} + \frac{1}{4} h^{\gamma\nu} \right), \frac{1}{8} \rightarrow \frac{1}{8} \left(4 \partial_{\gamma} h^{\mu\gamma} + 2 h^{\gamma\nu} \right) \quad (64)$$

*** bigger numerical

$$\left(\frac{1}{2} \partial_{\gamma} h^{\mu\gamma} + \frac{1}{4} h^{\gamma\nu} \right), 8 \rightarrow 8 \left(\frac{1}{16} \partial_{\gamma} h^{\mu\gamma} + \frac{1}{32} h^{\gamma\nu} \right) \quad (65)$$

*** smaller tensor

$$\left(X\partial_\gamma h^{\mu\gamma} A^\gamma + Y\partial_\nu h^{\mu\nu} A^\gamma\right), A^\gamma \rightarrow A^\gamma \left(X\partial_\gamma h^{\mu\gamma} + Y\partial_\nu h^{\mu\nu}\right) \quad (66)$$

*** not included tensor

$$X\partial_\mu\partial_\nu h^{\mu\nu} + Y\partial_\nu h^{\mu\nu}, A^{alpha} \rightarrow \left(X\partial_\mu\partial_\nu h^{\mu\nu} + Y\partial_\nu h^{\mu\nu}\right) \quad (67)$$

*** different index

(***decide if this should be the expected behaviour (tensor logic question))

$$\left(X\partial_\gamma h^{\mu\gamma} A^\gamma + Y\partial_\nu h^{\mu\nu} A^\gamma\right), A^\alpha \rightarrow \left(X\partial_\gamma h^{\mu\gamma} A^\gamma + Y\partial_\nu h^{\mu\nu} A^\gamma\right) \quad (68)$$

*** make sure it's recognizing not to factor out tensors under partials

$$\left(X\partial_\mu\partial_\nu h^{\mu\nu} + Y\partial_\nu h^{\mu\nu} A^\gamma\right), \partial_\nu h^{\mu\nu} \rightarrow \partial_\nu h^{\mu\nu} \left(Y A^\gamma\right) + \left(X\partial_\mu\partial_\nu h^{\mu\nu}\right) \quad (69)$$

*** what about if there's a different index with the same sum pattern, and what about coefficient of 1

$$\left(XA_\alpha^\alpha + A_\alpha^\alpha\right), A_\beta^\beta \rightarrow A_\beta^\beta (X + 1) \quad (70)$$

— variation, no brackets...

$$XA_\alpha^\alpha + A_\alpha^\alpha, A_\beta^\beta \rightarrow A_\beta^\beta (X + 1) \quad (71)$$

5 Replace

5.1 replace indices

Basic case:

*** single replacement

$$4X\partial_\mu\partial_\nu h^{\mu\nu} + Y\partial_\nu h^{\mu\nu} A^\gamma, (\mu \rightarrow \alpha) \rightarrow \left(4X\partial_\alpha\partial_\nu h^{\alpha\nu} + Y\partial_\nu h^{\alpha\nu} A^\gamma\right) \quad (72)$$

*** list replacement

$$4X\partial_\mu\partial_\nu h^{\mu\nu} + Y\partial_\nu h^{\mu\nu} A^\gamma, (\mu, \nu, \gamma \rightarrow \alpha, \beta, \xi) \rightarrow \left(4X\partial_\alpha\partial_\beta h^{\alpha\beta} + Y\partial_\beta h^{\alpha\beta} A^\xi\right) \quad (73)$$

*** replace index that doesn't exist

$$4X\partial_\mu\partial_\nu h^{\mu\nu} + Y\partial_\nu h^{\mu\nu} A^\gamma, (\mu, \nu, \xi \rightarrow \alpha, \beta, \chi) \rightarrow \left(4X\partial_\alpha\partial_\beta h^{\alpha\beta} + Y\partial_\beta h^{\alpha\beta} A^\gamma\right) \quad (74)$$

*** what if it's not there but it's one of the replacement indices

$$4X\partial_\mu\partial_\nu h^{\mu\nu} + Y\partial_\nu h^{\mu\nu} A^\gamma, (\mu, \nu, \alpha \rightarrow \alpha, \beta, \xi) \rightarrow \left(4X\partial_\alpha\partial_\beta h^{\alpha\beta} + Y\partial_\beta h^{\alpha\beta} A^\gamma\right) \quad (75)$$

*** long number of indices (mismatched list lengths)

5.2 replace terms

5.2.1 direct replacement, tensor \rightarrow tensor

- replacing a tensor on its own

Starting with the equation

$$A^{\alpha\gamma} + A^\gamma + C \quad (76)$$

replacing A^γ with B^γ

$$\left(A^{\alpha\gamma} + B^\gamma + C\right) \quad (77)$$

- replacing a tensor that is multiplied by other elements

Starting with the equation

$$4X\partial_\mu\partial_\nu h^{\mu\nu}B^{\alpha\gamma} + Y\partial_\nu h^{\alpha\nu}A^\gamma \quad (78)$$

replacing A^γ with B^γ

$$\left(4X\partial_\mu\partial_\nu h^{\mu\nu}B^{\alpha\gamma} + Y\partial_\nu h^{\alpha\nu}B^\gamma\right) \quad (79)$$

- replacing a tensor that has partial derivatives

- Replace with just a tensor

Starting with the equation

$$\partial_\mu\partial_\nu h^{\mu\nu} + \frac{3}{4}Y \quad (80)$$

replacing $\partial_\mu\partial_\nu h^{\mu\nu}$ with B_γ^γ

$$\left(B_\gamma^\gamma + \frac{3}{4}Y\right) \quad (81)$$

- Replace with a partial and tensor

Starting with the equation

$$\partial_\mu\partial_\nu h^{\mu\nu} + \frac{3}{4}Y \quad (82)$$

replacing $\partial_\mu\partial_\nu h^{\mu\nu}$ with $\partial_\gamma B^\gamma$

$$\left(\partial_\gamma B^\gamma + \frac{3}{4}Y\right) \quad (83)$$

- Includes a term that shouldn't be replaced

Starting with the equation

$$\left(\partial_\mu\partial_\nu h^{\mu\nu}\right)\left(\partial_\xi\partial_\beta h^{\chi\beta}\right) \quad (84)$$

replacing $\partial_\mu\partial_\nu h^{\mu\nu}$ with $\partial_\gamma B^\gamma$

$$\left(\partial_\gamma B^\gamma\right)\left(\partial_\xi\partial_\beta h^{\chi\beta}\right) \quad (85)$$

- replacing a tensor and some of its partial derivatives

- can replace either derivative

Starting with the equation

$$\partial_\mu\partial_\nu h^{\mu\nu} + \frac{3}{4}Y \quad (86)$$

replacing $\partial_\nu h^{\mu\nu}$ with B^μ

- must replace the derivative with the sum

Starting with the equation

$$\partial_\gamma\partial_\nu h^{\mu\nu} + \frac{3}{4}Y \quad (87)$$

replacing $\partial_\nu h^{\mu\nu}$ with B^μ

$$\left(\partial_\gamma B^\mu + \frac{3}{4}Y\right) \quad (88)$$

- must replace the derivative without the sum

Starting with the equation

$$\partial_\gamma \partial_\nu h^{\mu\nu} + \frac{3}{4}Y \quad (89)$$

replacing $\partial_\gamma h^{\mu\nu}$ with $B_\gamma^{\mu\nu}$

$$\left(\partial_\nu B_\gamma^{\mu\nu} + \frac{3}{4}Y \right) \quad (90)$$

- replacing a number of derivatives

Starting with the equation

$$\partial_\gamma \partial_\nu \partial^\epsilon h^{\mu\nu} + \frac{3}{4}Y \quad (91)$$

replacing $\partial_\nu \partial^\epsilon h^{\mu\nu}$ with $B^{\mu\epsilon}$

- including square

Starting with the equation

$$\partial_\gamma \partial_\nu \partial^\epsilon \square h^{\mu\nu} + \frac{3}{4}Y \quad (92)$$

replacing $\square h^{\mu\nu}$ with $B^{\mu\nu}$

- replacing a tensor that has partial derivatives and is multiplied by other elements

Starting with the equation

$$A^\zeta \partial_\mu \partial_\nu h^{\mu\nu} \square M_\zeta \quad (93)$$

replacing $\partial_\mu \partial_\nu h^{\mu\nu}$ with B_μ^μ

$$A^\zeta \square M_\zeta B_\mu^\mu \quad (94)$$

- replacing a tensor and some of its partial derivatives when it is multiplied by other elements

- can replace either derivative

Starting with the equation

$$A^\zeta \partial_\mu \partial_\nu h^{\mu\nu} \square M_\zeta \quad (95)$$

replacing $\partial_\nu h^{\mu\nu}$ with B^μ

- must replace the derivative with the sum

Starting with the equation

$$A^\zeta \partial_\gamma \partial_\nu h^{\mu\nu} \square M_\zeta \quad (96)$$

replacing $\partial_\nu h^{\mu\nu}$ with B^μ

$$A^\zeta \square M_\zeta \partial_\gamma B^\mu \quad (97)$$

- must replace the derivative without the sum

Starting with the equation

$$A^\zeta \partial_\gamma \partial_\nu h^{\mu\nu} \square M_\zeta \quad (98)$$

replacing $\partial_\gamma h^{\mu\nu}$ with $B_\gamma^{\mu\nu}$

$$A^\zeta \square M_\zeta \partial_\nu B_\gamma^{\mu\nu} \quad (99)$$

- replacing a tensor in a multigroup with derivatives

Starting with the equation

$$\partial_\mu \partial^\gamma \left(A^\zeta \partial_\gamma \partial_\nu h^{\mu\nu} \square M_\zeta \right) \quad (100)$$

replacing $h^{\mu\nu}$ with $B^{\mu\nu}$

$$\partial_\mu \partial^\gamma \left(A^\zeta \square M_\zeta \partial_\gamma \partial_\nu B^{\mu\nu} \right) \quad (101)$$

5.2.2 direct replacement, tensor \rightarrow sum

- replacing just a tensor

Starting with the equation

$$A^{\alpha\gamma} + A^\gamma + C \quad (102)$$

replacing A^γ with $B^\gamma + 8fG^\gamma$

$$(A^{\alpha\gamma} + C) + (B^\gamma + 8fG^\gamma) \quad (103)$$

- replacing a tensor that is multiplied by other elements

Starting with the equation

$$4X\partial_\mu\partial_\nu h^{\mu\nu}B^{\alpha\gamma} + Y\partial_\nu h^{\alpha\nu}A^\gamma \quad (104)$$

replacing A^γ with $B^\gamma + 8fG^\gamma$

`\begin{equation}`

`\(4 X B^{\{\alpha \gamma\}} \partial_{\mu} \partial_{\nu} h^{\mu \nu} a^{\} \)+Y \partial_{\nu} h^{\alpha \nu} a^{\}`

`\end{equation}`

- replacing a tensor that has partial derivatives

Starting with the equation

$$\partial_\mu\partial_\nu h^{\mu\nu} + \frac{3}{4}Y \quad (105)$$

replacing $\partial_\mu\partial_\nu h^{\mu\nu}$ with $\partial_\gamma B^\gamma + 5 + G^\mu_\mu$

$$\left(\frac{3}{4}Y\right) + \left(\partial_\gamma B^\gamma + 5 + G^\mu_\mu\right) \quad (106)$$

- replacing a tensor and some of its partial derivatives

Starting with the equation

$$\partial_\mu\partial_\nu h^{\mu\nu} + \frac{3}{4}Y \quad (107)$$

replacing $\partial_\mu\partial_\nu h^{\mu\nu}$ with $B^\gamma_\gamma + \frac{5}{4}V$

$$\left(\frac{3}{4}Y\right) + \left(B^\gamma_\gamma + \frac{5}{4}V\right) \quad (108)$$

- replacing a tensor that has partial derivatives and is multiplied by other elements

Starting with the equation

$$A^\zeta\partial_\mu\partial_\nu h^{\mu\nu}\Box M_\zeta \quad (109)$$

replacing $\partial_\mu\partial_\nu h^{\mu\nu}$ with $B^\mu_\mu + 16\Box B$

$$A^\zeta\Box M_\zeta\left(B^\mu_\mu + 16\Box B\right) \quad (110)$$

- replacing a tensor and some of its partial derivatives when it is multiplied by other elements

Starting with the equation

$$A^\zeta\partial_\gamma\partial_\nu h^{\mu\nu}\Box M_\zeta \quad (111)$$

replacing $\partial_\nu h^{\mu\nu}$ with $B^\mu + C^\mu$

$$A^\zeta\Box M_\zeta\partial_\gamma\left(B^\mu + C^\mu\right) \quad (112)$$

- replacing a tensor in a multigroup with derivatives

Starting with the equation

$$\partial_\mu \partial^\gamma \left(A^\zeta \partial_\gamma \partial_\nu h^{\mu\nu} \square M_\zeta \right) \quad (113)$$

replacing $h^{\mu\nu}$ with $B^{\mu\nu} + C^{\mu\nu}$

$$\partial_\mu \partial^\gamma \left(\right) + A^\zeta \square M_\zeta \partial_\gamma \partial_\nu \left(B^{\mu\nu} + C^{\mu\nu} \right) \quad (114)$$

a *** more complicated replacement term

$$4X \partial_\mu \partial_\nu h^{\mu\nu} B^{\alpha\gamma} + Y \partial_\nu h^{\alpha\nu} A^\gamma, \quad \partial_\nu h^{\alpha\nu} \rightarrow \partial_\zeta V^{\alpha\zeta} \quad (115)$$

$$4X \partial_\mu \partial_\nu h^{\mu\nu} + Y \partial_\nu h^{\mu\nu} A^\gamma,$$

$$\partial_\nu h^{\mu\nu} \rightarrow V^\mu \quad (116)$$

— ***probably one of the most complicated algorithms, need to look over in detail then finish cases***

6 Sort

6.1 combine like terms differing only by a numerical factor

Basic case:

$$3A^\gamma + \frac{5}{7}A^\gamma \rightarrow \left(\frac{26}{7}A^\gamma \right) \quad (117)$$

*** fraction simplification

$$3A^\gamma + \frac{4}{2}A^\gamma \rightarrow \left(5A^\gamma \right) \quad (118)$$

*** coefficient of 1 (implied)

$$A^\gamma + A^\gamma \rightarrow \left(2A^\gamma \right) \quad (119)$$

*** more complicated term

$$3\partial_\gamma A^\gamma + \frac{5}{7}\partial_\beta A^\beta \rightarrow \left(\frac{26}{7}\partial_\gamma A^\gamma \right) \quad (120)$$

$$3\partial_\gamma \partial^\mu A_{\nu\mu}^\gamma + 7\partial_\beta \partial^\zeta A_{\nu\zeta}^\beta \rightarrow \left(10\partial_\gamma \partial^\mu A_{\nu\mu}^\gamma \right) \quad (121)$$

*** not all terms combine

$$3\partial_\gamma A^\gamma + 4A^\chi + \frac{5}{7}\partial_\beta A^\beta \rightarrow \left(\frac{26}{7}\partial_\gamma A^\gamma + 4A^\chi \right) \quad (122)$$

*** things that shouldn't combine

*** different tensors

$$3A^\gamma + \frac{5}{7}B^\gamma \rightarrow \left(3A^\gamma + \frac{5}{7}B^\gamma \right) \quad (123)$$

*** not differing only by numerical factor

$$3XA^\gamma + 7A^\gamma \rightarrow \left(3XA^\gamma + 7A^\gamma \right) \quad (124)$$

***different free indices

$$A^\beta + A^\alpha \rightarrow \left(A^\beta + A^\alpha \right) \quad (125)$$

***different number of partials

$$3\partial_\gamma A_\nu^\gamma + 7\partial_\beta \partial^\zeta A_{\nu\zeta}^\beta \rightarrow \left(3\partial_\gamma A_\nu^\gamma + 7\partial_\beta \partial^\zeta A_{\nu\zeta}^\beta \right) \quad (126)$$

*** different position of free index

$$3\partial_\gamma\partial^\mu A_{\nu\mu}^\gamma + 7\partial_\beta\partial^\zeta A_{\zeta\nu}^\beta \rightarrow \left(3\partial_\gamma\partial^\mu A_{\nu\mu}^\gamma + 7\partial_\beta\partial^\zeta A_{\zeta\nu}^\beta\right) \quad (127)$$

*** (Unless A is a symmetric tensor, in which case):

$$3\partial_\gamma\partial^\mu A_{\nu\mu}^\gamma + 7\partial_\beta\partial^\zeta A_{\zeta\nu}^\beta \rightarrow \left(10\partial_\gamma\partial^\mu A_{\nu\mu}^\gamma\right) \quad (128)$$

6.2 combine like terms differing by any (numerical or symbolic) coefficient

*** basic case

$$3XA^\gamma + 7A^\gamma \rightarrow \left((3X + 7)A^\gamma\right) \quad (129)$$

*** more complicated term

$$3B\partial_\gamma A^\gamma + \frac{5}{7}V\partial_\beta A^\beta \rightarrow \left(\left(3B + \frac{5}{7}V\right)\partial_\gamma A^\gamma\right) \quad (130)$$

$$3B\partial_\gamma A^\gamma + \frac{5}{7}B\partial_\beta A^\beta \rightarrow \left(\frac{26}{7}B\partial_\gamma A^\gamma\right) \quad (131)$$

$$3M\partial_\gamma\partial^\mu A_{\nu\mu}^\gamma + 7X\partial_\beta\partial^\zeta A_{\nu\zeta}^\beta \rightarrow \left(\left(3M + 7X\right)\partial_\gamma\partial^\mu A_{\nu\mu}^\gamma\right) \quad (132)$$

*** not all terms combine

$$3V\partial_\gamma A^\gamma + 4ZA^x + \frac{5}{7}N\partial_\beta A^\beta \rightarrow \left(\left(3V + \frac{5}{7}N\right)\partial_\gamma A^\gamma + 4ZA^x\right) \quad (133)$$

*** shouldn't combine

$$3B\partial_\gamma A^\gamma + \frac{5}{7}V_\alpha^\alpha \partial_\beta A^\beta \rightarrow (I think this is what I want but should double check / see if an improvement combines these too \rightarrow would this be better) \quad (134)$$

*** different tensors

$$3CA^\gamma + \frac{5}{7}DB^\gamma \rightarrow \left(3CA^\gamma + \frac{5}{7}DB^\gamma\right) \quad (135)$$

*** different free indices

$$XA^\beta + A^\alpha \rightarrow \left(XA^\beta + A^\alpha\right) \quad (136)$$

*** different number of partials

$$3\partial_\gamma A_\nu^\gamma + 7M\partial_\beta\partial^\zeta A_{\nu\zeta}^\beta \rightarrow \left(3\partial_\gamma A_\nu^\gamma + 7M\partial_\beta\partial^\zeta A_{\nu\zeta}^\beta\right) \quad (137)$$

*** different position of free index

$$3V\partial_\gamma\partial^\mu A_{\nu\mu}^\gamma + 7L\partial_\beta\partial^\zeta A_{\zeta\nu}^\beta \rightarrow \left(3V\partial_\gamma\partial^\mu A_{\nu\mu}^\gamma + 7L\partial_\beta\partial^\zeta A_{\zeta\nu}^\beta\right) \quad (138)$$

*** (Unless A is a symmetric tensor, in which case):

$$3V\partial_\gamma\partial^\mu A_{\nu\mu}^\gamma + 7L\partial_\beta\partial^\zeta A_{\zeta\nu}^\beta \rightarrow \left(\left(3V + 7L\right)\partial_\gamma\partial^\mu A_{\nu\mu}^\gamma\right) \quad (139)$$

6.3 sort the tensors in each term by number of derivatives (least to greatest)

$$\partial_\gamma \square G^{\nu\gamma} \partial_\beta \partial^\xi M_\xi^\beta \square \square \partial_\chi X_\nu^{\kappa\zeta} \partial_\mu T^\mu \rightarrow \partial_\mu T^\mu \partial_\beta \partial^\xi M_\xi^\beta \partial_\gamma \square G^{\nu\gamma} \partial_\chi \square \square X_\nu^{\kappa\zeta} \quad (140)$$

*** What about with coefficients

$$4A\partial_\gamma \square G^{\nu\gamma} \partial_\beta \partial^\xi M_\xi^\beta \square \square \partial_\chi X_\nu^{\kappa\zeta} \partial_\mu T^\mu \rightarrow WORKING 4A\partial_\mu T^\mu \partial_\beta \partial^\xi M_\xi^\beta \partial_\gamma \square G^{\nu\gamma} \partial_\chi \square \square X_\nu^{\kappa\zeta} \quad (141)$$

*** What about multiple terms and coefficients

$$4A\partial_\gamma \square G^{\nu\gamma} \partial_\beta \partial^\xi M_\xi^\beta + \frac{7}{8}C \square \square \partial_\chi X_\nu^{\kappa\zeta} \partial_\mu T^\mu \rightarrow \left(4A\partial_\beta \partial^\xi M_\xi^\beta \partial_\gamma \square G^{\nu\gamma} + \frac{7}{8}C \partial_\mu T^\mu \partial_\chi \square \square X_\nu^{\kappa\zeta}\right) \quad (142)$$

*** And with brackets

$$\left(4A\partial_\gamma \square G^{\nu\gamma} \partial_\beta \partial^\xi M_\xi^\beta\right) \left(\frac{7}{8}C \square \square \partial_\chi X_\nu^{\kappa\zeta} \partial_\mu T^\mu\right) \rightarrow \left(4A\partial_\beta \partial^\xi M_\xi^\beta \partial_\gamma \square G^{\nu\gamma}\right) \left(\frac{7}{8}C \partial_\mu T^\mu \partial_\chi \square \square X_\nu^{\kappa\zeta}\right) \quad (143)$$

6.4 sort terms by number of derivatives (least to greatest)

$$\partial_\gamma \partial_\kappa A^\gamma + \partial^\chi B_\chi + C \rightarrow \left(C + \partial^\chi B_\chi + \partial_\gamma \partial_\kappa A^\gamma \right) \quad (144)$$

***with brackets

$$\left(\partial_\gamma \partial_\kappa A^\gamma + \partial^\chi B_\chi + C \right) \rightarrow \left(C + \partial^\chi B_\chi + \partial_\gamma \partial_\kappa A^\gamma \right) \quad (145)$$

***with coefficients

$$\left(9\partial^\chi B_\chi + \frac{1}{2}T\partial_\gamma \partial_\kappa A^\gamma + MC \right) \rightarrow \left(MC + 9\partial^\chi B_\chi + \frac{1}{2}T\partial_\gamma \partial_\kappa A^\gamma \right) \quad (146)$$

*** with multiplied terms

$$\left(\partial_\gamma \partial_\kappa A^\gamma + \partial_\omega G \right) \left(\partial^\chi B_\chi + C \right) \rightarrow \left(\partial_\omega G + \partial_\gamma \partial_\kappa A^\gamma \right) \left(C + \partial^\chi B_\chi \right) \quad (147)$$

** with multiple tensors per term (I think it goes by either least or greatest...)

$$\left(\partial_\gamma \partial_\kappa \square A^\gamma \partial_\omega G + \square \partial^\chi B_\chi + \square X_\xi + \square VC \right) \rightarrow \left(\square X_\xi + \square VC + \partial^\chi \square B_\chi + \partial_\gamma \partial_\kappa \square A^\gamma \partial_\omega G \right) \quad (148)$$

(must sort by total)

$$\partial_\xi H \partial_\omega G \partial^\xi M + \square A \rightarrow \left(\square A + \partial_\xi H \partial_\omega G \partial^\xi M \right) \quad (149)$$

*** interesting thing with brackets - should look into at some point

-! At the very least it should be a warning and not make it look like it worked...

*** same idea but with summation -! INTERESTING BRACKET STUFF - is it important??

$$(XA_\alpha^\alpha + YA_\alpha^\alpha) \rightarrow \text{NOTWORKING}(X+Y)A_\alpha^\alpha A_\alpha^\alpha \quad (150)$$

— variations on above:

$$(X\partial_\mu A_\alpha^\alpha + Y\partial_\mu A_\alpha^\alpha) \rightarrow \text{sameissue}(X+Y)\partial_\mu A_\alpha^\alpha \partial_\mu A_\alpha^\alpha \quad (151)$$

$$(X\partial_\alpha A^\alpha + Y\partial_\alpha A^\alpha) \rightarrow \text{ISSUE}(X+Y)\partial_\alpha A^\alpha \partial_\alpha A^\alpha \quad (152)$$

$$(X\partial_\alpha h^\alpha + Y\partial_\alpha h^\alpha) \rightarrow (X+Y)\partial_\alpha h^\alpha \partial_\alpha h^\alpha \quad (153)$$

$$(X\partial_\alpha h^{\alpha\beta} + Y\partial_\alpha h^{\alpha\beta}) \rightarrow (X+Y)\partial_\alpha h^{\alpha\beta} \partial_\alpha h^{\alpha\beta} \quad (154)$$