Economic Performance and Productivity Growth: The Australian "Miracle"

Jules Boudreau

August 18, 2019

Abstract

In this paper I explore the causes of Australia's uninterrupted spell of growth since 1990. After estimating total factor productivity using three complementary methods – index-based, econometric and non-parametric – I find that GDP growth has not been fuelled by productivity growth. Rather, it has been buoyed by an unsustainable capital accumulation spurred by a surging demand for its commodity exports. Furthermore, the growth in production has not translated into growth in real income and consumer utility due to high consumption inflation. These results suggest that for Australian consumers the economic expansion is more mirage than miracle and that growth shouldn't be expected to last for ever. I close with speculative evidence suggesting that the lack of land input data might explain a portion of the absence of measured productivity growth.

Contents

Introduction

In the last three decades, predictions of a slowdown of the Australian economy have all been proven wrong. Since suffering its last recession in 1990, Australia has avoided the Asian crash of 1997, the dotcom bubble burst of 2000 and the global financial crisis of 2008. It now boasts the longest uninterrupted spell of GDP growth in modern history among OECD countries. The strength and resilience of its economy are most often attributed to pro-market reforms in the 1990s – floating of the AUD, taming of inflation, deregulation of the financial sector, privatization of state-owned firms, slashing of tariffs and import quotas – that improved Australia's business environment and its ability to absorb external shocks. But this long spell of growth is also coincident with a surge in the quantity and price of its commodity exports, explained in large part by China's growing appetite for Australian metals. Can we expect this trend of solid growth to continue? To answer this question requires a deep dive into the causes and channels of that growth.

If hidden behind the GDP growth is a steady increase in productivity driven by the above-mentioned reforms, the probability that Australia stays in the fast lane is higher. Most OECD countries, especially in Europe, have been suffering from a stagnation in productivity, possibly due to bad business environments. If Australia does indeed benefit from a structural advantage stimulating its productivity growth, this augurs well for the future of its economy. If instead Australia's economy has been kept afloat by temporary factors such as high Chinese demand for its exports, it could risk sinking once the external demand fatigues.

Section 1 below will go over three different methods for estimating productivity:

index-based, econometric, and non-parametric. A consensus measure of productivity derived from these three methods provides reasonable evidence that productivity growth has not been the main factor behind the solid GDP growth of the last three decades. Section 2 will explore the macro data to find more convincing explanations to the "economic miracle" of Australia. It will also argue that Australian consumers are not as well off as the GDP numbers would indicate.

Except if noted elsewise, the theory presented in this paper is adapted from Prof. Diewert's measurement theory graduate lecture notes.[1] The entirety of the macro data comes from OECD Statistics or the Australian Bureau of Statistics. The paper does not go into detail about data gathering and cleaning to avoid bloat. For more information about these topics and details about model parameters, see the accompanying Shazam file. Interesting graphs not referenced in the paper are gathered in Appendix B.

1 Measuring Productivity

Accurately measuring productivity is crucial to understanding Australian growth. Because labour input is bounded above by working age population and most growth models assume a diminishing rate of return of capital, input accumulation cannot be a long-run source of per-capita production growth. Only changes in total factor productivity (TFP) – the quantity of outputs produced by a given quantity of inputs – allow GDP to grow sustainably.

[1] The lecture notes can be found at `https://economics.ubc.ca/faculty-and-staff/w-erwin-diewert`.

In a time-series setting, which is the relevant one for this case study of Australia, TFP growth is thus the residual growth in output after taking into account input growth. If output grows proportionally more than input, we have TFP growth. But if TFP is at its essence a residual, what causes it?

Let $f^t(x^t) = max\{y^t : y^t \in S^t(x^t)\}$ where x^t is period t input, y^t is period t output and $S(x^t)$ is the feasible production set in period t given input quantity x^t.[2] Consequently we assume that production is technically efficient in all periods. Let $t = 0, 1$. Then TFP growth between periods 0 and 1 is defined as

$$TFPG = \frac{y^1/x^1}{y^0/x^0} = \frac{y^1/y^0}{x^1/x^0} = \frac{f^1(x^1)/x^1}{f^0(x^0)/x^0} \tag{1}$$

The first component of TFP growth is technical progress (TP). It is a measure of the expansion of the feasible production set – and thus increase in $f^t()$ – from period 0 to 1 due to the adoption of technological or managerial innovations that originated either from research or experience. Taking period 0 inputs as base quantity, $TP = \frac{f^1(x^0)}{f^0(x^0)}$. The second component is returns to scale (RS). For a given $f^t()$, a change in input quantity can change the output/input ratio. In the case of increasing returns to scale (IRS), scaling up production units generates TFP gains by e.g. allowing for broader amortization of fixed costs, greater division of tasks or averaging out of risk. Taking period 0 production function as base quantity, $RS = \frac{f^1(x^1)/x^1}{f^1(x^0)/x^0}$, with $RS > 1$ for IRS. We then have that:

$$TFPG = TP \times RS \tag{2}$$

[2] We will show in Section 1.1 that x and y can represent an index of multiple outputs and inputs.

Unfortunately, production functions $f^t()$ are unknown to empirical researchers. Estimating TFP with time series data, it is impossible to do an accurate job of isolating TP from RS, as inputs and technology both generally increase monotonically with time. In estimating TFP for Australia, we therefore assume constant returns to scale (CRS). Technical progress will be the only source of expansions of the feasible production set.

The analysis above assumes that observed production is technically efficient. But because of input rigidities, observed production is not always on the boundary of the production set. Fixed inputs – land, capital – are treated as sunk costs in the event of an averse aggregate demand shock. They are kept in the production process, but utilized below capacity. Similarly, labour rigidities due to unionization or long-term retention of employees will cause to much labour to be kept as input during an economic downturn. Distortionary regulations can also cause inputs to rise faster than outputs, even outside of recessions. This factor of TFP growth is referred to as efficiency growth (EG). Obviously EG is not a long-term driver of TFP growth, as it can only bring back the output/input ratio up to the boundary of the production set following a downturn/distortion. But it can still have a major impact on TFP growth in the medium-run. Since the early 2000s, some European countries have actually seen their productivity *decline*. As it is unlikely that TP can be negative – it would imply that European firms forgot about certain technologies – the most reasonable explanation is that of misallocation of inputs. This misallocation is perhaps due to regulation and/or the demand shock of the Great Recession.

Another component of TFP growth is changes in the input mix (IM) caused by shifts in the relative costs of inputs. This component is generally unimportant.

If our measure x^t represents a single input or an index aggregating all possible inputs, then those four factors – TP, RS, EG and IM – would explain all TFP changes. Unfortunately, when empirically estimating a country's productivity, not all inputs can be accounted for. Typically, only labour and capital are reliably measured. Diewert (2000) identifies land, resources and inventories as crucial inputs missing from macro datasets. Therefore, TFP growth can also be explained by changes in these unobserved inputs (UI). For example, oil importing countries suffered a TFP slump after the oil crises of 1973 and 1979, due to their output falling. It is possible that this is in part explained by negative EG for the reasons explained above. But a measure of TFP with an incomplete input index would overstate the productivity loss in this case, as firms likely decreased their use of oil as an input in response to the price shock. If oil were included as an input in TFP, the drop in output in the numerator would have been mitigated by a drop in the denominator. This is a major issue, as this factor is fully unobservable. But until statistical agencies get good estimates of inputs other than labour and capital, it is one that cannot be solved. All the researcher can do is be cognizant of the possible role of unobserved inputs on TFP changes.

To summarize, TFP growth is composed of five factors, four if we assume CRS:

$$TFPG = TP \times EG \times IM \times UI \tag{3}$$

In the long-run, only TP should matter, but the three others are highly relevant when looking at past productivity in a 60-year window like this paper does. The next subsections describe how to empirically estimate time-series TFP.

1.1 Index-Based Method

From (1) we know that in the one-input one-output case TFP growth is period 1 output/input ratio divided by period 2 output/input ratio ($\frac{y^1/x^1}{y^0/x^0}$) or equivalently the two-period output ratio divided by the two-period input ratio ($\frac{y^1/y^0}{x^1/x^0}$). The latter equation suggests that to compute TFP growth for the multiple-input multiple-output case, we can replace the output ratio by a two-period output quantity index $Q(p^0, p^1, y^0, y^1)$ where p^t is a N-vector of output prices and y^t is a N-vector of outputs and the input ratio by a two-period input quantity index $Q^*(w^0, w^1, x^0, x^1)$ where w^t is a M-vector of input prices and x^t is a M-vector of inputs.

$$TFPG = \frac{Q(p^0, p^1, y^0, y^1)}{Q^*(w^0, w^1, x^0, x^1)} \tag{4}$$

Which functional form should we chose for Q and Q^*? The appropriate index form should be (a) a generalization of the one-input one-output case, (b) should have a sensible economic interpretation and (c) should have certain desirable properties.

Four quantity index functions are widely used in economics: Laspeyres, Paasche, Fisher and implicit Törnqvist-Theil. They are defined as:

$$Q_L\left(p^0, p^1, y^0, y^1\right) = \frac{p^0 \cdot y^1}{p^0 \cdot y^0} \tag{5}$$

$$Q_P\left(p^0, p^1, y^0, y^1\right) = \frac{p^1 \cdot y^1}{p^1 \cdot y^0} \tag{6}$$

$$Q_F\left(p^0, p^1, y^0, y^1\right) = \left[\frac{p^0 \cdot y^1}{p^0 \cdot y^0}\frac{p^1 \cdot y^1}{p^1 \cdot y^0}\right]^{1/2} \tag{7}$$

$$\ln Q_{IT}\left(p^0, p^1, y^0, y^1\right) = \frac{p^1 \cdot y^1}{p^0 \cdot y^0} / P_T\left(p^0, p^1, y^0, y^1\right) \tag{8}$$

$$\text{where } P_T\left(p^0, p^1, y^0, y^1\right) = (1/2)\prod^N \left(\frac{p_n^1}{p_n^0}\right)^{\frac{1}{2}\left(s_n^0+s_n^1\right)}$$

Replace p^t with w^t and y^t with x^t to get the corresponding input quantity indices $Q^*(p^0, p^1, y^0, y^1)$.

First, it is immediately obvious that all four of the functions simplify to $\frac{y^1}{y^0}$ if N=1. Similarly, the four corresponding Q^* input indices simplify to $\frac{x^1}{x^0}$ if M=1. Hence, TFP growth simplifies to the one-input one-output definition if N=M=1:

$$TFPG = \frac{Q\left(p^0, p^1, y^0, y^1\right)}{Q^*\left(w^0, w^1, x^0, x^1\right)} = \left(\frac{y^1/y^0}{x^1/x^0}\right) \tag{9}$$

Second, the Fisher and Törnqvist-Theil indices have a nice economic interpretation. They are both superlative indices, i.e. they are both exactly equal to the true quantity index $\frac{f(y^1)}{f(y^0)}$. Diewert (1976) shows that $P_F = \frac{f(y^1)}{f(y^0)}$ if the underlying preference function has a homogeneous quadratic form,

$$f(y) = [y^T BAy]^{1/2} \tag{10}$$

and that $P_T = \frac{f(y^1)}{f(y^0)}$ if the preference function has a translog form,

$$\ln f(y) = \alpha_0 + \sum_{i=1}^N \alpha_i \ln x_i + \sum_{i=1}^N \sum_{j=1}^N \gamma_{ij} \ln x_i \ln x_j. \tag{11}$$

The translog and quadratic forms are both *flexible functional forms*, a desirable feature meaning they that they do not arbitrarily restrict elasticities of supply and demand. They can provide a second-order approximation to any actual preference function. We will get back to the quadratic and translog functions in Section 1.2.

Third, the Fisher index has the most useful properties for an index. Diewert (1992) outlines 21 tests describing the desirability of an index, and proves that only the Fisher ideal indices P_F and Q_F pass all of them.

Hence, Q_F is the most appropriate index, although Diewert (1978) shows that Q_{IT} approximates Q_F to the second order around around an equal price and quantity point. Having determined that Q_F is ideal for aggregating Australian inputs and outputs series in the case with $t = 0, 1$, we are left with the question as to whether a fixed-based or a chained Fisher index should be used for extending the series to more than two periods:

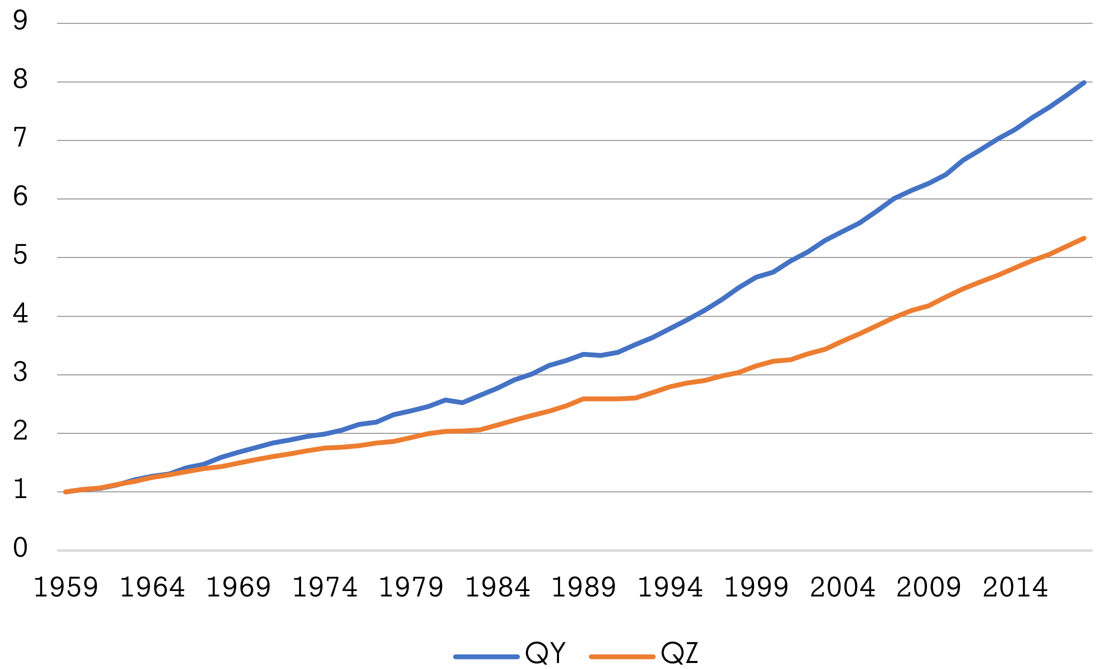
$$\text{Fixed-base: } Q_F^{t=1} = Q_F\left(p^0, p^1, y^0, y^1\right), Q_F^{t=2} = Q_F\left(p^0, p^2, y^0, y^2\right), \ldots \tag{12}$$

$$\text{Chained: } Q_F^{t=1} = Q_F\left(p^0, p^1, y^0, y^1\right), Q_F^{t=2} = Q_F\left(p^1, p^2, y^1, y^2\right), \ldots \tag{13}$$

Fixed-base breaks down if prices change over the long-run. In late period t there will be a large discrepancy between Q_L, which uses p^0 as base, and Q_P, which uses p^t as base. Q_F, the geometric mean of Q_L and Q_P, will then have a higher range of possible value to diverge from the true index. Chaining reduces the drift between Laspeyres and Paasche. Accordingly, Hill (1993) recommends chaining if prices and quantities are trending relatively smoothly across long periods of time. Fixed-base should be used for shorter periods with volatile prices, as chaining would exacerbate the short-term price jumps. This paper deals with annual series of highly aggregated data, hence chaining seems most appropriate.

From the Australian macro data we derive measures for five outputs – consumption (C), government expenses (G), investment (I), exports (X), imports (M) – and two inputs – labour (L), capital services (KS). Figure 1 shows the Fisher chained price indices for Australian inputs and outputs.

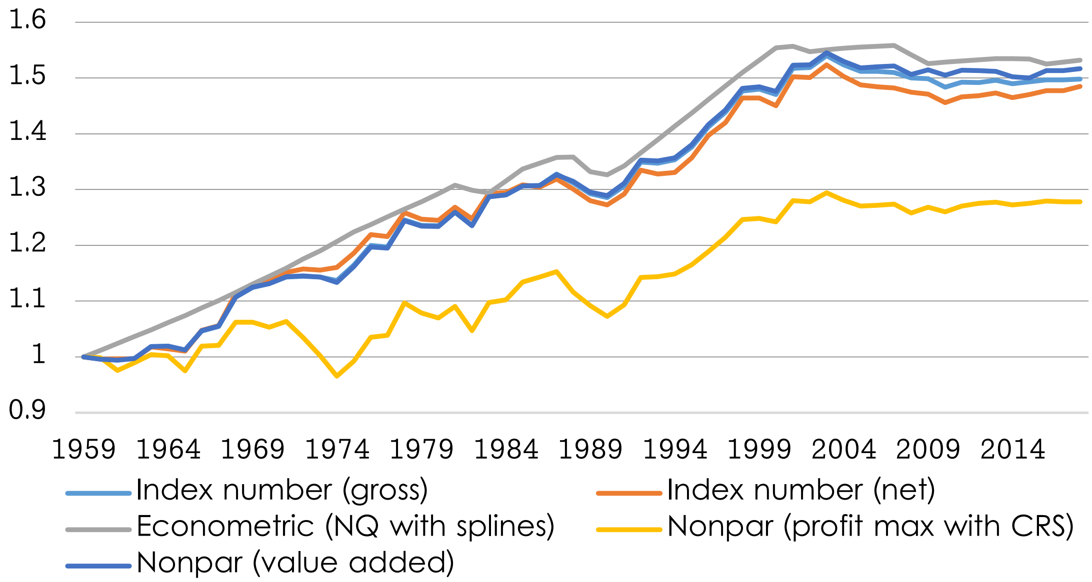
Figure 1: Fisher Chained Input and Output Quantity Indices



We can now compute Australian TFP growth from equation (4). The resulting TFPG index is shown as the light blue line in Figure 2. Average TFP growth is 0.69% per year.

The output and input quantity indices in Figure 1 and the TFP growth shown as the red line in Figure 2 are computed using gross investment as I and gross capital services as KS. In an economy, a share of the investment expenses of firms and governments goes towards increasing the stock of capital. But another share goes towards replacing obsolete capital. From use and passage of time, capital stocks depreciate, i.e. lose a proportion of their real value. In addition to the effect of depreciation, the real value of capital stocks also fluctuates through changes in real

Figure 2: Productivity Indices (1959 = 1)



asset price, PK/PC[3]. For instance, a decrease of PK/PC between the beginning and end of a period is equivalent to a negative obsolescence charge. Accordingly, real investment expense in that period, QI, has to compensate for the obsolescence from depreciation and asset price changes before resulting in net capital growth. But it is inadequate to treat the compensating share of investment in a period as income because in the long-run "consuming" that investment would lead to a full depreciation of the capital stock, dragging aggregate income down to 0. Thus we should adjust output by subtracting net depreciation (depreciation - asset price appreciation) from investment/output and from capital services input. This results in a smaller measure of both investment and capital services when depreciation

[3]The idea of treating variations in real asset prices as a source of changes in the real value of capital originates from Hayek (1935). It is commonly referred to as the maintenance of financial capital argument.

exceeds asset price appreciation.

In the gross framework, investment and capital services are:

$$VI_{gross} = PI \cdot QI \tag{14}$$

$$VKS_{gross} = PK \cdot R + PK \cdot D - PK \cdot (1 - D) \cdot IK \tag{15}$$

where the first term of KS_{gross} is capital waiting services, the second term is depreciation, and the third term is negative asset price appreciation.

In the net framework, we get instead:

$$VI_{net} = PI \cdot QI - PK \cdot D + PK \cdot (1 - D) \cdot IK \tag{16}$$

$$VKS_{gross} = PK \cdot R \cdot QK \tag{17}$$

The productivity growth computed from using these alternative definitions of investment and capital is shown as the orange line in Figure 2. The difference between net (0.68% mean growth) and gross (0.69% mean growth) productivity is slight, which is to be expected since both outputs and inputs decrease by the same amount under the net definitions.

See Appendix A for a sidebar on the implications of net vs. gross income for the evolution of labour shares in Australia.

1.2 Econometric Method

The second method employed to estimate technical progress is to add time trends in econometric models of producer supply and demand. Since we don't have a good way of isolating returns to scale from technical progress, we must assume CRS in order to get a reasonable estimate of technical progress. Thus the TFP growth obtained from the time trend "residuals" in an econometric model will be composed of the four factors from equation (3), technical progress being the only important one in the long-run.

To estimate producer supply and demand without arbitrarily restricting the resulting elasticities, we need to use cost or profit functions with flexible forms. Typically, these functions are computationally expensive as they have many parameters to allow for free determination of elasticities, but they are essential if we want our models to have any economic validity. We also require that the functions be homogeneous of degree 1 – i.e. such that $F(\lambda y) = \lambda F(y)$ – for CRS to hold. The two functions satisfying these criteria which we use are the translog and normalized quadratic functions[4]. Duality theory tells us that systems of supply and demand can be estimated with cost functions or variable profits functions, and that the latter can be derived from the former and vice versa. In this paper we use variable profit functions. We jump right to the profit functions, but note that historically these were derived from dual cost functions.

[4]See Chapter 9 of Prof. Diewert's lecture notes at `https://economics.ubc.ca/faculty-and-staff/w-erwin-diewert` for proofs of the flexibility of these forms

The translog unit variable profit function is defined as

$$\ln v^t(p) = \ln \frac{V^t(p,k)}{k^t} = \alpha_0 + \sum_{i=1}^{N} \alpha_i \ln p_i^t + \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \gamma_{ij} \ln p_i^t \ln p_j^t \tag{18}$$

where p is vector of output prices (in our case PC, PG, PI, PX and PM) and variable input prices (PL), and the parameters α_i and γ_{ij} satisfy the following restrictions:

$$\gamma_{ij} = \gamma_{ji} \tag{19}$$

$$\sum_{i=1}^{N} \alpha_i = 1 \tag{20}$$

$$\sum_{j=1}^{N} \gamma_{ij} = 0 \tag{21}$$

The restrictions ensure that the function is flexible and homogeneous in p. To capture technical change, we add time trends to equation (18):

$$\ln v^t(p) = \ln \frac{V^t(p,k)}{k^t} = \alpha_0 + \beta_0 t + \sum_{i=1}^{N} \alpha_i \ln p_i^t + \sum_{i=1}^{N} \beta_i t \ln p_i^t + \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \gamma_{ij} \ln p_i^t \ln p_j^t \tag{22}$$

where β_i satisfies the following additional restriction to ensure homogeneity in p:

$$\sum_{i=1}^{N} \beta_i = 0 \tag{23}$$

Even with restrictions guaranteeing homogeneity in p, estimating the variable profit function as a single linear equation leads to high collinearity, resulting in wide confidence intervals. More equations are needed. Hotelling's Lemma, $\frac{\partial V(p,k)}{\partial p_i} = y_i(p,k)$, allows us to add an equation for each output supply function y, with input L being treated as a negative output in the variable profit framework. Let the output share

s_i be defined as $s_i^t = p_i^t y_i^t / V^t$. Then,

$$
\begin{aligned}
s_i^t(k,p) &= \frac{p_i^t y_i^t(p,k)}{V^t(p,k)} && && (24)\\
&= p_i^t \frac{\partial V^t(p,k)}{\partial p_i} \frac{1}{V^t(p,k)} && \text{from Hotelling's Lemma} && (25)\\
&= p_i^t \frac{k \partial v^t(p)}{\partial p_i} \frac{1}{k v^t(p)} && \text{from CRS} && (26)\\
&= \frac{\ln v^t(p)}{\ln p_i} && && (27)\\
&= \alpha_i + \sum_{j \neq i}^{N-1} \gamma_{ij} \ln p_j^t + \beta_i t && && (28)
\end{aligned}
$$

with shares s_i adding up to 1. We thus need to drop the equation s_L from our model to ensure linear independence. Using the constraints from equations (19), (20), (21) and (23) above we can remove α_L, β_L, γ_{iL} and γ_{Lj} from our main equation (22). With these restrictions imposed and after adding error terms, we get our final system of six linear equations ready for econometric estimation:

$$
\begin{aligned}
\ln \frac{V^t(p,k)}{PL^t k^t} =& \alpha_0 + \beta_0 t + \sum_{i=1}^{5} \alpha_i \ln \frac{p_i^t}{PL^t} + \sum_{i=1}^{5} \beta_i t \ln \frac{p_i^t}{PL^t} + \frac{1}{2} \sum_{i=1}^{5} \gamma_{ij} (\ln \frac{p_i^t}{PL^t})^2 \\
&+ \sum_{i=1; i<j}^{5} \sum_{j=1}^{5} \gamma_{ij} \ln \frac{p_i^t}{PL^t} \ln \frac{p_j^t}{PL^t} + \epsilon_0^t
\end{aligned} \tag{29}
$$

$$
s_i^t(k,p) = \alpha_i + \sum_{j=1}^{5} \gamma_{ij} \ln p_j^t + \beta_i t + \epsilon_i^t \tag{30}
$$

where $p_1 = PC$, $p_2 = PG$, $p_3 = PI$, $p_4 = PX$, $p_5 = PM$. Modelling technical progress with non-varying coefficients is somewhat unrealistic, as exogenous sources of technical progress make it unlikely that it changes in a constant manner. Indeed, we observe strong autocorrelation of errors when we try to run equations (29) and (30). We can correct for this by adding linear splines to the time trends in the equations. The breakpoints for the splines are chosen as to limit autocorrelation.

Unfortunately, when estimating the six equations above with time trends, we find that curvature conditions do not hold. This means that the variable profit function $V(p,k)$ is not convex in p and so its estimation – with maximum likelihood in our case – does not ensure a solution that is a global optimum of the functions. Thus because we find that the Hessian of $V(p,k)$ w.r.t p is not positive semi-definite, the estimated parameters from a translog function cannot be trusted. Indeed, computing the own-price elasticities we see right away that something is awry. Table 1 shows the mean own-price elasticities for each component of $V(p,k)$, where both e_{CC} and e_{XX} have wrong signs.

Table 1: Own-price elasticities from translog estimation

	Mean
e_{CC}	-0.51
e_{GG}	0.26
e_{II}	0.67
e_{XX}	-0.54
e_{MM}	-1.10
e_{LL}	-0.36

We instead turn to the normalized quadratic form, which allows for the imposition of positive semi-definiteness to avoid the translog's curvature issues. The normalized quadratic unit variable profit function is defined as:

$$v^t(p) = \frac{V^t(p,k)}{k^t} = a^T p^t + b^T p^t t + \frac{1}{2}\left[\frac{p^{tT} B p^t}{\alpha^T p^t}\right] \tag{31}$$

where a and b are N-vectors of parameters and B is a NxN symmetric matrix of parameters with $Bp^* = 0_N$ for some p^*. $\alpha > 0$, $\sum_i^N \alpha_i = 1$ is a predetermined

N-vector for normalization of prices. From Hotelling's Lemma, and adding error terms, we can get one equation for each y_i, $i = 1, 6$:

$$\frac{y^t}{k^t} = \Delta_p V^t(p,k) = a + bt + B\overline{p}^t - \frac{1}{2}\overline{p}^{tT} B\overline{p}^t \alpha + e^t \tag{32}$$

where $\overline{p} = \frac{p^t}{\alpha^T p^t}$ is the vector or normalized prices.

For curvature conditions to hold, it is necessary and sufficient for the B-vector to be positive semi-definite.[5] To impose this, set $B = AA^T$ where $A^T p^* = 0_6$. Suppose that that $p^* = 1_6$ and B is a rank 1 matrix. Then, let $B = cc^T$ where c is a 6-vector of parameters to be estimated. So we need $c^T 1_6 = 0_6$, which implies $c_1 = -\sum_{i=2}^{N} c_i$.

We estimate the 6-equation model (32) iteratively, increasing the rank of matrix B until we lose convergence. At maximum rank 4, we add time trends in the B matrix add replace bt by linear splines as explained for the translog function. The resulting elasticities are displayed in Table 2.

Table 2: Own-price elasticities from NQ estimation

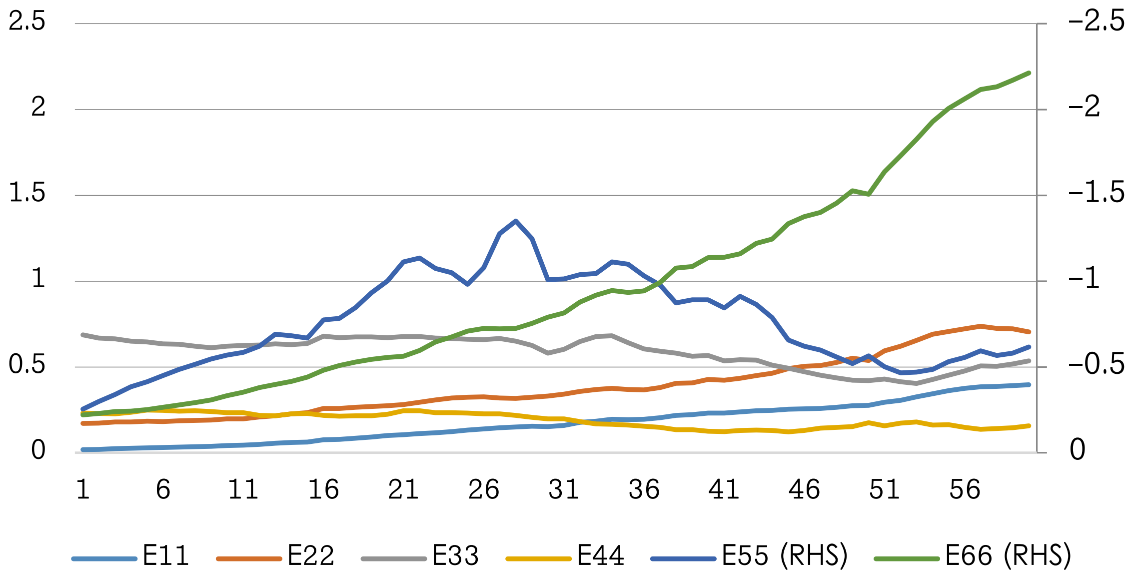
	Mean
e_{CC}	0.17
e_{GG}	0.38
e_{II}	0.59
e_{XX}	0.19
e_{MM}	-0.76
e_{LL}	-0.95

These all have the right size, as ensured by the imposition of a positive semi-definite Hessian. Additionally, the linear splines succeed in removing most of the

[5]See Diewert and Wales (1987) for proof.

autocorrelation in the residuals of our estimating equations. Figure 3 shows the historical trend of output supply and input demand elasticities. They seem to be relatively stable, except labour demand which becomes significantly more elastic across time, a trend consistent with globalization of supply chain. It is now easier to outsource tasks to other countries, so when wages increase in Australia firms can decrease their domestic hiring and replace Australian labour by foreign labour.

Figure 3: Mean Net Real Income Growth Factors



Having found a satisfying model, we can get back to the focus of this section and derive TFP growth estimates by differentiating the variable profit function w.r.t. t. It must be rescaled appropriately by for it to be comparable to our index number estimate:

$$
\begin{aligned}
TFPG^t &= \frac{\partial V^t(p,k)}{\partial t}\left(\frac{1}{V^{*t} - p_L^t y_L^{*t}}\right) \\
&= ((b^* \cdot p)k)\left(\frac{1}{V^{*t} - p_L^t y_L^{*t}}\right) \quad \text{with only a single time trend}
\end{aligned}
\tag{33}
$$

The exact equation becomes tedious once we add time trends in B and linear splines, so I don't reproduce it here.[6] The resulting TFPG index is shown as the grey line in Figure 2. It is somewhat higher (0.73% mean growth) than the index number TFPG, but still in the same range.

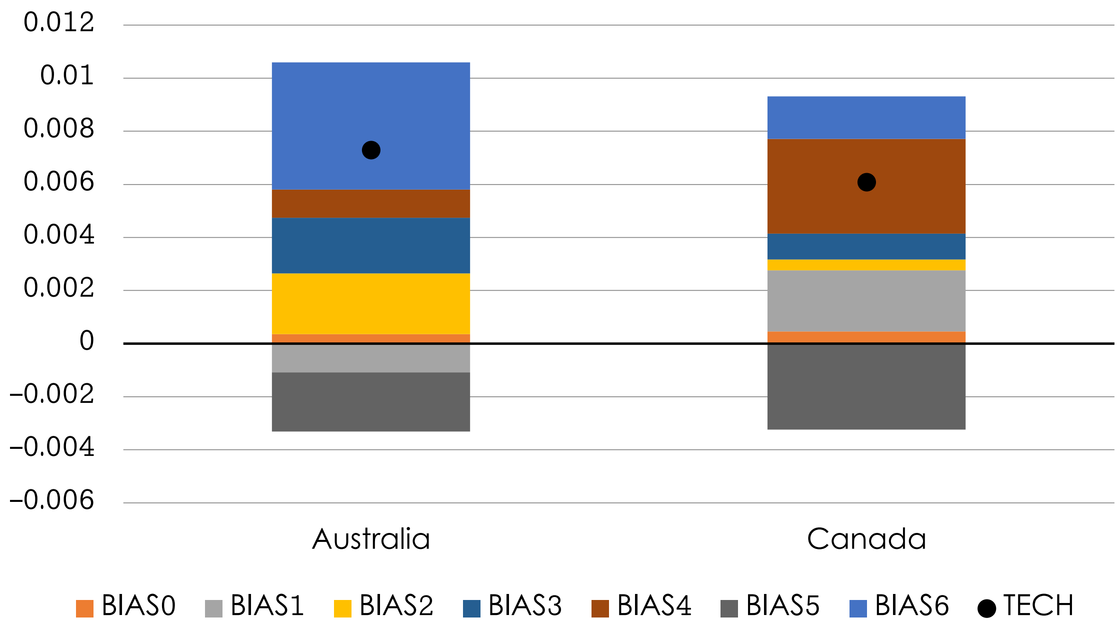
One of the benefits of estimating productivity growth through econometric rather than simple index number methods is that the structural model allows us to get a measure of the effect of technology growth on different components of outputs. Define $BIAS_i$ as the contribution of the change in net output i's price across time to overall TFPG. Then:

$$
BIAS_i = (b_i^* p_i k)\left(\frac{1}{V^{*t} - p_L^t y_L^{*t}}\right) \text{with only a single time trend} \tag{34}
$$

When we add time trends to the B matrix we will get a another bias term, $BIAS0$ that represents the contribution of changes in the B matrix to TFPG. We find this bias to be relatively small. Figure 4 shows the biases. We will come back to this figure in Section 2.

[6]See accompanying Shazam files for the detailed TFPG calculations.

Figure 4: TFP Price Biases



1.3 Nonparametric Method

The third method employed to measure productivity is the nonparametric approach to production theory introduced by Farrell (1957). Originally the method was meant to measure the relative efficiency of a cross-section of production units, yielding a ranking of the units by relative efficiency. In this paper, we apply the method in a different context. Rather than a cross-section of production units, we compare a unique "production unit" (the country of Australia) across multiple years. Assuming the country produces on the PPF every year[7], the difference in relative "efficiency" between two observations in time can only be explained by TFP growth. So while the cross-sectional context assumes shared technology but differences in efficiency,

[7] We will relax this assumption later in this section

in the time-series context we assume efficiency and attribute higher input-output ratios to worse technology. In what follows I keep the "efficiency" terminology from the original application, even though we should be talking about productivity rather than efficiency.

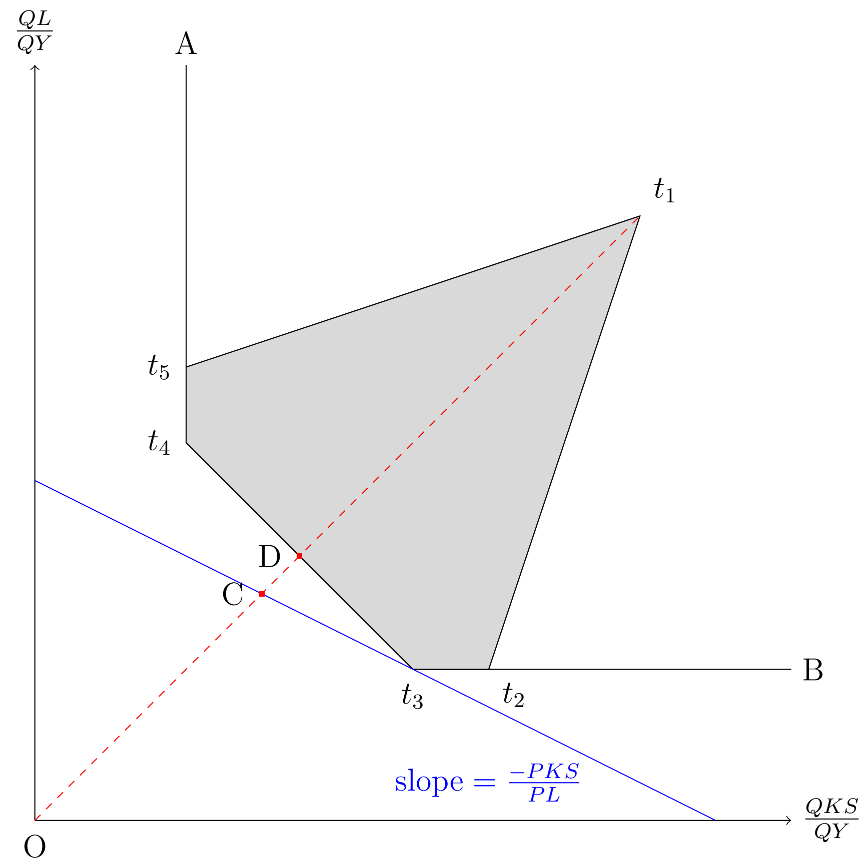
In our data, two inputs (KS, L) are used to produce five output types (C, G, I, X, M). For now suppose that we aggregate these five outputs into a quantity index Y. Let $\frac{QL^t}{QY^t}$ and $\frac{QKS^t}{QY^t}$ be the observed input-output coefficient for each input in year t. A lower ratio implies more output for the same input quantity. This only works assuming CRS, as each production-year can be fully described by input-output coefficients without regards to the size of the numerator (input) and denominator (output). This assumption will be relaxed later. Figure 5 illustrates an example distribution of five production years ($t = 1, ..., 5$) with $\frac{QL}{QY}$ as the y-axis and $\frac{QKS}{QY}$ as the x-axis.

The shaded set is the convex hull of the five observations t_1,...,t_5 and every point to the north and east of the convex hull – i.e. northeast of the A-B axis – is the convex free disposal hull. To be *technically efficient* a production-year must be on the frontier of the free disposal hull.[8] Otherwise, a production-year could produce the same quantity of output with less inputs by moving to the southwest towards the frontier.

Here only t_1 does not lie on the frontier. By moving to the feasible point D on the frontier, production-year t_1 achieves higher efficiency. $\frac{\mathrm{OD}}{\mathrm{O}t_1}$ is a measure of the

[8]An issue here is that points t_5 and t_4 are both on the frontier and are thus equally efficient in this context, even though t_4 is obviously preferable to t_5 as it produces the same output with less QL. We don't deal with this issue, which will become immaterial once we introduce price data.

Figure 5: Example Distribution of I-O Coefficients for Five Production Years

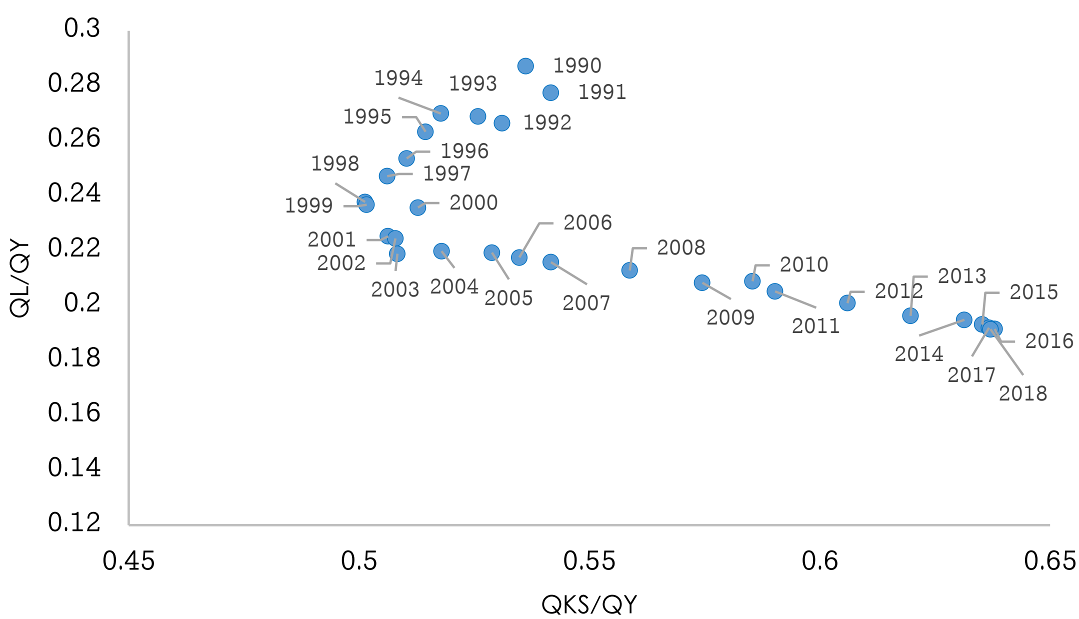


technical inefficiency of year $t = 1$.

This is the most stringent nonparametric efficiency measure we can apply for a two-input one-output framework with only quantity measures. Adding input price data to the mix, we can now discriminate between points on the frontier. Suppose we have a family of isocost curves parrallel to the blue line in Figure 5, which is the lowest of the isocost touching a feasible point in the hull. This tangent isocost curve shows that t_3 is the most efficient year, as it uses the least costly *mix* of inputs to produce its output. Hence, t_1 is not only technically inefficient, it is also *price inefficient*. Moving from t_1 to D would eliminate the technical inefficiency, but the resulting production process would still lie above the lowest isocost. $\frac{\text{OC}}{\text{OD}}$ is a measure of the price efficiency of year $t = 1$ and $\frac{\text{OC}}{\text{O}t_1}$ is measure of *overall efficiency*.

Before relaxing the one-output and CRS assumptions and going into the formal definitions, we give a visual representation of the Australian distribution of I-O coefficients. Figure 2 shows a Figure 5-type graph using our Australian data for 1990-2018. It looks like most production-years lie on the hull frontier, so we will probably not get good efficiency discrimination without using price data. We will come back to Figure 2 later, as it is an insightful illustration of the recent path of productivity.

Figure 6: Input-Output Coefficients for Australia (1990-2018)



The following formal extensions of Farrell's methodology can be found in Mendoza (1989). Let $\{y_1^t,\ldots,y_5^t,x_1^t,x_2^t\}=\{QC,QG,QI,QX,-QM,QL,QKS\}$. We define δ_i^* as the smallest possible fraction of the i^{th} input vector (x_1^t,x_2^t) such that $(y^t,\, \delta_t^{*CRS} x^t)$

is on the frontier spanned by the convex free disposal hull of the T observations. In Figure 5, δ_t^{*CRS} is a measure of the distance of the production-year i from the frontier. $\delta_t^{*CRS} = 1$ implies production-year t is on the frontier. At least one of the T observations will be on the frontier. The lower δ_t^{*CRS}, the higher the productivity gap between production-year t and the most productive year(s). δ_t^{*CRS} is obtained by solving the following linear programming problem.

$$\begin{aligned}\delta_t^{*CRS} = \min_{\delta_t \geq 0, \lambda_1 \geq 0, \ldots, \lambda_T \geq 0} \{\delta_t : \sum_{s=1}^{T} y_1^s \lambda_s \geq y_1^t; \sum_{s=1}^{T} y_2^s \lambda_s \geq y_2^t; \sum_{s=1}^{T} y_3^s \lambda_s \geq y_3^t; \\ \sum_{s=1}^{T} y_4^s \lambda_s \geq y_4^t; \sum_{s=1}^{T} y_5^s \lambda_s \geq y_5^t; \sum_{s=1}^{T} x_1^s \lambda_s \leq \delta_t x_1^t; \\ \sum_{s=1}^{T} x_2^s \lambda_s \leq \delta_t x_2^t; \}\end{aligned} \tag{35}$$

CRS is imposed as there are no restrictions on the λ_t's. To relax this assumption and allow for decreasing returns to scale we add the restriction $\sum_{s=1}^{T} \lambda_s = 1$.

$$\begin{aligned}\delta_t^{*} = \min_{\delta_t \geq 0, \lambda_1 \geq 0, \ldots, \lambda_T \geq 0} \{\delta_t : \sum_{s=1}^{T} y_1^s \lambda_s \geq y_1^t; \sum_{s=1}^{T} y_2^s \lambda_s \geq y_2^t; \sum_{s=1}^{T} y_3^s \lambda_s \geq y_3^t; \\ \sum_{s=1}^{T} y_4^s \lambda_s \geq y_4^t; \sum_{s=1}^{T} y_5^s \lambda_s \geq y_5^t; \sum_{s=1}^{T} x_1^s \lambda_s \leq \delta_i x_1^t; \\ \sum_{s=1}^{T} x_2^s \lambda_s \leq \delta_i x_2^t; \sum_{s=1}^{T} \lambda_s = 1\}\end{aligned} \tag{36}$$

The optimal solution for (36) is feasible for (35) as CRS is more restrictive, thus we have $\delta_t^{*CRS} \leq \delta_t^{*}$, but the increase in productivity discrimination by adding CRS is smaller in a time-series context than in a cross-sectional one as there are no jumps in scale from year to year. The first two columns of Table 3 show that even by imposing CRS, most production-years are estimated to lie on the frontier. We cannot get satisfactory results using quantity data only.

Two approaches can be used to estimate productivity nonparametrically using both quantity and price data. We begin by assuming that Australian production is determined by cost minimization and then by profit maximization.

We assume the producer faces input prices $\{w_1^t, w_2^t\} = \{PL^t, PKS^t\}$. Define ε_t^{*CRS} as the fraction of the input vector (x_1^t, x_2^t) such that $\varepsilon_t^{*CRS}(x_1^t, x_2^t)$ is on the minimum cost isocost line for the observation t, i.e. ε_t^* is analogue to the measure $\frac{\mathrm{OC}}{\mathrm{O}t_1}$ from Figure 5. Formally,

$$\begin{aligned}\varepsilon_t^{*CRS}[w_1^t x_1^t + w_2^t x_2^t] \equiv \min_{\lambda_1 \geq 0, \ldots, \lambda_T \geq 0} \{ & w_1^t (\sum_{s=1}^{T} x_1^s \lambda_s) + w_2^t (\sum_{s=1}^{T} x_2^s \lambda_s) : \sum_{s=1}^{T} y_1^s \lambda_s \geq y_1^t; \\ & \sum_{s=1}^{T} y_2^s \lambda_s \geq y_2^t; \sum_{s=1}^{T} y_3^s \lambda_s \geq y_3^t; \sum_{s=1}^{T} y_4^s \lambda_s \geq y_4^t; \sum_{s=1}^{T} y_5^s \lambda_s \geq y_5^t; \}\end{aligned} \tag{37}$$

The optimal solution for (35) is feasible for (37) so $\delta_t^{*CRS} \leq \varepsilon_t^{*CRS}$. Similarly to the quantity-only model above we can relax the CRS assumption by adding the restriction $\sum_{s=1}^{T} \lambda_s = 1$ to get the productivity fraction ε_t^*. The third and fourth columns of Table 3 show that cost minimization is still not restrictive enough to get meaningful differences in productivity. We turn to profit maximization.

We add output prices $\{p_1^t, ..., p_5^t\} = \{PC^t, PG^t, PI^t, PX^t, PM^t\}$ to the set of variables. Define α_t^{*CRS} as the the fraction of the input vector (x_1^t, x_2^t) such that $\alpha_t^{*CRS}(x_1^t, x_2^t)$ is on the minimum isocost line for the observation i, but taking into consideration changes in the relative importance of output components for aggregate output value due to variations in output prices. As opposed to δ_t and ε_t above I will start by exposing the non-CRS version of α_t. Formally,

$$\sum_{m=1}^{5} p_m^t y_m t - \alpha_t^*[w_1^t x_1^t + w_2^t x_2^t] \equiv \max_{\lambda_1 \geq 0, \ldots, \lambda_T \geq 0} \{ \sum_{m=1}^{5} p_m^t (\sum_{s=1}^{T} y_m^s \lambda_s) - \sum_{n=1}^{2} w_n^t (\sum_{s=1}^{T} x_n^s \lambda_s) : \sum_{s=1}^{T} \lambda_s = 1 \} \tag{38}$$

Proceeding as for δ_t and ε_t above and removing the restriction $\sum_{s=1}^{T} \lambda_s = 1$ from the constraint to impose CRS leaves the set of solutions unbounded. Instead we solve the conditional profit maximization problem that imposes capital to be fixed in the short run:

$$\begin{aligned} \sum_{m=1}^{5} p_m^t y_m^t - \alpha_t^{*CRS}[w_1^t x_1^t + w_2^t x_2^t] &\equiv \max_{\lambda_1 \geq 0, \ldots, \lambda_T \geq 0} \{ \sum_{m=1}^{5} p_m^t (\sum_{s=1}^{T} y_m^s \lambda_s) - \sum_{n=1}^{2} w_n^t (\sum_{s=1}^{T} x_n^s \lambda_s) : \sum_{s=1}^{T} x_2^s \lambda_s \leq x_2^t \} \\ &= \max_s \{ \left[\sum_{m=1}^{5} p_m^t y_m^s - (\sum_{n=1}^{2} w_n^t x_n^s) \right] [x_2^t / x_2^s] : s = 1, 2..., 25 \} \end{aligned} \tag{39}$$

It can be seen that the optimal solution for 35 is a feasible solution for 39. Hence $\delta_t^{*CRS} \geq \alpha_t^{*CRS}$ is established. The last two rows of Table 3 show that empirically both α_t^* and α_t^* result in better productivity discrimination between years than quantity-only and cost-minimization. To improve comparability with the results of our econometric model (see Section 1.2 above) which assumes CRS, we settle on α^{*CRS}. 2003 is found to be the only year on the frontier, which is consistent with our other productivity measurement methods. Estimating α^{*CRS} for the whole sample period (1959-2018) and converting it to a productivity index, we get TFPG as shown by the yellow line in Figure 2. It has the same shape as but is much lower than our other TFPG measures.

Table 3: Productivity Fractions for Australia (1995-2018)

	δ_i^* (Convex)	δ_i^{*CRS} (Convex/CRS)	ε_i^* (Convex/Cost)	ε_i^{*CRS} (Convex/Cost/CRS)	α_i^* (Convex/Profit)	α_i^{*CRS} (Convex/Profit/CRS)
1994	1	1	1	1	0.8452	0.88734
1995	1	1	1	1	0.8678	0.90053
1996	1	1	1	1	0.8952	0.91857
1997	1	1	1	0.9928	0.9237	0.93856
1998	1	1	1	1	0.9557	0.963
1999	1	1	1	0.9926	0.9594	0.96472
2000	1	1	1	1	0.9557	0.95991
2001	1	1	1	1	0.9885	0.98934
2002	1	1	1	1	0.9871	0.98756
2003	1	1	1	1	1	1
2004	1	1	0.9986	0.9986	0.9902	0.98969
2005	1	0.9959	0.9951	0.9951	0.9833	0.98169
2006	1	1	1	1	0.985	0.98278
2007	1	1	1	1	0.9871	0.98444
2008	1	0.998	1	0.996	0.9782	0.97214
2009	1	1	0.9993	0.9975	0.9852	0.98019
2010	1	1	1	0.9992	0.981	0.97344
2011	1	1	1	1	0.9873	0.98147
2012	1	1	1	1	0.9904	0.98522
2013	1	1	1	1	0.9918	0.98671
2014	1	1	0.9991	0.999	0.9901	0.98334
2015	1	1	1	1	0.9915	0.98526
2016	1	1	1	1	0.9936	0.98848
2017	1	1	1	1	0.9931	0.98739
2018	1	1	1	1	0.9933	0.98738

While the Farrell/Mendoza models use convexity assumptions to draw the productivity frontier and estimate the distance between production-years and that frontier, the last nonparametric model used in this paper relaxes this assumption. Diewert and Fox (2017) eschew convexity in favour of the assumption of a free disposal hull (FDH). Free disposability means if a specific pair of input and output is producible, any pairs of more input and less output are also producible. In our case, the FDH technology set S^t is made up of only current and past observations. We will see that because this set can only expand as time goes on, technical growth can only be positive. Under this assumption, for price vectors p and w and input quantity vector x, define the period t nonparametric cost-constrained value added function $R^t(p, w, x)$ for the production unit as follows:

$$r^t(p, w, x) \equiv \max_{\lambda_1 \geq 0, \ldots, \lambda_T \geq 0} \{p \cdot (\sum_{s=1}^{T} y^s \lambda_s) : w \cdot (\sum_{s=1}^{T} x^s \lambda_t) \leq w \cdot x\} \tag{40}$$

which is a nonparametric approximation to the cost-constrained value added function

$$R^t(p, w, x) \equiv \max_{y,z} \{p \cdot y : (y, z) \in S^t; w \cdot z \leq w \cdot x\} \tag{41}$$

In general $p^t \cdot y^t \leq r^t(p, w, x)$ with $p^t \cdot y^t = r^t(p, w, x)$ if the production-year is efficient. Accordingly, define value-added efficiency growth as:

$$\epsilon^t \equiv \frac{p^t \cdot y^t}{r^t(p^t, w^t, x^t)} / \frac{p^{t-1} \cdot y^{t-1}}{r^{t-1}(p^{t-1}, w^{t-1}, x^{t-1})} \tag{42}$$

Recall from equation (3) of Section 1 that excluding the effects of unobserved inputs, TFP growth can be described as the product of technical progress, efficiency growth, and input mix changes. Diewert and Fox (2017)'s methodology, which assumes CRS, allows us to obtain that decomposition. In fact, value-added efficiency growth ϵ in (42) is EG in (3). By separating value-added into its components, Diewert and Fox show that $TFPG$ can be decomposed in the following way[9]:

$$TFPG^t = \epsilon^t \gamma^t \tau^t \tag{43}$$

where efficiency growth ϵ^t is defined above, input mix change γ^t is defined as

$$\gamma^t \equiv \frac{R^t(p, w^t, x)}{R^t(p, w^{t-1}, x)} \tag{44}$$

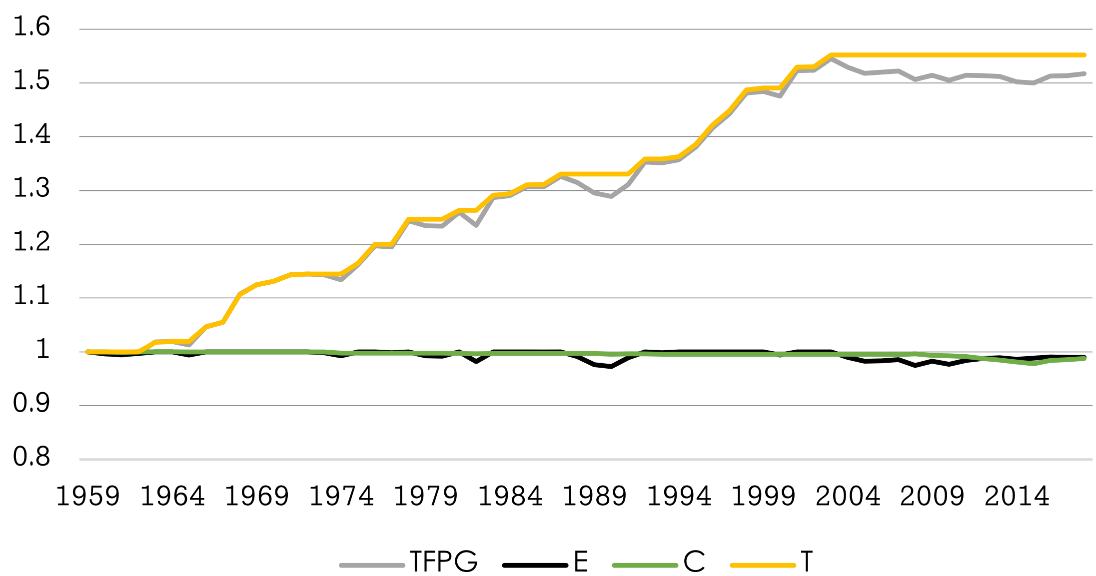
[9] I do not prove this here as the proof is intuitive but takes up space. See Diewert and Fox (2017) for the steps to obtain this decomposition.

and technical change τ^t is defined as

$$\tau^t \equiv \frac{R^t(p, w, x)}{R^{t-1}(p, w, x)} \tag{45}$$

Figure 7 shows the growth rates of the variables from (43) after converting them to a growth index. As imposed by the backward-looking FDH assumption, technical change index T is non-decreasing. The blue line in Figure 2 shows $TFPG$ from Figure 7 next to the other productivity measures derived above.

Figure 7: Nonparametric Value Added Decomposition Indices (1959 = 1)



In the end, every productivity measure in Figure 2 is within a small interval, somewhere around 0.70% average annual productivity growth. Only the convex profit-maximization nonparametric measure diverges. Being fairly confident in our productivity estimate of 0.70%, we can use it as a tool in the following Australian growth analysis.

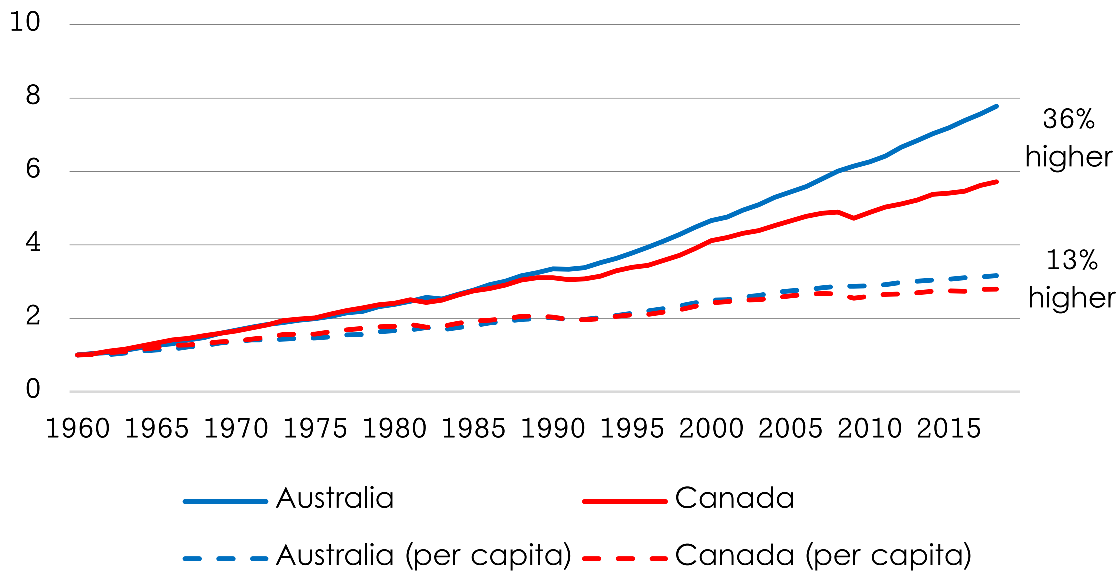
2 Sustainable Growth or Macro Mirage?

Figure 8 shows the growth of Australian GDP and GDP per capita. Canada, a small OECD open economy that has followed relatively smooth growth path, acts as a reference. Most noticeably, as previewed in the Introduction, Australian GDP has been increasing monotonically since 1990, the year of its last (minor) recession. In addition, more than simply avoiding recessions, Australian has been on a distinctly higher growth path than Canada (and most other OECD countries) after 1990, while its growth path perfectly traced that of Canada before 1990. While a portion of this higher growth is explained by stronger population growth, the dashed lines in Figure 8 show that GDP per capita, which removes the effect of population growth in the long-run, has also grown faster in Australia after 1990. The usual casual explanation behind the Australian economy's solid performance is that after years of distortionary import substitution policies, the Australian proceeded to a series of liberal policies in the late-1980s. These started to take effect in the 1990s, as the new dynamic business environment spurred investment and gave a boost to productivity. Is this accurate? And if so, has the effect carried to the 2000s and 2010s so as to explain the recent economic success?

2.1 Investment and Productivity

Turning to our measures of productivity in Figure 2, it is clear that productivity growth in the 1990s was indeed very strong. With an average annual TFP growth of 1.2%, the 1990s have the strongest TFP growth of any decades for which we have

Figure 8: Australian vs. Canadian GDP and GDP per capita growth (1960 = 1)



data.[10] This figure is also high above the average OECD TFP growth for the 1990s. For instance, Canada's was 0.9%. It seems like the reforms of the late-1980s did stimulate TFP growth in the medium-run. But Figure 2 also shows that the spurt was short-lived. In the 2000s and 2010s, TFP stopped growing. In 2018, TFP is still below its 2003 peak. Hence, productivity cannot explain the unusually high and smooth GDP per capita growth of the last two decades.

Figure 9 shows the growth path of labour and capital services inputs from 1960 to 2018. As suspected from looking at Figure 8, a portion of the Australian GDP growth is due to high labour growth. After decades of high immigration, Australia

[10] I refer to the gross index number TFP growth measure (light blue line), the median TFP measure from Section 1. The numbers would be similar with the other measures, except for the outlier convex profit-maximization nonparametric measure (yellow line).

has today the highest immigrant-to-population ratio among OECD countries.[11] But what seems to be a major factor in the substantial GDP *per capita* growth is the remarkably high capital input growth.[12] It seems that after TFP growth slowed down in the beginning of the 2000s, capital input growth sped up, concealing the negative effect of the productivity stagnation. Figure 6 illustrates this change in dynamics between the 1990s and 2000s. In the 1990s input-output ratios for both labour and capital services trended downwards, reflecting TFP growth. But starting in the 2000s $\frac{QKS}{QY}$ started increasing, growing from 0.51 in 2001 to 0.64 in 2018, while $\frac{QL}{QY}$'s downward trend waned somewhat. We can then conclude that contra expectations, the fast capital accumulation of the last 20 years did not significantly increase labour productivity. It seems hard to justify the capital accumulation in that case. Below, I make the case that the strong capital accumulation was driven by the commodity exports sector and that it could be one of the causes of the observed productivity stagnation.

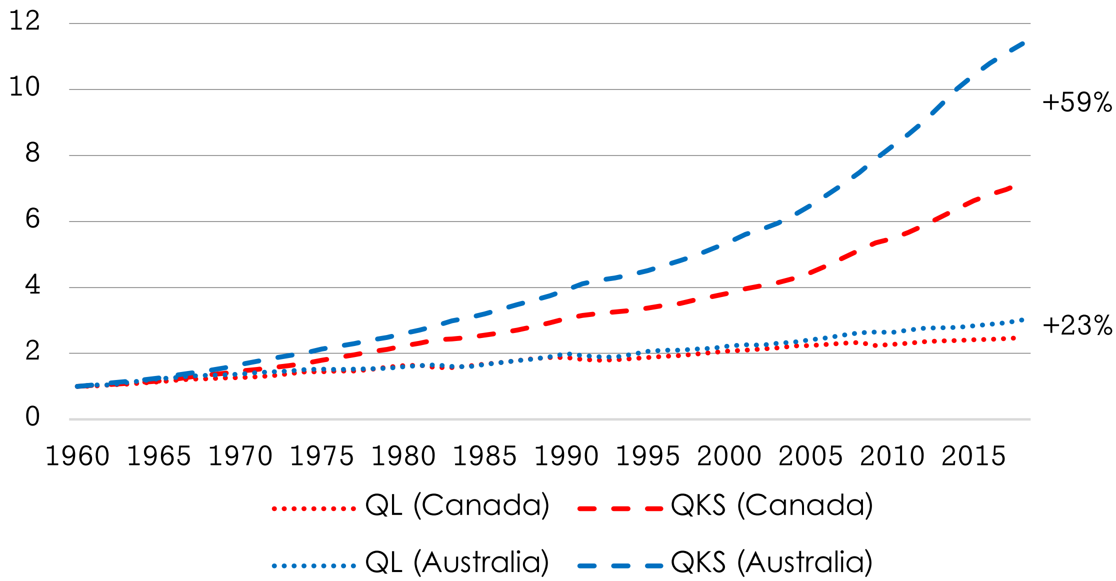
2.2 Commodity Prices

Since the beginning of our sample in 1960, trade has progressively become a major part of the Australian economy, undoubtedly helped along by the trade liberalization that start during the aforementioned reforms of the late-1980s. Figure 10 shows that growth of exports and exports has by far exceeded growth of the other three output components. Growth in export volumes was highest in the 1990s and late-2000s.

[11] United Nations (2015), *Trends in International Migrant Stock: The 2015 Revision.*

[12] Because the flow of capital services is approximately proportional to the productive capital stock in place, it is possible to talk interchangeably of growth in capital services and the same growth in productive stock.

Figure 9: Labour and Capital Services Volumes (1959 = 1)

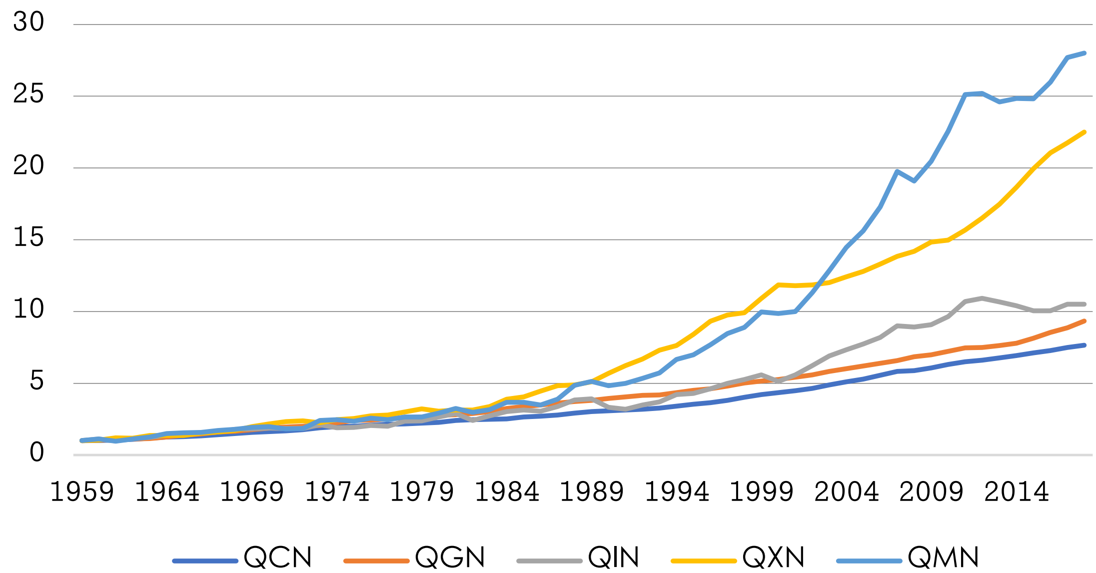


The lion's share of Australian exports is made up of commodity exports, as Figure 11 highlights. Metals and minerals make up two-thirds of all exports. Accordingly, Australia's economy is highly sensitive to demand patterns for metals. When the prices of iron and gold – two of Australia's main commodity products – surged in the late 2000s, the price of their overall exports followed. Figure 12 shows Australia's terms of trade (TOT) increasing by 50% between 2005 and 2011. For the income of a small open economy, an increase in terms of trade has an analogous effect to that of a productivity increase. So the jump in TOT undeniably contributed to Australia avoiding recession following the global financial crisis of 2008.[13]

There are some hints that the commodity exports sector is also responsible for the

[13]The financial crisis in fact contributed to the surge in Australia's TOT. Fears of deflation that accompanied the crisis spurred the rise in the price of gold.

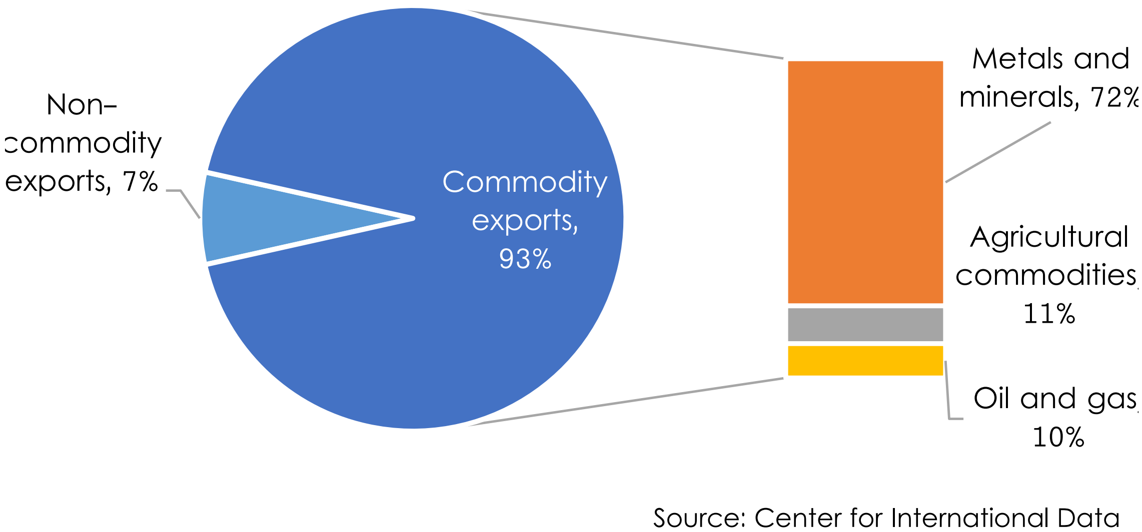
Figure 10: Output Volumes (1959 = 1)



exceptional capital input growth revealed in Section 1.1. First, the growth pattern of investment seems to be mirroring the path of TOT. High metals prices prompted firms to open new mines, which require huge initial capital outlay. Second, the growth in investment in the 2000s does not seem to have pushed downed real rates of return (RR), even if it did not result in higher labour productivity as exposed above. Figure 13 shows the path of RR.[14] The only explanation for increased investment not pushing down RR without productivity improvement is that firms' profits were buoyed by higher output prices. Dean Parham (2012) arrives at this conclusion by showing that the increase in profitability was mostly driven by an increase in

[14]See accompanying Shazam file for more detail on this and other variables' derivations. These rates of return are too high because they do not include inputs other than capital and labour (see beginning of section 1 for discussion). Here what matters to us is the change in RR rather than the absolute numbers, so the issue somewhat less problematic.

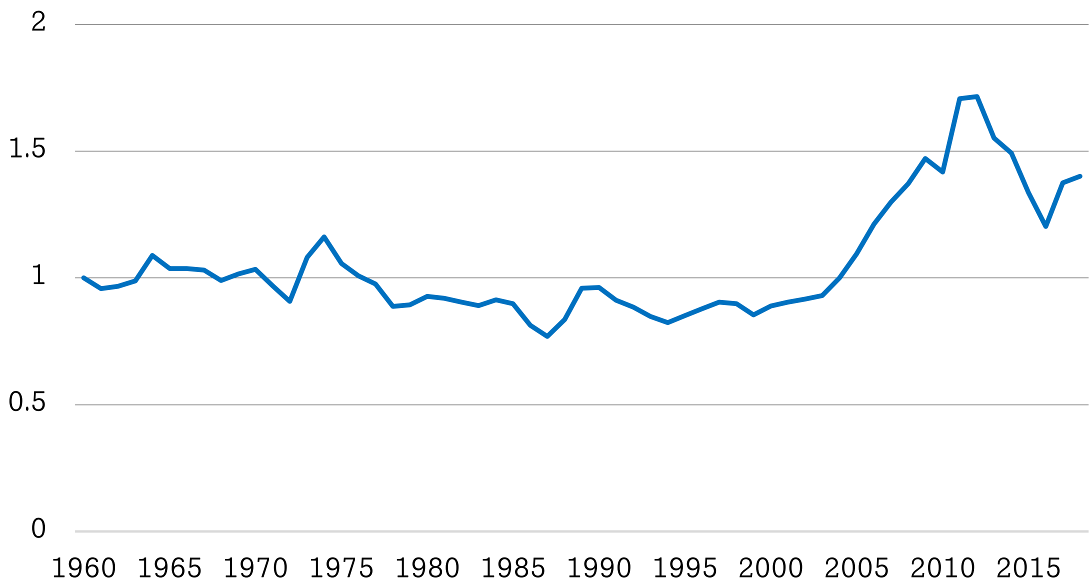
Figure 11: Australian Exports by Category (%)



the output price of the mining sector. He also provides industry-specific input data showing that mining increased its capital stock at annual average rate of 8.0% during the strongest of the boom period (2003-2009). The sector was responsible for 25% of aggregate capital accumulation during that period. Furthermore, because they are built on mining sites farther down the priority list (smaller size deposits, more remote sites, etc.), new mines are less efficient than old mines. Hence the opening of new mines decreases the overall productivity of the mining sector. This could explain a part of the productivity slowdown after 2003.

And of course building up capital stock in response to inflated output prices becomes an issue if these prices drop. This is another factor pointing towards Australia's growth being potentially unsustainable in the long-run.

Figure 12: Terms of Trade (1960 = 1)

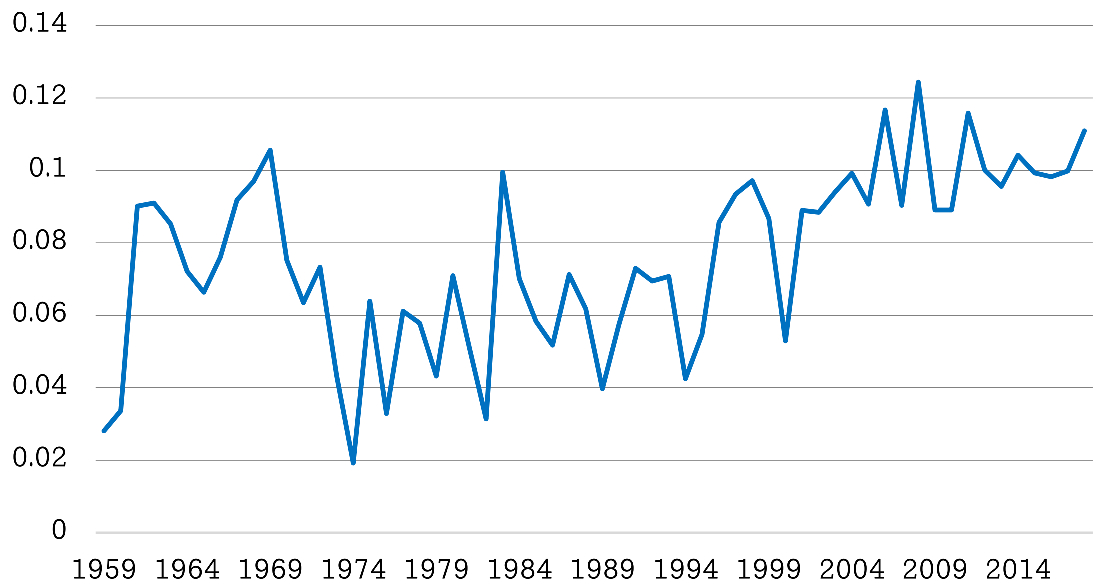


2.3 Real Income and Consumption Inflation

Australian GDP growth in our sample also overstates the gains in consumer well-being. Figure 1 shows Australian and Canadian growth in real income (RI), which measures the volume of consumption bundles afforded by a country's production.[15] We see that growth of RI per capita has been higher in Canada than Australia. Comparing this to GDP per capita growth in Figure 8, it is clear that RI growth has not followed GDP growth. Starting the graphs in 2000 would show the same pattern of subdued RI growth, albeit exhibiting less of a divergence with output growth. Thus while Australian national income has climbed steeply in our sample,

[15] Real income is nominal GDP deflated by the household expenditure deflator. Here we show the growth of net real income, having shown in Section 1.1 that it is preferable to gross income when measuring the consumption potential of a country. The pattern is similar with gross income.

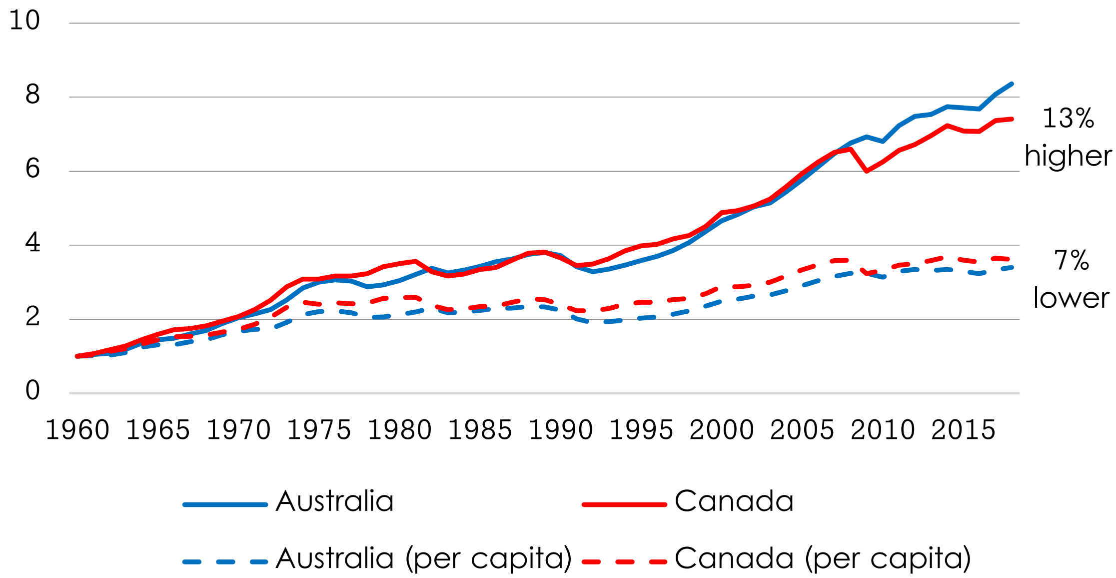
Figure 13: Real Rate of Return



the consumption potential of that income has been much less impressive.

Rising consumption prices are behind the lagging RI. Figure 15 shows the price growth of output components. Government output price inflation highest, with PG increasing 21-fold since 1959 (orange line). This is consistent with the experience of other OECD countries, as labour-intensive government services have suffered from the high labour price growth that accompanied the labour productivity gains and low fertility of developed countries in the last 70 years. But consumption inflation is not far behind, having increased 15-fold since 1959 (dark-blue line). This is more peculiar. For the majority of OECD countries, historical consumption inflation is in the same range as import, export and investment price inflation. But Australia exhibits PC growth well above that of these components. The divergence started in the 80s, but has accelerated since 2000.

Figure 14: Net Real Income (1960 = 1)

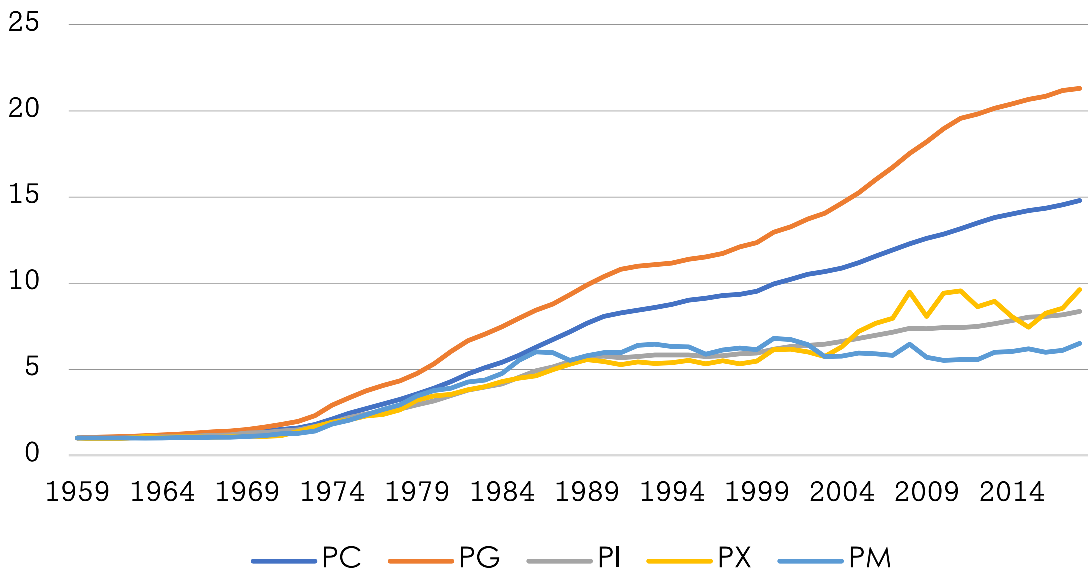


Diewert and Morrison (1986) and Kohli (1990) show that it is possible to decompose real income growth into its contributing factors with index number theory, making only light assumptions.[16] Figure 16 shows the Diewert-Morrison real income decompositions for Australia and Canada. The brown area represents capital input quantity growth and the dark blue area represents labour input quantity growth. Unsurprisingly after the analysis done in Section 2.1, the contribution of input quantity growth to real income growth is much higher for Australia than Canada. The contribution of productivity (orange area) is similar in both countries.

But the contribution of output price growth is lower. The inflation of output components other than consumption contributes positively to real income growth

[16] I do not go over the details of the method here. See Diewert and Morrison (1986) for complete proofs.

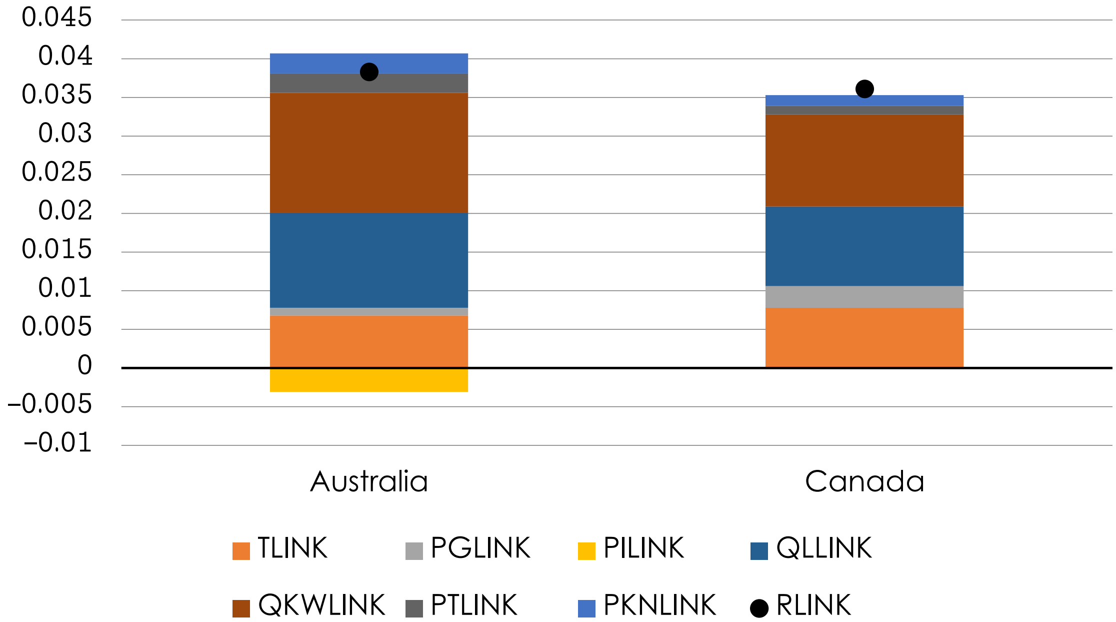
Figure 15: Output Prices (1959 = 1)



because an increase in the price of non-consumption expenses relative to consumption expenses implies that the economy can afford additional bundles of consumption from the income of other sectors of the economy. In Figure 16, the light grey area representing the contribution of PG is smaller, meaning its contribution to RI growth is weaker than in Canada. Most strikingly, the contribution of PI (yellow area) is negative. The dark grey area representing changes in terms of trade is bigger than in Canada, entirely explained by the large increase in TOT in the 2000s (see Section 2.2).

A final way of thinking of the high consumption inflation and its impact on the Australian economy is through the biases in technical change computed through our econometric TFPG estimation in Section 1.2. Figure 4 shows that the time trend coefficient for consumption is negative, i.e. that technical change has a negative

Figure 16: Mean Net Real Income Growth Factors



consumption bias for Australia (BIAS1). Hence technical change is consumption-diminishing.

We now know that consumption price inflation is responsible for the low RI growth. But which consumption category is most to blame? The next subsection explores disaggregated consumption expenses data to answer this question.

2.4 Land

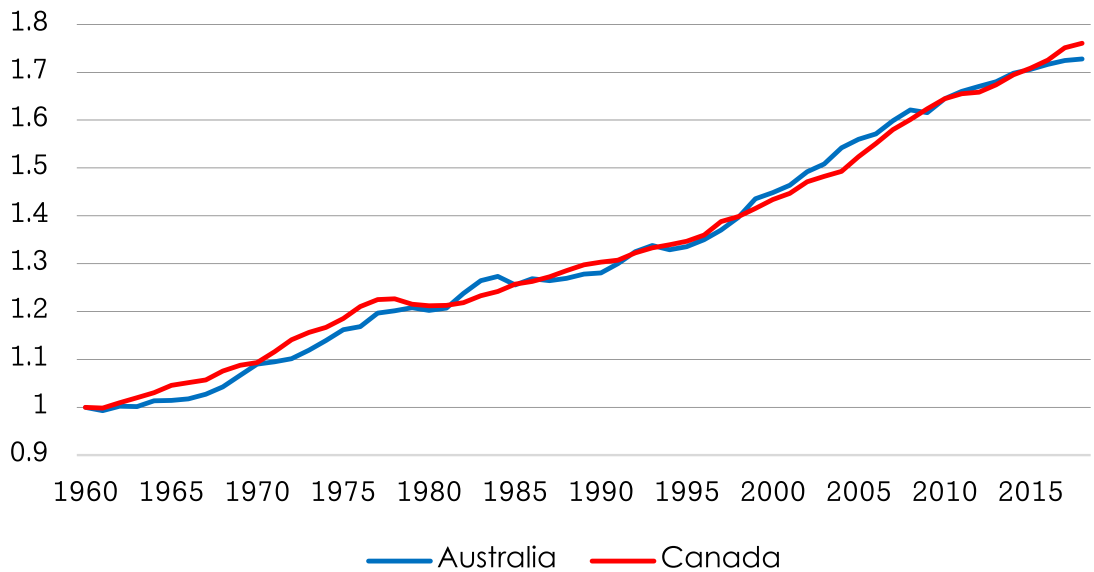
In addition to the producer econometric models characterized in Section 1.2, we estimate two consumer-side econometric models. The first is a consumption-leisure model and the second a 12-good disaggregated consumption model. We do not spell out the details of the estimating equations as they are mostly analogous to the producer model equations, replacing production with utility and input quantities and prices by consumption category quantities and prices. We use the same functional form as in Section 1.2, the normalized quadratic form, as it does not arbitrarily restrict elasticities and curvature conditions (concavity here) can be imposed. The only major difference between our producer and consumer models is that in the former we maximized profit while for the latter we solve the dual problem, minimizing cost instead. By Sheppard's Lemma, we get the same result from both optimization problems, but profit maximization was more appropriate to implement with our producer data.[17] We assume a given utility (income) level for each year, defined as a Fisher index of consumption and leisure. The growth index of the resulting utility series for Australia and Canada is shown in Figure 17. We see that similarly to RI, consumption-leisure utility per capita has risen more for Canada than Australia over the sample, even though GDP per capita growth has been much stronger in Australia. This reinforces the arguments from Section 2.3.

We should not expect a static time-series labour-leisure model with 70 years of data to yield great results, and indeed it doesn't. The resulting price elasticities are way too low, constrained by the imposed concavity. Furthermore, the income elasticity for leisure is found to be negative, making leisure an inferior good. While this is theoretically possible, it makes little intuitive sense, as it is unlikely that Australians would respond to extra income by working more. These elasticities are not reported here as they are of dubious quality, but they are displayed in Figures B.3 and B.4 of Appendix A.

Before exposing the 12-good model's results, we will explore the disaggregated

[17] See Chapter 9 of Prof. Diewert's lecture notes at `https://economics.ubc.ca/faculty-and-staff/w-erwin-diewert` for details.

Figure 17: Index Utility (1960 = 1)



consumption data. The Australian data does not fully cover our sample period (1959 -2018) as it only goes back to 1985 and ends in 2017, but it still provides crucial insights towards solving the PC inflation puzzle uncovered in Section 2.3. To find which consumption category j contributes most to the unusually high PC inflation, we subtract each category's average annual price growth by the average annual growth in PC. Define average annual excess inflation of category j ($AAEI_j$) as:

$$AAEI_j = \left[\frac{(P(j))_{2017}}{(P(j))_{1985}}\right]^{\frac{1}{32}} - \left[\frac{(PC)_{2017}}{(PC)_{1985}}\right]^{\frac{1}{32}} \tag{46}$$

We can then compare each category's $AAEI$ to Canada's measure for the same category to find which is responsible for the unusually high Australian consumption inflation. We do this in Table 4. The first column contains the name of the category. The second column contains the average of the weight of the category in Australian overall consumption. The third and fourth columns contain the category's $AAEI$ for Australia and Canada respectively.

Table 4: Average Annual Excess Growth of Consumption Components

Consumption category	Avg. share of j in VC (Australia)	$AAEI_j$ (Australia)	$AAEI_j$ (Canada)
Food and non-alcoholic beverages	10%	-0.01%	0.21%
Alcoholic beverages	2%	0.45%	0.42%
Tobacco	2%	6.07%	3.49%
Clothing and footwear	4%	-2.09%	-1.41%
Housing	26%	0.42%	-0.06%
Health	5%	0.59%	0.49%
Transportation	10%	-0.82%	0.17%
Communications	2%	-3.74%	-1.07%
Recreation and culture	11%	-1.67%	-0.89%
Education	3%	3.29%	2.89%
Services aggregate	21%	0.27%	0.32%
Expenses abroad (tourism)	2%	-1.33%	0.26%

Tobacco, housing, health, education have higher average excess inflation in Australia than in Canada. But tobacco, health and education each represent less than 6% of total consumption each and the excess inflation of health and education are only slight higher in Australia than Canada. These three categories are therefore not significant drivers of the high PC inflation. This leaves us with housing, which on average represents 26% of consumption expenses for Australians. The price of housing grew 0.42% faster than overall consumption in Australia, while it grew 0.06% slower than overall consumption in Canada.

Housing price growth outpacing consumption price growth by 0.42%/year over the entire sample with housing making up a quarter of overall consumption represents a huge pull on inflation, which in turn is a drag on real income growth. It thus seems that housing is almost single-handedly responsible for Australian consumers' utility lagging behind Australia's strong and steady GDP growth.

Having extracted this key insight from the disaggregated consumption data, we can get back to estimating the 12-good consumption model introduced above. The results are better than for the 2-good model, which is to be expected given that it allows us to have both a higher number of estimating equations (each category quantity represents a Hicksian demand equation) and more right-hand side variation in prices. Table 5 shows the estimated own-price and income elasticities for all twelve consumption categories. Most of the elasticities seem to be in line with intuition.

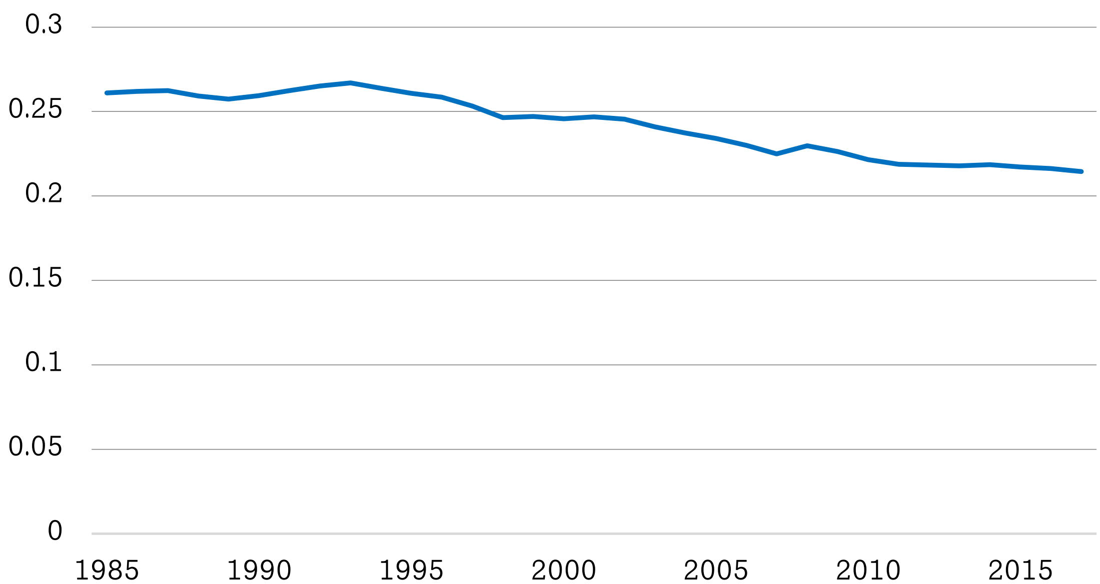
Table 5: Own-Price and Income Elasticities of Disaggregated Consumption Categories

Consumption category	Own-price elasticity (Australia)	Income elasticity (Australia)
Food and non-alcoholic beverages	-0.71	0.76
Alcoholic beverages	-1.22	2.16
Tobacco	-0.31	-1.34
Clothing and footwear	-1.22	0.70
Housing Aggregate	-0.22	0.91
Health	-0.57	1.07
Transportation	-0.18	1.08
Communications	-1.44	1.40
Recreation and culture	-0.54	1.82
Education	-0.20	0.88
Services aggregate	-1.95	0.71
Expenses abroad (tourism)	-1.27	2.88

Note the low own-price elasticity of housing. When the price of an inelastic good rises, the negative impact on consumers' utility is high because they don't substitute away from the good. Once again, this reinforces the idea that high housing inflation hurt Australian consumers.

Even given the low price elasticity of housing, Australians still reacted to rising housing prices by decreasing their consumption of housing. Figure 18 shows that housing's share of overall consumption volume decreased from 26% to 21% between 1985 and 2017.

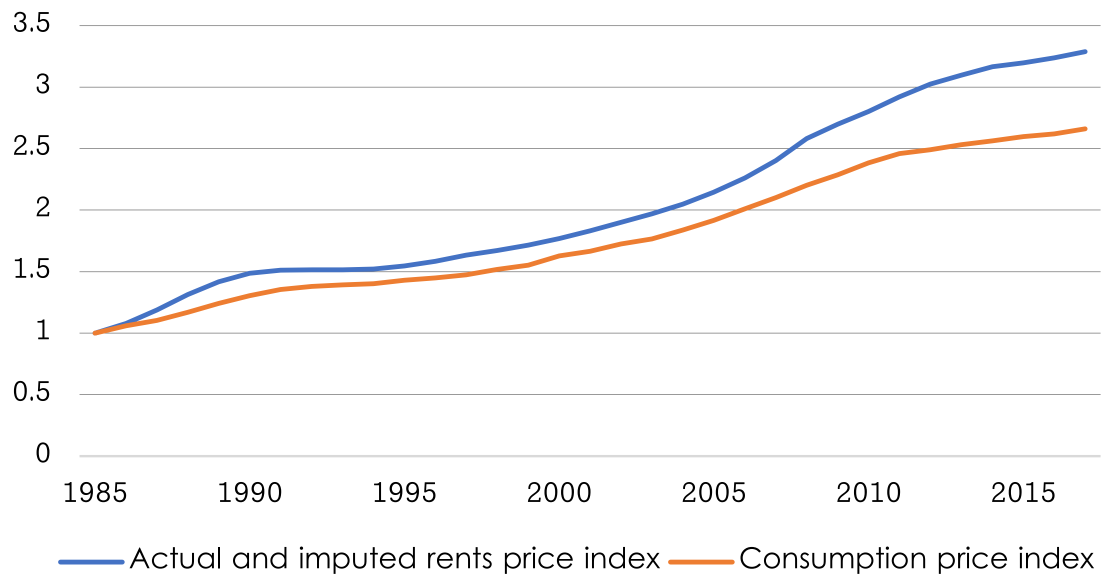
Figure 18: Housing's Share of Consumption Volume



As explained in the Introduction to this paper, a major issue with measuring productivity is the lack of price and quantity data for land. Housing might be the best proxy we have for land prices. In the Australian case, most of the excess inflation of the housing aggregate is due to imputed and actual rent inflation. Imputed and actual rent, which make up 70% of the volume of the housing aggregate, saw average annual inflation 0.81% higher than PC consumption, compared to 0.42% higher for the housing aggregate (as shown in Table 4).[18] Figure 19 shows the growth of rent prices vs. the growth of overall consumption.

[18]The other three components of housing are water, electricity and gas, and furnishing.

Figure 19: Rent Price Inflation Vs. Overall Consumption Inflation



Davis and Heathcote (2007) show that variations in land prices explain the majority of variations in house prices. This is especially true for expensive housing markets – Australia is one – where in general land value accounts for a higher proportion of house value. Thus we can somewhat speculatively assume that behind Australia's unusually high housing price inflation is an important land price inflation.

Going full circle, what does this unobserved land price inflation entail for Australian productivity? Our productivity measures from Section 1 show that productivity has stagnated since the beginning of the 2000s, and we showed above that capital accumulation accelerated in the 2000s without a sizable improvement in unit labour productivity (Figure 6). Figure 19 shows that rent inflation started to diverge from overall consumption inflation around the same period. A plausible hypothesis is that firms reacted to an increase in land prices by limiting their use of land – like consumers did with housing expenses – and increasing their use of other inputs to improve land productivity. Thus the observed capital accumulation did in fact result in increased TFP by increasing land productivity rather than labour productivity. But because we do not have data to measure land input, we don't measure the slow down in the growth of land input, and so do not observe the productivity improvement.

Conclusion

Section 1 shows that our three TFP-measurement methods – index-based, econometric and nonparametric – agree that Australian productivity has stagnated since the beginning of the 2000s. All methods but one result in a similar productivity path, giving us comfort in the relative accuracy of our estimated productivity growth. Only the nonparametric convex profit-maximization method disagrees.

Section 2 endeavours to explain how the Australian economy has grown steadily since 1990 without any productivity improvement in the last 15 years. It turns out that high investment growth is behind the strong GDP growth. We show that because the resulting capital accumulation of the 2000s and 2010s is accompanied by neither a significant improvement in labour productivity nor a fall in real rate of returns, it must be driven by output price growth. Commodity export prices are the main culprit. A surge in the price of metals in the 2000s drove up firm profits and incentivized Australian companies to increase mining capacity, requiring huge capital outlays. The resulting jump in terms-of-trade also helped Australia avoid a recession after the financial crisis.

Although GDP growth was surprisingly solid considering the productivity stagnation due to high investment, this didn't fully translate to improvements in consumer utility. Real income grew by less than GDP as a result of high consumption price inflation, mostly caused by housing prices.

Finally, house price inflation in the 2000s and 2010s suggests that land prices have also climbed in that period. This could explain the low measured productivity of capital and labour. The constraining input factor being land, firms focused on increasing their production with a limited increase in the use of land. If we had quantity and price data for land, we would observe an increase in land input productivity. Instead we measure a stagnation in the productivity of our two observed inputs, labour and capital. It is impossible to formally test this hypothesis without having quality land input. But it would go a long way towards solving the productivity puzzle. It seems unlikely that the opening of new, less efficient mines could explain the entirety of the 15-year Australian productivity stagnation. Part of it being due to unobserved land productivity gains would make this explanation more plausible.

References

Christensen et al. (1971), “Conjugate Duality and the Transcendental Logarithmic Production Fuction”, *Econometrica*, vol. 39, 255-256.

Davis, D.A. and J. Heathcote (2007). “The Price and Quantity of Residential Land in the United States”, *Journal of Monetary Economics*, Vol. 54:8, 2595-2620.

Diewert, W.E. (1976), “Exact and Superlative Index Numbers”, *Journal of Econometrics*, vol. 4, 114-145.

Diewert, W.E. (1978), “Superlative Index Numbers and Consistency in Aggregation”, *Econometrica*, vol. 46, 883-900.

Diewert, W.E. (1992), “Fisher Ideal Output, Input, and Productivity Indexes Revisited”, *The Journal of Productivity Analysis*, vol. 3, 211-248.

Diewert, W.E. (2000), “The Challenge of Total Factor Productivity Measurement”, *International Productivity Monitor*, vol. 1, 45-52.

Diewert, W.E. and T.J. Wales (1987), “Flexible Functional Forms and Global Curvature Conditions”, *Econometrica*, vol. 55, 43-68.

Diewert, W.E. and K.J. Fox (2017), “Decomposing productivity indexes into explanatory factors”, *European Journal of Operational Research*, vol. 256(1), 275-291.

Diewert W.E. and C.J. Morrison (1986), “Adjusting Output and Productivity Indexes for Changes in the Terms of Trade”, *The Economic Journal*, vol. 96:383, 659-679.

Hayek, F. (1935), "The Maintenance of Capital", *Economica*, vol. 2(7), 241-276.

Hill, T.P. (1993), "Price and Volume Measures", pp. 379-406 in *System of National Accounts 1993*, Eurostat, IMF, OECD, UN and World Bank, Luxembourg, Washington, D.C., Paris, New York, and Washington, D.C.

Farrell M.J. (1957), "The Measurement of Production Efficiency", *Journal of the Royal Statistical Society*, Series A, vol.120, 253-278.

Kohli, U. (1990), "Growth Accounting in the Open Economy: Parametric and Nonparametric Estimates", *Journal of Economic and Social Measurement*, vol. 16, 125-136.

Mendoza, M. (1989), *Essays in Production Theory: Efficiency Measurement and Comparative Statistics*, PhD Thesis, UBC.

Parham, D. (2012), "Australia's Productivity Growth Slump: Signs of Crisis, Adjustment or Both?", *Australian Productivity Commission*, 78 p.

A Labour Shares Under Net Income

The labour share of income is defined as:

$$SL = \frac{PL \times QL}{PY \times QY} = \frac{PL \times QL}{PZ \times QZ} \tag{47}$$

where PY and QY are the price and quantity of a Fisher index of aggregate outputs while PZ and QZ are the price and quantity of a Fisher index of aggregate inputs. By construction the nominal value of inputs is equal to the nominal value of outputs, as the price of capital services PKS is measured as residual so that $VY - VZ = 0$. Any net profits are are assigned to waiting services of capital $R \times PK$, a component of PKS.

The blue line in Figure A.1 shows that under the gross definition of income the labour share has been dropping steadily since the 1970s. Most of this decline is due to an increase in net depreciation, the difference between depreciation (blue line in Figure A.2) and capital inflation (orange line in Figure A.2). But as explained in Section 1.1, investment to replace net depreciation should not be removed from income (and similarly net depreciation should not be counted in VKS). When we subtract net depreciation from the denominator of Equation 47 it shrinks, resulting in a much higher labour share in 2018 as shown by the orange line in Figure A.1. Moreover, net labour share has not decreased since the 1970s. Instead, we see a large drop in the 1960s, as net depreciation dived due to high capital inflation, and a subsequent jump in the 1980s as capital inflation settled down.

Figure A.1: Labour Shares Under Gross and Net Income Definitions

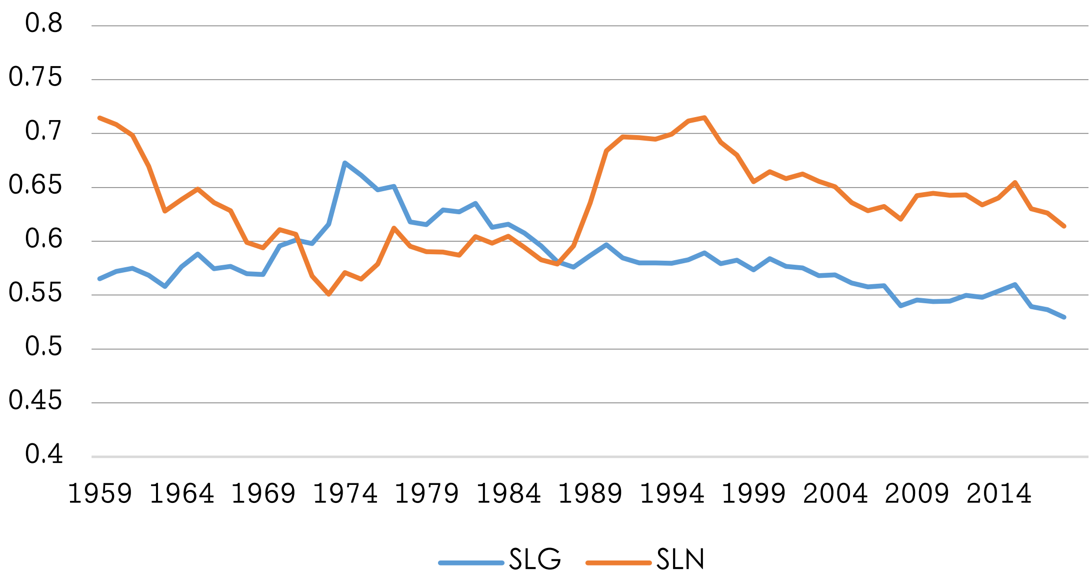
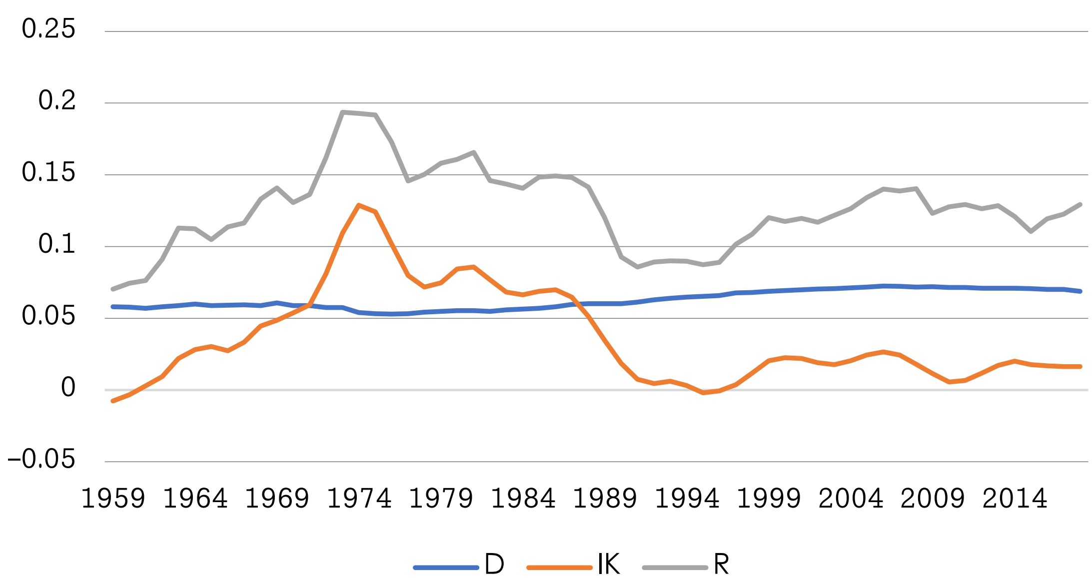


Figure A.2: Capital Measures



B Supplemental Graphs

Figure B.1: Tax Ratios

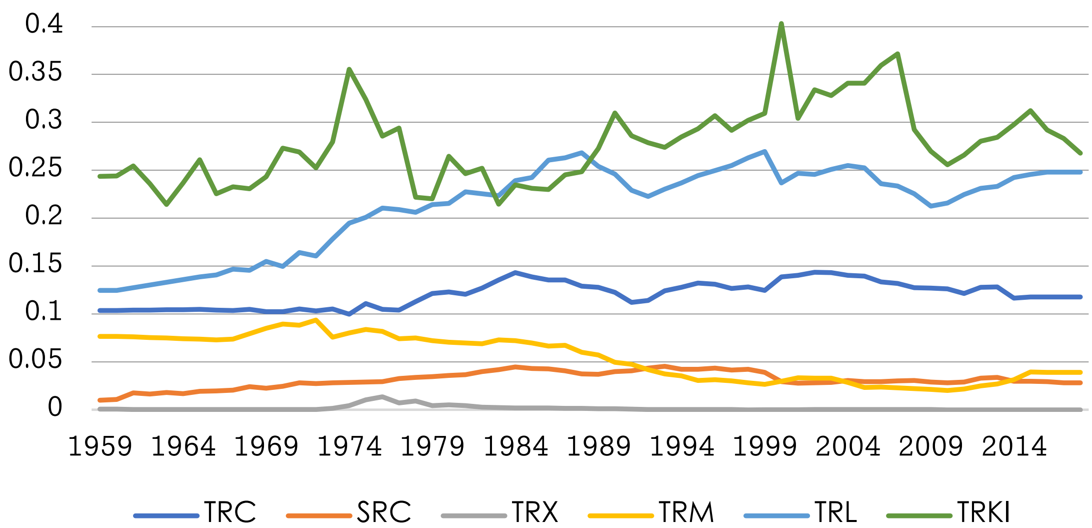


Figure B.2: Real Input Price Growth (1959 = 1)

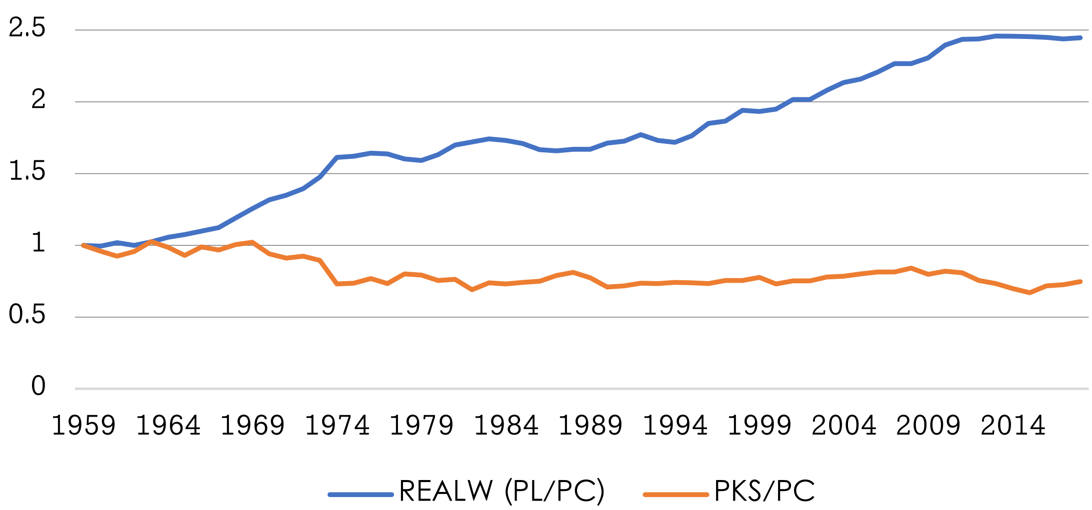


Figure B.3: Own-Price Elasticities of Consumption and Leisure

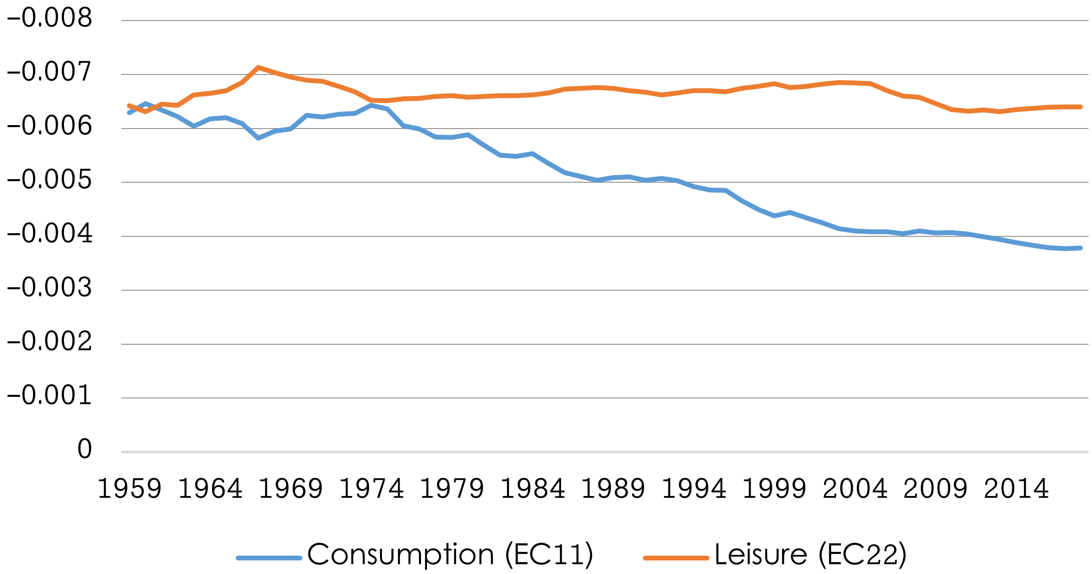


Figure B.4: Income Elasticities of Consumption and Leisure