Compensating a Power Amplifier using Iterative Learning Control : from Design to Realisation

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Thank You Note

Abstract

Contents

Chapter 1

Introduction

1.1 Why Digital Predistortion?

Power amplifiers are used in almost all wireless communication devices. They amplify the communication signal such that a good signal to noise ratio is obtained. They also are an important power consuming block in a communication chain. A power amplifier is often operated in a nonlinear operation mode to improve its efficiency. This nonlinear behavior should be compensated to reach the strict telecommunication requirements. A Digital Pre-Distortion (DPD) is a common technique to linearise the input-output behavior of a power amplifier. With DPD the input signal of the amplifier is modified such that the desired (i.e. linear) behavior is obtained.

In this work a novel technique to design DPDs is presented. By first extracting the Best Linear Approximation (BLA) from input/output measurements of a nonlinear system, and consequently using the BLA with Iterative Learning Control (ILC), an effective compensation is possible. In this chapter, the theoretical background of BLA and ILC is discussed. This serves as an introduction to the main idea of this thesis : using these two techniques to design a DPD.

1.2 Current Techniques of DPD

1.2.1 Direct and Indirect Learning

1.2.2 Nonlinear Models

1.2.3 Performance Assessment : Adjacent Channel Power Ratio

1.3 The Best Linear Approximation

1.3.1 The Paradigm

Linear systems are completely characterised by their impulse response function. The knowledge of the impulse response function allows one to predict the output of the system, given any input. Nonlinear systems do not share this elegant property. Because nonlinear modelling can be quite involved and time-consuming, one can wonder if it is possible to construct a linear approximation that is 'close enough' to the studied nonlinear system.

The *Best Linear Approximation* (BLA) is such an approximation. The BLA is a linear system that approximates the output of a nonlinear system with the response of a LTI model in mean square sense. The definition given below is in in time domain for ease of understanding. However, the BLA wil mainly be represented in the frequency domain which is a more convenient domain to study spectral regrowth.

Definition 1. *The BLA of a nonlinear system is the linear system $G_{bla}(q)$ that minimizes the mean square error:*

$$
\begin{aligned}
G_{bla}(q) &= \underset{G(q)}{arg\ min}\ E\left\{(\tilde{y}(n) - G(q)\tilde{u}(n))^2\right\}, \\
\tilde{u}(n) &= u(n) - E\{u(n)\}, \\
\tilde{y}(n) &= y(n) - E\{y(n)\},
\end{aligned}
$$

Where n is the time index, q is the time shift operator ($qx(n) = x(k+1)$) and the expected value $E\{.\}$ is taken with respect to the random realisations of $u(n)$. Zero-mean input/output signals will be assumed from now on and thus the tilde symbol will be left out.

Unlike the impulse response of a linear system, the BLA depends on the probability density function and power spectrum of the chosen input signal. The chosen class of signals in this work are excitations with a Gaussian pdf. The power spectrum is a design choice specific to each application. More specifically, random phase multisines will be used. These are periodic signals consisting of a sum of sines with a random phase, which have very nice properties to measure the BLA.

Definition 2. *A multisine $u(t)$ can be defined in continuous time as :*

$$
u(t) = \sum_{k=0}^{N-1} |A_k| cos(2\pi f_0 t + \phi_k) \tag{1.1}
$$

And computed in discrete time with the inverse discrete-time fourier transform :

$$
u_d(n) = \sum_{k=0}^{N-1} |A_k| e^{j\phi_k} e^{jn\frac{k}{N}} \tag{1.2}
$$

Where A_k is the amplitude spectrum of the multisine and ϕ_k is the phase, chosen such that $E\{e^{j\phi_k}\} = 0$. f_0 is called the frequency resolution.

The BLA allows to represent the output of a nonlinear system in the frequency domain as the sum of a linear contribution and a nonlinear distortion term $Y_s(k)$.

$$
Y(k) = G_{bla}(k)U(k) + Y_s(k) \tag{1.3}
$$

Interestingly, $Y_s(k)$ is asymptotically zero-mean normally distributed. The variance of $Y_s(k)$ is a thus an useful measure for the quality of the linear approximation. To have a relative measure of the quality of approximation, the variance of $\frac{Y_s(k)}{U(k)}$ is often considered, and called the nonlinear stochastic variance of $G_{bla}(k)$.

As a concluding but very important remark: the BLA is only fit approximate PISPO (Period In, Same Period Out) systems meaning that the input and output signals have the same period length. An in-depth and more rigourous study of the BLA paradigm can be found in [1].

1.3.2 Detection of Nonlinear Distortions

Now the BLA has been defined, one wants to estimate it from input/output measurements.

Coherent Contributions

A coherent contribution is a contribution to the output spectrum $\angle Y(\omega)$ of a nonlinear system where the phase shift between input and output is constant, and can be modelled as a linear contribution. Because the input signals are random phase multisines, all contributions that are non-coherent will have a random phase shift. Because $E\{e^{j\phi_k}\} = 0$, if different multisines are fed to the system and the output spectrum is averaged over these different realisations, non-coherent contributions will be averaged to zero. This last properties is at the core of the robust measurement method that is presented in the next section.

Robust Measurement Approach

To measure the BLA, one can measure the steady-state response over P periods of M multisines with different random phase realisations. Because of previous properties, the sample variance over the P periods only depends on the output noise, while the sample variance over the M realisations depends on both the stochastic nonlinear distortions and the output noise. This allows to discriminate between nonlinear distortions and noise, and to measure the BLA. Figure 1.1 shows a visual aid to understand the measurement method. Choosing a high M decreases the nonlinear distortion variance and increasing P decreases the noise variance. This is the only measurement method that will be used in this work.

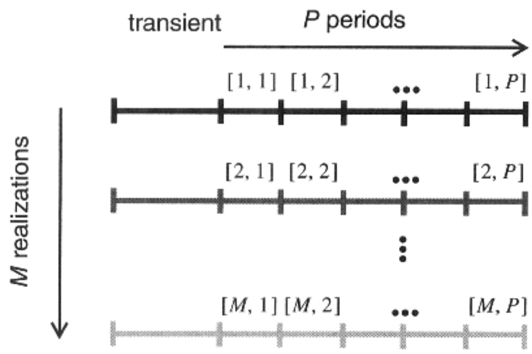

Figure 1.1: The robust measurement procedure. Averaging i/o spectra over periods at each realisation gives an estimated FRF $\hat{G}_m$ and an estimate for the noise variance $\hat{\sigma}_{n,m}$. Further averaging over the realisations gives an estimate of the nonlinear variance $\hat{\sigma}_s$, and an improved noise variance $\hat{\sigma}_n$.

1.3.3 Out of Band BLA and the Tickler Tone

A nonlinear system will

1.4 Iterative Learning Control

1.4.1 The Algorithm

1.4.2 Properties

Chapter 2

Using the BLA in ILC for DPD

2.1 Introduction

The plethora of abbreviations used in this title only hides a simple concept. What if one uses the BLA in the system inversion algorithm in ILC?

2.2 The Blocky Tought Experiment

A nonlinear dynamic system can alternatively be represented by the combination of a linear transfer function G_{BLA} and a nonlinear function F.

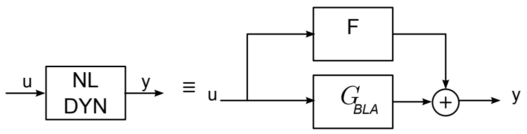

Figure 2.1: Alternative representations of a nonlinear system.

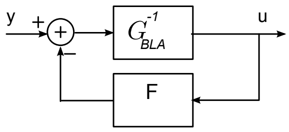

Figure 2.2: Switching the input and output, creating the inverse of the nonlinear system.

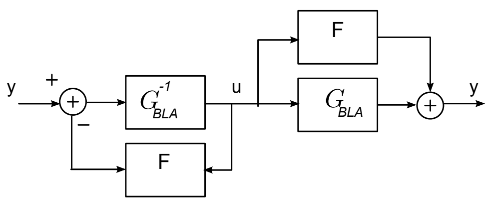

Figure 2.3: Connecting the inverse and the original system together.

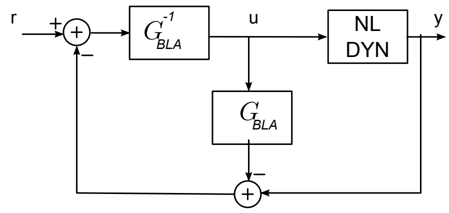

Figure 2.4: Getting creative with the blocks.

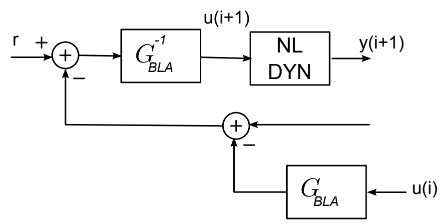

Figure 2.5: Cut the loop!

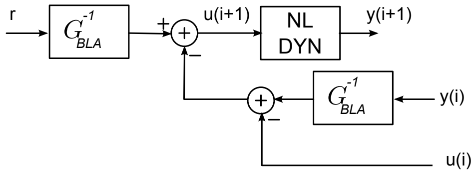

Figure 2.6: Reorganise the blocks one last time.

2.3 Why can it work?

2.4 Creation of a standalone DPD

2.5 Estimating the Preinverse

Chapter 3

Simulation Results

3.1 ILC with BLA

3.1.1 Proof of Concept

Before constructing a solid framework for predistortion using ILC, a small number of simulations on test systems will be run to estimate the performance of this technique. A description of the systems will be given, followed by the measurement of their BLAs. The ILC compensation algorithm.

Presentation of Test Systems

The Static Nonlinearity that obeys the function `y = 10*tanh(x/5)`

The Wiener System is a linear dynamic system followed by a static nonlinearity. The linear block is a Chebyshev filter and the nonlinearity is a $tanh()$ function. The MATLAB parameters of this system are as follows:

1. Input Block : `cheby1(1,1,2*1/15)`
2. Nonlinearity : `5*tanh(x/5)`

The Wiener Hammerstein System which is a static nonlinearity sandwiched between two linear dynamic blocks. This particular system consists of two discrete-time Chebyshev filters and a $tanh()$ function as nonlinearity. The MATLAB parameters of this system are as follows:

1. Input Block : `cheby1(1,1,2*1/15)`
2. Nonlinearity : `5*tanh(x/1.2)`
3. Output Block : `cheby1(3,1,2*1/20)`

Additionally, measurement noise is simulated only on this system by adding zero-mean gaussian noise with $\sigma = 3.10^{-3}$

Measuring the Best Linear Approximation

In figure 3.1, the BLA of each test system is plotted. All measurements had the same parameters that can be found in table 25 independent realisations, 2 measurement periods, 1 transient period and input signal rms of 0.3.

Table 3.1: Parameters of multisine for robust measurement.

Realizations M	10
Periods P	2
Excited Bandwith	0 to 0.1 $\frac{cycles}{sample}$
Number of samples N	4096
Multisine rms	0.3

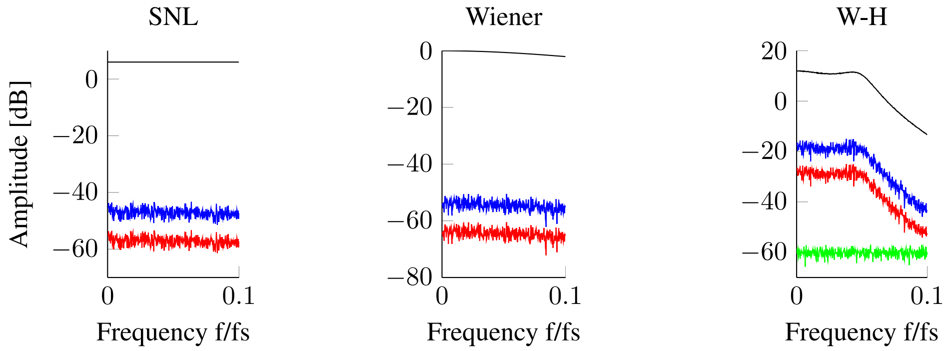

Figure 3.1: Best Linear Approximation of the test systems. BLA in black, total variance in red, stochastic nonlinear distortion in blue and noise variance in green.

Using the Best Linear Approximation in Iterative Learning

The BLA measured in the previous section are now used in the system inversion ILC algorithm. The BLAs are not fitted to models, but instead the FRF is just inverted at each frequency. Because the FRF is not known outside of the measured frequencies, the inverse is put to zero there so it does not influence the learning [1]. This is equivalent to having an ideal Q-filter of 1 in the band of interest and 0 outside. As known (**ref to intro**), this is helpful for the stability and robustness of the algorithm.

Figure 3.2 shows the output spectrum after applying ILC for 10 iterations giving as wanted output a multisine filtered (bandwith = 0 to $0.05f/f_s$) trough the BLA. The nonlinear spectral regrowth of the first iteration (red) is totally removed in the last iteration (green). Outside of the linear band, the amplitude reaches the noise levels or computer precision in the noiseless cases. **SHOW ILC ERROR DECREAS-**

[1]This means that outside of the measured frequencies, no compensation can occur. Therefore, it is only sensible to look at the results in the measured band.

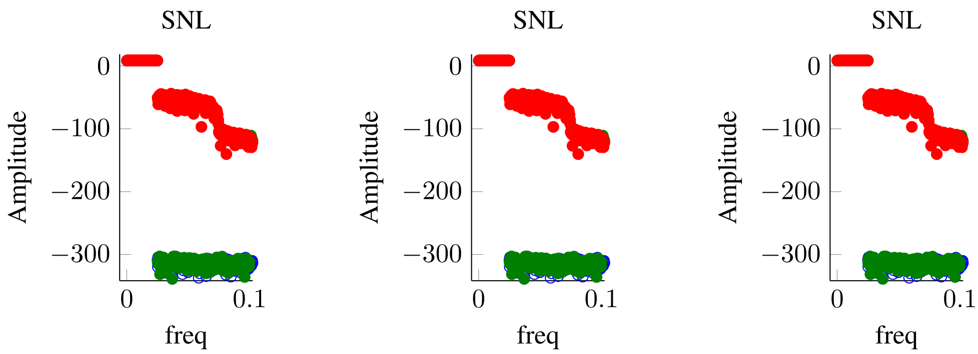

Figure 3.2: Comparison of system output before and after applying ILC to the input signal in the frequency domain. Uncompensated output in red and compensated output in green.

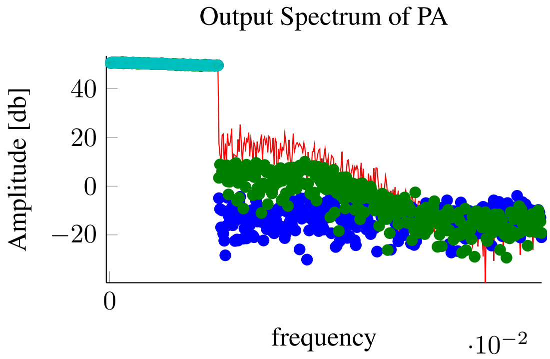

Figure 3.3: Output spectrum of system.

ING at each iterations

Estimating the Digital Predistorter

Iterative Learning created in the previous step inputs that give a linear output to the system. A iterative algorithm is not well suited for application in predistortion. A digital predistorter is a system, not an iterative algorithm. In this step, the whole ILC algorithm will be modelled as a nonlienar system. This is using the ILC reference outputs as input of the DPD, and the converged ILC inputs as output of the DPD.

A Wiener system has been estimated for the DPD with the results plotted in figure 3.3. There is a gain with respect to non-compensated input (about $-10dB$), but the it is not as effective as the real ILC.

3.1.2 Influence of Noise

3.1.3 Study of Convergence

3.1.4 Compensate to Static Gain or BLA?

3.2 Standalone DPD

Chapter 4

Application on a Audio Valve Amplifier

4.1 The Synchronisation Challenge

4.2 Measure the BLA

4.3 Apply ILC

4.4 Estimate DPD

4.5 Results

Chapter 5

Conclusion

Bibliography

[1] R. Pintelon, J. Schoukens, *System Identification : A Frequency Domain Approach - 2nd edition.* IEEE Press (2012).

[2] J. Schoukens, R. Pintelon, Y. Rolain , *Mastering System Identification in 100 Exercises.* IEEE Press (2012), 183-238.