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Global Quantitative & Derivatives Strategy
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# Harvesting Volatility Risk Premia With Machine Learning

## Volatility Spread Strategy Using Dynamic Linear Model

### Volatility spread: A hedge for equities

Volatility spread strategies deliver mostly positive returns when equity markets suffer from large drawdowns, and could serve as a hedge for equity portfolios. To systematically harvest volatility risk premia, we propose a machine learning approach using a class of State Space models.

### Modelling the PnL of VIX/VSTOXX: Volatility change + carry

We decompose the PnL of volatility futures into two components: (1) change in volatility; and (2) carry (aka roll cost). While volatility is mean-reverting, carry tends to be stable and is negatively correlated to changes in volatility. We construct a Dynamic Linear Model to capture these relationships, and predict the PnL using the Kalman Filter algorithm. Using the expected PnL as a trading signal, we adjust our long/short positions in the VIX/VSTOXX volatility spread strategy.

### VIX/VNKY strategy: Hedging for a spike in VNKY

Investors concerned about a spike in VNKY could consider our VIX/VNKY strategy that shorts VIX and longs VNKY, with systematically adjusted exposures based on our Dynamic Linear Model. This strategy offers protections during volatile periods in the NIKKEI and earns decent carry from the short VIX position during tranquil markets.

**Global Quantitative and Derivatives Strategy**

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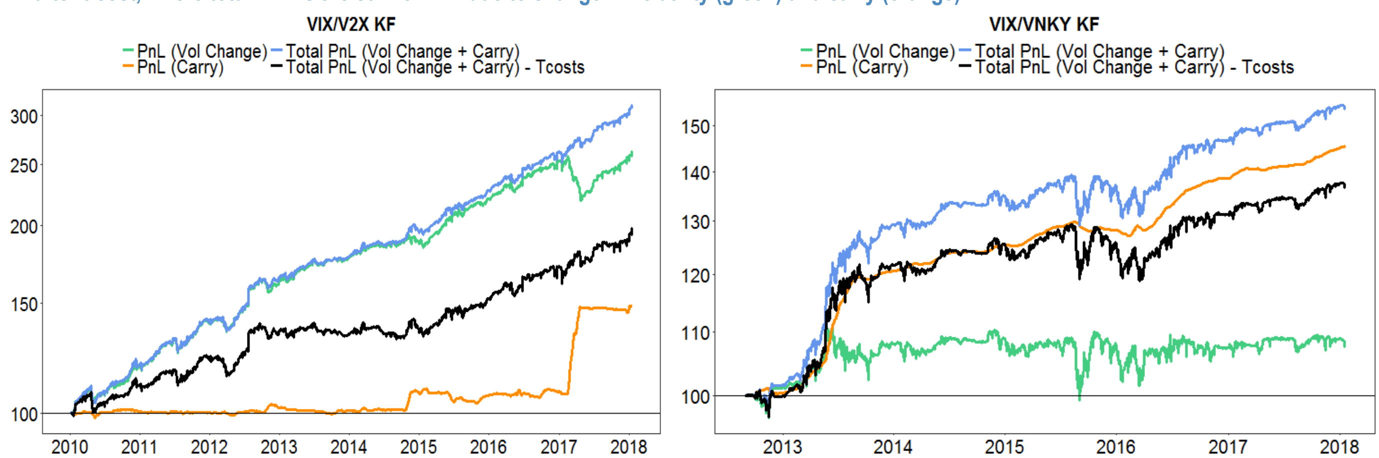
**Cumulated PnL (in vol points) of our systematic VIX/VSTOXX strategy (left) and VIX/VNKY strategy (right): The black line shows the total PnL after t-cost, where total PnL is the sum of PnL due to change in volatility (green) and carry (orange)**




Source: J.P. Morgan Quantitative and Derivatives Strategy, Bloomberg.

**See page 47 for analyst certification and important disclosures, including non-US analyst disclosures.**

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- Equity Risk Premia Strategies: Risk Factor Approach to Portfolio Management
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# Introduction

In this report, we examine some of the popular systematic equity volatility strategies, analyze their properties and propose a novel statistical model for volatility spread trading. This report is helpful even for investors who may not be familiar with the trading of volatility futures, as we provide plenty of details in the Appendix. For a primer on VIX risk premia strategies, see Kolanovic, M., et al. (2014). For basic information on various volatility indices and futures, see Naito, et al. (2011) (esp. for VNKY) and Lee, et al. (2012) (esp. for VHSI).

**Interest in volatility risk premia has surged**

We have seen immense interest in volatility risk premia from investors in the past few years. In the recent low-volatility environment, short volatility strategies have provided decent carry, where equities could be the best vehicle for vol carry due to steep vols (Bouquet & Kolanovic (2017)). Another potential application of volatility strategies lies in portfolio diversification. Some studies show that adding long volatility exposure to an equity portfolio can provide an efficient approach to manage risk (Guobuzaite (2011), Stanescu (2014)). Although some papers argue that out-of-sample performance may not be superior due to the high roll costs involved (Alexander & Korovilas (2011)), including volatility risk premia in a diversified portfolio has gained popularity and become one of the major topics in research.

J.P. Morgan offers a comprehensive suite of investable volatility indices that attempt to harvest carry (e.g., implied to realized volatility risk premia and the roll-down volatility term premia), provide hedging (e.g., put ratios, VIX convexity), generate income (e.g., put underwriting, call overwriting) and exploit relative value between volatility pairs. This report provides a detailed analysis of a volatility spread strategy, which falls under the relative value trading paradigm.

**Modelling volatility**

Volatility spread strategies depend on the mean-reversion of spreads, and hence we need a framework for deciding whether the spread will narrow or widen. A simple, yet common approach is to model the spread with a rolling average. In Peng, et al. (2015), we find that the VIX/VSTOXX volatility spread can be largely explained by relative valuation (i.e., earnings yield) between S&P 500 and STOXX 50. On the other hand, Stanescu, S. and Tunaru, R. (2014) consider a GARCH model for the volatility spread. The authors use the model to forecast the spread and devise a statistical arbitrage strategy between VIX and VSTOXX[1].

Of course, there is no reason to restrict ourselves to equity volatility futures. In fact, there have been ample studies showing that volatility does spill over across markets and asset classes. Spillover effects can be quantified by a Vector Autoregressive (VAR) model through variance decomposition (Diebold (2009)). In Ribeiro, et al. (2012), we fit a VAR model for VIX, VSTOXX and VNKY, and find positive cross-autocorrelations across the volatilities[2]. In general, past volatilities in one market could help to forecast volatilities in another market, and short volatility strategies conditional on these expectations deliver higher Sharpe ratios (Ribeiro, et al. (2012)).

1 Another angle to invest in volatility futures is via portfolio allocation (Peng, et al. (2017)).
2 Although interactions between Europe and Japan are not too consistent.

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**Why do we consider a Dynamic Linear Model?**

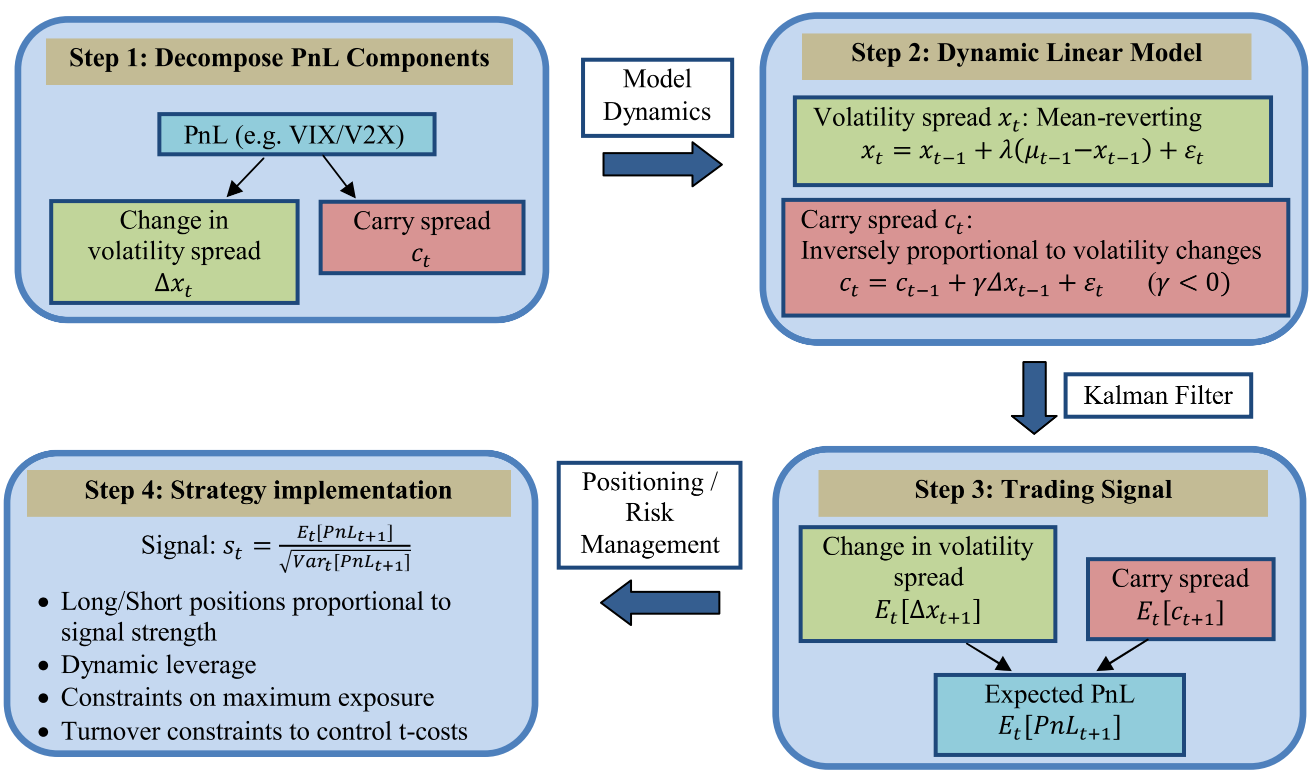
Our approach of volatility spread trading using a Dynamic Linear Model (a.k.a. State Space model) is novel and interesting in several respects:

- **Modeling both components of PnL:**
  The model captures not only the dynamics of volatility spread, but also the dynamics of the carry component. This could be useful, as the total PnL of a volatility spread strategy depends on both components (see p.43 in Appendix).
- **A Bayesian approach:**
  We use the Kalman Filter algorithm to obtain the predictions from the Dynamic Linear Model, which is more adaptive than simple rolling regressions. It is because the Kalman Filter is a Bayesian approach, where we update our predictions by combining our prior knowledge of the system together with the latest observations (see p.40 in Appendix).

With the popularity of using Machine Learning for systematic strategies (Kolanovic, M., et al. (2017)), this report is another example of applying statistical models (i.e., Dynamic Linear Model and Kalman Filter) to systematic risk premia investing.

Figure 1 illustrates our framework of constructing a volatility spread strategy using a Dynamic Linear Model, highlighting the major ideas and details on strategy implementation.

Figure 1: Framework of our systematic volatility spread strategy, based on a Dynamic Linear Model and Kalman Filter




Source: J.P. Morgan Quantitative and Derivatives Strategy

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## PnL = Volatility change + carry

First, let us look at the PnL in volatility futures. The underlying of a volatility future is the volatility index, which measures the forward implied volatility of the equity market at a certain horizon (usually 30 days), based on near-term options. Hence, it is common to consider a constant maturity by holding a weighted combination of futures with different expiry dates.

### 1M constant-maturity volatility futures

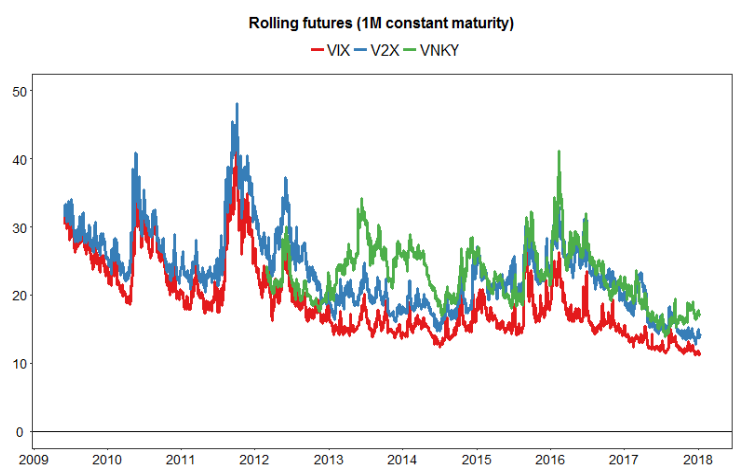
In this report, we consider a constant 1M volatility future[3], which can be obtained by holding a weighted combination of the front-month contract and the next-month contract (see p.42 in Appendix). For instance, 1M VIX is given by

$$VIX_{1M,t} = \omega_{1,t} UX_{1,t} + \omega_{2,t} UX_{2,t} \qquad \omega_{1,t} + \omega_{2,t} = 1$$

Figure 2 shows the prices of constant 1M volatility futures for VIX, VSTOXX (V2X) and VNKY. Clearly, the volatilities of all the major equity markets have declined to historical lows since 2016. Also, we note that

- In general, V2X is higher than VIX, as the number of stocks in STOXX is lower than that in S&P.
- VNKY was significantly higher than VIX and V2X during the spike in 2013.

Figure 2: Volatility of 1M constant expiry based on daily rolling: The table shows the statistics of 1M volatility (since 2009-06 for VIX and V2X, since 2012-02 for VNKY)




| | 1M VIX | 1M V2X | 1M VNKY | 1M VIX – 1M V2X Volatility Spread | 1M VIX – 1M VNKY Volatility Spread |
|---|---|---|---|---|---|
| Mean | 19.12 | 23.46 | 22.64 | -4.34 | -6.32 |
| Standard deviation | 5.56 | 5.82 | 4.17 | 2.06 | 3.47 |
| Median | 17.60 | 22.79 | 22.19 | -4.02 | -5.87 |
| Median absolute deviation | 4.41 | 5.72 | 4.43 | 2.21 | 3.76 |
| Minimum | 11.24 | 12.79 | 13.82 | -10.30 | -17.05 |
| Maximum | 42.14 | 48.12 | 41.12 | -0.03 | 1.74 |
| Range | 30.91 | 35.32 | 27.30 | 10.26 | 18.79 |
| Skew | 1.03 | 0.80 | 0.39 | -0.44 | -0.24 |
| Kurtosis | 0.48 | 0.78 | -0.02 | -0.56 | -0.59 |

Source: J.P. Morgan Quantitative and Derivatives Strategy, Bloomberg

[3] One can also consider constant 60-day volatility futures, as in Peng, et al. (2015).

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**Decomposing PnL into two components**

The PnL of holding any volatility future can be decomposed into two components (for details see p.43 in Appendix):

**PnL = Change in volatility + carry**

- **Change in volatility:**
  If volatility increases, a long position will give a positive PnL.
- **Carry (aka term structure roll-down):**
  As futures will expire, we need to roll our positions from the front contract to the next contract. Volatility term structures are usually in Contango (i.e., upward-sloping), where the price of the next contract is in general higher than that of the front contract. This incurs a cost of carry (for a long position in volatility) when we roll into the next contract.

  Let $P_{1,t}$ and $P_{2,t}$ be the prices of the front contract and the next contract, respectively, and $\Delta\omega_{2,t}$ be the proportion of rolling into the next contract. Then, the PnL due to carry is

$$\text{Carry} = -\Delta\omega_{2,t}(P_{2,t} - P_{1,t})$$

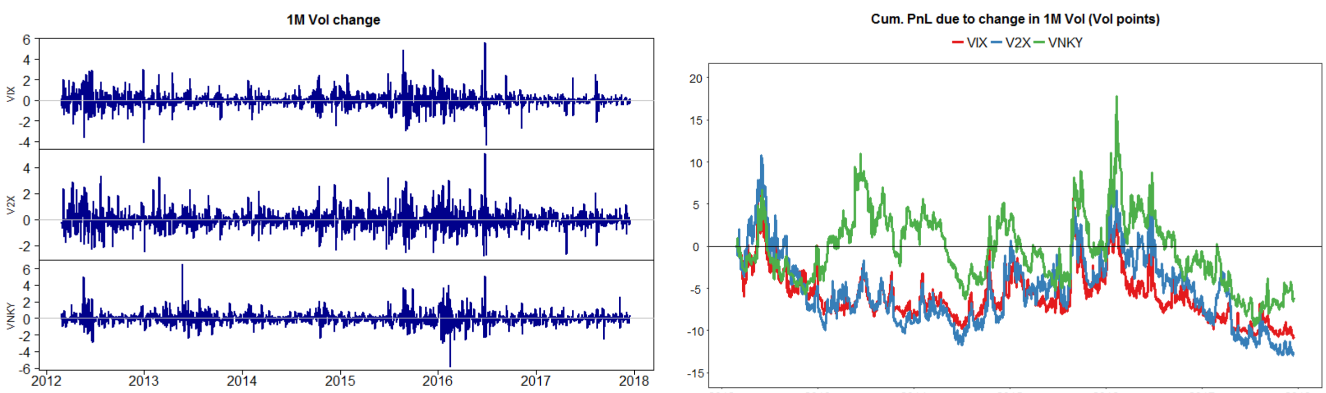
This "carry" component explains most of the premia in equity short vol strategies during tranquil markets. However, those strategies will suffer from large drawdowns when volatility spikes, where the first component dominates.

Suppose we hold a long position in 1M volatility. Our PnL depends on volatility change and carry, which exhibit very different properties, as below.

**Long 1M vol: PnL due to volatility tends to mean-revert**

Figure 3 shows the daily change in 1M volatility for VIX, V2X and VNKY. We see that daily changes of volatility exhibit clustering. On the right chart, we show the cumulated PnL due to the volatility component only. As volatility tends to mean-revert, so does the corresponding cumulated PnL.

Figure 3: Volatility tends to mean-revert, and daily changes of volatility exhibit clustering – the right chart shows the cumulated PnL (due only to changes in volatility) of a long position in 1M volatility

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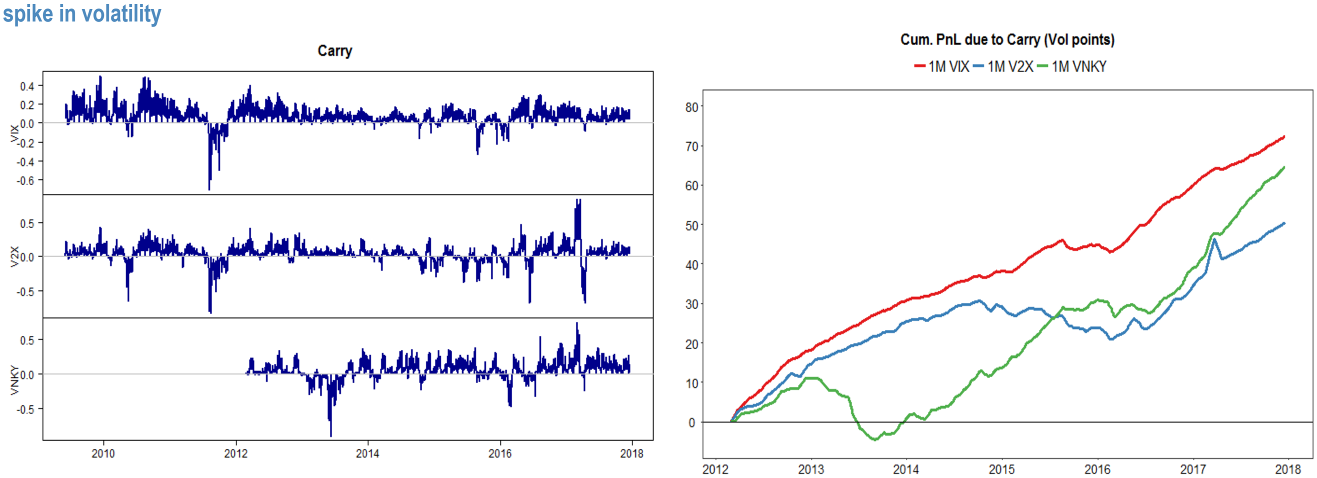
**Long 1M vol: Cost of carry is consistently positive**

In Figure 4, we see that carry is positive for most of the time and turns negative only occasionally during stress periods (when volatility spikes and the term structure is inverted). Another example of a large negative carry occurred in March 2017 for V2X due to uncertainties about the outcome of the French presidential election. The right chart shows the cumulated PnL of a short volatility position due to the carry component only, which is clearly trending upward for most of the time.

The summary statistics for carry in Figure 4 highlight the reason for the popularity of short VIX strategies: They earn an attractive and stable carry (over 1 vol point on average) during relatively calm periods, with a standard deviation smaller than that of V2X or VNKY. One of the reasons is the large investment into systematic long-volatility ETNs such as VXX, where the VIX term structure could be distorted by supply and demand (Kolanovic, M., et al. (2014)).

For VNKY, as the term structure tends to be steep, carry may sometimes be larger than that of VIX. This can be observed from the steeper slope of the cumulated PnL for VNKY around 2014-2015, and from 2017 onward.

Figure 4: Carry cost of longing a 1M volatility future is in general positive (left) – the right chart shows the cumulated PnL (due only to carry) of a short position in 1M volatility; short volatility strategies can harvest the carry premia (right) and have done well unless there is a large spike in volatility




Statistics of carry (since 2009-06 for VIX and V2X, since 2012-02 for VNKY)

| | VIX Carry (UX2 - UX1) | V2X Carry (FVS2 - FVS1) | VNKY Carry (JVI2 – JVI1) | VIX - V2X Carry Spread | VIX – VNKY Carry Spread |
|---|---|---|---|---|---|
| Mean | 1.14 | 0.80 | 0.95 | 0.35 | 0.09 |
| Standard deviation | 1.18 | 1.89 | 1.98 | 1.38 | 1.96 |
| Median | 1.19 | 1.00 | 1.20 | 0.25 | -0.14 |
| Median absolute deviation | 0.82 | 1.26 | 1.63 | 0.74 | 1.71 |
| Minimum | -6.99 | -9.15 | -8.45 | -8.46 | -7.10 |
| Maximum | 5.05 | 9.95 | 8.95 | 8.08 | 9.25 |
| Range | 12.04 | 19.10 | 17.40 | 16.54 | 16.35 |
| Skew | -1.27 | -0.25 | -0.73 | -1.11 | 0.76 |
| Kurtosis | 5.84 | 5.33 | 2.43 | 11.40 | 1.83 |

Source: J.P. Morgan Quantitative and Derivatives Strategy, Bloomberg

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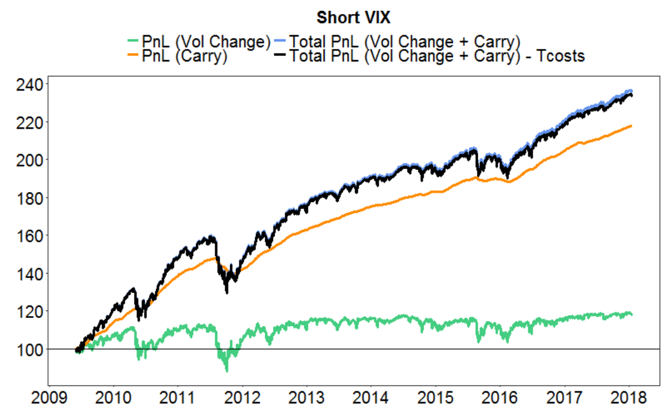
## Short volatility strategies

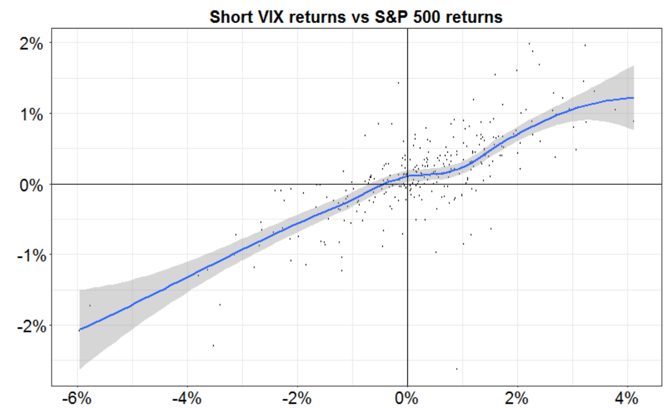
As shown above, carry is the main source of PnL for a simple short vol strategy. In Figure 5, we show the cumulated PnL of a strategy that always shorts 1 unit of 1M volatility future. Using S&P 500 as an example, we point out the high correlations between short vol returns and market returns. We show four cumulated PnL in each chart:

- PnL due only to change in volatility
- PnL due only to carry
- Total PnL (volatility change + carry)
- Total PnL minus transaction costs (VIX: 0.02, V2X: 0.03, VNKY: 0.5 vol points)

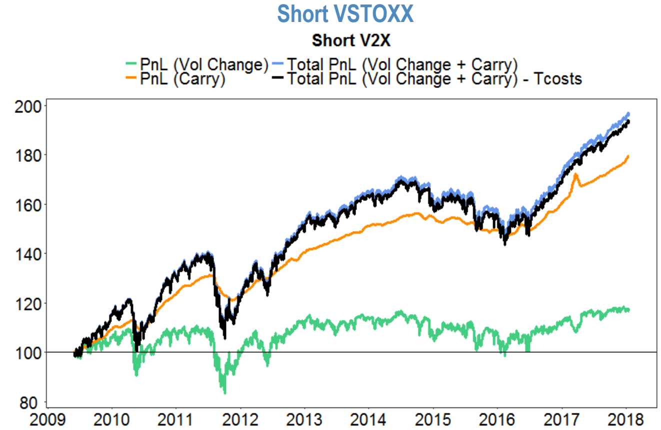
**Figure 5: Cumulated PnL of short 1M volatility futures in volatility points: The table shows the daily return statistics – Transaction costs are in volatility points; Sortino ratio is the returns scaled by downside volatility; Calmer ratio is the returns scaled by maximum drawdown**

**Short VIX**

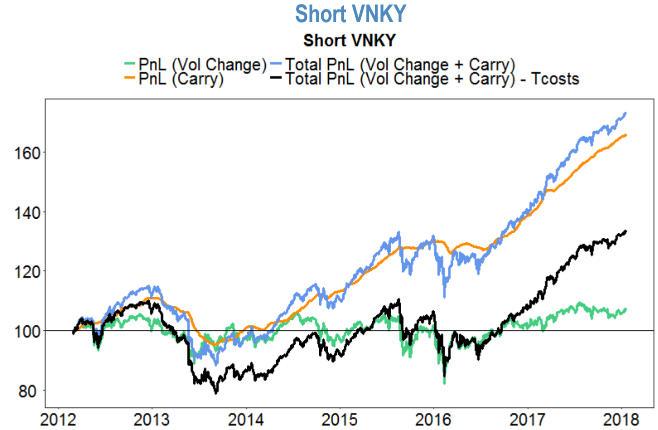



**Short VSTOXX**




**Short VNKY**




| Short 1M Vol | Start | End | t-cost (vol points) | Annualized returns | Annualized Vol | IR | Max Drawdown | Hit Ratio | Sortino Ratio | Calmer Ratio |
|---|---|---|---|---|---|---|---|---|---|---|
| Short VIX | 2009-06-03 | 2018-01-19 | 0.02 | 10.5% | 8.6% | 1.21 | 18.9% | 56.8% | 0.11 | 0.55 |
| Short V2X | 2009-06-03 | 2018-01-19 | 0.03 | 8.0% | 11.1% | 0.72 | 24.7% | 55.4% | 0.06 | 0.32 |
| Short VNKY | 2012-02-29 | 2018-01-19 | 0.5 | 5.3% | 13.3% | 0.40 | 28.3% | 55.5% | 0.04 | 0.19 |

Source: J.P. Morgan Quantitative and Derivatives Strategy, Bloomberg

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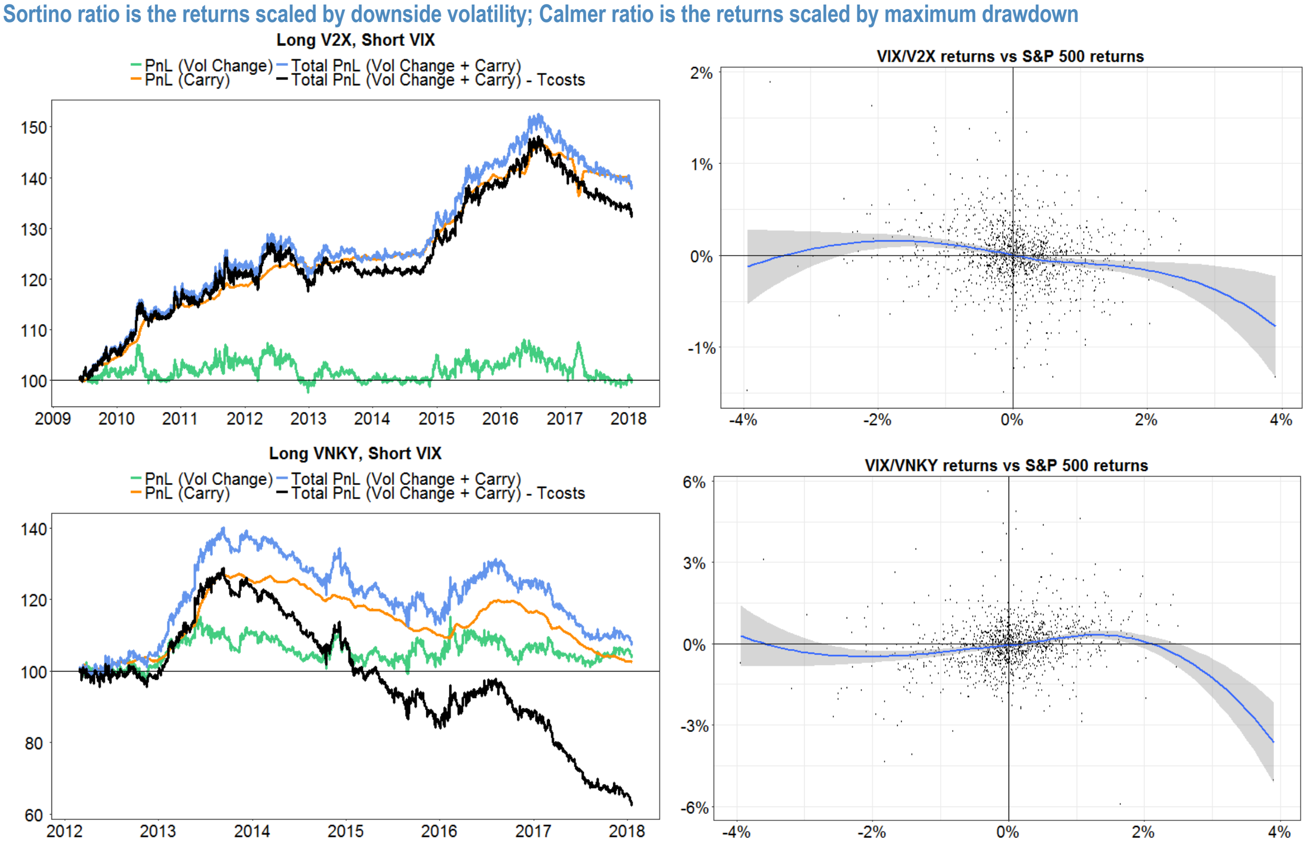
## Volatility spread strategies

Apart from earning carry from short vol strategies, another popular play is the long/short vol spread strategy. It is a relative value strategy that depends on the mean-reversion of volatility spread between two volatility futures. The economic rationale behind the mean-reversion arises from the fact that markets are interconnected. For instance, a volatility spike in VIX is likely to be followed by an increase in V2X (Shore, 2013).

Figure 6 shows a static version of volatility spread strategy, where we always short 1M VIX and long another volatility future (either 1M V2X or VNKY). Historically, it has always been preferable to short VIX due to its consistent positive carry (see Figure 4). VIX is also less volatile than V2X and VNKY.

The top right chart in Figure 6 shows that the VIX/V2X strategy is quite defensive, as it is slightly negatively correlated with S&P 500 returns. Apparently, the long exposure in V2X has helped to hedge against adverse market conditions. For VIX/VNKY, the returns are slightly positively correlated with S&P 500 returns.

**Figure 6: Long/short volatility spread strategies: Here, we always short 100% 1M VIX and long 100% 1M V2X (top) or long 1M VNKY (bottom); the cumulated PnL are in volatility points: The table shows the daily USD returns statistics of vol spread strategies with transaction costs; Sortino ratio is the returns scaled by downside volatility; Calmer ratio is the returns scaled by maximum drawdown**




| Vol Spread (After t-cost) | Start | End | Annualized returns | Annualized Vol | IR | Max Drawdown | Hit Ratio | Sortino Ratio | Calmer Ratio |
|---|---|---|---|---|---|---|---|---|---|
| VIX / V2X | 2009-06-03 | 2018-01-19 | 3.4% | 6.4% | 0.53 | 10.6% | 51.1% | 0.05 | 0.32 |
| VIX / VNKY | 2012-02-28 | 2018-01-19 | -7.9% | 15.2% | -0.52 | 51.4% | 47.6% | -0.04 | -0.15 |

Source: J.P. Morgan Quantitative and Derivatives Strategy, Bloomberg

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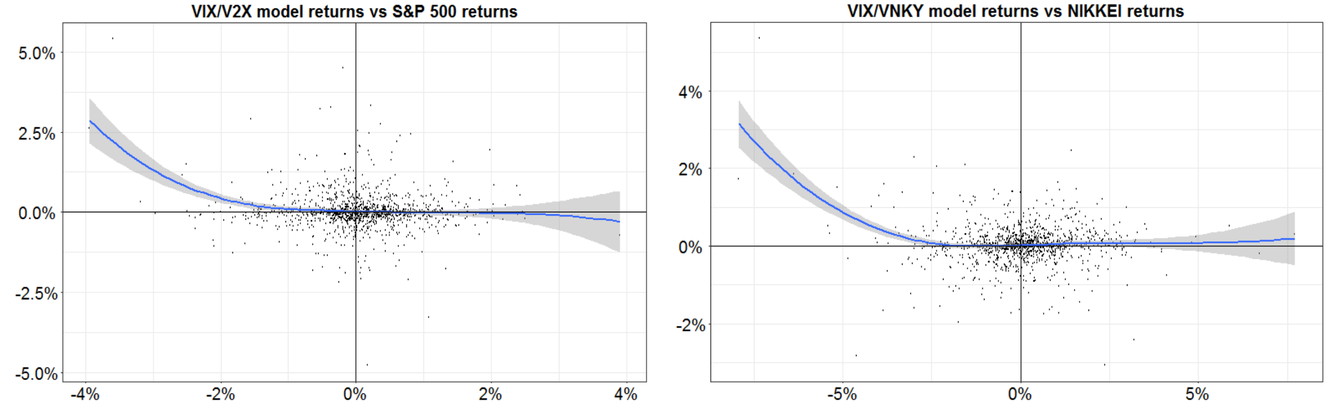
## Defensive risk premium

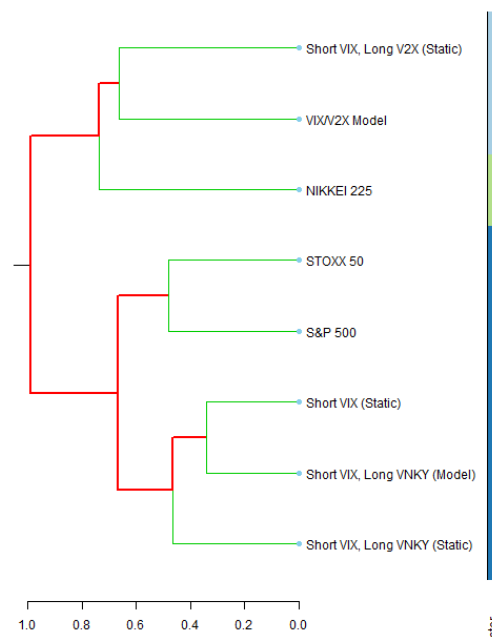
As we have seen in Figure 6, volatility spread strategies can be quite defensive: When the equity market is down, volatility spread returns tend to be positive, thus providing a hedge to adverse market conditions. This is due partly to the nature of the vol spread strategy: As a mean-reversion strategy, it attempts to profit from dislocations in volatilities across related equity markets.

In Figure 7, we show that our systematic vol spread strategies (to be detailed in the next sections) are long convexity and can be regarded as a defensive risk premia. When the market goes down, our strategies provide positive returns, acting as a hedge for equity portfolios. In the bottom, we highlight the low correlations between the vol spread strategies with equity markets.

**Figure 7: Vol spread strategies could serve as a hedge to the equity market as they long convexity; they are also lowly correlated to market returns: We show our model strategies, as well as the "static" versions (as in the previous sections) that always take the same positions**




| | Short VIX, Long VNKY (Static) | Short VIX, Long VNKY (Model) | Short VIX (Static) | S&P 500 | STOXX 50 | NIKKEI 225 | VIX/V2X Model | Short VIX, Long V2X (Static) |
|---|---|---|---|---|---|---|---|---|
| Short VIX, Long VNKY (Static) | 1 | 0.72 | 0.52 | 0.27 | 0.33 | -0.17 | 0.05 | 0.16 |
| Short VIX, Long VNKY (Model) | 0.72 | 1 | 0.77 | 0.37 | 0.52 | 0.07 | 0.07 | 0.12 |
| Short VIX (Static) | 0.52 | 0.77 | 1 | 0.42 | 0.64 | 0.23 | 0.08 | 0.14 |
| S&P 500 | 0.27 | 0.37 | 0.42 | 1 | 0.54 | 0.12 | -0.02 | -0.16 |
| STOXX 50 | 0.33 | 0.52 | 0.64 | 0.54 | 1 | 0.23 | | -0.37 |
| NIKKEI 225 | -0.17 | 0.07 | 0.23 | 0.12 | 0.23 | 1 | | -0.07 |
| VIX/V2X Model | 0.05 | 0.07 | 0.08 | -0.02 | | | 1 | 0.12 |
| Short VIX, Long V2X (Static) | 0.16 | 0.12 | 0.14 | -0.16 | -0.37 | -0.07 | 0.12 | 1 |




Source: J.P. Morgan Quantitative and Derivatives Strategy, Bloomberg

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# Dynamics of volatility and carry

While the PnL of volatility futures can be decomposed into two components (change in volatility and carry), these components are closely related. In the following, we look at some of their properties. We attempt to put these dynamics into consideration when we construct a structural model for trading volatility spreads.

## Mean reversion of volatility

As volatility cannot drop below zero nor rise indefinitely, common perceptions tell us that volatility tends to mean-revert, i.e., there exists a long-term mean, and any deviations away from the "norm" should eventually revert. There are various ways to quantify this behavior, e.g., variance ratio tests, Hurst exponent, etc. One of the standard measures is the Augmented Dickey-Fuller (ADF) test. Suppose $x_t$ has a mean of zero and is autoregressive such that

$$x_t = (1-\lambda)x_{t-1} + \varepsilon_t$$

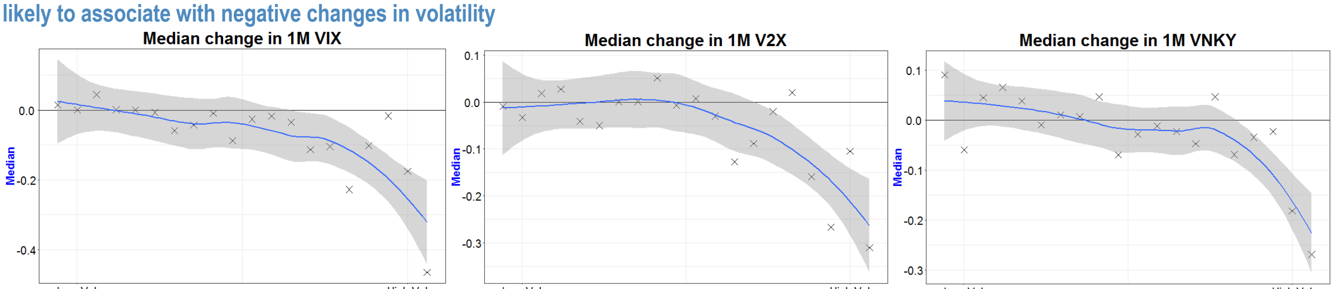
for some coefficient $1-\lambda$. We want to test for the null hypothesis of $\lambda = 0$ against the alternative hypothesis of $\lambda > 0$. If $\lambda = 0$, $x_t$ is a random walk and the series is non-stationary. Alternatively, if $\lambda > 0$, then $x_t$ is stationary and mean-reverting. To see this, note that $\Delta x_t = x_t - x_{t-1}$ is inversely proportional to $x_t$:

$$\Delta x_t = -\lambda x_{t-1} + \varepsilon_t$$

In other words, if $x_{t-1}$ is positive (away from mean zero), the change $\Delta x_t$ is expected to be negative, such that the value reverts toward zero.

Figure 8 exhibits the above relationships for VIX, V2X and VNKY, where we sort the volatility levels from low to high along the horizontal axis. When volatility is high, the change in volatility is very negative, showing strong reversion toward the mean. Conversely, for extremely low volatility levels, changes in volatility tend to be positive, although with a smaller magnitude.

**Figure 8: Median change in volatility (y-axis) against the level of volatility (x-axis): Volatility tends to mean-revert, as high volatility is more likely to associate with negative changes in volatility**

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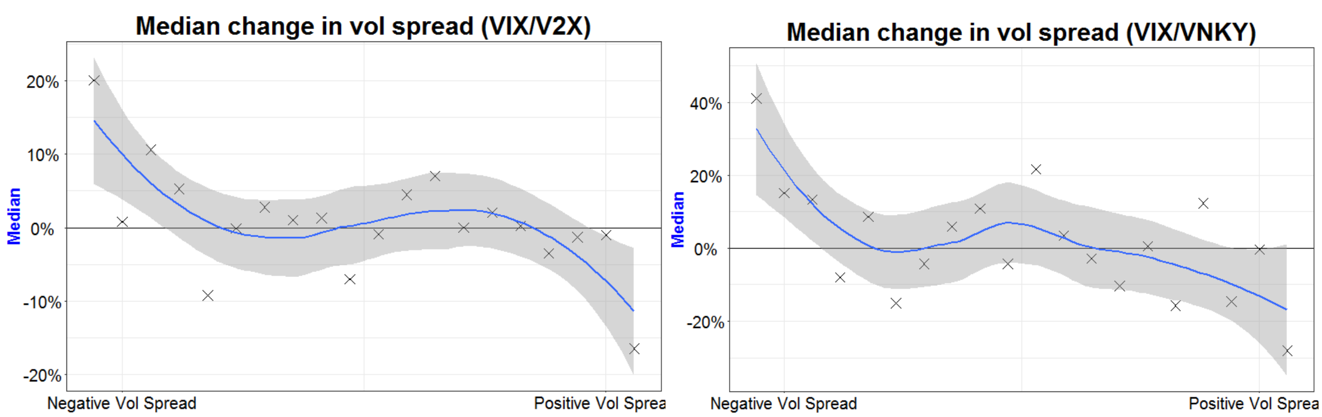
**Volatility spread may exhibit stronger mean reversion**

Figure 9 shows the median changes in volatility spread at different values of the spreads. Apparently, mean reversion of volatility spreads may be stronger than that of volatility itself. When the spread is positive (or wide), the change in spread tends to be negative (i.e., the spread contracts). On the other hand, when the spread is negative (or narrow), the change in spread tends to be positive (i.e., the spread expands)

Figure 9: Median change in volatility spread (y-axis) against volatility spread (x-axis): Volatility spread tends to mean-revert since positive volatility spread is more likely to associate with a decrease in volatility spread; on the other hand, negative volatility spread is more likely to associate with an increase in volatility spread




Source: J.P. Morgan Quantitative and Derivatives Strategy, Bloomberg

## Change in carry and change in volatility: An inverse relationship

When there is a spike in volatility, the term structure often becomes inverted and carry may even turn negative. Using 1M VIX as an example, we have

$$VIX_{1M,t} = \omega_{1,t} UX_{1,t} + \omega_{2,t} UX_{2,t} \qquad \omega_{1,t} + \omega_{2,t} = 1$$

where $UX_{1,t}$ and $UX_{2,t}$ are the prices of the first and second contract, respectively, and $\omega_{1,t}$, $\omega_{2,t}$ are the rolling weights, respectively. We can look at the relationship between the change in 1M volatility:

$$\Delta VIX_{1M,t} = VIX_{1M,t} - VIX_{1M,t-1}.$$

and the change in carry:

$$\Delta Carry_t = \Delta\left(UX_{2,t} - UX_{1,t}\right)$$

When there is a spike in volatility (i.e., $\Delta VIX_{1M,t}$ is large), the price of the front-month contract ($UX_{1,t}$) tends to rise sharply and may even be higher than the price of the next contract ($UX_{2,t}$). As a result, carry ($UX_{2,t} - UX_{1,t}$) in general decreases or even becomes negative.

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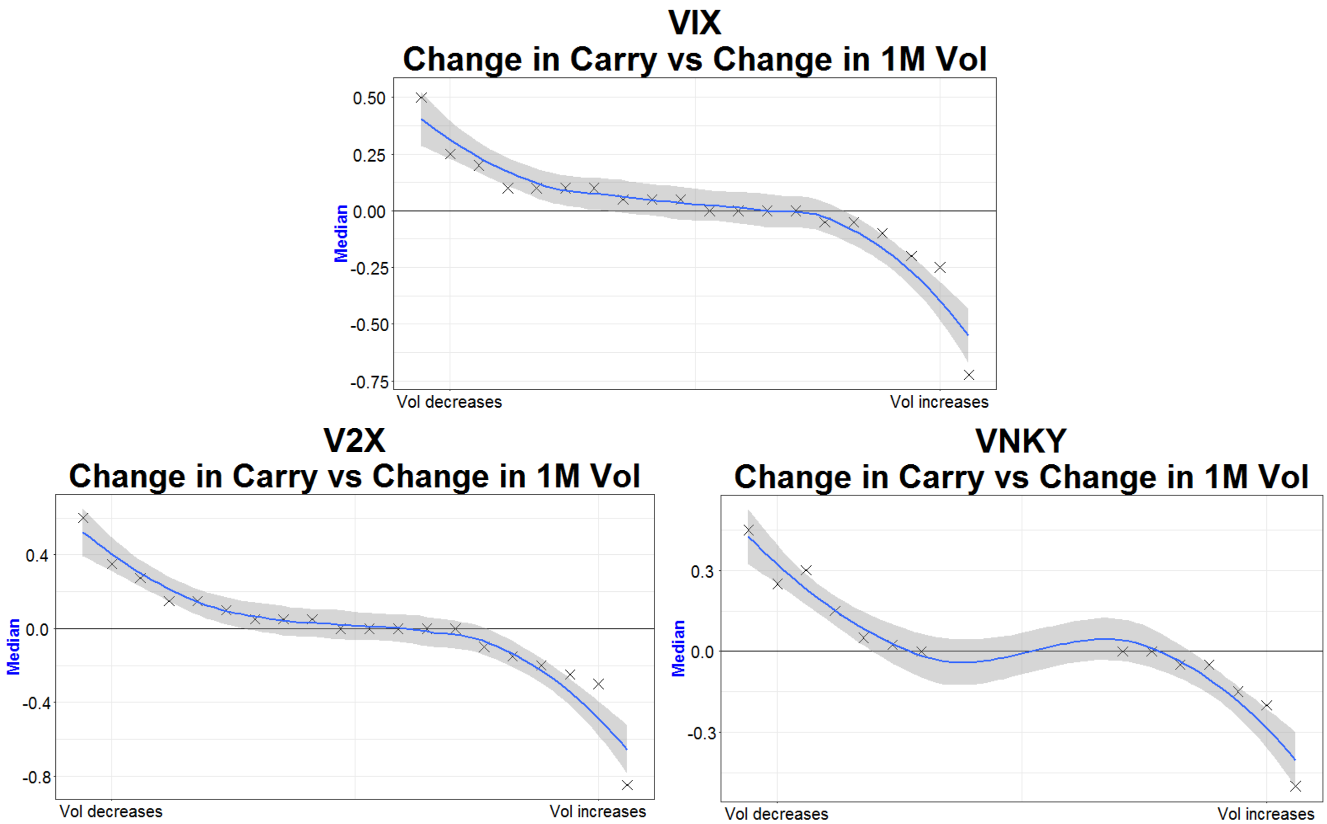
In Figure 10, we show such an inverse relationship. We first sort the changes in 1M volatility ( $\Delta VIX_{1M,t}$ ) into 20 buckets and calculate the median change in carry ( $\Delta Carry_t$ ) within each bucket. Apart from VIX, we also look at V2X and VNKY and find that an increase in volatility in general corresponds to a decrease in carry.

**Figure 10: Relationship between changes in carry (y-axis) and changes in volatility (x-axis): We sort the changes in 1M volatility into 20 buckets and calculate the median change in carry within each bucket – In general, an increase in volatility corresponds to a decrease in carry**




Source: J.P. Morgan Quantitative and Derivatives Strategy, Bloomberg

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**Changes in spreads are also inversely related**

We also observe an inverse relationship between volatility spreads and carry spreads, which are analogous to the relationship as shown in Figure 10 for changes in volatility and carry.

Using VIX/V2X as an example, volatility spread is given by[4]

$$x_t = VIX_{1M,t} - V2X_{1M,t}$$

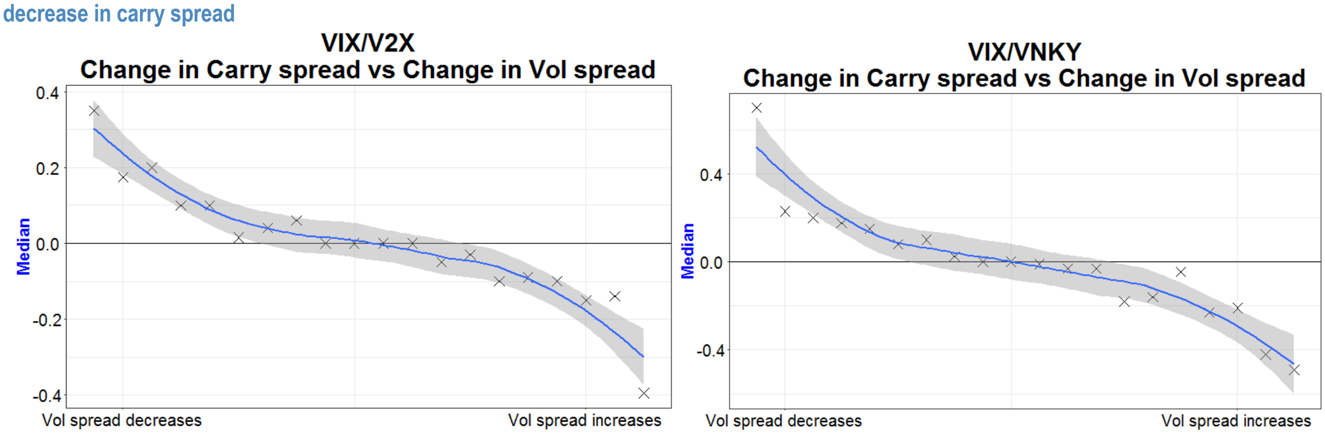
Carry spread is the difference between VIX carry and V2X carry:

$$c_t = \left(UX_{2,t} - UX_{1,t}\right) - \left(FVS_{2,t} - FVS_{1,t}\right)$$

where $UX_{1,t}$, $UX_{2,t}$, $FVS_{1,t}$ and $FVS_{2,t}$ are the prices of the first and second contracts, respectively, for VIX and V2X futures.

In Figure 11, we look at the relationship between the changes in volatility spread, $\Delta x_t = x_t - x_{t-1}$ and the changes in carry spread, $\Delta c_t = c_t - c_{t-1}$. We sort the changes in volatility spread into 20 buckets and calculate the median change in carry spread within each bucket. For both cases in VIX/V2X and VIX/VNKY, we see that an increase in volatility spread corresponds to a decrease in carry spread.

**Figure 11: Relationship between changes in carry spread and changes in volatility spread: We sort the changes in volatility spread into 20 buckets and calculate the median change in carry spread within each bucket – In general, an increase in volatility spread corresponds to a decrease in carry spread**




Source: J.P. Morgan Quantitative and Derivatives Strategy, Bloomberg

[4] In this report, we take the convention that volatility spread is VIX over V2X (and VIX over VNKY). As VIX is usually lower than V2X or VNKY, our spread is in general negative. Other studies may take the opposite direction to ensure the spread is positive.

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# Building a Dynamic Linear Model

As we have seen, the PnL of volatility futures can be decomposed into two components (volatility change and carry) that exhibit very different dynamics:

- Volatility is mean-reverting.
- Carry is stable and mostly positive, except during spikes in volatility.

In the following, we attempt to model these components with a structural model and use the predictions of PnL as our trading signal in the volatility spread strategy. Here, we use VIX/V2X as an example, but the logic applies to other pairs of volatility futures. In the next section, we will build a similar model for VIX/VNKY.

## Modelling the PnL components

The PnL (in USD) of a VIX/V2X strategy can be decomposed into two components, if we assume EURUSD is constant (see p.44 in Appendix):

$$\text{PnL in USD} = \left(\Delta VIX_{1M,t} - \Delta V2X_{1M,t}\right) + \left(Carry_{VIX,t} - Carry_{V2X,t}\right)$$

with $$Carry_{VIX,t} = -\Delta\omega_{2,t}^{VIX}\left(UX_{2,t} - UX_{1,t}\right)$$

$$Carry_{V2X,t} = -\Delta\omega_{2,t}^{V2X}\left(FVS_{2,t} - FVS_{1,t}\right)$$

Assuming the % of rolling into the next contract is the same: $\Delta\omega = \Delta\omega_{2,t}^{VIX} = \Delta\omega_{2,t}^{VIX}$

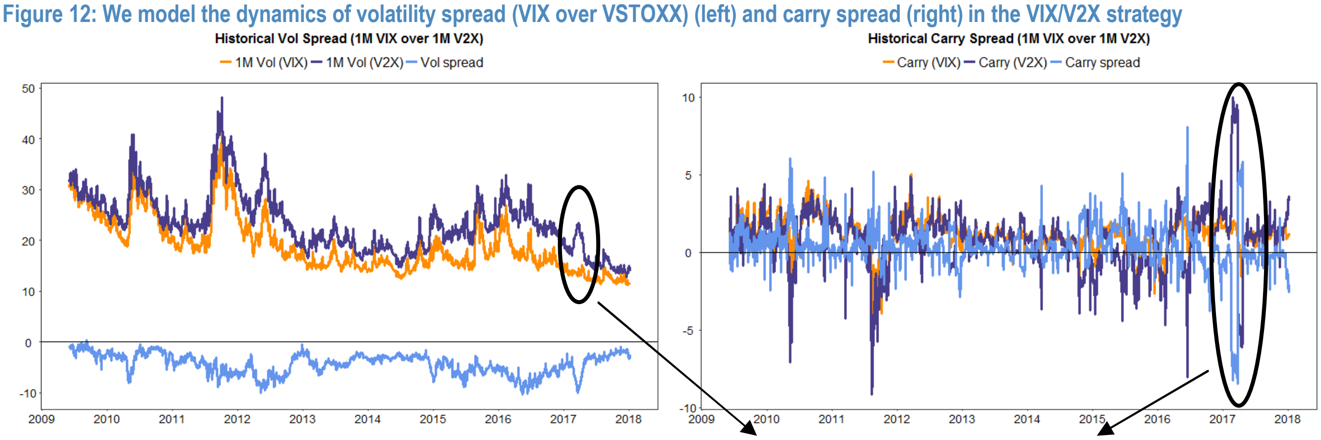
We have

$$\text{PnL in USD} = \left(\Delta VIX_{1M,t} - \Delta V2X_{1M,t}\right) - \Delta\omega \times \left[\left(UX_{2,t} - UX_{1,t}\right) - \left(FVS_{2,t} - FVS_{1,t}\right)\right]$$
$$= \Delta x_t - \Delta\omega \times c_t$$

Figure 12 shows the dynamics of volatility spread $x_t$ and carry spread $c_t$ :

- **Volatility spread**: $x_t = VIX_{1M,t} - V2X_{1M,t}$
- **Carry spread**: $c_t = \left(UX_{2,t} - UX_{1,t}\right) - \left(FVS_{2,t} - FVS_{1,t}\right)$

Figure 12: We model the dynamics of volatility spread (VIX over VSTOXX) (left) and carry spread (right) in the VIX/V2X strategy




Source: J.P. Morgan Quantitative and Derivatives Strategy, Bloomberg

Large swings in volatility and carry for V2X prior to French presidential election in 2017

In Figure 12, we see that VIX over V2X volatility spread is negative[5], i.e., 1M V2X is higher than 1M VIX. Also, there could be large swings in carry, e.g., for V2X prior to the French presidential election in 2017, that are accompanied by large swings in volatility.

**Dynamics of volatility spread $x_t$**

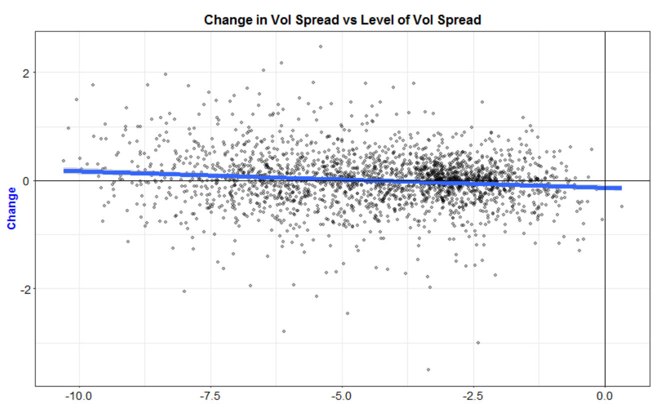
As volatility spread tends to be mean-reverting (see Figure 9), we model it using the Ornstein-Uhlenbeck (OU) process. In discrete time, it is simply an AR(1) model of the form:

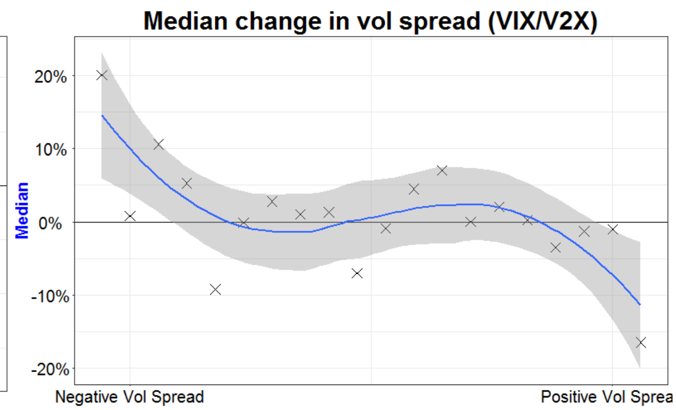
1. **Dynamics for Vol Spread:** $x_t = x_{t-1} + \lambda(\mu_{t-1} - x_{t-1}) + \varepsilon_t$

where $x_t$ is the volatility spread, $\lambda > 0$ is the rate of mean reversion, $\mu_t$ is the long term mean, and $\varepsilon_t$ is the residual.

For example, if volatility spread is lower than the long-term mean (i.e., $\mu_{t-1} > x_{t-1}$), then we expect the value to increase by $\Delta x_t = \lambda(\mu_{t-1} - x_{t-1})$ in the next period. Figure 13 shows the change in volatility spread against the level of volatility spread, indicating the above mean-reversion property.

Figure 13: Volatility spread is mean-reverting: Change in volatility spread is negatively proportional to the level of volatility spread: The two charts show the same information – the left one shows the scatter plot for all datapoints, while the right one shows a smooth curve by aggregating similar levels of volatility spread into 20 bins along the x-axis




Source: J.P. Morgan Quantitative and Derivatives Strategy, Bloomberg

We further assume that the long-term mean of the volatility spread follows a random walk instead of being a constant, i.e.,

2. **Dynamics for long-term mean (of vol spread):** $\mu_t = \mu_{t-1} + \varepsilon_t$

[5] Some studies may define the spread as V2X over VIX to ensure it is positive.

**Dynamics of carry spread** $c_t$

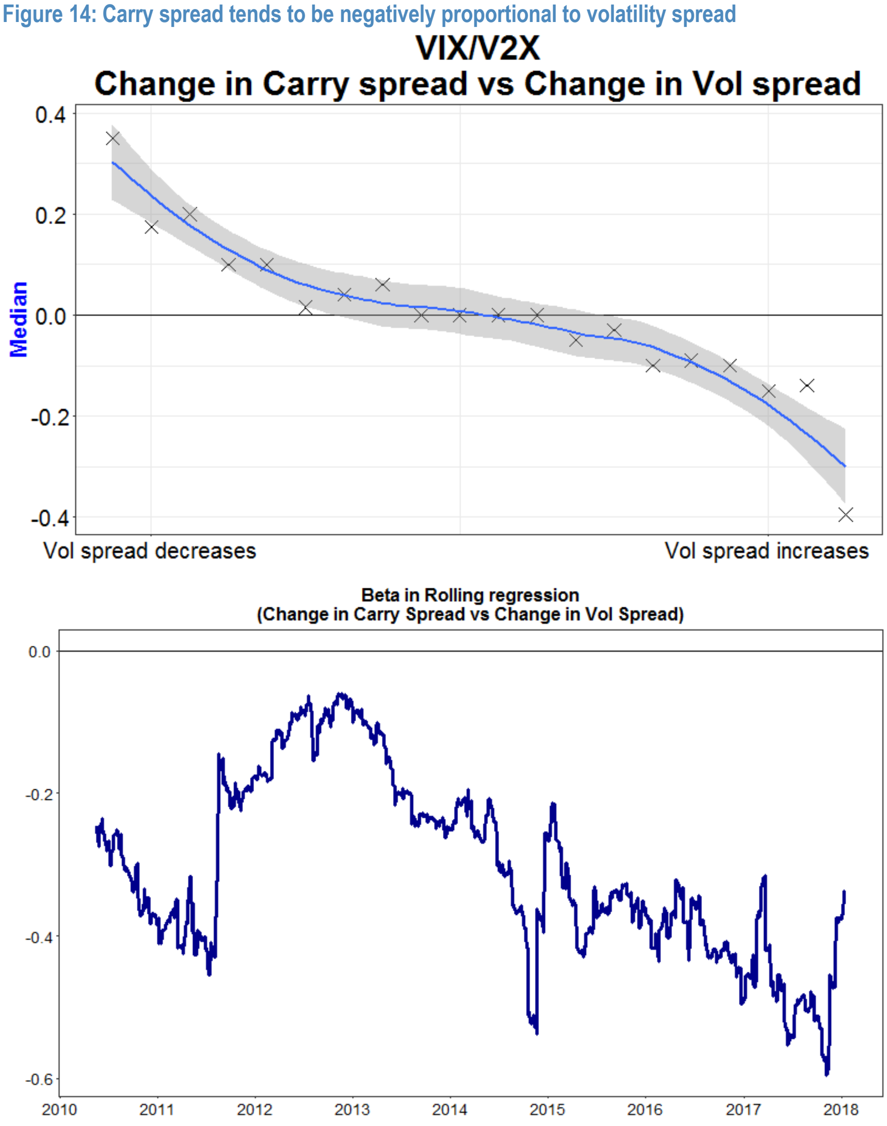
As seen earlier in Figure 11, changes in volatility spread are closely related to changes in carry spread: An increase in volatility spread typically corresponds to a decrease in carry spread. Hence, we model the carry spread as below:

**3. Dynamics for carry spread:** $c_t = c_{t-1} + \gamma(x_{t-1} - x_{t-2}) + \varepsilon_t$

With the inverse relationship, we expect the coefficient γ to be negative. It turns out that the estimated parameter is indeed mostly negative.

Figure 14 shows the relationship between changes in carry spread and volatility spreads. The bottom panel gives the coefficient γ if we estimate with a simple rolling regression.

Figure 14: Carry spread tends to be negatively proportional to volatility spread




Source: J.P. Morgan Quantitative and Derivatives Strategy, Bloomberg

## State space representation

Combining the above dynamics in (1) – (3), a state space model can be obtained. The below gives the matrix representations that define our dynamic system. Readers interested in the final strategy can go directly to p.21.

**Unobserved state vector**

Let us consider the hidden "state" of the system as a vector with four components:

$$\theta_t = (x_t \quad x_{t-1} \quad \mu_t \quad c_t)'$$

where $x_t$ is the volatility spread, $\mu_t$ is the long-term mean of volatility spread, and $c_t$ is the carry spread. At a glance, it may look strange to include $x_{t-1}$ in the state vector, but we will see that the inclusion is for the dynamics of $c_t$.

**Evolution of the state**

The evolution of the state vector can be written in matrix form as

$$\theta_t = G\theta_{t-1} + w_t \qquad w_t \sim N(0, W)$$

where $W$ is a diagonal covariance matrix governing the uncertainties of the state variables. If $W$ is large, a lot of information is lost when the state evolves from $\theta_{t-1}$ to $\theta_t$, i.e., the decay of information is fast (see the Kalman Gain on p.40 in Appendix). Based on the dynamics in the previous section, we have

$$x_t = x_{t-1} + \lambda(\mu_{t-1} - x_{t-1}) + \varepsilon_{1,t}$$
$$\mu_t = \mu_{t-1} + \varepsilon_{2,t}$$
$$c_t = c_{t-1} + \gamma(x_{t-1} - x_{t-2}) + \varepsilon_{3,t}$$

Hence, we get

$$G = \begin{pmatrix} 1-\lambda & 0 & \lambda & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \gamma & -\gamma & 0 & 1 \end{pmatrix}$$

**Measurements driven by the state**

In a state space model, we assume that the measurements are driven by the unobserved state vector. Now, our measurement $z_t$ is a vector with two components, i.e., the volatility spread $x_t$ and the carry spread $c_t$:

$$z_t = (x_t \quad c_t)'$$

This means that our observations are simply noisy measurements of the state.

In terms of matrix equations, we write

$$z_t = F\theta_t + v_t \qquad v_t \sim N(0, V)$$

where $V$ is a diagonal covariance matrix governing the measurement uncertainties. Similar to $W$, it determines how much information is to be kept within the new observations. Since $\theta_t = (x_t \quad x_{t-1} \quad \mu_t \quad c_t)'$, it is trivial that

$$F = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

## Rolling estimation of model parameters

With the model described above, we can use the Kalman Filter algorithm (see p.40 in Appendix) to efficiently obtain the optimal predictions for the state vector $\theta_t$ and the measurement $z_t$. However, we first need to estimate the model parameters. To avoid look-ahead bias, we use a rolling estimation that maximizes the likelihood of the data, as follows:

- At the end of each month (from 2009-12-31 onward), we consider an expanding window of all past observations[6] for the daily change in 1M volatility spread and carry spread. This is our measurement vector $z_t$.
- With the observations $z_t$, we estimate the parameters $\lambda$, $\gamma$ in the state evolution matrix $G$, as well as the diagonal terms in the state covariance matrix $W$ and the observation covariance matrix $V$. We use the R package "dlm" to estimate these parameters based on maximum likelihood[7].

**Fixing the variances of long-term mean**

Recall that our state vector consists of the long-term mean of volatility spread $\mu_t$, which follows a random walk: $\mu_t = \mu_{t-1} + \varepsilon_{2,t}$.

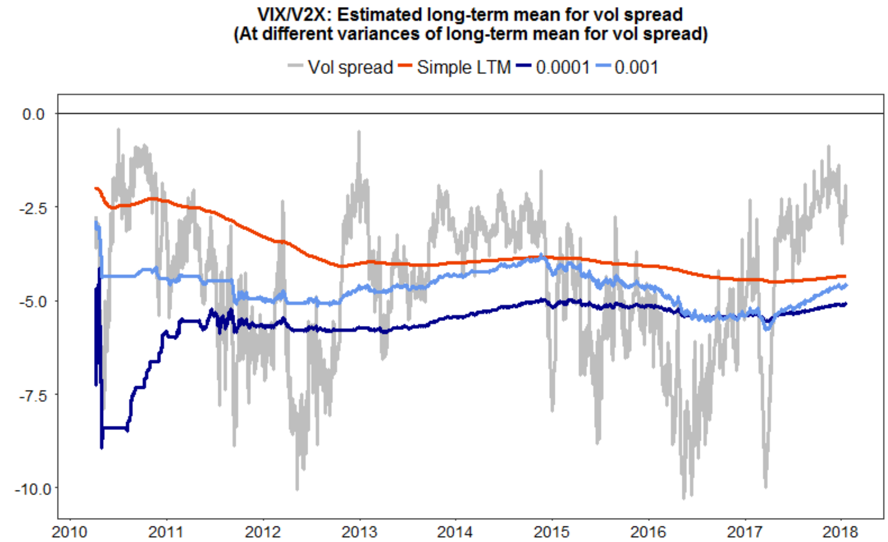
In the model, we need to determine $Var(\varepsilon_{2,t})$ in the state covariance matrix $W$. We find that Maximum Likelihood Estimation (MLE) will lead to large values of $Var(\varepsilon_{2,t})$, giving a noisy long-term mean, which is undesirable. After all, we expect the long-term mean to evolve slowly.

6 We also tried rolling one-year and two-year windows, and results are qualitatively comparable. This is expected, as the Kalman Filter algorithm puts a decaying weight on past observations, and hence should not differ too much if we include all history.

7 We use the "L-BFGS-B" algorithm for maximizing the likelihood, which could handle box constraints if necessary (e.g., $\lambda > 0$, $\gamma < 0$). However, we do not impose constraints in the estimation.

To handle this issue, we decide to fix this variance parameter manually, instead of estimating it with MLE. Figure 15 shows the estimated long-term mean $\mu_t$ at different chosen values of variance parameters. For larger variances (light blue), the long-term mean fluctuates more. The red line is obtained by simple moving average.

**Figure 15: Estimated long-term mean for volatility spread at different values of variances**

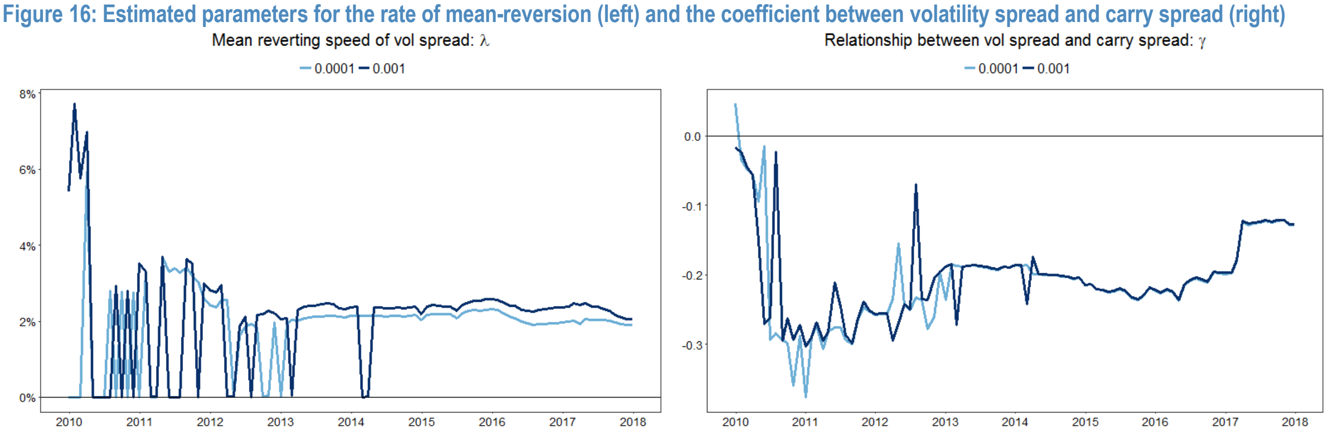



Source: J.P. Morgan Quantitative and Derivatives Strategy, Bloomberg

To avoid having a volatile long-term mean $\mu_t$, we fix its variance at different values (e.g., 0.0001, 0.0005, etc.) and estimate all other parameters based on MLE. Figure 16 shows the monthly estimated parameters for the rate of mean-reversion $\lambda > 0$ and the coefficient $\gamma$ (relationship between volatility spread and carry spread) in the state evolution matrix $G$.

We find that volatility spread is not always strongly mean-reverting: There are periods with $\lambda \approx 0$. On the other hand, the coefficient $\gamma$ is mostly negative, matching our expectations, as in Figure 11.

**Figure 16: Estimated parameters for the rate of mean-reversion (left) and the coefficient between volatility spread and carry spread (right)**




Source: J.P. Morgan Quantitative and Derivatives Strategy, Bloomberg

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# Volatility spread strategy: VIX/VSTOXX

With a statistical model that describes the dynamics of volatility spread and carry spread (i.e., the PnL components), we are set to derive a VIX/V2X trading strategy.

## Trading signal

Our trading signal is the predicted PnL, which is a function of the one-day-ahead forecast of $z_t = (x_t \quad c_t)'$. Recall that the PnL of a volatility spread strategy (long VIX, short V2X) can be decomposed into two components (see p.44 in Appendix):

$$\text{PnL in USD} = \left(\Delta VIX_{1M,t} - \Delta V2X_{1M,t}\right) - \Delta\omega \times \left[(UX_{2,t} - UX_{1,t}) - (FVS_{2,t} - FVS_{1,t})\right]$$
$$= \Delta x_t - \Delta\omega \times c_t$$

where $\Delta\omega$ is the % of daily rolling into the next contract. Using the Kalman Filter algorithm (see p.40 in Appendix), we can make predictions for the volatility spread $E[x_{t+1}]$ and the carry spread $E[c_{t+1}]$, as well as their uncertainties. We then obtain the forecasts for each PnL component:

- Expected change in volatility spread: $E[\Delta x_{t+1}] = E[x_{t+1}] - x_t$
- Expected carry spread: $\Delta\omega \times E[c_{t+1}]$

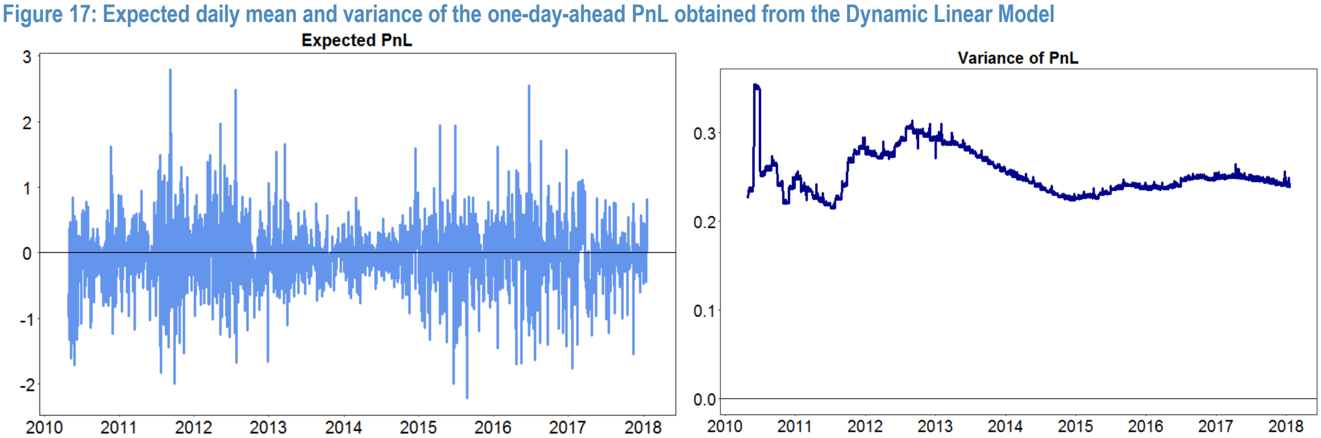
The expected mean and the variance of one-day-ahead PnL is then given by

$$E[PnL_{t+1}] = E[x_{t+1}] + \Delta\omega \times E[c_{t+1}]$$
$$Var[PnL_{t+1}] = Var[x_{t+1}] + (\Delta\omega)^2 \times Var[c_{t+1}]$$

The trading signal $s_t$ is the expected PnL scaled by uncertainty:

$$s_t = \frac{E_t[PnL_{t+1}]}{\sqrt{Var_t[PnL_{t+1}]}}$$

Figure 17: Expected daily mean and variance of the one-day-ahead PnL obtained from the Dynamic Linear Model




Source: J.P. Morgan Quantitative and Derivatives Strategy

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## Strategy backtest

We backtest a volatility spread strategy in VIX and V2X futures, using the signal of predicted PnL based on our Dynamic Linear Model.

### Strategy positions

Our daily positions $w_t$ on 1M VIX and 1M V2X volatility futures are:

- If $s_t > 0$: Long VIX, short V2X

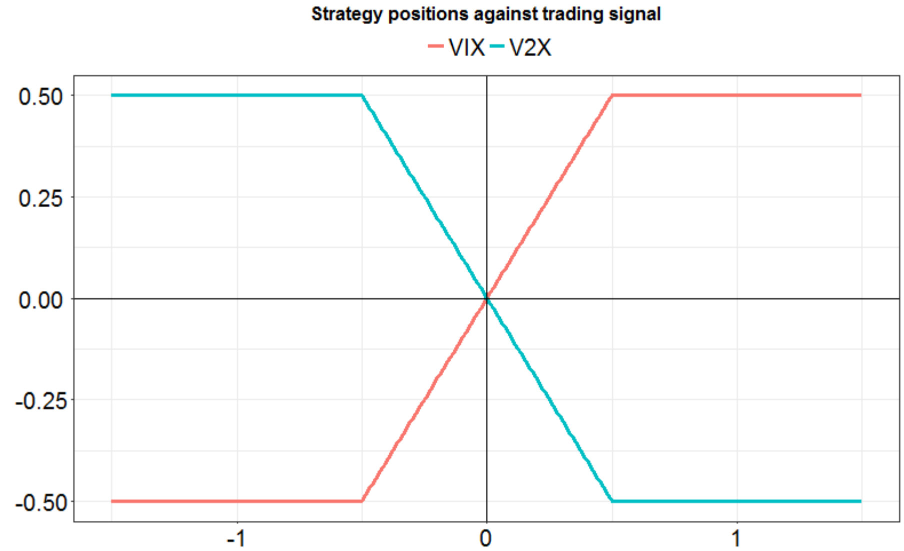
$$w_{VIX} = \min(s_t, M) \qquad w_{V2X} = \max(-s_t, -M)$$

- If $s_t < 0$: Short VIX, long V2X

$$w_{VIX} = \max(s_t, -M) \qquad w_{V2X} = \min(-s_t, M)$$

where $M = 0.5$ is the maximum weight we impose on each leg. Figure 18 depicts the trading positions on VIX and V2X, depending on the signal $s_t$.

Figure 18: Strategy positions for VIX and V2X, based on our trading signal




Source: J.P. Morgan Quantitative and Derivatives Strategy, Bloomberg

### Time synchronization: Mid-morning VIX prices

Since VIX and V2X futures trade on different exchanges, it is preferred to synchronize the observations. Fortunately, this is achievable because we have the mid-morning TWAP prices for VIX futures, calculated as of London market close at 4:30pm GMT. The prices are published by CBOE and are available on Bloomberg under the tickers TWLVIX*n* <Index> for the $n^{th}$ VIX futures.

Hence, we always calculate the signal and rebalance the trade at 4:30pm GMT, using the settlement price of V2X futures and the mid-morning TWAP prices for VIX futures.

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**Turnover constraints and transaction costs**

We further impose the following constraints and transaction costs:

- Maximum weight on each future: 50%
- Daily absolute change in the weight for each future to be within 0.05 and 0.2:

$$0.05 \leq |\Delta w| \leq 0.2$$

- Transaction cost for VIX futures is 0.02 vol points, and that for V2X futures is 0.03 vol points.
  Note that even if there is no change in our positions, i.e., $\Delta w = 0$, we still have transaction costs due to the daily rolling of volatility futures (if $\Delta \omega \neq 0$).

**Dynamic leverage**

Without leverage, the weights on 1M VIX and 1M V2X are given by

$$w = (w_{VIX}, w_{V2X})$$

where $w_{VIX} = -w_{V2X}$ and $0 \leq |w_{VIX}| + |w_{V2X}| \leq 1$.

We could leverage our exposure dynamically so as to match the strategy's PnL (in dollar terms) with our risk target. One simple way to do so is to ensure that the magnitude of the PnL is proportional to our wealth.

Suppose that the prices of 1M VIX and 1M V2X are $P_t^{VIX}$ and $P_t^{V2X}$, respectively. If our cumulated PnL is labelled as $Wealth_t$, then we leverage our exposure by $w_t \to N_t w_t$, where

$$N_t = \frac{Wealth_t}{\max(P_t^{VIX}, P_t^{V2X})}$$

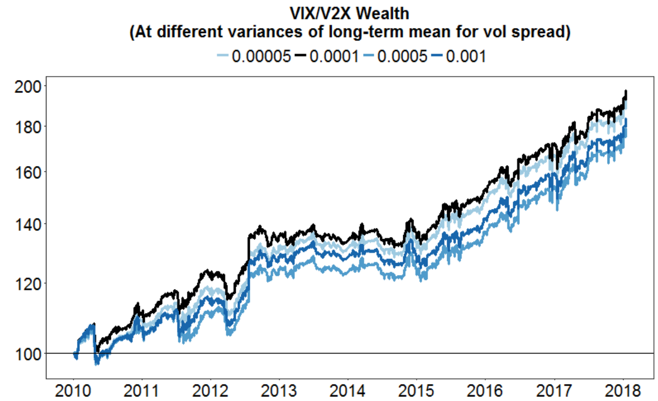
**Backtest results**

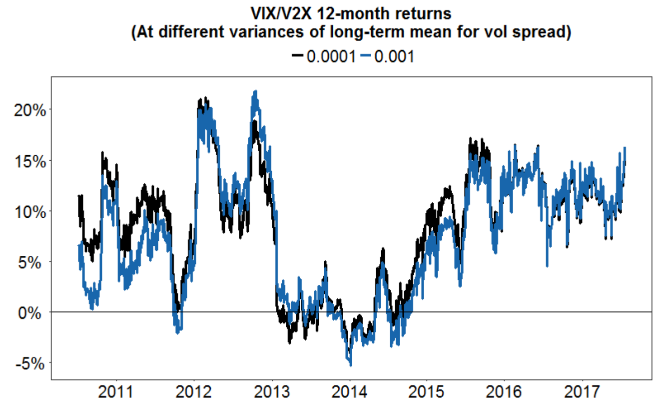
Figure 19 shows the cumulated PnL of the VIX/V2X volatility spread strategy. As we explained on p.19, we need to manually fix a value for the variances of long-term mean. As such, we show the backtest results at different parameters.

Apparently, using different parameters as the variances of long-term mean gives comparable returns, except in the beginning of the backtest. We find that setting the variance parameter to 0.0001 gives the best historical performance (black line). Such a small value corresponds to a stable long-term mean of volatility spread, as we saw in Figure 15.

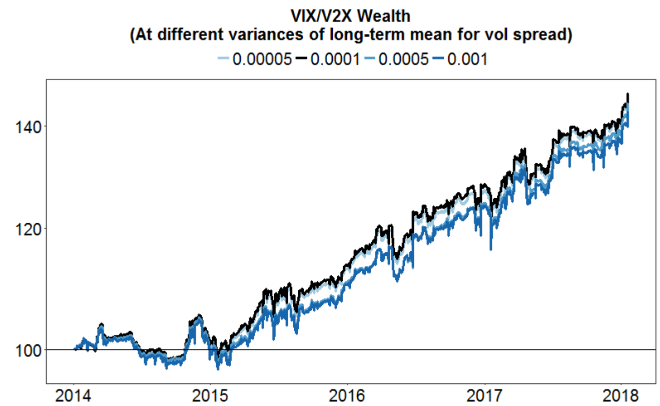
**Figure 19: Wealth curves (left) and rolling 12-month returns (right) of the VIX/V2X strategy at different values of variances of long-term mean for volatility spread: Setting the variance of long-term mean to 0.0001 gives the highest IR since 2010 (black line); the strategy is rebalanced daily, and we have accounted for transaction costs**

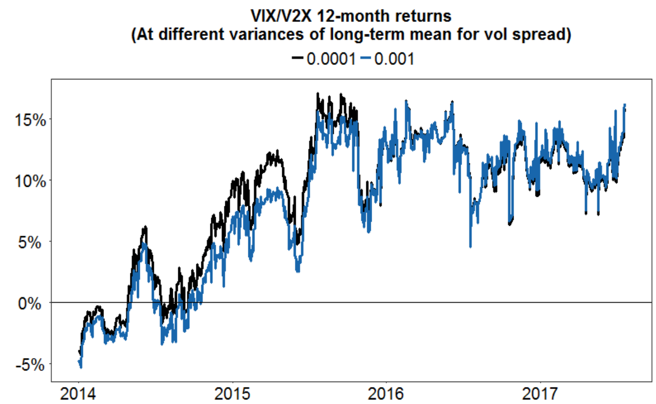
**Since 2010**




**Since 2014**




| Variance of long-term mean for volatility spread | Start | End | Annualized Returns | Annualized Vol | IR | Max Drawdown | Hit Ratio | Sortino Ratio | Calmer Ratio |
|---|---|---|---|---|---|---|---|---|---|
| | | | | **Since 2010** | | | | | |
| 0.00005 | 2010-01-05 | 2018-01-19 | 8.6% | 9.2% | 0.93 | 8.4% | 45.8% | 0.10 | 1.02 |
| **0.0001** | **2010-01-05** | **2018-01-19** | **8.9%** | **9.2%** | **0.96** | **7.4%** | **45.9%** | **0.10** | **1.21** |
| 0.0005 | 2010-01-05 | 2018-01-19 | 7.6% | 9.2% | 0.83 | 8.9% | 45.5% | 0.09 | 0.85 |
| 0.001 | 2010-01-05 | 2018-01-19 | 7.9% | 9.3% | 0.85 | 9.6% | 45.3% | 0.09 | 0.83 |
| | | | | **Since 2014** | | | | | |
| 0.00005 | 2014-01-02 | 2018-01-19 | 10.0% | 10.1% | 0.99 | 6.5% | 46.2% | 0.10 | 1.55 |
| **0.0001** | **2014-01-02** | **2018-01-19** | **10.2%** | **10.2%** | **1.00** | **6.4%** | **45.9%** | **0.10** | **1.59** |
| 0.0005 | 2014-01-02 | 2018-01-19 | 9.7% | 10.0% | 0.97 | 7.1% | 45.1% | 0.10 | 1.36 |
| 0.001 | 2014-01-02 | 2018-01-19 | 9.4% | 10.1% | 0.93 | 7.2% | 45.2% | 0.10 | 1.30 |

Source: J.P. Morgan Quantitative and Derivatives Strategy, Bloomberg

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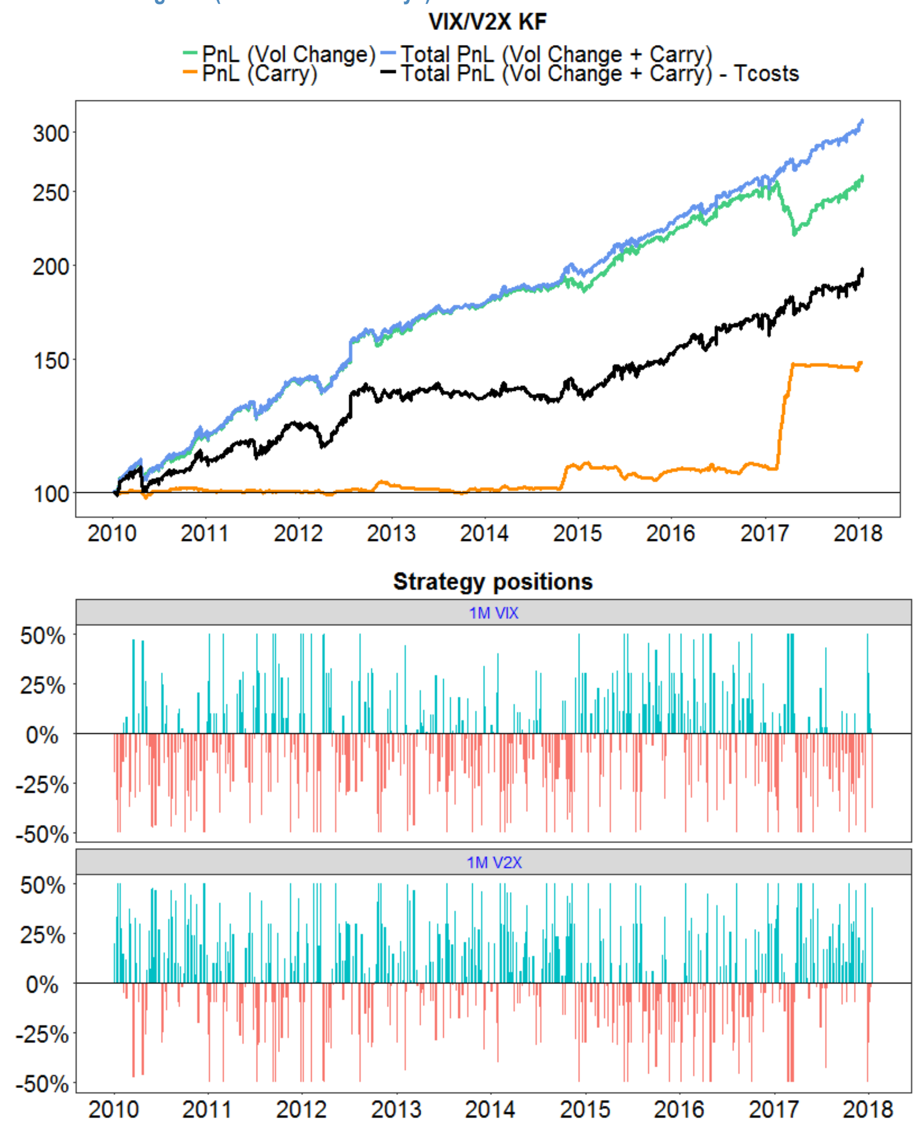
**Mean reversion of volatility spread delivers most of the returns**

Let us look closer at the strategy returns by fixing the variance parameter at 0.0001. In Figure 20, we decompose the returns into two sources: PnL due to change in volatility spread (green) and PnL due to carry spread (orange). The black line gives the total PnL after transaction costs.

We find that the strategy harvests mainly the returns due to changes in volatility spread. This is expected, as carry depends on the daily rolling, which tends to have a smaller magnitude. Moreover, in a long/short vega-neutral strategy, carry on VIX and carry on V2X may largely cancel out. In the bottom of Figure 20, we show the long/short exposures to VIX and V2X. For about 60% of days, the strategy longs V2X and shorts VIX.

Figure 20: Cumulated PnL (top) and long/short positions before leverage (bottom) of the VIX/V2X strategy (variance parameter chosen at 0.0001): The final PnL after t-costs is in black – we mostly short VIX and long V2X (for about 60% of days)




Source: J.P. Morgan Quantitative and Derivatives Strategy, Bloomberg

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**VIX leg earns decent carry**

In Figure 21, we show the cumulated PnL of each leg in the VIX/V2X strategy. We find that the VIX portion delivers positive PnL after costs, as it earns a stable and positive carry by spending a majority of days with a short exposure. Alternatively, the V2X portion has been paying for the carry cost for most of the time with its long exposure, except in 2017 prior to the French presidential election.

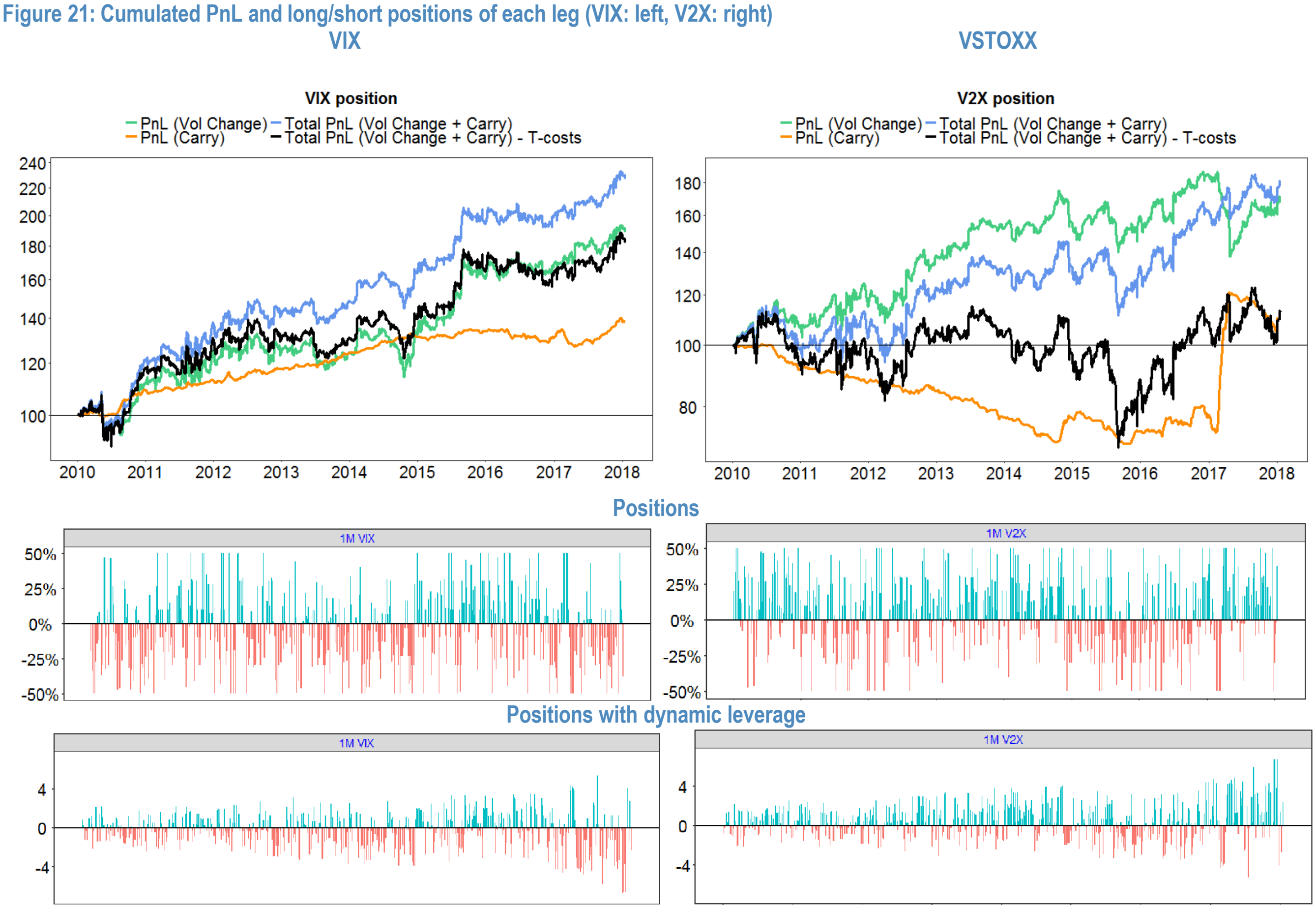
The PnL due to a change in volatility spread (i.e., a play of mean reversion) is mostly positive for both legs in VIX and V2X.

**Carry also matters…**

Interestingly, around March 2017 (prior to the French presidential election on 23 April), the strategy has a large short position in V2X. This earns a significant carry that offsets the loss due to a drop in volatility.

This is a good example showing that one should consider both volatility and carry (i.e., term structure) in a volatility spread strategy, as opposed to the intuition that the mean reversion of volatility plays the major role.

Figure 21: Cumulated PnL and long/short positions of each leg (VIX: left, V2X: right)




Source: J.P. Morgan Quantitative and Derivatives Strategy, Bloomberg

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# Volatility spread strategy: VIX/VNKY

In this section, we look into another application of the model for volatility spread trading using VIX/VNKY futures.

### A strategy to hedge for a rise in VNKY

In the recent low-volatility regime, investors concerned about a rise in volatility in the Japanese equity market may want to hedge their positions by having a long exposure to VNKY. However, due to its steep term structure, holding VNKY futures in general incurs high costs of carry.

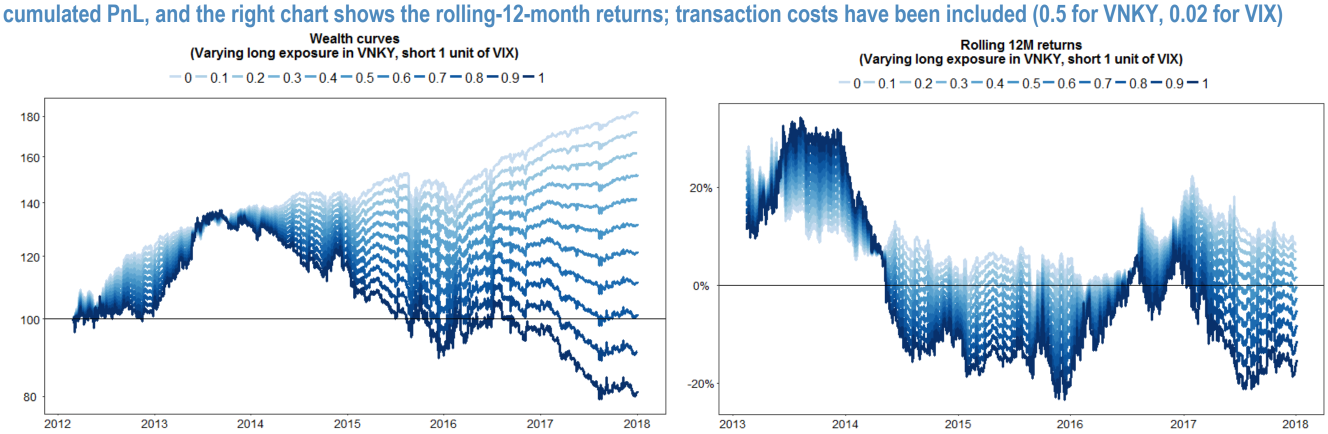
One solution to the above problem is to trade a VIX/VNKY spread strategy that mainly shorts VIX and longs VNKY[8]. Due to the stable and positive carry in shorting VIX (see Figure 4), we could use this VIX carry to fund part of our long position in VNKY futures. During tranquil markets, we expect the carry of the VIX/VNKY strategy to be positive and large enough to cover the loss even if VNKY drops (as we have a long position). When there is a spike in volatility in the NIKKEI, our long exposure in VNKY could provide protection.

Our idea is to use the Dynamic Linear Model to estimate the expected PnL of the VIX/VNKY strategy and use it to adjust an optimal long exposure to VNKY. In general, it is expensive to long the VNKY future due to the carry cost. As such, we want to have a significant long exposure only when we have a strong view on the increase in VNKY.

### Short VIX, long VNKY benchmarks

In Figure 22, we show the cumulated PnL of the portfolios that short 1 unit of VIX and long $k$ unit of VNKY futures, where $k \in [0,1]$. In general, increasing long exposure ($k$) to VNKY is costly due to the expensive carry, except during 2013 and early 2016, when VNKY went up significantly.

**Figure 22: Varying long exposures of VNKY in a long-VNKY, short-VIX strategy (0%: light blue, 100%: dark blue): The left chart shows the cumulated PnL, and the right chart shows the rolling-12-month returns; transaction costs have been included (0.5 for VNKY, 0.02 for VIX)**




Source: J.P. Morgan Quantitative and Derivatives Strategy, Bloomberg

[8] One could argue whether our strategy should be flexible so as to allow a short position in VNKY if it is preferable. We will look at a long/short strategy on p.37, but it turns out that high transaction costs in VNKY futures will hurt performance.

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**Volatility spread and carry spread**

As in the case for VIX/V2X, we now consider a Dynamic Linear Model for the volatility spread $x_t$ and the carry spread $c_t$ in the VIX/VNKY strategy:

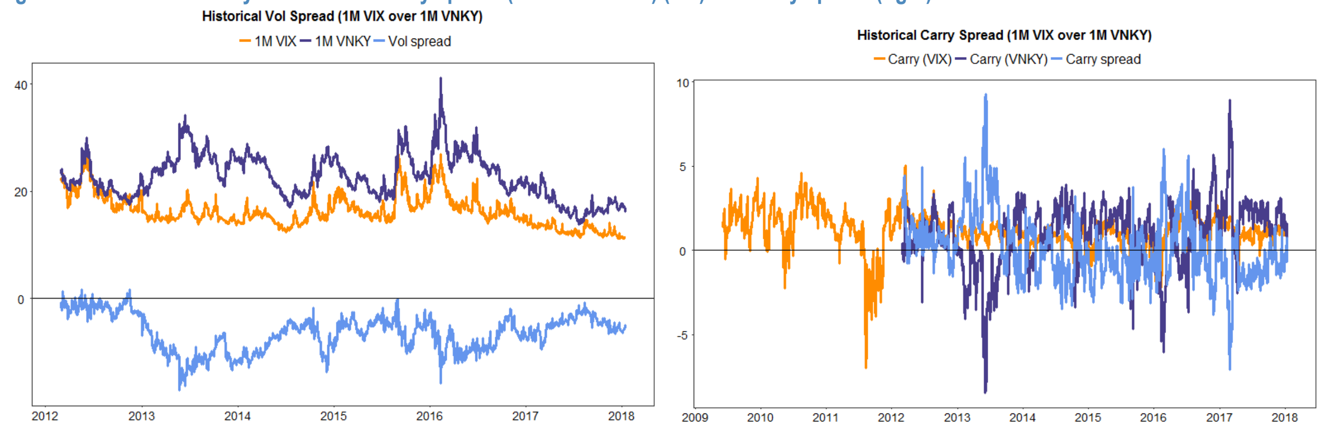
- **Volatility spread**: $x_t = VIX_{1M,t} - VNKY_{1M,t}$
- **Carry spread**: $c_t = \left(UX_{2,t} - UX_{1,t}\right) - \left(JVI_{2,t} - JVI_{1,t}\right)$

With these two components, we can model the PnL of a volatility strategy (see p.15 for the case of VIX/V2X or p.44 in the Appendix):

$$\begin{aligned}\text{PnL in USD} &= \left(\Delta VIX_{1M,t} - \Delta VNKY_{1M,t}\right) - \Delta\omega \times \left[(UX_{2,t} - UX_{1,t}) - (JVI_{2,t} - JVI_{1,t})\right] \\ &= \Delta x_t - \Delta\omega \times c_t\end{aligned}$$

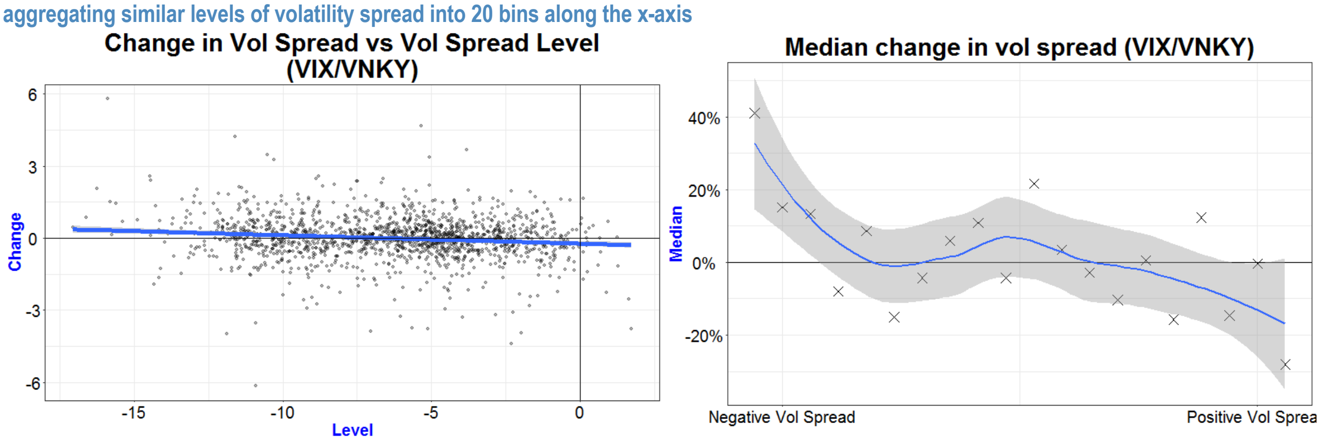
Figure 23 shows the volatility and carry for VIX and VNKY, as well as their spreads. Figure 24 illustrates the mean-reverting property in VIX/VNKY volatility spreads.

Figure 23: We model the dynamics of volatility spread (VIX over VNKY) (left) and carry spread (right) in VIX/VNKY




Source: J.P. Morgan Quantitative and Derivatives Strategy, Bloomberg

Figure 24: Volatility spread is mean-reverting: Changes in volatility spread are negatively proportional to the level of volatility spread; the two charts show the same information – the left one shows the scatter plot for all datapoints, while the right one shows a smooth curve by aggregating similar levels of volatility spread into 20 bins along the x-axis




Source: J.P. Morgan Quantitative and Derivatives Strategy, Bloomberg

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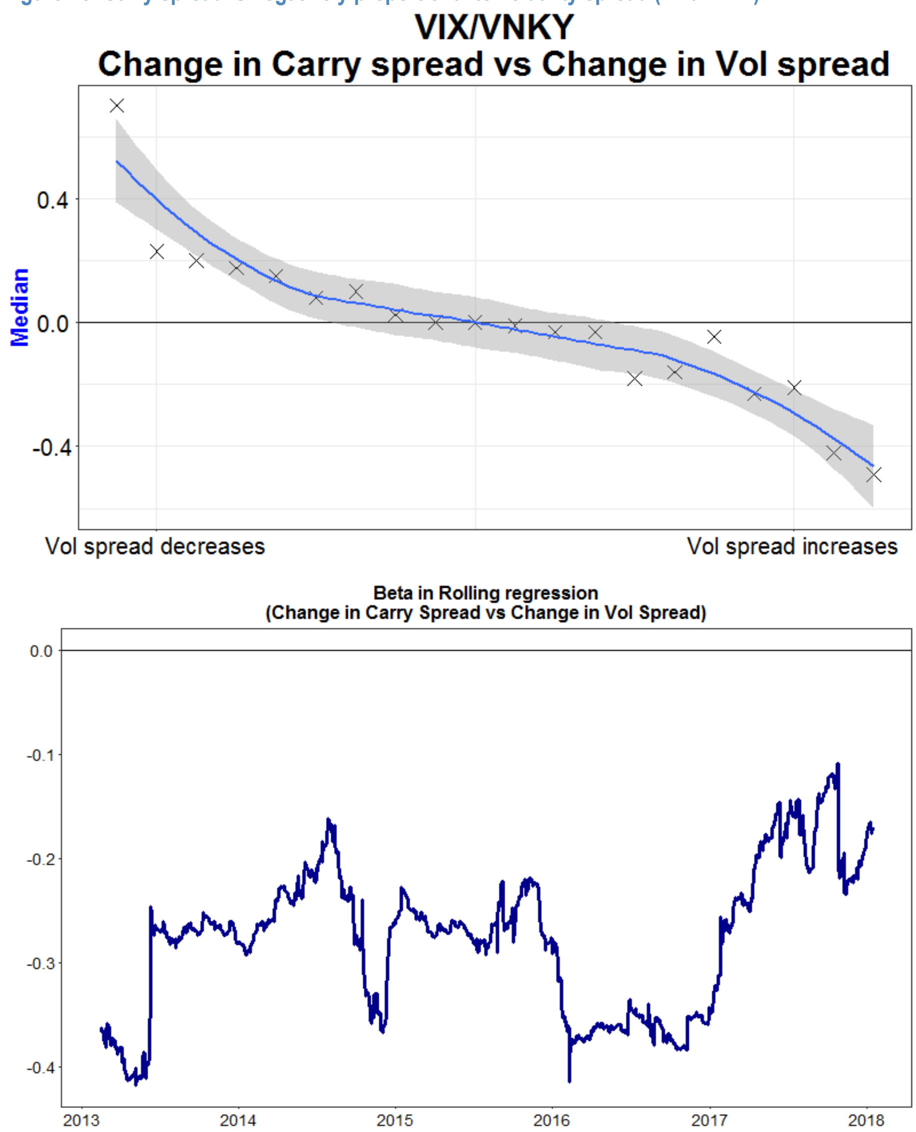
Also, as seen in Figure 11, carry spread tends to be negatively proportional to volatility spread. Recall our model for carry spread:

$$c_t = c_{t-1} + \gamma(x_{t-1} - x_{t-2}) + \varepsilon_t$$

We repeat the analysis in Figure 25. The bottom panel gives the coefficient $\gamma$ if we estimate with a simple rolling regression.

Figure 25: Carry spread is negatively proportional to volatility spread (VIX/VNKY)

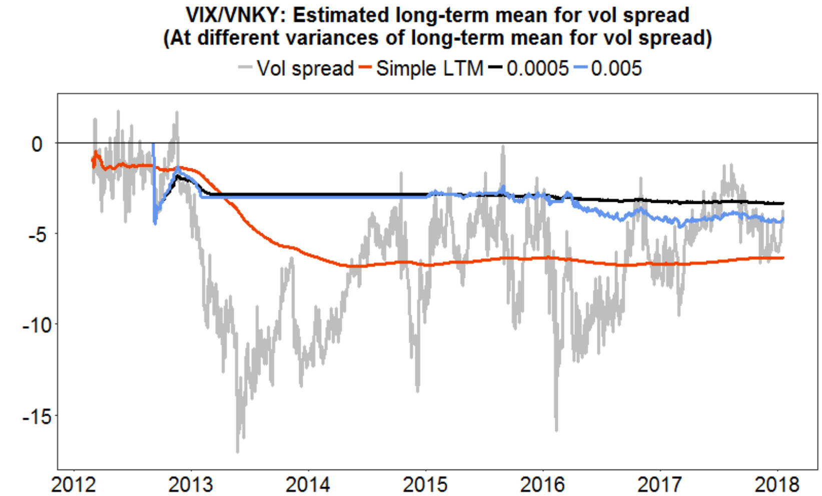



Source: J.P. Morgan Quantitative and Derivatives Strategy, Bloomberg

### Estimated parameters

We use the same methodology (see p.19) to estimate the parameters in the model for VIX/VNKY. For consistency, we also consider an expanding lookback window[9]. As explained earlier, we need to fix the variance parameter for the long-term mean of volatility spread, so as to ensure its smooth variation over time. Figure 26 shows the estimated long-term mean for volatility spread. At a chosen parameter of 0.0005 (dark blue), we have a very stable long-term mean that is higher than the one estimated with simple moving average (red).

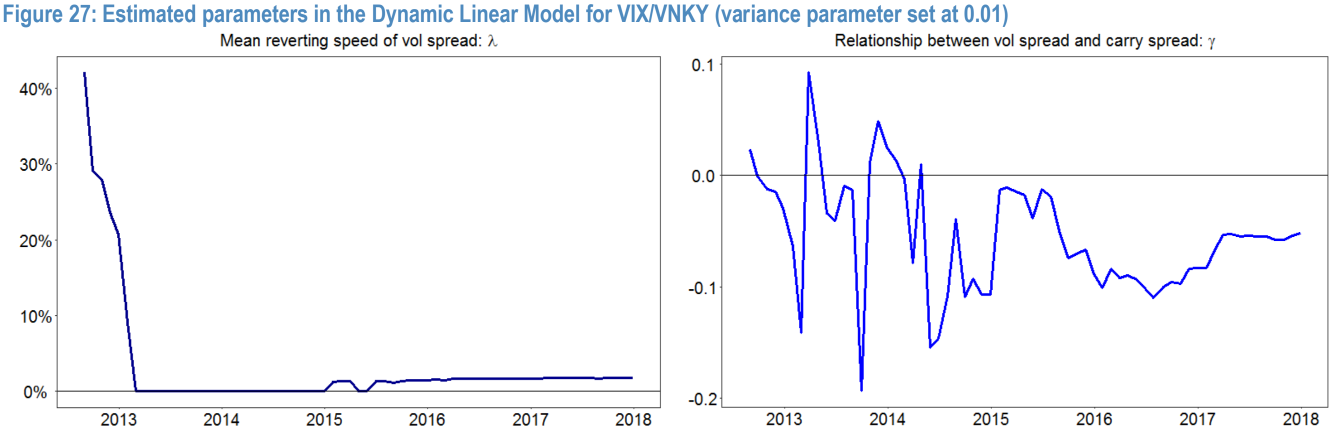
Figure 26: Estimated long-term mean for volatility spread at different values of variances




Source: J.P. Morgan Quantitative and Derivatives Strategy, Bloomberg

Figure 27 shows that the rate of mean reversion $\lambda$ is very large in late 2012, which is not surprising, as volatility spread has dropped way below average. Between 2013 and mid-2014, we do not see strong mean reversion, and we have $\lambda \approx 0$. We also show the coefficient $\gamma$, which governs the relationship between changes in carry and changes in volatility spread. As expected, its value is mostly negative.

Figure 27: Estimated parameters in the Dynamic Linear Model for VIX/VNKY (variance parameter set at 0.01)




Source: J.P. Morgan Quantitative and Derivatives Strategy, Bloomberg

9 Actually, we get a slightly better performance with a two-year lookback window.

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## Trading signal

Our trading signal for the VIX/VNKY strategy is the forecasted PnL based on our Dynamic Linear Model.

**Monthly rebalancing**

As VNKY futures are illiquid, with high transaction costs (about 0.5 vol points), we rebalance only at the end of every month[10]. At every month-end, we update the model and use it to predict the total PnL (of a long-VIX, short-VNKY position) in the next 22 days (approximately one month of trading days). As we will hold the position for a month, what matters to us is one-month-ahead PnL.

**Monte Carlo simulations**

It is quite straightforward to obtain multi-step ahead predictions based on our Dynamic Linear Model. We can draw the error terms from the Gaussian distribution and evolve the state vector using the evolution matrix $G$. We use the function "dlmForecast" in the R package "dlm" to run 100 simulations and obtain the predictions for volatility spread $x_t$ and carry spread $c_t$ in the next 22 days:

- Expectations on volatility spread: $E[x_{t+1}],\ldots,E[x_{t+22}]$
- Expectations on carry spread: $E[c_{t+1}],\ldots,E[c_{t+22}]$

The expected PnL at $t+1$ for the position that longs 1 unit of 1M VIX and shorts 1 unit of VNKY is (see p.44 in the Appendix)

**PnL in USD at (t+1)**

$$= \left(\Delta VIX_{1M,t+1} - \Delta VNKY_{1M,t+1}\right) - \Delta\omega_{2,t+1}\left[(UX_{2,t+1} - UX_{1,t+1}) - (JVI_{2,t+1} - JVI_{1,t+1})\right]$$

$$= \Delta E[x_{t+1}] - \Delta\omega_{2,t+1} E[c_{t+1}]$$

Hence, we can calculate the total expected PnL (of a long-VIX, short-VNKY position) in the next 22 days as a summation:

$$\textbf{Total PnL in USD (t) in the next 1-22 days} = \sum_{j=1}^{22}\left(\Delta E[x_{t+j}] - \Delta\omega_{2,t+j} E[c_{t+j}]\right)$$

where we assume that the percentage of daily rolling for each future is roughly the same and equal to $\Delta\omega_{2,t}$. As we have run 100 simulations for the predicted PnL in the next 22 days, we calculate the mean and variance of the total PnL from the 100 samples. The month-end trading signal is

$$s_t = \frac{E_t[PnL(t+1,t+22)]}{\sqrt{Var_t[PnL(t+1,t+22)]}}$$

[10] Note that we cannot avoid the transaction costs due to rolling of the VNKY futures.

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## Strategy backtest: Short VIX/long VNKY

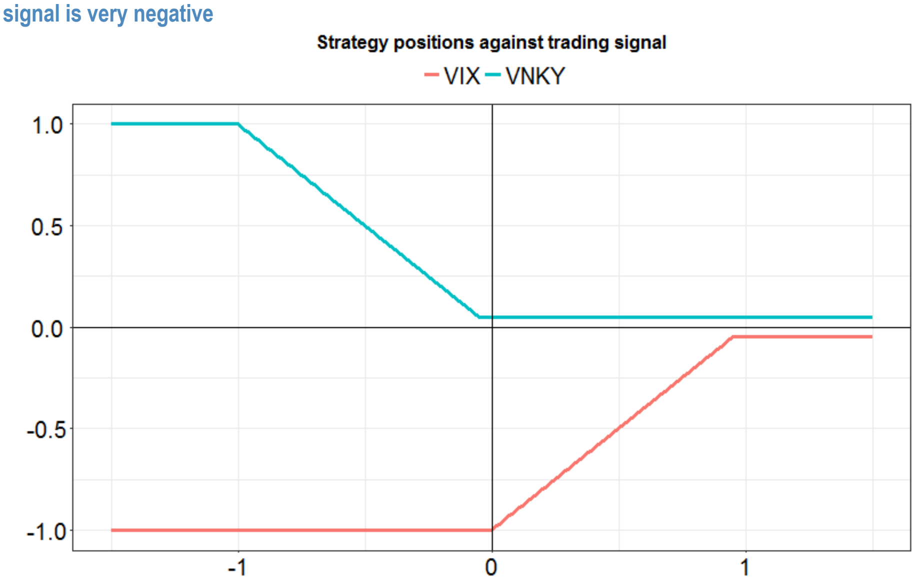
Since the carry in VIX is in general positive and relatively stable compared to that of VNKY, it is usually preferable to fix the short leg to VIX. We first backtest such a strategy because fixing the direction can reduce turnover and, hence, transaction costs. On p.36, we examine the case where we allow the strategy to long VIX and short VNKY.

Let us adjust the short exposure to VIX and the long exposure to VNKY based on our trading signal $s_t$. We set a minimum exposure of 5% and a maximum of 100%:

- If $s_t > 0$: $w_{VIX} = \min(-0.05, s_t - 1)$, $w_{VNKY} = 0.05$
  As expected PnL is positive, we prefer to long VIX and short VNKY. However, since we only allow long positions in VNKY, we put the VNKY exposure to a minimum of 5%.
  For VIX, we adjust the short exposure based on the magnitude of the signal. If the signal is too large, we reduce the short exposure until it reaches a minimum of 5%.
- If $s_t < 0$: $w_{VIX} = -1$, $w_{VNKY} = \min(1, \max(0.05, |s_t|))$
  As expected PnL is negative, we prefer to short VIX and long VNKY. In this case, we fix our short VIX exposure to 100%. The long exposure to VNKY is proportional to the magnitude of the signal $|s_t|$, while being constrained to lie between 5% and 100%.

We hold the position from the beginning of the month until next month-end.

**Figure 28: Strategy positions at different levels of the trading signal (x-axis): Minimum exposure is set at 5% and maximum at 100% – we always short VIX and long VNKY; we scale down the short exposure in VIX if the signal is very positive and scale up the long exposure in VNKY if the signal is very negative**




Source: J.P. Morgan Quantitative and Derivatives Strategy, Bloomberg

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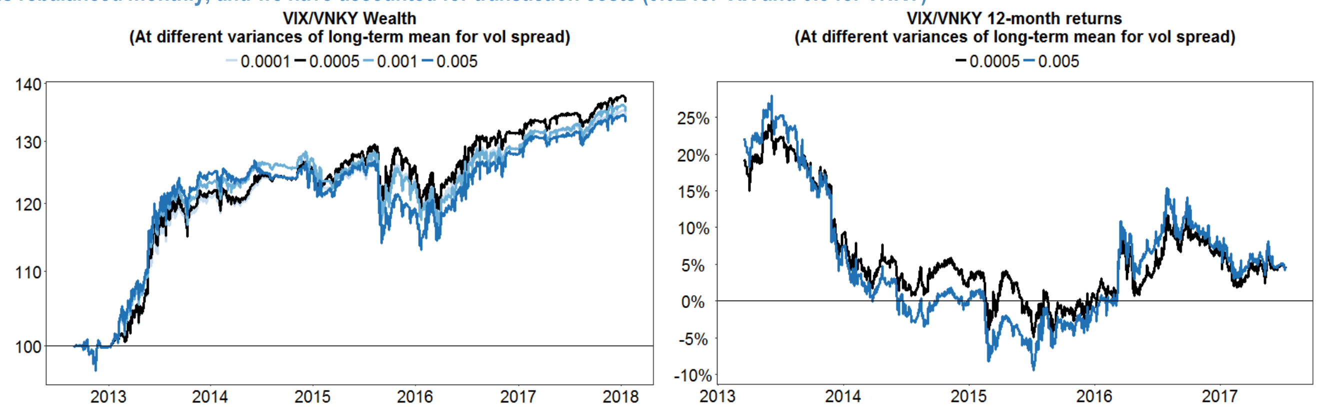
Figure 29 shows the cumulated PnL and the 12-month returns of the VIX/VNKY strategy.

**Figure 29: Cumulated PnL in vol points (left) and rolling 12-month returns (right) of the VIX/VNKY strategy, at different values of variance parameters for the long-term mean for volatility spread: We choose a value of 0.0005, which gives a high IR and lower drawdown; the strategy is rebalanced monthly, and we have accounted for transaction costs (0.02 for VIX and 0.5 for VNKY)**

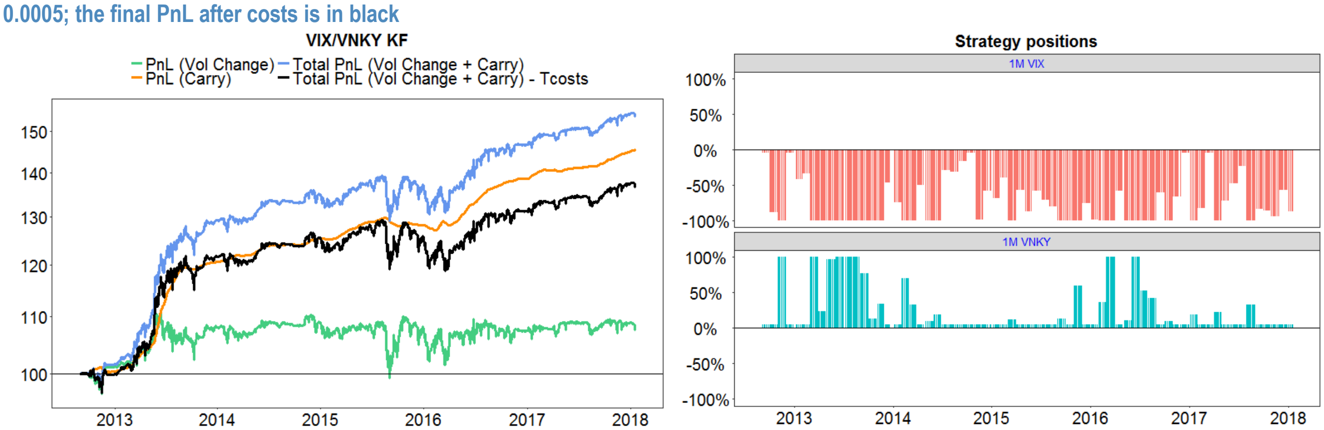



| Variance of long-term mean for volatility spread | Start | End | Annualized Returns | Annualized Vol | IR | Max Drawdown | Hit Ratio | Sortino Ratio | Calmer Ratio |
|---|---|---|---|---|---|---|---|---|---|
| | | | **Since 2012-09** | | | | | | |
| 0.0001 | 2012-09-04 | 2018-01-19 | 6.1% | 8.3% | 0.74 | 8.0% | 53.3% | 0.07 | 0.76 |
| **0.0005** | **2012-09-04** | **2018-01-19** | **6.5%** | **8.5%** | **0.76** | **8.2%** | **53.9%** | **0.07** | **0.79** |
| 0.001 | 2012-09-04 | 2018-01-19 | 6.2% | 8.6% | 0.72 | 9.3% | 54.2% | 0.07 | 0.67 |
| 0.005 | 2012-09-04 | 2018-01-19 | 5.9% | 8.9% | 0.67 | 10.8% | 54.2% | 0.06 | 0.55 |
| | | | **Since 2014-01** | | | | | | |
| 0.0001 | 2014-01-06 | 2018-01-19 | 2.8% | 7.6% | 0.37 | 8.0% | 53.6% | 0.04 | 0.36 |
| **0.0005** | **2014-01-06** | **2018-01-19** | **3.2%** | **7.7%** | **0.42** | **8.2%** | **54.2%** | **0.04** | **0.39** |
| 0.001 | 2014-01-06 | 2018-01-19 | 2.6% | 7.7% | 0.33 | 9.3% | 54.5% | 0.03 | 0.28 |
| 0.005 | 2014-01-06 | 2018-01-19 | 1.9% | 8.1% | 0.23 | 10.8% | 54.5% | 0.02 | 0.17 |

Source: J.P. Morgan Quantitative and Derivatives Strategy, Bloomberg

Figure 30 shows the cumulated PnL of the VIX/VNKY strategy, fixed at a variance parameter of 0.0005. As expected, the strategy earns a stable and positive carry (in orange) via the short VIX portion.

**Figure 30: Cumulated PnL in vol points (left) and long/short positions (right) of the VIX/VNKY strategy at a chosen variance parameter of 0.0005; the final PnL after costs is in black**




Source: J.P. Morgan Quantitative and Derivatives Strategy, Bloomberg

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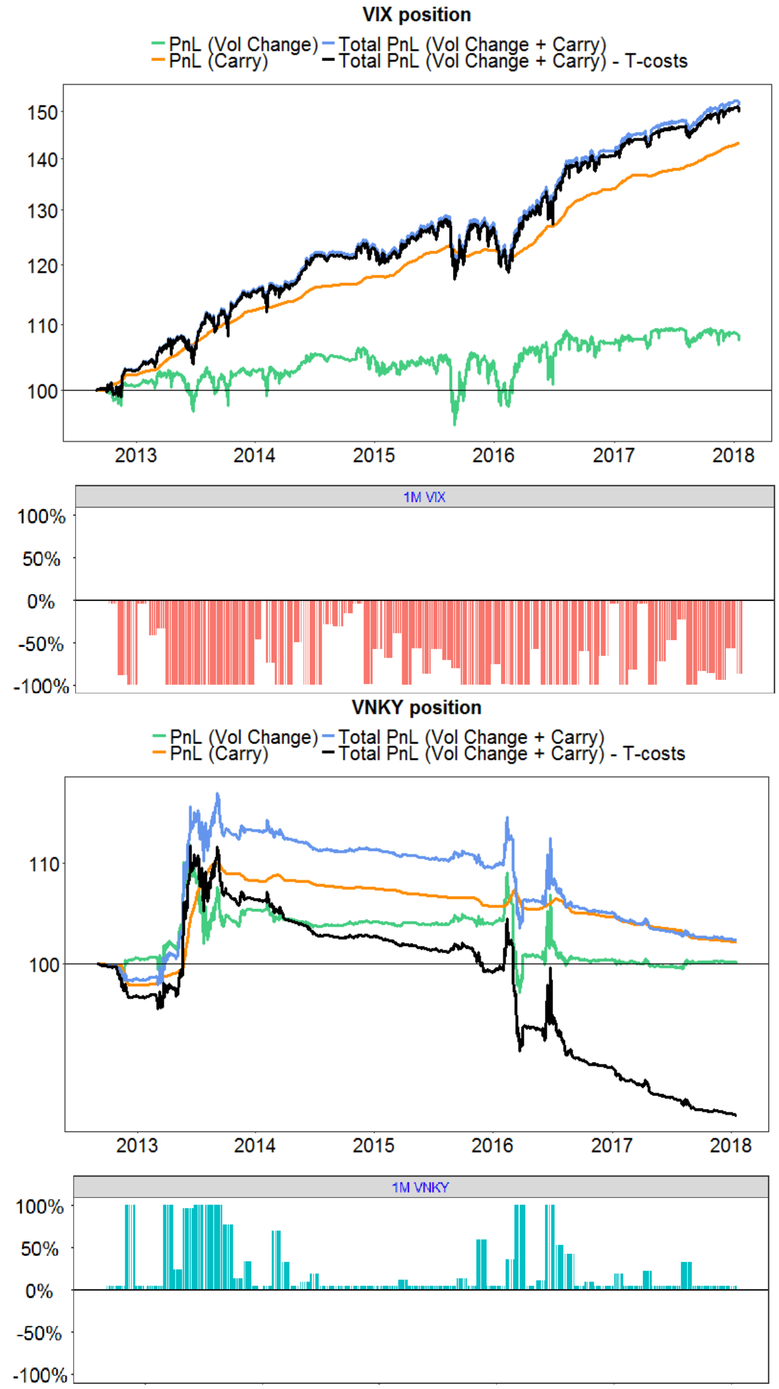
**Long VNKY: A hedge funded by carry in short VIX**

Figure 31 shows the cumulated PnL of each leg. As we saw in Figure 5, simply shorting VIX has earned a steady and positive carry over the years. For the long leg on VNKY, we obtain some positive returns in 2013 and 2016, as VNKY has increased significantly (see Figure 2). As such, this strategy could serve as a hedge for a rise in VNKY, while earning some carry during calm markets.

Figure 31: Cumulated PnL (in vol points) and positions of each leg (VIX: top, VNKY: bottom)




Source: J.P. Morgan Quantitative and Derivatives Strategy, Bloomberg

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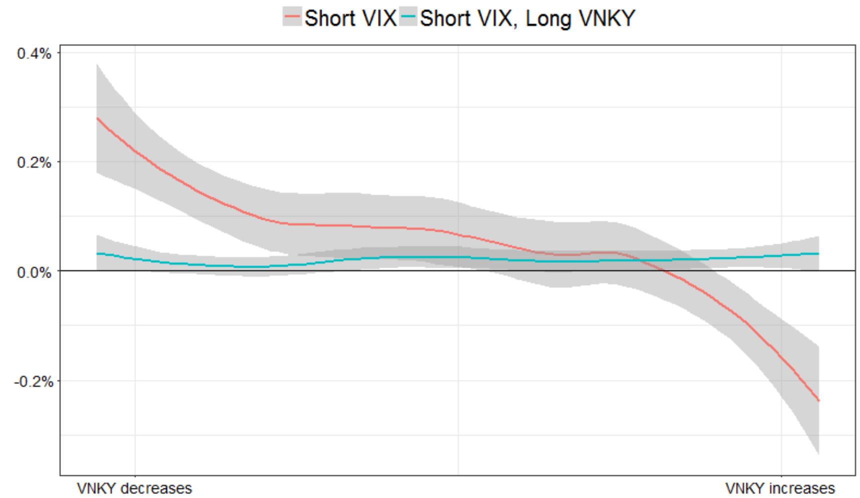
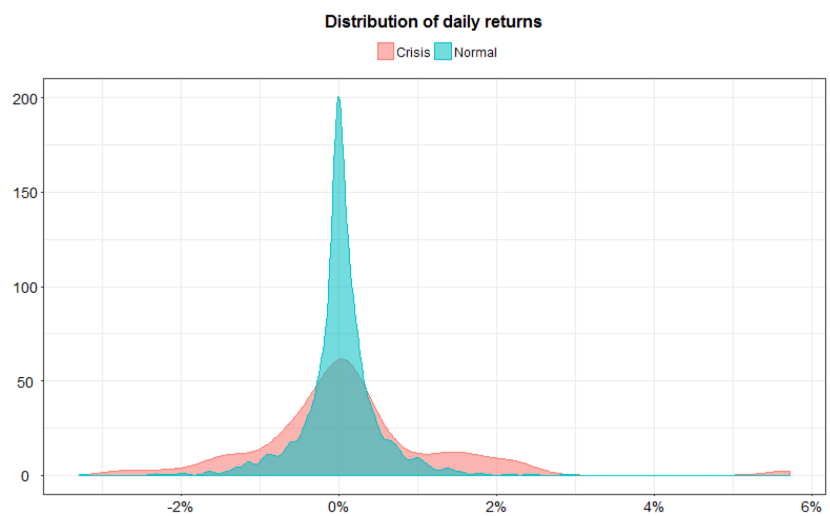
## Hedging a rise in VNKY

Our short VIX / long VNKY strategy can be regarded as a systematic way to hedge for volatility in the NIKKEI. When VNKY increases significantly, our strategy tends to provide positive returns due to the long exposure in VNKY. Figure 32 shows the median daily returns of our strategy (blue) at different daily changes in VNKY.

The bottom panel shows the distributions of returns for our strategy. On average, it earns 15 bps per day during “crisis,” periods when the daily increase in VNKY is within the top 5% in history. This is compared to 2 bps during normal trading days.

**Figure 32: Median daily returns of the VIX/VNKY strategy vs. changes in VNKY (top), and distribution of daily returns in crisis periods vs. normal periods (bottom)**




| VIX/VNKY daily returns (crisis days, i.e., change of VNKY in top 5%) | | Daily returns (normal days) |
|---|---|---|
| Minimum | -2.71% | -3.28% |
| 1st quantile | -0.45% | -0.13% |
| Median | 0.04% | 0.01% |
| **Mean** | **0.15%** | **0.02%** |
| 3rd quantile | 0.48% | 0.20% |
| Maximum | 5.73% | 2.91% |

Source: J.P. Morgan Quantitative and Derivatives Strategy, Bloomberg

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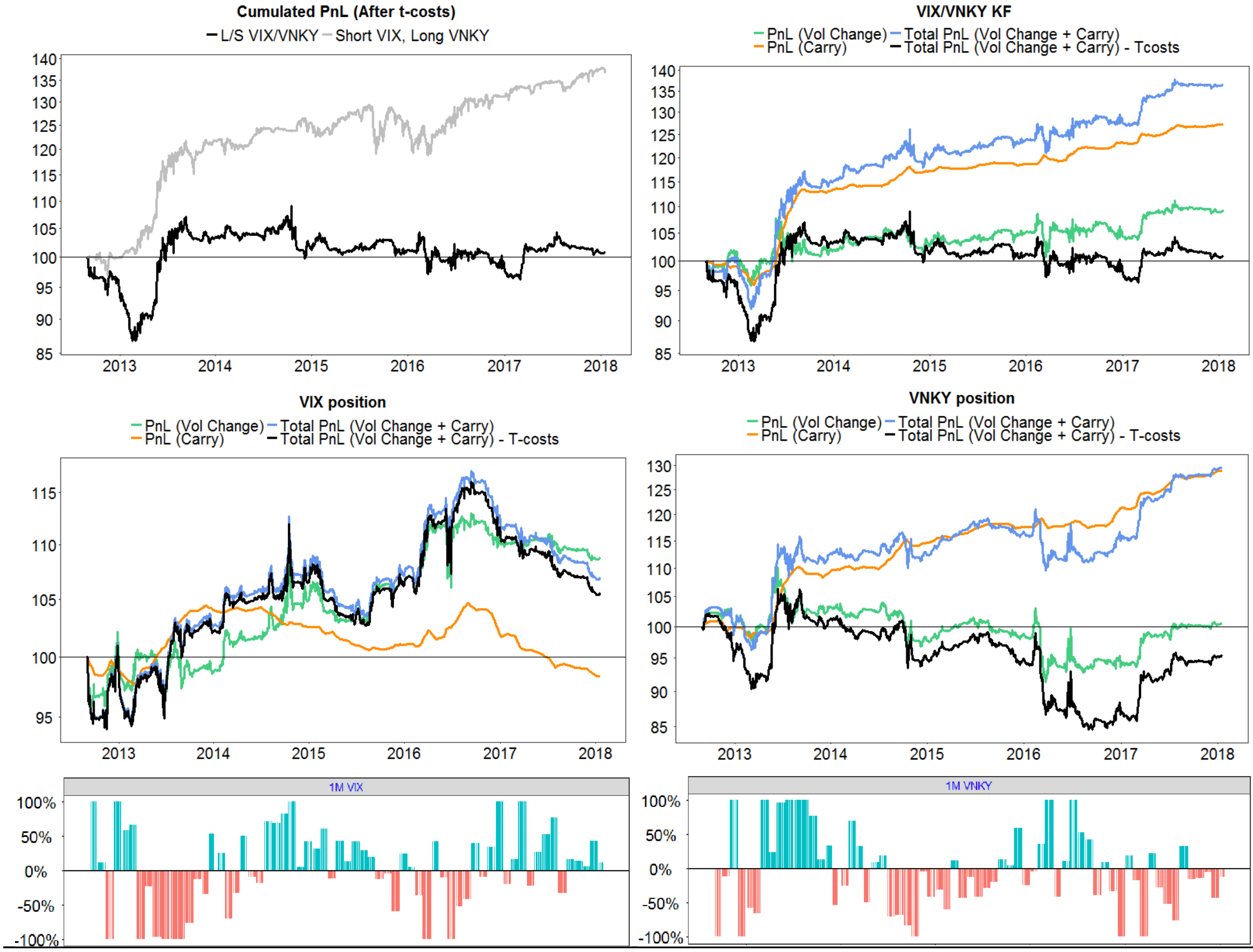
## VIX/VNKY: Long/short strategy

Previously, we considered a "one-sided" version of the VIX/VNKY strategy such that we always short VIX and long VNKY. We use the signal to adjust the long/short exposure, but we do not allow a long VIX / short VNKY position.

What if we use the signal to decide which leg to long or short, i.e., devise a strategy similar to the case in VIX/V2X? We find that the strategy PnL before cost looks decent. Unfortunately, due to higher turnover for VNKY in this long/short strategy, its high transaction cost (0.5 vol points) consumes a significant portion of the profits made by the strategy.

Figure 33: Cumulated PnL of the VIX/VNKY pairs strategy where we allow both long or short positions of each leg, rather than shorting the VIX only: High transaction costs for VNKY eat up most of the returns




| Variance parameter (0.0005) | Start | End | Annualized Returns | Annualized Vol | IR | Max Drawdown | Hit Ratio | Sortino Ratio | Calmer Ratio |
|---|---|---|---|---|---|---|---|---|---|
| **Before T-costs** | | | | | | | | | |
| Since 2012 | 2012-09-04 | 2018-01-19 | 6.3% | 7.8% | 0.81 | 8.8% | 51.5% | 0.08 | 0.72 |
| Since 2014 | 2014-01-06 | 2018-01-19 | 4.5% | 6.0% | 0.75 | 6.6% | 52.2% | 0.07 | 0.68 |
| **After T-costs** | | | | | | | | | |
| Since 2012 | 2012-09-04 | 2018-01-19 | 0.2% | 9.0% | 0.02 | 13.2% | 47.9% | 0.01 | 0.01 |
| Since 2014 | 2014-01-06 | 2018-01-19 | -0.7% | 7.3% | -0.10 | 11.8% | 48.8% | -0.01 | -0.06 |

Source: J.P. Morgan Quantitative and Derivatives Strategy, Bloomberg

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# Further research ideas

In this report, we propose a novel Dynamic Linear Model for volatility spread trading, in particular, using VIX/V2X and VIX/VNKY futures. We show that such strategies could systematically harvest the volatility risk premia and provide protection to large drawdowns in equity markets.

Here, we want to highlight some ideas for further research:

1. **Variance swaps:** As VNKY futures are illiquid, one might need to replicate the exposure using vanilla options. Instead of trading VNKY futures, we could look into the same strategy using variance swaps. Variance swaps are more flexible instruments that are traded OTC, offering much better liquidity. Another advantage of using NKY variance swaps is a longer history: Backtest simulations could start before 2012.

2. **Exploiting different parts of the term structure:** Instead of using constant 1M volatility futures, we could consider a portfolio of strategies based on different horizons, e.g., 30, 60 or 90 days. This has the advantage of exploiting the whole volatility term structure. For instance, if the term structure roll-down from the third contract to the second contract is steepest (i.e., carry is largest), we could trade a constant 60-day volatility future to harvest this term premia.

3. **Volatility regimes:** It is well known that volatility exhibits clustering, and its dynamics could behave differently under low- and high-volatility regimes. One could define regimes heuristically by some threshold. Another way is to estimate regimes based on Hidden Markov Models (HMM). We could modify the strategy such that it takes into account the volatility regime (e.g., the mean reversion parameters could be different when volatility is low as opposed to high).

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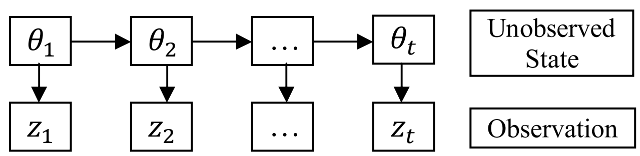
# Appendix

## Kalman Filter: A recursive algorithm

The Kalman Filter is a recursive algorithm to compute the optimal mean and covariance of the hidden state, under the assumption that the variables follow a multivariate Gaussian distribution and their dynamics governed by a linear model. The algorithm applies to a state space model where the observations are driven by the unobserved states, and involves two evolution equations: (1) updating equation for the state; and (2) measurement equation connecting the observations with the state:

- $\theta_t$: State variable (which are unobserved)
- $z_t$: Observation

$$
\begin{array}{lll}
\text{Updating equation:} & \theta_t = \mathbf{G}_t \theta_{t-1} + \omega_t & \omega_t \sim N(\mathbf{0}, \mathbf{W}_t) \\
\text{Measurement equation:} & \mathbf{z}_t = \mathbf{F}_t \theta_t + \nu_t & \nu_t \sim N(\mathbf{0}, \mathbf{V}_t)
\end{array}
$$




Let $D_t$ denote the information up to time *t*. Given the state space model, the state and the observations follow Gaussian distributions that evolve as below:

1. **Initialize state variable at time t = 0:**

- $a_{t|t}$ and $P_{t|t}$ are the filtered mean and variance of the state vector $\theta_t$ at time *t*
- At time *t* = 0, we initialize the distribution of the state variable. For instance, take $a_{0|0} = 0$ and $P_{0|0} = diag(10^9)$ as a non-information prior with mean zero:

$$\theta_0 | D_0 \sim N(a_{0|0}, P_{0|0})$$

2. **Predict the state:**
   Get the one-step-ahead prediction for the state variable at time *t* > 0, given information up to *t*-1:

$$\theta_t | D_{t-1} \sim N(a_{t|t-1}, P_{t|t-1})$$

   From the state evolution equation, we have

$$
\begin{aligned}
a_{t|t-1} &= G_t a_{t-1|t-1} \\
P_{t|t-1} &= G_t P_{t-1|t-1} G'_t + W_t
\end{aligned}
$$

3. **Predict the measurement:**
   Get the one-step-ahead prediction for the observations at time *t* > 0, given information up to *t*-1:

$$z_t | D_{t-1} \sim N(z_{t|t-1}, \Sigma_{t|t-1})$$

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From the measurement equation, we have

$$z_{t|t-1} = F_t a_{t|t-1}$$
$$\Sigma_{t|t-1} = F_t P_{t|t-1} F'_t + V_t$$

4. **Filter the state:**
Obtain the filtered state variable at time *t*:

As we observe $z_t$, we get new information and can use it to update our state variable and predictions. Denote our prediction error as

$$e_t = z_t - z_{t|t-1}$$

We want to update the state distribution

$$\theta_t | D_t \sim N(a_{t|t}, P_{t|t})$$

From the evolution equations, we have

$$\underbrace{a_{t|t}}_{\text{Posterior}} = \underbrace{\boldsymbol{a_{t|t-1}}}_{\text{Prior}} + \underbrace{\boldsymbol{P_{t|t-1}F'_t\Sigma_t^{-1}}}_{\text{Kalman gain: Relative uncertainty between prior and new information}} \underbrace{\boldsymbol{e_t}}_{\text{New information}}$$

$$P_{t|t} = P_{t|t-1} - \boldsymbol{P_{t|t-1}F'_t\Sigma_t^{-1}} F_t P_{t|t-1}$$

i.e., if our prediction error is zero ($e_t = 0$), then the state remains at the same value (as we do not receive "new" information to update our prior belief).

One may notice that the last set of equations resembles the ones for the conditional mean and variance for a multivariate Gaussian distribution. Indeed, looking at the joint distribution of the state and the observations,

$$\begin{pmatrix} \theta \\ z \end{pmatrix} \sim N\left( \begin{pmatrix} \hat{\theta} \\ \hat{z} \end{pmatrix}, \begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix} \right)$$

The distribution of the state variable $\theta_t$ conditional on the latest observation in $z_t$ is $\theta_t | z_t \sim N(\mu, \Omega)$, where

$$\mu = \hat{\theta} + \boldsymbol{\Omega_{12}\Omega_{22}^{-1}}(z - \hat{z})$$

This gives the Bayesian interpretation in the Kalman Filter algorithm (where we highlight the Kalman Gain in purple).

5. **Repeat Steps (2) and (3) to predict state variables and observations, and use Step (4) to filter the state variable when new observations arrive.**

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## Futures rolling mechanism

Futures contracts, unlike stocks, have expiry dates. Hence, for strategies involving futures, the choice of rolling mechanism can severely impact returns. For instance, one can roll into the next contract one day or one week before the expiry of the current contract.

For volatilities futures, it is common to use a continuously rolling mechanism so as to maintain at a fixed 1M horizon. Using VIX futures as an example, let $UX_{1,t}$ and $UX_{2,t}$ be the prices of the first contract and the second contract, respectively. A constant 1M VIX future is a weighted combination of the first and second contracts:

$$VIX_{1M,t} = \omega_{1,t} UX_{1,t} + \omega_{2,t} UX_{2,t} \qquad \omega_{1,t} + \omega_{2,t} = 1$$

At the end of the day before expiry, we hold 100% of the second contract, $UX_{2,t}$. On expiry date, the second contract will become the first contract, and we do not need any rolling. Table 1 shows an example of the rolling schedule for 1M VIX futures using a weighted combination of UX1 and UX2. The column *"% Roll"* shows the increase in weight in UX2 (proportional to calendar days). In our notations, we have $\% roll = \Delta\omega_{2,t}$, and a long position in 1M VIX futures incurs a cost of carry:

$$Carry = -\Delta\omega_{2,t}\left(UX_{2,t} - UX_{1,t}\right)$$

Table 1: Daily rolling schedule for 1M VIX futures: Every day we roll from UX1 to UX2, which incurs a negative carry in general

| Date | VIX expiry date | UX1 weight | UX2 weight | % Roll $\Delta\omega_2$ | UX1 Index | UX2 Index | 1M VIX | Volatility Change Δ1M VIX | Carry $-\Delta\omega_2$(UX2 - UX1) | Daily PnL (Vol change + Carry) |
|---|---|---|---|---|---|---|---|---|---|---|
| **2017-11-14** | | | **100.0%** | **3.7%** | **12.05** | **12.675** | **12.675** | **-0.057** | **-0.023** | **-0.081** |
| **2017-11-15** | **Expiry** | **100.0%** | | | **13.05** | **13.175** | **13.175** | **0.500** | | **0.500** |
| 2017-11-16 | | 97.1% | 2.9% | 2.9% | 12.675 | 13.975 | 12.713 | -0.462 | -0.038 | -0.500 |
| 2017-11-17 | | 94.1% | 5.9% | 2.9% | 12.625 | 13.875 | 12.699 | -0.015 | -0.037 | -0.051 |
| 2017-11-20 | | 85.3% | 14.7% | 8.8% | 12.075 | 13.425 | 12.274 | -0.425 | -0.119 | -0.544 |
| 2017-11-21 | | 82.4% | 17.6% | 2.9% | 11.625 | 12.975 | 11.863 | -0.410 | -0.040 | -0.450 |
| 2017-11-22 | | 79.4% | 20.6% | 2.9% | 11.475 | 12.875 | 11.763 | -0.100 | -0.041 | -0.141 |
| 2017-11-23 | | 76.5% | 23.5% | 2.9% | 11.475 | 12.875 | 11.763 | 0.000 | -0.041 | 0.000 |
| 2017-11-24 | | 73.5% | 26.5% | 2.9% | 11.425 | 12.825 | 11.796 | 0.032 | -0.041 | -0.050 |
| 2017-11-27 | | 64.7% | 35.3% | 8.8% | 11.325 | 12.875 | 11.872 | 0.076 | -0.137 | -0.060 |
| 2017-11-28 | | 61.8% | 38.2% | 2.9% | 11.275 | 12.725 | 11.829 | -0.043 | -0.043 | -0.085 |
| 2017-11-29 | | 58.8% | 41.2% | 2.9% | 11.475 | 12.925 | 12.072 | 0.243 | -0.043 | 0.200 |
| 2017-11-30 | | 55.9% | 44.1% | 2.9% | 11.675 | 13.025 | 12.271 | 0.199 | -0.040 | 0.159 |
| 2017-12-01 | | 52.9% | 47.1% | 2.9% | 11.875 | 13.275 | 12.534 | 0.263 | -0.041 | 0.222 |
| 2017-12-04 | | 44.1% | 55.9% | 8.8% | 11.925 | 13.225 | 12.651 | 0.118 | -0.115 | 0.003 |
| 2017-12-05 | | 41.2% | 58.8% | 2.9% | 11.875 | 13.175 | 12.640 | -0.012 | -0.038 | -0.050 |
| 2017-12-06 | | 38.2% | 61.8% | 2.9% | 11.725 | 13.075 | 12.559 | -0.081 | -0.040 | -0.121 |
| 2017-12-07 | | 35.3% | 64.7% | 2.9% | 11.175 | 12.675 | 12.146 | -0.413 | -0.044 | -0.457 |
| 2017-12-08 | | 32.4% | 67.6% | 2.9% | 10.875 | 12.325 | 11.856 | -0.290 | -0.043 | -0.332 |
| 2017-12-11 | | 23.5% | 76.5% | 8.8% | 10.325 | 11.975 | 11.587 | -0.269 | -0.146 | -0.415 |
| 2017-12-12 | | 20.6% | 79.4% | 2.9% | 10.375 | 11.975 | 11.646 | 0.059 | -0.047 | 0.012 |
| 2017-12-13 | | 17.6% | 82.4% | 2.9% | 10.525 | 11.975 | 11.719 | 0.074 | -0.043 | 0.031 |
| 2017-12-14 | | 14.7% | 85.3% | 2.9% | 10.375 | 11.875 | 11.654 | -0.065 | -0.044 | -0.109 |
| 2017-12-15 | | 11.8% | 88.2% | 2.9% | 9.925 | 11.475 | 11.293 | -0.362 | -0.046 | -0.407 |
| 2017-12-18 | | 2.9% | 97.1% | 8.8% | 9.875 | 11.325 | 11.282 | -0.010 | -0.128 | -0.138 |
| **2017-12-19** | | | **100.0%** | **2.9%** | **10.075** | **11.325** | **11.325** | **0.043** | **-0.037** | **0.006** |
| **2017-12-20** | **Expiry** | **100.0%** | | | **9.6** | **11.425** | **11.425** | **0.100** | | **0.100** |
| 2017-12-21 | | 96.4% | 3.6% | 3.6% | 11.225 | 12.225 | 11.261 | -0.164 | -0.036 | -0.200 |

Source: J.P. Morgan Quantitative and Derivatives Strategy, Bloomberg

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## Decomposing PnL components

In the following, we decompose the PnL of a volatility future as a sum of:

- Change in volatility
- Carry

We first show the PnL of a long-only position on a constant 1M volatility future (with VIX as an example). Using the results, we show the PnL of a long/short position (with VIX/V2X as an example). Note that the results hold with any rolling schedules, although we always use a daily rolling for 1M futures in this report.

**Long 1M VIX**

We show the PnL of a long position in a 1M VIX. Denoting the prices of the first and second contracts as $P_{1,t}$ and $P_{2,t}$ (so $P_{1,t} = UX_{1,t}$ for VIX), we have

$$VIX_{1M,t} = \omega_{1,t} P_{1,t} + \omega_{2,t} P_{2,t} \qquad \omega_{1,t} + \omega_{2,t} = 1$$

where $\omega_{1,t} > 0$ and $\omega_{2,t} > 0$ are the weights on the first and second contracts, respectively. The PnL of this position is

$$\begin{aligned} \text{Total PnL(t)} &= \text{PnL of } 1^{st} \text{contract} + \text{PnL of } 2^{nd} \text{contract} \\ &= w_{1,t-1}(P_{1,t} - P_{1,t-1}) + w_{2,t-1}(P_{2,t} - P_{2,t-1}) \end{aligned}$$

Note that the daily change in 1M VIX is

$$\begin{aligned} VIX_{1M,t} - VIX_{1M,t-1} &= (w_{1,t}P_{1,t} + w_{2,t}P_{2,t}) - (w_{1,t-1}P_{1,t-1} + w_{2,t-1}P_{2,t-1}) \\ &= w_{1,t-1}(P_{1,t} - P_{1,t-1}) + (w_{1,t} - w_{1,t-1})P_{1,t} \\ &\quad + w_{2,t-1}(P_{2,t} - P_{2,t-1}) + (w_{2,t} - w_{2,t-1})P_{2,t} \\ &= \text{Total PnL(t)} + \Delta w_{1,t}P_{1,t} + \Delta w_{2,t}P_{2,t} \end{aligned}$$

The terms in brown can be simplified to

$$\begin{aligned} \Delta w_{1,t}P_{1,t} + \Delta w_{2,t}P_{2,t} &= \Delta w_{2,t}(P_{2,t} - P_{1,t}) + (\Delta w_{1,t} + \Delta w_{2,t})P_{1,t} \\ &= \Delta w_{2,t}(P_{2,t} - P_{1,t}) \end{aligned}$$

since we have $\Delta\omega_{1,t} + \Delta\omega_{2,t} = 0$. As a result,

$$\begin{aligned} \text{Total PnL(t)} &= \Delta VIX_{1M,t} - \Delta w_{2,t}(P_{2,t} - P_{1,t}) \\ &= \text{Increase in vol points} - \text{cost of carry} \end{aligned}$$

On expiry dates, the above decomposition also holds, but we have a special case where the total PnL depends only on the second contract (as we hold 100%), and the cost of carry is zero:

$$\begin{aligned} \text{Total PnL(t = expiry date)} &= \text{Increase in vol points} \\ &= P_{2,t} - P_{2,t-1} \end{aligned}$$

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**Long 1M VIX / short 1M V2X**

As VIX is traded in USD and V2X in euros, we need to convert the PnL of the V2X position into USD.

Consider a long/short portfolio of 1M VIX and 1M V2X with weights

$$w_t = \left(w_{VIX,t}, w_{V2X,t}\right)$$

Hence, a \$1 USD long VIX and \$1 USD short V2X position is denoted as $w_t = (1, -1)$. For V2X, we convert the weight based on the exchange rate FX = EURUSD:

$$w_{V2X,t} \rightarrow \frac{w_{V2X,t}}{FX_t}$$

The PnL of a volatility spread strategy is given by the PnL of each leg:

$$\text{PnL in USD (t)} = w_{VIX,t-1} \times \text{PnL of 1M VIX in USD (t)} + \frac{w_{V2X,t-1}}{FX_{t-1}} \times \text{PnL of 1M V2X in EUR (t)} \times FX_t$$

Referring to the last section, each PnL is further decomposed into a change in volatility and carry:

$$\text{PnL in USD (t)} = w_{VIX,t-1} \times \left[\Delta VIX_{1M,t} - \Delta\omega_{2,t}^{VIX}(UX_{2,t} - UX_{1,t})\right] + w_{V2X,t-1} \times \left[\Delta V2X_{1M,t} - \Delta\omega_{2,t}^{V2X}(FVS_{2,t} - FVS_{1,t})\right] \times \frac{FX_t}{FX_{t-1}}$$

For a long/short position $w_t = \left(w_{VIX,t}, w_{V2X,t}\right) = (1,-1)$, and assuming that the exchange rate $FX_t = EURUSD$ is constant,

$$\begin{aligned}
\textbf{PnL in USD} &= w_{VIX,t-1}\left(\Delta VIX_{1M,t} + Carry_{VIX,t}\right) + w_{V2X,t-1}\left(\Delta V2X_{1M,t} + Carry_{V2X,t}\right) \\
&= \left(\Delta VIX_{1M,t} + Carry_{VIX,t}\right) - \left(\Delta V2X_{1M,t} + Carry_{V2X,t}\right) \\
&= \left(\Delta VIX_{1M,t} - \Delta V2X_{1M,t}\right) + \left(Carry_{VIX,t} - Carry_{V2X,t}\right) \\
&= \textbf{Change in volatility spread + carry spread}
\end{aligned}$$

where we define carry as $Carry = -\Delta\omega_{2,t}\left(P_{2,t} - P_{1,t}\right)$. Assuming the % of daily rolling is the same, where $\Delta\omega = \Delta\omega_{2,t}^{VIX} = \Delta\omega_{2,t}^{VIX}$, we have

$$\textbf{PnL in USD} = \left(\Delta VIX_{1M,t} - \Delta V2X_{1M,t}\right) - \Delta\omega \times \left[(UX_{2,t} - UX_{1,t}) - (FVS_{2,t} - FVS_{1,t})\right]$$

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Table 2 shows the volatility, carry and PnL of the VIX leg (top panel), the V2X leg (middle panel) and the long VIX / short V2X spread (bottom panel).

**Table 2: An illustration of the decomposition of PnL into change in volatility and carry: The top panel shows the PnL for VIX, the middle panel shows the PnL for V2X, and the bottom panel shows the PnL for a long VIX / short V2X volatility spread**

**1M VIX**

| | TWLVIX1 | TWLVIX2 | 1M VIX | 1M VIX Change | VIX Carry | VIX roll | VIX Carry Cost | Long VIX PnL (1M VIX Change + VIX Carry Cost) |
|---|---|---|---|---|---|---|---|---|
| 2017-12-04 | 11.39 | 12.88 | 12.22 | -1.33 | 1.49 | 8.8% | -0.13 | -1.46 |
| 2017-12-05 | 11.48 | 12.88 | 12.30 | 0.08 | 1.4 | 2.9% | -0.04 | 0.04 |
| 2017-12-06 | 11.81 | 13.08 | 12.59 | 0.29 | 1.27 | 2.9% | -0.04 | 0.25 |
| 2017-12-07 | 11.33 | 12.78 | 12.27 | -0.33 | 1.45 | 2.9% | -0.04 | -0.37 |
| 2017-12-08 | 11.03 | 12.5 | 12.02 | -0.24 | 1.47 | 2.9% | -0.04 | -0.29 |
| 2017-12-11 | 10.68 | 12.23 | 11.87 | -0.16 | 1.55 | 8.8% | -0.14 | -0.30 |
| 2017-12-12 | 10.38 | 11.88 | 11.57 | -0.29 | 1.5 | 2.9% | -0.04 | -0.34 |
| 2017-12-13 | 10.38 | 11.88 | 11.62 | 0.04 | 1.5 | 2.9% | -0.04 | 0.00 |
| 2017-12-14 | 10.43 | 11.88 | 11.67 | 0.05 | 1.45 | 2.9% | -0.04 | 0.01 |
| 2017-12-15 | 9.97 | 11.57 | 11.38 | -0.28 | 1.6 | 2.9% | -0.05 | -0.33 |
| 2017-12-18 | 9.78 | 11.28 | 11.24 | -0.15 | 1.5 | 8.8% | -0.13 | -0.28 |
| 2017-12-19 | 10.03 | 11.43 | 11.43 | 0.19 | 1.4 | 2.9% | -0.04 | 0.15 |
| 2017-12-20 | 11.28 | 12.23 | 12.23 | 0.80 | 0.95 | 0.0% | 0.00 | 0.80 |
| 2017-12-21 | 11.38 | 12.23 | 11.41 | -0.82 | 0.85 | 3.6% | -0.03 | -0.85 |
| 2017-12-22 | 11.23 | 12.18 | 11.30 | -0.11 | 0.95 | 3.6% | -0.03 | -0.15 |

**1M V2X**

| | FVS1 | FVS2 | 1M V2X | 1M V2X Change | V2X Carry | V2X roll | V2X Carry Cost | Long V2X PnL (1M V2X Change + V2X Carry Cost) |
|---|---|---|---|---|---|---|---|---|
| 2017-12-04 | 13.2 | 14.7 | 14.04 | -1.06 | 1.5 | 8.8% | -0.13 | -1.20 |
| 2017-12-05 | 12.95 | 14.55 | 13.89 | -0.15 | 1.6 | 2.9% | -0.05 | -0.19 |
| 2017-12-06 | 13.4 | 15 | 14.39 | 0.50 | 1.6 | 2.9% | -0.05 | 0.45 |
| 2017-12-07 | 13.15 | 14.75 | 14.19 | -0.20 | 1.6 | 2.9% | -0.05 | -0.25 |
| 2017-12-08 | 12.95 | 14.6 | 14.07 | -0.12 | 1.65 | 2.9% | -0.05 | -0.17 |
| 2017-12-11 | 12.7 | 14.3 | 13.92 | -0.14 | 1.6 | 8.8% | -0.14 | -0.28 |
| 2017-12-12 | 12.1 | 14 | 13.61 | -0.31 | 1.9 | 2.9% | -0.06 | -0.37 |
| 2017-12-13 | 12.35 | 14.1 | 13.79 | 0.18 | 1.75 | 2.9% | -0.05 | 0.13 |
| 2017-12-14 | 12.35 | 14.1 | 13.84 | 0.05 | 1.75 | 2.9% | -0.05 | 0.00 |
| 2017-12-15 | 11.95 | 13.65 | 13.45 | -0.39 | 1.7 | 2.9% | -0.05 | -0.44 |
| 2017-12-18 | 10.9 | 12.85 | 12.79 | -0.66 | 1.95 | 8.8% | -0.17 | -0.83 |
| 2017-12-19 | 11.15 | 13 | 13.00 | 0.21 | 1.85 | 2.9% | -0.05 | 0.15 |
| 2017-12-20 | 11.11 | 13.6 | 13.60 | 0.60 | 2.49 | 0.0% | 0.00 | 0.60 |
| 2017-12-21 | 13.3 | 15.85 | 13.39 | -0.21 | 2.55 | 3.6% | -0.09 | -0.30 |
| 2017-12-22 | 13.35 | 15.95 | 13.54 | 0.14 | 2.6 | 3.6% | -0.09 | 0.05 |

**Volatility Spread (Long VIX, Short V2X)**

| | Volatility Spread PnL (VIX PnL – V2X PnL) | Vol Spread | Change in Vol Spread | Carry Spread | % Roll | Volatility Spread PnL (Change in Vol Spread – % Roll × Carry Spread) |
|---|---|---|---|---|---|---|
| 2017-12-04 | -0.26 | -1.82 | | -0.01 | 8.8% | |
| 2017-12-05 | 0.23 | -1.59 | 0.23 | -0.20 | 2.9% | 0.23 |
| 2017-12-06 | -0.20 | -1.79 | -0.21 | -0.33 | 2.9% | -0.20 |
| 2017-12-07 | -0.12 | -1.92 | -0.12 | -0.15 | 2.9% | -0.12 |
| 2017-12-08 | -0.12 | -2.04 | -0.12 | -0.18 | 2.9% | -0.12 |
| 2017-12-11 | -0.01 | -2.06 | -0.02 | -0.05 | 8.8% | -0.01 |
| 2017-12-12 | 0.03 | -2.04 | 0.02 | -0.40 | 2.9% | 0.03 |
| 2017-12-13 | -0.13 | -2.18 | -0.14 | -0.25 | 2.9% | -0.13 |
| 2017-12-14 | 0.01 | -2.18 | 0.00 | -0.30 | 2.9% | 0.01 |
| 2017-12-15 | 0.11 | -2.07 | 0.11 | -0.10 | 2.9% | 0.11 |
| 2017-12-18 | 0.55 | -1.56 | 0.51 | -0.45 | 8.8% | 0.55 |
| 2017-12-19 | 0.00 | -1.57 | -0.01 | -0.45 | 2.9% | 0.00 |
| 2017-12-20 | 0.20 | -1.37 | 0.20 | -1.54 | 0.0% | 0.20 |
| 2017-12-21 | -0.55 | -1.98 | -0.61 | -1.70 | 3.6% | -0.55 |
| 2017-12-22 | -0.20 | -2.24 | -0.26 | -1.65 | 3.6% | -0.20 |

Source: J.P. Morgan Quantitative and Derivatives Strategy, Bloomberg

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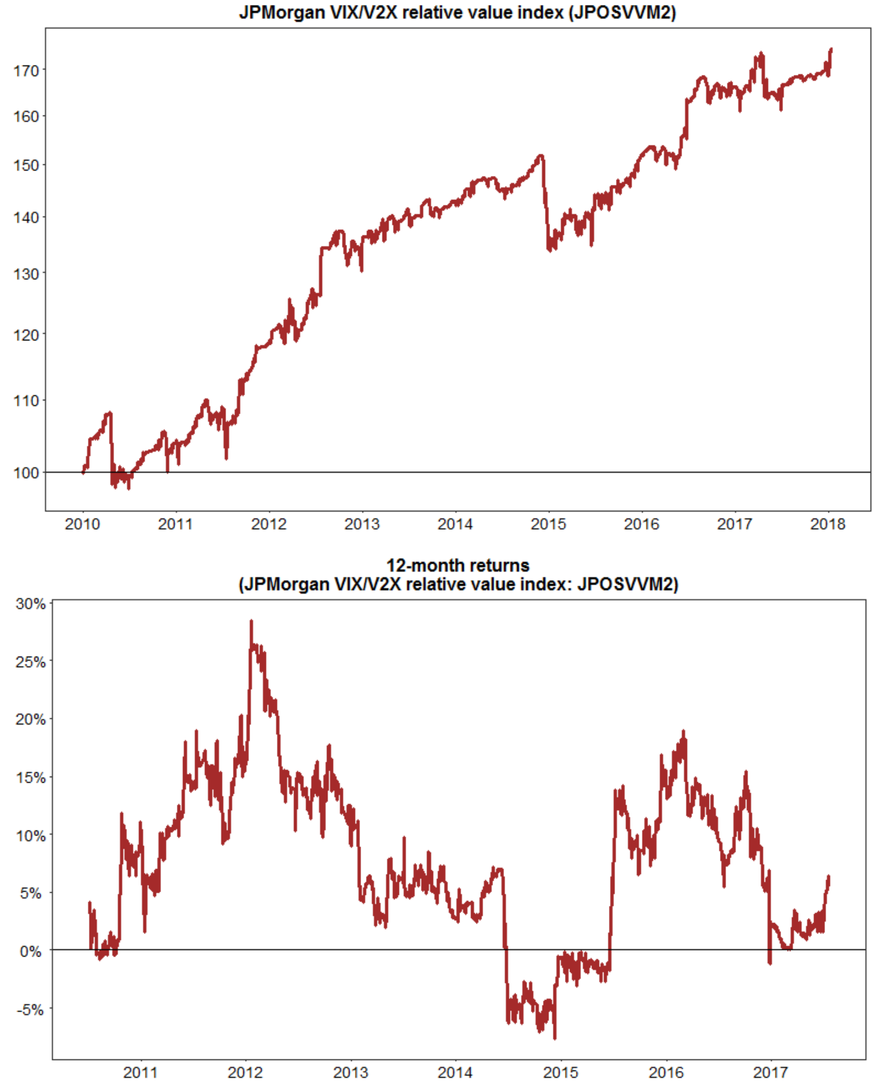
## J.P. Morgan Investible Volatility Spread Index

J.P. Morgan has a suite of investable indices on equities volatility that aims to harvest carry and income, provide hedges or extract relative value.

The JPOSVVM2 Index is a VIX/V2X volatility spread strategy aiming to harvest relative value. This vega-neutral strategy has a long/short position on VIX and V2X, where the exposures depend on the rolling average of the volatility spread.

**Figure 34: J.P. Morgan VIX/V2X investable index (JPOSVVM2), after transaction costs**




| JPOSVVM2 | Start | End | Annualized Returns | Annualized Vol | IR | Max Drawdown | Hit Ratio | Sortino Ratio | Calmer Ratio |
|---|---|---|---|---|---|---|---|---|---|
| Since 2010 | 2010-01-05 | 2018-01-19 | 7.3% | 8.9% | 0.82 | 11.8% | 45.8% | 0.08 | 0.62 |
| Since 2014 | 2014-01-02 | 2018-01-19 | 5.1% | 8.7% | 0.59 | 11.8% | 45.5% | 0.06 | 0.44 |

Source: J.P. Morgan Quantitative and Derivatives Strategy, Bloomberg

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