A challenge on twitter

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The challenge is to give an elegant proof of this: https://twitter.com/JDHamkins/status/1340346873236905985.

Use the fact that if $(x_0, y_0), \ldots, (x_n, y_n)$ are the vertices of a polygon where $(x_0, y_0) = (x_n, y_n)$, then its area is:

$$A = \frac{1}{2} \sum_{i=0}^{n} \det \begin{pmatrix} x_i & x_{i+1} \\ y_i & y_{i+1} \end{pmatrix} \tag{1}$$

This formula holds true regardless of the position of the vertices if we use signed area.

In our case $y_i = 0$ or $y_i = 1$ and we get a telescoping sum so the answer is $\frac{1}{2}(x_r - x_l)$ regardless of whether lines cross, if we use signed area.