Determinant formulas for symmetric polynomials of eigenvalues

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Abstract

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1 Introduction

Theorem 1.1. *Let $\mathbb{K}$ be a finite index set, and let $A^{(k)}$ be $n \times n$ matrices for $k \in \mathbb{K}$. Then the quantity*

$$\sum_{K \in \mathbb{K}^{n \times n}} p_K \det_{ij}(A_{ij}^{(K_{ij})}) \tag{1}$$

is independent of the basis of the $A^{(k)}$ if p_K is symmetric (i.e. $p_K = p_{K'}$ if K' is the same as K up to a row and column permutation).

Theorem 1.2. *Let $\mathbb{K}$ be a finite index set, and let $A^{(k)}$ be commuting $n \times n$ matrices with eigenvalues $a_i^{(k)}$ for $k \in \mathbb{K}$, then*

$$\sum_{K \in \mathbb{K}^{n \times n}} p_K \det_{ij}\left(A_{ij}^{(K_{ij})}\right) = \sum_{K \in \mathbb{K}^{n \times n}} p_K \prod_{i=1}^{n} a_i^{(K_{ii})} \tag{2}$$

if p_K is symmetric ($p_K = p_{K'}$ if K' is the same as K up to a row and column permutation).

Let A be a 2×2 matrix with eigenvalues a_1, a_2, then

$a_1^n a_2^k + a_1^k a_2^n = \det(A_1^n, A_2^k) + \det(A_1^k, A_2^n)$

where A_i^n is the i-th column of A^n.