

# Serializability in Programmable Networking Services

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Add abstract here

## **1 Introduction**

continue..

## **2 Problem Definition**

The problem..

## **3 Example: NetKAT + Global Variables + Yields**

NetKAT...

## **4 Formulations**

The..

## **5 Proofs**

The theorems..

## **6 Implementation**

The implementation..

## 7 Related Work

Serializability was first formalized by Eswaran et al. [17] as a correctness condition for concurrent transaction execution. Their work also introduced conflict serializability, a stricter variant that requires equivalence to a serial schedule through solely reordering non-conflicting operations. Papadimitriou [35] later proved that determining serializability even for a given, single interleaving is NP-hard. Moreover, although conflict serializability is more conservative measure than serializability, it is easier to enforce during runtime by various approaches. These approaches are typically categorized as either *pessimistic* locking approaches, e.g., 2-Phase Locking [6], or alternatively — *optimistic* locking approaches, e.g., Optimistic Concurrency Control (OCC) [10, 25]. Both approaches ensure acyclicity in the conflict graph — a necessary and sufficient condition for conflict serializability. However, because these approaches *ignore program semantics*, these may incorrectly reject executions that, although are not conflict-serializable, they are still valid serializable executions, i.e., execution equivalent to a serial execution.

Alur et al. [2] established the complexity of deciding conflict serializability for bounded transaction systems. Bouajjani et al. [7] later extended this result to unbounded systems, proving the problem remains decidable and EXPTIME-complete. Their key insight reveals that while the conflict graph becomes infinite, cycle detection, and thus conflict serializability, is independent of transaction count. By modeling transactions via vector addition systems (equivalent to Petri Nets), they provide a finite framework for analyzing infinite behaviors. This approach inspired our use of Petri Nets to capture  $\text{Int}(S)$ .

A separate research direction attempts to *directly* verify serializability, without limiting it to conflict restrictions, as done in other works. Towards this end, the expressive Temporal Logic of Actions (TLA) [29] is used to encode a formal specification that validates that only serializable executions occur. While TLA can naturally encode serializability (based on final-state equivalence), existing approaches [21, 38] remain limited to bounded transaction systems. This limitation stems from TLA/TLA+ model checkers like TLC and Apalache [23, 40], which require finite-state verification and cannot handle unbounded transaction counts.

While these contributions represent significant advances, to our knowledge, our work is the first to: (i) Decide serializability universally — *considering all executions* purely through program semantics and final states, independent of read/write conflicts; (ii) Support *unbounded* transaction systems; and (iii) Provide a complete end-to-end implementation.

Several works have proposed relaxations of the (strong) consistency notion of serializability guarantees. Rastogi et al. [36] introduced predicate-wise serializability (PWSR), which preserves database invariants while permitting non-atomic transactions. Other approaches focus on weaker consistency models: Lamport’s causal consistency [28], later generalized to shared memory as causal memory [1] and implemented in systems like COPS [30], has inspired extensive research on model checking and complexity analysis [8, 26, 41]. Recent work by Brutschy et al. [9] further bridges these concepts by statically detecting non-serializable behaviors in causally consistent databases.

Our work also builds upon both theoretical literature, as well as practical results, pertaining to Petri Nets [14, 34, 37]. Firstly, our undecidability result is based on a classic result by Hack [18, 19], showing that, given two Petri Nets, it is undecidable to answer whether they have equivalent reachability sets. Hack based his result on the work of Rabin (which was never published). These undecidability results follow from a series of reductions, originating from Hilbert’s 10th problem, i.e., deciding if a Diophantine polynomial has an integer root (a problem that was proved undecidable by Matijasević [31]). Later, Jančar [22] proved this result by demonstrating that Petri Nets can simulate 2-counter Minsky Machines [33], which are universally computable and hence undecidable. Moreover, Jančar strengthened the original result and proved that reachability equivalence is undecidable even for Petri Nets with five unbounded places [22].

Our decision procedure itself is based on an algorithm for deciding whether a given marking is reachable, for a Petri Net. Mayr [32] was the first to put forth an algorithm for this problem given a (potentially, unbounded) Petri Net (note that for a bounded case this is straightforward, as you can enumerate all reachable markings.) Mayr’s reachability algorithm was later improved and simplified by Kosaraju [24], and then again by Lambert [27]. Very recently, this problem was also proven to be Ackermann complete [11], implying that, although decidable, it is practically infeasible to solve on large nets. Furthermore, these theoretical algorithms have inspired various tools, such as K-Reach [13], DICER [39], MARCIE [20], and others. Specifically, our tool employs SMPT [4], a state-of-the-art Petri Net reachability tool, which employs an SMT-based approach [3, 5]. SMPT curtails the search space by reducing the reachability problem to a satisfiability query (that is subsequently dispatched to

the Z3 solver [12]) and inferring invariants on the net's structure. We refer the reader to a survey by Esparza and Nielsen [15] (recently republished in [16]) for a comprehensive summary on additional decidability results pertaining to Petri Nets.

**[TODO]** decide over 1992 paper: *Modeling Serializability via Process Equivalence in Petri Nets*

## **8 Discussion**

### **8.1 Conclusion**

To conclude..

### **8.2 Future Work**

Next..

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