COMP4107 - Assignment 1

Student Name: Yunkai Wang
Student Number: 100968473

Student Name: Jules Kuehn
Student Number: 100661464

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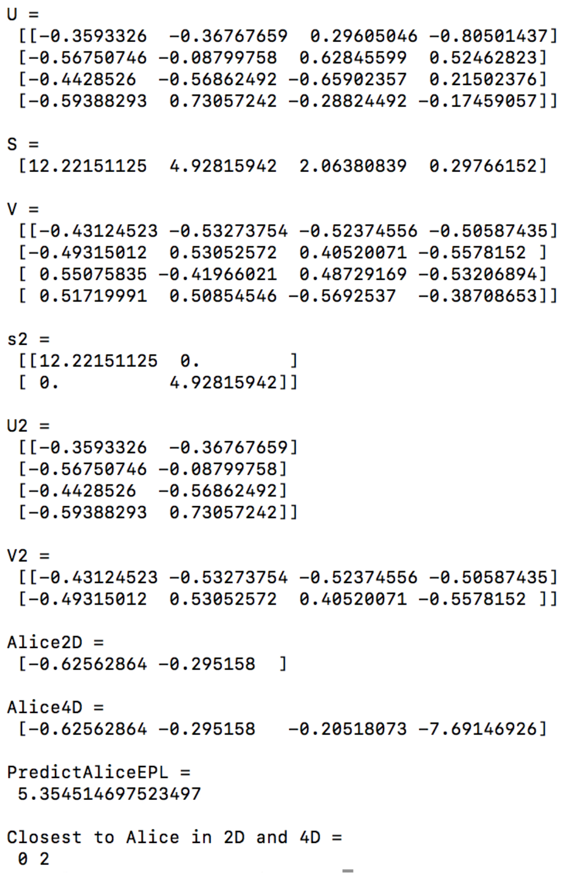
1. Question 1 Implementation for question 1 can be found in q1.py. The result is

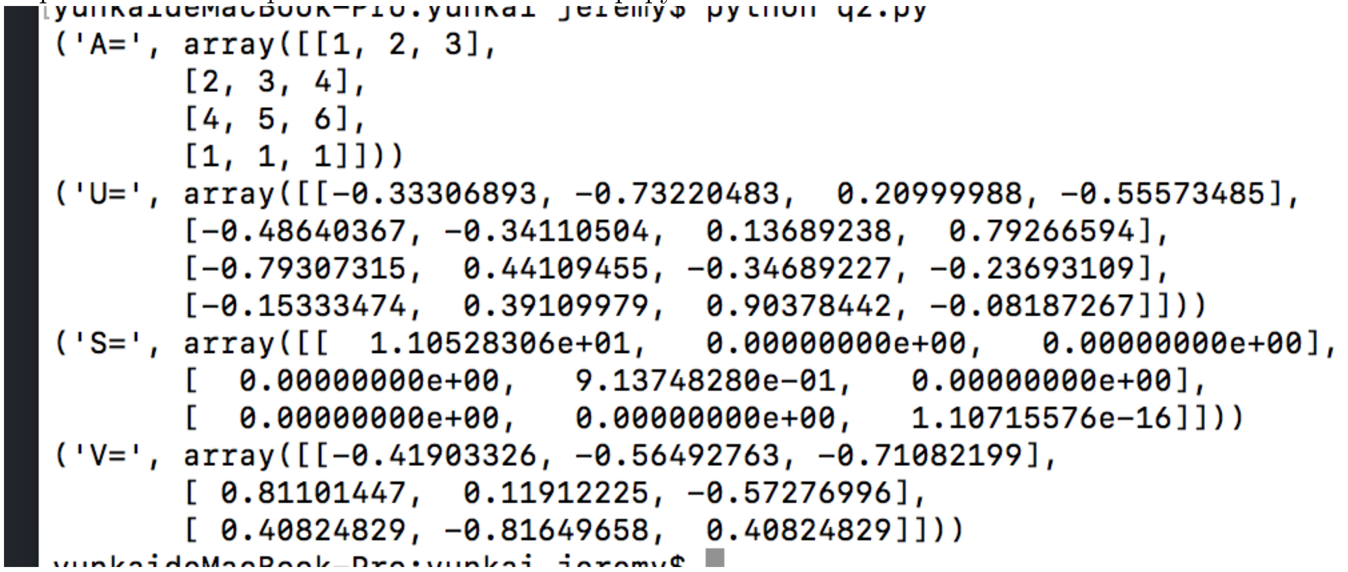


```
U =
 [[-0.3593326  -0.36767659  0.29605046 -0.80501437]
 [-0.56750746 -0.08799758  0.62845599  0.52462823]
 [-0.4428526  -0.56862492 -0.65902357  0.21502376]
 [-0.59388293  0.73057242 -0.28824492 -0.17459057]]

S =
 [12.22151125  4.92815942  2.06380839  0.29766152]

V =
 [[-0.43124523 -0.53273754 -0.52374556 -0.50587435]
 [-0.49315012  0.53052572  0.40520071 -0.5578152 ]
 [ 0.55075835 -0.41966021  0.48729169 -0.53206894]
 [ 0.51719991  0.50854546 -0.5692537  -0.38708653]]

s2 =
 [[12.22151125  0.        ]
 [ 0.          4.92815942]]

U2 =
 [[-0.3593326  -0.36767659]
 [-0.56750746 -0.08799758]
 [-0.4428526  -0.56862492]
 [-0.59388293  0.73057242]]

V2 =
 [[-0.43124523 -0.53273754 -0.52374556 -0.50587435]
 [-0.49315012  0.53052572  0.40520071 -0.5578152 ]]

Alice2D =
 [-0.62562864 -0.295158  ]

Alice4D =
 [-0.62562864 -0.295158   -0.20518073 -7.69146926]

PredictAliceEPL =
 5.354514697523497

Closest to Alice in 2D and 4D =
 0 2
```

Notice that the matrices U, V are different with the matrices in the slide, however, in order to generate the same result, we have to take the transpose of the original matrix, which is not something that we decided to go with since I don't think we are allowed to do that. The closest user in 2D is Alicia, and the closest user in 4D is Mary.

2. Question 2

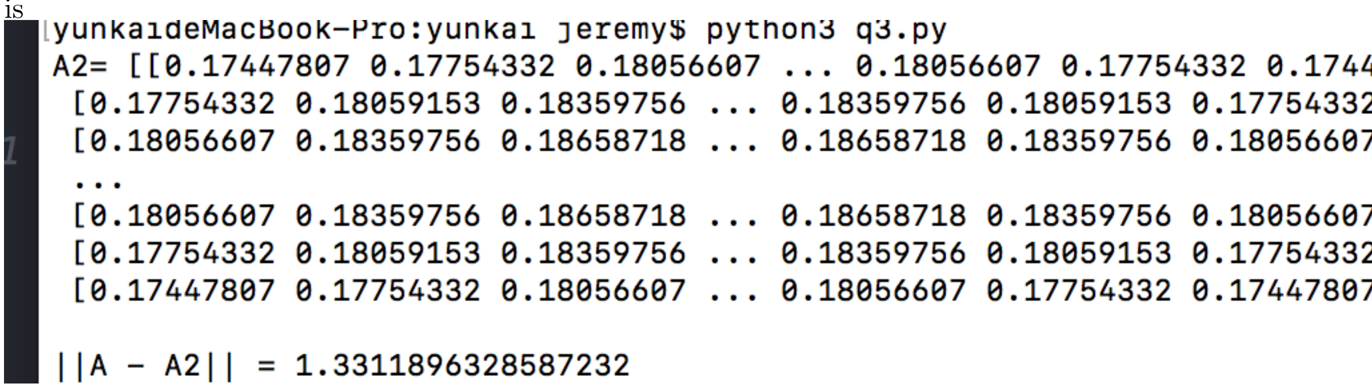
Implementation for question 2 can be found in q2.py. The results is



```
yunkaideMacBook-Pro:yunkai jeremy$ python q2.py
('A=', array([[1, 2, 3],
       [2, 3, 4],
       [4, 5, 6],
       [1, 1, 1]]))
('U=', array([[-0.33306893, -0.73220483,  0.20999988, -0.55573485],
       [-0.48640367, -0.34110504,  0.13689238,  0.79266594],
       [-0.79307315,  0.44109455, -0.34689227, -0.23693109],
       [-0.15333474,  0.39109979,  0.90378442, -0.08187267]]))
('S=', array([[  1.10528306e+01,   0.00000000e+00,   0.00000000e+00],
       [  0.00000000e+00,   9.13748280e-01,   0.00000000e+00],
       [  0.00000000e+00,   0.00000000e+00,   1.10715576e-16]]))
('V=', array([[-0.41903326, -0.56492763, -0.71082199],
       [ 0.81101447,  0.11912225, -0.57276996],
       [ 0.40824829, -0.81649658,  0.40824829]]))
yunkaideMacBook-Pro:yunkai jeremy$
```

3. Question 3

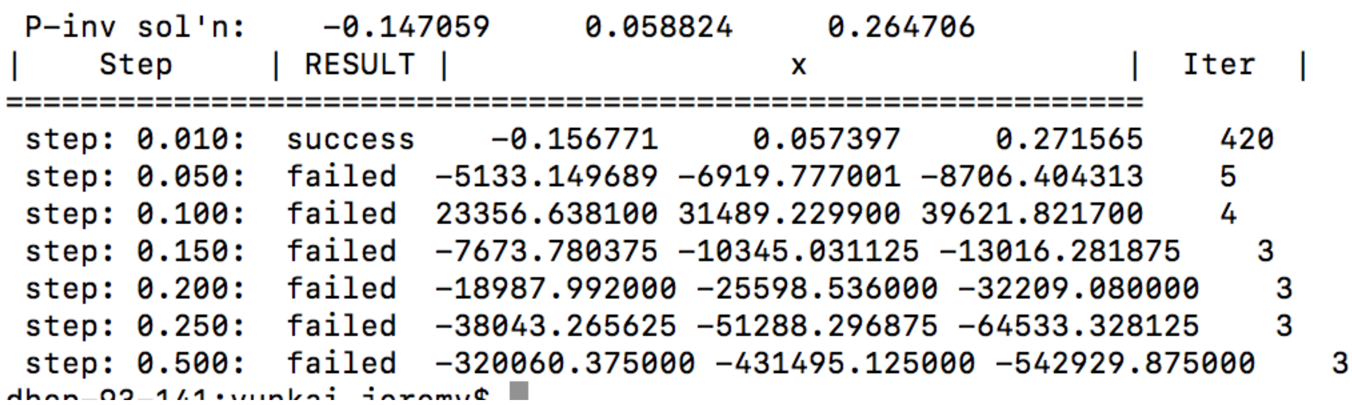
   Implementation for question 3 can be found in q3.py. The rank-2 approximation and $||A - A_2||$ is



```
yunkaideMacBook-Pro:yunkai jeremy$ python3 q3.py
A2= [[0.17447807 0.17754332 0.18056607 ... 0.18056607 0.17754332 0.1744
 [0.17754332 0.18059153 0.18359756 ... 0.18359756 0.18059153 0.17754332
 [0.18056607 0.18359756 0.18658718 ... 0.18658718 0.18359756 0.18056607
 ...
 [0.18056607 0.18359756 0.18658718 ... 0.18658718 0.18359756 0.18056607
 [0.17754332 0.18059153 0.18359756 ... 0.18359756 0.18059153 0.17754332
 [0.17447807 0.17754332 0.18056607 ... 0.18056607 0.17754332 0.17447807

||A - A2|| = 1.3311896328587232
```

4. Question 4

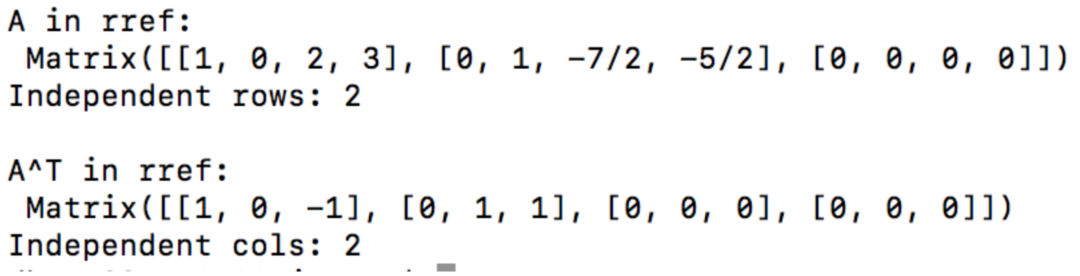
   Implementation for question 4 can be found in q4.py. The only learning rate that will work is when $\varepsilon = 0.01$, which will lead to the correct result with ≈ 420 iterations. The other ones won't work as we are descending too quickly, and therefore we will miss the correct answer and failed to come back. We set the program to stop once the norm of the vector is greater than 100000, which is obviously too big and therefore it's impossible to get the result.



```
 P-inv sol'n:    -0.147059     0.058824     0.264706
|    Step     | RESULT |                  x                   |  Iter  |
==================================================================
 step: 0.010:  success    -0.156771     0.057397     0.271565    420
 step: 0.050:  failed  -5133.149689 -6919.777001 -8706.404313    5
 step: 0.100:  failed  23356.638100 31489.229900 39621.821700    4
 step: 0.150:  failed  -7673.780375 -10345.031125 -13016.281875    3
 step: 0.200:  failed  -18987.992000 -25598.536000 -32209.080000    3
 step: 0.250:  failed  -38043.265625 -51288.296875 -64533.328125    3
 step: 0.500:  failed  -320060.375000 -431495.125000 -542929.875000    3
```

5. Question 5

   The implementation for q5 can be found in q5.py. $A = \begin{bmatrix} 3 & 2 & -1 & 4 \\ 1 & 0 & 2 & 3 \\ -2 & -2 & 3 & 3 \end{bmatrix}$. It's clear that the rows of A are not linearly independent. To see this, we reduce A to its RREF form and we will get $A = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & -7/2 & -5/2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, rank of rows of A equals to 2. The columns of A are not linearly independent as well. Since as we reduce A^T to RREF form, we will get $A = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, clearly that rank of the columns of A equal to 2, so the columns are not linearly independent. The inverse of A cannot be calculated as A is not a square matrix, and the rows and columns of A are not linearly independent.

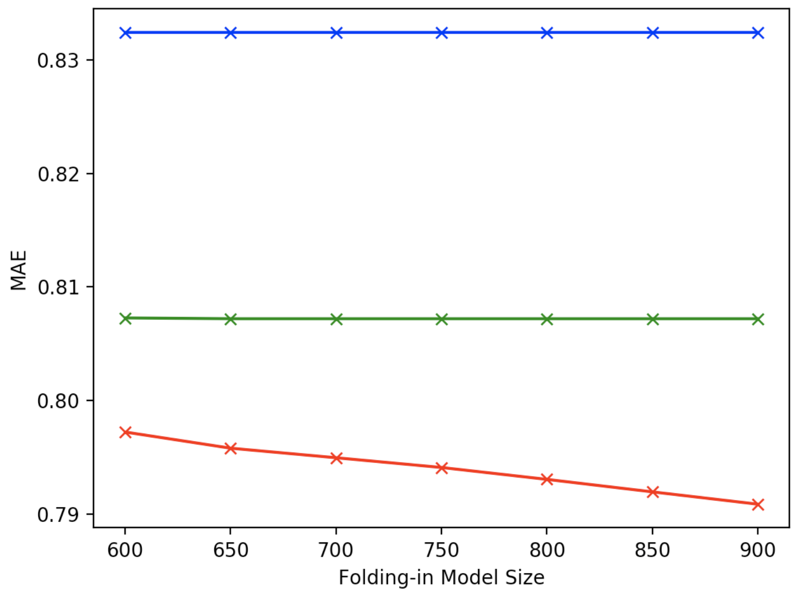


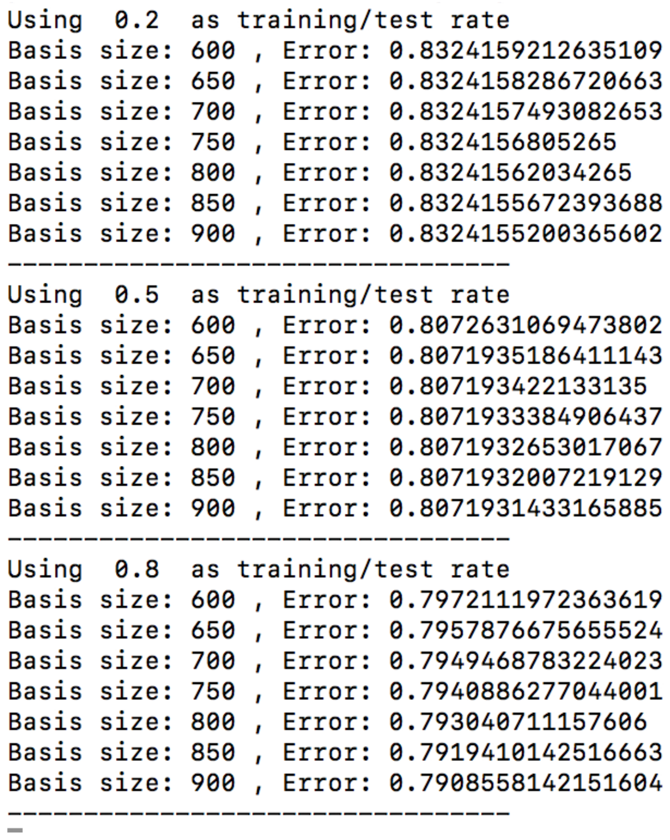
```
A in rref:
 Matrix([[1, 0, 2, 3], [0, 1, -7/2, -5/2], [0, 0, 0, 0]])
Independent rows: 2

A^T in rref:
 Matrix([[1, 0, -1], [0, 1, 1], [0, 0, 0], [0, 0, 0]])
Independent cols: 2
```

6. Question 6

7. Question 7

   To accomplish question 7, we read another paper online which can be found at here. We used the normalization method and the folding in method mention in the paper and are able to produce these following graphs.





```
Using  0.2  as training/test rate
Basis size: 600 , Error: 0.8324159212635109
Basis size: 650 , Error: 0.832415828672066З
Basis size: 700 , Error: 0.832415749З082653
Basis size: 750 , Error: 0.8324156805265
Basis size: 800 , Error: 0.83241562034265
Basis size: 850 , Error: 0.8324155672393688
Basis size: 900 , Error: 0.8324155200365602
---------------------------------
Using  0.5  as training/test rate
Basis size: 600 , Error: 0.8072631069473802
Basis size: 650 , Error: 0.8071935186411143
Basis size: 700 , Error: 0.807193422133135
Basis size: 750 , Error: 0.8071933384906437
Basis size: 800 , Error: 0.8071932653017067
Basis size: 850 , Error: 0.8071932007219129
Basis size: 900 , Error: 0.8071931433165885
---------------------------------
Using  0.8  as training/test rate
Basis size: 600 , Error: 0.7972111972363619
Basis size: 650 , Error: 0.7957876675655524
Basis size: 700 , Error: 0.7949468783224023
Basis size: 750 , Error: 0.7940886277044001
Basis size: 800 , Error: 0.793040711157606
Basis size: 850 , Error: 0.7919410142516663
Basis size: 900 , Error: 0.7908558142151604
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```

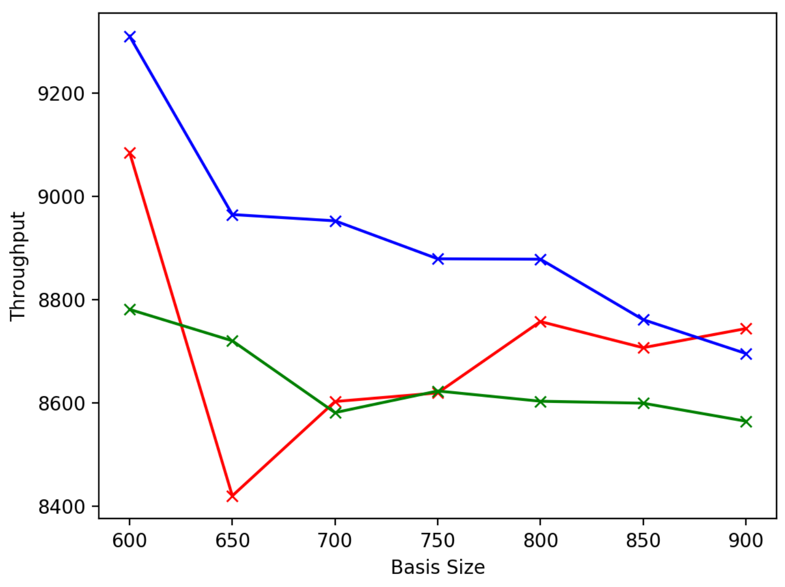

9200
9000
8800
8600
8400
Throughput
600
650
700
750
800
850
900
Basis Size