Practice Final Examination

Logic
Leiden University

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Problem 1 Refute the following argument by means of a counterexample. A counterexample suffices. You do not need to show the premises to be true and the conclusion to be false.

(a) $\forall x \exists y (Rxy \wedge Ryx) \not\models \exists y \forall x (Rxy \wedge Ryx)$

Problem 2 Prove the following using natural deduction:

(a) $P \vee (Q \to R) \vdash (Q \wedge \neg R) \to P$
(b) $\vdash \neg((P \vee \neg Q) \wedge (\neg P \wedge Q))$
(c) $\neg \exists x Px \vdash \forall x \neg Px$
(d) $\forall x (Px \to Qx), \exists x \neg Qx \vdash \exists x \neg Px$

Problem 3 Formalize each of the following sentences in as much detail as possible, explicitly specifying your dictionary. If there are any non-extensional expressions in the English sentence, demonstrate their failure of extensionality using examples, and explain what this means for the formalization. Note any ambiguities and provide alternative formalizations:

(a) Every cat destroys a Christmas tree ornament. [Note: you should formalize 'Christmas tree ornament' as a single predicate]
(b) Some philosophers admire only obscure philosophers.
(c) Leonard wishes he could adopt a cat, but he believes he probably can't.
(d) A coward shot Jesse James.
(e) A train robber needs a gun.
(f) A crime was committed by every outlaw.

Problem 4 Show that the following argument is not valid by providing a counterexample:

> Some but not all desks are wooden and some but not all desks are old-fashioned. Therefore, there is at least one wooden desk that is old-fashioned.

Problem 5 Formalize the following argument into sentences of $\mathcal{L}_2$ with as much detail as possible, and then prove that it is valid using natural deduction.

> Every number comes after another number. Six is a number. Therefore six comes after another number, which also comes after another number.

Answers:

Answers:

Problem 1 (a) One such model would be the following:
Domain $= \{1, 2\}$
$R = \{\langle 1, 1\rangle, \langle 2, 2\rangle\}$

Problem 2
(a)

$$\dfrac{P \lor (Q \to R) \qquad \dfrac{[P]}{(Q \land \neg R) \to P} \qquad \dfrac{\dfrac{\dfrac{[Q \to R] \qquad \dfrac{[Q \land \neg R]}{Q}}{R} \qquad \dfrac{[Q \land \neg R]}{\neg R}}{P}}{(Q \land \neg R) \to P}}{(Q \land \neg R) \to P}$$

(b)

$$\dfrac{[((P \lor \neg Q) \land (\neg P \land Q))] \qquad \dfrac{\dfrac{[((P \lor \neg Q) \land (\neg P \land Q))]}{P \lor Q} \qquad \dfrac{[P] \qquad \dfrac{\dfrac{[((P \lor \neg Q) \land (\neg P \land Q))]}{\neg P \land Q}}{\neg P}}{\neg((P \lor \neg Q) \land (\neg P \land Q))} \qquad \dfrac{[\neg Q] \qquad \dfrac{\dfrac{[((P \lor \neg Q) \land (\neg P \land Q))]}{\neg P \land Q}}{Q}}{\neg((P \lor \neg Q) \land (\neg P \land Q))}}{\neg((P \lor \neg Q) \land (\neg P \land Q))}}{\neg((P \lor \neg Q) \land (\neg P \land Q))}$$

(c)

$$\dfrac{\dfrac{\dfrac{[Pa]}{\exists x Px}\,(\exists Intro) \qquad \neg\exists x Px}{\neg Pa}\,(\neg Intro)}{\forall x \neg Px}\,(\lor Intro)$$

(d)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\forall x(Px \to Qx)}{Pa \to Qa} \qquad [Pa]}{Qa} \qquad [\neg Qa]}{\neg Pa}}{\exists x \neg Px} \qquad \exists x \neg Qx}{\exists x \neg Px}$$

Problem 3 (a) This is ambiguous between interpretations on which there every cat destroys some Christmas ornament or another and one on which there is a single Christmas ornament that is destroyed by every cat. They would be formalized as follows:
$\forall x(Cx \to \exists y(Oy \land Dxy))$ and
$\exists y(Oy \land \forall x(Cx \to Dxy))$
Dictionary: C: ... is a cat; O ... is an ornament; D ... destroys ...

(b) $\exists x(Px \land \forall y((Py \land Axy) \to Oy))$. Dictionary: P: ... is a philosopher; A: ... admires ...; O ... is obscure. This might be ambiguous between a reading where every philosopher who is admired by these philosophers is obscure and one where these philosophers admire no one else at all besides obscure philosophers. If interpreted in the second way, it would be formalized as $\exists x(Px \land \forall y((Axy) \to (Py \land Oy))$ with the same dictionary.

(c) $Pa \wedge Qa$: ...wishes he could adopt a cat; Q ... believes he probably can't; a: Leonard. Neither '...wishes he could adopt a cat' nor '... believes he probably can't' is extensional. We may imagine an astronomer incorrectly thinking that Hesperus and Phosphorus are distinct celestial bodies, and consequently wishing that she would see Hesperus tomorrow without believing that she will see Phosphorus tomorrow; likewise, she might believe that it is likely that she won't see Hesperus tomorrow without believing it to be likely that she won't see Phosphorus tomorrow.

(d) A coward shot Jesse James.
$\exists x(Px \wedge Qxa)$
P: ... is a coward
Q: ... shot ...
a: Jesse James

(e) A train robber needs a gun.
$\forall x(Px \to \exists y(Qxy \wedge Ry))$
P: ... is a train robber
Q: ... needs ... (it could be formalized as ...needs a gun, but that would have less detail)
R: ... is a gun

(f) A crime was committed by every outlaw.
Ambiguity: is there one crime that was committed by all outlaws, or does every outlaw commit some crime but not necessarily the same one?
$\exists x(Px \wedge \forall y(Qy \to Ryx))$
P: ... is a crime
Q ... is an outlaw
R: ... was committed by ...
Or: $\forall y(Qy \to \exists x(Px \wedge Ryx))$

Problem 4 We first formalize the argument as follows:
$\exists x(Bx \wedge Wx) \wedge \neg\forall x(Bx \wedge Wx) \wedge \exists x(Bx \wedge Ox) \wedge \neg\forall x(Bx \wedge Ox) \models \exists x(Bx \wedge (Wx \wedge Ox))$
where B: ...is a desk; W: ...is wood; O: ...is old-fashioned.

To see that this argument is not valid, we can construct a counterexample as follows. Let our domain $= \{1,2\}$; $B = \{1,2\}$; $W = \{1\}$; $O = \{2\}$.

Problem 5 Every number comes after another number. Six is a number. Therefore six comes after another number, which also comes after another number.

Dictionary:
$P : \ldots$ is a number
$Q : \ldots_1$ comes after $\ldots_2$
a : Six
proof on next page

With this, the argument can be formalized as the following valid argument of $\mathcal{L}_2$:

$$\forall x(Px \to \exists y(Py \land Rxy)), Pa \vdash \exists x(Px \land Rax \land \exists y(Py \land Rxy))$$

$$\dfrac{\dfrac{Pa \quad \dfrac{\forall x(Px \to \exists y(Py \land Rxy))}{Pa \to \exists y(Py \land Ray)}}{\exists y(Py \land Ray)} \quad \dfrac{\dfrac{[Pb \land Rab] \quad \dfrac{\dfrac{[Pb \land Rab]}{Pb} \quad \dfrac{\forall x(Px \to \exists y(Py \land Rxy))}{Pb \to \exists y(Py \land Rby)}}{\exists y(Py \land Rby)}}{Pb \land Rab \land \exists y(Py \land Rby)}}{\exists x(Px \land Rax \land \exists y(Py \land Rxy))}}{\exists x(Px \land Rax \land \exists y(Py \land Rxy))}$$