

# Practice Midterm

Logic  
Leiden University

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- Problem 1 (a) Express in set-theoretic notation the binary relation *... squared is ...* (as in '2 squared is 4') defined over the set of the positive real numbers. Determine if this relation is:
- (i) reflexive over the set of positive real numbers
  - (ii) symmetric
  - (iii) asymmetric
  - (iv) antisymmetric
  - (v) transitive
  - (vi) a function?
- (b) Consider the relation containing the ordered pairs  $\langle \text{Square}, \text{Triangle} \rangle$ ,  $\langle \text{Pentagon}, \text{Triangle} \rangle$ ,  $\langle \text{Pentagon}, \text{Square} \rangle$ ,  $\langle \text{Hexagon}, \text{Triangle} \rangle$ ,  $\langle \text{Hexagon}, \text{Square} \rangle$ ,  $\langle \text{Hexagon}, \text{Pentagon} \rangle$ , and no other ordered pairs. Draw a diagram for this relation. Then determine whether this relation is:
- (i) reflexive
  - (ii) symmetrical
  - (iii) transitive
  - (iv) a function?

Problem 2 Add quotation marks to make the following true English sentences. Try to find all the correct answers.

- (a) The second letter of this is the same as the second letter of the first word of this sentence.
- (b) P.J. Veth denotes P.J. Veth.
- (c) This statement is false is false.

Problem 3 Show that the following sentences are contradictions, using truth tables.

- (i)  $\neg(P \vee \neg P)$
- (ii)  $\neg((P \rightarrow (Q \rightarrow R)) \rightarrow ((P \rightarrow Q) \rightarrow (Q \rightarrow R)))$
- (iii)  $(P \vee \neg P) \rightarrow (Q \wedge \neg Q)$

Problem 4 Determine the scopes of the underlined occurrences of connectives in the following sentences, which have been abbreviated in accordance with the bracketing conventions.

- (a)  $\neg(P \wedge Q) \rightarrow P \underline{\vee} Q \vee R$
- (b)  $\underline{\neg}(P \wedge Q \rightarrow \neg Q)$
- (c)  $P \wedge Q \underline{\rightarrow} P \vee Q$
- (d)  $(P \wedge Q) \underline{\vee} R \rightarrow (P \vee Q) \vee (Q \vee R)$

Problem 5 Is the following argument in English valid? Is it propositionally valid?

Everyone who rides a bicycle gets wet. Hans rides a bicycle. Therefore, Hans gets wet.

Problem 6 Is the following argument propositionally valid? If so, show this. If not, explain why not.

If Abbey Road has a great song, then it is a good album. *Abbey Road* has *Here Comes The Sun*, which is a nice song but not great. Also, Ringo doesn't write very good songs. Ringo wrote *Octopus's Garden*, which is utter garbage. That song is on Abbey Road too. Still Abbey Road is a good album, because *Come Together* is a great song, and that's also on Abbey Road. And if *Come Together* is on Abbey Road, then Abbey Road has a great song.

Problem 7 Formalize the following sentences into sentences of  $\mathcal{L}_1$  with as much detail as possible. Note any difficulties that arise.

- (i) It is necessary that laws of nature are exceptionless.
- (ii) Lisa will come to the party only if her train is on time.
- (iii) Sarah is a frightened swimmer but a brave mountaineer.

**Answers****Problem 1(a)**  $\{\langle d, e \rangle : d^2 = e\}$ .

- (i) reflexive NO
- (ii) symmetrical NO
- (iii) assymetric NO
- (iv) antisymmetric YES
- (v) transitive NO
- (vi) a function? YES

**Problem 1(b)**

- (i) reflexive NO
- (ii) symmetrical NO
- (iii) transitive YES
- (iv) a function? NO

**Problem 2**

- (a) The second letter of ‘this’ is the same as the second letter of the first word of this sentence;
- (b) ‘P.J. Veth’ denotes P.J. Veth;
- (c) T‘This statement is false’ is false.

**Problem 3** All are contradictions.**Problem 4**

- (a)  $\neg(P \wedge Q) \rightarrow \underbrace{((P \vee Q) \vee R)}$
- (b)  $\neg \underbrace{(P \wedge Q \rightarrow \neg Q)}$
- (c)  $\underbrace{P \wedge Q \rightarrow P \vee Q}$
- (d)  $\underbrace{(P \wedge Q) \vee R} \rightarrow (P \vee Q) \vee (Q \vee R)$

**Problem 5** It is valid in English but not propositionally valid.**Problem 6**Formalised as:  $P \rightarrow Q, R, P_1, P_1 \rightarrow P \models Q$  $P$  : Abbey Road has a great song $Q$  : Abbey Road is a good album $R$  : Come together is a great song $P_1$ : Come together is on Abbey Road**Problem 7**

- (i)  $P$   
Dictionary:  $P$ : It is necessary that laws of nature are exceptionless.
- (ii)  $L \rightarrow T$ .  
Dictionary:  $L$ : Lisa will come to the party;  
 $T$ : Lisa's train is on time.
- (iii)  $P \wedge Q$ .  
Dictionary:  $P$ : Sarah is a frightened swimmer;  
 $Q$ : Sarah is a brave mountaineer.