

ETHICS

*Spinoza prepared to publish the Ethics, the comprehensive account of his philosophical system, in 1674. The work and its five parts had been completed after over a decade's labor, and after the turmoil of the years since the Short Treatise and the publication of the Principles of Cartesian Philosophy. The time had come but at the advice of friends, Spinoza felt the danger and the risks too deeply. As he reported to Henry Oldenburg in the fall of 1675, he was attacked both by theologians and by Cartesians and felt compelled to halt publication (Ep68; see Jonathan Israel, *Radical Enlightenment*, 286–7). Indeed, the work—one of the classics of Western philosophy—was only finally published in 1677 after Spinoza's death, in the *Opera Posthuma*, edited by his friends and published by Jan Rieuwerts. Within a year, on 25 June 1678, it was censored by the States of Holland and West-Friesland as a “profane, atheistic, and blasphemous book.”*

Some scholars believe that the appendix to the Short Treatise, probably composed in 1661 or early 1662, including seven axioms about substance, its attributes, and causality, together with four demonstrations about substance, was already an early version of the mathematically, geometrically organized content of the first book of the Ethics. By late 1662 or early 1663, with Spinoza in Rijnsburg, his Amsterdam friends had a copy of an early chapter of Part I “On God.” Pieter Balling had delivered it to Simon de Vries, and it soon became the topic of meetings in Amsterdam where it was read and discussed. On and off, then, from 1661 to 1674, Spinoza worked on the Ethics, his magnum opus, paying the promissory note made in the TIE and setting out the details of his philosophical account of nature, mind, and the good life.

By June 1665, Spinoza seems to have had a complete draft in hand, a work of three parts, most likely following the design of the Short Treatise—“on God, man, and his well-being.” Eventually, by 1675, of course, the Ethics had been revised and expanded, taking on its now famous five-part structure—on God, humankind and human epistemology, the passions, human bondage to the passions, and rational freedom. A June 1665 letter to Johan Bouwmeester, an Amsterdam friend and associate of Lodewijk Meyer, suggests that the original Part III was nearly complete and ready to be translated from Latin into Dutch, perhaps by Bouwmeester himself (Ep28). This third part contained much of what is found in Parts IV and V of the version we now have. Hence, by the time Spinoza turned, that autumn of 1665, to the Theological-Political Treatise, his system was complete.

A remarkable work it was. The *Ethics*'s five parts famously lay out a system in the style of Euclid's geometry—starting from definitions and axioms and working through theorems or propositions with corollaries, notes or scholia, appendices, and more. The axiomatic style mirrors the system's rationality and exemplifies the way knowledge should be grasped. As the system proceeds from metaphysics through its account of human nature, knowledge, and emotion, to its understanding of human flaws and aspirations, and finally to the ethical goal of human life (a life of freedom and understanding), the work both grounds itself and motivates its readers to conduct their lives according to the best conception of what human life can and should be. In short, Spinoza's magnum opus earns its title.

The book's contents are, in broad terms, well known. Spinoza's is an early modern naturalism, a set of principles underlying a rational, scientific view of religion, nature, psychology, and ethics. In Part I he defines crucial terms such as substance, attribute, mode, eternity, and God. He demonstrates that only one substance, with infinite attributes, exists; it does so necessarily, and every mode that follows from it occurs with precise and necessary determination. This one eternal, necessary, determinate substance is God, and hence nature or the natural world is either identical to it or to certain ways of understanding it. Modes of substance are not properties of substance, as in classical philosophy, but rather things in the world existing in precise states or ways. Modes are manifestations of substance and its attributes, which might be thought of as regulative natural forces.

In Part II, Spinoza introduces the two attributes by which we understand substance and in terms of which substance is manifest to our experience—thought and extension—and builds an account of the mental and physical dimensions of nature. This account leads to a set of propositions about human experience and cognition and, in Part III, of human emotions, feelings, and more, all as the psychological correlates of physical states of the human body. The causal structure of physical bodies, determined by their proportion of motion and rest, and influenced by the lawful interactions of bodies, is correlated with mental states, some cognitive, others affective, in all of nature and in particular in the minds of human beings. Spinoza's psychology is grounded in his physics and in the conception of *conatus*, the striving of each being to persevere and to manifest its essence; here is the dynamic element in Spinoza's vitalistic conception of nature. In human beings, the *conatus* takes on certain predictable psychological features. Ultimately, people seek to satisfy desires, feel joy and pleasure, and enhance their well-being, and these goals require increasing harmonious activity within nature and the diminishing of the passions, which mark a person's subordination to beings external to it and failure to satisfy its own preservation. This goal requires as complete and perfect a knowledge of nature as one can attain, a knowledge that corresponds in the mind to the maximizing of life-enhancing physical states on the body's part. Later in the *Ethics*, Spinoza calls this cognitive goal the "intellectual love of God" or "blessedness," and, in the notorious concluding section of Part V, he associates it with the mind's eternity and thereby with the traditional notion of the immortality of the soul.

Within the confines of this naturalistic system, Spinoza installs some claims that, even in his own day, became famous and even notorious. He also took some steps that have remained perplexing, if not confusing. Spinoza's natural world, for example, is not created, nor does it permit contingency or the existence of miracles. Furthermore, insofar as extension is an attribute of substance, Spinoza's God is physically extended; Spinoza could be and was charged with a kind of atheistic materialism. His natural world is also wholly determined and without goals or purposes. While Spinoza's God is material, human beings—unities of the physical and psychological—are as necessary and determined as God or nature. For this reason, Spinoza denies the existence of free will but not the existence of freedom, which he regards as a feature of actions which are active and rational, performed with a minimum of constraint and external coercion. In this sense, moreover, God is the only perfect being and human life an effort of imitatio dei. People are free, to the degree that they love God, understand God, and indeed emulate God, but for Spinoza these activities and aspirations are no different from seeking to understand nature and to live in harmony with natural law.

There are many obvious outcomes of this ethic of rational self-discipline and peace of mind. One is a life of democratic republicanism in which all citizens equally collaborate in a lawful society aimed at enhancing the well-being of all rational citizens and restraining harmful self-interest in behalf of this goal. In his last years Spinoza would turn, out of a sense of urgency, to an elaboration of these political implications.

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PART I

CONCERNING GOD

Definitions

1. By that which is self-caused I mean that whose essence involves existence; or that whose nature can be conceived only as existing.

2. A thing is said to be finite in its own kind [*in suo genere finita*] when it can be limited by another thing of the same nature. For example, a body is said to be finite because we can always conceive of another body greater than it. So, too, a thought is limited by another thought. But body is not limited by thought, nor thought by body.

3. By substance I mean that which is in itself and is conceived through itself; that is, that the conception of which does not require the conception of another thing from which it has to be formed.

4. By attribute I mean that which the intellect perceives of substance as constituting its essence.

5. By mode I mean the affections of substance, that is, that which is in something else and is conceived through something else.

6. By God I mean an absolutely infinite being, that is, substance consisting of infinite attributes, each of which expresses eternal and infinite essence.

Explication I say “absolutely infinite,” not “infinite in its kind.” For if a thing is only infinite in its kind, one may deny that it has infinite attributes. But if a thing is absolutely infinite, whatever expresses essence and does not involve any negation belongs to its essence.

7. That thing is said to be free [*liber*] which exists solely from the necessity of its own nature, and is determined to action by itself alone. A thing is said to be necessary [*necessarius*] or rather, constrained [*coactus*], if it is determined by another thing to exist and to act in a definite and determinate way.

8. By eternity I mean existence itself insofar as it is conceived as necessarily following solely from the definition of an eternal thing.

Explication For such existence is conceived as an eternal truth, just as is the essence of the thing, and therefore cannot be explicated through duration or time, even if duration be conceived as without beginning and end.

Axioms

1. All things that are, are either in themselves or in something else.
2. That which cannot be conceived through another thing must be conceived through itself.

3. From a given determinate cause there necessarily follows an effect; on the other hand, if there be no determinate cause, it is impossible that an effect should follow.

4. The knowledge of an effect depends on, and involves, the knowledge of the cause.

5. Things which have nothing in common with each other cannot be understood through each other; that is, the conception of the one does not involve the conception of the other.

6. A true idea must agree with that of which it is the idea [*ideatum*].

7. If a thing can be conceived as not existing, its essence does not involve existence.

PROPOSITION 1

Substance is by nature prior to its affections.

Proof This is evident from Defs. 3 and 5.

PROPOSITION 2

Two substances having different attributes have nothing in common.

Proof This too is evident from Def. 3; for each substance must be in itself and be conceived through itself; that is, the conception of the one does not involve the conception of the other.

PROPOSITION 3

When things have nothing in common, one cannot be the cause of the other.

Proof If things have nothing in common, then (Ax. 5) they cannot be understood through one another, and so (Ax. 4) one cannot be the cause of the other.

PROPOSITION 4

Two or more distinct things are distinguished from one another either by the difference of the attributes of the substances or by the difference of the affections of the substances.

Proof All things that are, are either in themselves or in something else (Ax. 1); that is (Defs. 3 and 5), nothing exists external to the intellect except substances and their affections. Therefore, there can be nothing external to the intellect through which several things can be distinguished from one another except substances or (which is the same thing) (Def. 4) the attributes and the affections of substances.

PROPOSITION 5

In the universe there cannot be two or more substances of the same nature or attribute.

Proof If there were several such distinct substances, they would have to be distinguished from one another either by a difference of attributes or by a difference

of affections (Pr. 4). If they are distinguished only by a difference of attributes, then it will be granted that there cannot be more than one substance of the same attribute. But if they are distinguished by a difference of affections, then, since substance is by nature prior to its affections (Pr. 1), disregarding therefore its affections and considering substance in itself, that is (Def. 3 and Ax. 6), considering it truly, it cannot be conceived as distinguishable from another substance. That is (Pr. 4), there cannot be several such substances but only one.

PROPOSITION 6

One substance cannot be produced by another substance.

Proof In the universe there cannot be two substances of the same attribute (Pr. 5), that is (Pr. 2), two substances having something in common. And so (Pr. 3) one cannot be the cause of the other; that is, one cannot be produced by the other.

Corollary Hence it follows that substance cannot be produced by anything else. For in the universe there exists nothing but substances and their affections, as is evident from Ax. 1 and Defs. 3 and 5. But, by Pr. 6, it cannot be produced by another substance. Therefore, substance cannot be produced by anything else whatsoever.

Another Proof This can be proved even more readily by the absurdity of the contradictory. For if substance could be produced by something else, the knowledge of substance would have to depend on the knowledge of its cause (Ax. 4), and so (Def. 3) it would not be substance.

PROPOSITION 7

Existence belongs to the nature of substance.

Proof Substance cannot be produced by anything else (Cor. Pr. 6) and is therefore self-caused [*causa sui*]; that is (Def. 1), its essence necessarily involves existence; that is, existence belongs to its nature.

PROPOSITION 8

Every substance is necessarily infinite.

Proof There cannot be more than one substance having the same attribute (Pr. 5), and existence belongs to the nature of substance (Pr. 7). It must therefore exist either as finite or as infinite. But it cannot exist as finite, for (Def. 2) it would have to be limited by another substance of the same nature, and that substance also would have to exist (Pr. 7). And so there would exist two substances of the same attribute, which is absurd (Pr. 5). Therefore, it exists as infinite.

Scholium 1 Since in fact to be finite is in part a negation and to be infinite is the unqualified affirmation of the existence of some nature, it follows from Proposition 7 alone that every substance must be infinite.

Scholium 2 I do not doubt that for those who judge things confusedly and are not accustomed to know things through their primary causes it is difficult to grasp

the proof of Proposition 7. Surely, this is because they neither distinguish between the modification of substances and substances themselves, nor do they know how things are produced. And so it comes about that they ascribe to substances a beginning which they see natural things as having; for those who do not know the true causes of things confuse everything. Without any hesitation they imagine trees as well as men talking and stones as well as men being formed from seeds; indeed, any forms whatsoever are imagined to change into any other forms. So too, those who confuse the divine nature with human nature easily ascribe to God human emotions, especially so long as they are ignorant of how the latter are produced in the mind. But if men were to attend to the nature of substance, they would not doubt at all the truth of Proposition 7; indeed, this Proposition would be an axiom to all and would be ranked among universally accepted truisms. For by substance they would understand that which is in itself and is conceived through itself; that is, that the knowledge of which does not require the knowledge of any other thing. By modifications they would understand that which is in another thing, and whose conception is formed from the thing in which they are. Therefore, in the case of nonexistent modifications we can have true ideas of them since their essence is included in something else, with the result that they can be conceived through that something else, although they do not exist in actuality externally to the intellect. However, in the case of substances, because they are conceived only through themselves, their truth external to the intellect is only in themselves. So if someone were to say that he has a clear and distinct—that is, a true—idea of substance and that he nevertheless doubts whether such a substance exists, this would surely be just the same as if he were to declare that he has a true idea but nevertheless suspects that it may be false (as is obvious to anyone who gives his mind to it). Or if anyone asserts that substance is created, he at the same time asserts that a false idea has become true, than which nothing more absurd can be conceived. So it must necessarily be admitted that the existence of substance is as much an eternal truth as is its essence.

From here we can derive in another way that there cannot be but one [substance] of the same nature, and I think it worthwhile to set out the proof here. Now to do this in an orderly fashion I ask you to note:

1. The true definition of each thing involves and expresses nothing beyond the nature of the thing defined. Hence it follows that—
2. No definition involves or expresses a fixed number of individuals, since it expresses nothing but the nature of the thing defined. For example, the definition of a triangle expresses nothing other than simply the nature of a triangle, and not a fixed number of triangles.
3. For each individual existent thing there must necessarily be a definite cause for its existence.
4. The cause for the existence of a thing must either be contained in the very nature and definition of the existent thing (in effect, existence belongs to its nature) or must have its being independently of the thing itself.

From these premises it follows that if a fixed number of individuals exist in Nature, there must necessarily be a cause why those individuals and not more or

fewer exist. If, for example, in Nature twenty men were to exist (for the sake of greater clarity I suppose that they exist simultaneously and that no others existed in Nature before them), in order to account for the existence of these twenty men, it will not be enough for us to demonstrate the cause of human nature in general; it will furthermore be necessary to demonstrate the cause why not more or fewer than twenty men exist, since (Note 3) there must necessarily be a cause for the existence of each one. But this cause (Notes 2 and 3) cannot be contained in the nature of man, since the true definition of man does not involve the number twenty. So (Note 4) the cause of the existence of these twenty men, and consequently of each one, must necessarily be external to each one, and therefore we can reach the unqualified conclusion that whenever several individuals of a kind exist, there must necessarily be an external cause for their existence. Now since existence belongs to the nature of substance (as has already been shown in this Scholium) the definition of substance must involve necessary existence, and consequently the existence of substance must be concluded solely from its definition. But the existence of several substances cannot follow from the definition of substance (as I have already shown in Notes 2 and 3). Therefore, from the definition of substance it follows necessarily that there exists only one substance of the same nature, as was proposed.

[...]

APPENDIX

I have now explained the nature and properties of God: that he necessarily exists, that he is one alone, that he is and acts solely from the necessity of his own nature, that he is the free cause of all things and how so, that all things are in God and are so dependent on him that they can neither be nor be conceived without him, and lastly, that all things have been predetermined by God, not from his free will or absolute pleasure, but from the absolute nature of God, his infinite power. Furthermore, whenever the opportunity arose I have striven to remove prejudices that might hinder the apprehension of my proofs. But since there still remain a considerable number of prejudices, which have been, and still are, an obstacle—indeed, a very great obstacle—to the acceptance of the concatenation of things in the manner which I have expounded, I have thought it proper at this point to bring these prejudices before the bar of reason.

Now all the prejudices which I intend to mention here turn on this one point, the widespread belief among men that all things in Nature are like themselves in acting with an end in view. Indeed, they hold it as certain that God himself directs everything to a fixed end; for they say that God has made everything for man's sake and has made man so that he should worship God. So this is the first point I shall consider, seeking the reason why most people are victims of this prejudice and why all are so naturally disposed to accept it. Secondly, I shall demonstrate its falsity; and lastly I shall show how it has been the source of misconceptions about good and bad, right and wrong, praise and blame, order and confusion, beauty and ugliness, and the like.

However, it is not appropriate here to demonstrate the origin of these misconceptions from the nature of the human mind. It will suffice at this point if I take as my basis what must be universally admitted, that all men are born ignorant of the causes of things, that they all have a desire to seek their own advantage, a desire of which they are conscious. From this it follows, firstly, that men believe that they are free, precisely because they are conscious of their volitions and desires; yet concerning the causes that have determined them to desire and will they do not think, not even dream about, because they are ignorant of them. Secondly, men act always with an end in view, to wit, the advantage that they seek. Hence it happens that they are always looking only for the final causes of things done, and are satisfied when they find them, having, of course, no reason for further doubt. But if they fail to discover them from some external source, they have no recourse but to turn to themselves, and to reflect on what ends would normally determine them to similar actions, and so they necessarily judge other minds by their own. Further, since they find within themselves and outside themselves a considerable number of means very convenient for the pursuit of their own advantage—as, for instance, eyes for seeing, teeth for chewing, cereals and living creatures for food, the sun for giving light, the sea for breeding fish—the result is that they look on all the things of Nature as means to their own advantage. And realizing that these were found, not produced by them, they come to believe that there is someone else who produced these means for their use. For looking on things as means, they could not believe them to be self-created, but on the analogy of the means which they are accustomed to produce for themselves, they were bound to conclude that there was some governor or governors of Nature, endowed with human freedom, who have attended to all their needs and made everything for their use. And having no information on the subject, they also had to estimate the character of these rulers by their own, and so they asserted that the gods direct everything for man's use so that they may bind men to them and be held in the highest honor by them. So it came about that every individual devised different methods of worshipping God as he thought fit in order that God should love him beyond others and direct the whole of Nature so as to serve his blind cupidity and insatiable greed. Thus it was that this misconception developed into superstition and became deep-rooted in the minds of men, and it was for this reason that every man strove most earnestly to understand and to explain the final causes of all things. But in seeking to show that Nature does nothing in vain—that is, nothing

that is not to man's advantage—they seem to have shown only this, that Nature and the gods are as crazy as mankind.

Consider, I pray, what has been the upshot. Among so many of Nature's blessings they were bound to discover quite a number of disasters, such as storms, earthquakes, diseases and so forth, and they maintained that these occurred because the gods were angry at the wrongs done to them by men, or the faults committed in the course of their worship. And although daily experience cried out against this and showed by any number of examples that blessings and disasters befall the godly and the ungodly alike without discrimination, they did not on that account abandon their ingrained prejudice. For they found it easier to regard this fact as one among other mysteries they could not understand and thus maintain their innate condition of ignorance rather than to demolish in its entirety the theory they had constructed and devise a new one. Hence they made it axiomatic that the judgment of the gods is far beyond man's understanding. Indeed, it is for this reason, and this reason only, that truth might have evaded mankind forever had not Mathematics, which is concerned not with ends but only with the essences and properties of figures, revealed to men a different standard of truth. And there are other causes too—there is no need to mention them here—which could have made men aware of these widespread misconceptions and brought them to a true knowledge of things.

I have thus sufficiently dealt with my first point. There is no need to spend time in going on to show that Nature has no fixed goal and that all final causes are but figments of the human imagination. For I think that this is now quite evident, both from the basic causes from which I have traced the origin of this misconception and from Proposition 16 and the Corollaries to Proposition 32, and in addition from the whole set of proofs I have adduced to show that all things in Nature proceed from all eternal necessity and with supreme perfection. But I will make this additional point, that this doctrine of Final Causes turns Nature completely upside down, for it regards as an effect that which is in fact a cause, and vice versa. Again, it makes that which is by nature first to be last; and finally, that which is highest and most perfect is held to be the most imperfect. Omitting the first two points as self-evident, Propositions 21, 22, and 23 make it clear that that effect is most perfect which is directly produced by God, and an effect is the less perfect in proportion to the number of intermediary causes required for its production. But if the things produced directly by God were brought about to enable him to attain an end, then of necessity the last things for the sake of which the earlier things were brought about would excel all others. Again, this doctrine negates God's perfection; for if God acts with an end in view, he must necessarily be seeking something that he lacks. And although theologians and metaphysicians may draw a distinction between a purpose arising from want and an assimilative purpose,² they still admit that God has acted in all things for the sake of himself, and

² [Spinoza alludes here to a late scholastic distinction between two kinds of purposes, or goals: (1) a purpose that satisfies some internal need or lack (*fines indigentiae*); and (2) a purpose that aims to share what one already has with others who lack it (*fines assimilationis*). In the present case, this

not for the sake of the things to be created. For prior to creation they are not able to point to anything but God as a purpose for God's action. Thus they have to admit that God lacked and desired those things for the procurement of which he willed to create the means—as is self-evident.

I must not fail to mention here that the advocates of this doctrine, eager to display their talent in assigning purpose to things, have introduced a new style of argument to prove their doctrine, i.e., a reduction, not to the impossible, but to ignorance, thus revealing the lack of any other argument in its favor. For example, if a stone falls from the roof on somebody's head and kills him, by this method of arguing they will prove that the stone fell in order to kill the man; for if it had not fallen for this purpose by the will of God, how could so many circumstances (and there are often many coinciding circumstances) have chanced to concur? Perhaps you will reply that the event occurred because the wind was blowing and the man was walking that way. But they will persist in asking why the wind blew at that time and why the man was walking that way at that very time. If you again reply that the wind sprang up at that time because on the previous day the sea had begun to toss after a period of calm and that the man had been invited by a friend, they will again persist—for there is no end to questions—"But why did the sea toss, and why was the man invited for that time?" And so they will go on and on asking the causes of causes, until you take refuge in the will of God—that is, the sanctuary of ignorance. Similarly, when they consider the structure of the human body, they are astonished, and being ignorant of the causes of such skillful work they conclude that it is fashioned not by mechanical art but by divine or supernatural art, and is so arranged that no one part shall injure another.

As a result, he who seeks the true causes of miracles and is eager to understand the works of Nature as a scholar, and not just to gape at them like a fool, is universally considered an impious heretic and denounced by those to whom the common people bow down as interpreters of Nature and the gods. For these people know that the dispelling of ignorance would entail the disappearance of that astonishment, which is the one and only support for their argument and for safeguarding their authority. But I will leave this subject and proceed to the third point that I proposed to deal with.

When men become convinced that everything that is created is created on their behalf, they were bound to consider as the most important quality in every individual thing that which was most useful to them, and to regard as of the highest excellence all those things by which they were most benefited. Hence they came to form these abstract notions to explain the natures of things: Good, Bad,

distinction implies that when God does something purposively, he acts not to fulfill a need he has, but to benefit creatures. In their commentaries on the *Ethics*, both Lewis Robinson and Harry Wolfson refer to the seventeenth-century Dutch theologian A. Heereboord as Spinoza's source for this distinction (L. Robinson, *Kommentar zu Spinoza's Ethik* (Leipzig, 1928), pp. 234–235; H. Wolfson, *The Philosophy of Spinoza* (New York, 1969), vol. 1, p. 432).

The theologians derided by Spinoza hoped to avoid by means of this distinction the suggestion that if God acts purposively, he does so because of a need on his part.]

Order, Confusion, Hot, Cold, Beauty, Ugliness; and since they believed that they are free, the following abstract notions came into being: Praise, Blame, Right, Wrong. The latter I shall deal with later on after I have treated of human nature; at this point I shall briefly explain the former.

All that conduces to well-being and to the worship of God they call Good, and the contrary, Bad. And since those who do not understand the nature of things, but only imagine things, make no affirmative judgments about things themselves and mistake their imagination for intellect, they are firmly convinced that there is order in things, ignorant as they are of things and of their own nature. For when things are in such arrangement that, being presented to us through our senses, we can readily picture them and thus readily remember them, we say that they are well arranged; if the contrary, we say that they are ill arranged, or confused. And since those things we can readily picture we find pleasing compared with other things, men prefer order to confusion, as though order were something in Nature other than what is relative to our imagination. And they say that God has created all things in an orderly way, without realizing that they are thus attributing human imagination to God—unless perchance they mean that God, out of consideration for the human imagination, arranged all things in the way that men could most easily imagine. And perhaps they will find no obstacle in the fact that there are any number of things that far surpass our imagination, and a considerable number that confuse the imagination because of its weakness.

But I have devoted enough time to this. Other notions, too, are nothing but modes of imagining whereby the imagination is affected in various ways, and yet the ignorant consider them as important attributes of things because they believe—as I have said—that all things were made on their behalf, and they call a thing's nature good or bad, healthy or rotten and corrupt, according to its effect on them. For instance, if the motion communicated to our nervous system by objects presented through our eyes is conducive to our feeling of well-being, the objects which are its cause are said to be beautiful, while the objects which provoke a contrary motion are called ugly. Those things that we sense through the nose are called fragrant or fetid; through the tongue, sweet or bitter, tasty or tasteless; those that we sense by touch are called hard or soft, rough or smooth, and so on. Finally, those that we sense through our ears are said to give forth noise, sound, or harmony, the last of which has driven men to such madness that they used to believe that even God delights in harmony. There are philosophers who have convinced themselves that the motions of the heavens give rise to harmony. All this goes to show that everyone's judgment is a function of the disposition of his brain, or rather, that he mistakes for reality the way his imagination is affected. Hence it is no wonder—as we should note in passing—that we find so many controversies arising among men, resulting finally in skepticism. For although human bodies agree in many respects, there are very many differences, and so one man thinks good what another thinks bad; what to one man is well ordered, to another is confused; what to one is pleasing, to another is displeasing, and so forth. I say no more here because this is not the place to treat at length of this

subject, and also because all are well acquainted with it from experience. Everybody knows those sayings: “So many heads, so many opinions,” “everyone is wise in his own sight,” “brains differ as much as palates,” all of which show clearly that men’s judgment is a function of the disposition of the brain, and they are guided by imagination rather than intellect. For if men understood things, all that I have put forward would be found, if not attractive, at any rate convincing, as Mathematics attests.

We see therefore that all the notions whereby the common people are wont to explain Nature are merely modes of imagining, and denote not the nature of anything but only the constitution of the imagination. And because these notions have names as if they were the names of entities existing independently of the imagination I call them “entities of imagination” [*entia imaginationis*] rather than “entities of reason” [*entia rationis*]. So all arguments drawn from such notions against me can be easily refuted. For many are wont to argue on the following lines: If everything has followed from the necessity of God’s most perfect nature, why does Nature display so many imperfections, such as rottenness to the point of putridity, nauseating ugliness, confusion, evil, sin, and so on? But, as I have just pointed out, they are easily refuted. For the perfection of things should be measured solely from their own nature and power; nor are things more or less perfect to the extent that they please or offend human senses, serve or oppose human interests. As to those who ask why God did not create men in such a way that they should be governed solely by reason, I make only this reply, that he lacked not material for creating all things from the highest to the lowest degree of perfection; or, to speak more accurately, the laws of his nature were so comprehensive as to suffice for the production of everything that can be conceived by an infinite intellect, as I proved in Proposition 16.

These are the misconceptions which I undertook to deal with at this point. Any other misconception of this kind can be corrected by everyone with a little reflection.

PART II

OF THE NATURE AND ORIGIN OF THE MIND

I now pass on to the explication of those things that must necessarily have followed from the essence of God, the eternal and infinite Being; not indeed all of them—for we proved in Proposition 16, Part I that from his essence there must follow infinite things in infinite ways—but only those things that can lead us as it were by the hand to the knowledge of the human mind and its utmost blessedness.

Definitions

1. By “body” I understand a mode that expresses in a definite and determinate way God’s essence insofar as he is considered as an extended thing. (See Cor. Pr. 25, I.)

2. I say that there pertains to the essence of a thing that which, when granted, the thing is necessarily posited, and by the annulling of which the thing is necessarily annulled; or that without which the thing can neither be nor be conceived, and, vice versa, that which cannot be or be conceived without the thing.

3. By idea I understand a conception of the Mind which the Mind forms because it is a thinking thing.

Explication I say “conception” rather than “perception” because the term perception seems to indicate that the Mind is passive to its object whereas conception seems to express an activity of the Mind.

4. By an adequate idea I mean an idea which, insofar as it is considered in itself without relation to its object, has all the properties, that is, intrinsic characteristics, of a true idea [*ideatum*].

Explication I say “intrinsic” so as to exclude the extrinsic characteristic—to wit the agreement of the idea with that of which it is an idea.

5. Duration is the indefinite continuance of existing.

Explication I say “indefinite” because it can in no wise be determined through the nature of the existing thing, nor again by the thing’s efficient cause which necessarily posits, but does not annul, the existence of the thing.

6. By reality and perfection I mean the same thing.

7. By individual things [*res singulares*] I mean things that are finite and have a determinate existence. If several individual things concur in one act in such a way as to be all together the simultaneous cause of one effect, I consider them all, in that respect, as one individual.

Axioms

1. The essence of man does not involve necessary existence; that is, from the order of Nature it is equally possible that a certain man exists or does not exist.

2. Man thinks.

3. Modes of thinking such as love, desire, or whatever emotions are designated by name, do not occur unless there is in the same individual the idea of the thing loved, desired, etc. But the idea can be without any other mode of thinking.

4. We feel a certain body to be affected in many ways.

5. We do not feel or perceive any individual things except bodies and modes of thinking. [N.B.: For Postulates, see after Proposition 13.]

PROPOSITION 1

Thought is an attribute of God; i.e., God is a thinking thing.

Proof Individual thoughts, or this and that thought, are modes expressing the nature of God in a definite and determinate way (Cor. Pr. 25, I). Therefore, there belongs to God (Def. 5, I) an attribute the conception of which is involved in all individual thoughts, and through which they are conceived. Thought, therefore, is one of God's infinite attributes, expressing the eternal and infinite essence of God (Def. 6, I); that is, God is a thinking thing.

Scholium This Proposition is also evident from the fact that we can conceive of an infinite thinking being. For the more things a thinking being can think, the more reality or perfection we conceive it to have. Therefore, a being that can think infinite things in infinite ways is by virtue of its thinking necessarily infinite. Since therefore by merely considering Thought we conceive an infinite being, Thought is necessarily one of the infinite attributes of God (Defs. 4 and 6, I), as we set out to prove.

PROPOSITION 2

Extension is an attribute of God; i.e., God is an extended thing.

Proof This Proposition is proved in the same way as the preceding proposition.

PROPOSITION 3

In God there is necessarily the idea both of his essence and of everything that necessarily follows from his essence.

Proof For God can (Pr. 1, II) think infinite things in infinite ways, or (what is the same thing, by Pr. 16, I) can form the idea of his own essence and of everything that necessarily follows from it. But all that is in God's power necessarily exists (Pr. 35, I). Therefore, such an idea necessarily exists, and only in God (Pr. 15, I).

Scholium By God's power the common people understand free will and God's right over all things that are, which things are therefore commonly considered as contingent. They say that God has power to destroy everything and bring it to nothing. Furthermore, they frequently compare God's power with that of kings. But this doctrine we have refuted in Cors. 1 and 2, Pr. 32, I; and in Pr. 16, I, we proved that God acts by the same necessity whereby he understands himself; that is, just as it follows from the necessity of the divine Nature (as is universally agreed) that God understands himself, by that same necessity it also follows that God acts infinitely in infinite ways. Again, we showed in Pr. 34, I that God's power is nothing but God's essence in action, and so it is as impossible for us to conceive that God does not act as that God does not exist. Furthermore if one wished to pursue the matter, I could easily show here that the power that common people assign to God is not only a human power (which shows that they conceive God as a man or like a man) but also involves negation of power. But I am reluctant to hold forth

so often on the same subject. I merely request the reader most earnestly to reflect again and again on what we said on this subject in Part I from Proposition 16 to the end. For nobody will rightly apprehend what I am trying to say unless he takes great care not to confuse God's power with a king's human power or right.

¹ [I e , their existence as ideas — M L.M]

PROPOSITION 7

The order and connection of ideas is the same as the order and connection of things.

Proof This is evident from Ax. 4, I; for the idea of what is caused depends on the knowledge of the cause of which it is the effect.

Corollary Hence it follows that God's power of thinking is on par with his power of acting. That is, whatever follows formally from the infinite nature of God, all this follows from the idea of God as an object of thought in God according to the same order and connection.

Scholium At this point, before proceeding further, we should recall to mind what I have demonstrated above—that whatever can be perceived by infinite intellect as constituting the essence of substance pertains entirely to the one sole substance. Consequently, thinking substance and extended substance are one and the same substance, comprehended now under this attribute, now under that. So, too, a mode of Extension and the idea of that mode are one and the same thing, expressed in two ways. This truth seems to have been glimpsed by some of the Hebrews,² who hold that God, God's intellect, and the things understood by God are one and the same. For example, a circle existing in Nature and the idea of the existing circle—which is also in God—are one and the same thing, explicated through different attributes. And so, whether we conceive Nature under the attribute of Extension or under the attribute of Thought or under any other attribute, we find one and the same order, or one and the same connection of causes—that is, the same things following one another. When I said that God is the cause, e.g., of the idea of a circle only insofar as he is a thinking thing, and of a circle only insofar as he is an extended thing, my reason was simply this, that the formal being of the idea of a circle can be perceived only through another mode of thinking as its proximate cause, and that mode through another, and so ad infinitum, with the result that as long as things are considered as modes of thought, we must explicate the order of the whole of Nature, or the connection of causes, through the attribute of Thought alone; and insofar as things are considered as modes of Extension, again the order of the whole of Nature must be explicated through the attribute of Extension only. The same applies to other attributes. Therefore God, insofar as he consists of infinite attributes, is in fact the cause of things as they are in themselves. For the present, I cannot give a clearer explanation.

² [The reference is most likely to Moses Maimonides, *The Guide of the Perplexed*, Part 1, Chapter 68 —S F]

PROPOSITION 17

If the human body is affected in a way [modo] that involves the nature of some external body, the human mind will regard that same external body as actually existing, or as present to itself, until the human body undergoes a further modification which excludes the existence or presence of the said body.

Proof This is evident; for as long as the human body is thus affected, so long will the human mind (Pr. 12, II) regard this affection of the body; that is (by the preceding Proposition), so long will it have the idea of a mode existing in actuality, an idea involving the nature of an external body; that is, an idea which does not exclude but posits the existence or presence of the nature of the external body. So the mind (Cor. 1 of the preceding proposition) will regard the external body as actually existing, or as present, until . . . etc.

Corollary The mind is able to regard as present external bodies by which the human body has been once affected, even if they do not exist and are not present.

Proof When external bodies so determine the fluid parts of the human body that these frequently impinge on the softer parts, they change the surfaces of these softer parts (Post. 5). Hence it comes about (Ax. 2 after Cor. Lemma 3) that the fluid parts are reflected therefrom in a manner different from what was previously the case; and thereafter, again coming into contact with the said changed surfaces in the course of their own spontaneous motion, they are reflected in the same way as when they were impelled toward those surfaces by external bodies. Consequently, in continuing this reflected motion they affect the human body in the same manner, which manner will again be the object of thought in the mind (Pr. 12, II); that is (Pr. 17, II), the mind will again regard the external body as present. This will be repeated whenever the fluid parts of the human body come into contact with those same surfaces in the course of their own spontaneous motion. Therefore, although the external bodies by which the human body has once been

affected may no longer exist, the mind will regard them as present whenever this activity of the body is repeated.

Scholium So we see how it comes about that we regard as present things which are not so, as often happens. Now it is possible that there are other causes for this fact, but it is enough for me at this point to have indicated one cause through which I can explicate the matter just as if I had demonstrated it through its true cause. Yet I do not think that I am far from the truth, since all the postulates that I have assumed contain scarcely anything inconsistent with experience; and after demonstrating that the human body exists just as we sense it (Cor. Pr. 13, II), we may not doubt experience.

In addition (preceding Cor. and Cor. 2 Pr. 16, II), this gives a clear understanding of the difference between the idea, e.g., of Peter which constitutes the essence of Peter's mind, and on the other hand the idea of Peter which is in another man, say Paul. The former directly explicates the essence of Peter's body, and does not involve existence except as long as Peter exists. The latter indicates the constitution of Paul's body rather than the nature of Peter; and so, while that constitution of Paul's body continues to be, Paul's mind will regard Peter as present to him although Peter may not be in existence. Further, to retain the usual terminology, we will assign the word "images" [*imagines*] to those affections of the human body the ideas of which set forth external bodies as if they were present to us, although they do not represent shapes. And when the mind regards bodies in this way, we shall say that it "imagines" [*imaginari*].

At this point, to begin my analysis of error, I should like you to note that the imaginations of the mind, looked at in themselves, contain no error; i.e., the mind does not err from the fact that it imagines, but only insofar as it is considered to lack the idea which excludes the existence of those things which it imagines to be present to itself. For if the mind, in imagining nonexistent things to be present to it, knew at the same time that those things did not exist in fact, it would surely impute this power of imagining not to the defect but to the strength of its own nature, especially if this faculty of imagining were to depend solely on its own nature; that is (Def. 7, I), if this faculty of imagining were free.

PROPOSITION 18

If the human body has once been affected by two or more bodies at the same time, when the mind afterward imagines one of them, it will straightway remember the others too.

Proof The mind imagines (preceding Cor.) any given body for the following reason, that the human body is affected and conditioned by the impressions of an external body in the same way as it was affected when certain of its parts were acted upon by the external body. But, by hypothesis, the human mind was at that time conditioned in such a way that the mind imagined two bodies at the same time. Therefore, it will now also imagine two bodies at the same time, and the mind, in imagining one of them, will straightway remember the other as well.

Scholium Hence we clearly understand what memory is. It is simply a linking of ideas involving the nature of things outside the human body, a linking which occurs in the mind parallel to the order and linking of the affections of the human body. I say, firstly, that it is only the linking of those ideas that involve the nature of things outside the human body, not of those ideas that explicate the nature of the said things. For they are in fact (Pr. 16, II) ideas of the affections of the human body which involve the nature both of the human body and of external bodies. Secondly, my purpose in saying that this linking occurs in accordance with the order and linking of the affections of the human body is to distinguish it from the linking of ideas in accordance with the order of the intellect whereby the mind perceives things through their first causes, and which is the same in all men.

Furthermore, from this we clearly understand why the mind, from thinking of one thing, should straightway pass on to thinking of another thing which has no likeness to the first. For example, from thinking of the word “pomum” [apple] a Roman will straightway fall to thinking of the fruit, which has no likeness to that articulated sound nor anything in common with it other than that the man’s body has often been affected by them both; that is, the man has often heard the word “pomum” while seeing the fruit. So everyone will pass on from one thought to another according as habit in each case has arranged the images in his body. A soldier, for example, seeing the tracks of a horse in the sand will straightway pass on from thinking of the horse to thinking of the rider, and then thinking of war, and so on. But a peasant, from thinking of a horse, will pass on to thinking of a plough, and of a field, and so on. So every person will pass on from thinking of one thing to thinking of another according as he is in the habit of joining together and linking the images of things in various ways.

PROPOSITION 26

The human mind does not perceive any external body as actually existing except through the ideas of affections of its own body.

Proof If the human body is not affected in any way by an external body, then (Pr. 7, II) neither is the idea of the human body—that is (Pr. 13, II), the human mind—affected in any way by the idea of the existence of that body; i.e., it does not in any way perceive the existence of that external body. But insofar as the human body is affected in some way by an external body, to that extent it perceives the external body (Pr. 16, II, with Cor. 1).

Corollary Insofar as the human mind imagines [*imaginatur*] an external body, to that extent it does not have an adequate knowledge of it.

Proof When the human mind regards external bodies through the ideas of affections of its own body, we say that it imagines [*imaginatur*] (see Sch. Pr. 17, II), and in no other way can the mind imagine external bodies as actually existing (preceding Pr.). Therefore, insofar as the mind imagines external bodies (Pr. 25, II), it does not have adequate knowledge of them.

PROPOSITION 27

The idea of any affection of the human body does not involve adequate knowledge of the human body.

Proof Any idea whatsoever of any affection of the human body involves the nature of the human body only to the extent that the human body is considered to be affected in some definite way (Pr. 16, II). But insofar as the human body is an individual thing that can be affected in many other ways, the idea . . . etc. (see Proof Pr. 25, II).

PROPOSITION 35

Falsity consists in the privation of knowledge which inadequate ideas, that is, frag-mentary and confused ideas, involve.

Proof There is nothing positive in ideas which constitutes the form [*forma*] of falsity (Pr. 33, II). But falsity cannot consist in absolute privation (for minds, not bodies, are said to err and be deceived), nor again in absolute ignorance, for to be ignorant and to err are different. Therefore, it consists in that privation of knowledge which inadequate knowledge, that is, inadequate and confused ideas, involves.

Scholium In Sch. Pr. 17, II I explained how error consists in the privation of knowledge, but I will give an example to enlarge on this explanation. Men are de-ceived in thinking themselves free, a belief that consists only in this, that they are conscious of their actions and ignorant of the causes by which they are deter-mined. Therefore, the idea of their freedom is simply the ignorance of the cause of their actions. As to their saying that human actions depend on the will, these are mere words without any corresponding idea. For none of them knows what the will is and how it moves the body, and those who boast otherwise and make up stories of dwelling places and habitations of the soul provoke either ridicule or disgust.

As another example, when we gaze at the sun, we see it as some two hundred feet distant from us. The error does not consist in simply seeing the sun in this way but in the fact that while we do so we are not aware of the true distance and the cause of our seeing it so. For although we may later become aware that the sun is more than six hundred times the diameter of the earth distant from us, we shall nevertheless continue to see it as close at hand. For it is not our ignorance of its true distance that causes us to see the sun to be so near; it is that the affec-tion of our body involves the essence of the sun only to the extent that the body is affected by it.

PROPOSITION 36

Inadequate and confused ideas follow by the same necessity as adequate, or clear and distinct, ideas.

Proof All ideas are in God (Pr. 15, I), and insofar as they are related to God, they are true (Pr. 32, II) and adequate (Cor. Pr. 7, II). So there are no inadequate or confused ideas except insofar as they are related to the particular mind of some-one (see Prs. 24 and 28, II). So all ideas, both adequate and inadequate, follow by the same necessity (Cor. Pr. 6, II).

PROPOSITION 37

That which is common to all things (see Lemma 2 above) and is equally in the part as in the whole does not constitute the essence of any one particular thing.

Proof If this is denied, conceive, if possible, that it does constitute the essence of one particular thing, B. Therefore, it can neither be nor be conceived without B (Def. 2, II). But this is contrary to our hypothesis. Therefore, it does not pertain to B's essence, nor does it constitute the essence of any other particular thing.

PROPOSITION 38

Those things that are common to all things and are equally in the part as in the whole can be conceived only adequately.

Proof Let A be something common to all bodies, and equally in the part of any body as in the whole. I say that A can be conceived only adequately. For its idea (Cor. Pr. 7, II) will necessarily be in God both insofar as he has the idea of the human body and insofar as he has the ideas of affections of the human body, affections which partly involve the natures of both the human body and external bodies (Prs. 16, 25, and 27, II). That is (Prs. 12 and 13, II), this idea will necessarily be adequate in God insofar as he constitutes the human mind; that is, insofar as he has the ideas which are in the human mind. Therefore, the mind (Cor. Pr. 11, II) necessarily perceives A adequately, and does so both insofar as it perceives itself and insofar as it perceives its own body or any external body; nor can A be perceived in any other way.

Corollary Hence it follows that there are certain ideas or notions common to all men. For (by Lemma 2) all bodies agree in certain respects, which must be (preceding Pr.) conceived by all adequately, or clearly and distinctly.

PROPOSITION 39

Of that which is common and proper to the human body and to any external bodies by which the human body is customarily affected, and which is equally in the part as well as in the whole of any of these bodies, the idea also in the mind will be adequate.

Proof Let A be that which is common and proper to the human body and to any external bodies and which is equally in the human body as in those same external bodies, and which is finally equally in the part of any external body as in the whole. There will be in God an adequate idea of A (Cor. Pr. 7, II) both insofar as he has the idea of the human body and insofar as he has ideas of those posited external bodies. Let it now be supposed that the human body is affected by an external body through that which is common to them both, that is, A. The idea of this affection will involve the property A (Pr. 16, II), and so (Cor. Pr. 7, II) the idea of this affection, insofar as it involves the property A, will be adequate in God insofar as he is affected by the idea of the human body; that is (Pr. 13, II), insofar as he constitutes the nature of the human mind. So this idea will also be adequate in the human mind (Cor. Pr. 11, II).

Corollary Hence it follows that the mind is more capable of perceiving more things adequately in proportion as its body has more things in common with other bodies.

PROPOSITION 40

Whatever ideas follow in the mind from ideas that are adequate in it are also adequate.

Proof This is evident. For when we say that an idea follows in the human mind from ideas that are adequate in it, we are saying no more than that there is in the divine intellect an idea of which God is the cause, not insofar as he is infinite nor insofar as he is affected by ideas of numerous particular things, but only insofar as he constitutes the essence of the human mind.

Scholium 1 I have here set forth the causes of those notions that are called “common,” and which are the basis of our reasoning processes. Now certain axioms or notions have other causes which it would be relevant to set forth by this method of ours; for thus we could establish which notions are useful compared with others, and which are of scarcely any value. And again, we could establish which notions are common to all, which ones are clear and distinct only to those not laboring under prejudices [*praejudiciis*] and which ones are ill-founded. Furthermore, this would clarify the origin of those notions called “secondary”—and consequently the axioms which are based on them—as well as other related questions to which I have for some time given thought. But I have decided not to embark on these questions at this point because I have set them aside for another treatise,³ and also to avoid wearying the reader with too lengthy a discussion of this subject. Nevertheless, to omit nothing that it is essential to know, I shall briefly deal with the question of the origin of the so-called “transcendental terms,” such as “entity,” “thing,” “something” [*ens, res, aliquid*].

These terms originate in the following way. The human body, being limited, is capable of forming simultaneously in itself only a certain number of distinct images. (I have explained in Sch. Pr. 17, II what an image is.) If this number be exceeded, these images begin to be confused, and if the number of distinct images which the body is capable of forming simultaneously in itself be far exceeded, all the images will be utterly confused with one another. This being so, it is evident from Cor. Pr. 17 and Pr. 18, II that the human mind is able to imagine simultaneously and distinctly as many bodies as there are images that can be formed simultaneously in its body. But when the images in the body are utterly confused, the mind will also imagine all the bodies confusedly without any distinction, and will comprehend them, as it were, under one attribute, namely, that of entity, thing, etc. This conclusion can also be reached from the fact that images are not always equally vivid, and also from other causes analogous to these, which I need not here explicate. For it all comes down to this, that these terms signify ideas confused in the highest degree.

³ [This is Spinoza's incomplete essay, *On the Improvement of the Understanding*]

Again, from similar causes have arisen those notions called “universal,” such as “man,” “horse,” “dog,” etc.; that is to say, so many images are formed in the human body simultaneously (e.g., of man) that our capacity to imagine them is surpassed, not indeed completely, but to the extent that the mind is unable to imagine the unimportant differences of individuals (such as the complexion and stature of each, and their exact number) and imagines distinctly only their common characteristic insofar as the body is affected by them. For it was by this that the body was affected most repeatedly, by each single individual. The mind expresses this by the word “man,” and predicates this word of an infinite number of individuals. For, as we said, it is unable to imagine the determinate number of individuals.

But it should be noted that not all men form these notions in the same way; in the case of each person the notions vary according as that thing varies whereby the body has more frequently been affected, and which the mind more readily imagines or calls to mind. For example, those who have more often regarded with admiration the stature of men will understand by the word “man” an animal of upright stature, while those who are wont to regard a different aspect will form a different common image of man, such as that man is a laughing animal, a featherless biped, or a rational animal. Similarly, with regard to other aspects, each will form universal images according to the conditioning of his body. Therefore, it is not surprising that so many controversies have arisen among philosophers who have sought to explain natural phenomena through merely the images of these phenomena.

Scholium 2 From all that has already been said it is quite clear that we perceive many things and form universal notions:

1. From individual objects presented to us through the senses in a fragmentary [*mutilate*] and confused manner without any intellectual order (see Cor. Pr. 29, II); and therefore I call such perceptions “knowledge from casual experience.”
2. From symbols. For example, from having heard or read certain words we call things to mind and we form certain ideas of them similar to those through which we imagine things (Sch. Pr. 18, II).

Both these ways of regarding things I shall in future refer to as “knowledge of the first kind,” “opinion,” or “imagination.”

3. From the fact that we have common notions and adequate ideas of the properties of things (see Cor. Pr. 38 and 39 with its Cor., and Pr. 40, II). I shall refer to this as “reason” and “knowledge of the second kind.”

Apart from these two kinds of knowledge there is, as I shall later show, a third kind of knowledge, which I shall refer to as “intuition.” This kind of knowledge proceeds from an adequate idea of the formal essence of certain attributes of God to an adequate knowledge of the essence of things. I shall illustrate all these kinds of knowledge by one single example. Three numbers are given; it is required to find a fourth which is related to the third as the second to the first. Tradesmen have no hesitation in multiplying the second by the third and dividing the product by the first, either because they have not yet forgotten the rule they learned

without proof from their teachers, or because they have in fact found this correct in the case of very simple numbers, or else from the force of the proof of Proposition 19 of the Seventh Book of Euclid, to wit, the common property of proportionals. But in the case of very simple numbers, none of this is necessary. For example, in the case of the given numbers 1, 2, 3, everybody can see that the fourth proportional is 6, and all the more clearly because we infer in one single intuition the fourth number from the ratio we see the first number bears to the second.