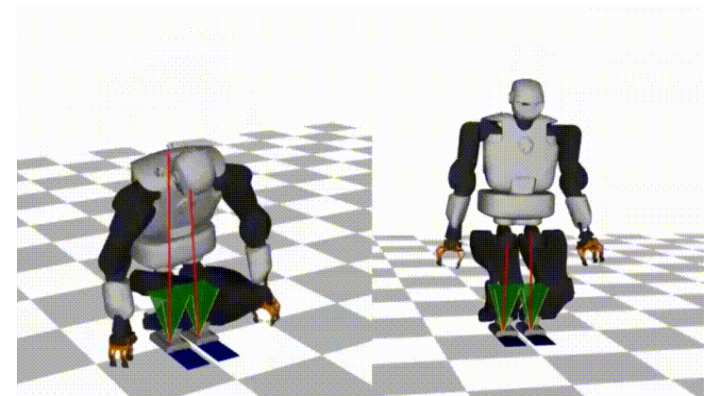


# Highly-Dynamic Movements of a Humanoid Robot Using Whole-Body Trajectory Optimization

Master Thesis Presentation  
Julian Eßer

Time: Apr – Sep 2020  
Supervisors: Prof. Kirchner,  
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Mentors: Dr. Kumar,  
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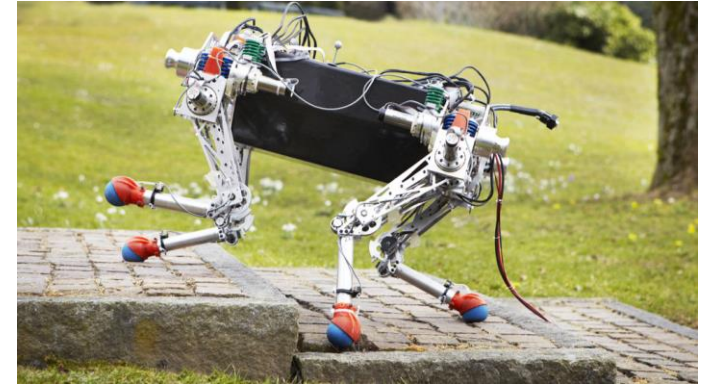
<https://github.com/loco-3d/crocodyl>



1. Introduction
2. Contact Stability Constrained DDP
3. Application to a Humanoid Robot
  - Bipedal Walking Variants
  - Highly-Dynamic Movements
4. Validation of the Planned Motions
  - Physics Simulation
  - Real-World Experiments
5. Conclusion and Outlook

- Why Legged Robots? <sup>[1]</sup>

- Improved mobility
- Step over obstacles
- Adapt to environment



<https://rsl.ethz.ch/robots-media/starlet/pictures.html>

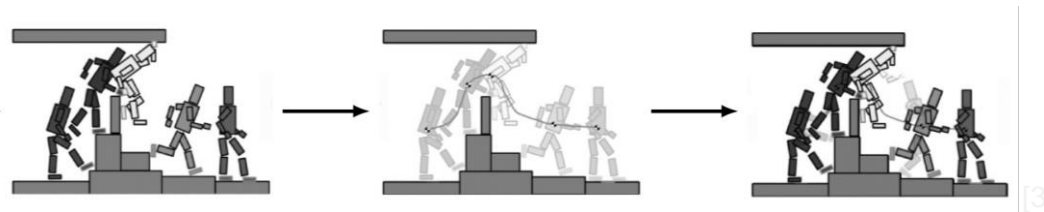
- Why Humanoid Robots? <sup>[2]</sup>

- Inspired by human capabilities
- Human-tailored environments
- Intuitive collaboration



<https://robotik.dfki-bremen.de/en/research/projects/transfit.html>

- **Characteristic:** Decoupled Base and Multibody Dynamics
- Reasons for **Complexity**
  - High-dimensional systems
  - Trivial underactuation
  - Nonlinear, hybrid dynamics
- **Decomposition** of Motion Planning into Subproblems

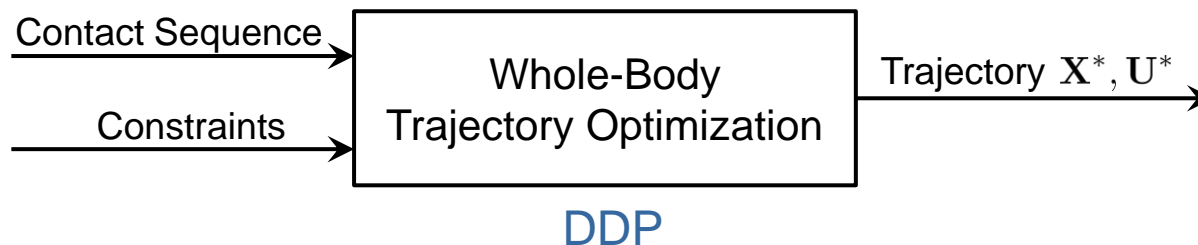


- **Optimization-Based Planning** for Efficient Motions



- **Goal:** Compute Optimal Trajectories
  - Minimize a given cost function
  - Satisfy a set of constraints
- **Classes of TO Algorithms**
  - Direct methods (e.g. SQP): constrained but slow
  - Indirect methods (e.g DDP): fast but unconstrained
- **Usage** for Legged Locomotion Planning
  - TO based on reduced centroidal dynamics
  - Whole-body TO for efficient motions

- **Goal:** Generate Balanced and Efficient Motion Plans
- **Approach:** DDP-Based Whole-Body TO



- **Contributions of this Thesis:**
  - **C1:** Contact Stability Constrained DDP
  - **C2:** Experimental pipeline for whole-body TO
  - **C3:** Physical limitations of RH5 humanoid

- Involved Frameworks



<https://github.com/loco-3d/crocodyl>

(Optimal Control)



<https://github.com/stack-of-tasks/pinocchio>

(Robot Dynamics)



<https://robotik.dfki-bremen.de/en/research/softwaretools/hyrodyn/>

(Parallel Mechanisms)

- RH5 Humanoid Robot

- Biologically inspired (200 cm, 32 DoFs)
- Lightweight (62 kg)
- Series-parallel hybrid robot [4]
- Tree-type robot model



(Quelle: DFKI GmbH)



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- Finite Horizon Optimal Control Problem [5]

$$\mathbf{X}^*, \mathbf{U}^* = \arg \min_{\mathbf{X}, \mathbf{U}} l_N(x_N) + \sum_{k=0}^{N-1} \int_{t_k}^{t_k + \Delta t} l_k(\mathbf{x}, \mathbf{u}) dt$$

- Cost at One Knot of the OC Problem

$$l_k = \sum_{c=1}^C \alpha_c \Phi_c(\mathbf{q}, \dot{\mathbf{q}}, \boldsymbol{\tau})$$

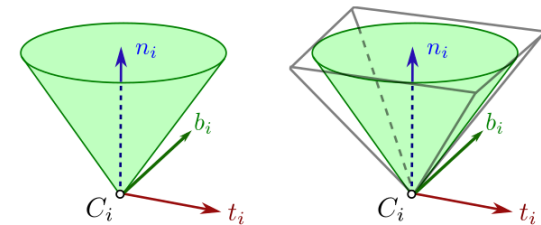
- Multi-Contact Dynamics as Holonomic Constraints [6]

$$\begin{bmatrix} \dot{\mathbf{v}} \\ -\boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{M} & \mathbf{J}_c^\top \\ \mathbf{J}_c & \mathbf{0} \end{bmatrix}^{-1} \begin{bmatrix} \boldsymbol{\tau}_b \\ -\mathbf{a}_0 \end{bmatrix}$$

$$\mathbf{J}_c = \frac{\delta \phi}{\delta \mathbf{q}}, \quad \phi(\mathbf{q}) = 0$$

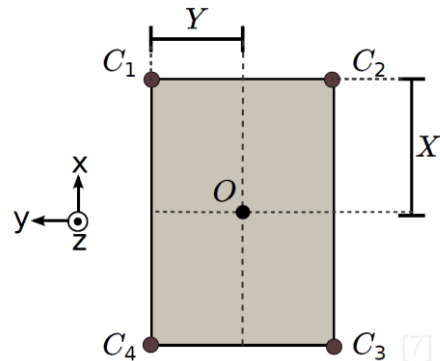
- **Idea:** Constrain Contact Stability for Each Contact
- **Conditions** for Contact Stability
  - I. Unilaterality of the forces
  - II. Forces inside friction cone
  - III. Center of Pressure (CoP) inside contact area
- **Constraints** for Unilaterality (I) and Friction Cone (II)

$$\begin{aligned} f_i^z &> 0 \\ |f_i^x| &\leq \mu f_i^z \\ |f_i^y| &\leq \mu f_i^z \end{aligned}$$



<https://scaron.info/teaching/friction-cones.html>

- CoP Stability Conditions



$$\begin{aligned} -X &\leq C_x \leq X \\ -Y &\leq C_y \leq Y \end{aligned}$$

- CoP Computation

$$p_{CoP} = \frac{n \times \tau_O^c}{f^c \cdot n}$$

- Constraints for CoP (III)

$$\underbrace{\begin{bmatrix} Xn_0 & Xn_1 & Xn_2 & 0 & -n_2 & n_1 \\ Xn_0 & Xn_1 & Xn_2 & 0 & n_2 & -n_1 \\ Yn_0 & Yn_1 & Yn_2 & n_2 & 0 & -n_0 \\ Yn_0 & Yn_1 & Yn_2 & -n_2 & 0 & n_0 \end{bmatrix}}_A \underbrace{\begin{bmatrix} f^x \\ f^y \\ f^z \\ \tau^x \\ \tau^y \\ \tau^z \end{bmatrix}}_W \geq \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$



- **Goal:** Integrate CoP Constraints into a Novel Cost Function
- **Residual** and **Cost** Computation

$$\mathbf{r} = \mathbf{A}\mathbf{w} \geq \mathbf{0}$$

$$\Phi_{CoP} = \begin{cases} \frac{1}{2} \mathbf{r}^T \mathbf{r} & | \text{lb} > \mathbf{r} > \text{ub} & \text{(Outside contact area)} \\ 0 & | \text{lb} \leq \mathbf{r} \leq \text{ub} & \text{(Inside contact area)} \end{cases}$$

$$\text{lb} = \mathbf{0}, \text{ub} = \infty$$

- Analytical Computation of **Derivatives**



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- Robot **Tasks**

$$\Phi_{\text{foot}} = || \mathbf{f}(t) - \mathbf{f}^{\text{ref}}(t) ||_2^2$$

$$\Phi_{\text{CoM}} = || \mathbf{c}(t) - \mathbf{c}^{\text{ref}}(t) ||_2^2$$

- Inequality Constraints for **Physical Consistency**

$$l_k = \sum_{c=1}^C \alpha_c \Phi_c(\mathbf{q}, \dot{\mathbf{q}}, \boldsymbol{\tau})$$

$$\Phi_{\text{CoP}}, \Phi_{\text{friction}}, \Phi_{\text{joints}} = \begin{cases} \frac{1}{2} \mathbf{r}^T \mathbf{r} & | \text{lb} > \mathbf{r} > \text{ub} \\ 0 & | \text{lb} \leq \mathbf{r} \leq \text{ub} \end{cases}$$

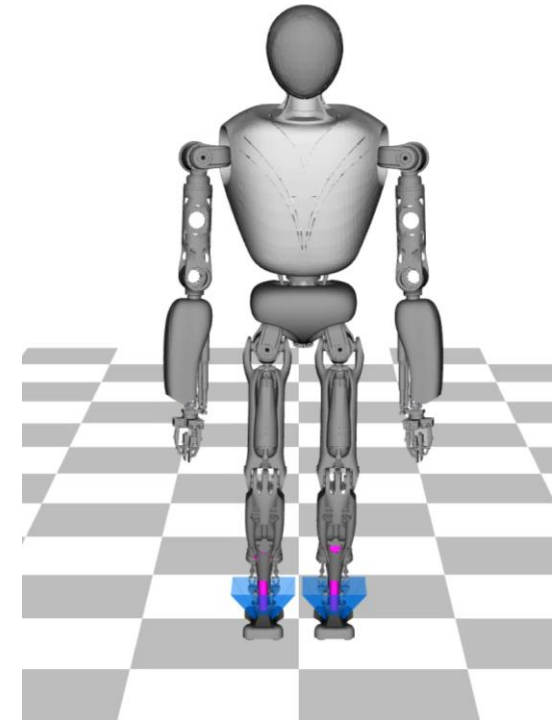
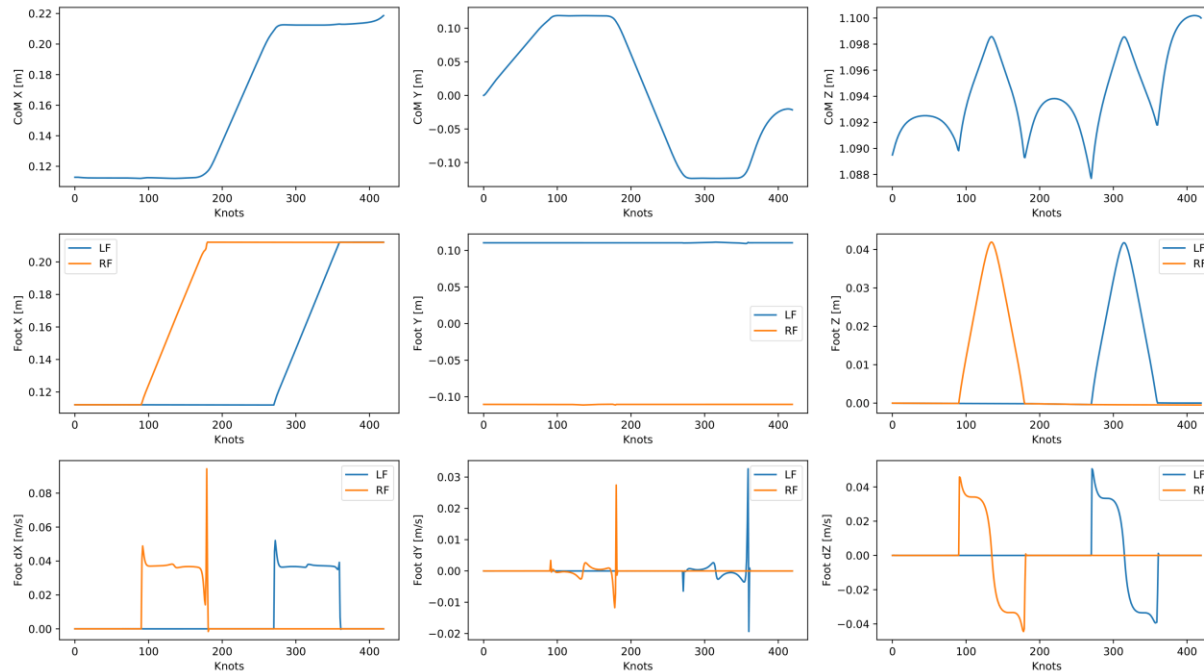
- Further **Regularization** Terms

$$\Phi_{\text{torque}} = || \boldsymbol{\tau}(t) ||_2^2$$

$$\Phi_{\text{posture}} = || \mathbf{q}(t) - \mathbf{q}^{\text{ref}}(t) ||_2^2$$

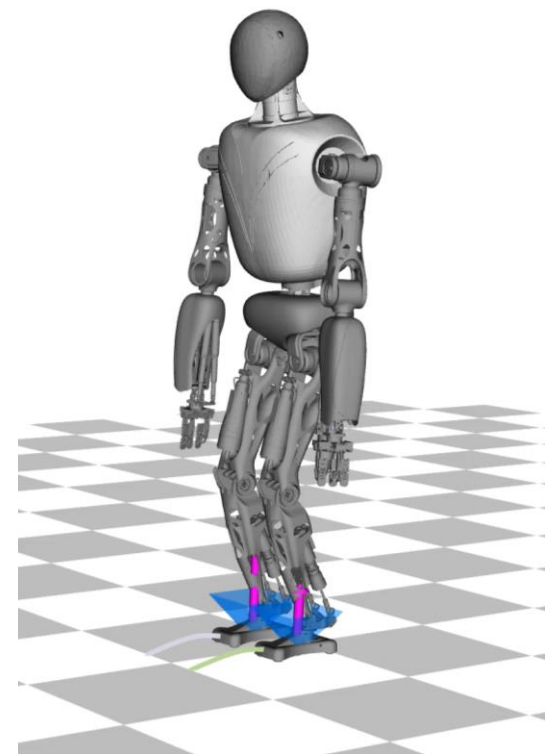
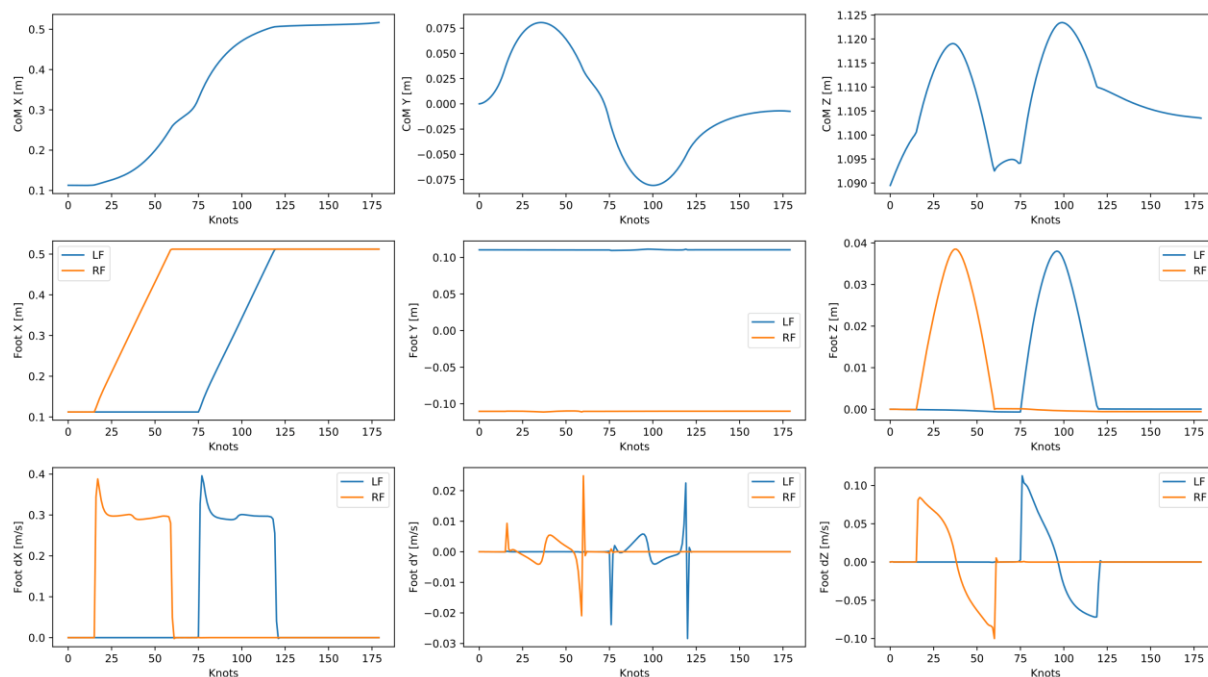
Gait Characteristics		Optimization Constraints	
Step length:	10 cm	Tasks:	$\Phi_{\text{foot}}, \Phi_{\text{CoM}}$
Step height:	5 cm	Stability:	$\Phi_{\text{friction}}$
Time:	12 s	Limits:	$\Phi_{\text{joint}}, \text{torques}$
Step size:	0.03 s	Regularization:	$\Phi_{\text{posture}}, \Phi_{\text{torque}}$

- Static Stability Criterion



Gait Characteristics		Optimization Constraints	
Step length:	40 cm	Tasks:	$\Phi_{\text{foot}}$
Step height:	5 cm	Stability:	$\Phi_{\text{CoP}}, \Phi_{\text{friction}}$
Time:	2 s/step	Limits:	$\Phi_{\text{joint}}, \text{torques}$
Step size:	0.03 s	Regularization:	$\Phi_{\text{posture}}, \Phi_{\text{torque}}$

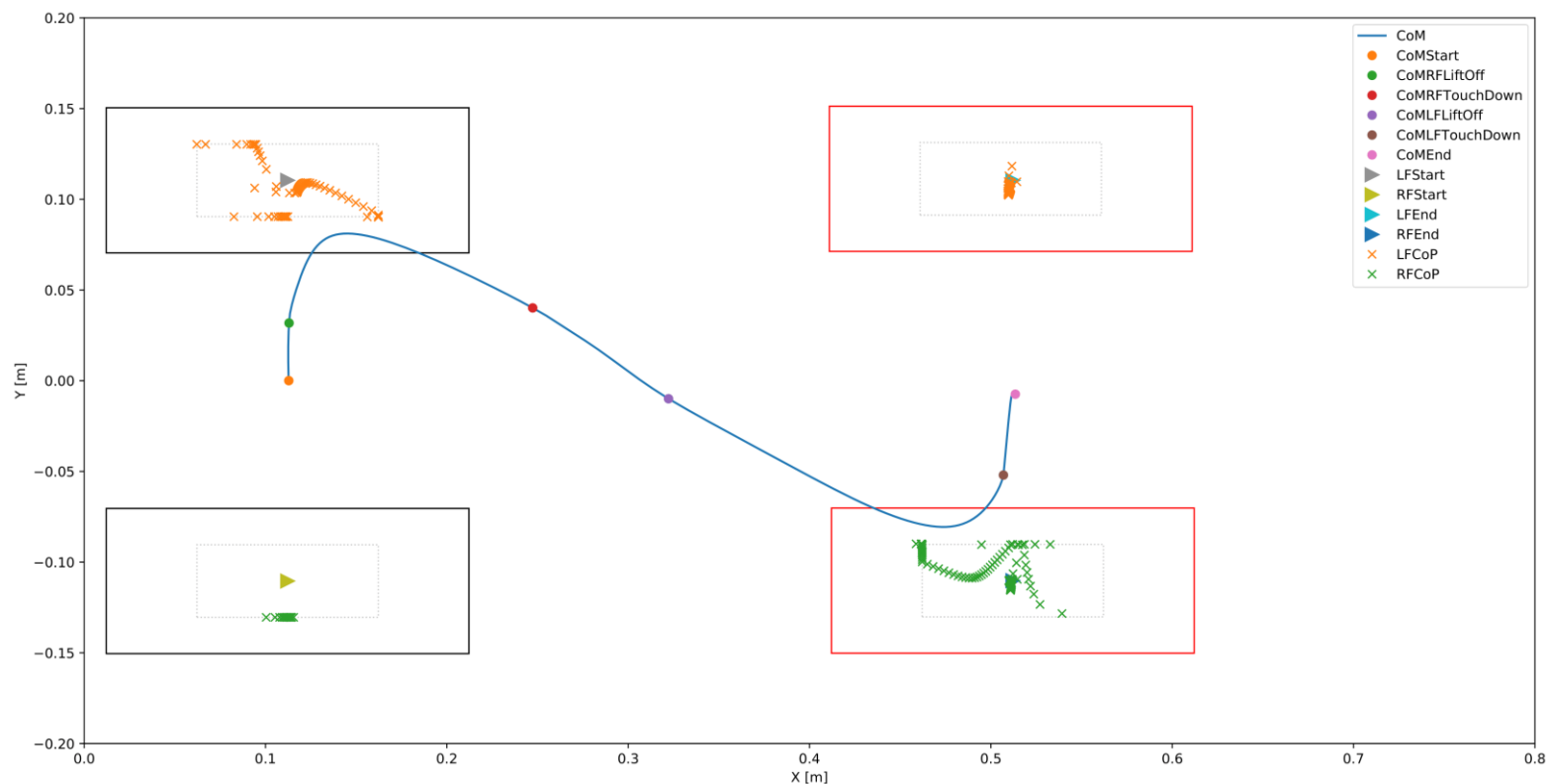
- Contact Stability Constrained DDP





- Stability Analysis for Dynamic Walking

(500% F<sub>500</sub> Area)



- Finding:** Proposed Approach Yields Balanced Motions



- **Flight Phases:** Physical Consistency of Contact Timings
  - Falling time given by physics
  - Derive other timings
- **Numerical Drift** in the Holonomic Constraints
  - Baumgarte stabilization
  - Reduce integration step size
- **Multi-Phase** Optimal Control Problem

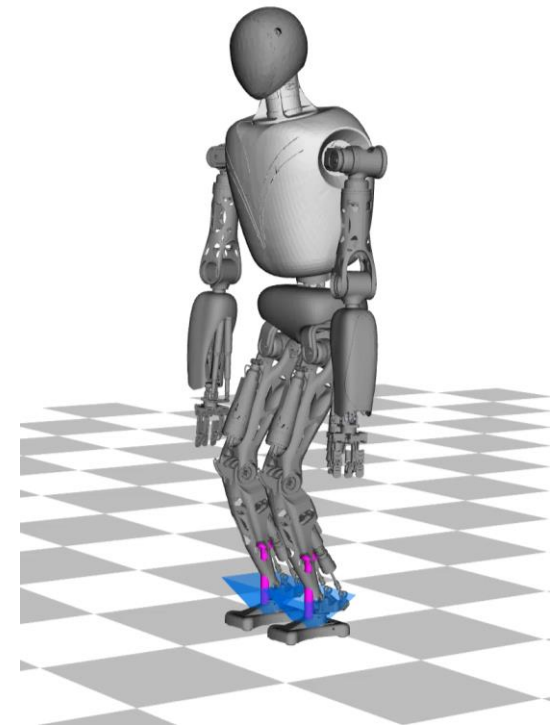
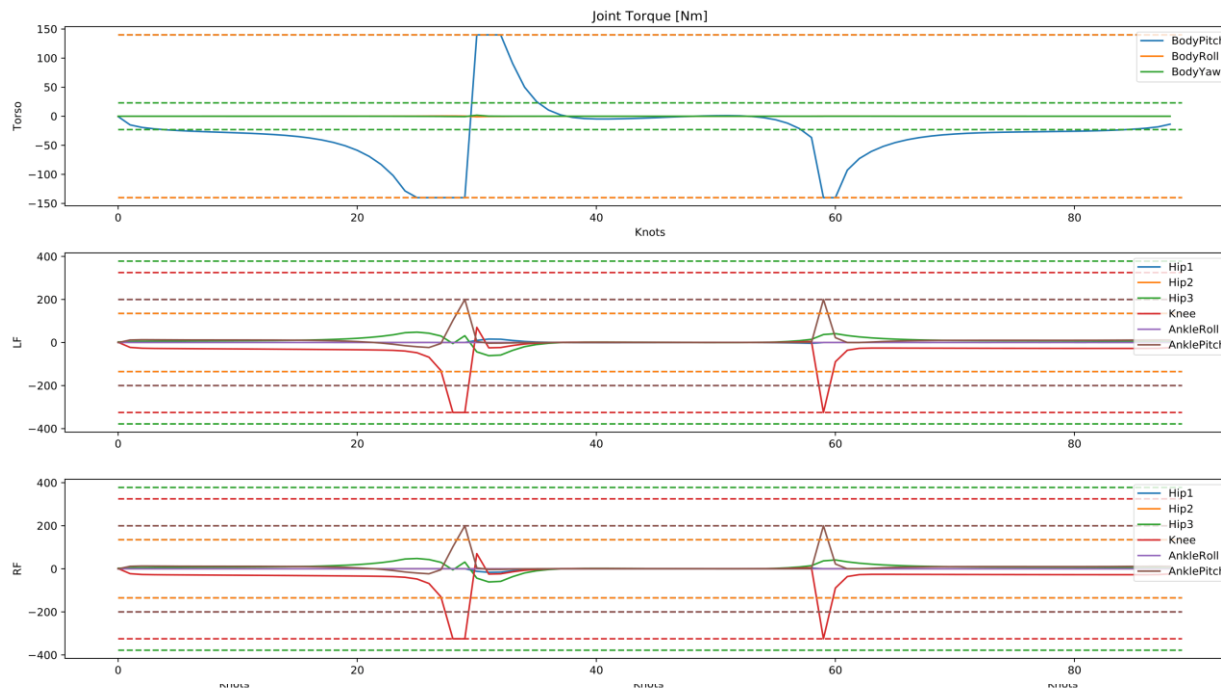
$$\mathbf{X}^*, \mathbf{U}^* = \arg \min_{\mathbf{X}, \mathbf{U}} \sum_{p=0}^P \sum_{k=0}^N \int_{t_k}^{t_k + \Delta t} l_p(\mathbf{x}, \mathbf{u}) dt$$

# RESULTS: VERTICAL JUMP



Jump Characteristics		Optimization Constraints	
Jump length:	0 cm	Tasks:	$\Phi_{\text{foot}}$
Jump height:	10 cm	Stability:	$\Phi_{\text{CoP}}$ , $\Phi_{\text{friction}}$
Total time:	0.9 s	Limits:	Torques
Step size:	0.01 s	Regularization:	$\Phi_{\text{posture}}$ , $\Phi_{\text{torque}}$

- Analysis of System Limits

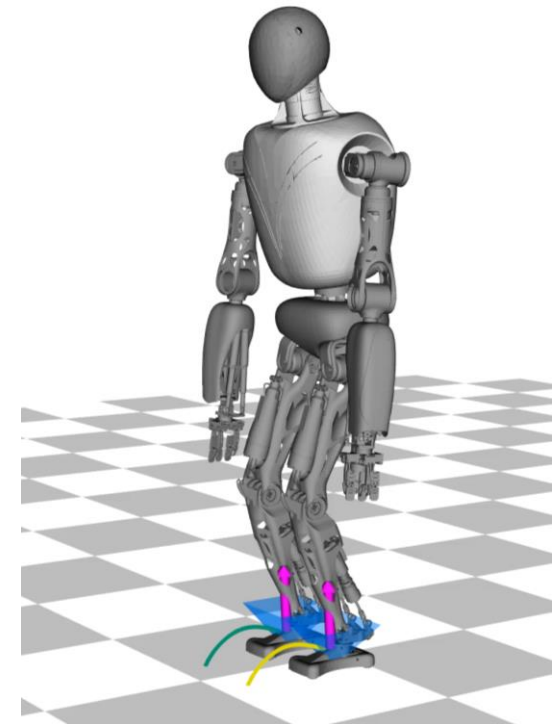
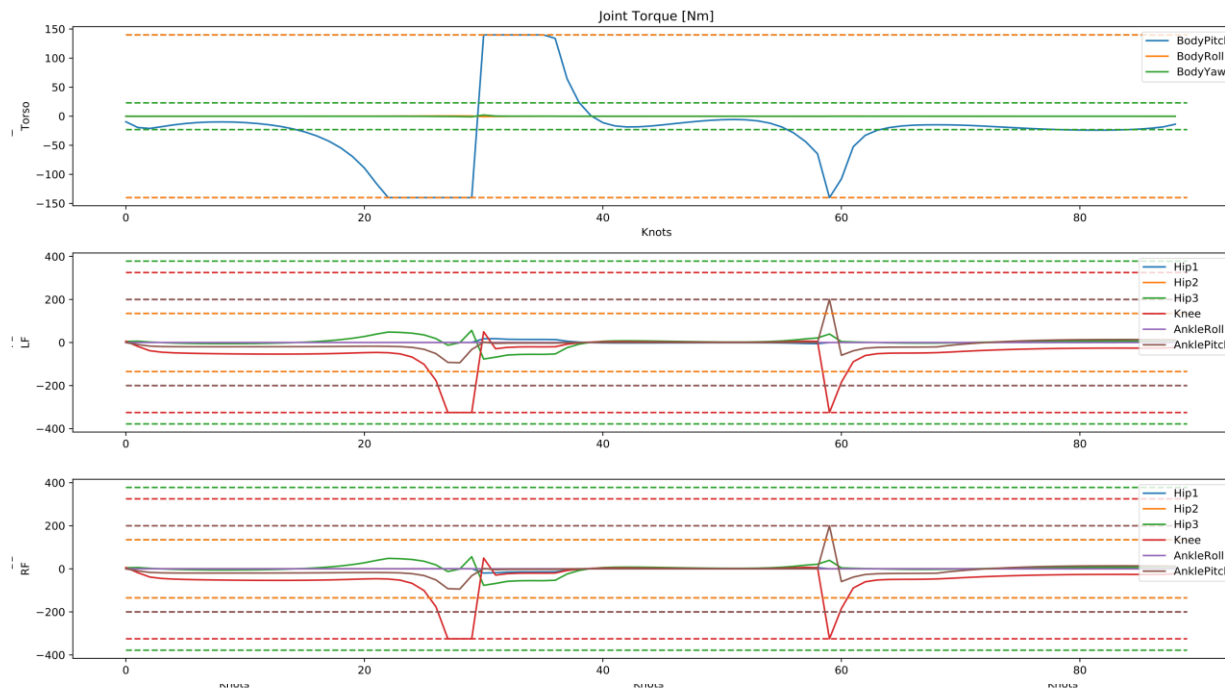


# RESULTS: FORWARD JUMP



Jump Characteristics		Optimization Constraints	
Jump length:	30 cm	Tasks:	$\Phi_{\text{foot}}$
Jump height:	10 cm	Stability:	$\Phi_{\text{CoP}}$ , $\Phi_{\text{friction}}$
Total time:	0.9 s	Limits:	Torques
Step size:	0.01 s	Regularization:	$\Phi_{\text{posture}}$ , $\Phi_{\text{torque}}$

- Analysis of System Limits

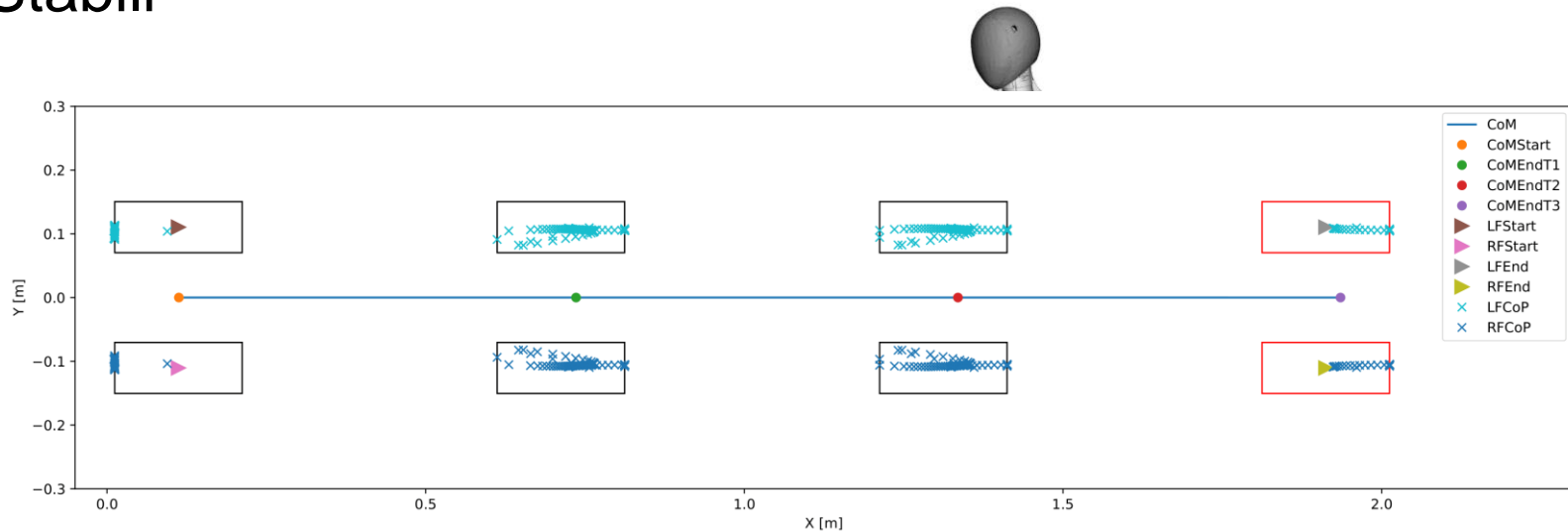


# RESULTS: MULTIPLE OBSTACLES JUMPS

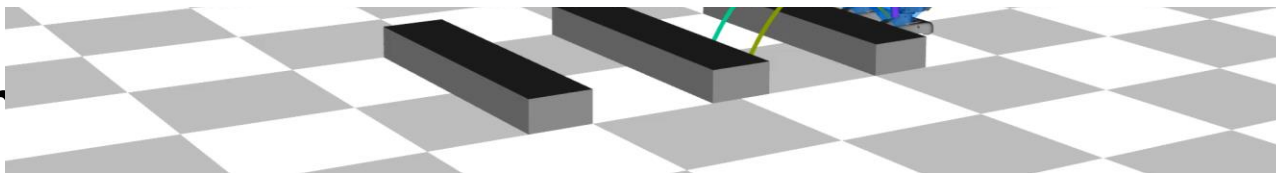


Jump Characteristics	Optimization Constraints
Jump length: 60 cm	Tasks: $\Phi_{\text{foot}}$
Jump height: 25 cm	Stability: $\Phi_{\text{CoP}}$ , $\Phi_{\text{friction}}$
Total time: 0.9 s / jump	Limits: -
Step size: 0.01 s	Regularization: $\Phi_{\text{posture}}$ , $\Phi_{\text{torque}}$

## • Stability



## • Finding





- Case Studies of Increasing Complexity

	Position Limits	Torque Limits	Velocity Limits
Vertical Jump ( $l = 0$ cm)			
$h = 1$ cm	✓	✓	✓
$h = 5$ cm	✓	✓	✗ <sub>3</sub>
$h = 10$ cm	✓	(✓)	✗ <sub>3</sub>
$h = 20$ cm	✓	(✓)	✗ <sub>5</sub>
$h = 30$ cm	✓	(✓)	✗ <sub>7</sub>
Forward Jump ( $h = 10$ cm)			
$l = 10$ cm	✓	(✓)	✗ <sub>7</sub>
$l = 20$ cm	✓	(✓)	✗ <sub>7</sub>
$l = 30$ cm	✓	(✓)	✗ <sub>7</sub>
$l = 40$ cm	✓	(✓)	✗ <sub>7</sub>
$l = 50$ cm	✓	(✓)	✗ <sub>7</sub>
Obstacle Jump ( $h = 25$ cm)			
$l = 60$ cm	✓	✗ <sub>5</sub>	✗ <sub>7</sub>

- Critical Joint Velocities: Body, Knee, Hip, Shoulder
- Result:** Guidelines for Next Design Iteration



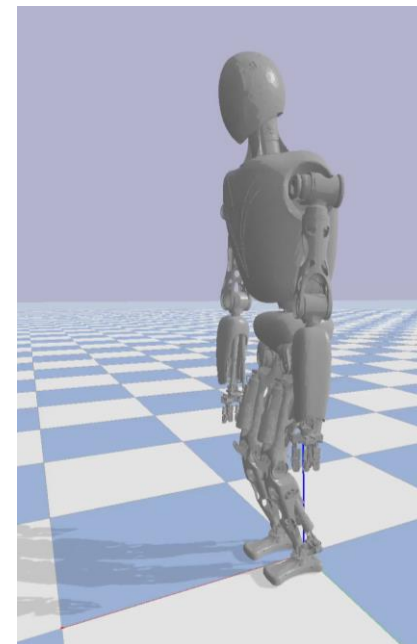
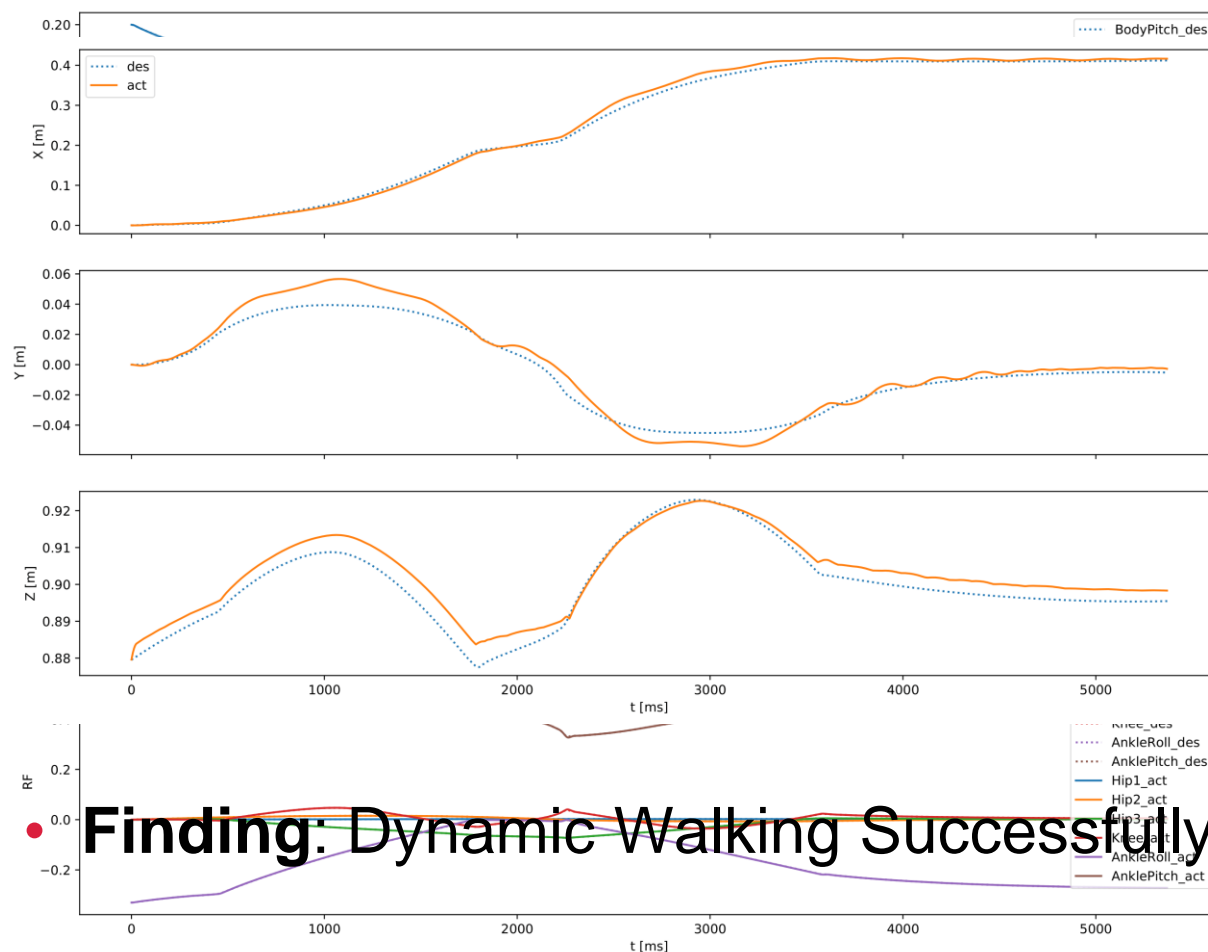
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- **Goal:** Online Stabilization of Planned Motions
- Real-Time Physics Simulation: **PyBullet**
  - Rigid contact model
  - Collision detection
- **Pipeline** Comparable to Real Robot
  - Trajectory from file
  - Cubic spline interpolation (1 kHz)
- **Control** Architecture
  - Joint space level
  - PD position/velocity control



## Resulting Performance Metrics (Union Speed)

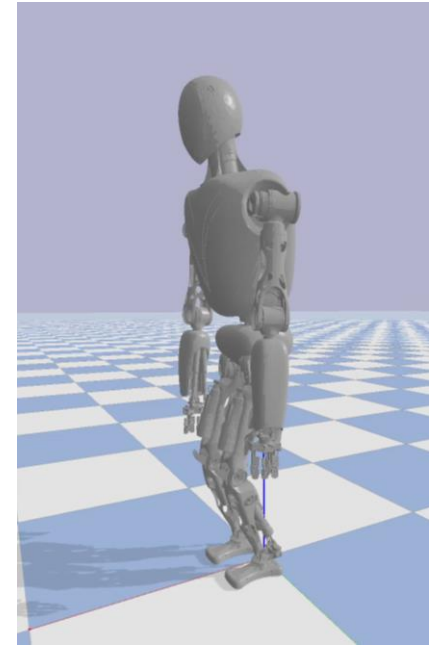
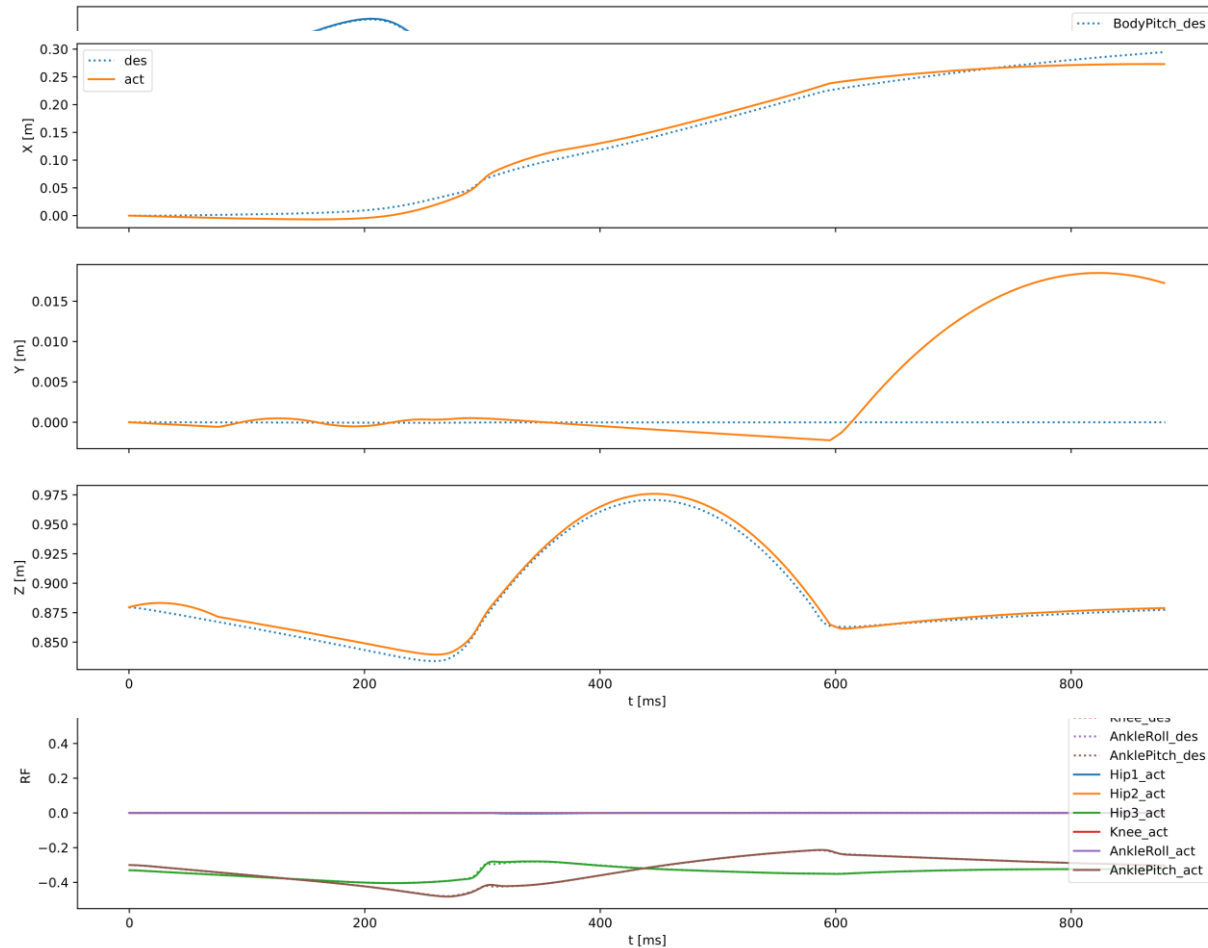


**Finding: Dynamic Walking Successfully Stabilized**

# RESULTS: FORWARD JUMPING

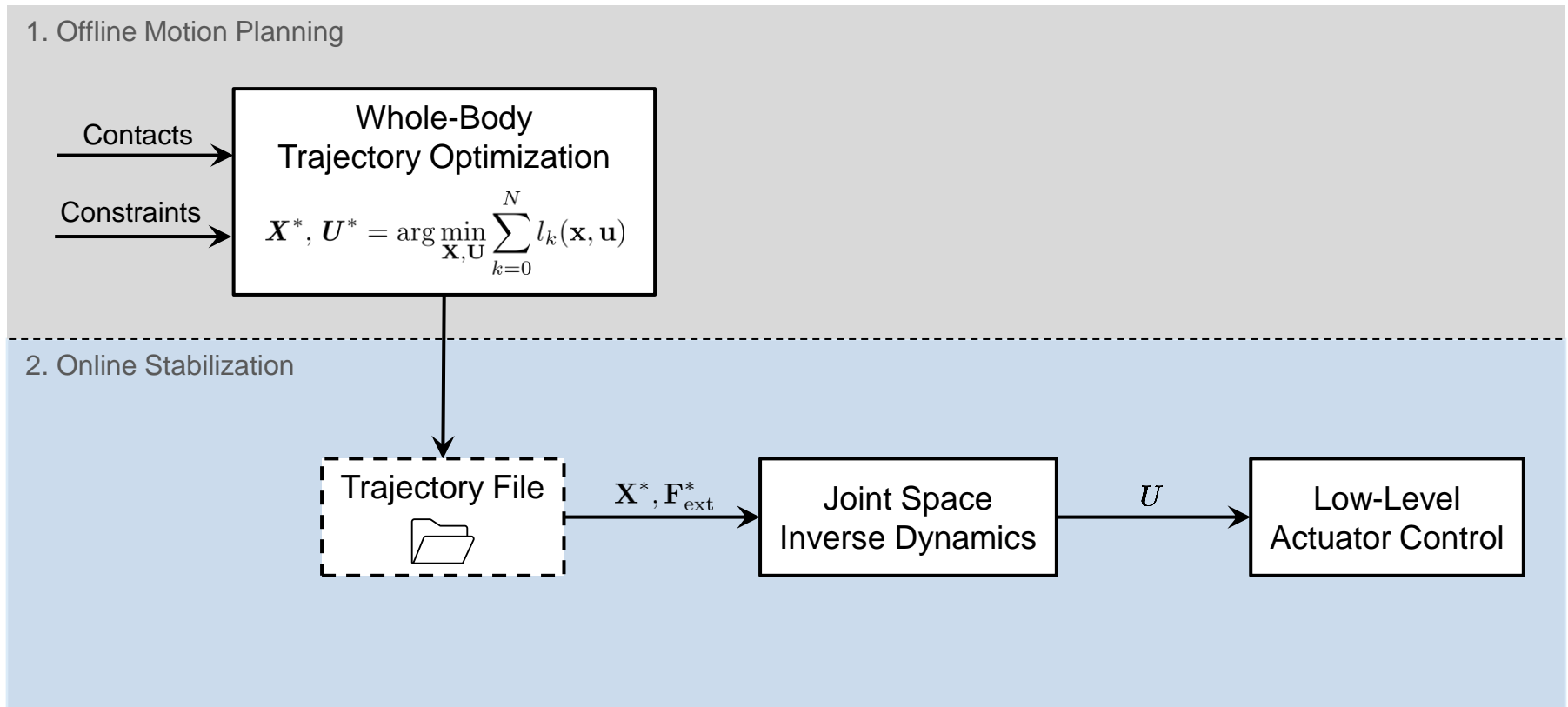


## Resulting Performance Metrics (Union Split)



Stabilized

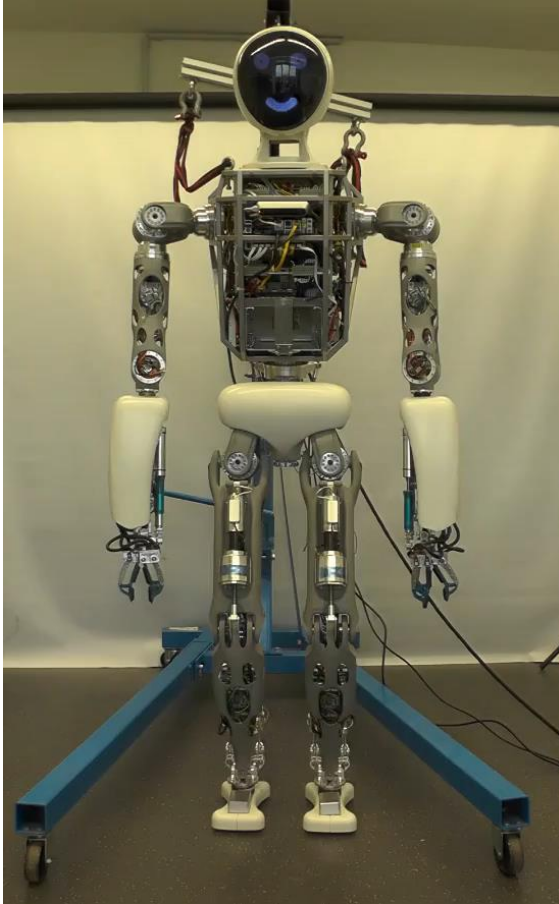
- **Goal:** Online Stabilization of Planned Motions



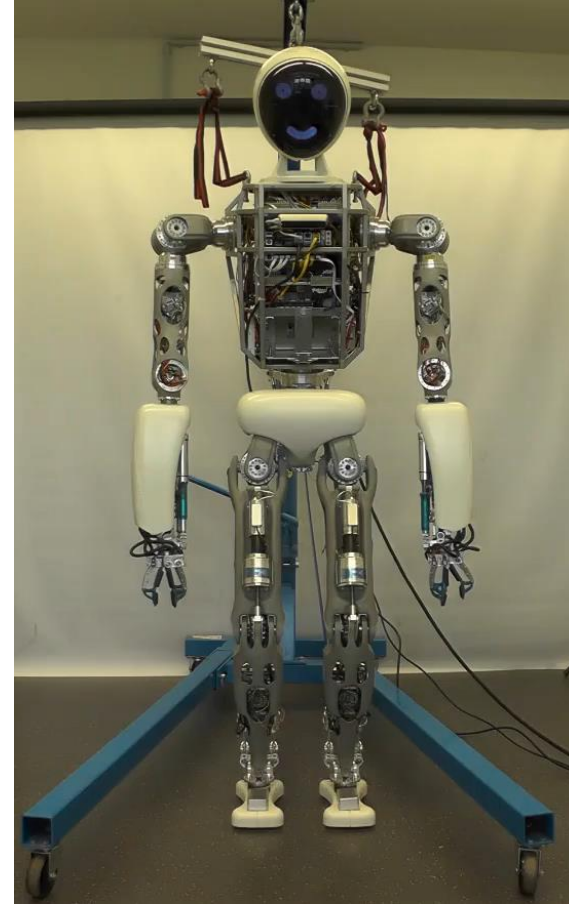
- Four Experiments of Increasing Difficulty

	I	II	III	IV
	Balancing	Static Walk	Fast Squats	Dynamic Walk
Surface contacts	✓	✓	✓	✓
Base motion	✓	✓	✓	✓
Swing foot motion	✓	✓	✗	✓
Step sequence	✗	✓	✗	✓
Impacts	✗	✓	✗	✓
Dynamic forces	✗	✗	✓	✓
Flight-phases	✗	✗	✗	✗
Success	✓	(✓)	✓	✗

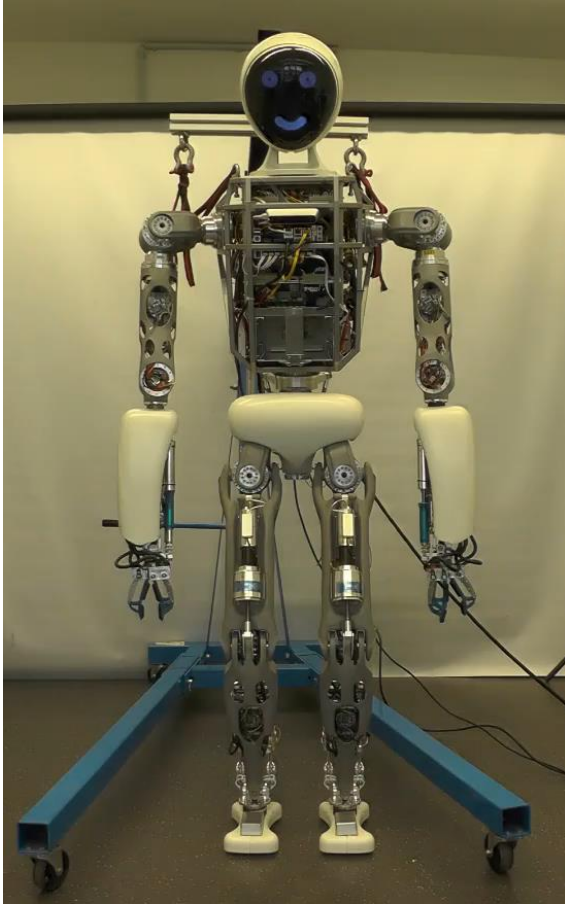
- I: One-Leg Balancing ✓



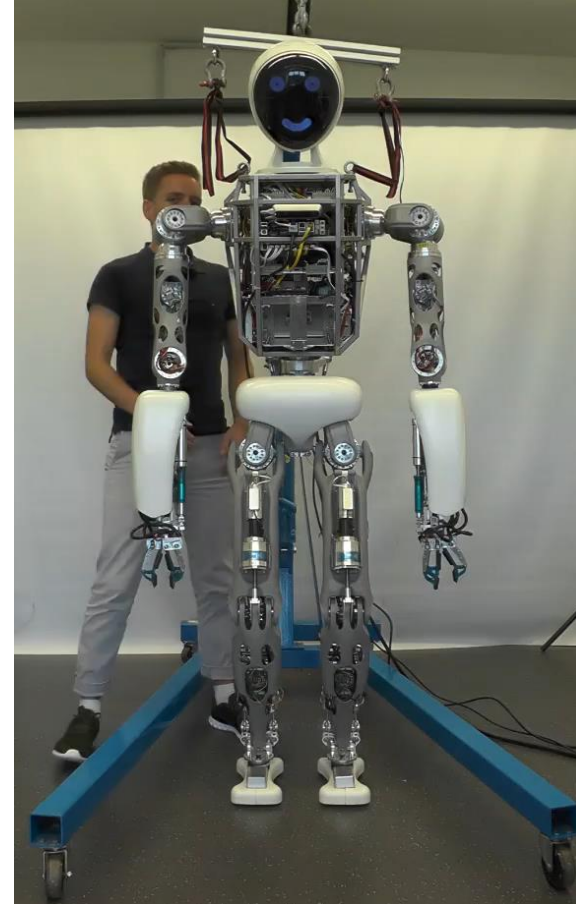
## II: Static Walking (✓)



- III: Fast Squats ✓



- IV: Dynamic Walking ✗



- **Stability** of the Motions
- **Related Issues**
  - Tracking performance
  - Handling impulses
  - Model discrepancies
  - Mechanical deficiencies
- **Actions for Improvement**
  - Task space control
  - System identification
  - Hardware upgrade

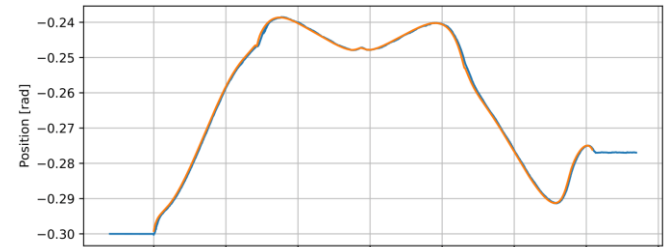


Fig. 1: Ankle Roll Tracking

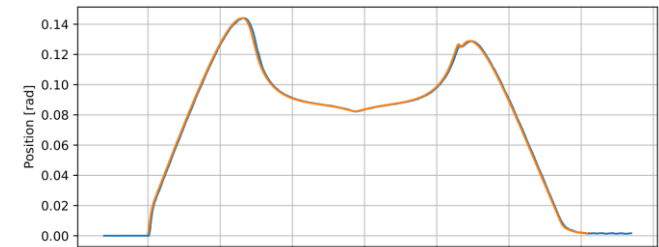


Fig. 2: Ankle Pitch Tracking

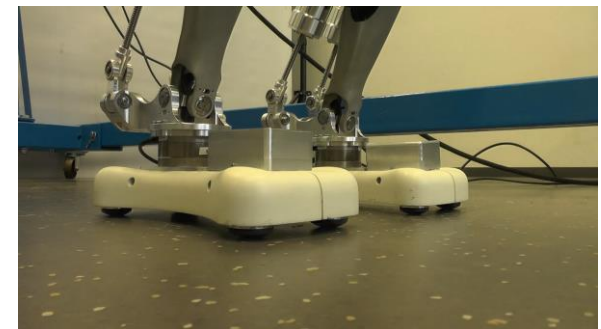


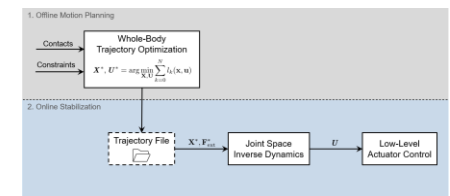
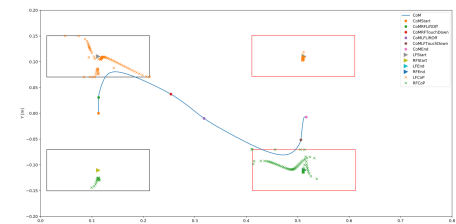
Fig. 3: Deviation in Task Space



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- **Motivation:** Physically Consistent and Efficient Motion Plans
- **Approach:** DDP-Based Whole-Body TO
- **Idea:** Contact Stability Constrained DDP
  - Evaluation: inherently balanced motions
  - Validation: simple control architecture
- **Experimental Pipeline**
- **Physical Limitations** of RH5 Humanoid

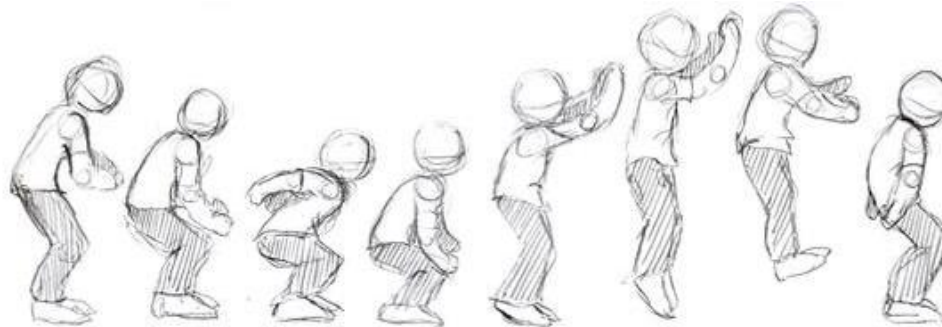


	Position Limits	Torque Limits	Velocity Limits
Vertical Jump ( $l = 0$ cm)			
$h = 1$ cm	✓	✓	✓
$h = 5$ cm	✓	✓	✗ <sub>5</sub>
$h = 10$ cm	✓	(✓)	✗ <sub>5</sub>
$h = 20$ cm	✓	(✓)	✗ <sub>5</sub>
$h = 30$ cm	✓	(✓)	✗ <sub>7</sub>
Forward Jump ( $h = 10$ cm)			
$l = 10$ cm	✓	(✓)	✗ <sub>7</sub>
$l = 20$ cm	✓	(✓)	✗ <sub>7</sub>
$l = 30$ cm	✓	(✓)	✗ <sub>7</sub>
$l = 40$ cm	✓	(✓)	✗ <sub>7</sub>
$l = 50$ cm	✓	(✓)	✗ <sub>7</sub>
Obstacle Jump ( $h = 25$ cm)			
$l = 60$ cm	✓	✗ <sub>5</sub>	✗ <sub>7</sub>



- **Assessment:** Large Potential of Whole-Body TO
  - Reduced handcrafted components
  - High-level tasks formulation
  
- 1. Algorithmic Perspective**
  - Inequality constraints embedded in DDP
  - Solving internal closed loops
  
- 2. Control Perspective**
  - Task space control
  - Model predictive control

# Thanks for your attention!



<https://www.deviantart.com/crystalstarspirit/art/Animation-Jump-Sequence-105385147>



- [1] Marc H Raibert. *Legged robots that balance*. MIT press, 1986.
- [2] Bruno Siciliano and Oussama Khatib. *Springer handbook of robotics*. Springer, 2016.
- [3] Kevin Giraud, Pierre Fernbach, Gabriele Buondonno, Carlos Mastalli, Olivier Stasse, et al. Motion planning with multi-contact and visual servoing on humanoid robots. 2020.
- [4] Shivesh Kumar. *Modular and Analytical Methods for Solving Kinematics and Dynamics of Series-Parallel Hybrid Robots*. PhD thesis, Universität Bremen, 2019.
- [5] Carlos Mastalli, Rohan Budhiraja, Wolfgang Merkt, Guilhem Saurel, Bilal Ham-moud, Maximilien Naveau, Justin Carpentier, Ludovic Righetti, Sethu Vijayaku-mar, and Nicolas Mansard. Crocoddyl: An Efficient and Versatile Framework for Multi-Contact Optimal Control. In *IEEE International Conference on Robotics and Automation (ICRA)*, 2020.
- [6] Rohan Budhiraja, Justin Carpentier, Carlos Mastalli, and Nicolas Mansard. Dif-ferential dynamic programming for multi-phase rigid contact dynamics. In *2018 IEEE-RAS 18th International Conference on Humanoid Robots (Humanoids)*, pages 1–9. IEEE, 2018.
- [7] Stéphane Caron, Quang-Cuong Pham, and Yoshihiko Nakamura. Stability of sur-face contacts for humanoid robots: Closed-form formulae of the contact wrench cone for rectangular support areas. In *2015 IEEE International Conference on Robotics and Automation (ICRA)*, pages 5107–5112. IEEE, 2015.