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Towards a unified understanding of basic notions and terms in humanoid robotics

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SUMMARY
The intention of this paper is to contribute towards a unified understanding of the basic notions and terms in the domain of humanoid robotics, having in mind that the same notions are sometimes interpreted in different ways (some interpretations are contradictory, and some even erroneous). Hence, the first part of the paper is devoted to defining some basic notions, walk and gait being among the first. Then, the paper deals with the notion of dynamic balance and stability, particularly the difference between them, since these essentially different notions are often confused and, rarely, regarded as identical. As dynamic balance is directly related to the notion of zero-moment point (ZMP), it was necessary to touch upon some misunderstandings concerning the ZMP. Gait stability is an especially delicate category, as humanoid locomotion systems have certain specific features that are not possessed by other systems. Namely, because of external disturbances, there may appear unpowered (passive) degrees of freedom that cause loss of dynamic balance. Hence, these unpowered degrees of freedom cannot be overlooked in the stability analysis. As the stability of motion of humanoid robots is inseparably linked with control, it was also necessary to pay due attention to this notion. Finally, the paper ends with a discussion of posture and postural stability with all their specificities. The authors hope that this paper will contribute to a clearer understanding of the basic notions of humanoid robotics, especially concerning robots with high dynamic and control performances.

KEYWORDS: Humanoid robotics; Dynamic balance; Stability; Control; Gait; Walk; Posture.

1. Introduction

In the recent years, we have witnessed an explosive development of humanoid robotics, which has finally become a distinctly differentiated scientific discipline within robotics itself. Part of 'roboticist community' might be surprised by such an abrupt growth of interest in humanoid robotics. However, a careful chronicler could observe that from the very beginning of modern robotics, and even from the ideas of science fiction that preceeded it, the basic goal of robotics has been to copy to a highest possible degree particular capabilities of humans, whereby the future period of 'robot's entering the living and working space of man' will be marked with an even greater degree of anthropomorphism.[1–3] This direction in the research of robotics development has been obvious to a lower or higher extent, but it was always present. Particular periods that have been dominated by different aspects of robotics, such as industrial robots, multi-leg vehicles, etc., appeared to be a 'side work', but the results gained and accumulated knowledge, and technological advancements have enabled an effective switchover to the 'mainstream work'—humanoid robotics.

Now, when we ascertain the reality of the emergence of a special scientific discipline—humanoid robotics—we should say that artificial walk and skeletal activities in general, represent very active research directions, so that it is necessary to address some formal aspects that may be of importance for a further development of high-performance humanoid robots. We refer to the presence of some kind of inconsistency in the usage of some basic notions in the sense of the definitions and terms used. For example, there are no clear definitions of walk, gait, and even posture. Also, there is a pretty great diversity in using the terms stability, dynamic balance and dynamic equilibrium. A majority of these notions and terms have been tacitly accepted, probably because they have been considered to be intuitively sufficiently clear. Hence, they have never been formally defined in the robotic literature. Because of this we think it would be purposeful to define these notions in one place, although it seems that the lack of formal definitions has not essentially caused widespread misunderstanding. However, there are a number of important notions that have been defined by various authors in different ways, and the terminology has not been brought into concordance. Hence, it would be necessary to propose the appropriate definitions and terms in order to attain a unified understanding.

The need for unified definitions of some basic notions and corresponding terms can be illustrated by one of the fundamental concepts in biped locomotion, i.e. zero-moment point (ZMP). Since it appeared more than 35 years ago, the ZMP method[1,2,4–8] has provided a practical basis for the synthesis of dynamic nominals of biped gait and of the control that has the task of preserving dynamic balance of the system as a whole. In the course of time that has elapsed since then, the ZMP method has been widely accepted and used in a great number of scientific papers and realizations, and it has become a 'common thing'. Thus, there appeared papers in which the method was used without giving a reference, because, we believe, the authors considered it as something already generally known, whose source need not be cited. However, such degree of acceptance of a method may just be the cause of a superficial grasping of its source meaning, a consequence of which may be its insufficiently precise, or even erroneous, interpretation.

In view of all the above, we thought it useful to write a paper in which we would propose unified definitions and terms for some fundamental concepts in humanoid robotics. Thus, we hope to initiate their further elaboration that would lead to a generally accepted, well-ordered and unified system of notions that should be characteristic of every mature scientific discipline.

In the very beginning, we would like to point out that the subject of our consideration is *humanoid locomotion*.(1) While speaking about a *humanoid robot* it is usual to simply say *humanoid*. Since the biomechanics of human motion is an important source of information for humanoid robotics, and since the main definitions and conclusions in this text apply to both humans and humanoids, the designation *human/humanoid* will be used as well.

2. Basic Definitions Related to Humanoid Robots Locomotion

2.1. Walk and gait

Let us consider first the definitions of some basic notions that appear in the area of biped locomotion. These notions have been tacitly accepted, probably because they represent basic notions, so that everybody has thought for himself (and from this stemmed also the collective acceptance) that they are understandable. Hence, these basic notions have never been formally defined in the robotic literature. We think that these notions have still to be defined and, although the lack of definitions caused no serious confusion, it can be noted that a number of very important notions have been defined by various authors in different ways, so that it would be desirable to have a unified terminology.

For a mathematical treatment, we will need some notations. Let the human/humanoid body involve n one-degree-of-freedom (one-DOF) joints. Let $\mathbf{q} = (q_1, \ldots, q_n)$ be the vector of joint angles (the so-called internal coordinates). Let each joint be actuated and let $\boldsymbol{\tau} = (\tau_1, \ldots, \tau_n)$ be the vector of joint drives.

Walk. According to ref. 9, walk is understood as the '*movement by putting forward each foot in turn, not having both feet off the ground at once*'. From this definition it comes out that walk is characterized by such displacement of legs in which both feet are not separated from the ground at the same time, and which ensures that the body as a whole moves in space—usually forward, though it is possible to consider a backward walk too. We think that this definition, though it did not originate from technical literature, satisfies the needs of humanoid robotics.

Gait. It is known from experience that the walk of every individual is specific and that a man walks differently in different situations. Each of these particular ways of walking represents a particular gait. Therefore, it can be said that gait represents the '*manner of walking or running*'.[9] Hence, any walk is realized by a certain gait. By recording the time history $\mathbf{q}(t)$, one is recording in fact a particular gait. The basic notions that are related to gait and that should be considered here are step and repeatability conditions, periodicity and symmetry.

- *Step*. When speaking of gait we should point out the fact that has been indirectly pronounced by the formulation '...by putting forward each foot in turn...'.[9] It suggests that in leg locomotion, even in the most general case, there exists a certain kind of repeatability: *in the direction of motion, during the contact with the ground, the leg from the front position with respect to the trunk comes to the rear position, then it is deployed from the ground and in the transfer phase moves to the front position, to make again contact with the ground, and the cycle is repeated.* The described sequence of actions represents a basic cycle of walk, and it is called a *step*. Step duration should not be necessarily constant. If t_i is the time instant of the beginning of the ith step, then the duration of one step is $T_i = t_{i+1} - t_i$, and it can vary. It should be noted that the instant from which we observe a step within this cycle can be arbitrarily selected (we need not start as in the above example with the contact of the 'front' leg with the ground). The described repetition of movements is the basis of locomotion activity. Still, we should emphasize that each step can be generally different, an example being the stuggering of a drunk man. Although a step can be divided into a large number of phases, we think that each step consists of at least two phases: *a single-support phase, when only one foot is in contact with the ground (during this time period the supporting leg from the front position with respect to the trunk comes to the rear position, while the swing leg from the rear position comes to the front position), a and double-support phase in which both feet are simultaneously on the ground*. The two phases alternate regularly.
- *Periodic gait*. If the gait is realized by repeating the same step in an identical way, then we speak of a periodic gait. In that case the relative position of legs' links is repeated periodically. This does not mean that the other parts of the body (e.g. the arms or the head) behave obligatorily in a periodic manner, but it still should be pointed out that it is the most common case. Mathematically, the periodicity condition is expressed via the change of the internal coordinates q_j (joint angles): $q_j(t + T) = q_j(t), \forall t$, $j = 1, \ldots,$ where j changes as per each legs' joint of the locomotion mechanism, while T represents the constant step duration. *Periodicity of the motion of legs' joints is a necessary and sufficient condition for a periodic gait.*

[1] In their previous works, the authors almost exclusively used the term biped locomotion system or biped locomotion mechanism (sometimes simply referred to as biped).

The majority of papers in the area of biped locomotion consider periodic gait. Let us add that periodic gait can be realized only if the motion is performed on the ground surface of appropriate characteristics that allow periodicity.

- *Repeatability conditions*. In some literature sources, the attributes *periodic* and *repeatable* are considered synonymous. However, the term *periodic* has its firm footing in the mathematical definition of a periodic function, whereas *repeatability* and *repeatability conditions* require certain explanation. Namely, *to attain periodicity, a necessary condition is the equality of the system state (more precisely, the state of the lower extremities) at the beginning of each step*. However, this condition is not sufficient because a periodic gait (in the sense of the above definition) will be realized only under the conditions that the humanoid performs each step on an identical ground, obeying the same control law, and in the absence of disturbances. Mathematically, repeatability conditions can be expressed in the following way: $q_j(t_i) = q_j(t_{i-1})$, $\dot{q}_j(t_i) = \dot{q}_j(t_{i-1})$, $j = 1, \ldots,$ where *j* changes as per each joint of the locomotion mechanism legs, and t_i represents the instant of the beginning of the *i*th step. Therefore, the identity of the state at the beginning of each step offers the possibility to repeat the preceding step, and thus realize a periodic gait. In the literature,[10] we can find a somewhat different definition of repeatability conditions, requiring that *the state at the end of a step is equal to the state at its beginning*. Such formulation is identical to the previous one, with an additional explanation that the end of a step coincides with the beginning of the next one. Here, we should comment the fact that the walk segment that represents a step can be chosen so that it ends by the foot touching the ground. At this instant, the impact occurs that might cause discontinuity in the system state (instantaneous change in velocities). The above definition assumes that the impact and potential change in the state are an integral part of a step.

  A natural question is posed as to whether the notion of periodicity is synonymous with repeatability. Thus, why should a human/humanoid abandon repeating the previous step provided it could be realized? Is this a purely academic issue or something that can happen in reality? We think that it is a real possibility. For example, there may arise a situation in which (because of a certain disturbance) the robot performing periodic gait has to change the motion of one leg in some step phase (e.g. the foot has to be lifted somewhat higher because of the presence of an obstacle on the ground) and, after this 'intervention', return again to the previous trajectory, to complete the step with a state identical to the one at its beginning.

  It should also be mentioned that repeatability conditions are unavoidable in gait synthesis, where the walk is formed by synthesizing one step that is then repeated.[7,8,10]

- *Symmetric step and gait*. Symmetry is a characteristic of a step, but a gait, being a sequence of symmetric steps, can also be called symmetric. A prerequisite for a symmetric step and gait is the symmetry of the extremities, i.e. of the left and right legs (which is almost always considered as being fulfilled). *If a step can be divided in two equal time periods, and if the left leg in one period behaves as the right leg in the other then we speak of a symmetric gait*. The half-period and the motion realized in it are termed *half-step*. The symmetry condition can be mathematically expressed as: $q_j^{\text{right}}(t + T/2) = q_j^{\text{left}}(t), \forall t,\ j = 1, \ldots,$ where *j* denotes the symmetric joints of the right and the left leg, and *T*/2 is the half-step duration. For a symmetric gait, this necessary and sufficient condition should be fulfilled during the gait. If all the body joints move symmetrically,[2] we speak of a symmetric motion of human/humanoid, and the above mathematical expression expands to hold for the entire body. Symmetry assumes a straight-line gait, but it is also important to emphasize the need for the 'symmetry' of the support (ground), i.e. the equality of support conditions for the left and the right legs. With a symmetric gait, all kinematic and dynamic analyses can be carried out on one half-step.

It should be pointed out that a periodic and repeatable gait need not be symmetric, and that symmetry does not necessarily assume either periodicity or repeatability.

It is important to note that all the above definitions assume implicitly the gait continuation, i.e. the human/humanoid is not going to fall. Hence, let the gait that is realized with two legs[3] and for which there are no any additionally preset conditions (symmetry, repeatability, etc.) except for its continuation be called *sustained gait*.

Regular gait. Under the notion *regular gait* is understood as a periodic gait in which *the leg in the single-support phase is in contact with the ground by the whole foot area or the area of its front part* (the toes link with the two-link foot), and in the case of the double-support phase the requirement applies to at least one foot. It should be noted that regular gait can, but not necessarily, be symmetric (e.g. when the robot performs a turn and the 'internal' leg passes the shorter way). The gait consisting of the parts that are all regular is also regular. For example, climbing the staircases, straight-line gait forward, turning, etc., considered as a whole, also represent regular gaits.

Ideal gait. Ideal gait is a purely academic notion, and it represents a regular gait for which the repeatability and symmetry conditions can be mathematically checked. In view of the fact that there is always some difference between the data used in mathematical treatment (mechanism parameters, time changes of joints angles, characteristics of the ground on which the humanoid is walking, etc.) and real data, the data used in mathematical treatment are called ideal. Ideal gait is often used as a reference motion that the system is attempting to realize. Although the ideal gait coincides greatly (almost in full) with regular gait—the differences being in the level of 'refinement', the authors consider this notion necessary,

[2] For the joints that have no 'symmetric pair' (waist and neck), the condition of gait symmetry is somewhat different: $q_j(t + T/2) = -q_j(t)$.

[3] The motion by crawling or staggering while using hands to hold on to something, cannot be considered a gait.

and the term as being appropriate, because such gait is most often used in all theoretical investigations in this area.

Support area.[4] This is the surface determined by the contact of the foot and the ground. With regular gait, there is always a support area of a finite size: *In the single-support phase the support area coincides with the area of the foot in contact with the ground, whereas in the double-support phase, the support area is a convex area determined by the areas of the feet and the ground and common tangents, so that the encompassed area is maximized.* Support area does not exist only in the case when both the feet are off the ground (runing or jumping) or when the contact area has degenerated to a point or a line (this, however, means that the rigid foot rotates about an axis or point and that the mechanism as a whole is overturning). In the case of the occurrence of any of the two instances, the gait of the humanoid cannot be considered regular.

In this paper, we will confine ourselves to *regular* gait only. Pathological and other kinds of deformed gait (people with some disabilities, drunk persons, a walk on irregular surfaces, etc.) and other gaits that have not been encompassed by the above definitions will be called *irregular*. Although irregular gaits are not presently that interesting to humanoid roboticists, it can be expected that the interest in the analysis of the gaits that are characterized by asymmetry and aperiodicity will certainly increase in the future.

2.2. Dynamic balance and the ZMP

The main task of humanoid in bipedal gait is to avoid overturning. However, this requirement has not yet been terminologically defined in a proper way. For a long time, the usual term for the gait that proceeds continuously has been *stable gait*.[5] The term made it possible for someone to grasp the problem intuitively but it might also cause misunderstanding. The term stability, in this context, is not an appropriate choice because stability in system theory and automatic control has already been clearly and precisely defined, and it strongly concerns the system control. (Hence, stability issue for biped location systems will be introduced in an appropriate and precise way later when discussing the control of humanoid.) Because of that, a new term is needed to define the conditions that the gait is sustained, i.e. continued.

In the course of the single-support phase, a human/humanoid represents complex, multi-link, actuated inverted pendula. To prevent the humanoid from falling during a walk, a necessary and sufficient condition is to ensure that the foot–ground contact in each instant is a surface[6] and not a line or a point (regardless of whether the full foot or its front link is in contact only, representing the toes). By the leg's touching the ground, the system passes to the double-support phase and thus forms an expanded support area. In this phase, it is necessary again to maintain the support area, and prevent its degeneration to a line or point. Therefore, in any gait phase, transformation of the support area to a line or point leads to the rotation and overturning of the humanoid. In other words, *to avoid overturning in real conditions, the human/humanoid must 'maintain balance', both dynamic and static.* It should be mentioned that the term 'balance' is used here in the sense of 'maintaining an upright position of the overall humanoid', and it must not be confused with 'equilibrium' in the sense of the D'Alambert principle. Namely, a humanoid that is falling by the rotation about its foot edge cannot be considered as balanced in the sense of, e.g. upright position, although the D'Alambert principle still holds (even for a system that is falling) for a point on the foot edge where the pressure force acts. Since the term 'dynamic equilibrium' is common in the formulation of the D'Alambert principle (which holds always and for any system) we suggest the use of the term 'balance' (as we defined it) for maintaining the humanoid in an upright position. It should be emphasized that these two terms (balance and equilibrium) are not equivalent and that a clear distinction of their meanings may help to avoid misunderstandings, especially having in mind that the term

[4] Commonly used term is support polygon. This came out from the fact that all realized walking robots had feet of a rectangular shape. However, the future robots need not have such feet, which might be even of a shape close to that of human, and thus far from a rectangle or a polygon, so that the support area will not be of a polygonal shape.

[5] There are a lot of examples in which gait stability is understood as the prevention of overturning of the humanoid as a whole. We will give only two. Thus, for example, in ref. 11 (p. 3346) one can read: 'Since a biped robot tends to tip over easily, stable and reliable biped walking is a very important achievement', whereas in ref. 12 (p. 65) we find 'Since biped robot easily tips over, it is necessary to take stability into account when determining a walking pattern'.

[6] In ref. 13 (p. 2009) we can read: 'Robotics literature often equates the location of the CoP/ZMP in the interior of the foot with the stability of the robot. This is not an accurate description of the physical phenomenon, mainly because there is no agreed upon definition of stability. For example, a robot can maintain a flat foot and still hit the ground with its trunk. On the other hand, the foot can start to turn on its toe and still manage to bring itself back without falling'. It should be noted that both the above examples are rather superfluous and without clear footing related to the conditions that are valid. Let us, consider just the first example stating that 'a robot can maintain a flat foot and still hit the ground with its trunk'. If we assume that the humanoid is realizing a gait or a static posture (the term posture will be discussed in detail later), then in order to 'hit the ground with its trunk' a perturbation must act on it. If, however, after perturbation the foot is still in full contact with the ground this means that the perturbation was not strong enough to overturn the humanoid, and the gait will be continued. If the perturbation was strong enough to overturn the humanoid, it will collapse and the flat foot contact will not be maintained. On the other hand, it is theoretically possible to *synthesize* such motion in which the system as a whole will rotate about its ankle joint and 'hit the ground with its trunk',[13] whereby the foot will be in contact with the ground by its full area. However, such motion cannot be classified as locomotion, and such example is not acceptable. On the other hand, if the system subjected to strong perturbation rotates about the foot edge, its total collapse can be prevented in several ways: by strong compensational action of the arms or trunk, by striding and passing to the new double-support phase, etc. If the system manages to avoid its total collapse, the fact that it was actually in the state of falling cannot be changed. Hence, such motion, irrespective of the lack of the 'agreed upon definition of stability' could in no case be considered as either stable or dynamically balanced. However, several lines further the authors state that they: 'will maintain the full controllability of the robot. We will do so by regulating the CoP to keep it in the interior of the foot, away from the edge', by which we come back to the fact from the beginning of the text that the ZMP being within the support area (i.e. the foot is in full contact with the ground) is a crucial prerequisite for the existence of dynamic balance and gait continuation.

balance has been increasingly used.[14,15] Having in mind that the humanoid is under the influence of inertial forces, whose direction and intensity change during the gait, we suggest the term *dynamic balance* as more appropriate.

We can now define this notion.

Dynamic balance. The human/humanoid gait is dynamically balanced *if there is no rotation of the supporting foot (or feet) about its (or their common) edge during walking*.

The authors are of the opinion that the term *dynamic balance* is appropriate for describing a regular gait, as it reflects the essence of the notion to which it is related, and can also be intuitively grasped.

Here, we need to take a deeper look into the essence of dynamic balance of a humanoid robot. *For the dynamic balance, it is necessary and sufficient that the resultant of the normal pressure forces from the foot (or feet) to the ground act at a point that is inside the support area (excluding the edges).* The mechanical contact of the foot (or feet) and the ground, considered along the normal onto the contact surface, has a unilateral character. In one direction the relative motion is constrained, while in the opposite direction the relative motion is possible. Thus, the reaction forces along the normal are all in one direction (from the ground towards the foot) and accordingly, cannot make a couple. If we find a point where only the resultant of normal forces act, the absence of any couple means that the moments will be equal to zero for any axis passing through this point and being tangential to the ground. That is how we come to the well-known notion of ZMP. This point is sometimes referred to as Center of Pressure[(7)] (CoP).[6,16]

ZMP. Let us introduce a Cartesian frame with the origin at the mentioned point where the resultant pressure force is acting, the two axes (x and y) being tangential to the ground and the third (z) being normal. Now, a *mathematical expression for dynamic balance is:*[(8)] $\Sigma M_x = 0$ *and* $\Sigma M_y = 0$. The moments include gravity, inertial forces and other external forces acting on the humanoid body (like wind, strike, etc.). Note that the sum of all the moments about the z-axis need not necessarily be zero since it is compensated for by the sufficient friction. Finally, one may define ZMP: *ZMP is the point on the support area (excluding the edge) for which* $\Sigma M_x = 0$ *and* $\Sigma M_y = 0$, or more precisely:

$$\begin{aligned}&\Sigma M_x^{\substack{\text{inertial}\\ \text{(without foot)}}} + \Sigma M_x^{\substack{\text{gravitational}\\ \text{(without foot)}}} + M_x^{\substack{\text{gravitational}\\ \text{(of immobile foot)}}} \\ &\quad + \Sigma M_x^{\text{exterenal}} = 0 \\ &\Sigma M_y^{\substack{\text{inertial}\\ \text{(without foot)}}} + \Sigma M_y^{\substack{\text{gravitational}\\ \text{(without foot)}}} + M_y^{\substack{\text{gravitational}\\ \text{(of immobile foot)}}} \\ &\quad + \Sigma M_y^{\text{exterenal}} = 0.\end{aligned} \tag{1}$$

whereby the external moments result from all the external forces (wind, push, etc.).

Let us note that the requirement for the constant existence of a contact area implies that the supporting foot (or feet) is (are) not moving with respect to the support (ground). So, if the support is immobile, there will be no inertia of the foot (feet) in the sum of moments. If the support moves, there will be no relative acceleration of the foot (and the corresponding inertia), but there will exist the transfer acceleration and inertia.

We recall that the ZMP should not come to the edge[(9)] of the support area. If this happens, rotation about the edge will occur and this means that the system will fall down. This is not considered a balanced motion and the notion of ZMP loses its true essence.[6,17] Therefore, let us repeat that the CoP and ZMP coincide while being inside the support area. At the edge, for a system that is falling, the term ZMP should not be used (in fact, ZMP does not exist in such an instance), while CoP might still be used. None of them can be found outside the support area.

Let us mention only in brief[(10)] the confusions that may be found in relation to the notion of ZMP. We will refer first to the 'complete and unconditional identity'[19] of the ZMP and CoP. Although, as mentioned earlier, the CoP and ZMP coincide while the dynamic balance is being preserved, in the case of the rotation of the system as a whole about the foot edge the CoP still exists (as long as the foot–ground contact exists), but the ZMP does not exist any more because dynamic balance has been lost. To make it more intelligible, let us have a look at the equations $\Sigma M_x = 0$ and $\Sigma M_y = 0$, but now for the case when the foot is rotating about the front edge conciding with the x-axis. Then it holds

$$\begin{aligned}&\Sigma \widehat{M}_x^{\substack{\text{inertial}\\ \text{(without foot)}}} + \Sigma \widehat{M}_x^{\substack{\text{gravitational}\\ \text{(without foot)}}} + M_x^{\substack{\text{inertial}\\ \text{(of rotating foot)}}} \\ &\quad + M_x^{\substack{\text{gravitational}\\ \text{(of rotating foot)}}} + \Sigma \widehat{M}_x^{\text{exterenal}} = 0 \\ &\Sigma \widehat{M}_y^{\substack{\text{inertial}\\ \text{(without foot)}}} + \Sigma \widehat{M}_y^{\substack{\text{gravitational}\\ \text{(without foot)}}} + M_y^{\substack{\text{gravitational}\\ \text{(of rotating foot)}}} \\ &\quad + \Sigma \widehat{M}_y^{\text{exterenal}} = 0\end{aligned} \tag{2}$$

and the difference from Eq. (1) is straightforward. Please note that elements of eq. (2) denoted by $\widehat{M}$ differ from corresponding elements in eq. (1) due to rotation of the whole system about foot edge.

Besides, it is also necessary to touch upon the notions such as 'virtual or fictitious ZMP—FZMP' and 'foot rotation indicator (FRI) point'.

The notion of FZMP was mentioned first in ref. 6. It is a point *outside the support area* for which the ZMP conditions

[7] It should be noted that in the early papers from this area it was still implicitly pointed out that the ground reaction force is formed as a sum of pressure forces of the ground on the foot. (See, for example ref. 8 (Fig. 3, p. 501) or ref. 5 (Fig. 1.2, p. 19)).

[8] It is possible[6,10] to prescribe also the motion at $n-3$ joints. Then, the rotational motion of the trunk about its vertical axis ensures that under the humanoid foot (in addtion to $M_x = 0$ and $M_y = 0$), the condition $M_z = 0$ is also fulfilled. The conditions $M_x = 0$ and $M_y = 0$ define the dynamic balance of the humanoid, whereas $M_z = 0$ prevents its deflection from the motion course in the case of insufficient friction between the foot and the ground.

[9] In the works from the domain of biped locomotion, the foot links are mainly considered to be rigid. In that case, in a strict mathematical sense, when the ZMP would come infinitesimally close to the edge of the support area from the 'inner' side, the dynamic balance would still exist theoretically. However, it is not practically possible to maintain the ZMP position during the motion exactly at the edge of the support area. This is possible to achieve if a 'safety zone' within the support area is formed and the ZMP is kept within it.

[10] Misunderstandings concerning the ZMP notion have been dealt with in detail in ref. 18.

($\Sigma M_x = 0$, $\Sigma M_y = 0$) are fulfilled. It should be noted that the FZMP position is determined in the same way as the ZMP (using Eq. (1)). However, at the instant when the FZMP appears, the assumption that the foot is immobile with respect to the ground does not hold any more. Namely, when the existence of FZMP is observed, the system falling has already begun, and because of the changed foot–ground contact (contact area degenerated to a line—foot rotation axis), one cannot get information on the system dynamics by measuring the ground reaction forces involved. Because of this the authors in ref. 6 point out the fictitious (imaginary) character of FZMP and the impossibility of its utilization in humanoid gait control.

The notion of foot rotation indicator (FRI) point, was proposed in ref. 16 as a 'generlization' of the ZMP notion with additional capability to give information about the 'severity of disbalance' with respect to the dynamically balanced mechanism. The proposed definition[16] reads: 'The FRI point is that point on the foot/ground surface, within or outside the support polygon, where the net ground-reaction force would have to act to keep the foot stationary'. However, it is evident that as long as the FRI is out of the support area, it represents a fictitious quantity, since the ground reaction force cannot act on the system because there is no contact with it. Therefore, within the support area FRI≡ZMP; when outside FRI≡FZMP. Normally, a real and a fictitious notion cannot be termed the same. Besides, in view of refs. 19 and 16 there may arise confusion that a notion related to dynamic balance (ZMP, FRI) in one case can and in the other cannot be outside the support polygon.

It should be noted that there may exist such a walk in which, as part of a step, the phase of the overall system rotation appears about the foot edge. To illustrate this, let us consider the terminal part of the single-support phase, in which the leg in the swing phase is already stretched forward and prepared for the double-support phase. If at that instant there appears rotation about the front edge of the supporting leg foot, then there will be no undesired consequence to the humanoid and to the continuation of its gait. Such a situation can be conditionally termed 'controlled utilization of unpowered DOFs' (controlled fall), as the humanoid, by bringing the front leg to a predetermined position, has already been prepared for such situation. Rotation will surely be stopped by the contact of the front leg with the ground, and the walk will be continued.

In view of the above definition of walk[9] as a '*move by putting forward each foot in turn, not having both the feet off the ground at once*', it is quite clear that there are no reasons that would prevent the existence of a step (or half-step) portion in which the system would not be dynamically balanced. 'Preventive' preparation of the humanoid by assuming a defined configuration prior to 'controlled utilization of unpowered DOFs' allows the avoidance of the loss of the system motion control.

It should be noted that, to our knowledge, the gaits of humanoid robots with feet that have been realized up to now were dynamically balanced in full, and contained no phase of 'controlled utilization of unpowered DOFs'. Although we are not going to dwell further upon this issue, we still mention it because we believe that this phase of humanoid robots gait will certainly be present in the gait synthesis and realization in the future.

Semi-inverse method for the reference motion synthesis. In the realization of artificial walk, the first step is to synthesize the motion that is to be reproduced. Such a motion synthesized for the absence of any disturbances is called the reference motion. Here, we will describe in brief the semi-inverse method[7,8,10] for the synthesis of the reference motion,[(11)] a functional and dynamically balanced gait.

The model of system dynamics relates the independent motion coordinates $\boldsymbol{q}'$ and joint drives $\boldsymbol{\tau}$. In the single-support phase, all joint coordinates are independent and hence $\boldsymbol{q}' = \boldsymbol{q}$, while in the double-support phase the number of independent coordinates is reduced due to the contact of the supporting leg with the ground. Depending on the character of contact (heel-strike, flat-foot, etc.) the degree of reduction may vary. In the current discussion, we concentrate on the single-support phase without compromising the generality of conclusions. Since $\boldsymbol{q}' = \boldsymbol{q}$, the dynamic model is

$$\boldsymbol{\tau} = \boldsymbol{H}(\boldsymbol{q})\,\ddot{\boldsymbol{q}} + \boldsymbol{h}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \boldsymbol{G}(\boldsymbol{q}) \tag{3}$$

where $\boldsymbol{H}$ is the system inertia matrix, $\boldsymbol{h}$ is a vector comprising all 'velocity' influences (Coriolis, centrifugal, etc.) and $\boldsymbol{G}$ is a vector of moments due to gravitational forces. The model is used for the synthesis of reference motion and the feed-forward control.

In order to ensure functional movements, the motion for one part of the system should be known in advance. The remaining part of the system should be solved in such a way so as to maintain the system's dynamic balance for the prescribed boundary conditions (i.e. repeatability conditions). This is why the term 'semi-inverse' method is used. Let us explain this in more detail.

- One may assume that the motions of leg joints are prescribed when a desired gait is selected. Let there be n_l such joints: $\boldsymbol{q}^l(\boldsymbol{t})$. With humans, these are learned walking patterns. Human motions could be recorded and applied to the humanoids.
- If some manipulation or any other arm activity is to be performed along with walking, then the arm joints motions are considered prescribed. Let there be n_a such joints: $\boldsymbol{q}^a(\boldsymbol{t})$.
- The neck has to orient the head in a prescribed direction involving n_n DOFs: $\boldsymbol{q}^n(\boldsymbol{t})$.
- Thus, at $n_l + n_a + n_n$ joints, the motion is prescribed. The waist motion is still undefined.

Let there be $n_w = 3$ waist DOFs: $\boldsymbol{q}^w$. Two of them are utilized to maintain the dynamic balance. These are the trunk rotations left–right and forward–backward. To achieve the balance, these motions are calculated to ensure a desired position of the ZMP inside the support area by applying the two scalar conditions (Eq. (1)). The rest waist motion, rotation about the vertical axis, can be either prescribed (if

[11] The motion thus synthesized was first called in refs. 5 and 10 as nominal motion.

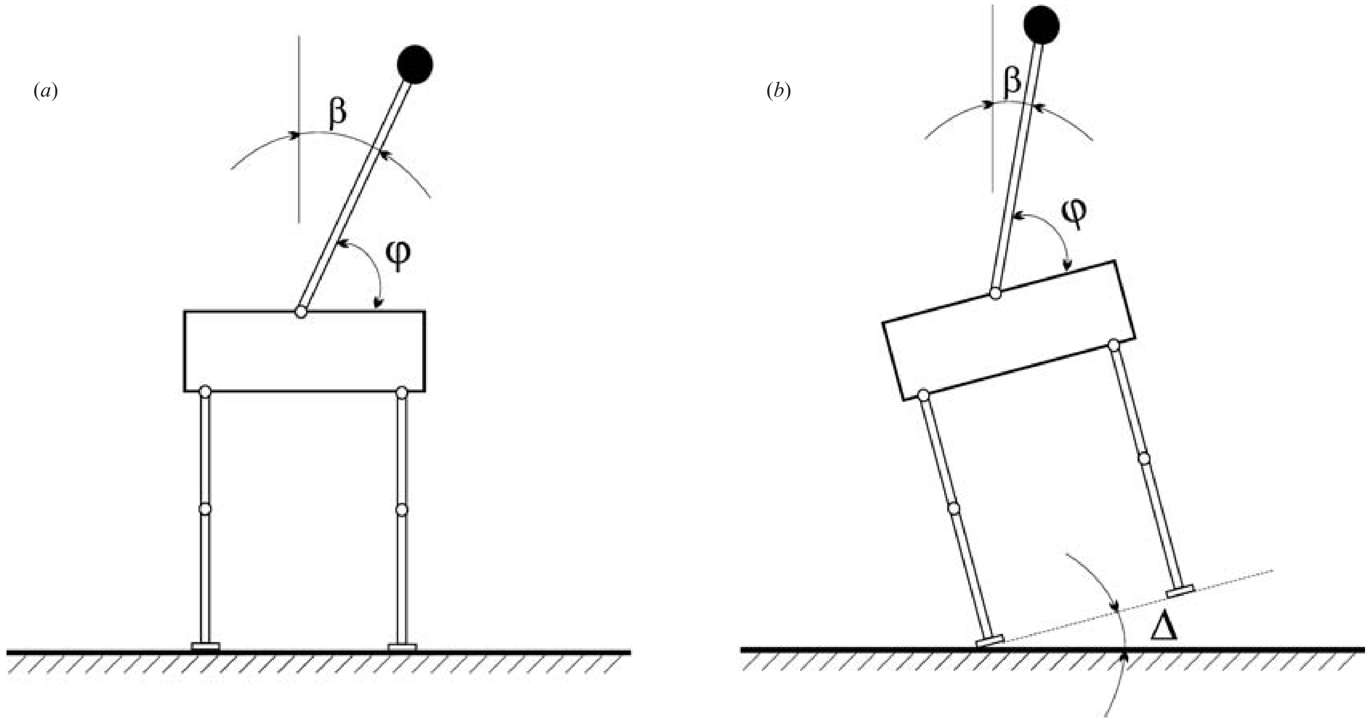

Fig. 1. Illustration of the internal and external synergies. (a) Undisturbed motion. (b) Effect of the appearance of an unpowered DOF.

some given task requires it) or can be calculated so as to reduce the friction torque between the foot and the ground. Note that in the situation when the arms are not engaged in any functional activity, they usually move so as to assist the waist in reducing the friction torque under the foot (feet). In this way, all $n = n_l + n_a + n_n + n_w$ joint motions (comprising reference motion) can be determined.

3. Control and Stability

It is quite clear that humanoid motion is realized by changing joints angles in the manner that will yield the desired relative position of the humanoid in its environment. To the periodic laws of change of joint angles $\varphi_i(t)$, correspond the periodic laws of change $\beta_i(t)$ (Fig. 1(a)), where β_i denotes the angular positions of the links with respect to the absolute coordinate frame. We call the laws of change $\varphi_i(t)$ as *internal synergy* and $\beta_i(t)$ as *external synergy.*[(12)4,2]. In order to control humanoid motion, the realization of internal synergy has to influence the external synergy in a unique way.

As is known, one of the basic characteristics of a humanoid system is that this unique relationship can be lost because of potential rotation of the system about the foot edge and its overturning. To clarify the difference between the internal and external synergies, let us consider a highly simplified model in which reference motion is performed with the pelvis being constantly parallel to the horizontal plane and the legs moving in the planes that are always perpendicular to the ground. A sketch of this model (front view) is given in Fig. 1. If the system rotates about the foot edge (and falls), this means that the coordinate Δ (Fig. 1(b)) has arisen, which is 'uncontollable' because the corresponding DOF has no actuator.

Under ideal conditions, the realization of the reference internal synergy $\varphi_i^0(t)$ means simultaneous realization of the reference external synergy $\beta_i^0(t)$, whereby the ZMP is also in the reference position. However, perturbations are always present. Compensation for the perturbation is possible only by changing the action of the actuators of powered coordinates (φ_i, $i = 1, \ldots, n$), whereby the overall system dynamics is changed.

Besides, in order to ensure that the external synergy depend only on the internal synergy, the reference relationship between the humanoid and its environment has to be preserved ($\Delta = 0$), i.e. there must exist a contact area (not degenerating to a line or point) between the foot and ground, as presented in Fig. 1(a). This requirement means that the condition of dynamic balance is fulfilled, which will be true if the ZMP is within the support area.[(13)] In the cases of disturbance under which $\Delta = 0$ still holds, this condition is fulfilled, so that the compensational action at the mechanism joints can directly influence the humanoid motion to return to the reference external synergy.

In the case of larger disturbances, even if exact internal synergy is realized, the external synergy can be disturbed. For example, as a result of the influence of external disturbances, the locomotion mechanism may come to the position shown in Fig. 1(b), in which the support and contact with the ground are realized only at the edge of the supporting leg foot. In

[12] The term synergy, in the sense of a concerted and simultaneous action of the joints involved, was introduced into the terminology of locomotion biodynamics by Bernsteins.[20]

[13] To ensure dynamic balance, the ZMP should be within the support area (the edge excluded), *and only if the ZMP is in the reference position* (i.e. in the motion synthesis), *such dynamic balance is termed reference balance*.

such case, even if desired internal synergy $\varphi_i^0(t)$ is performed perfectly, external synergy $\beta_i^0(t)$ will deviate from desired one (the angles $\beta_i(t)$ will differ from the $\beta_i^0(t)$ i.e. reference ones) due to the rotation of the complete locomotion system about foot edge.

In the absence of disturbances, the external synergy β depends only on the internal synergy angles φ. To illustrate this, let us cosider, for example, only one coordinate. In Fig. 1, the trunk angle corresponding to the instantaneous external synergy is denoted as β_{TRUNK}, and the one corresponding to the internal synergy as φ_{TRUNK}. In the absence of disturabnces, for the humanoid trunk from Fig. 1(a), we can write

$$\beta_{\mathrm{TRUNK}} = \frac{\pi}{2} - \varphi_{\mathrm{TRUNK}}.$$

If the disturbance is acting (Fig 1(b)), the angle β_{TRUNK} for the upper part is not determined only by the angle φ_{TRUNK}, but also by the angle Δ, so that the previous expression becomes

$$\beta_{\mathrm{TRUNK}} = \frac{\pi}{2} - (\varphi_{\mathrm{TRUNK}} + \Delta).$$

This (very simplified) example well illustrates how the external synergy can be upset even if the internal synergy has been exactly realized. If, for any reason, the fulfillment of internal synergy is disturbed, this will reflect on the external synergy too. On the other hand, the external and not internal synergy, determines the conditions of gait repeatability with respect to the absolute coordinate frame.

The overall mechanism motion that fully corresponds to all the preset requirements the real system tends to attain, is called reference motion. Reference motion can be defined in the following way:

Reference motion.[(14)] It is the *ideal motion of the locomotion system that satisfies all the desired functionality conditions (e.g. realization of the desired gait along with the desired arms movements), including the maintenance of dynamic balance*. It is assumed that reference motion represents the motion to which the mechanism tends when walking is done in real conditions. The synthesis of such motion trajectory has been discussed in Section 2.

3.1. Control—Basic issues

Under real conditions, the realization of motion is always accompanied by disturbances, and some deviation from the ideal (reference) motion always exists. The deviation in tracking reference trajectories at the joints causes deviation of the ZMP from its reference position. An especially undesired situation arises when the ZMP comes too close to the edge of the support area, when the humanoid could lose its balance, could start to rotate about the support area edge and could collapse. The control has to ensure the approaching to the reference trajectories of all the powered joints, while constantly preserving dynamic balance.

In order to better comprehend the complex issue related to control and later to define the stability of humanoid motion, let us consider three characteristic cases of gait. In all three cases it is assumed that the humanoid in the beginning of the considered period peforms reference motion and then disturbances begin to act.[(15)]

1. Because of the action of external disturbance force of smaller intensity, ideal tracking of joint trajectories is disturbed, as well as the ZMP position, but in such a manner that it still remains within a 'safety zone'. Appropriate control actions can return the system to the reference trajectory.
2. Assume now that a disturbance force of medium intensity is involved. In this case too, the force disturbs the ideal tracking of joint trajectories, and increases the deviation of the ZMP from its reference position. To preserve dynamic balance, the humanoid must undertake a 'more resolute' action (e.g. arms swinging), in order to ensure that the ZMP remains within the 'safety zone' and the system returns to the reference motion.
3. In the case of a high-intensity disturbance, as mentioned earlier, unpowered (passive) DOFs arise, and the system as a whole starts to rotate about the edge of the support area. The attempt to minimize deviations of joint trajectories and resume reference motion is senseless if the system has lost the dynamic balance. Hence, it is of highest interest to preserve (or re-establish) dynamic balance, taking no care at the moment of joints trajectories tracking. Because of that, the humanoid has to abandon the realization of the previous reference motion and, for example, step by one leg in the direction of falling, support on it, and—in the next several steps—return to the reference trajectory.

In all three cases, the task of control is to minimize the deviations of the real humanoid state from the reference one for all the joints, while preserving the dynamic balance, whereby in case 2, and especially in case 3, the priority task is to prevent the system from falling (i.e. preserve its dynamic balance).

In practice, small disturbances are most common, so that the problem of gait control reduces to case 1, which has been investigated most thoroughly. Hence, we are not going to elaborate it in detail and we give only a brief account of it, just for the sake of completeness of our discussion. In this case, the control has to minimize the deviations at all joints and, when the attained motion is close to the reference one, the real ZMP will be close to its reference position. However, compensational actions[(16)] at joints have as a side-effect the change of links' accelerations (differing from the reference ones), and this further leads to the change of the intensity and direction of inertial forces (and consequently the ZMP position). Thus, by correcting the internal synergy,

14 Reference motion can also be precisely defined as the motion that satisfies simultaneously the desired internal and external synergies.

15 Although disturbed motion can arise because of any kind of the previously mentioned disturbances, to make the explanation clearer, the disturbances in all three cases are represented as external forces only.

16 Compensational (corrective) actions can be realized only at the mechanism joints, whereby all the deviations of the internal and external synergies have to be minimized simultaneously. As long as the dynamic balance is preserved (ZMP is within the support polygon), the fulfillment of these requirements reduces to the minimization of deviations of the reference trajectory tracking.

the external one can be upset. To prevent the appearance of unpowered DOFs, it is necessary to constantly measure[4] *the real ZMP position*[(17)] and endeavour to bring it closer to its reference position. Information about the ZMP position is fundamental for both the synthesis of reference motion (reference ZMP position) and control of the real motion of the humanoid (real ZMP position).

In the case of small disturbances, the deviations can be compensated for in two ways as follows:

(a) the local feedbacks at each joint attempt to bring the trajectory of joint motion as close as possible to the reference one, whereby care is taken of the compensation 'intensity' (i.e. of the generated accelerations because of which the real inertial forces will differ from the reference ones), in order to avoid the side-effect of jeopardizing the dynamic balance, and

(b) apart from the local feedbacks acting at each joint, an additional task can be assigned to a joint or a group of joints to prevent inadmissible excursions of the ZMP, even at the expense of 'spoiling' the quality of tracking reference trajectories at joints. Compensational actions to prevent the loss of dynamic balance of the humanoid are defined solely on the basis of measuring the real ZMP position.

Cases 2 and 3 differ from case 1 only in respect of disturbance intensity; the larger is the disturbance, the more endangered is the dynamic balance. Hence, the most urgent task is to prevent the humanoid from falling and, when this danger is eliminated, the system can turn to minimizing joint deviations and bringing the motion as close as possible to the reference one. Because of that, in the first phase of compensating larger disturbances, the humanoid's part that is not directly involved in the motion (e.g. arms) have to abandon tracking of the reference trajectories for a while, and, by an energetic action (e.g. of the arms or trunk) 'ensure' dynamic balance, whereby the legs' links do not significantly change their trajectories. After that the system switches to the algorithm similar to that described for case 1, to return fully to the reference motion. In other words, part of the system abandons the realization of internal synergy to prevent overturning and preserve the possibility of walk continuation, and then returns to tracking the external reference synergy.

If the disturbance is of such intensity that it causes instantaneous appearance of unpowered DOFs and overturning of the humanoid, then the overall system has to abandon tracking of internal synergy and attempt to resume dynamic balance, e.g. to step in the direction of system's falling, support on that leg, re-establish dynamic balance equilibrium, and, in the next several steps, return to the reference motion.[(18)]

It should be noted that the disturbance described in case 3 results in the degeneration of the humanoid–ground contact area to a line or point, and the sensory information about contact force is lost. The ZMP, then, does not exist. In order to take any decision about compensational action, even on passing to a new internal synergy, some other sensory information about the relevant coordinates of the instantaneous state of the system is needed, especially about the relative position of the humanoid in its environment. In a similar situation, man receives the necessary information from the equilibrium sensors, visual system, and the like, while in the case of the humanoid similar information could be obtained from, e.g. gyroscope,[22] vision, inclinometer, etc. In any case, additional sensory information is needed.

3.2. *Stability—Basic issues*

Testing the stability of biped locomotion systems is an extremely complex task.[(19)] A special problem represents the pronounced nonlinearity, high dimensionality and strong coupling between the locomotion robot DOFs. As a rule, one can find in the literature examples of stability study of simplified locomotion systems (planar mechanisms with a small number of DOFs), whereby the model is most often linearized.[25] It should be pointed out that stability of the real systems with a large number of DOFs has often been regarded as being identical to the preservation of dynamic balance.[26–31] However, in these cases no attention is paid to the deviations in the tracking of prescribed trajectories at the joints. It should be especially pointed out that dynamic balance may be preserved although the robot does not realize the planned motion. Hence, the authors of this paper think that these two notions (stability and dynamic balance) are not the same.

In view of the fact that the results cannot be directly applied onto real systems, studies of the stability of simplified humanoid models is only justified in the sense of presenting a new method or qualitatively investigating biped locomotion systems. Hence, it is of practical interest to develop methods whose results would be applicable onto real humanoid systems.

Let us summarize the basic characteristics of biped humanoid robots that are important for stability analysis. Of course, one of the most important features is the potential

[17] Such control logic (though with the difference in terminology and designations) can be found in a number of papers. Thus, for example, in ref. 21 we can read: 'The combination of the ideal walking pattern's inertia force and gravity force is called the 'desired total inertia force'. The point on the ground at which the moment of the desired total inertia force becomes zero is called the 'desired zero moment point' or 'desired ZMP'. A ground reaction force acts on both feet of the robot. The point on the ground where the moment of the ATGRF becomes zero is called the 'center of actual total ground reaction force' or 'C-ATGRF.' If the robot is walking in ideal conditions, the desired ZMP and the C-ATGRF will be at the same point. In reality, however, terrain is often irregular. This means that the C-ATGRF may differ from the desired ZMP...'.

[18] The first phase of the robot's compensational action represents in fact the action in a hazardous situation, and one of the ways in which this complex action can be realized[10] is that the biped is prepared in advance for such a case. This can be achieved by preparing in advance a whole series of altenative synergies. When the system is forced to abandon the reference trajectory, the most suitable of them (e.g. the one corresponding to the instantaneous state) is selected, and the system switches to it to re-establish dynamic balance.

[19] An excellent survey of the research in the domain of biped gait modeling, study of stability and control synthesis has been given in refs. 23 and 24. Special attention is given to the problem of including the impact that arises in the transition of the locomotion system from the single-support to the double-support phase in gait modeling and using Poincaré maps for stability analysis.

presence of unpowered DOFs, which cannot and must not be avoided in the stability analysis.

Besides, it should be noted that the specificity of biped gait is the periodic change of the humanoid–environment relationship at the appearance of the impact[(20)] in the transition from the single-support to the double-support gait phase. The basic time cycles within which the system structure remains unchanged are the single-support and the double-support phases, and the two taken together make a step. The phases alternate successively, and the duration of each phase is limited. Therefore, because of this time limitation of the single-support and the double-support phases it can happen sometimes that the disturbances 'have no time' to go out from an admissible range, and because of the change of the step phase (e.g. from the single-support phase the system passes to the double-support phase) the change in the configuration has a stabilizing effect on the system.

To provide an additional illustration of this case, let us assume that at the end of the single-support phase FZMP appear, i.e. the system in a short time interval rotates around the supporting foot toes and 'freely falls' forward. However, the leg that is in the transfer phase is at that instant already stretched and prepared for contact with the ground, so that the system readily 'meets' and realizes the double-support phase, and the walk is normally continued. In this case, the 'unstable phase' of the gait has been practically planned in advance; the system is ready for its realization, so that, strictly speaking, the system is constantly following the reference motion.

The previous case should be distinguished from the appearance of the FZMP as a consequence of the disturbance when the phase of activation of unpowered DOFs has not been planned in advance and occurs unexpectedly. If a large disturbance results in an unplanned appearance of the FZMP and the tracking of the reference motion has to be abandoned, the system's primary concern would be to preserve its dynamic balance. This could be done by an unplanned stepping out, to realize the double-support phase, stabilize the system's state, and return again to tracking the reference motion.

Therefore, it comes out that the stability of biped locomotion systems has to be considered on limited time intervals because each phase and step as a whole has a limited duration. Besides, the alternation of gait phases changes the configuration of the system, so that the stability criterion has to include the changes in the system's configuration.

When speaking of humanoid motion stability it should be primarily borne in mind that this notion is always defined with respect to a certain reference motion.[(21)] Hence, one

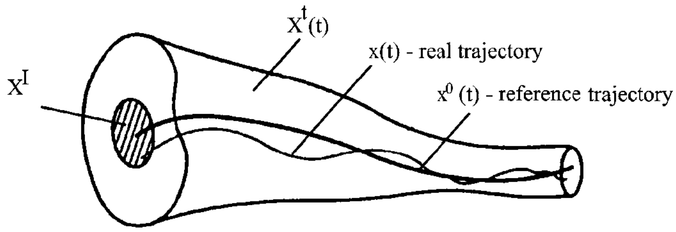

Fig. 2. Finite region in state space around reference trajectory.

can speak of stability only in the case of small disturbances (case 1), as the system constantly follows the reference motion, and for each state coordinate we can define a zone around the reference trajectory that the system must not abandon in the considered time interval.

In cases 2 and 3, to preserve the dynamic balance (which is the task of primary importance) the system (at least in part) *temporarily abandons the tracking of the reference trajectories*. Hence, such motion (even though the dynamic balance has been preserved) cannot be considered as stable. We think that one can say only that the humanoid has preserved its dynamic balance.

In our opinion, of the known definitions of stability, the most appropriate to biped locomotion is the definition of practical stability.[32–35] Practical stability was introduced by Michel,[32] and then adapted and adopted by Vukobratović and Coworkers,[33,35] to investigate stability of strongly coupled manipulation robots with all powered joints.

Here, we describe in brief the notion of practical stability. Let the desired trajectory in the state space be denoted by $x^0(t)$, as shown in Fig. 2. In general, the task of the control system can be defined as the task of transferring the system state from an arbitrary initial state to a defined point in the state space in the course of a predefined time interval. However, in most of the cases, the initial state can belong only to a bounded region in the system state space $\mathbf{X}^\mathbf{I}$, and it is not necessary to transfer the system state to a point. It suffices to transfer it to a bounded region in the state space around the desired point $\mathbf{X}^\mathbf{F}$. The system is considered in the predefined time interval τ, and it is required that the state is transferred from $\mathbf{X}^\mathbf{I}$ to $\mathbf{X}^\mathbf{F}$ for $\tau_S \leq \tau$. It is also required that during the transfer, the system state remains within the bounded region $\mathbf{X}^\mathbf{t}(t)$. Accordingly, the system is considered to be *practically stable* if $\forall x(0) \in \mathbf{X}^\mathbf{I}$ implies $x(t) \in \mathbf{X}^\mathbf{I}$ $\forall t \in \mathbf{T_s}$, where $\mathbf{T_s} = \{t\!:\!t \in (\tau_s, \tau)\}$ and $x(t) \in \mathbf{X}^\mathbf{t}(t)$, $\forall t \in \mathbf{T}$, where $\mathbf{X}^\mathbf{I} \subseteq \mathbf{X}^\mathbf{t}(t)$, and $\mathbf{X}^\mathbf{F} \subseteq \mathbf{X}^\mathbf{t}(t)$. This definition of practical stability is sufficiently broad to include all other feasible definitions of practical stability.

On the basis of the above definition of stability, a method has been developed for the analysis of locomotion systems.[5,35] Its basis is the aggregation-decomposition

[20] In ref. 10, impact arising at the contact of the heel with the ground is called the 'heel strike'.

[21] Let us emphasize once more that the common walk of humans takes place as a fully automatic action, so that even small deviations from ideal motion are compensated for without explicit involvement of conciousness. Such ideal motion is termed 'reference motion'. If the disturbances, which are inevitable in gait realization, are compensated for in the way intending to return the system to the previous reference trajectory, they are considered to be 'small'. In the case of such disturbances, it is possible to examine the system's stability. However, if, due to the action of disturbance the system is forced to switch to a new reference trajectory (e.g. by stepping aside), to eliminate the danger of overturning with the aim of resuming the tracking of the original reference motion, we speak of 'large' disturbances. In this case, one cannot speak about motion stability but only about preserving dynamic balance. In a number of papers, including refs. 23 and 24, no distinction is made between small and large disturbances, which directly leads to equating the notions of dynamic balance and stability.

method via the Lyapunov vector functions in bounded regions of state space, which was originally developed for manipulation robots having all DOFs powered. For application onto the locomotion mechanisms, the method was modified so as to incorporate the models of unpowered DOFs (one powered and the associated unpowered DOF[22] form the model of a, so-called, composite subsystem). In this way, the complete mechanism is included into stability analysis.

It should be emphasized that stability analysis has been performed for the single-support phase only. If we wish to expand the area of investigating the stability of the humanoid robot gait, it is necessary to consider a step as a whole, so that both phases, as well as the transition from one phase to the other are included. Besides, a question regarding the simultaneous realization of additional tasks arises. Very often when the gait is only 'an indispensable auxiliary means' in the realization of a certain manipulation task, e.g. carrying an object, special conditions are imposed on the object's orientation. Fulfillment of the additional tasks by hands in the course of motion is a real assumption. Although it is clear that a prerequisite for a successful realization of the manipulation task is the simultaneous realization of the locomotion task, there is a difference in the 'stringency' of the imposed constraints. For the part of the system that is in direct contact with the manipulated object the requirements imposed on the deviations of the system state coordinates from the reference values differ from those imposed on the part of the system that is 'auxiliary' in the task considered. In the example mentioned, the motion of the legs has a significantly higher freedom of deviation from the reference motion than the hands carrying the object and, what should be especially pointed out, the deviations are defined with respect to the external coordinate frame. This means that, in addition to the stability of the internal synergy, it is necessary to ensure the stability of realization of external synergy, especially in the part of the system that is of particular interest to the task considered.

4. Posture and Postural Stabilization

Posture belongs to the class of notions that seems intuitively quite clear, so that its definition can be rarely found in the literature. In the research during the last several decades, the issue of posture has not occupied a prominent place because a regular gait has always been of primary concern. Besides, there are significant differences in understanding this notion. However, it is to be expected that the importance of posture will naturally be growing in the future, as research and implementation will be increasingly devoted to combined locomotion–manipulation tasks, where posture is of an essential importance.

[22] When defining control of the humanoid, in addition to the need that each powered DOF should tend to its reference trajectory, some powered DOFs are allotted the additional task of preserving the dynamic balance, i.e. preventing the ZMP from becoming FZMP. By their activity, these DOFs should directly influence the behaviour of the unpowered DOFs. In the stability analysis, the models of the unpowered DOFs and the 'belonging' powered DOFs, are united into a composite subsystem, so that the stability of the two DOFs involved is analyzed as a whole.

The main difference in the interpretation of the notion of posture in the literature is that some consider posture as a static[36] and some as a dynamic (in refs. 37 and 38 *posture dynamics* and *posture behavior* are used) notion. Besides, in some cases posture is understood as a certain relative fixed position of humanoid links that move as a whole. Thus, in ref. 39 posture is referred to as the configuration that the humanoid has to assume in the case of falling, to minimize potential damage because of the impact with the ground. An example of posture definition can be found in ref. 38, where it is defined in the following way: 'the posture (is) the robot's self configuration', whereas in ref. 40, posture is defined as 'the position or bearing of the body, whether characteristic or assumed, for a special purpose'. From this undoubtedly follows the dilemma whether posture should encompass all the links or only a part of the humanoid mechanism. Although a certain posture represents relative position of all the mechanism links, some of them can be of particular importance for the character of the posture. Namely, posture can have a certain main feature such as, for example, the upright trunk or a certain position of the arms, for example, head–arms–torso in ref. 41. For all the postures having the same main characteristic they are said to belong to the same type.

Similar to the existence of different interpretations of the notion of posture, the problem of posture control can be considered from different aspects. Certainly, the basic issue when considering posture *per se* is how to preserve it under the action of disturbances. This will be discussed later. Another class of problems appears when posture is considered within the realization of another task, for example, manipulation. Thus in ref. 37, posture is considered in the frame of manipulation tasks, whereby the problem of posture control is defined in the following way: how to change the instantaneous mechanism posture in the desired direction so that the induced accelarations caused by posture change do not affect the task that is realized simultaneously?

From all the above, it comes out that there is a significant nonuniformity in the understanding and interpretation of the notion of posture. Hence, we consider it useful to point out to this diversity and even more useful might be a further unification of the terminology.

Although posture is intuitively associated with a static issue, the authors of this work think that the definition of posture should encompass dynamic case too. Hence, posture in the domain of humanoid robotics should be defined as the *relative position of the links of statically or dynamically balanced humanoid*. Under static balance, we understand the requirement that the projection of the humanoid mass center to the ground plane should be inside the support area, whereas dynamic balance assumes that the ZMP is within this area. Here, a statically balanced posture is called a *static posture* and the dynamically balanced one is called a *dynamic posture*. It should be noted that the definition of static posture does not assume the immobility of the humanoid mechanism (it can move to preserve the static balance, and a dynamically balanced posture can exist only in the case of humanoid motion. Thus, it comes out that a dynamically balanced gait is just a seqeunce of a dynamically balanced postures. Also, the transition from one static posture to another proceeds via the realization of a series of dynamically balanced postures.

Of special interest is the task of posture control. We consider only the posture preservation that is not simultaneously coupled with the realization of some other task, e.g. manipulation. In our understanding, the task of posture control is the preservation of the posture (static or dynamic) under perturbations. Very frequently, this task is called posture stabilization. In view of the above discussion of stability we think it is more correct to use the term 'balance preservation'.

In the case of static posture, the posture control task is reduced to the preservation of static balance, and in the case of dynamic posture to preserving dynamic balance. Preserving a dynamically balanced posture (in fact this is gait control) has already been considered in Section 3.1.

Let us suppose that the system is in the *statically balanced posture*.[(23)] Because of the perturbation action, the position of the projection of the system mass center to the ground surface will be displaced from its balanced position. As in gait stabilization, we can consider different ways of compensating for the disturbance, depending on the disturbance intensity. In gait control, we consider the ways of compensating disturbances of small, medium and high intensities, whereas in the case of posture it is more appropriate to consider two cases: the case when the balance is not lost (low-intensity perturbation) and the case when the humanoid loses its static balance (high-intensity disturbance).

In the case of low-intensity disturbances, the displacement of the projection of the system mass center onto the ground surface is within the 'static-balance margin'[(24)] of the considered posture, so the humanoid need not undertake any action by which the effect of disturbance would be annulled. When the perturbation stops, the humanoid automatically returns to the previous balanced position.

In the case when the perturbation displaces the projection of the system mass center out of the static-balance margin, the control system has to switch to the stabilization algorithm that will ensure that the projection of the system mass center again lies within the static-balance margin. Forms of compensational movements can be different (performed only by the arms, legs, torso or by several links simultaneously in a synergistic manner), and detailed investigations in this domain are to be expected. One of the ways of compensation that has already been known in theory is by striding.[(25)]

Postural control under large disturbances demands the projection of the system mass center to be displaced to a new position, where the static balance is ensured. It can be achieved by changing the stride of the posture. In accordance with our approach to the synthesis of artificial gait, the angle trajectories that bring the system back to static balance (conditionally, it can be called 'postural synergies'), have to be calculated in advance. In the 'postural synergies', the angular trajectories of the legs are prescribed, to obtain an anthropomorphic form of striding, while the dynamic balance of the system is ensured by calculating the trajectories of the upper part of the body in such a way as to ensure that the ZMP position is within the safety zone. By changing the stride, the static-balance margin of the system is changed too, and, when the system finds itself in the new appropriately balanced position, the initial perturbations will be automatically compensated for.[42]

Let us consider now the potential role of posture in the case of the occurrence of large disturbances. Suppose that the humanoid is in the single-support phase and that because of the disturbance the humanoid is not capable of tracking the reference trajectory, and there is a danger of losing its dynamic balance. There are at least two ways in which this problem can be resolved.

In the first way, the humanoid 'practically' stops the motion, re-establishes static balance (provided it is capable of doing that) in the posture that is closest to the position it was when the disturbance occurred (in fact, the current position becomes a posture), and then, from the set of predefined trajectories, the system selects the one that is closest to its current disturbed state and, by tracking it, returns to the reference trajectory, to continue the motion.[42] In other words, after the perturbation, the system passes to a statically balanced posture, and then, through a series of dynamically balanced postures, returns to the reference motion. An alternative solution (provided the system is not capable of realizing a static posture) is that the instantaneous configuration (which is not statically balanced) is 'frozen' and then, by the controlled use of unpowered DOFs, the system passes to a new statically balanced double-support posture, from which, following the new trajectory, it will return to the reference motion.

In the second way, the system after perturbation from its instantaneous state (i.e. instantaneous dynamic posture) passes without stopping through a series of dynamically balanced postures (which belong to the synergy that is selected from the set of the predefined postural synergies) and again reaches the reference trajectory. For the sake of illustration, in Fig. 3 is shown an example (taken from ref. 10, Fig. 2.29) of the set of predefined synergies (only for the ankle and knee joints of the supporting leg) parameterized with respect to the parameters S and T (stride and cadence) that are presented by dashed lines. The actual trajectory of the system is presented by solid line. It is easy to notice the instant of the disturbance occurrence and passing to a new synergy selected from the set of the existing synergies prepared in advance.

Finally, let us summarize the above discussion. Posture, whose study recently has been somewhat neglected, can be of a twofold character: static and dynamic. Although the difference between the static and dynamic postures seems to be quite obvious, it appears that they are also closely related in the case of humanoid locomotion, and especially to the case of combined locomotion–manipulation tasks that have not yet been sufficiently theoretically treated. Hence, this discussion can be considered only as an initial step, whereby

[23] In the case of the posture that has a certain main characteristic, the primary task in its stabilization is to preserve it. To relative positions of the other links, which are not of essential importance, more significant deviations from the initial position may be allowed, whereby the posture is considered preserved.

[24] This term is an analogy to the term 'stability margin'.

[25] Striding is the process of changing the stride (usually making it larger or shorter) of the posture.

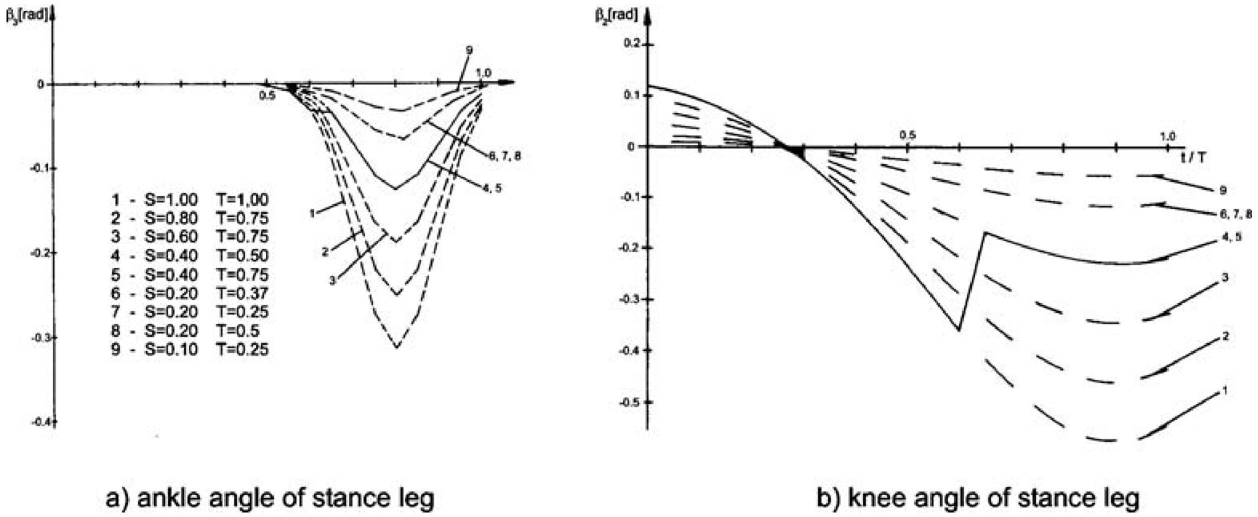

Fig. 3. Prescribed synergies (dashed lines) for the ankle (a) and knee (b). Solid line represents actual system trajectory. (Example taken from [10]).

significant research results are to be expected in the near future.

5. Conclusion

The gait and skeletal activities of humanoids in general, represent a very important and extremely active research direction in humanoid robotics. Hence, it was necessary to address the existence of certain incongruities concerning some basic notions in the sense of the definitions and terms used in this area. For example, there are no clear definitions of walk, gait and even posture. Also, one should also point out the existence of a pretty great diversity in the usage of the terms such as stability, dynamic balance, dynamic equilibrium, postural stabilization, etc.

For a number of years, robots have been in existence not only in industrial plants, at a time their traditional workspace, but also more increasingly in the close living and working environment of humans. This has inevitably imposed the need of the 'working coexistence' of man and robot and their sharing of the living and working space. In view of the fact that no significant rearrangement of human environment are to be expected as a consequence of the presence of robots, they will have to further 'adapt' to the environment previously occupied only by humans. However, in the times to come, one has to anticipate the necessity of joint action of man and robot and take a step in the direction of increasing the comfort of their cooperation. Besides, it is expected that the robots cooperating with humans will exhibit operation efficiency as close as possible to that of humans. Also, the working and living environment, adapted to humans, imposes on robots with their mechanical-control structure at least two classes of tasks: manipulating various objects from the human environment and moving in a specific environment with obstacles like staircases, thresholds, multi-level floors, etc.

In view of the above, this work was intended to contribute to a unified understanding of the basic notions and terms in the domain of humanoid robotics. Despite the fact that humanoid robotics began almost 40 years ago, it has been noticed that some basic notions have been differently (sometimes in an insufficiently correct manner) interpreted by different authors, and that the same terms have been used in different ways. The majority of these terms have been tacitly accepted, probably becuase they were thought of as being intuitively sufficiently clear. Hence, some of them have never been formally defined in the robotic literature. Because of that, the first part of the paper was devoted to defining some basic notions (walk, gait, posture, etc.) since, as far as we know, these notions, despite their fundamental importance in yet understanding gait as a phenomenon, have not yet been explicitely defined. Hence, among others, we considered the notions of dynamic balance and stability; more precisely, we explained the difference between them, as these two essentially very different notions have frequently been equated in the literature. Bearing in mind that dynamic balance is directly related to the ZMP, it was necessary to address some misunderstandings related to this notion. Since motion stability of humanoid robots cannot be considered without including their control, it was necessary to pay due attention to this issue, especially from the aspect of the appearance of unpowered DOFs, which have to be unavoidably included into the analysis of stability of humanoid systems. We sincerely hope that this paper will contribute to a clearer and more uniform understanding of the basic notions in the domain of humanoid robotics, which will certainly lead to reducing the existing and potential future misunderstandings.

The authors were of the opinion that, parallel to indicating some terminological inconsistencies in the extremly rapidly developing domain of high-performance humanoid robots, it was necessary to pay attention to particular examples from the domain of control synthesis, dynamic balance and stability of humanoid robots, whose further development and wider application are yet to come.

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