A CLASSIFICATION OF STABILITY MARGINS FOR WALKING ROBOTS

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ABSTRACT

Throughout the history of walking robots several static and dynamic stability criteria have been defined. Nevertheless, different applications may require different stability criteria and, up to the authors' best knowledge, there is no qualitative classification of such stability measurements. Controlling a robot gait by means of using the wrong stability criterion may prevent the task from succeeding. By the other hand, if the optimum criterion is found the robot gait can also be optimized. In this work, the stability criteria that have been applied to walking robots with at least four legs are examined attending to the stability margin on different static and dynamic situations. As a result, a qualitative classification of stability criteria for walking machines is proposed so that the proper criterion can be chosen for every desired application.

1 INTRODUCTION

Robot stability must be carefully controlled during gait generation. The first generation of walking machines were huge mechanisms composed of heavy limbs too difficult to control [13]. The adoption of statically stable gaits [7] could simplify their control. However, during the motion of the heavy limbs and body some inertial effects and other dynamic components (friction, elasticity, etc.) arise. For this reason, low and constant velocity movements must be performed. Thus, the adoption of static stability limits speed of motion.

Little effort has been done to solve for the dynamic effects that limit statically stable machines' performance [3][5][6][12][15]. However, one of the main goals of the research on legged locomotion is the application of walking robots into industrial processes and transport areas, and such kind of robots are not intended to trot or gallop but walk.

The few dynamic stability criteria defined for quadrupedal walking seem to give different form and name to a unique idea: the sign of the moment around the edges of the support polygon caused by dynamic effects acting over the center of mass of the vehicle. The suitability of each criterion for each particular application (i.e. manipulation forces and moment present, uneven terrain, etc.) is not clear at all. Nevertheless, the use of a stability criterion not suitable for the current application may prevent the task from succeeding. Therefore a qualitative classification of the existing static and dynamic stability criteria for robots of four or more legs is absolutely required.

In this work, the existing stability criteria are briefly reviewed in Section 2. Then, Section 3 describes a comparative study of their stability margins attending to their suitability to measure stability for a number of representative situations. Also, the final qualitative classification of the stability criteria is proposed. Finally, Section 4 presents some conclusions.

2 A REVIEW ON STABILITY CRITERIA

2.1 Static Stability Margins

The first static stability criterion for an ideal machine walking at constant speed along a constant direction and over flat and even terrain was proposed by McGhee and Frank in 1968 [7]. The CG Projection Method claims that the vehicle is statically stable if the horizontal projection of its center of mass (c.o.m.) is inside the support polygon (defined as the convex polygon formed by connecting footprints). Later, this criterion was extended to uneven terrain [8] by redefining the support polygon as the horizontal projection of the real support pattern. The Static Stability Margin, SSM was defined for a given support polygon as the smallest of the distances from the c.o.m. projection to the edges of the support polygon. Approximations to the SSM are the Longitudinal Stability Margin, LSM [17] and the Crab Longitudinal Stability Margin, CLSM [18].

The above stability criteria are all based on geometric concepts; the SSM, LSM and CLSM are independent of the c.o.m. height and do not consider either kinematic or dynamic parameters. It is intuitive that the stability of a non-ideal walking machine should depend on those parameters.

A more convenient stability measurement was proposed by Messuri in 1985 [9]. He defines the Energy Stability Margin, ESM, as the minimum potential energy required to tumble the robot around the edges of the support polygon, that is:

$$S_{ESM} = \min_{i}^{l_s}(mgh_i) \qquad (2.1)$$

where i denotes the segment of the support polygon considered as rotation axis and l_s the number of supporting legs. h_i is the variation of c.o.m. height during the tumble, which comes from:

$$h_i = R_i(1-\cos\theta)\cos\psi \qquad (2.2)$$

where R_i is the distance from the c.o.m. to the rotation axis, θ is the angle that R forms with the vertical axis and ψ is the inclination angle of the rotation axis relative to the horizontal plane.

The ESM gives a qualitative idea of the amount of impact energy supported by the vehicle. Extensions of the ESM were proposed by Nagy in 1991 [10] to consider the foot sinkage on soft and compliant terrain (the Compliant Energy Stability Margin, CESM) and the stabilizing effect of a leg of a foot that is in the air (the Tipover Energy Stability Margin, TESM). For most walking machines the ESM and the TESM coincide because the non-supporting legs are too far from the floor to enhance stability. Only frame-based vehicles will find this stability margin an advantage.

Finally, Hirose *et al.* in 1998 normalize the ESM to the robot weight and propose the Normalized Energy Stability Margin, NESM , defined as [4]:

$$S_{NESM} = \frac{S_{ESM}}{mg} = \min_{i}^{l_s} h_i \tag{2.3}$$

2.2 Dynamic Stability Margins

The first dynamic stability criterion for quadrupeds using crawl gaits was proposed in 1976 by Orin as an extension of the CG Projection Method. The Centre of Pressure Method, COP [11] declares that a robot is dynamically stable if the projection of the c.o.m. along the direction of the resultant force acting on the c.o.m. is inside the support polygon. The Dynamic Stability Margin is thus defined as the smallest distance from the COP to the edges of the support polygon (see also [3]).

Kang *et al.* (1997) lately renamed the COP as the Effective Mass Center, EMC [5], and redefine it as the point on the support plane where the resultant moment due to terrain-reaction forces and moments vanishes, which in the literature of biped robots is commonly named as the Zero Moment Point, ZMP [16].

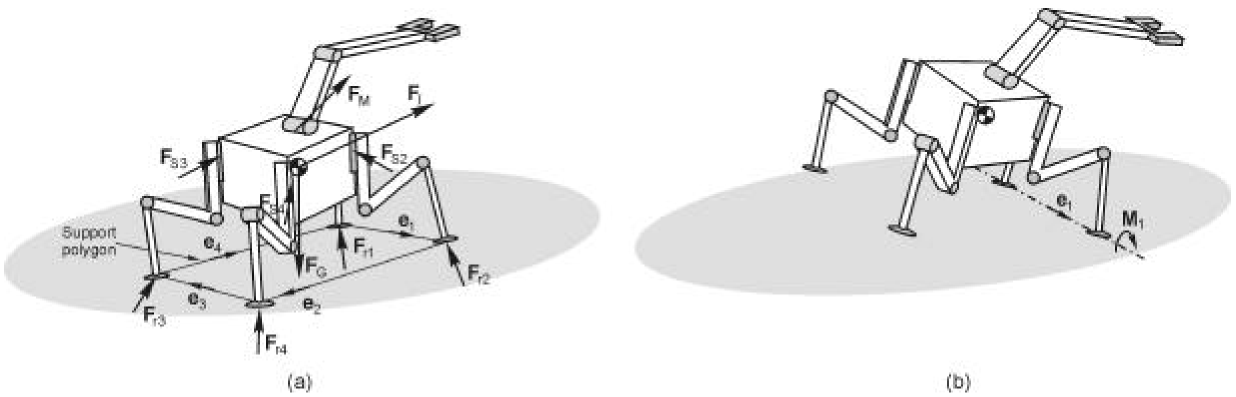


Figure 1: (a) Forces acting on a robot + manipulator system. (b) Robot tumbling around the rotation axis.

At the same time, some momentum-based stability criteria have been defined. Here, only the most meaningful ones are reviewed. The statement is as follows: given a robot + manipulator system as shown in Figure 1(a), the forces and moments acting over the c.o.m. may destabilize it, making the system tumble. The dynamic equilibrium at the c.o.m. requires:

$$\mathbf{F_I} = \mathbf{F_S} + \mathbf{F_G} + \mathbf{F_M} \tag{2.4}$$

$$\mathbf{M_I} = \mathbf{M_S} + \mathbf{M_G} + \mathbf{M_M} \tag{2.5}$$

where subscripts **I**, **S**, **G** and **M** denote inertia, support, gravitational and manipulation effects respectively.

During the tumble the robot loses most of the support feet, remaining only those that conform a rotation axis (see Figure 1(b)). An interaction force $\mathbf{F_R}$ and moment $\mathbf{M_R}$ between robot and terrain exist as the addition of reaction forces at every foot ($\mathbf{F_{ri}}$) and momentum they generate around the c.o.m., respectively. This reaction force and moment generate a moment $\mathbf{M_i}$ about the rotation axis i that must compensate for the destabilizing forces and moments to ensure the system stability. When such compensation is not enough, the system is said to be dynamically unstable.

Based on this statement, Lin and Song (1993) [Lin and Song 1993] redefine the Dynamic Stability Margin, DSM, as the smallest of all moments M_i for every rotation axis in the support polygon normalized by the weight of the system, that is:

$$S_{DSM} = \min_i \frac{\mathbf{e_i} \cdot (\mathbf{F_R} \times \mathbf{P}_i + \mathbf{M_R})}{mg} \tag{2.6}$$

where $\mathbf{P_i}$ is the position vector from the c.o.m. to the i-th support foot and $\mathbf{e_i}$ is the unit vector that goes round the support boundary n the clockwise sense. If all moments are positive (relative to $\mathbf{e_i}$) then the system is stable.

Note that the term DSM is used for both Orin's dynamic stability margin and Lin and Song's criterion, but in this paper, the term DSM will be reserved for Lin and Song's criterion, while Orin's dynamic stability margin will be referred to as the EMC.

Yoneda and Hirose (1997) [Yoneda and Hirose 1997] propose the Tumble Stability Judgment, TSJ, based on the same statement. In the dynamic equilibrium of the system they assume massless legs, so leg-support and foot-reaction forces coincide. Therefore they obtain the resultant reaction force $\mathbf{F_R}$ and moment $\mathbf{M_R}$ from:

$$\mathbf{F_R} = \mathbf{F_I} - \mathbf{F_G} - \mathbf{F_M} \tag{2.7}$$

$$\mathbf{M_R} = \mathbf{M_I} - \mathbf{M_G} - \mathbf{M_M} \tag{2.8}$$

Thus, the moment M_i around the rotation axis is calculated as follows:

$$M_i = \mathbf{M_R} \cdot \mathbf{e_i} + \mathbf{F_R} \times \mathbf{P_i} \cdot \mathbf{e_i} \tag{2.9}$$

Note that the moment calculated by eq. (2.9) is exactly the same that is used in eq. (2.6).

The TSJ claims that the system is dynamically stable if there exists any support foot j in the direction of rotation that prevents the system from tumbling. Then, the Tumble Stability Margin, TSM becomes:

$$S_{TSM} = \min_i \frac{M_i}{mg} \tag{2.10}$$

Recently, Zhou *et al*. (2000) [19] proposed the Leg-end Supporting Moment criterion, LSM. This stability margin is exactly the same as the TSM, but they obtain the resultant force $\mathbf{F_R}$ and moment $\mathbf{M_R}$ from force sensors at the feet.

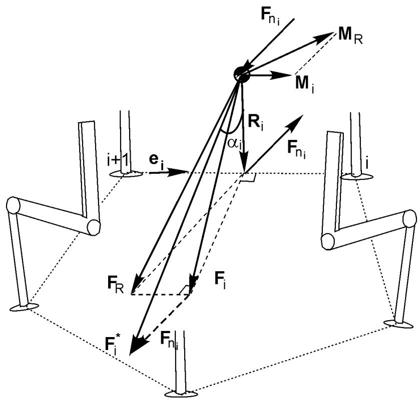


Figure 2: Geometric problem of the Force-Angle stability margin.

Apart from ZMP-based and momentum-based stability criteria a different criterion was proposed by Papadopoulos and Rey (1996) [12]. The Force-Angle stability criterion finds the angle, α_i, between the resultant force acting from the c.o.m. on the ground (the opposite to the reaction force $\mathbf{F_R}$) and the vector $\mathbf{R_i}$, normal to the rotation axis from the c.o.m. (see Figure 2). The system gets unstable when this angle becomes zero. The stability margin is the product of the angle times the resultant force $\mathbf{F_R}$, that is:

$$S_{FASM} = \min_i (a_i \left|\mathbf{F_R}\right|) \tag{2.11}$$

3 COMPARATIVE ANALYSIS OF STABILITY MARGINS

A comparative analysis of the reviewed stability margins has been performed throughout simulation. For this purpose the Yobotics! Simulation construction Set® [14] has been chosen as it provides the suitable tools for dynamic simulations. The model of the SILO4 walking robot has been used [1]. The goal of the analysis is a qualitative classification of stability margins to determine which one is more suitable for each given application.

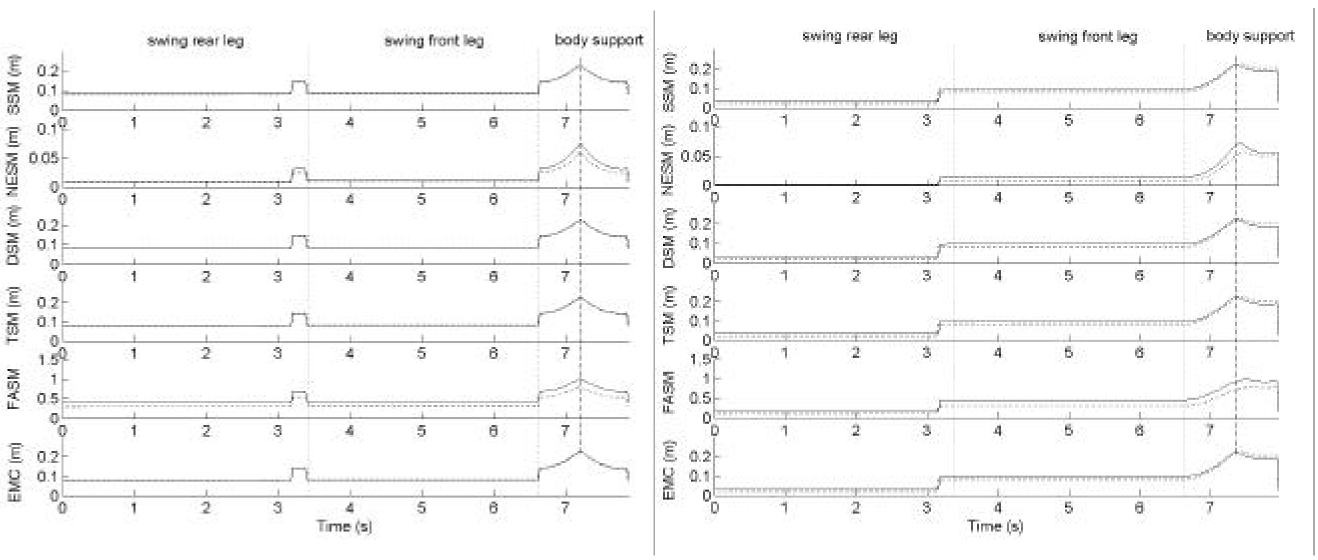
The stability margins that have been selected for the analysis are the SSM, the NESM, the DSM, the TSM, the FASM and the EMC. They have been computed while the robot walks a two-phases discontinuous gait [2] under the following six different terrain and dynamic situations:

CASE 1: Horizontal and even terrain in the absence of dynamics.
CASE 2: Uneven terrain in the absence of dynamics.
CASE 3: Horizontal and even terrain. Inertial and elastic effects arise.
CASE 4: Uneven terrain. Inertial and elastic effects arise.
CASE 5: Horizontal and even terrain. Inertial, elastic and manipulation dynamics arise.
CASE 6: Uneven terrain. Inertial, elastic and manipulation dynamics arise.

The above six case studies represent different situations that a robot can meet during real applications.

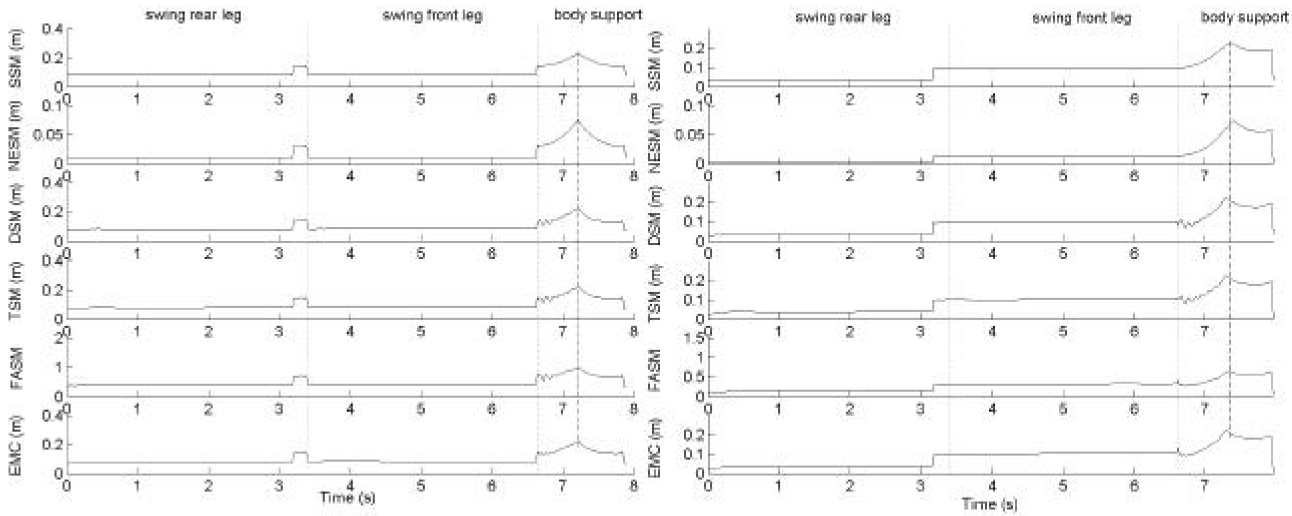
Figure 3(a) and (b) shows one half of the gait cycle for the cases 1 and 2 respectively. The gait phases corresponding to the swing of the rear and front legs precede the body support phase. From both figures it can be observed that the margins SSM, DSM, TSM and EMC coincide. Also, for a different height of the c.o.m. (dotted line) those margins coincide. It is relevant that in the first case study (Figure 3(a)), when the terrain is horizontal and even, these four margins do not vary with the c.o.m. height. This is obviously a drawback of these four criteria because the increase of the c.o.m. height is a destabilizing effect. However, in the second case study, for uneven terrain (see Figure 3(b)), the six margins consider the c.o.m. height. The vertical dashed line inside the body support phase interval points to the instant when the SSM stability margin is maximum, which, over horizontal terrain is one half of the support phase interval. For both case studies that instant coincides for all margins. The NESM and FASM are the only margins that reflect the effect of body height increase over horizontal and even terrain. Thus, they are the only margins that give a successful stability measurement.

Over an inclined surface (see Figure 3(b)), the NESM and FASM differ from the others in the instant of maximum stability. The maximum NESM takes place after SSM, DSM, TSM and EMC (which coincide). Also the maximum FASM occurs even later than NESM. Which of the margins is the best? The NESM is defined as the measurement of impact energy that the system can absorb during the tumble. Thus, when the c.o.m. is placed in the maximum NESM point the possibility of tumbling downhill is equal to uphill (see also [15]). Therefore, the instant of maximum NESM is the optimum one.



(a) CASE 1 (b) CASE 2

Figure 3: Different stability margins in the absence of system dynamics. Solid line for a robot height of 320 mm and dotted line for a robot height of 420 mm. (a) CASE 1: horizontal terrain, (b) CASE 2: terrain inclined 10º from the horizontal plane.



(a) CASE 3 (b) CASE 4

Figure 4: Different stability margins when inertial and elastic effects arise. (a) CASE 3: horizontal terrain, (b) CASE 4: terrain inclined 10º from the horizontal plane.

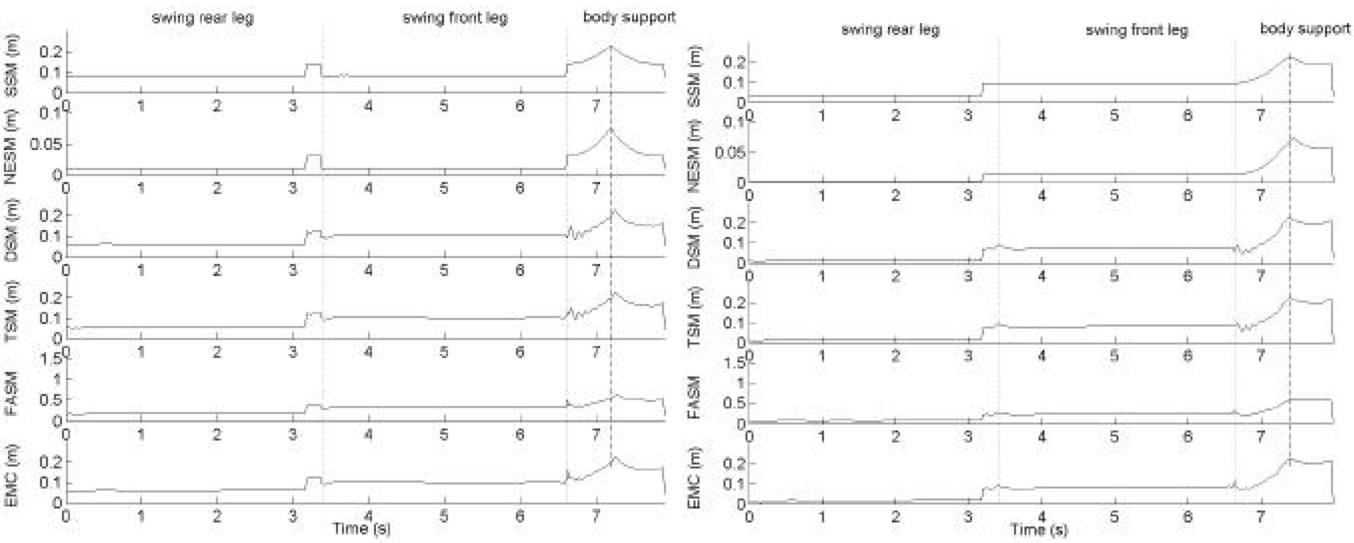
Figure 4(a) and (b) shows one half of the gait cycle for the cases 3 and 4 respectively, which correspond to the existence of inertial effects when the robot walks over horizontal and inclined terrain respectively. Also elastic effects are introduced due to joint elasticity and ground contact effects.

Over horizontal terrain (see Figure 4(a)) all the instants of maximum stability still coincide. However, the DSM, TSM, FASM and EMC reflect some oscillation of the margin due to elasticity. Also inertial effects during the legs transfer phase and body support are reflected. Those dynamic effects are not reflected by the SSM and NESM (just because they are static stability margins).

Therefore, only dynamic stability criterions are valid to judge stability when inertial and elastic effects arise (which is obvious). However, as over horizontal terrain the DSM, TSM and EMC do not consider the effect of height changes, only the FASM is suitable for the case study 3. Nevertheless, if the robot height is not presumed to change, also DSM, TSM and EMC are suitable.

Figure 4(b) represents stability margins for the fourth case study, when the terrain is inclined and inertial and elastic effects arise. While the SSM and NESM do not reflect any reduction of the stability margin due to the existing dynamics (as they are the same as in Figure 4(b)), the DSM, TSM, FASM and EMC reflect an stability decrease. Also their instants of maximum stability occur some time before than in the case study 2, when no dynamics exist.

The maximum stability instant of the FASM occurs later than DSM, TSM and EMC. However, the optimum criterion is the one whose maximum stability instant takes place before FASM and after DSM, TSM and EMC. Therefore, no optimum stability criterion exists for the case study 4.



(a) CASE 5 (b) CASE 6

Figure 5: Different stability margins when inertial, elastic and manipulation effects arise as a 20 N constant force opposing to motion. (a) CASE 5: horizontal terrain, (b) CASE 6: terrain inclined 10º from the horizontal plane.

Figure 5(a) and (b) shows one half of the gait cycle for the cases 5 and 6 respectively, which correspond to the existence of manipulation effects when the robot walks over horizontal and inclined terrain respectively. Also inertial and elastic effects are considered. Both figures show that manipulation forces opposing to motion cause an stability decrease at the rear leg's swing phase and increase at the front leg's swing phase. Also, a delay of the maximum stability instant on the DSM, TSM, FASM and EMC can be observed. It is obvious from the figure that if the manipulation force is increased the robot could be destabilized during the swing of the rear leg. This will never be previewed by the SSM and NESM.

The instant of maximum FASM takes place after the instant of maximum DSM, TSM and EMC. However, the optimum criterion should meet the maximum stability position after DSM and before FASM. Thus, no optimum criteria exists for the fifth case study.

The same can be stated for the last case, when manipulation forces exist and the robot walks over uneven terrain.

Table I resumes a classification of the studied stability margins. The symbol Ö denotes that the criterion is "valid", the symbol ´ denotes "not valid" and the symbol * denotes "optimum". As

Table I: Classification of the existing stability criteria.

Uneven terrain	Inertial effects	Manipulation forces	SSM	NESM	DSM	TSM	FASM	EMC
no	no	no	√	*	√	√	*	√
no	yes	no	×	×	√	√	*	√
no	yes	yes	×	×	√	√	√	√
yes	no	no	√	*	√	√	√	√
yes	yes	no	×	×	√	√	√	√
yes	yes	yes	×	×	√	√	√	√

static stability margins only NESM provide the optimal measurement. However all of them are valid. As dynamic stability margins the SSM and NESM are not valid. When inertial effects arise over horizontal terrain only the FASM provides the optimum measurement, however the rest of dynamic stability criteria are valid. When any other dynamics are present, as manipulation forces and moments, over horizontal or uneven terrain, there is no criterion that provides the optimum margin. Therefore there is no criterion that can assure the stability of a machine under those conditions. Another conclusion of the study is that the DSM, TSM and EMC provide the same measurement for every studied situation.

4 CONCLUSIONS

In this work a comparative analysis of the existing stability criteria has been performed. For this purpose, six case studies have been considered, which cover all the situations that can occur during real industrial applications of legged robots.

As a result, a classification of stability criteria has been proposed. It shows that no optimum criterion exists for every situation studied. Also it has been showed that every momentum-based stability criteria provide the same stability margin, and they are never optima.

This classification enables the election of the proper stability criterion for each real application

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