Comparison of Optimal Control Software Frameworks in the Context of Bipedal Walking

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CROCODDYL - LAAS-CNRS

2.1 Introduction

2.1.1 Motivation

Crocoddyl is an **optimal control library for robot control under contact sequence**. Its solver is based on an efficient Differential Dynamic Programming (DDP) algorithm. Crocoddyl computes optimal trajectories along with optimal feedback gains. It uses Pinocchio for fast computation of robot dynamics and its analytical derivatives [1].

Crocoddyl is focused on multi-contact optimal control problem (MCOP) which has the form:

$$\mathbf{X}^*, \mathbf{U}^* = \begin{Bmatrix} \mathbf{x}_0^*, \cdots, \mathbf{x}_N^* \\ \mathbf{u}_0^*, \cdots, \mathbf{u}_N^* \end{Bmatrix} = \arg\min_{\mathbf{X},\mathbf{U}} \sum_{k=1}^{N} \int_{t_k}^{t_k+\Delta t} l(\mathbf{x}, \mathbf{u}) dt$$

subject to

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}),$$
$$\mathbf{x} \in \mathcal{X}, \mathbf{u} \in \mathcal{U}, \boldsymbol{\lambda} \in \mathcal{K}.$$

where

- the state $\mathbf{x} = (\mathbf{q}, \mathbf{v})$ lies in a manifold, e.g. Lie manifold $\mathbf{q} \in SE(3) \times \mathbb{R}^{n_j}$, n_j being the number of degrees of freedom of the robot.
- the system has underactuacted dynamics, i.e. $\mathbf{u} = (\mathbf{0}, \boldsymbol{\tau})$,
- $\mathcal{X}$, $\mathcal{U}$ are the state and control admissible sets, and
- $\mathcal{K}$ represents the contact constraints.

Note that $\boldsymbol{\lambda} = \mathbf{g}(\mathbf{x}, \mathbf{u})$ denotes the contact force, and is dependent on the state and control.

2.1.2 Features

According to the description in the Github repository [1], it comprises the following features:
Crocoddyl is **versatible**:

- various optimal control solvers (DDP, FDDP, BoxDDP, etc) - single and multi-shooting methods
- analytical and sparse derivatives via Pinocchio
- Euclidian and non-Euclidian geometry friendly via Pinocchio
- handle autonomous and nonautomous dynamical systems
- numerical differentiation support

Crocoddyl is **efficient** and **flexible**:

- cache friendly,
- multi-thread friendly
- Python bindings (including models and solvers abstractions)
- C++ 98/11/14/17/20 compliant
- extensively tested

2.2 How-To

2.2.1 Install

2.2.1.1 Two ways to go

Basically there are existing two ways of installing Crocoddyl:

- Option 1: Installation via the *robotpkg* package manager
- Option 2: Installation from source

I personally would recommend the installation through *robotpkg*, since it preserves you from dealing with the multiple dependencies of Crocoddyl and therefore seems to be the faster approach. Generally you should decide beforehand which python version you want to use. This effects the robotpkg version as well as the export of the PYTHONPATH variable.

2.2.1.2 Installation via robotpkg (preferred)

Steps for installing via robotpkg according to the installation section of [1]

1. Add robotpkg as source repository to apt:

```
sudo tee /etc/apt/sources.list.d/robotpkg.list <<EOF
deb [arch=amd64] http://robotpkg.openrobots.org/wip/packages/debian/pub $(lsb_release -sc) robotpkg
deb [arch=amd64] http://robotpkg.openrobots.org/packages/debian/pub $(lsb_release -sc) robotpkg
EOF
```

2. Register the authentication certificate of robotpkg:

```
curl http://robotpkg.openrobots.org/packages/debian/robotpkg.key | sudo apt-key add -
```

3. You need to run at least once apt update to fetch the package descriptions:

   ```
   sudo apt-get update
   ```

4. The installation of Crocoddyl:

   ```
   sudo apt install robotpkg-py27-crocoddyl # for Python 2
   sudo apt install robotpkg-py36-crocoddyl # for Python 3
   ```

5. Finally you will need to configure your environment variables (watch out for the python version!), e.g.:

   ```
   export PATH=/opt/openrobots/bin:$PATH
   export PKG_CONFIG_PATH=/opt/openrobots/lib/pkgconfig:$PKG_CONFIG_PATH
   export LD_LIBRARY_PATH=/opt/openrobots/lib:$LD_LIBRARY_PATH
   export PYTHONPATH=/opt/openrobots/lib/python3.6/site-packages:$PYTHONPATH
   ```

2.2.1.3 (Installation from source)

If you prefer installing Crocoddyl from source, the following steps should do the work:

```
git clone https://github.com/loco-3d/crocoddyl.git
git submodule update --init
mkdir build && cd build
export PKG_CONFIG_PATH=/opt/openrobots/lib/pkgconfig
cmake -DCMAKE_INSTALL_PREFIX=/opt/openrobots  ..
make
sudo make install
```

Additionally you will have to install the dependent libraries (i.e. pinocchio, example-robot-data (optional for examples, install Python loaders), gepetto-viewer-corba (optional for display), jupyter (optional for notebooks) and matplotlib (optional for examples) and fix the incude paths.

2.2.1.4 For Displaying Results: Additionally Install Gepetto-Viewer

In order to see not just 2-dimensional plots, but also observe the 3D robot behaviour, you additionally have to install the gepetto-viewer.

```
sudo apt install robotpkg-py36-qt4-gepetto-viewer-corba
```

2.2.2 Running the Examples

Since the installation through robotpkg did not provide you with the examples from the git repository, you should clone the repo [1] for getting the data. You do not have to build the library, since it already is installed. In the cloned repository go to */examples*. For running e.g. the bipedal walking example, just type

```
python3 bipedal_walk.py
```

and you will see the calculations for optimal gait trajectories running in the console. The examples provide a *plot* and *display* argument. In order to display the 3D results and also plot some data, just do

```
gepetto-gui
```

for starting the 3D environment. Then, in another terminal, run the example:

```
python3 bipedal_walk.py display plot
```

2.3 Abstract Workflow

For each node (i.e. each timestep) of the optimal control problem,

1. Load robot data (URDF, SRDF, Meshes)
2. Define Action Models (Dynamics+Costs) for running and terminal states
   - Setup a cost model
   - Add the desired cost functions (state, control, frame-placement etc.)
   - Calculate Integrated & Differential Action Model (IAM/DAM) based on the model
3. Define the optimal control problem (knots+IAMs, initPose)
4. Solve the Shooting Problem

2.4 Issues and Insights

2.4.1 Issues Encountered

Since Crocoddyl currently is under active development, there frequently will occure smaller incompatibilities because of versioning issues.This is a brief overview of emerged difficulties:

- Python versioning errors in the examples.

  The examples most often are written for python2, which means that if you are under python3, you will have to adapt some commands (e.g. lists handling, matplotlib, print).

- Crocoddyl versioning errors in the examples.

  Since Crocoddyl depends on other libraries (i.e. Pinnochio, example_robot_data), there sometimes occured errors with the class because they were not updated.

- Confusions displaying the results via the Gepetto-Viewer

  The Gepetto-Viewer is used for displaying the robots and resulting trajectories from optimization. The examples only contain out of the box solutions. If one wants to simply display a robot in some specified pose (e.g. the initial pose) the following, quite unintuitive, commands have to be applied:

```
display = crocoddyl.GepettoDisplay(rh5_legs, 4, 4, frameNames=[rightFoot, leftFoot])
display.display(xs=[x0])
```

2.4.2 Limitations

- No possibility to model 4 contact points per foot since the solver can't properly handle redundant constraints.

2.4.3 Cost Functions

Notes:

- The cost function can contain multiple *cost items* (i.e state/control error, frame displacements or center of mass tracking).
- Weights are considered in the costs via scalar multiplication with the identity matrices (Ix, Ixx etc.) of the according cost item.
- These weighted matrices of cost items are simply summed up within a *costModelContainer*.

2.4.4 Joint Limits

- Input Data: Within the URDF file, for each joint there are specified the
  - torque limit (effort),
  - position limits (lower, upper),
  - velocity limit.
- These limits are not automatically taken into account in Crocoddyl when solving a shooting problem, but explicitly have to be adressed.
- **Torque Limits**: Require the use of specific solvers, standard ddp is not sufficient. Implemented solvers that can handle torque limits explicitly are:
  - BoxDDP (Compare Tassa method [4])
  - BoxFDDP (Novel solver that is under development at LAAS)

  State Limits (Pos/Vel): Position and Velocity limits can be handled via penalization, i.e. added as cost terms to the optimization problem.

2.4.5 Introduction: Bipedal Walking in Crocoddyl

- The desired foot step plan simply is an equally distributed line of knots specified via the desired stepheight and steplength.
- A long walk consists of multiple gaitphases, each phase is a single shooting problem.
- These problems are generated with *createWalkingProblem()* involving one left and one right foot step.
- Each shooting problem contains various locomotion phases
  - Double support at beginning (both legs on ground) via *createSwingFootModel()*
  - Right step (Swing-up and swing-down phase equally distributed) init via *createFootstepModels()*
  - Double support again
  - Left step
- In the end, all knots of all phases are basically one *swingFootModel*. They only vary in the adressed foot, and if a CoM task or a swingFootTask is set.
- The *swingFootModel* is an IAM containing a

- 6D multi-contact model,
- Cost model and
- Differentiation (DAM) and Integration (IAM) routines.

2.4.6 Cost Function for Bipedal Walking

The main cost items (weights in brackets)utilized in the example are

- (1e6) CoM tracking
- (1e6) Foot tracking
- (1e1) Friction cone per contact point
- (1e1) State regression, i.e. penalize joint variance
- (1e-1) Control input regression, i.e. penalize control effort

2.4.7 Multi-Contact Model

2.4.7.1 Previous: Assume foot to have a point contact

- Each *swingFootModel*, meaning each knot of the OC, includes at least one individual contact model.
- The number of contact models depends is specified via *supportFootIds*, i.e. the number of foots that currently are not in the air (e.g. biped: 1 for swing-phase, 2 for double-support).
- For each point of one foot that actually does touch the ground, a contact has to be added to the according contact model (specified via frame ID)
- In the bipedal examples they assume a point-contact, i.e. adding only one contact per supporting foot.
- The point-contact is specified via the name of the according link from the URDF.

2.4.7.2 [aborted] Extension: Consider 4 point contact per foot

The key implementation idea:

- Each contact model for one supporting foot now contains four contact items instead of one
- The cost model contains now four cost items for the four friction cones instead of just one

The simulation did not run at all with this kind of model, neither did the solver showed any convergence. An explanation might be the following:

By providing 4/8 contact items (one support foot, double support while standing) with each 6 constraints (position, rotation) we overconstrain the dynamics of the system. The underlaying dynamics solver from the crocoddyl library does not seem to be able to handle these redundant constraints properly and fails.

2.5 Working with the Examples

This section contains a brief overview of some of the examples that have been modified. Please note that many **additional explanations have been added in the commentaries of the examples**. All figures and additionaly videos are contained within the /media directory of the repo [2].

As stated in the ReadMe of the Repo, Crocoddyl comes with some introductory examples that are written as Jupyter notebooks (.ipynb). While

`examples/notebooks/introduction_to_crocoddyl.ipynb`

offers a more conceptual overview about crocoddyl, other ones represent basic underactuated systems (e.g. Cartpole swing-up, Bipdeal Walking).

2.5.1 Manipulator: Multi-Point Trajectory

The task in the tutorial

`examples/notebooks/arm_manipulation.ipynb`

was to find an optimal trajectory for a manipulator from an initial configuration to a target point (red ball).

2.5.1.1 Extending the Example

I extended this example to a multi-point optimal control problem by considering four targets to reach in a row. This extended example can be found in

`/examples/arm_manipulation_trajectory.py`

Differences to the existing one-target example include:

- Defining an array of the four targets in space
- Setup individual cost functions for each of the sequences
- Setup running and terminal models for the sequences and finally
- Define the shooting problem as row of these sequences
- Optimize the weights of the cost functions successively for the four sequences.

2.5.1.2 Results: Multiple Targets

2.5.2 Talos Legs: Bipedal Walking

The Crocoddyl library offers an example for bipedal walking with the lower body of the Talos [3] Robot. The results of the solved trajectory can be found in Figure 2.2.

2.6 Results: RH5 Legs

Please note once again that all related plots and additional videos can be found in high-quality in the related github repository [2] of this project.

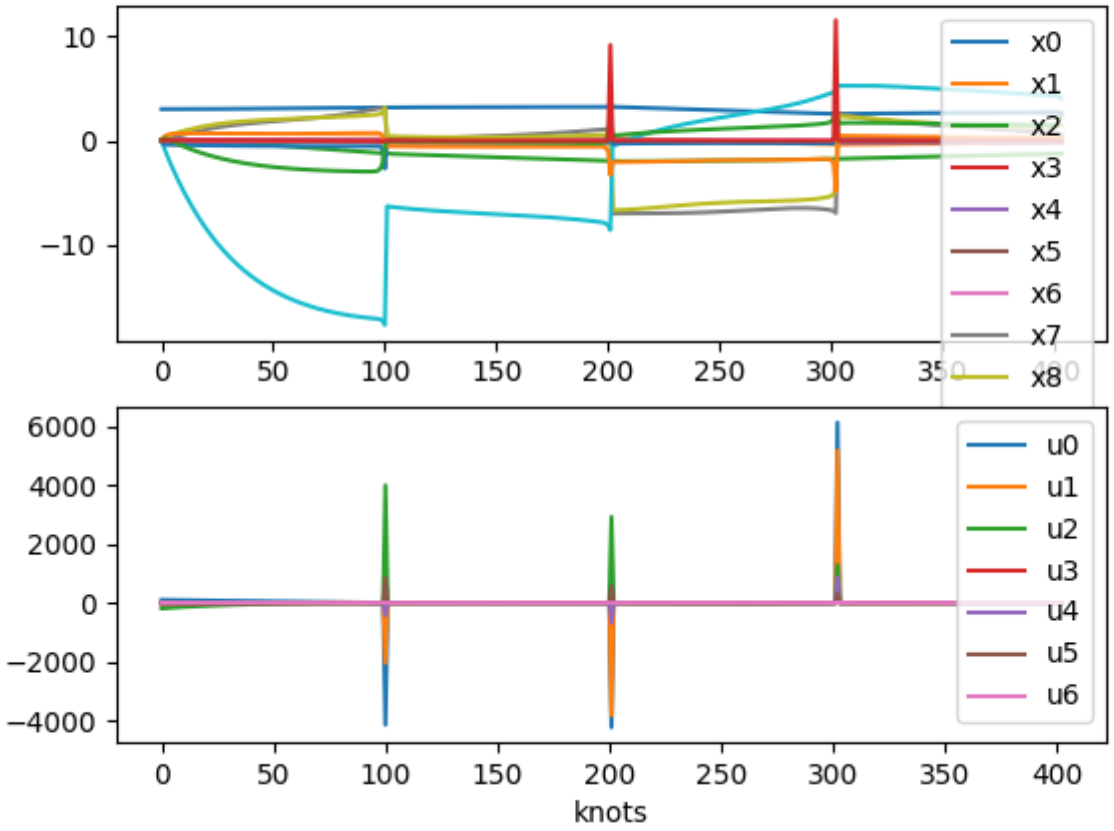


Figure 2.1: Multi-Point Optimal Control of the Manipulator for reaching four targets. The high-control peaks could be handled by a more advanced solver (e.g. box-ddp), but were not performed here.

2.6.1 Navigating the Files

The main file for testing RH5 offers several options for setting up, analyzing and constraining the optimization problem of bipedal walking including

- Choosing a desired URDF
- Specifying gait length and variations in step length/height
- Vary the initial pose of the robot
- Constraining the torque input (Use different solver)
- Defining singular or multiple point contacts per foot (Use different class from biped utils)

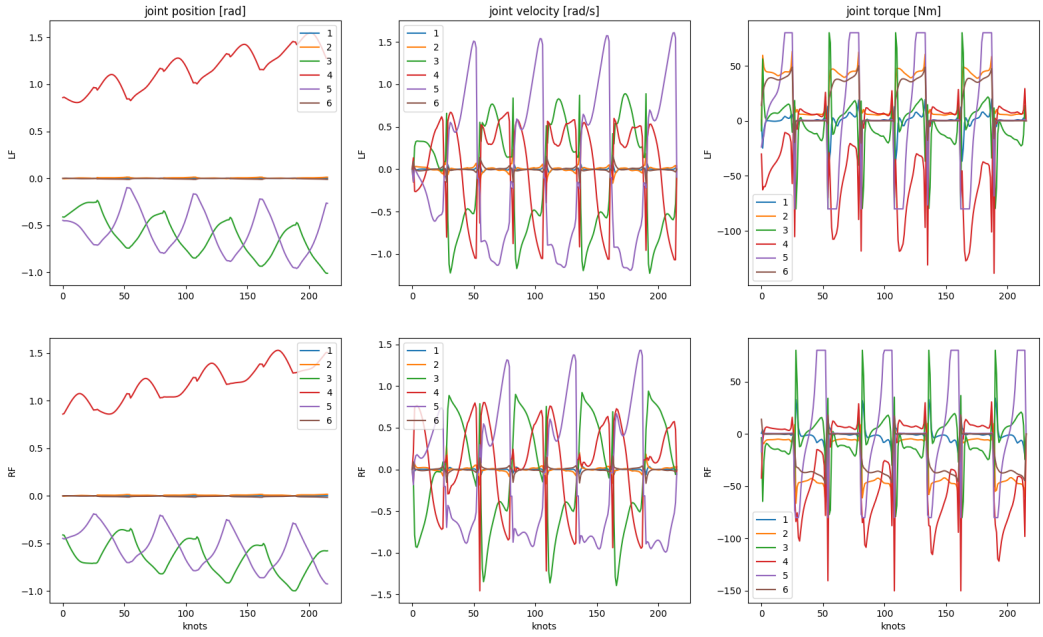


Figure 2.2: Results for the talos legs performing a walk with three gait-phases.

2.6.2 Integration of the RH5 Legs into Crocoddyl

The main issue that had to be solved was related to the underlying URDF file of the robot. In particular:

- Choosing one of our several files (abstract-smurf)
- Cutting out everything apart from the legs and the root joint
- Fixing the mesh file paths:
  We usually define the path relative to the URDF file location, e.g.

  `"../meshes/stl/RH5_Root_Link.001.stl".`

  The integrated URDF parser in Crocoddyl instead, expects the path specified via the package URI, e.g.

  `"package://abstract-smurf/meshes/stl/RH5_Root_Link.001.stl".`

- Adjust the contact frames for the Walking Problem.

2.6.3 Performing a Full Gait

Results for a full gait (6 consecutive steps) are shown in Figure 2.3.

2.6.4 Torque-Constrained Full Gait

It is possible to constrain the input torques via the limits that are parsed from the URDF. However, the correct solver (box-ddp) needs to be applied to the OC problem for considering these limits. Results are shown in Figure 2.4

2.6.5 Initial Pose Variants: Starting Near the Zero Configuration

The full gait has been initialized according to the pose specifications from the RH5 DFKI smurf file. In this and the following section, two solutions for other initial poses are presented: One near the zero configuration and one at the zero configuration.

2.6.6 Initial Pose Variants: Starting At the Zero Configuration

Initializing at the zero configuration turns out to be more critical. Sometimes the solver converges to a feasable solution (Figure 2.6), sometimes it does not find a proper solution (Figure 2.7).

2.6.7 Towards Periodic Joint Trajectories

All previous results share a common pattern: While the trajectories for joint velocity and input torque are mostly periodic, the joint trajectories appear to be not. Especially, the absolute value of several joints increases over time. Investigations have shown that this pattern can be prevented by modifying the cost function. When it comes to joint movements, the state regression penalization is crucial.

Increasing the state regression cost item by one dimension (from 1e1 to 1e2) leads to

- Periodic joint trajectories and
- Bounded joint limits.
- Not reaching the desired step height.

Consequently, this approach is appeared not to be the right way to go. Alternative approaches could include setting up an equality constraint for the initial and terminal pose. The (preliminary) results are shown in Figure 2.8.

2.7 Results: RH5 Legs + Torso

The results from the last section give a first impression, of how optimal control for bipedal walking applied to RH5 can look like. The natural next step is to extend the analysed robot model with the torso. This an especially useful step towards first real-world experiments on the robot, since the computational unit is integrated within the torso and it therefore would be difficult to only control the RH5 legs.

Note on the walking speed of videos and related CoM shifting:

The real walking speed (0.75m/s) is slowed down within the videos for proper analysis of the systems behavior. Since the desired speed is this high, the resulting CoM nearly doesn't shift at all, which is one of the key properties of dynamic walking. The same parameters hold for the simulation results for just the RH5 legs.

2.7.1 Necessary Adjustments

Including the torso in our analysis, we in particular include three additional DoFs (Body-Roll, Body-Pitch, Body-Yaw) that need to be considered. The following adjustments were taken:

- Create an appropriate URDF version based on the abstract-urdf
- Consider the dimensional changes of the joint space within the implementation
- Adjust the initial pose of the robot
- Extend the plotting functionalities for further analysis

2.7.2 Adopted Findings from Previous Analysis of the Legs

Within the previous lection different aspects have been covered. For the continuing analysis we will cherry-pick and apply the most promising ones. In particular, this means to

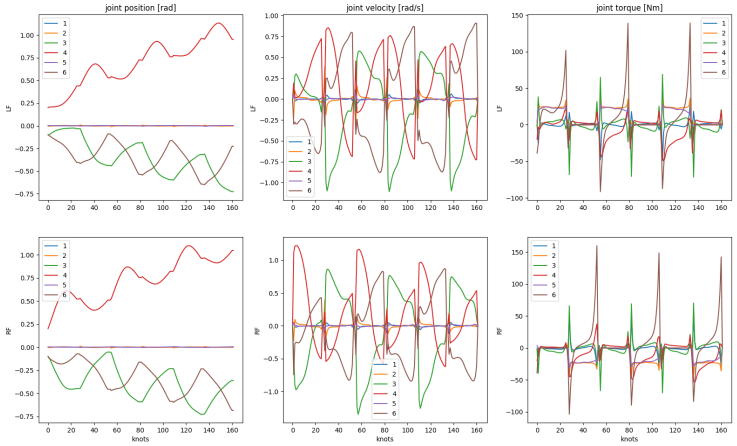
- Performing a full gait (6 consecutive steps),
- Consider bounded input torques, but only to the extend defined in the URDF (no additional artificially reduction)
- Choose an initial pose that is as close as possible to the zero configuration
- Have special emphisis on the state weights in order to achieve periodic joint movements.

2.7.3 Performing a Full Gait

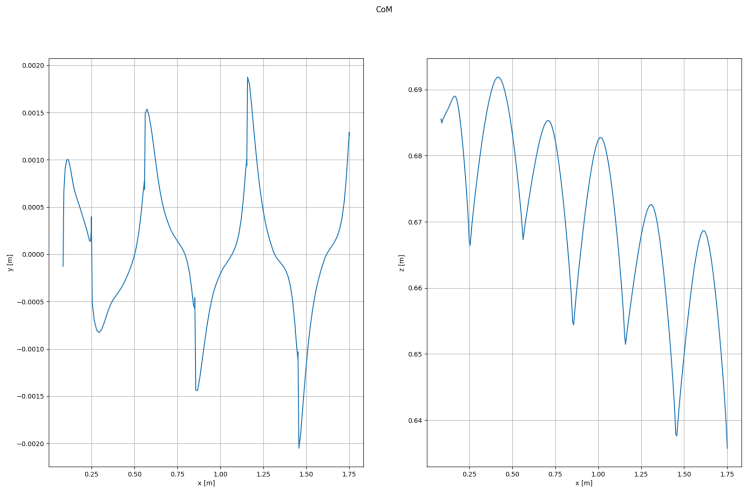
Results for a full gait (6 consecutive steps) are shown in Figure 2.10. This first shot without proper adjustment of the cost functions revealed an interesting pattern: Although stable, the body pitch (joint 2 in the upper plot) increases within each step and therefore is not periodically. As can be seen in the related video, this means the torso is going to 'fall' with each step more into the right direction.

2.7.4 Towards a Stable Torso

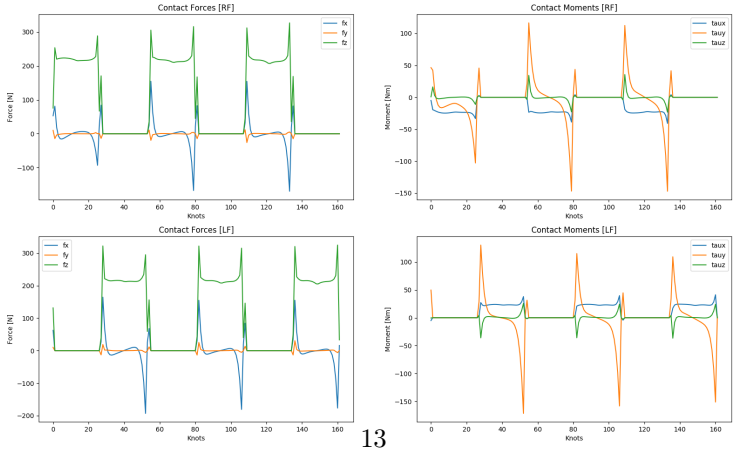
In order to handle the appeared problems with the torso drift during the walk, we have to adjust the cost functions. Further analysis revealed, that a modification of the individual weights for the state regression cost item can lead to a stabilised upper body.



(a) Solution for states and torques.

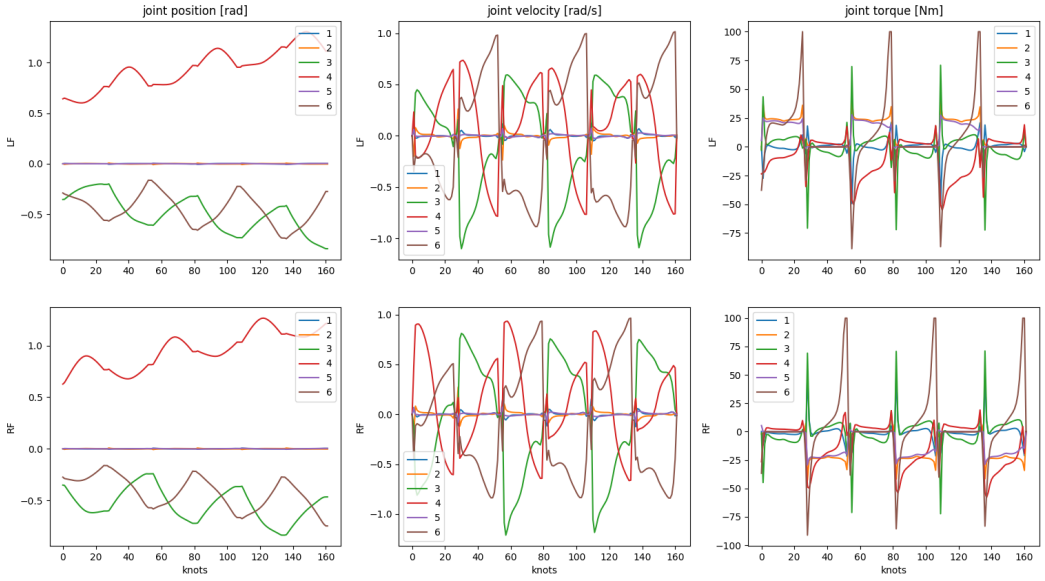


(b) CoM results.

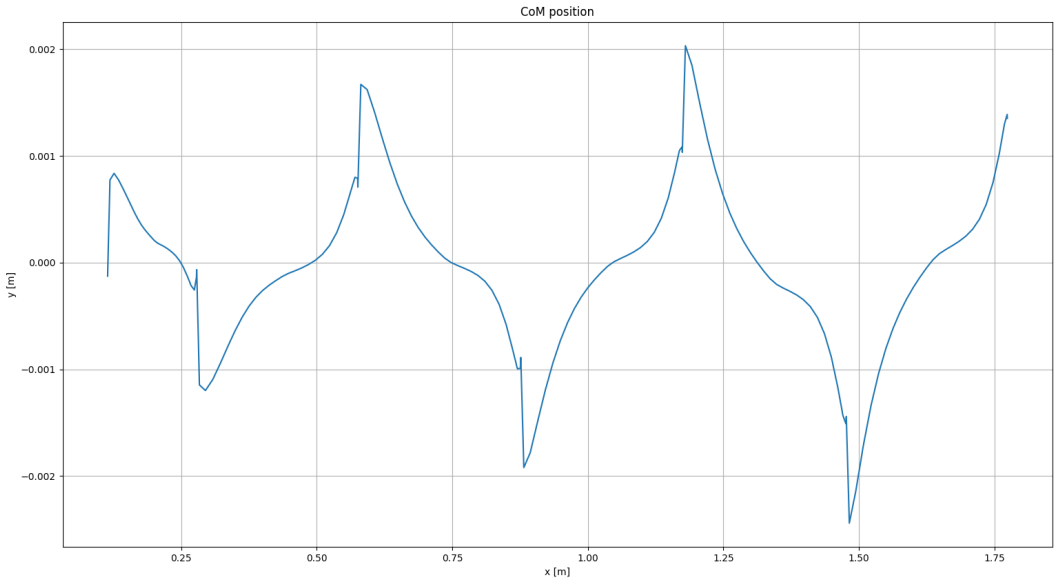


(c) Contact wrenches.

Figure 2.3: Results for a walk with three gait-phases. The walk is implemented as a sequence of three consecutive shooting problems similar to the 2 steps task.

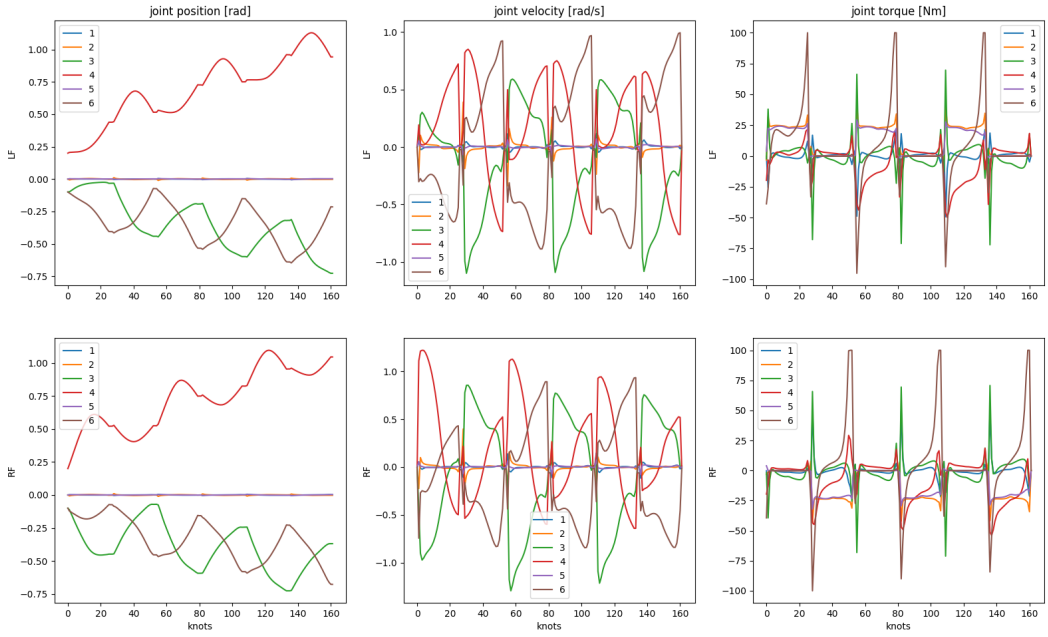


(a) Solution for states and torques.



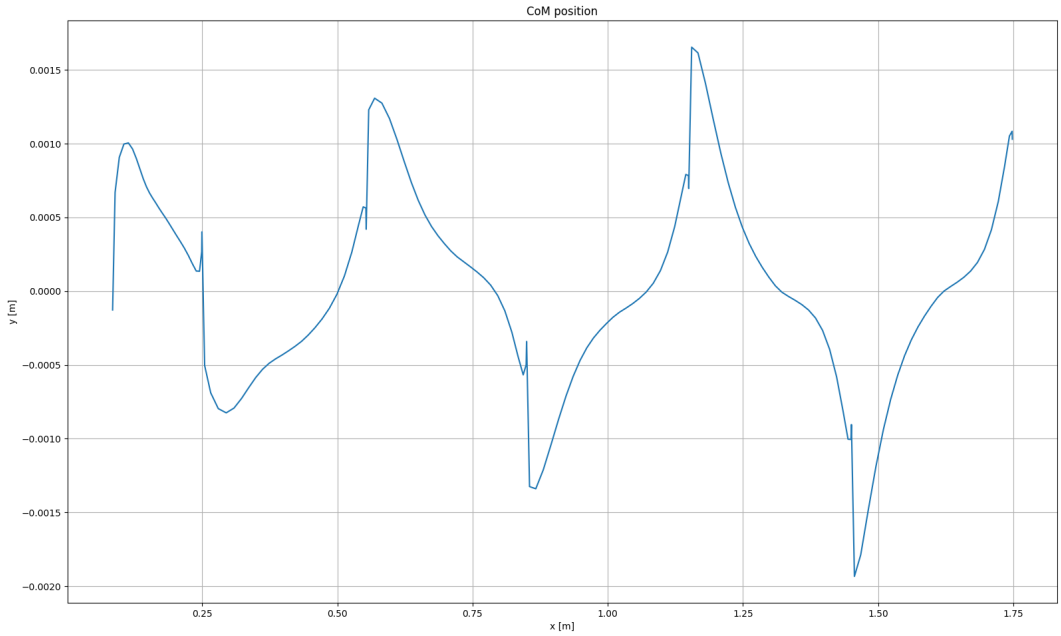
(b) CoM results.

Figure 2.4: Results for full gait with bounded input torques. To be even more restrictive, the available torques form the URDF have been reduced artificially by 50 percent. The effect becomes clear for e.g. the AnkleFT.



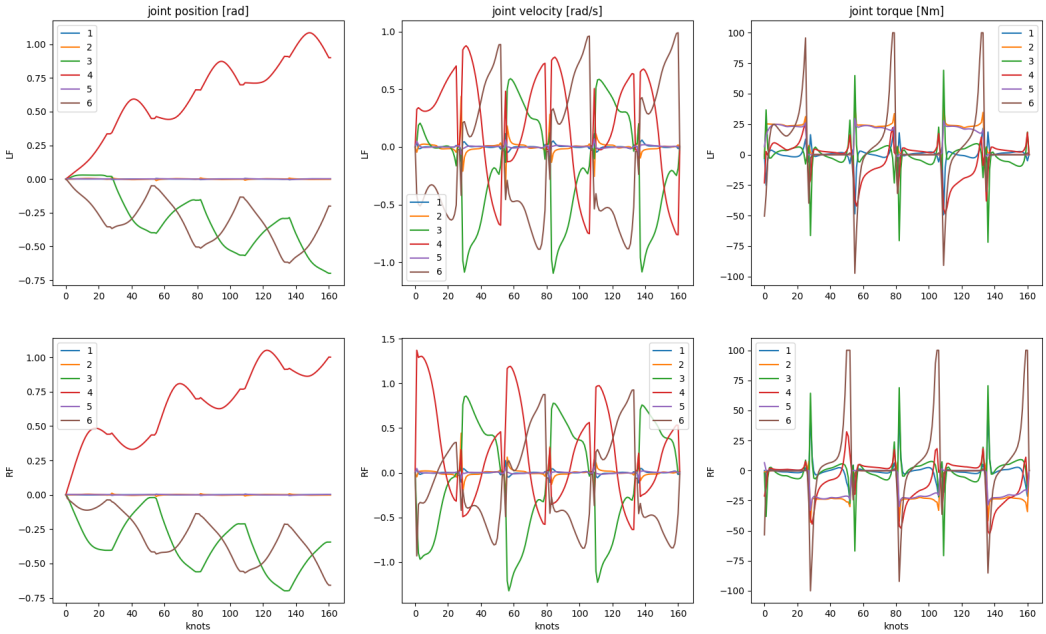
(a) Solution for states and torques

.



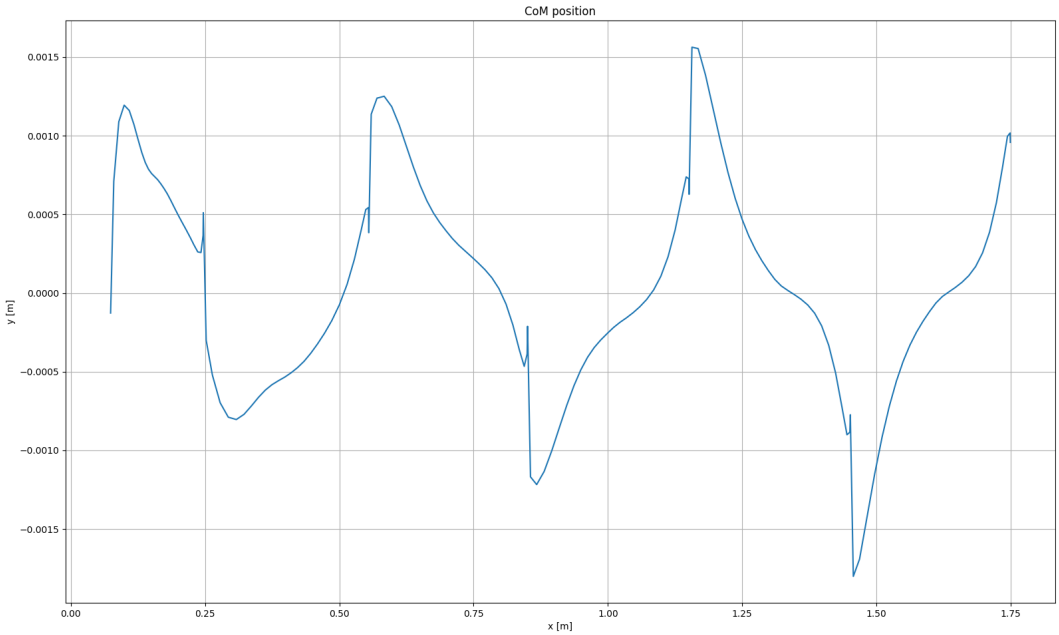
(b) CoM results.

Figure 2.5: Results for the full gait with initial pose **near** the zero configuration. q0=[0,0,-0.1,0.2,0,-0.1, 0,0,-0.1,0.2,0,-0.1] where n=12 and represents the joint angles [rad] for the left and right leg.



(a) Solution for states and torques

.



(b) CoM results.

Figure 2.6: Results for the full gait with initial pose **at** the zero configuration.

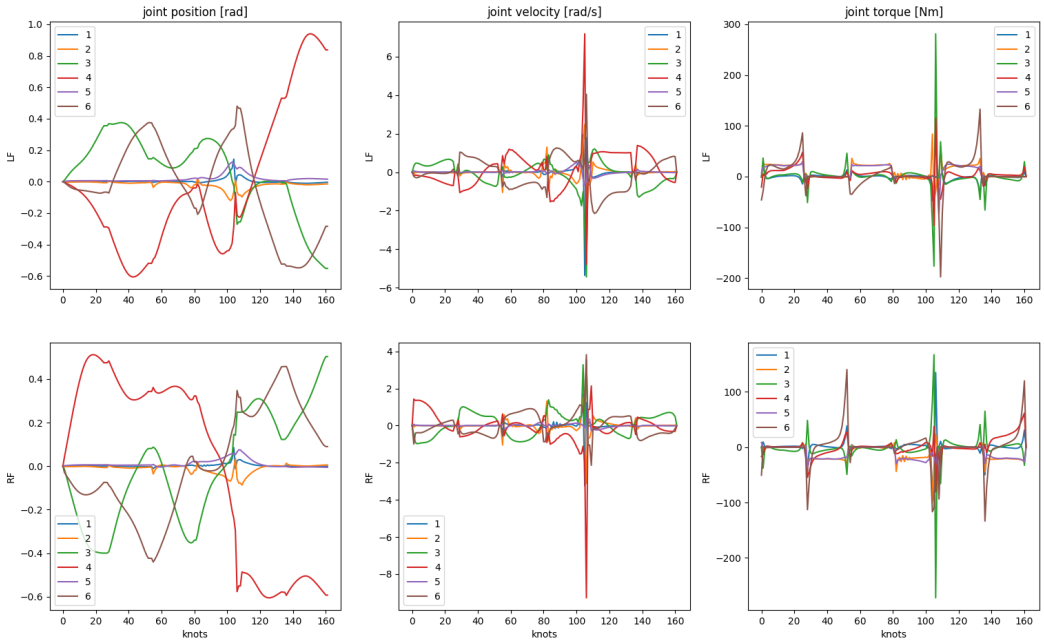
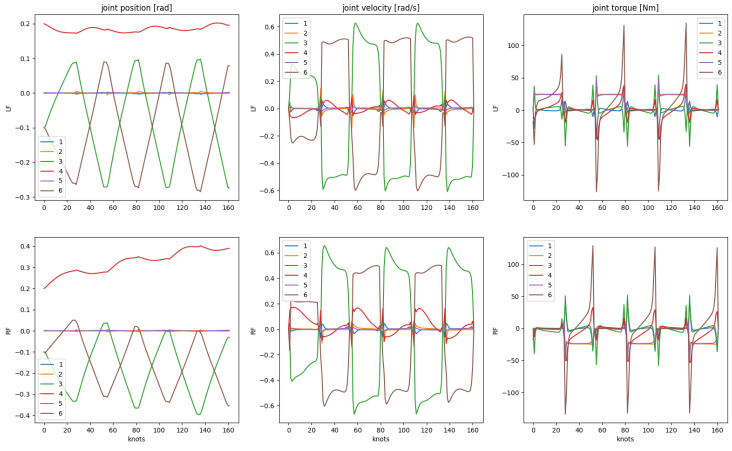
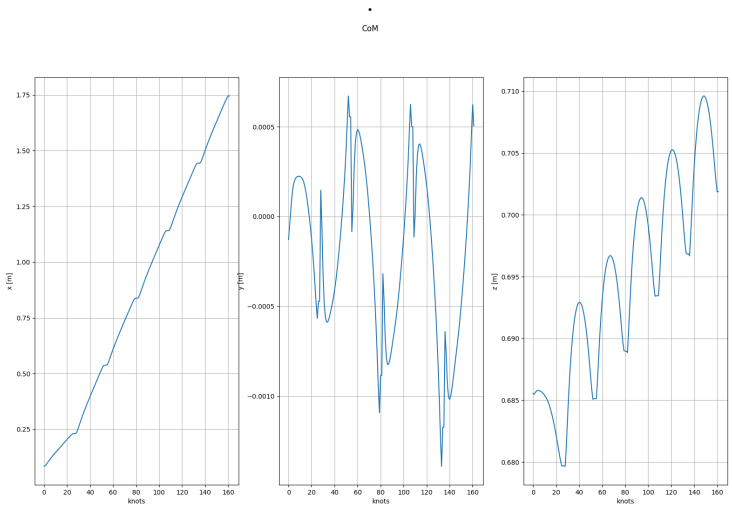


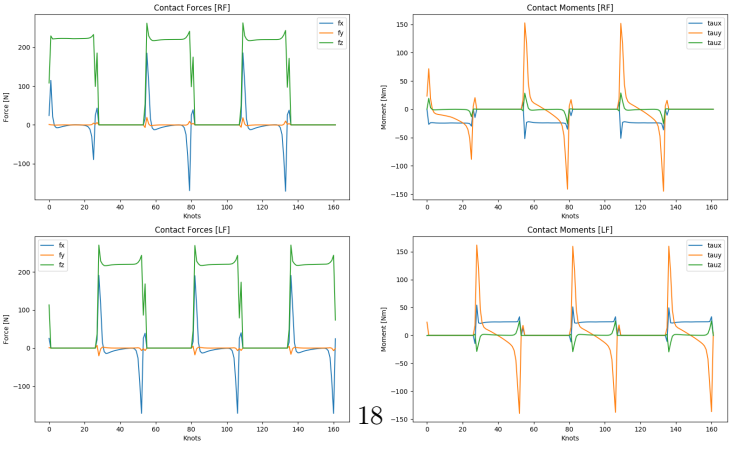
Figure 2.7: Another result for the full gait with initial pose **at** the zero configuration. The solution appears to be instable and would exceed the torque limits (see video).



(a) Solution for states and torques

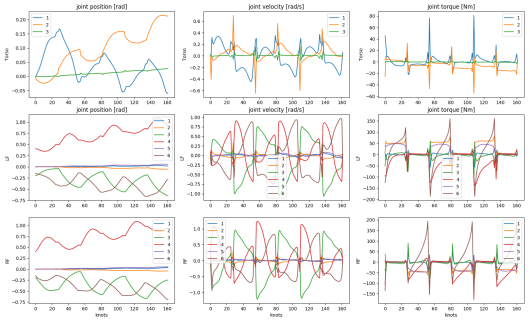


(b) CoM results.

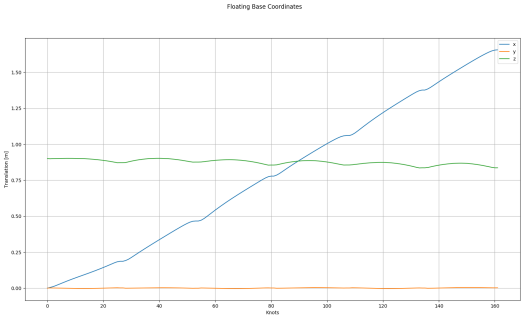


(c) Contact wrenches.

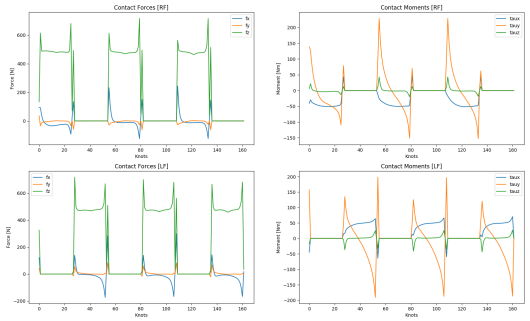
Figure 2.8: Increasing the state regression cost item leads to a more periodic and bounded solution regarding the joint position trajectories.



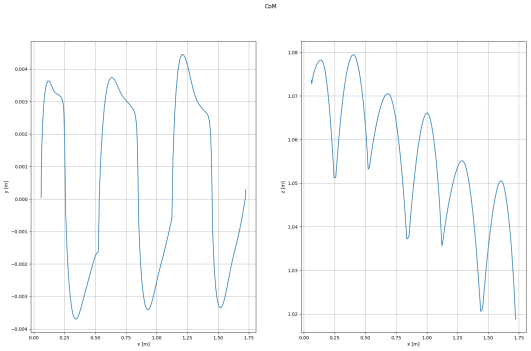
(a) Solution for states and torques.



(b) Floating base.

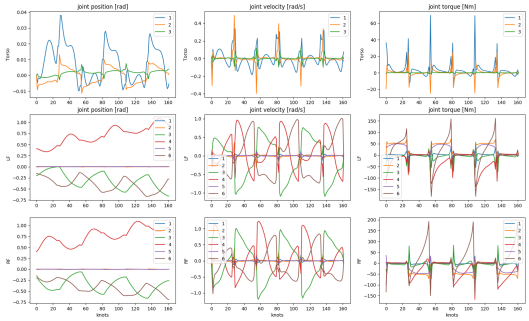


(c) Contact wrenches.

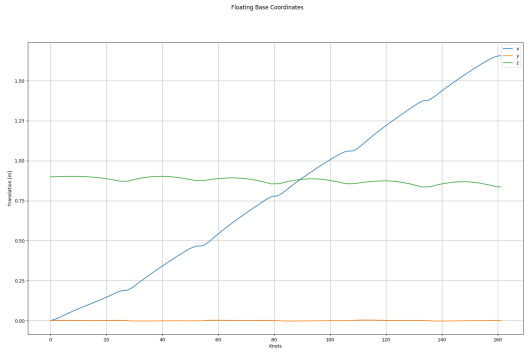


(d) Center of mass.

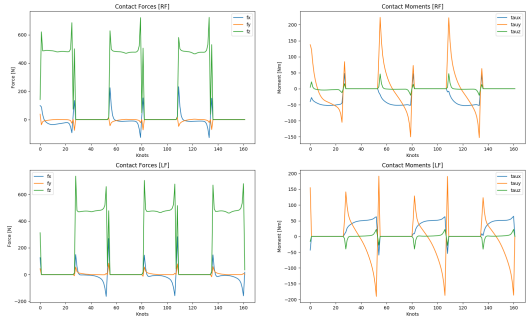
Figure 2.9: Results for the RH5 legs + torso for a full gait (6 steps).



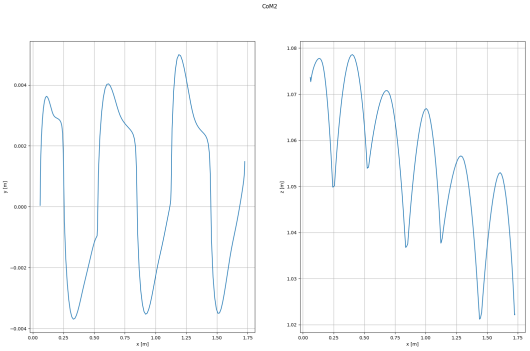
(a) Solution for states and torques.



(b) Floating base.



(c) Contact wrenches.



(d) Center of mass.

Figure 2.10: Results for the RH5 legs + torso for a full gait (6 steps).

Chapter 3

DRAKE - MIT CSAIL

Chapter 4

COMPARISON

Chapter 5

CONCLUSION AND OUTLOOK

Appendix A

Background: Crocoddyl Workflow

This chapter contains details on the workflow in Crocoddyl and presents some of the underlying math. This information can be found in [1] within examples/notebooks/introduction_to_crocoddyl.ipynb.

A.1 Define an Action Model (Dynamics+Costs)

In crocoddyl, an action model combines dynamics and cost models. Each node, in our optimal control problem, is described through an action model. In order to describe a problem, we need to provide ways of computing the dynamics, the cost functions and their derivatives. All these are described inside the action model.

To understand the mathematical aspects behind an action model, let's first get a locally linearize version of our optimal control problem as:

$$\mathbf{X}^*(\mathbf{x}_0), \mathbf{U}^*(\mathbf{x}_0) = \arg\max_{\mathbf{X},\mathbf{U}} = cost_T(\delta\mathbf{x}_N) + \sum_{k=1}^{N} cost_t(\delta\mathbf{x}_k, \delta\mathbf{u}_k)$$

subject to

$$dynamics(\delta\mathbf{x}_{k+1}, \delta\mathbf{x}_k, \delta\mathbf{u}_k) = \mathbf{0},$$

where

$$cost_T(\delta\mathbf{x}) = \frac{1}{2}\begin{bmatrix}1\\ \delta\mathbf{x}\end{bmatrix}^\top \begin{bmatrix}0 & \mathbf{l_x}^\top\\ \mathbf{l_x} & \mathbf{l_{xx}}\end{bmatrix}\begin{bmatrix}1\\ \delta\mathbf{x}\end{bmatrix}$$

$$cost_t(\delta\mathbf{x}, \delta\mathbf{u}) = \frac{1}{2}\begin{bmatrix}1\\ \delta\mathbf{x}\\ \delta\mathbf{u}\end{bmatrix}^\top \begin{bmatrix}0 & \mathbf{l_x}^\top & \mathbf{l_u}^\top\\ \mathbf{l_x} & \mathbf{l_{xx}} & \mathbf{l_{ux}}^\top\\ \mathbf{l_u} & \mathbf{l_{ux}} & \mathbf{l_{uu}}\end{bmatrix}\begin{bmatrix}1\\ \delta\mathbf{x}\\ \delta\mathbf{u}\end{bmatrix}$$

$$dynamics(\delta\mathbf{x}_{k+1}, \delta\mathbf{x}_k, \delta\mathbf{u}_k) = \delta\mathbf{x}_{k+1} - (\mathbf{f_x}\delta\mathbf{x}_k + \mathbf{f_u}\delta\mathbf{u}_k)$$

where an action model defines a **time interval** of this problem:

- $actions = dynamics + cost$

Important notes:

- An action model describes the dynamics and cost functions for a node in our optimal control problem.

- Action models lie in the discrete time space.
- For debugging and prototyping, we have also implemented numerical differentiation (NumDiff) abstractions.

These computations depend only on the definition of the dynamics equation and cost functions. However to asses efficiency, crocoddyl uses **analytical derivatives** computed from Pinocchio.

A.2 Differential Action Model

Optimal control solvers require the time-discrete model of the cost and the dynamics. However, it's often convenient to implement them in continuous time (e.g. to combine with abstract integration rules). In crocoddyl, this continuous-time action models are called "Differential Action Model (DAM)". And together with predefined "Integrated Action Models (IAM)", it possible to retrieve the time-discrete action model.
At the moment, we have:

- a simpletic Euler and
- a Runge-Kutte 4 integration rules.

An optimal control problem can be written from a set of DAMs as:

$$\mathbf{X}^*(\mathbf{x}_0), \mathbf{U}^*(\mathbf{x}_0) = \arg\max_{\mathbf{X},\mathbf{U}} = cost_T(\delta\mathbf{x}_N) + \sum_{k=1}^{N} \int_{t_k}^{t_k+\Delta t} cost_t(\delta\mathbf{x}_k, \delta\mathbf{u}_k)dt$$

subject to

$$dynamics(\delta\mathbf{x}_{k+1}, \delta\mathbf{x}_k, \delta\mathbf{u}_k) = \mathbf{0},$$

where

$$cost_T(\delta\mathbf{x}) = \frac{1}{2}\begin{bmatrix}1\\ \delta\mathbf{x}\end{bmatrix}^\top \begin{bmatrix}0 & \mathbf{l_x}^\top\\ \mathbf{l_x} & \mathbf{l_{xx}}\end{bmatrix}\begin{bmatrix}1\\ \delta\mathbf{x}\end{bmatrix}$$

$$cost_t(\delta\mathbf{x}, \delta\mathbf{u}) = \frac{1}{2}\begin{bmatrix}1\\ \delta\mathbf{x}\\ \delta\mathbf{u}\end{bmatrix}^\top \begin{bmatrix}0 & \mathbf{l_x}^\top & \mathbf{l_u}^\top\\ \mathbf{l_x} & \mathbf{l_{xx}} & \mathbf{l_{ux}}^\top\\ \mathbf{l_u} & \mathbf{l_{ux}} & \mathbf{l_{uu}}\end{bmatrix}\begin{bmatrix}1\\ \delta\mathbf{x}\\ \delta\mathbf{u}\end{bmatrix}$$

$$dynamics(\delta\dot{\mathbf{x}}, \delta\mathbf{x}, \delta\mathbf{u}) = \delta\dot{\mathbf{x}} - (\mathbf{f_x}\delta\mathbf{x} + \mathbf{f_u}\delta\mathbf{u})$$

Optimal control solvers often need to compute a quadratic approximation of the action model (as previously described); this provides a search direction (computeDirection). Then it's needed to try the step along this direction (tryStep).
Typically calc and calcDiff do the precomputations that are required before computeDirection and tryStep respectively (inside the solver). These functions update the information of:

- **calc**: update the next state and its cost value

$$\delta\dot{\mathbf{x}}_{k+1} = \mathbf{f}(\delta\mathbf{x}_k, \mathbf{u}_k)$$

- **calcDiff**: update the derivatives of the dynamics and cost (quadratic approximation)

$$\mathbf{f_x}, \mathbf{f_u} \quad (dynamics)$$

$$\mathbf{l_x}, \mathbf{l_u}, \mathbf{l_{xx}}, \mathbf{l_{ux}}, \mathbf{l_{uu}} \quad (cost)$$

A.3 Integrated Action Model

General speaking, the system's state can lie in a manifold M where the state rate of change lies in its tangent space $T_{\mathbf{x}}M$. There are few **operators that needs to be defined** for different rutines inside our solvers:

- $\mathbf{x}_{k+1} = integrate(\mathbf{x}_k, \delta\mathbf{x}_k) = \mathbf{x}_k \oplus \delta\mathbf{x}_k$
- $\delta\mathbf{x}_k = difference(\mathbf{x}_{k+1}, \mathbf{x}_k) = \mathbf{x}_{k+1} \ominus \mathbf{x}_k$

where $\mathbf{x} \in M$ and $\delta\mathbf{x} \in T_{\mathbf{x}}M$.
And we also need to defined the **Jacobians** of these operators with respect to the first and second arguments:

- $\frac{\partial \mathbf{x} \oplus \delta\mathbf{x}}{\partial \mathbf{x}}, \frac{\partial \mathbf{x} \oplus \delta\mathbf{x}}{\partial \delta\mathbf{x}} = Jintegrante(\mathbf{x}, \delta\mathbf{x})$
- $\frac{\partial \mathbf{x}_2 \ominus \mathbf{x}_2}{\partial \mathbf{x}_1}, \frac{\partial \mathbf{x}_2 \ominus \mathbf{x}_1}{\partial \mathbf{x}_1} = Jdifference(\mathbf{x}_2, \mathbf{x}_1)$

For instance, a state that lies in the Euclidean space will have the typical operators:

- $integrate(\mathbf{x}, \delta\mathbf{x}) = \mathbf{x} + \delta\mathbf{x}$
- $difference(\mathbf{x}_2, \mathbf{x}_1) = \mathbf{x}_2 - \mathbf{x}_1$
- $Jintegrate(\cdot, \cdot) = Jdifference(\cdot, \cdot) = \mathbf{I}$

These defines are encapsulated inside the State class. **For Pinocchio models, we have implemented the StatePinocchio class which can be used for any robot model.**

A.4 Solving the Optimal Control Problem

Our optimal control solver interacts with a defined ShootingProblem. A **shooting problem** represents a **stack of action models** in which an action model defines a specific node along the OC problem.
First we need to create an action model from DifferentialFwdDynamics. We use it for building terminal and running action models. In this example, we employ an simpletic Euler integration rule.
Next we define the set of cost functions for this problem. One could formulate

- Running costs (related to individual states)
- Terminal costs (related to the final state)

in order to penalize, for example, the state error, control error, or end-effector pose error.
Onces we have defined our shooting problem, we create a DDP solver object and pass some callback functions for analysing its performance.

A.5 Application to Bipedal Walking

In crocoddyl, we can describe the multi-contact dynamics through holonomic constraints for the support legs. From the Gauss principle, we have derived the model as:

$$\begin{bmatrix} \mathbf{M} & \mathbf{J}_c^\top \\ \mathbf{J}_c & \mathbf{0} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{v}} \\ -\boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\tau} - \mathbf{h} \\ -\dot{\mathbf{J}}_c \mathbf{v} \end{bmatrix}$$

.

This DAM is defined in "DifferentialActionModelFloatingInContact" class.

Given a predefined contact sequence and timings, we build per each phase a specific multi-contact dynamics. Indeed we need to describe **multi-phase optimal control problem**. One can formulate the multi-contact optimal control problem (MCOP) as follows:

$$\mathbf{X}^*, \mathbf{U}^* = \begin{Bmatrix} \mathbf{x}_0^*, \cdots, \mathbf{x}_N^* \\ \mathbf{u}_0^*, \cdots, \mathbf{u}_N^* \end{Bmatrix} = \arg\min_{\mathbf{X},\mathbf{U}} \sum_{p=0}^{P} \sum_{k=1}^{N(p)} \int_{t_k}^{t_k+\Delta t} l_p(\mathbf{x}, \mathbf{u}) dt$$

subject to

$$\dot{\mathbf{x}} = \mathbf{f}_p(\mathbf{x}, \mathbf{u}), \text{for } t \in [\tau_p, \tau_{p+1}]$$

$$\mathbf{g}(\mathbf{v}^{p+1}, \mathbf{v}^p) = \mathbf{0}$$

$$\mathbf{x} \in \mathcal{X}_p, \mathbf{u} \in \mathcal{U}_p, \boldsymbol{\lambda} \in \mathcal{K}_p.$$

where $\mathbf{g}(\cdot, \cdot, \cdot)$ describes the contact dynamics, and they represents terminal constraints in each walking phase. In this example we use the following **impact model**:

$$\mathbf{M}(\mathbf{v}_{next} - \mathbf{v}) = \mathbf{J}_{impulse}^T$$

$$\mathbf{J}_{impulse}\mathbf{v}_{next} = \mathbf{0}$$

$$\mathbf{J}_c\mathbf{v}_{next} = \mathbf{J}_c\mathbf{v}$$

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