Methods of finding electric potential maps to interpolated flow data

Jules van Irsel

October 6, 2023

This document outlines three choices of fitting an electric potential map to a linearly interpolated, topside plasma flow field, $\mathbf{v}$. This interpolated flow field is generated by some means, in this case via the replication, translation, scaling, and rotation of isinglass flow data. The three choice are as follows:

1. **Potential fit**: A least-squares fitting algorithm (MATLAB's `lsqcurvefit` from its *Optimization Toolbox*) that fits a potential map, $\phi(x, y)$, such that

$$\min_{\phi} \|-\nabla\phi - \mathbf{E}\|_2^2 = \min_{\phi} \sum_{i,j} \Big(-(\nabla\phi)_{ij} - \mathbf{E}_{ij}\Big)^2 \tag{1}$$

   where $\mathbf{v} = \mathbf{E} \times \mathbf{B}/B^2$.

2. **Averaged path-integrated**: The average of several path-integrated potential maps in an attempt to drown out the non-conservative part of the interpolated field, i.e.

$$\phi_i(x, y) = \int_{x_i}^{x} \mathrm{d}x' \mathbf{E}(x', y_i) \cdot \hat{x} + \int_{y_i}^{y} \mathrm{d}y' \mathbf{E}(x, y') \cdot \hat{y} \tag{2}$$

$$\tag{3}$$

   where we take $\phi(x, y) = \langle \phi_i(x, y) \rangle$.

3. **Helmholtz decomposition**: We use the Fourier transform method for Helmholtz decomposition where we define $\mathbf{G}(k_x, k_y)$ to be the Fourier transform of $\mathbf{E}$, i.e.

$$\mathbf{E}(x, y) = \int_{-\infty}^{+\infty} \mathrm{d}k_x \int_{-\infty}^{+\infty} \mathrm{d}k_y \mathbf{G}(k_x, k_y) e^{i(xk_x + yk_y)} \tag{4}$$

   and we define $G_\phi(k_x, k_y)$ by the wave vector projection of

$$G_\phi(k_x, k_y) = i\frac{\mathbf{k} \cdot \mathbf{G}(k_x, k_y)}{\|\mathbf{k}\|^2} \tag{5}$$

   such that

$$\phi(x, y) = \int_{-\infty}^{+\infty} \mathrm{d}k_x \int_{-\infty}^{+\infty} \mathrm{d}k_y G_\phi(k_x, k_y) e^{i(xk_x + yk_y)} \tag{6}$$

   Note: This solution picks out a potential map whose gradient is a non-unique irrotational electric field, i.e. $-\nabla\phi = \mathbf{E} + \nabla f$ where $\nabla^2 f = 0$.

Along with the interpolated flow field (column 1), these three options are shown in Figure 1 (columns 2-3). The pros and cons with each methods are as follows:

Method	**Pros**	**Cons**
Potential fit	Easiest to justify being the "best" fit.	Can take up to 5 hours on Discovery with 128 × 256 cells.
Averaged path-int.	Easy to explain and faster than Potential fit.	Assumption of random-noise divergence is poor.
Helmholtz decomp.	Fastest of the three and it matches closest to Potential fit.	We're not sure why it fits so well.

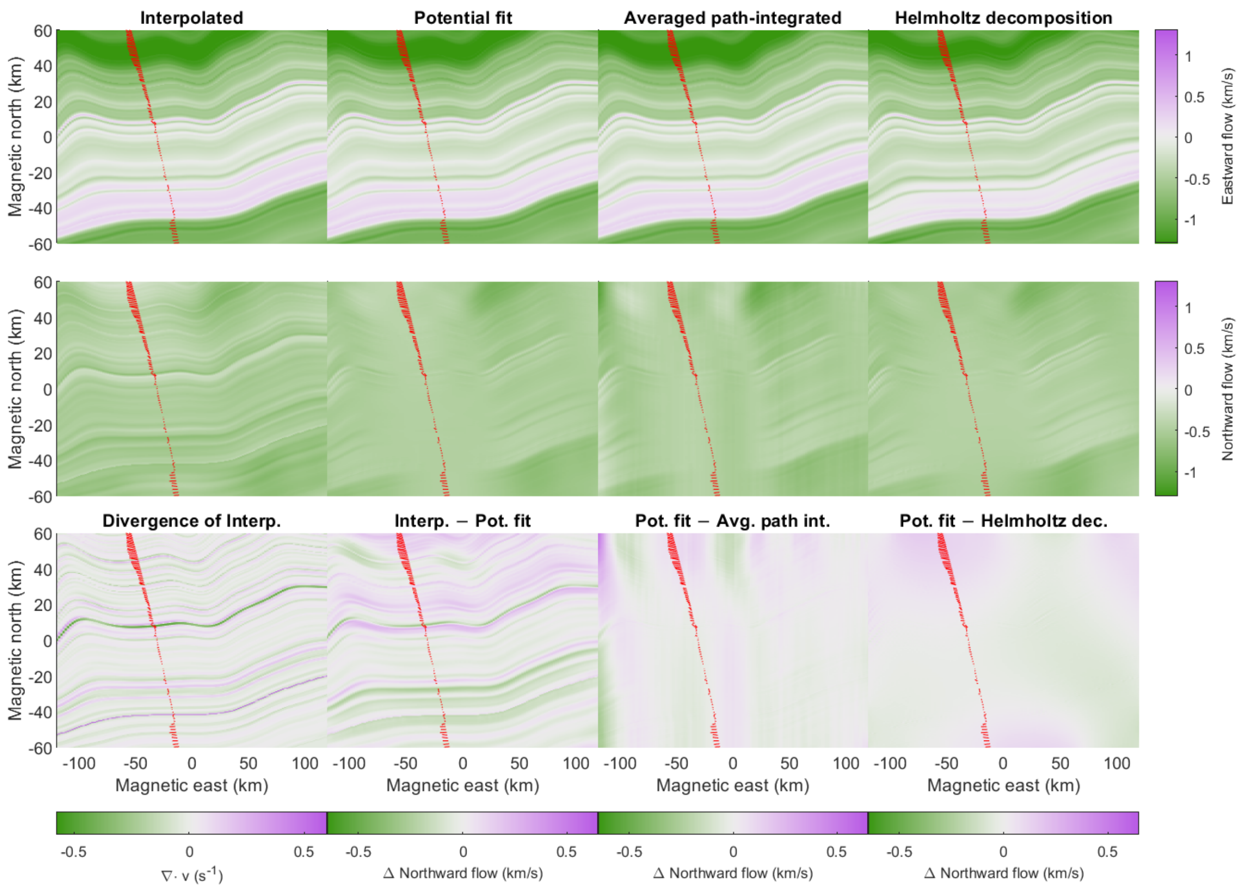

Figure 1: **Top row**: Eastward flow resulting from the interpolation and the three different potential finding methods. **Middle row**: Same as the top row but the northward flow. **Bottom row**: Divergence of the interpolated field and the difference between a) the potential fit and the interpolated field, b) the averaged path-integrated and the potential fit, and c) the Helmholtz decomposition and the potential fit. The original background removed version of the Isinglass flow data is shown in red.