

# 1 Current Closure and Joule Heating in Data-Driven 3-D 2 Auroral Arc Simulations

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## 10 Key Points:

- 11 Understanding current closure in discrete auroral arc systems requires data-driven,  
three-dimensional ionospheric simulations
- 12 Large-scale convection fields play a significant role in determining auroral arc cur-  
rent closure morphology and associated Joule heating
- 13 • Details of precipitating electron energy distributions can significantly affect cur-  
rent closure and Joule heating in auroral arc systems

17 **Abstract**

18 Discrete auroral arc systems, despite many symmetries, are three-dimensional in nature,  
 19 encapsulating latitude and longitude variations in precipitation and field-aligned currents  
 20 combined with important altitude variations in conductivities, hence closure currents.  
 21 This study presents data-driven, 3-D numerical simulations of these processes based on  
 22 a coordinated campaign of heterogeneous measurements collected from the Poker Flat  
 23 Research Range during a sequence of Swarm spacecraft overpasses. These measurements  
 24 include field-aligned current, global-scale convection flow, and auroral emissions, which  
 25 are used to create top-boundary drivers for auroral arc simulations. Six conjunctions be-  
 26 tween the spacecraft, all-sky imagers, and radars are investigated and their measurements  
 27 are used to simulate auroral arcs through multiple iterations per conjunction event. We  
 28 look at different estimates of the background convection flow, assumptions about the en-  
 29 ergy distributions of electron precipitation, and along-arc structures in field-aligned cur-  
 30 rent, and see what effect they have on current closure and Joule heating in auroral arc  
 31 systems. Across the six conjunction events, 11 comparisons of auroral arc systems are  
 32 presented, covering a catalog of 17 simulations in total. These comparisons allow us to  
 33 look at the sensitivity of auroral arc systems to input parameters and envelop the sim-  
 34 ulations in a qualitative confidence interval. Our results suggest that discrete aurorae  
 35 should be studied in three dimensions to fully understand field-aligned current closure  
 36 and, by extension, Magnetosphere-Ionosphere-Thermosphere coupling. Additionally, our  
 37 results demonstrate that both large-scale convection flows and specifics about the en-  
 38 ergy distributions of auroral precipitation can significantly affect current closure and Joule  
 39 heating in auroral arc systems.

40 **Plain Language Summary**

41 The aurora, or northern and southern lights, are embedded within a system of in-  
 42 teracting electric and magnetic fields, and charged particles, the more energetic of which  
 43 produce the lights themselves by exciting the neutral atmosphere. This brings about a  
 44 three-dimensional current system and resistive heating, known as Joule heating. These  
 45 currents enter and exit the atmosphere along the Earth's magnetic field, and can only  
 46 close their circuit between altitudes of 80 – 150 km, where the current carriers collide  
 47 with the atmosphere. This paper outlines the importance of simulating aurorae in three-  
 48 dimensions, and looks at how sensitive these simulations are to various input choices by  
 49 observing the resulting differences in current connectivity and Joule heating. We look  
 50 at collections of measurements from six different events and simulate them multiple times  
 51 with different combinations covering 17 simulations in total. This allows us to gain in-  
 52 sight into how much confidence can be had in our auroral arc simulations, and, by ex-  
 53 tension, what aspects are important to get right when studying auroral arcs. We con-  
 54 clude that large-scale plasma motion and the distribution of energies of the light-producing  
 55 electrons both significantly affect the auroral system, and that current connectivity should  
 56 be studied in three dimensions.

57 **1 Introduction**

58 Laws governing the physics of auroral arc systems are intrinsically three-dimensional—  
 59 the conservation of mass, momentum, and energy density, in conjunction with Maxwell's  
 60 equations, outline a system whose across-arc, along-arc, and field aligned directions are  
 61 coupled. In the last decade or two, interest in three-dimensional (3-D) studies of the au-  
 62 roral ionosphere has slowly picked up (Amm et al., 2008; Fujii et al., 2011, 2012; Marghitu,  
 63 2012; M. Zettergren & Snively, 2019; Clayton et al., 2019, 2021; Lynch et al., 2022; Yano  
 64 & Ebihara, 2021; van Irsel et al., 2024), and we continue this trend by investigating quiet,  
 65 discrete auroral forms in 3-D. Specifically, this paper looks at how electric current clo-  
 66 sure and Joule heating are affected by global-scale electric fields, the energy distributions

of precipitating electrons, and along-arc structure in field-aligned currents (FAC), to provide insight into the geophysical domain of auroral arc systems.

The conductivity of the ionospheric volume surround auroral arcs is highly sensitive to impact ionization from electron precipitation (Fang et al., 2008, 2010). This ionization increases with increased energy flux, varies horizontally depending on arc structure, and varies in altitude depending on the energy distribution of the precipitation. Furthermore, the overarching, large-scale convection electric field guides the current continuity solution and directly affects the Joule heating of the system. For these reasons, to better understand auroral arc system currents, it is crucial that such systems are studied in 3-D.

Auroral-arc-scale science plays an important role in interpreting magnetosphere-ionosphere-thermosphere (MIT) coupling. The ionospheric end plays a non-passive role in this coupling (Marghitu, 2012, & references therein) and is involved in an ongoing sequence of system science studies (Wolf, 1975; Seyler, 1990; Cowley, 2000; Lotko, 2004; Fujii et al., 2011, 2012; Marghitu, 2012; Khazanov et al., 2018; Clayton et al., 2019, 2021; Yano & Ebihara, 2021; Lynch et al., 2022; Enengl et al., 2023; Wang et al., 2024; van Irsel et al., 2024). Such MIT studies require *F*-region ionospheric maps of FAC and electric potential to be consistent with a 3-D ionospheric conductivity volume created by sunlight and charged-particle, auroral precipitation. However, what is often looked at is the two-dimensional (2-D) perspective of auroral arc systems, whether that is north-up or east-north. In this case of the horizontal ( $\perp \mathbf{B}$ ) perspective, high-latitude electrostatic coupling assumes the height-integrated relation between quasi-static electric field, FAC, and conductances given by Kelley (2009, Equation 8.15):

$$j_{\parallel}(x, y) = \Sigma_P \nabla_{\perp} \cdot \mathbf{E} + \mathbf{E} \cdot \nabla_{\perp} \Sigma_P + (\mathbf{b} \times \mathbf{E}) \cdot \nabla_{\perp} \Sigma_H, \quad (1)$$

where  $(x, y)$  is the plane orthogonal to the local magnetic field,  $j_{\parallel}$  is the ionospheric topside FAC,  $\Sigma_{P,H}$  are the height-integrated Pedersen and Hall conductivities, i.e. conductances,  $\mathbf{E}$  is the ionospheric electric field, and  $\mathbf{b} = \mathbf{B}/B$  is the magnetic field direction. Yano and Ebihara (2021) (among others, Marghitu, 2012; Fujii et al., 2012) however, have pointed out that integrating out altitudinal effects can hide significant information regarding polar ionospheric systems, especially in terms of current closure. They use simplified 3-D Hall-magnetohydrodynamic simulations, taking into account ion-neutral collisions, to show that 2-D FAC closure assumed by the thin-layer approximation of the ionosphere is fundamentally different from the 3-D description, if alone for the fact that current streamlines can pass underneath one another.

The electric field solution from Equation 1 can be separated it into a constant, large-scale electric field,  $\bar{\mathbf{E}}$ , and a perturbation field,  $\delta\mathbf{E}$ , which gives two FAC contributions:  $j_{\parallel} = \bar{j}_{\parallel} + \delta j_{\parallel}$  where

$$\bar{j}_{\parallel}(x, y) = \bar{\mathbf{E}} \cdot \nabla_{\perp} \Sigma_P + (\mathbf{b} \times \bar{\mathbf{E}}) \cdot \nabla_{\perp} \Sigma_H, \quad (2)$$

and

$$\delta j_{\parallel}(x, y) = \Sigma_P \nabla_{\perp} \cdot \delta\mathbf{E} + \delta\mathbf{E} \cdot \nabla_{\perp} \Sigma_P + (\mathbf{b} \times \delta\mathbf{E}) \cdot \nabla_{\perp} \Sigma_H. \quad (3)$$

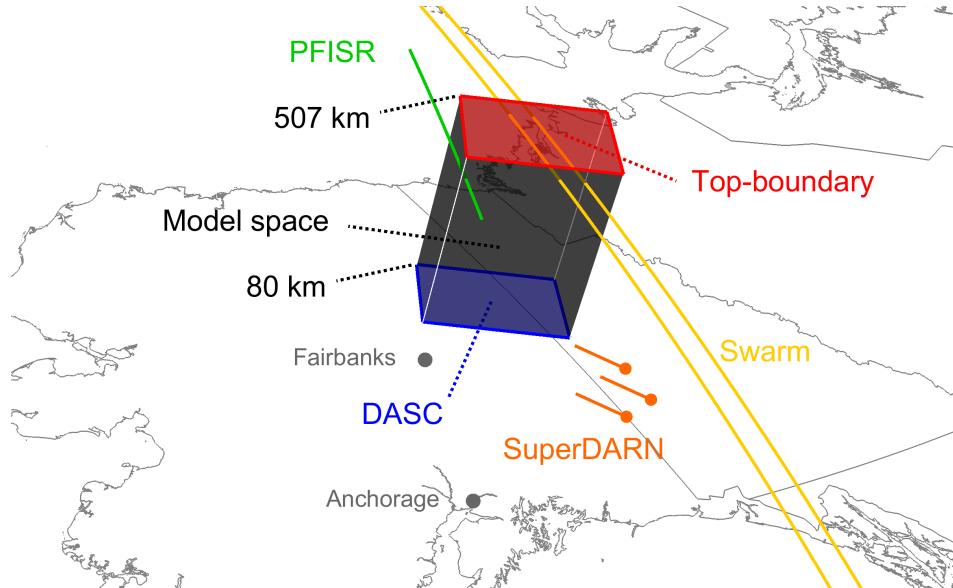
After calculating and height-integrating the conductivities at a particular point in time, one can subtract  $\bar{j}_{\parallel}$  from a specified *F*-region map of FAC,  $j_{\parallel}$ , with which  $\delta\mathbf{E}$  can be determined, i.e. solving current continuity and ionospheric Ohm's law with source term  $\delta j_{\parallel} = j_{\parallel} - \bar{j}_{\parallel}$ . In this sense, the electrostatic drivers are  $j_{\parallel}$  and  $\bar{\mathbf{E}}$ , and the ionosphere responds by introducing polarization fields to provide the remaining FAC. In other words,  $\delta\mathbf{E}$  is a result from local polarization charge densities within the ionospheric volume, while  $\bar{\mathbf{E}}$  is an electric field external to our auroral-arc-scale system. With this perspective, a constant global estimate of the background flow,  $\bar{\mathbf{v}} = \bar{\mathbf{E}} \times \mathbf{b}/B$ , from either SuperDARN or PFISR, is an additional current driver and thus should be accounted for when interpreting FAC observations. Both Equation 1 and topics discussed in this paper deal with self-consistency, not causal relationships, when finding solutions to auroral current continuity.

Marghitu (2012) reviews sequentially more complex descriptions of auroral arcs, the first of which takes on a band of enhanced uniform conductance with negligible altitudinal thickness and polarization electric fields that are fully in the across-arc direction. Having no along-arc gradients whatsoever results in FAC closure which relies only on Pedersen currents (see Equation 1), while the electrojet current flows underneath, but plays no part in FAC closure. The second description introduces an along-arc component in the electric field which can greatly enhance the auroral electrojet current by means of the Cowling effect (Cowling, 1932). With a partial Cowling channel (one with some FAC blockage), Amm et al. (2008) point out that this requires taking into account the ionospheric thickness when looking at current continuity. This is because, as Yano and Ebihara (2021) have also pointed out, divergence-free currents cannot flow through one another. Amm et al. (2011); Fujii et al. (2011, 2012) therefore take on a finite length Cowling channel model, which includes a thin Pedersen layer on top of a thin Hall layer, allowing for primary and secondary Pedersen and Hall currents to connect. The third description by Marghitu (2012) only ignores the along-arc variation in the electric field, but does take on gradients of conductance along the arc. To understand FAC closure with this description, Marghitu (2012) uses 2-D (east-north) modeling given the non-linear nature of this problem. Marghitu (2012) concludes, however, that, even though various one- or two-dimensional descriptions of auroral arcs capture a substantial interpretation, a complete 3-D description is necessary to fully understand, even sheet-like, auroral arc systems.

This paper builds from work done by Clayton et al. (2021), who study auroral arc systems and, to do so, developed new methods for driving simulations with 2-D maps of auroral data to study the surrounding ionosphere in 3-D. Similar to the work presented in this paper, they use multi-spectral auroral imagery from the Poker Flat DASC to both (a) infer the electron precipitation energetics and (b) replicate one-dimensional, in situ measurements of plasma flow, creating continuous 2-D driver maps. Their plasma flow measurements are provided by the Isinglass sounding rocket campaign and the replication methods are described by Clayton et al. (2019). In this paper, we use replication methods by van Irsel et al. (2024), which expand upon these ideas, yet altered slightly in order to use in situ FAC data from orbital spacecraft (Swarm) instead of plasma flow data. With these tools, and given an abundance of observational datasets from the winter months of 2023, we explore the dependencies of current closure paths and Joule heating in auroral arc systems to different values of  $\bar{E}$ , forms of electron precipitation spectra, and top-boundary FAC structures.

In this paper, we aim to determine geophysical, self-consistent solutions to ionospheric current continuity in non-ideal discrete auroral arcs that possess structure in across-arc, along-arc, and field aligned directions. In doing so, we explore how to properly drive 3-D simulations of auroral arc systems using 2-D electrostatic, continuous top-boundary conditions from distributed, multi-platform datasets: all-sky, multi-spectral imagery, in situ FAC data, and radar-based background convection flow data. Additionally, we study the sensitivity of current continuity solutions to various driver parameters, particularly background convection flow and precipitation parameters, in order to envelop auroral arc simulations in a form of qualitative confidence estimates. This provides a better understanding of the dominant physics behind auroral current closure and Joule heating for different situations. Ancillary to this, this study provides a catalog of auroral arc simulations covering six conjunction events with multiple modeling iterations per event, as well as driver and visualization tools to facilitate future studies of auroral arc systems.

In Section 2 we outline the instrumentation used in this work, a brief description of the ionospheric model used to produce our simulations, along with methods for imagery inversion, the replication technique, the implementation of precipitating electron impact ionization, and our use of flux tubes for 3-D visualization of current closure. Section 3 summarizes the 6 conjunction events and Section 4 covers the simulation results



**Figure 1.** Geographical context of our simulations, using the February 10, 2023 conjunction as an example, showing the model space (black), the Swarm A and C crossings (yellow), the PFISR track (green), the top-boundary for the driver maps (red), the approximate location of the imagery from below (blue), and a symbolic depiction of some flow vectors from the SuperDARN global data map (orange) on top of Alaska.

and comparisons thereof. We conclude our findings and discuss possible improvements and future uses of our work in Section 5. Appendix A covers the derivation of the differential hemispherical number flux of accelerated Maxwellian precipitation, and figures of simulations not included in this paper are in the Supporting Information, along with other supporting figures and descriptions.

## 2 Observational Data, Instrumentation, & Methodologies

The data products we use are of six conjunction events that are part of the *Swarm-over-Poker-2023* campaign. This campaign facilitated simultaneous observations in February – March, 2023, of a variety of auroral arcs during times when the European Space Agency’s (ESA) Swarm spacecraft orbited overhead of the Poker Flat Research Range in Alaska. These observations are of key ionospheric electromagnetic parameters including, but not limited to, (1) the ESA Swarm mission’s ion flow data from the Thermal Ion Imagers (TII, Knudsen et al., 2017) and (2) FAC data derived from its magnetometers (Ritter et al., 2013), (3) convection flow data from AMISR’s Poker Flat Incoherent Scatter Radar (PFISR, Kelly & Heinselman, 2009; Nicolls & Heinselman, 2007; Heinselman & Nicolls, 2008), (4) global convection flow maps from the Super Dual Auroral Radar Network (SuperDARN, Greenwald et al., 1995), and (5) multi-spectral, all-sky imagery from the Poker Flat Digital All-Sky Camera (DASC, Conde et al., 2001). Figure 1 shows the geographical context of the February 10, 2023 conjunction event. In this section we cover the details surrounding these data products and any methodologies applied to them, as well as the model used to create our auroral arc simulations.

## 2.1 Poker Flat Digital All-Sky Cameras & Imagery Inversion

The all-sky, multi-spectral auroral imagery we use comes from the University of Alaska Fairbanks Geophysical Institute's Poker Flat Digital All-Sky Cameras (DASC, Conde et al., 2001) located at  $212.57^{\circ}$  east and  $65.12^{\circ}$  north (geographic). From this imagery we use a Python-based routine and the GLobal airglOW model (GLOW, Solomon, 2017) to produce estimated maps of both total precipitating energy flux,  $Q_p$ , and expected energy,  $\langle E \rangle$ . In this work, the expected energy is either the characteristic energy,  $E_0$ , or acceleration potential,  $U_a$  (see Section 2.7).

As shown by Rees and Luckey (1974), and later expanded on by several others (Strickland et al., 1989; Janhunen, 2001; Hecht et al., 2006; Grubbs II, Michell, Samara, Hampton, Hecht, et al., 2018; Grubbs II, Michell, Samara, Hampton, & Jahn, 2018), the ratio of green line (558 nm) to red line (630 nm) intensity for emissions driven by electron precipitation mostly depends on  $\langle E \rangle$ , while the blue line (428 nm) intensity mostly depends on  $Q_p$ . Roughly following Grubbs II, Michell, Samara, Hampton, and Jahn (2018), we use GLOW, driven with ionospheric background conditions, to generate a lookup table of emission line intensities for a variety of driving precipitation energy spectra. Each energy spectrum in the table is parameterized by its values of  $Q_p$  and  $\langle E \rangle$ , and GLOW simulates emission line intensities separately for each.

After denoising and calibrating the imagery, mapping each color to its rough emission altitude, and removing background brightness, we apply a simple Python routine (<https://github.com/317Lab/asispectralinversion>) that uses the lookup tables to invert each usable pixel of the image to a value of  $Q_p$  and  $\langle E \rangle$ , along with rough error bars associated with the inversion. After inversion, all precipitation maps are Gaussian smoothed in the magnetic northward direction with a window size of 32 km ( $\sigma \approx 5.3$  km).

## 2.2 Swarm Spacecraft

The European Space Agency's Swarm mission consists of three satellites which were launched into nearly polar, low Earth orbits on 22 November, 2013, with the goal of providing highly detailed measurements of variations in the Earth's magnetic field. We use their version 0401, level 2 FAC data derived down to 1 Hz from the Vector Field Magnetometer (VFM, Ritter et al., 2013) data, along with their version 0302, level 1B Electric Field Instruments data, specifically the 2 Hz TII ion drift measurements (Knudsen et al., 2017; Burchill & Knudsen, 2022). The TII data, like the precipitation maps, are Gaussian smoothed to 32 km, while the FAC data are smoothed to 16 km ( $\sigma \approx 2.7$  km) to account for the differential relationship between the  $\mathbf{E}$  and  $\Sigma_{P,H}$  maps, and  $j_{\parallel}$  (see Equation 1). The ion drift measurements have a 100 – 200 m/s one-sigma accuracy, and are used only in our discussions (Section 5) as a point of comparison with our simulation results.

We note that the choice of smoothing window, an important and carefully deliberated choice, strongly affects the science scales we can investigate. The specific smoothing window is chosen to match and align the available input data scales; we know that driving the model with inconsistent drivers (i.e., fine-scale fields data and large-scale imagery) leads to spurious signatures. For this study, therefore, we have not fully characterized the dependence on this scale choice. Instead we focus our studies on permutations of input parameters at these scales (i.e. on/off or from instrument A versus instrument B and so on).

## 2.3 Poker Flat Incoherent Scatter Radar

The Poker Flat Incoherent Scatter Radar (PFISR, Kelly & Heinselman, 2009; Nicolls & Heinselman, 2007; Heinselman & Nicolls, 2008) is an Advanced Modular Incoherent

Scatter Radar facility and has been operational since 2007. PFISR is located at the Poker Flat Research Range ( $212.53^{\circ}$  E,  $65.13^{\circ}$  N), which is owned by the University of Alaska Fairbanks Geophysical Institute, and the radar is maintained for the US National Science Foundation by SRI International. The antenna boresight points at an azimuth of  $15^{\circ}$  east-of-north and elevation of  $74^{\circ}$ . In this paper, we take single-value, uniform averages of plasma drift velocity within the latitude ranges of our simulation regions, and use these averages as large-scale background flow estimates. We use their resolved vector velocity (“vvels”) data based on long pulse experiments with a five minute integration time. These data products are produced by Python scripts found at <https://zenodo.org/records/10892410>. We use these data to provide one plasma drift velocity average per conjunction event.

## 250      2.4 Super Dual Auroral Radar Network

The Super Dual Auroral Radar Network (SuperDARN) is comprised of 35+ HF and VHF radars located across the northern and southern hemispheres and is operated by 20 institutions across 10 nations. This paper uses plasma convection flow estimates over Poker Flat, AK—one global estimate per conjunction event—that are interpolated by the pyDARN open-source python library. Greenwald et al. (1995) describe the SuperDARN global-scale network and the pyDARN repository can be found at <https://zenodo.org/records/14796490>. SuperDARN convection map data shown in this paper was processed using the FITACF3 algorithm with a spectral width-based Heppner-Maynard Boundary. Both the order and degree of the fit was 6.

## 260      2.5 FAC Replication

Our simulations require spatially continuous, topside ionospheric FAC maps. van Irsel et al. (2024) outline how this can be done for electrostatic plasma convection maps. Here we have adjusted their methods for FAC maps instead. The replications can be done using distributed optical data, provided by all-sky, multi-spectral imagery, combined with FAC data tracks, provided by spacecraft or sounding rockets. We first invert the imagery using methods outlined in Section 2.1, from which preliminary estimates of the height-integrated conductivities (conductances) are gathered. The conductance maps are then queried for two iso contours at user-defined conductance values which are the primary and secondary arc boundaries. With these boundaries, the replication process is as follows:

- 271      1. The original FAC data track is translated in the east-north plane by some amount  
272      following the primary arc boundary such that the original and replicated data are  
273      equal at the primary boundary-track intersections.
- 274      2. The replicated data track is scaled in the along-track direction such that the orig-  
275      inal and replicated data are equal at the secondary boundary-track intersections.
- 276      3. This replication is repeated for multiple translations along the arc until the top-  
277      boundary is filled with FAC values at a sufficient replication density.
- 278      4. The replicated FAC data map is then interpolated onto the simulation grid, pro-  
279      viding the top-boundary simulation driver.

280      For replications whose data lie just outside of the simulation region, the arc boundaries  
281      are extrapolated, ensuring sensible matching between FAC and precipitation.

## 282      2.6 GEMINI Simulations

283      Simulations for this study use the Geospace Environment Model of Ion-Neutral Inter-  
284      actions (GEMINI, M. D. Zettergren & Semeter, 2012; M. Zettergren & Snively, 2019).  
285      GEMINI solves for 3-D electrostatic current continuity and ionospheric Ohm’s law, ac-

counting for changes in state parameters which affect conductivities as it steps forward in time; it calculates the electric field that is consistent with how the top-boundary FAC requirements connect through the ionospheric volume—one whose conductivity is highly sensitive to impact ionization from electron precipitation, which is implemented into GEMINI using methods by Fang et al. (2008, 2010).

GEMINI is a multi-fluid (electrons and six ion species), quasi-electrostatic model with its calculations of particle continuity consisting of chemical production/loss and photo/impact ionization. Calculations of local densities, plasma flows, and temperatures are treated self-consistently and the model includes thermal conduction heat flux, collisional heating, thermoelectric electron heat flux, and inelastic cooling/heating from photoelectrons. This is supplemented with Maxwell's equations and, at the time of writing, includes no displacement current or magnetic induction. With this, the system is solved through enforcing divergence-free currents, curl-free electric fields, and invoking Ohm's law. GEMINI can be driven with (aside from maps of precipitation energetics handling impact ionization) a map of FAC or electric potential at the top-boundary. When driving GEMINI with a top-boundary map of FAC, a user-specified background electric field,  $\bar{\mathbf{E}}$ , is input separately. GEMINI assumes equipotential magnetic field lines, providing horizontal electric fields that are constant in altitude (Farley Jr., 1959). For a full description of the governing equations solved by GEMINI, see M. D. Zettergren and Snively (2015, Appendix A).

## 2.7 Electron Precipitation Methods

### 2.7.1 Electron Precipitation Energy Spectra

For auroral arc systems, electron precipitation energy spectra,  $\phi(E)$ , are often assumed to be of a standard unaccelerated Maxwellian form (Fang et al., 2008) whose differential number flux,  $\phi_u(E)$ , is

$$\phi_u(E) = \frac{Q_p}{2E_0^2} \frac{E}{E_0} \exp\left(-\frac{E}{E_0}\right), \quad (4)$$

where  $Q_p$  is the total precipitating energy flux,  $E_0$  is the characteristic energy, and  $E$  is the precipitation energy. This has its flux peak at an energy of  $E_0$ , representing the arc energy, however, it also incurs an energy spread of

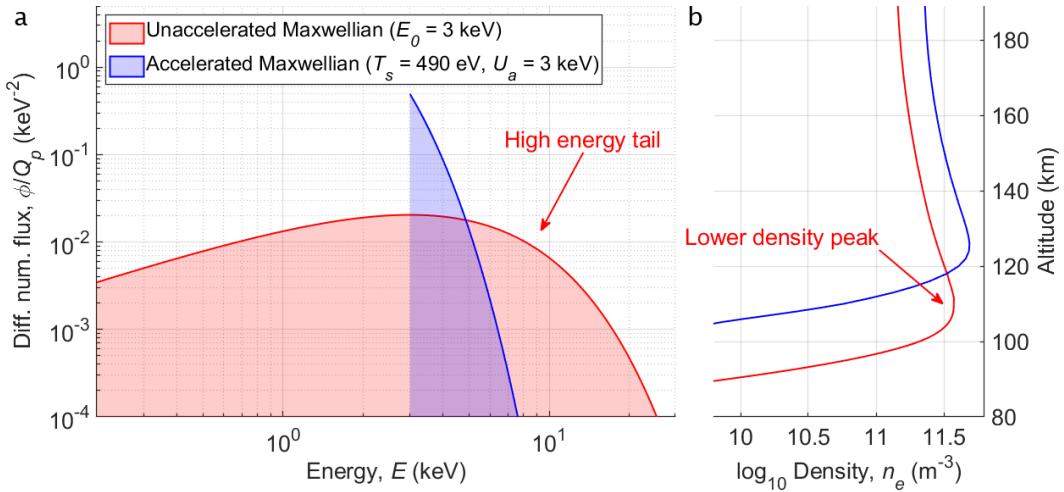
$$\sqrt{\langle (E - E_0)^2 \rangle} = \sqrt{\frac{\int_0^\infty (E - E_0)^2 \phi_u(E) dE}{\int_0^\infty \phi_u(E) dE}} = \sqrt{3}E_0. \quad (5)$$

In contrast to this formulation, in auroral situations, there is often an accelerated signature (Evans, 1968; Paschmann et al., 2003), where the energy spread is related to the source region thermal motions, while the peak energy is related to the auroral acceleration region (Evans, 1974). Therefore, we look at an alternative  $\phi(E)$ ; that of an accelerated Maxwellian whose differential number flux,  $\phi_a(E)$ , is (see Appendix A)

$$\phi_a(E) = \frac{Q_p}{T_s^2 + (T_s + U_a)^2} \frac{E}{T_s} \exp\left(-\frac{E - U_a}{T_s}\right), \quad E \geq U_a, \quad (6)$$

where  $T_s$  is now the source region characteristic energy, and  $U_a$  is the auroral acceleration region potential drop. With  $U_a/T_s \sim 3$ , which is not untypical, this has an energy spread of  $\sqrt{3}T_s$ . This choice for  $\phi(E)$  has decoupled the energy spread and peak energy, which in this case is  $U_a$  when  $U_a > T_s$ , which is the case for all our conjunction events.

Relationships between the acceleration potential and the source region/ionospheric characteristic energy exists via the FAC this system holds (Knight, 1973; Rönnmark, 2002), but these are not the focus of this paper. Equation 6 is implemented into GEMINI using methods described by Fang et al. (2010). Both the GLOW model and the methods



**Figure 2.** Comparison between unaccelerated and accelerated Maxwellian electron precipitation spectra. (a) Normalized energy spectra of  $\phi_u(E)/Q_p$  (red) and  $\phi_a(E)/Q_p$  (blue). Note that both spectra peak at 3 keV. (b) Electron density altitude profiles modeled by GLOW (Solomon, 2017) with the same color scheme.

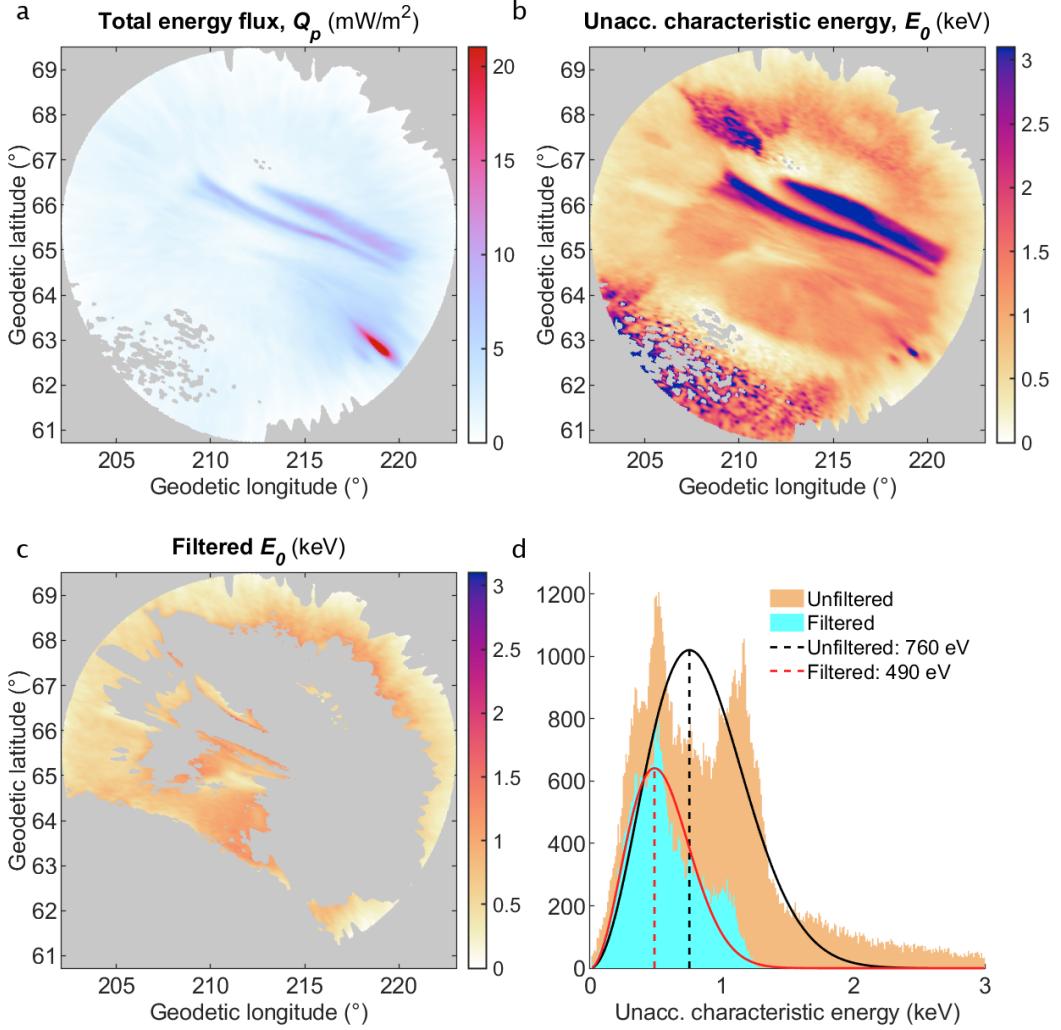
described by Fang et al. (2008, 2010) take into account secondary and back-scattering electrons (Evans, 1974).

Figure 2 shows examples of  $\phi_u(E)$  and  $\phi_a(E)$  (Equation 4 and 6) with  $U_a = E_0 = 3$  keV and  $T_s = 490$  eV. Both these spectra have the same integrated energy flux,  $Q_p$ , and both peak at 3 keV, yet the accelerated Maxwellian has a significantly lower energy spread: 0.8 keV compared to 5.2 keV in the unaccelerated case. Along with this, their respective electron density altitude profiles are shown, determined using the GLOW transport model (Solomon, 2017). It is evident that the assumption of  $\phi_u(E)$  can overestimate the electron density at lower altitudes given the high energy tail of these spectra. It is noted that a choice of  $T_s = U_a = E_0$  has  $\phi_a/\phi_u = 2e/5 \approx 1.09$ , which therefore does not change the spectral shape, but merely scales the total energy flux. This suggests that, when using  $\phi_u(E)$ , one inadvertently is making the choice of  $T_s = U_a$  with  $\phi_a(E)$ . Additionally, with  $U_a = 0$ , i.e. no auroral acceleration, we have  $\phi_a/\phi_u = 1$ , which covers the relatively low energy background precipitation surrounding auroral arcs. This fact is what we use to determine  $T_s$ .

### 2.7.2 Determining Source Region Characteristic Energies, $T_s$

The differential number flux for an accelerated Maxwellian population approaches that of the unaccelerated population as  $U_a$  approaches zero. In this limit  $T_s$  becomes analogous to  $E_0$ , thus, in order to find an estimate for  $T_s$ , we first invert the imagery (see Section 2.1) assuming an unaccelerated population, which provides a map of  $E_0$ . Figure 3, panels a – b, show this map of  $E_0$  and the total energy flux,  $Q_p$ , for our February 10, 2023 event.

Next, assuming that  $U_a$  vanishes outside of discrete auroral arcs, we filter the arc region out of this map of  $E_0$  by removing pixels corresponding to the top 40th percentile of  $Q_p$ . We also remove the lower 30th percentile of the red emissions, as the inversion to  $E_0$  performs sub-optimally for lower red intensities. This is shown in Figure 3c. We then look at the histogram of the remaining  $E_0$  values and fit a Gaussian magnitude distribution to it, the peak of which is selected as the source region characteristic energy.



**Figure 3.** Steps in determining the source region characteristic energy. (a) The total precipitating electron energy flux,  $Q_p$ , inverted assuming unaccelerated Maxwellian energy spectra. (b) The characteristic energy,  $E_0$ , inverted assuming unaccelerated Maxwellian energy spectra. (c)  $E_0$  filtered by removing the top 40th percentile of  $Q_p$  and the lower 30th percentile of the red line emissions. (d) Histograms of data in panels b (orange) and c (light blue) along with Gaussian magnitude fits (black and red respectively) and their peaks (dashed). Data source: DASC (2025).

In this case, we have  $T_s = 490$  eV, as is shown in panel d. This panel also shows the unfiltered distribution which shows two distinct populations, suggesting different physics behind them—presumably that of the accelerated electrons and that of the unaccelerated precipitation.

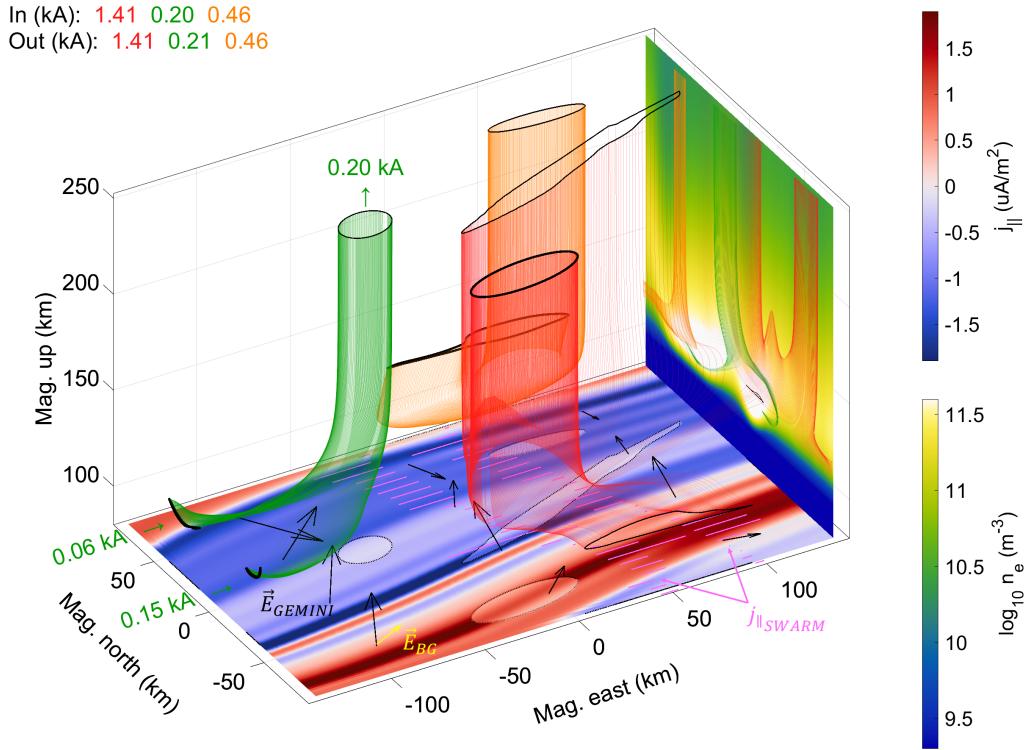
The percentiles used in filtering are chosen by simultaneously minimizing the 95% confidence range and maximizing the adjusted R-squared value of the fits. The different choices for these percentiles raise a rough precision of around  $\pm 10 - 20\%$  surrounding the  $T_s$  estimations.

We assume this value of  $T_s$  to be constant over the relevant source region and use it in Equation 6, with which we perform the inversion described in Section 2.1. This inversion now happens over a  $(Q_p, U_a)$  parameter space, for a given  $T_s$ , instead of  $(Q_p, E_0)$ , when creating lookup tables, providing 2-D maps of  $Q_p$  and  $U_a$ . This is all done for each of the six conjunction events. Reassuringly, we find that inversions of these six events done with either the  $\phi_u(E)$  or  $\phi_a(E)$  assumptions provide nearly identical maps of  $Q_p$ ; however, as we will show, they imply quite different conductivity and current density distributions through the ionosphere.

## 2.8 Current Flux Tube Visualization

In order to visualize current closure in GEMINI output data, we show flux tubes of electric current. GEMINI enforces  $\nabla \cdot \mathbf{j} = 0$ , where  $\mathbf{j}$  is the current density, which makes the usage of flux tubes as a visualization tool sensible. We have developed tools to generate current flux tubes starting at user-defined ellipses contained inside the GEMINI simulation volume. From these ellipses, a number of current vector streamlines are sourced, which, by definition, are tangent to  $\mathbf{j}$  throughout the simulation volume. This ensures the current flux through such ellipses is equal to the flux through the orientable surface enclosed by the curve connected by the streamline endpoints. Current fluxes are calculated for tubes that meet flat exit surfaces and are compared against entry fluxes as a check for numerical error. Streamline endpoints that are too far apart, or that meet at a corner of the simulation volume, are locations where the flux tube splits into multiple tubes. In this case, the fluxes of each tube are provided separately. This method of visualization is part of the toolset available at [https://github.com/317Lab/aurora\\_gemini](https://github.com/317Lab/aurora_gemini).

Figure 4 shows three example current flux tubes. This  $425 \times 288 \times 384$  cell (up, east, north) magnetically aligned volume contains a GEMINI calculated 3-D current density from which the flux tubes are derived. In this paper, simulations are all located in the northern hemisphere and magnetic east, north, and up refer to a locally orthonormal basis with up being anti-parallel to the local magnetic field, east in the direction of increasing modified apex longitudes, and north completing the set. The simulation in Figure 4 is driven by a top-boundary map of FAC which is plotted at the bottom for visualization purposes. The colormap of FAC has red associated with the downward, parallel-to- $\mathbf{B}$  (in the northern hemisphere) current vector, also referred to hereinafter as return current (i.e. “red is return”). The blue represents the upward current (downward-moving electrons in the Northern hemisphere) where, often, the accelerated auroral electron precipitation is found. On the eastern wall, a central cut of electron density is plotted. The density perturbations, which are in most part the result of the top-boundary precipitation driver maps, govern the 3-D conductivity volume and thus, in part, the current closure. The black arrows plotted on the FAC map are a sparse sample of the GEMINI calculated electric field—the second aspect governing the current closure—and the yellow arrow is the imposed constant, background convection electric field,  $\bar{\mathbf{E}}$ . The pink lines indicate the FAC current data from, in this example, Swarm A and C, that are footpointed down to the top-boundary and plotted at the bottom as well (these form the basis of the replicated FAC map in red and blue).



**Figure 4.** Example of a current flux tube plot using an example February 10 simulation. The top-boundary FAC driver is plotted at the bottom for visualization purposes. Similarly, a central cut of electron density is plotted at the eastern wall. The current flux tubes are color-coded for distinction purposes and start/end at the bold/thin black solid curves. The black dashed lines are their counterparts projected on top of the FAC map. The pink lines indicate FAC data from Swarm A (right track) and C (left track) with parallel being right. The black arrows are a sparse sample of the electric field calculated by GEMINI and the yellow arrow indicates the constant background electric field. Data sources: Swarm (2025), SuperDARN (2025), and Simulations (2025).

The current flux tubes are color-coded for easy distinction. In this example, the red flux tube originates from an ellipse at the top-boundary inside the southernmost downward, return current sheet. It carries 1.4 kA down through the volume, splitting in three, finding its way out through the top-, south-, and east-boundary. The influx and outflux regions are outlined by bold and thin closed black curves, and shadows of these curves are projected to the bottom to visualize which portion of the FAC map they embody. The green flux tube has its user-defined ellipse in the upward current and is calculated in reverse. It carries around 0.2 kA from two sources on the western wall, combines into a single tube, and connects with the top-boundary. Lastly, the orange flux tube (also calculated in reverse) is sourced at the northern boundary and also connects to the upward FAC. Figure 4, and similar figures in the remainder of this paper, display in- and out-fluxes to two decimal places and illustrate the degree of precision of the flux tube calculations. Most current flux tubes in this paper are precise up to one decimal place, with a few exceptions of more complex current flux tubes or ones with higher amperage ( $>10$  kA).

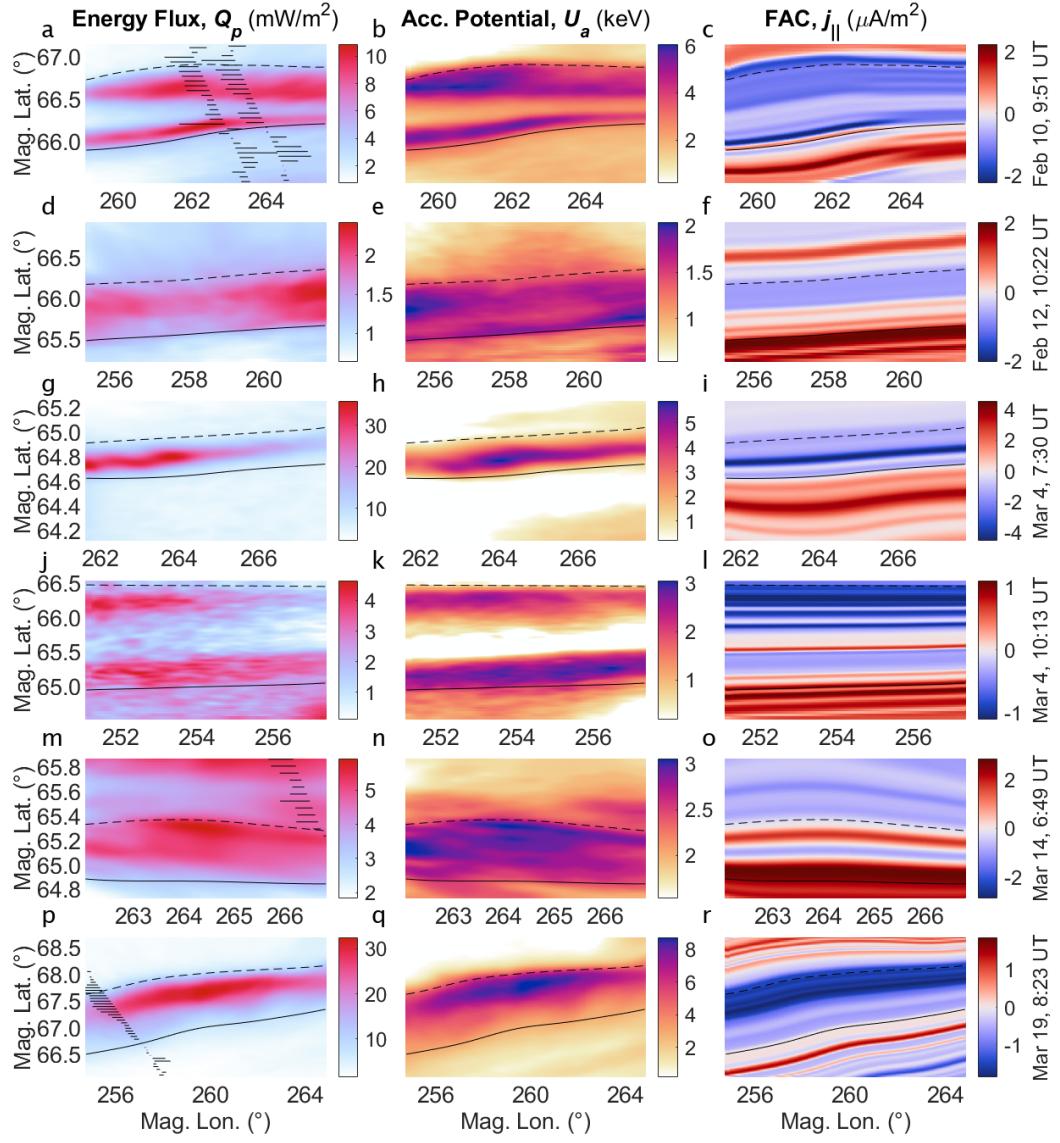
### 3 Conjunction Events

This study uses a total of six conjunction events ranging from February 10 to March 19, 2023, from the Swarm-over-Poker-2023 campaign (Poker Flat Research Range, AK). As a summary of the conjunctions used in this work, Figure 5 shows the top-boundary simulation data-drivers for each of the six events: the total energy flux of the precipitating electrons,  $Q_p$ , the acceleration potential,  $U_a$ , and the FAC maps,  $j_{||}$ , replicated from the Swarm data. Driver maps of  $E_0$  or those of  $j_{||}$  using fewer than all available spacecraft are not shown. Also plotted are the primary and secondary boundaries used in the replication process (see section 2.5) and the FAC data tracks themselves. In addition, Table 1 displays information regarding which Swarm spacecraft are part of the conjunction, the activity levels, the PFISR and SuperDARN background flow estimates, and the rough peak values of the simulation top-boundary drivers for each event. The distance from Poker Flat to the nearest SuperDARN plasma flow estimate,  $\bar{v}_{SD}$ , is denoted  $d_{SD}$ .

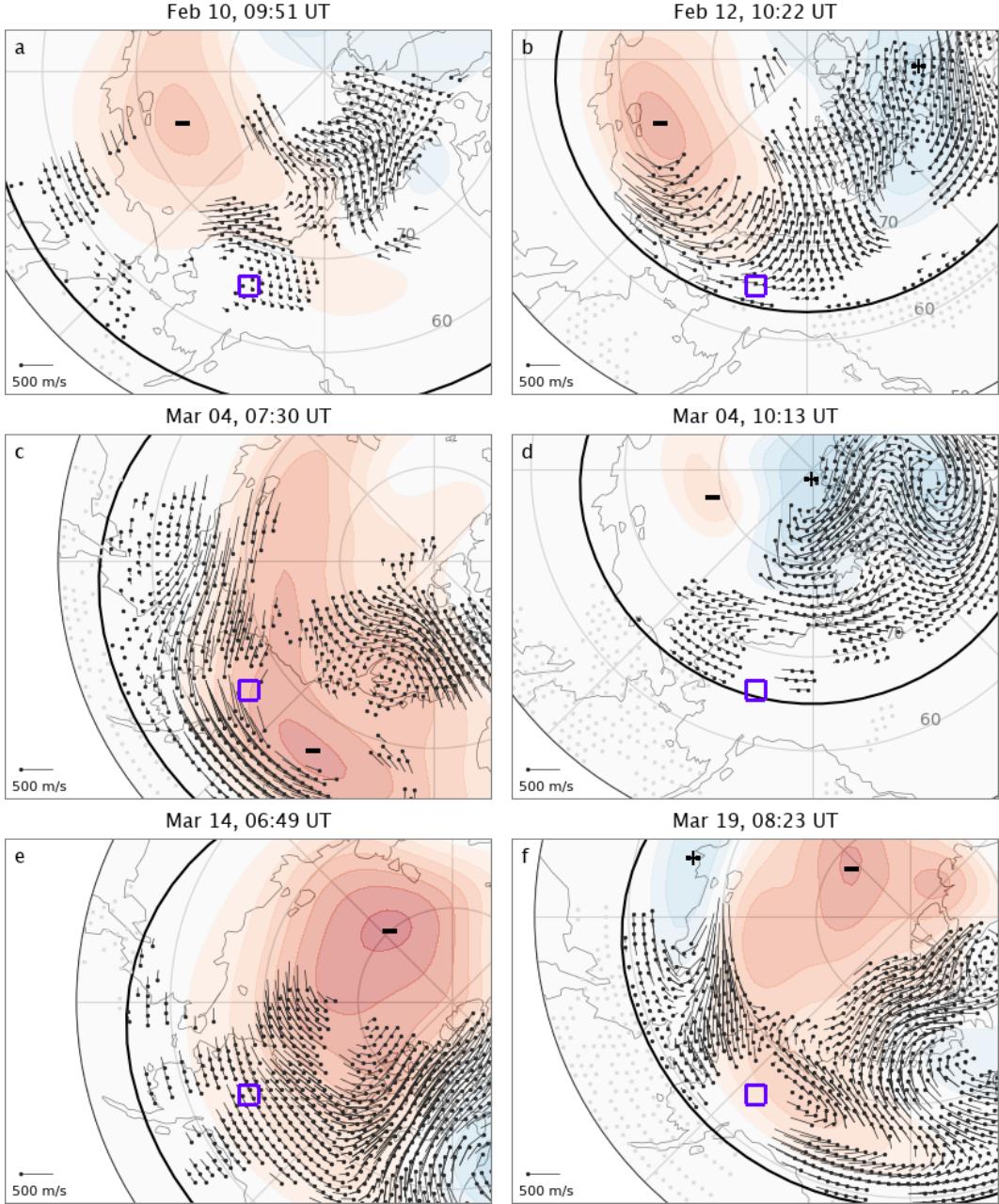
Not all events have PFISR data tracks available because either they are too far from their respective, chosen simulation regions, or the data are considered inadequate for our purposes. Also, not all of the events have a simulation using the unaccelerated assumption for  $\phi(E)$ . Determining plausible arc boundaries requires meticulous care and determines where the simulation boundaries are, which is why, for several conjunction events, the FAC data track(s) lie(s) just east or west of simulation region. In such cases, the arc boundaries are extrapolated to the data tracks. Following are brief synopses of each of the six conjunction events after which, in Section 4, we cover their simulation results.

#### 3.1 February 10, 9:51:27 UT

Figure 5a – c: This event includes both Swarm A and C cutting through the center of the simulation around 47 km apart. It has a curved double arc precipitation pattern with each peaking around a total energy flux of  $Q_p = 10.0 \text{ mW/m}^2$  and acceleration potential of  $U_a = 5.8 \text{ keV}$ . The precipitation is collocated with the FAC replication where the precipitating and return current sheets are between  $j_{||} = -2.3$  to  $2.0 \mu\text{A/m}^2$ . The PFISR convection flow data are positioned at the western edge of the simulation space and estimate a strong magnetic westward flow of  $\bar{v}_{PF} = (-343, 2) \text{ m/s}$ . In contrast, SuperDARN estimates a nearly stagnant flow of  $\bar{v}_{SD} = (-14, 29) \text{ m/s}$ . The Magnetic Local Time (MLT) is 23.1, however, as is shown in Figure 6a, the event occurs 3 – 4 hours duskside of the Harang discontinuity.



**Figure 5.** Top-boundary drivers of conjunction events. (a) The total precipitating electron energy flux,  $Q_p$ , for the February 10, 9:51 UT event. (b) The acceleration potential,  $U_a$ , for the same event. (c) The replicated FAC map,  $j_{\parallel}$ , for the same event. (d-r) Same format for remaining events. Note that the respective colorbars change per event. The solid black feather plot indicates the Swarm FAC data tracks with right being parallel. Not all Swarm data tracks are within the simulation volume and are thus not shown. Data sources: Swarm (2025) and DASC (2025).



**Figure 6.** SuperDARN convection maps of conjunction events. Panels a – f represent event IDs 1 – 6 (see Table 1). Purple boxes are approximately centered on Poker Flat, AK and are on the order of the simulation sizes. The bold black line is the Heppner-Maynard Boundary. The colormap shows the electric potential and the “+” and “–” symbols indicate the maximum and minimum potential points. Local magnetic midnight is at the bottom and the dusk side is left. Data source: SuperDARN (2025).

**Table 1.** Summary of conjunction events with input map values<sup>a</sup>.

Event ID	1	2	3	4	5	6
Date	Feb 10	Feb 12	Mar 4	Mar 4	Mar 14	Mar 19
Time (UT) <sup>b</sup>	9:51:27	10:22:11	7:30:12	10:13:49	6:49:07	8:23:30
MLT (Hours)	23.1	23.3	20.7	22.9	20.1	21.4
Region (km) <sup>c</sup>	290 × 182	290 × 189	290 × 126	290 × 225	220 × 126	432 × 291
Swarm S/C	A + C	C	C	B	A + C	B
Ap	15	7	16	16	18	9
F10.7 (a) (s.f.u.)	208 (175)	200 (175)	182 (161)	182 (161)	138 (162)	143 (162)
̄v <sub>SD</sub> (m/s) <sup>c</sup>	-14, 29	-170, -31	-323, 269	-45, 0	-200, -9	-494, 96
d <sub>SD</sub> (km) <sup>d</sup>	51	51	184	373	51	375
̄v <sub>PF</sub> (m/s) <sup>c</sup>	-343, 2	-237, -17	-	-	-418, -44	178, -68
Q <sub>p</sub> (mW/m <sup>2</sup> ) <sup>e</sup>	10.0	2.3	32.3	4.1	5.8	31.3
U <sub>a</sub> (keV) <sup>e</sup>	5.8	1.9	5.4	2.9	3.0	8.5
T <sub>s</sub> (eV)	490	580	800	860	240	680
E <sub>0</sub> (keV) <sup>e</sup>	4.2	1.4	4.0	2.3	-	-
j <sub>  </sub> (μA/m <sup>2</sup> ) <sup>e</sup>	-2.3, 2.0	-0.7, 1.9	-4.5, 3.8	-1.1, 1.0	-1.2, 2.8	-1.9, 1.4

<sup>a</sup>Variables ̄v<sub>SD</sub>, ̄v<sub>PF</sub>, Q<sub>p</sub>, U<sub>a</sub>, T<sub>s</sub>, E<sub>0</sub>, and j<sub>||</sub> are defined in-text.

<sup>b</sup>Times indicate the spacecraft crossing approximately through the simulation center.

<sup>c</sup>Regions and flows are in GEMINI magnetic coordinates/components.

<sup>d</sup>Distances from Poker Flat to nearest SuperDARN data point.

<sup>e</sup>Values for Q<sub>p</sub>, U<sub>a</sub>, E<sub>0</sub>, and j<sub>||</sub> are 99% quantiles of maps within a 10 cell border.

### 455 3.2 February 12, 10:22:11 UT

456 Figure 5d – f: This is a low flux, low energy, and generally inactive event with a  
 457 Swarm A conjunction roughly 153 km west of the simulation space and with a PFISR  
 458 data cut through the center. It has a single, blurry but straight arc of around Q<sub>p</sub> = 2.3  
 459 mW/m<sup>2</sup> and U<sub>a</sub> = 1.9 keV, with the FAC sheets ranging from j<sub>||</sub> = -0.7 to 1.9 μA/m<sup>2</sup>.  
 460 Both PFISR and SuperDARN suggest a large westward flow of ̄v<sub>PF</sub> = (-237, -17) and  
 461 ̄v<sub>SD</sub> = (-170, -31) m/s respectively. The MLT is 23.3—roughly 1 hour prior to the  
 462 Harang discontinuity.

### 463 3.3 March 4, 7:30:12 UT

464 Figure 5g – i: In contrast to the previous event, this one has an intense arc of Q<sub>p</sub> =  
 465 32.3 mW/m<sup>2</sup> and U<sub>a</sub> = 5.4 keV with a Swarm C crossing around 141 km eastward of  
 466 the simulation space and FAC data of j<sub>||</sub> = -4.5 to 3.8 μA/m<sup>2</sup>. This arc has reason-  
 467 able along-arc structure; the total energy flux ranges from its peak to around 20 mW/m<sup>2</sup>  
 468 going from west to east. Unfortunately, this event does not have usable PFISR data, but  
 469 SuperDARN shows a very strong northwestern flow of ̄v<sub>SD</sub> = (-323, 269) m/s. This  
 470 strong, skewed flow is the result of a skewed two-cell convection pattern determined by  
 471 pyDARN v4.1 (Greenwald et al., 1995) as shown in Figure 6c. The event's MLT is 20.7,  
 472 but this convection pattern places it around 5 – 7 hours before the two-cell split.

### 473 3.4 March 4, 10:13:49 UT

474 Figure 5j – l: This event, just under three hours later than the previous at an MLT  
 475 of 22.9, has a straight double arc pattern at Q<sub>p</sub> = 4.1 mW/m<sup>2</sup> and U<sub>a</sub> = 2.9 keV with

476 Swarm B an average of 94 km westward of the simulation. This event has  $T_s = 860$  eV,  
 477 which is 60 eV higher than 2.75 hours earlier, and the currents have now subsided down  
 478 to  $j_{\parallel} = -1.1$  to  $1.0 \mu\text{A}/\text{m}^2$ . Again, this event includes no PFISR data, while Super-  
 479 DARN now estimates a stagnant flow of  $\bar{\mathbf{v}}_{SD} = (-45, 0)$  m/s. Compared to the pre-  
 480 vious event, Figure 6d shows a much subdued convection pattern with the Harang re-  
 481 gion sits right around local magnetic midnight.

### 482 3.5 March 14, 6:49:07 UT

483 Figure 5m – o: This event is distinct in that it has its precipitation collocated with  
 484 downward, rather than upward, FAC. There is a down-up-down FAC sheet set ranging  
 485 from  $j_{\parallel} = 2.8$  to  $-1.2$  to  $2.0 \mu\text{A}/\text{m}^2$  centered around a  $Q_p = 5.8 \text{ mW/m}^2$ ,  $U_a = 3.0$   
 486 keV precipitation pattern. It is also the second event with both Swarm A and C conjunc-  
 487 tions. Swarm A sits around 44 km east of the model space, while the Swarm C cross-  
 488 ing is just inside at the northeastern corner, and the southernmost PFISR data point  
 489 is located around 100 km west of the simulation. The direction of both the PFISR and  
 490 SuperDARN convection flow estimates are very similar, however, the PFISR flow esti-  
 491 mate of  $\bar{\mathbf{v}}_{PF} = (-418, -44)$  m/s is around twice as strong as the SuperDARN estimate  
 492 of  $\bar{\mathbf{v}}_{SD} = (-200, -9)$  m/s. This 20.1 MLT event sits at around 2 hours duskside to the  
 493 Harang discontinuity.

### 494 3.6 March 19, 8:23:30 UT

495 Figure 5p – r: The last event, and the second Swarm B conjunction, is unaligned  
 496 to magnetic latitudes and has strong precipitation with along-arc structure; the energy  
 497 flux peaks at around  $Q_p = 31.3 \text{ mW/m}^2$  and subsides to around  $20 \text{ mW/m}^2$  at the east-  
 498 ern and western boundaries. The acceleration potential is the highest among our events,  
 499 peaking at around  $U_a = 8.5$  keV and the FAC data range from around  $j_{\parallel} = -1.9$  to  
 500  $1.4 \mu\text{A}/\text{m}^2$ . PFISR cuts through the center and estimates a flow of  $\bar{\mathbf{v}}_{PF} = (178, -68)$ ,  
 501 where SuperDARN estimates  $\bar{\mathbf{v}}_{SD} = (-494, 96)$  m/s. The MLT is 21.4, however, Fig-  
 502 ure 6f shows a multi-cell convection pattern which gives a relatively nonstandard con-  
 503 text.

## 504 4 Simulation Results

505 The six conjunction events are each simulated multiple times, iterating through dif-  
 506 ferent parameters, allowing the simulations to be systematically compared. This high-  
 507 lights and isolates the relevant physics involved and allows us to study sensitivities to  
 508 these parameters. Table 2 provides the list of simulation comparisons covered in this pa-  
 509 per (and its Supporting Information), labeled IDs I-XI, where individual simulations are  
 510 denoted Ia, Ib, IIa, and so on. The comparisons are divided into three categories of fea-  
 511 ture permutations: (1) background convection flow and its source, (2) the assumption  
 512 of unaccelerated versus accelerated Maxwellian precipitation spectra, and (3) single ver-  
 513 sus double spacecraft replications, highlighting along-arc FAC structure.

514 Each simulation has  $425 \times 288 \times 384$  cells in the magnetic up, east, and north di-  
 515 rections respectively and are simulated for 60 seconds with static drivers. The altitudi-  
 516 nal extent is 80 – 507 km, with cell heights of 0.3 – 10 km respectively, and the magnetic  
 517 east/north extents are given in Table 1 and Figure 5. Horizontal cell dimensions settle  
 518 at 700 – 1400 m in the magnetic east direction, and 238 – 700 m in the magnetic north  
 519 direction. Unless otherwise stated, all simulations default to SuperDARN background  
 520 flow estimates, accelerated Maxwellian precipitation, and FAC replication using max-  
 521 imal data tracks. The simulations can be found at <https://rcweb.dartmouth.edu/LynchK/Gemini3D>.

**Table 2.** Summary of event comparisons<sup>a</sup>

Category	ID	Datetime (UT) <sup>b</sup>	BG flow (m/s) <sup>c</sup>		BG source		Acc.	Swarm		
			a	b	a	b				
Background flow	I	Feb 10, 9:51	(-14, 29)	(-343, 2)	SD	PFISR	Y	-	AC	-
	II	Feb 12, 10:22	(-170, -31)	(-237, -17)	SD	PFISR	Y	-	C	-
	III	Mar 4, 7:30	(-323, 269)	(0, 0)	SD	None	Y	-	C	-
	IV	Mar 14, 6:49	(-200, -9)	(-418, -44)	SD	PFISR	Y	-	AC	-
	V	Mar 19, 8:23	(-494, 96)	(178, -68)	SD	PFISR	Y	-	B	-
								a	b	
Accelerated vs. unaccelerated	VI	Feb 10, 9:51	(-14, 29)	-	SD	-	Y	N	AC	-
	VII	Feb 12, 10:22	(-170, -31)	-	SD	-	Y	N	C	-
	VIII	Mar 4, 7:30	(-323, 269)	-	SD	-	Y	N	C	-
	IX	Mar 4, 10:14	(-45, 0)	-	SD	-	Y	N	B	-
								a	b	
Along-arc structure	X	Feb 10, 9:51	(-14, 29)	-	SD	-	Y	-	AC	A
	XI	Mar 14, 6:49	(-200, -9)	-	SD	-	Y	-	AC	A

<sup>a</sup>Comparisons are labeled I-XI with individual simulations labeled Ia, Ib, IIa, etc.

<sup>b</sup>Times indicate the spacecraft crossing approximately through the simulation center.

<sup>c</sup>PFISR and SuperDARN background flows are in GEMINI magnetic east/north components.

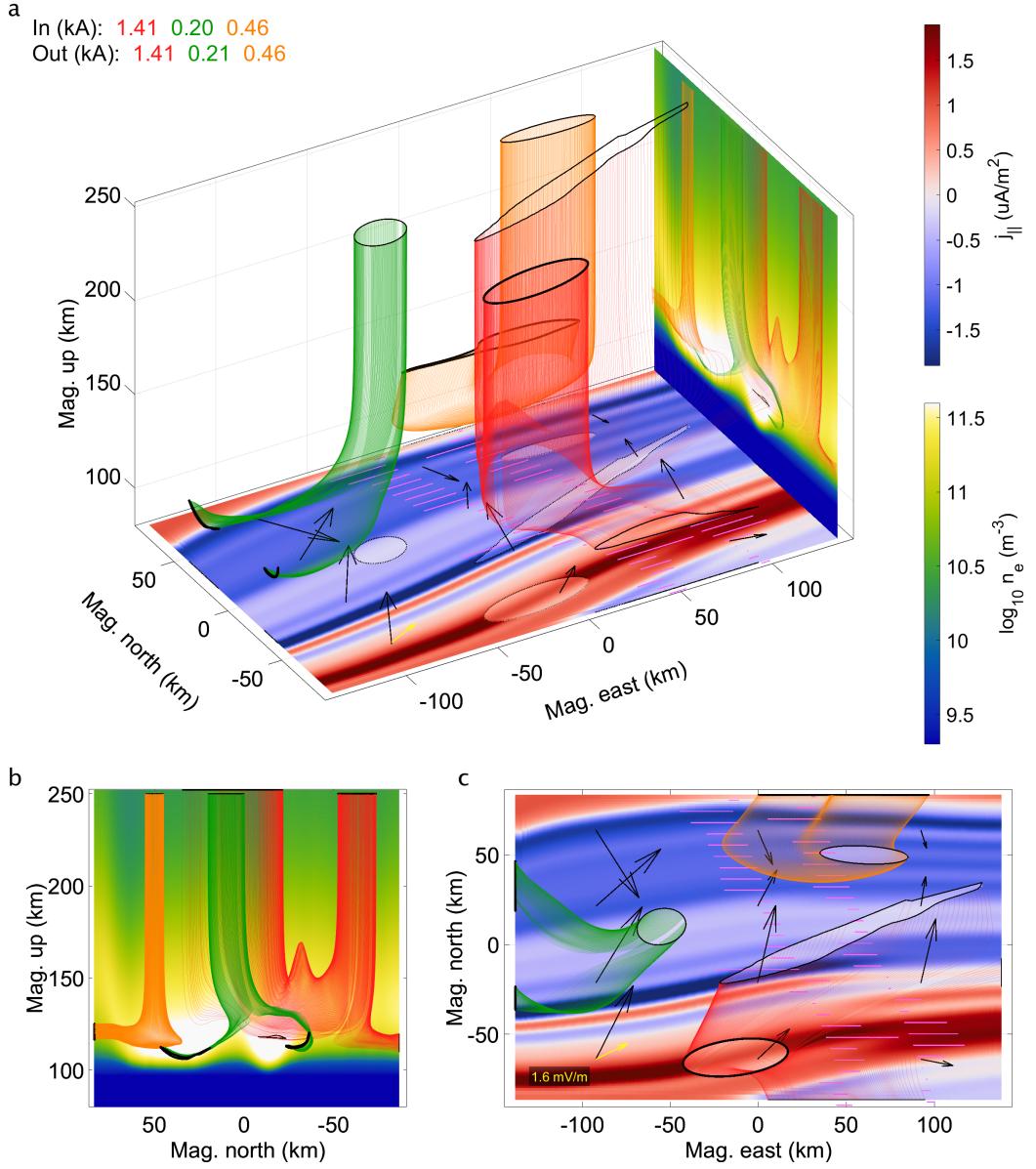
523

#### 4.1 Background Flow & Electric Field

524 There are two factors which dictate the existence of closure currents: (1) the Pedersen and Hall conductivities, and (2) the strength of the electric field. The conductivities require enhanced ionization at closure altitudes which is largely dictated by electron precipitation—enhanced energy fluxes,  $Q_p$ , increase the conductivity overall, while stronger acceleration potentials,  $U_a$ , give preference to Hall over Pedersen closure. Adding 525 to this, spatial structure in the precipitation means that these conductivities have 3-D 526 structure, affecting current closure in all directions. The magnitude of the electric field, 527 however, dictates the magnitude of closure currents overall. We argue that strong electric 528 fields can render the need for Hall closure to be negligible. We therefore begin by 529 looking at comparisons of simulations that have different background electric field 530 assumptions.

531 Figure 7 shows three view angles of the results for Simulation Ia, referenced in Table 532 2, where Section 2.8 explains the format of this figure. It uses FAC data from Swarm 533 A and C, the accelerated Maxwellian precipitation assumption, and a background plasma 534 flow estimate from SuperDARN. In this first example, the background flow of  $\bar{v}_{SD} =$  535  $(-14, 29)$  m/s amounts to a constant background electric field of 1.6 mV/m directed roughly 536 26 degrees north-of-east (geomagnetic).

537 What follows are descriptions of three of our five comparisons (see Table 2) that 538 outline the sensitivity of auroral current closure to the constant background electric field, 539  $\bar{\mathbf{E}}$ , around which GEMINI solves current continuity and Ohm's law for  $\mathbf{E} = \bar{\mathbf{E}} + \delta\mathbf{E}$ . 540 The remaining comparisons, along with their associated figures and descriptions, can be 541 found in the Supporting Information.



**Figure 7.** Isometric (a), side (b), and top (c) view of the GEMINI results for Simulation Ia. For plot details, see Section 2.8. Data sources: Swarm (2025), SuperDARN (2025), and Simulations (2025).

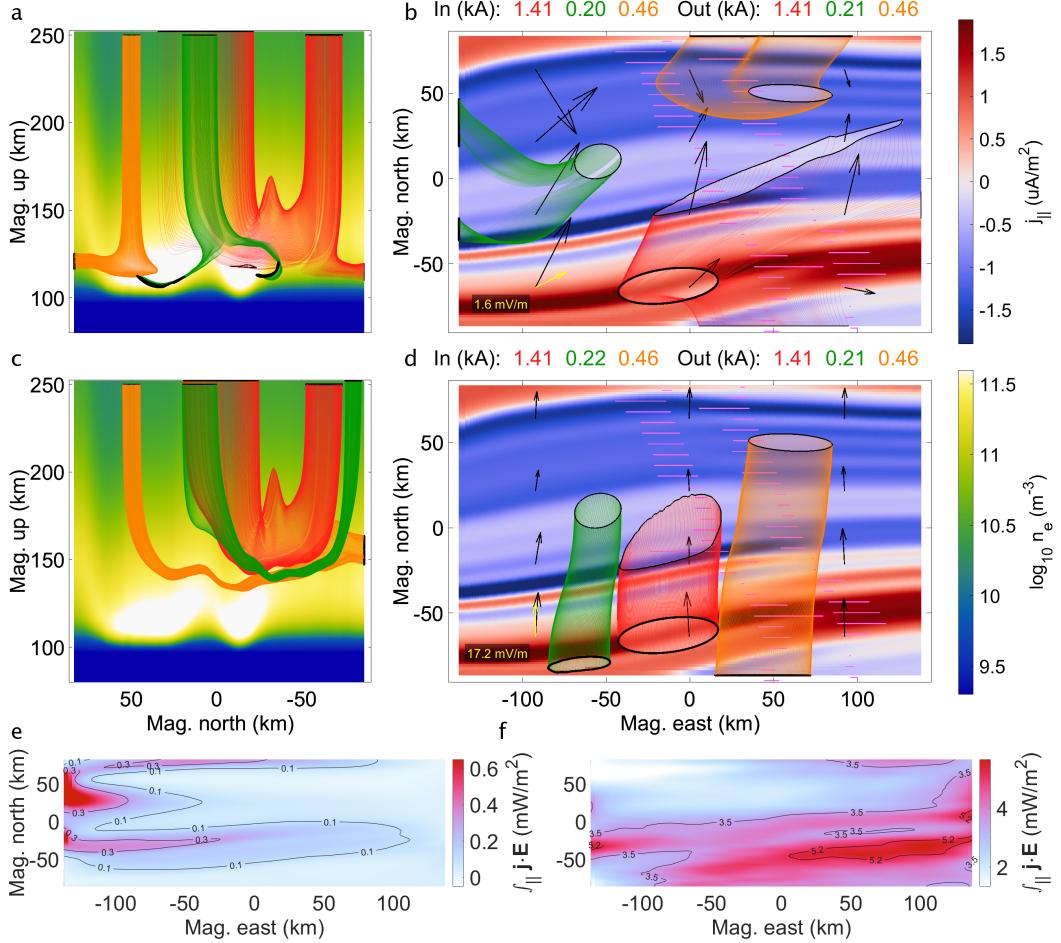
546      **4.1.1 Comparison I: Background Flow**

547      In Comparison I, we compare and contrast the use of SuperDARN derived back-  
 548      ground flow against using the PFISR observed background flow. Figure 7 illustrates three  
 549      current closure paths of Simulation Ia, which assumes the SuperDARN background flow,  
 550      and shows the complexity of current closure in a reasonably typical discrete auroral arc  
 551      system. The red current flux tube, carrying 1.4 kA, starts at the center of southernmost  
 552      return current sheet and rotates to closure currents at an altitude range of 110 – 150 km.  
 553      The bulk of the current continues northward, however, 0.2 kA exits through the south-  
 554      ern boundary and >0.1 kA exists through the eastern boundary. Focusing on the remain-  
 555      ing 1.2 kA, panel c shows that this segment opens up to the northeast, aligning the tube  
 556      with the electric field at first, i.e. Pedersen closure. The relatively weak strength of the  
 557      electric field, however, renders the Pedersen closure infective and requires the tube to  
 558      traverse through lower altitudes to find sufficient paths for closure. At these lower al-  
 559      titudes, the Hall currents dominate and thus this portion of the tube rotates perpendic-  
 560      ular to the electric field. This increases the length it has to travel while crossing into the  
 561      upward FAC region and stretches the overall current closure morphology in the along-  
 562      arc direction. The portion which exits through the eastern wall, presumably, would fol-  
 563      low this same pattern somewhere outside the simulation volume, but this is speculative.  
 564      More notably, however, the remaining unclosed portion on the southern part of the tube  
 565      traverses southward, but this is for the same reason: the tube rotates in the Hall layer  
 566      looking for upward FAC somewhere outside the simulation.

567      The green tube is sourced from the western boundary with two ends, both carry-  
 568      ing around 0.1 kA, which combine into a single, 0.2 kA upward segment of the tube clos-  
 569      ing in between the two precipitation current sheets. Panel a shows how they cling to the  
 570      higher density, i.e. higher conductivity, regions caused by the double-arc precipitation;  
 571      they wrap around these density enhancements in the northward direction following the  
 572      local electric field.

573      The orange tube is sourced from the northern boundary with 0.5 kA and travels  
 574      southward, somewhat aligned to the electric field, before it hits an electric field conver-  
 575      gence. Thus, to avoid going against the electric field, the flux tube lowers in altitude,  
 576      in search of Hall conductivity, and abruptly turns to the east. This outlines the self-consistency  
 577      aspect of the nature of auroral current closure—the flux tube (a) lowers in altitude where  
 578      (b) the density is higher, (c) the electric field converges, and (d) the Hall conductivity  
 579      increases allowing for an eastward turn, all spatially coincident. Finally, the tube fur-  
 580      ther rotates to gain just enough Pedersen current, and hence altitude, to allow for a con-  
 581      nection with the upward FAC sheet. This current flux tube, along with the previous two,  
 582      highlights a set of 3-D considerations needed when trying to understand current closure  
 583      morphology, and thus MIT coupling. This is especially true when Hall currents are re-  
 584      quired in this closure, which is the case for Simulation Ia, given its weaker electric field.

585      In contrast, Figure 8 shows three current flux tubes for Simulation Ib (panels c –  
 586      d) that capture the same FAC regions, whether at the start or end of each tube. The  
 587      only change here is that the simulation now assumes the PFISR derived constant back-  
 588      ground flow of  $\bar{\mathbf{v}}_{PF} = (-343, 2)$  m/s, which amounts to 17.2 mV/m directed nearly north-  
 589      ward compared to the northeasterly 1.6 mV/m from Simulation Ia (panels a – b). This  
 590      larger background flow drastically changes the current closure morphology of all three  
 591      flux tubes. Given the tenfold increase in the electric field magnitude, on top of a more  
 592      direct Pedersen pathway across the arcs, the Pedersen closure has become significantly  
 593      more effective. Panels a and c show an increase in closure altitudes of 110 – 150 to 130  
 594      – 180 km, which means the Hall layer is virtually untouched by these Simulation Ib clo-  
 595      sure patterns. Panel d solidifies this idea, as all three tubes follow the electric fields al-  
 596      most directly. This outlines the ability of the background electric field,  $\bar{\mathbf{E}}$ , to actively drive  
 597      auroral arc systems in conjunction with the top-boundary map of  $j_{\parallel}$ .



**Figure 8.** Comparison I (February 10, 9:51 UT): Top and side views of Simulation Ia with SuperDARN derived background flow (a, b) versus Simulation Ib with PFISR derived background flow (c, d) along with height-integrated Joule heating for Simulation Ia (e) and Ib (f). For plot details, see Section 2.8. Data sources: Swarm (2025), SuperDARN (2025), PFISR (2025), and Simulations (2025).

To emphasize the sensitivity to the background electric field from the perspective of energy dissipation, panels e – f of Figure 8 show the height-integrated Joule heating for Simulations Ia – b respectively. They show the extent to which this auroral arc system can be an electrostatic load, and how  $\bar{\mathbf{E}}$  can change this greatly; aside from having an order-of-magnitude higher electric field strength, Simulation Ib also closes mostly in Pedersen currents—parallel to the electric field—both facts favoring higher  $\mathbf{j} \cdot \mathbf{E} = \sigma_P |\mathbf{E}|^2$  values throughout. Not only does this increase the Joule heating for Simulation Ib, it also relocates a bulk portion of it equatorward of the precipitation.

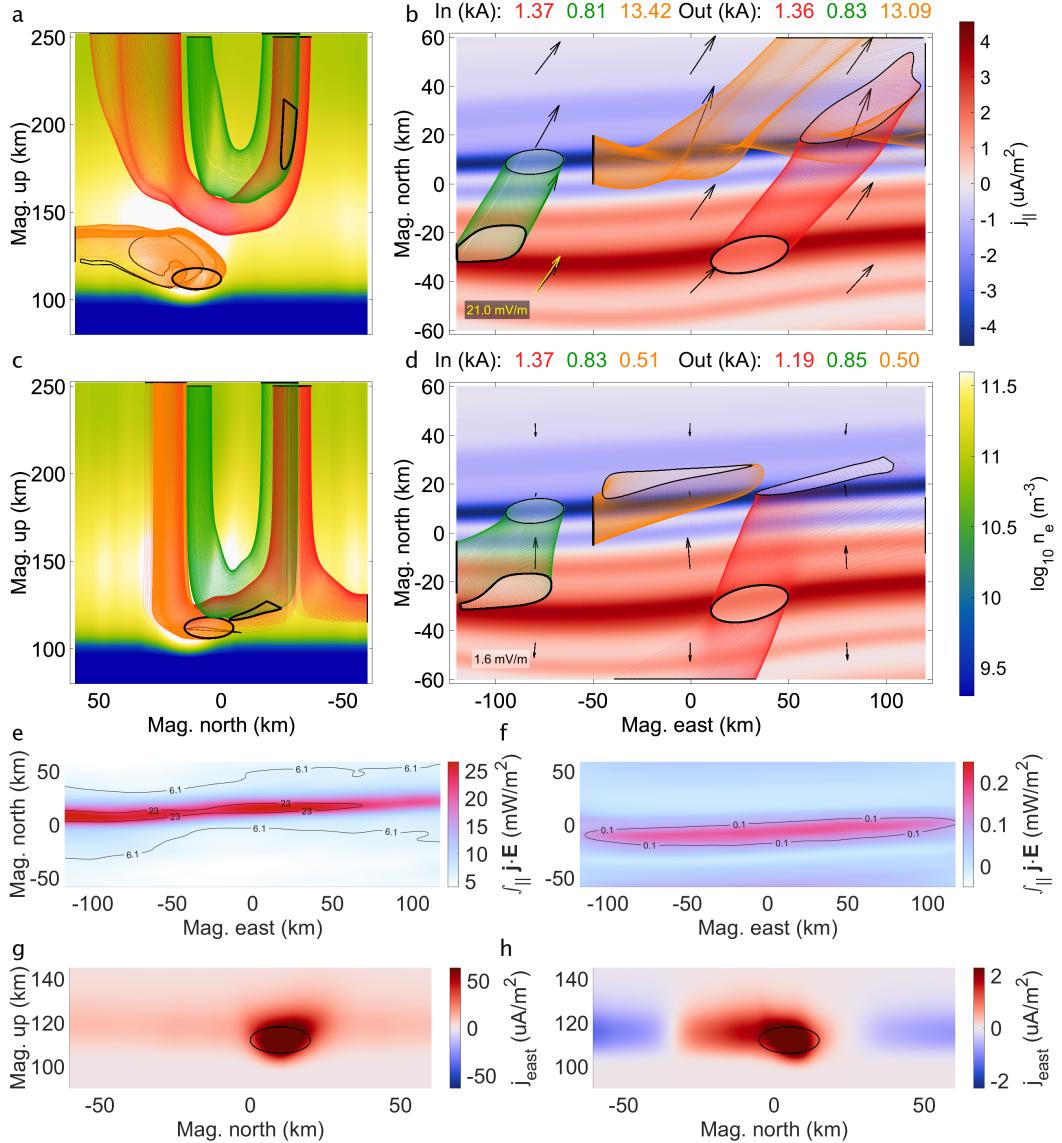
The simulations in the next comparison, Comparison III, have a similar disparity in electric field strengths, yet both have higher FAC requirements, dictating a larger need for closure currents. However, they both also have more precipitation; a factor which partially fulfills this need for additional closure.

#### 4.1.2 Comparison III: Background Flow

The conjunction event for Comparison III, unfortunately, occurs too far from the PFISR field-of-view and therefore has no PFISR-deduced background flow estimate. Nevertheless, Figure 9 demonstrates the sensitivity to the choice of background flow by looking at Simulation IIIa, where the SuperDARN derived background convection amounts to 21.0 mV/m directed 40 degrees east of north (first row), and comparing it to Simulation IIIb, which has the background flow set to zero, as there is no estimate for it (second row). As explained in Section 1, having zero background electric field amounts to assuming most of the top-boundary FAC,  $j_{\parallel}$ , comes from electric fields caused by local polarization,  $\delta\mathbf{E}$ , alone. This comparison shows how much such an assumption affects current closure. Note that, with  $|\bar{\mathbf{E}}| = 0$ , for illustration purposes, the electric field label (black here) indicates the magnitude of the GEMINI calculated electric field vector shown nearest the label.

Comparisons I and III both look at simulations with an order-of-magnitude difference in their electric field strengths and both cover conjunction events whose acceleration potentials peak at around  $U_a = 5$  keV. Comparison III, however, has the precipitation energy flux more than triple, and FAC requirements roughly double, with respect to Comparison I (see Tables 1 and 2). This creates a higher need for current closure—a need partially fulfilled by increased conductivity at all altitudes and the strong electric field strength. Hindering these needs, however, is the less direct path for Pedersen closure given the roughly 40 degrees angle at which the electric field crosses the arc in Simulation IIIa. The combination of these features allows us to look at how the sensitivity to electric field strength is affected by a different arrangement of auroral arc parameters.

Simulation IIIb, with  $\bar{\mathbf{E}} = 0$ , depicts a typical perspective of discrete aurora (Marghitu, 2012)—an arc-aligned line of diverging electric field at the downward current sheet, and a converging one at the upward current sheet, as suggested by Equations 1 – 3. In this simulation, this is the result of the absence of a background electric field causing current continuity and Ohm's law to be solved with electric fields from local polarization alone. The red flux tube in Simulation IIIb digs deep into the Hall layer while closing and is forced to split when bottoming out. This causes 0.4 kA to exit through the southern wall, 0.7 kA through the top-boundary, and >0.1 kA through the eastern wall. (Note that this tube loses around 0.2 kA throughout its path which is a result of edge effects at the eastern wall). In contrast, the order-of-magnitude higher electric field in Simulation IIIa means that its red flux tube carries that 1.4 kA from the return current sheet across to the precipitation sheet all throughout Pedersen altitudes and, thus, its closure is directed almost completely in the electric field direction. Contrarily, the green flux tubes for both simulations close largely with Pedersen currents given that their ends are relatively near one-another. Even though the green Simulation IIIb flux tube finds its clos-



**Figure 9.** Comparison III (March 4, 7:30 UT): Top and side views of Simulation IIIa with SuperDARN derived background flow (a, b) versus Simulation IIIb with no derived background flow (c, d) along with height-integrated Joule heating for Simulation IIIa (e) and IIIb (f). (g, h) North-up slices of the magnetic eastward current component for Simulations IIIa – b respectively taken at 50 km west from center with the start curves of their respective orange flux tubes (solid black). For plot details, see Section 2.8. Data sources: Swarm (2025), SuperDARN (2025), and Simulations (2025).

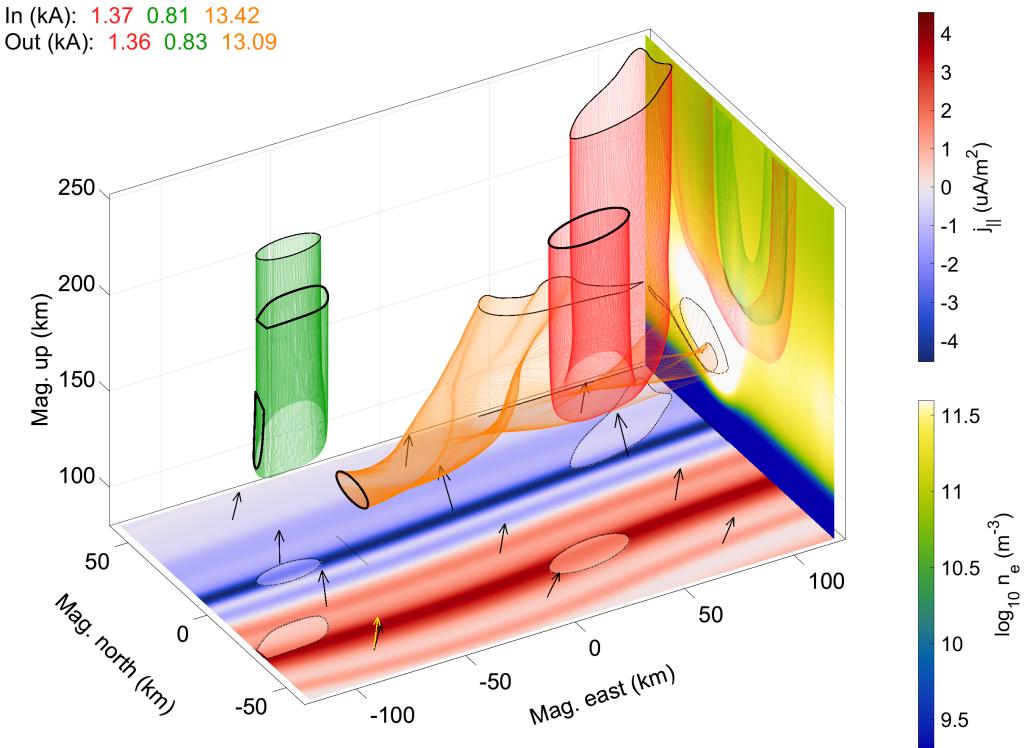
649       ing currents at lower altitudes, it is still mostly dominated by Pedersen conductivity through-  
 650       out; only the bottom apex of this tube veers to the across-**E** direction.

651       Morphologically speaking, the most striking difference between Simulations IIIa and  
 652       IIIb lies in their connection to the electrojet current. Figure 9, panels g – h, for Simu-  
 653       lations IIIa – b respectively, show a slice of the magnetic eastward component of **j** taken  
 654       at 50 km west-from-center, along with the intake ends of their respective orange flux tubes  
 655       in panels a – d. With its stronger electric field, Simulation IIIa has a much higher elec-  
 656       trojet current. This makes this auroral arc system closely resemble a 3-D version of the  
 657       description from Section 4 by Marghitu (2012): A “thick uniform 2-D arc” whose cur-  
 658       rent closure is separated into a thin Pedersen and Hall layer as shown by Fujii et al. (2012).  
 659       Expanding on this description, here we show how current flux tubes can navigate around  
 660       each other in a coherent and self-consistent way by venturing into the 3-D perspective.

661       Given the more complex shape of the orange flux tube in Simulation IIIa, Figure  
 662       10 shows the isometric view of the simulation results, in addition to the side and top views  
 663       in panels a – b from Figure 9. Here we see the almost helical shape of the orange Sim-  
 664       ulation IIIa current flux tube, resembling that of Example 3 by Mallinckrodt (1985) but  
 665       in 3-D. This tube captures 13.4 kA of the electrojet current, while its Simulation IIIb  
 666       counterpart carries around 0.5 kA. Both intake ellipses have the same dimensions and  
 667       are centered on their respective peaks of magnetic eastward currents slices. The simu-  
 668       lations both have the same relatively strong precipitation arc ( $Q_p = 32.3 \text{ mW/m}^2$ ,  $T_s =$   
 669       800 keV) around 10 – 20 km north, resulting in a high amount of impact ionization at  
 670       relatively lower altitudes. This provides plenty of Hall conductivity and, thus, has both  
 671       simulations susceptible to strong electrojet currents. These currents, however, are still  
 672       proportional to the electric field strength which is why the order-of-magnitude increase  
 673       in electric field results in a similarly increased electrojet current.

674       The enhanced electrojet current in Simulation IIIa does not partake in parallel cur-  
 675       rent closure, whereas the Simulation IIIb electrojet current *is* required in the coupling  
 676       of magnetospheric currents. As mentioned before, the lower electric field strength over-  
 677       all renders all closure currents less effective, hence the FAC has to rely on enhanced conductivity—  
 678       Pedersen and then Hall—to connect. Naturally, Simulation IIIa is a more energetic con-  
 679       figuration in terms of Joule heating; the integrated Joule heating peaks are at around  
 680       26.6 mW/m<sup>2</sup> and 0.17 mW/m<sup>2</sup> for Simulations IIIa – b respectively, as shown in pan-  
 681       els e – f of Figure 9. This is consistent with the order-of-magnitude difference in elec-  
 682       tric field strengths, given the  $|\mathbf{E}|^2$  relationship. Given that Hall currents are dissipation-  
 683       less (Kaepller et al., 2012), Simulation IIIb is thus able to rely on the electrojet currents  
 684       for closure instead. In Simulation IIIa, the electrojet largely is assumed to follow the global-  
 685       scale convection pattern D-shaped Hall currents instead, and is much less involved in au-  
 686       roral FAC closure.

687       As with Comparison I (as well as II and IV in the Supporting Information), here,  
 688       yet again, we see that a sufficiently large background electric field has FAC close with  
 689       Pedersen currents, and thus in the direction of the electric field. Even with the less-direct  
 690       Pedersen pathway for closure and the higher FAC requirements, the strong electric field  
 691       and relatively large precipitation energy flux provides sufficient conductivity at higher  
 692       altitudes and renders Pedersen closure to be the dominant method in MIT coupling for  
 693       Simulation IIIa. Furthermore, the Simulation IIIb solution features a distinct  $\nabla \cdot \mathbf{E}$  sig-  
 694       nature. In Simulation IIIa, however, this signature is masked by the its large background  
 695       electric field (compared to no background field in Simulation IIIb). This emphasizes the  
 696       dominance of the  $\nabla \Sigma_{P,H}$  terms in balancing the FAC map for auroral systems with large  
 697       electric fields. Next, we move onto Comparison V whose simulations both have strong  
 698       electric fields, yet in severely different orientations.



**Figure 10.** Isometric view of the GEMINI results for Simulation IIIa. For plot details, see Section 2.8. Data sources: Swarm (2025), SuperDARN (2025), and Simulations (2025).

699            **4.1.3 Comparison V: Background Flow**

700            Simulation Va assumes a background electric field of 25.2 mV/m directed 11 degrees  
 701            east of magnetic north as estimated by SuperDARN. In the almost complete op-  
 702            posite direction to this, Simulation Vb has PFISR estimate 9.5 mV/m directed 21 de-  
 703            grees west of south. This results in drastic differences in both current closure mor-  
 704            phology and Joule heating, as depicted by Figure 11. Both the red and green flux tubes  
 705            completely flip directions in their current closure. The green flux tube, in its attempt to  
 706            connect to the broad, primary precipitation current sheet, changes from sourcing its roughly  
 707            5.3 kA from the southwest corner in Simulation Va, to doing so from the northern end  
 708            in Simulation Vb. The red flux tube, closing the southern primary downward current  
 709            sheet, simply flips direction by following the electric field, and, interestingly, in both sim-  
 710            ulations it ends up skipping over an adjacent, lesser downward current sheet when clos-  
 711            ing its 0.8 kA.

712            As shown in panels g – h of Figure 11, the orange flux tube, like in Comparison III,  
 713            captures the electrojet current for both Simulations Va – b. (Here, the user-defined el-  
 714            lipse sits at 0 km east and the tube is calculated in both directions.) As expected, this  
 715            flux tube also flips its orientation, with the current flowing from west-to-east in Simu-  
 716            lation Va, and east-to-west in Simulation Vb. However, like in Comparison III (Figure  
 717            9) but to a lesser extent, the weaker electric field strength in Simulation Vb requires the  
 718            need of this electrojet current to help close some of the FAC, 0.3 kA in this case. The  
 719            62% weaker field also has reduced this Hall current flux tube by about 52%.

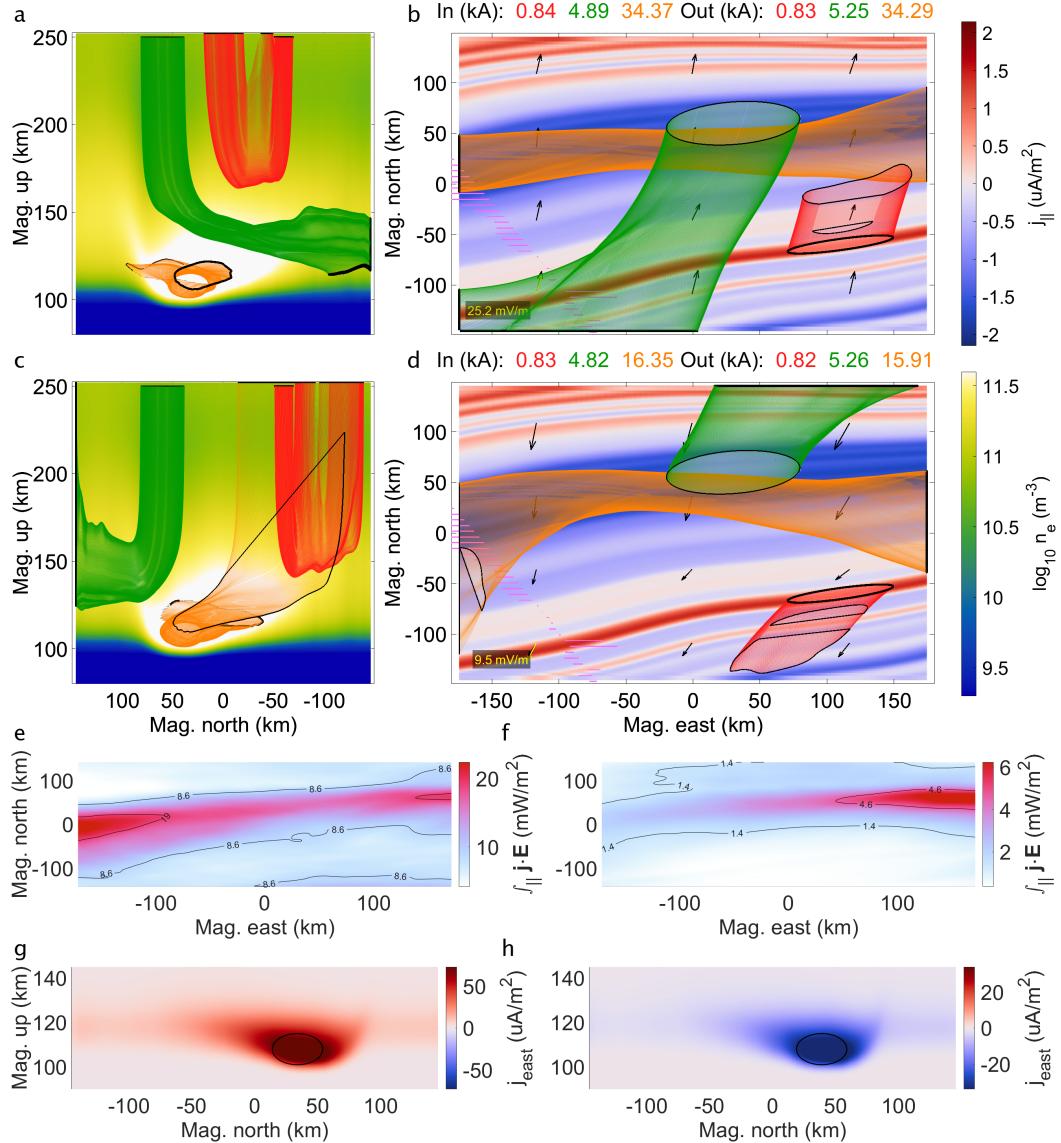
720            As in Comparisons I and III, the height-integrated Joule heating shown in panels  
 721            e – f of Figure 11 varies roughly in proportion to the electric field strength squared. One  
 722            notable difference, however, lies in the tapering off of this Joule heating in simulation  
 723            Vb. This indicates that the western boundary of this simulation relies more on Hall clo-  
 724            sure; an idea supported by the electrojet usage in FAC closure depicted by the orange  
 725            flux tube in panels c – d. Regardless of the reasoning behind this, Comparison V has shown  
 726            that a mere directional change in the background electric field can create different dis-  
 727            sipation characteristics of an auroral system. Moreover, Comparison V highlights how  
 728            the direction of the background electric field completely changes the connectivity of a  
 729            given map of FAC. It is tempting to assume that a precipitation current sheet connects  
 730            with its closest adjoining return current sheet, but as we have shown here, knowledge  
 731            of the global-scale convection has considerable influence when it comes to FAC connec-  
 732            tivity.

733            **4.1.4 Summary: Background Flow**

734            Auroral arc systems are very sensitive to the electric field in matters of current clo-  
 735            sure. Given that there are many self-consistent solutions for  $\mathbf{E}$  in Equation 1 that can  
 736            be considered geophysical, we have shown here that it is crucial to get a good estimate  
 737            of the global background flow in order to properly interpret behavior at auroral arc scales.  
 738            In terms of simulation confidence, we can have more trust in simulations whose sources  
 739            for background electric field estimates agree, such as Comparisons II and IV. However,  
 740            when attempting to best understand the auroral arc system pertaining to a particular  
 741            conjunction event, more certainty is needed for systems like those shown in Comparisons  
 742            I, III, or V. Future conjunction campaigns will therefore benefit greatly from dedicated,  
 743            multi-platform observations of large-scale convection flow—observations of comparable  
 744            importance to in situ FAC measurements.

745            **4.2 Electron Precipitation Spectra**

746            As discussed in Section 2.7, the choice of precipitating electron energy spectra can  
 747            affect the impact ionization rate at different altitudes; an unaccelerated Maxwellian pro-



**Figure 11.** Comparison V (March 19, 8:23 UT): Top and side views of Simulation Va with SuperDARN derived background flow (a, b) versus Simulation Vb with PFISR derived background flow (c, d) along with height-integrated Joule heating for Simulation Va (e) and Vb (f). (g, h) Central north-up slices of the magnetic eastward current component for Simulations Va – b respectively with the start curves of their respective orange flux tubes (solid black). For plot details, see Section 2.8. Data sources: Swarm (2025), SuperDARN (2025), PFISR (2025), and Simulations (2025).

file, Equation 4, often carries an erroneous high-energy tail which overestimates the  $E$ -region density enhancement from electron precipitation. Moreover, choosing to use unaccelerated Maxwellian spectra in inverting multi-spectral imagery results in a de facto source region characteristic energy equal to the accelerating potential drop, i.e.  $T_s = U_a = E_0$ . This is not unlike how a choice of  $\bar{\mathbf{E}} = 0$  carries hidden assumptions about  $j_{\parallel}$ . With an accelerated Maxwellian profile, Equation 6, we estimate  $T_s$  prior to multi-spectral image inversion which allows for much “colder” source populations and, we argue, more geophysical precipitating electron modeling.

Below are two comparisons which look at how decoupling the source region characteristic energy from the auroral acceleration potential changes auroral current closure. See Table 2 for details on these comparisons and Table 1 for the (peak) values for  $Q_p$ ,  $U_a$ ,  $T_s$ , and  $E_0$ .

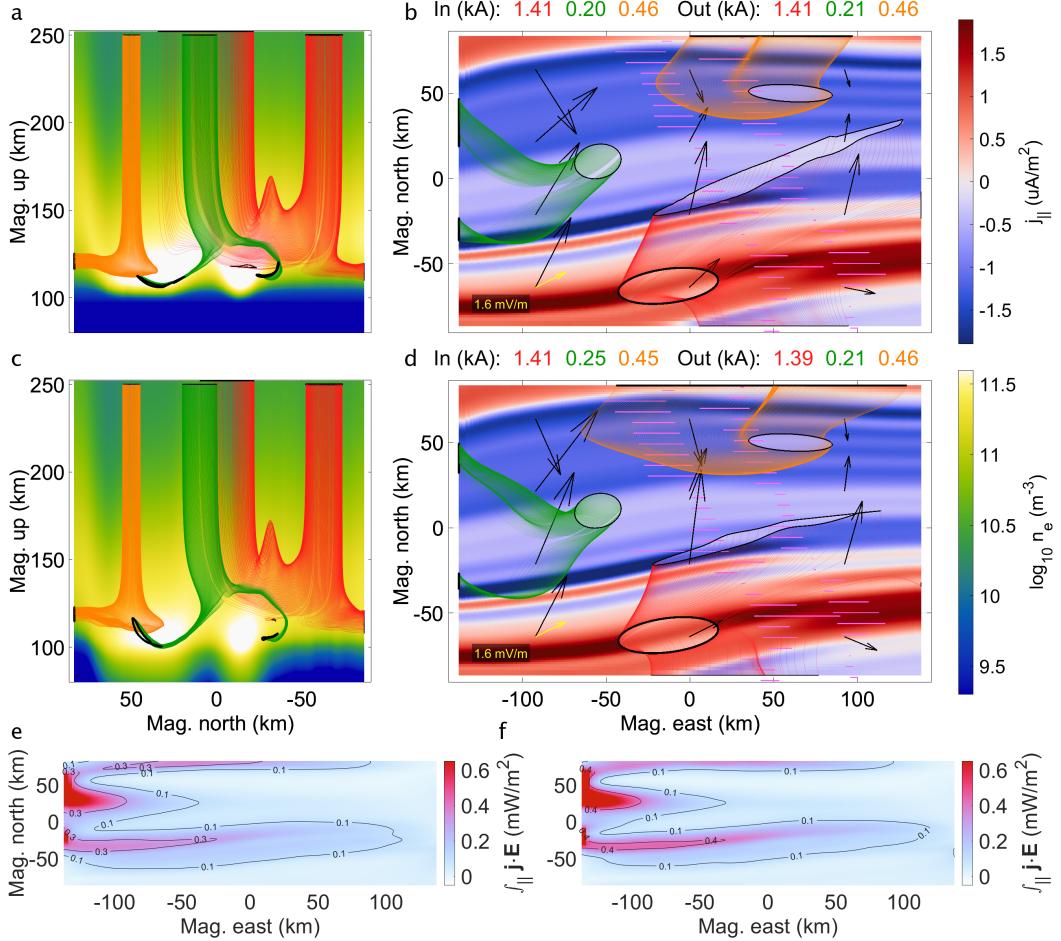
#### 4.2.1 Comparison VI: Precipitation Spectra

Returning back to the February 10 conjunction event from Comparison I, Figure 12 depicts Comparison VI which looks at the differences between Simulation VIa (also named Ia) with the accelerated precipitation spectra assumption given by  $\phi_a(E)$  (first row), and Simulation VIb which assumes  $\phi_u(E)$  instead (second row). The first feature to point out is the central, north-up electron density slices shown in panels a and c: simulation VIa has both precipitation arc induced density enhancements tucked above 100 km in altitude, while the use of  $\phi_u(E)$  in Simulation VIb has these same two arcs increasing their electron density enhancements to the bottom of the simulation volume. This limits the closure paths of current flux tubes in Simulation VIa, compared to Simulation VIb.

Panels a and c show that all three current flux tubes are squished to higher altitudes in Simulation VIa, compared to Simulation VIb, forcing them to have a preference of Pedersen, over Hall, current closure. Panels b and d show how this affects the curvature of the flux tubes from a topside view. The red flux tube in Simulation VIa extends significantly further north given its preferred direction of that of the electric field, whereas the Simulation VIb red flux tube stays more parallel to the arc, traveling perpendicular to the electric field. Note that, despite the morphology being more along-arc, this flux tube does not extend much further east compared to the one in Simulation VIa, as it is able to capture higher upward FAC densities in this direction. Subsequently, its end region has a smaller overall area needed to capture 1.5 kA of upward FAC. Similar to the red flux tubes, the Simulation VIb orange current flux tube travels more often in the direction perpendicular to  $\mathbf{E}$  compared to its Simulation VIa tube, again because the flux tube is able to traverse lower altitudes.

Panels e – f show that, in this instance, the height-integrated Joule heating increases by around 30% with the unaccelerated, over the accelerated, Maxwellian precipitation assumption. This can be counterintuitive when considering the Pedersen closure preference of Simulation VIa. Looking at panels b and d of Figure 12, however, tells us that the electric fields (black arrows) surrounding the arcs are higher in strength with the unaccelerated assumption which, evidently, is consistent with an increase in Joule heating. Ultimately, along with having the same FAC and background electric field drivers, both simulations have near identical maps of total precipitating electron energy, even though their imagery inversions assume two different spectral shapes. This implies that the altitudinal distribution of impact ionization alters the energy accounting, and thus the electric load characteristic of this auroral arc system.

Both Simulations VIa – b assume the relatively weak, SuperDARN derived  $|\bar{\mathbf{E}}| = 1.6$  mV/m, which makes them more susceptible to changes in the Hall closure layer as we have shown in Section 4.1. Adding to this susceptibility, the precipitation arcs have relatively high values of  $U_a = 5.8$  and  $E_0 = 4.2$  keV respectively. This deposits the



**Figure 12.** Comparison VI (February 10, 9:51 UT): Top and side views of Simulation VIa with accelerated Maxwellian electron precipitation (a, b) versus Simulation VIb with unaccelerated Maxwellian electron precipitation (c, d) along with height-integrated Joule heating for Simulation VIa (e) and VIb (f). For plot details, see Section 2.8. Data sources: Swarm (2025), SuperDARN (2025), and Simulations (2025).

799 impact ionization to lower altitudes, rendering the Hall layer more important still. This,  
 800 along with the altered Joule heating, puts emphasis on the energy distribution shape of  
 801 precipitating electrons in such auroral systems. Up next, we look at Comparison VIII  
 802 whose simulations have both much stronger background electric fields and significantly  
 803 higher total precipitation energy flux, which, along this line of reasoning, implies both  
 804 Simulations VIIIA – b are less reliant on Hall closure in MIT coupling.

#### 805 4.2.2 Comparisons VIII: Precipitation Spectra

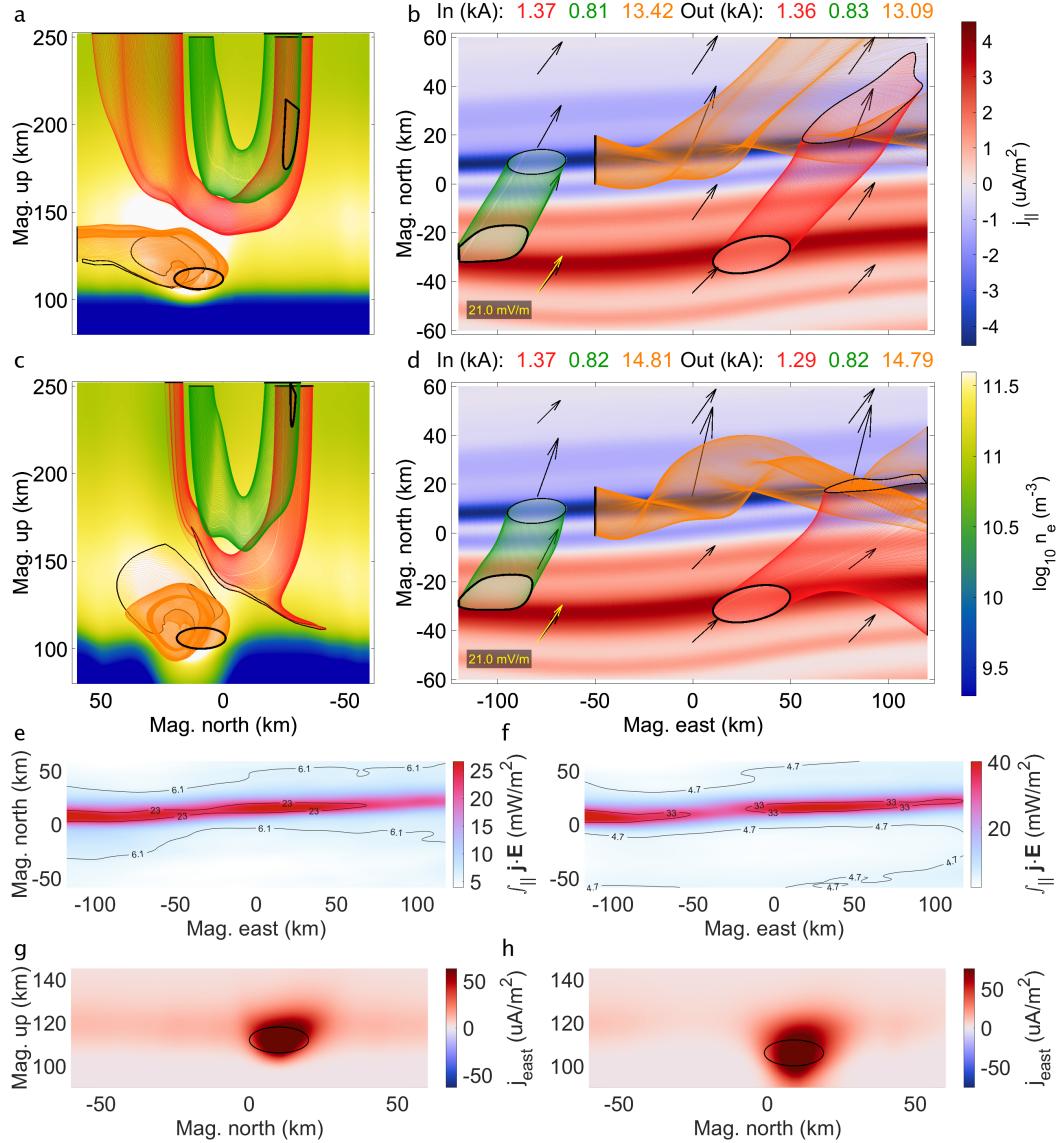
806 The precipitation arcs in Simulation VIIIA (also named IIIA) assume an acceleration  
 807 potential peaking at around  $U_a = 5.4$ , and the characteristic energy for the arcs  
 808 in Simulation VIIIB reach around  $E_0 = 4.0$  keV—similar to the values from Compar-  
 809 ison VI. However, with respect to Comparison VI, the simulations in Comparison VIII  
 810 have more than three times the energy flux,  $Q_p = 32.3$  mW/m<sup>2</sup>, background electric  
 811 fields that are more than 13 times stronger,  $|\mathbf{E}| = 21.0$  mV/m, and FAC sheets whose  
 812 magnitudes around double,  $|j_{\parallel}| = 3.8 - 4.5 \mu\text{A}/\text{m}^2$ . Additionally, at  $T_s = 800$  eV, the  
 813 source region characteristic energy for simulation VIIIA also nearly doubles that of Sim-  
 814 ultations VIa. Figure 13 shows how unaccelerated Maxwellian precipitation at these more  
 815 energetic parameters compares to accelerated Maxwellian precipitation.

816 By proxy of the electrojet currents shown in panels g – h, the unaccelerated Maxwellian  
 817 precipitation deposits ionization to both lower altitudes—around 6 km lower compared  
 818 to Simulation VIIIA—and to a larger altitudinal range given the nearly four times higher  
 819 energy spread of the unaccelerated energy spectra. We focus on these electrojet currents  
 820 by looking at both orange flux tubes, which capture similar values of 13.1 and 14.8 kA  
 821 for Simulations VIIIA – b respectively. As before, the density volume resulting from the  
 822 accelerated Maxwellian assumption is restricted to above around 100 km, forcing the re-  
 823 spective orange tube to take on more Pedersen current. This means the electrojet in Sim-  
 824 ulation VIIIA veers to the northeast, directed toward the electric field (see panel b). The  
 825 orange flux tube in Simulation VIIIB, being overall at lower altitudes, travels more east-  
 826 erly, staying relatively orthogonal to the electric field.

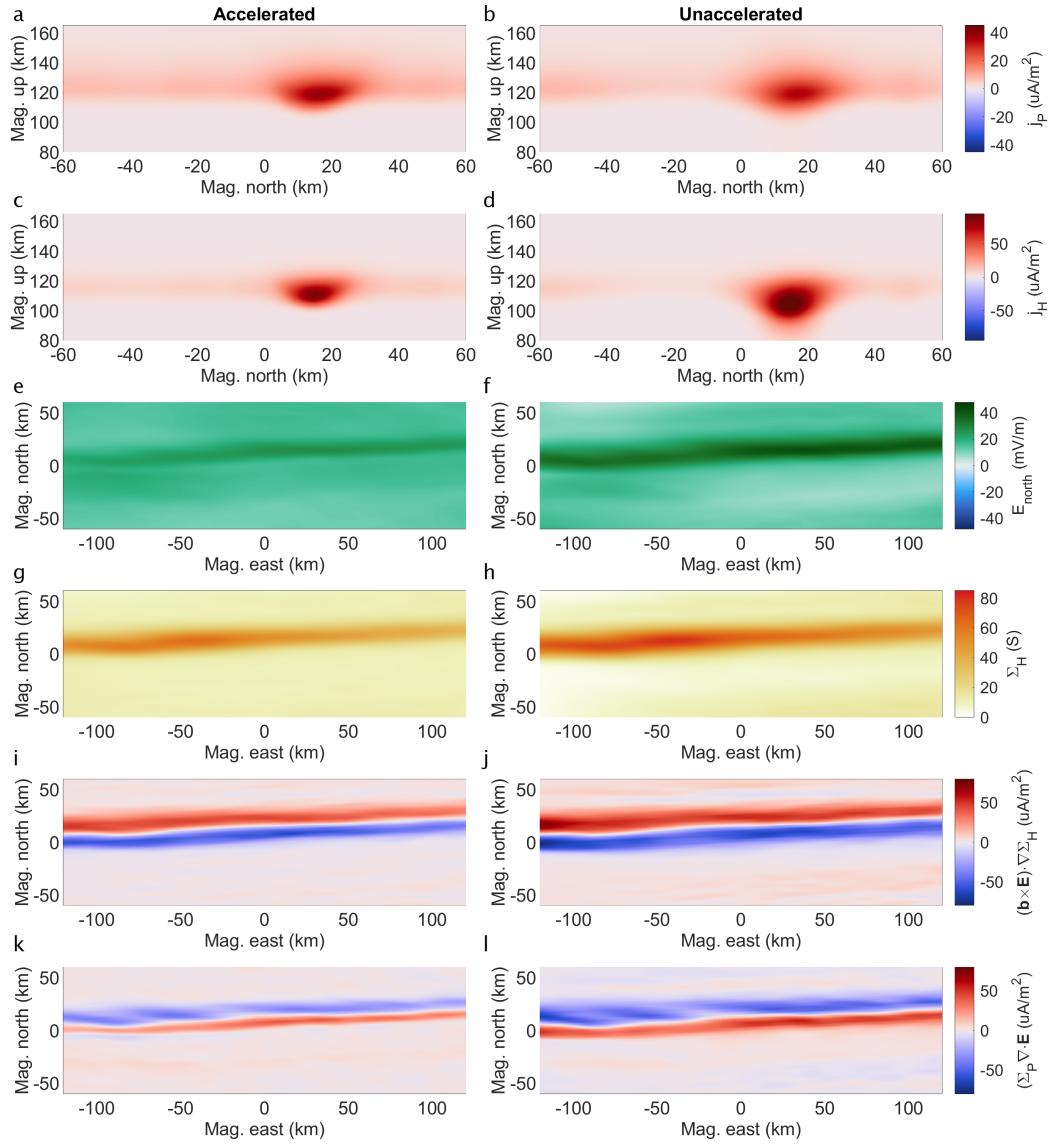
827 As shown in panels a – b, the red flux tube in Simulation VIIIA takes advantage  
 828 of the energy deposition at higher altitudes and the large electric field strength, and finds  
 829 closure through Pedersen alone. In Simulation VIIIB, however, only around 0.7 of the  
 830 1.4 kA is able to connect with the FAC, while the remainder exists through the eastern  
 831 boundary. Interestingly, the existence of the electrojet current in Simulation VIIIB ap-  
 832 pears to push the red flux tube away from the highest densities, subsequently squeez-  
 833 ing it to lower altitudes. The green flux tube, having to travel a shorter horizontal dis-  
 834 tance compared to the other tubes, remains at altitudes where the  $\phi_u(E)$  versus  $\phi_a(E)$   
 835 assumption matters much less, and so it barely changes its morphology and amperage  
 836 across the two simulations.

837 Panels e – f of Figure 13 show a band of enhanced Joule heating just equatorward  
 838 of the precipitating arc in both simulations, yet Simulation VIIIA has this band peak at  
 839 around 26.6 mW/m<sup>2</sup>, while Simulation VIIIB peaks closer to 40.0 mW/m<sup>2</sup>—around a  
 840 50% increase. Between the two simulations, the Pedersen current density remains fairly  
 841 similar; it is the significantly varying Hall current density that creates the different mor-  
 842 phologies (see Figure 14, panels a – d). This points to the electric field strength; in Sim-  
 843 ulation VIIIA there is a band of enhanced across-arc electric field collocated with the Joule  
 844 heating and peaks at around 20 mV/m, while the same is true for Simulation VIIIB ex-  
 845 cept that it peaks around 40 mV/m (see panels e – f).

846 The band of precipitation enhanced Hall conductance for Simulations VIIIA – b peak  
 847 at around 60 and 80 S respectively, as shown in panels g – h. Now, since their spatial  
 848 morphology comes from the same imagery, it implies that this increase in peak value also  
 849 increases  $\nabla_{\perp}\Sigma_H$ , enhancing its associated FAC contributions as per Equation 1. Pan-



**Figure 13.** Comparison VIII (March 4, 7:30 UT): Top and side views of Simulation VIIIA with accelerated Maxwellian electron precipitation (a, b) versus Simulation VIIIB with unaccelerated Maxwellian electron precipitation (c, d) along with height-integrated Joule heating for Simulation VIIIA (e) and VIIIB (f). (g, h) North-up slices of the magnetic eastward current component for Simulations VIIIA – b respectively taken at 50 km west from center with the start curves of their respective orange flux tubes (solid black). For plot details, see Section 2.8. Data sources: Swarm (2025), SuperDARN (2025), and Simulations (2025).



**Figure 14.** Factors that play a role in enhancing Joule heating for Simulation VIIIb over VI-IIa. (a, b) Central up-north cuts of Pedersen current for Simulation VIIIa – b. (c, d) Same for Hall current. (e, f) East-north plots of electric field’s magnetic north component from Simulation VIIIa – b. (g, h) Same for Hall conductance. (i, j) East-north plots of the third term in Equation 1 for Simulation VIIIa – b. (k, l) Same for the first term in Equation 1. Data sources: Simulations (2025).

els i – j show that these contributions, in this case, are in the opposing direction with respect to the total FAC driver—the third term in Equation 1 creates an upward current sheet where the driver map expects a downward sheet, and vice-versa. Given that the second term,  $\mathbf{E} \cdot \nabla_{\perp} \Sigma_P$ , can only help balance this by increasing  $|\mathbf{E}|$ , it would do so equally to that third term,  $(\mathbf{E} \times \mathbf{b}) \cdot \nabla_{\perp} \Sigma_H$ . This leaves the local polarization to help balance the FAC, as is evident in panels k – l of Figure 14. As before, all the input maps have nearly the same spatial morphology for both simulations, hence, to increase  $\nabla_{\perp} \cdot \mathbf{E}$ , the simulation assuming an unaccelerated spectrum has a higher peak electric field, resulting in enhanced Joule heating despite the dissipationless Hall current enhancement.

In all, even though Simulations VIIa – b both have high total energy flux and strong electric field strengths, the large FAC requirements and the higher electron energy distribution peaks mean that these systems do touch on the Hall layer in their current closure. In contrast, Comparisons VII and IX (see Supporting Information) both pertain to auroral arc systems whose FAC requirements, precipitation energy fluxes, and energy distribution peaks are relatively low. These combinations of parameters, even in the case of a weaker electric field in Comparison IX, results in simulations whose assumption of electron energy distributions matter less in both current closure and Joule heating as a result of FAC source term balancing.

#### 4.2.3 Summary: Precipitation Spectra

We have shown that, if a particular auroral arc system requires Hall currents for FAC closure, choosing unaccelerated Maxwellian energy spectra for precipitating electrons is too restrictive when attempting to best represent the resulting impact ionization. Decoupling the energy spread from the most probable energy allows the modeling of auroral arcs whose electrons are accelerated from much colder source regions compared to their acceleration potential. Even for relatively “hot” accelerated precipitation, such as that from Comparisons VIII and IX ( $T_s = 800 - 860$  eV), the alternate, unaccelerated choice of  $U_a = T_s = E_0$  still grossly overestimates the depth reached by the electron density enhancements. Holding FAC demands constant, this matters most when the average electric field strength is sufficiently weak, and/or the precipitation is low-reaching, i.e. any factor that puts emphasis on the Hall conductivity layer. Furthermore, unaccelerated Maxwellian electron distributions can overestimate the Hall currents as a whole, as well as the height-integrated Joule heating.

We have shown that specific assumptions of electron precipitation spectra can change the interpretation of auroral arc systems. Aptly, recent increases in the availability of multi-spectral, over white-light, all-sky imagery allows the community to move away from the assumption of unaccelerated Maxwellian precipitation spectra, and toward energy distributions which decouple the energy spread from the peak energy, allowing for more flexibility in modeling electron precipitation.

### 4.3 Along-Arc FAC Structure

Of our six conjunction events, two have a double-spacecraft arc crossing. This gives us an opportunity to look at two sensitivities: (1) how does along-arc structure in FAC affect current closure, and (2) how much confidence can be had in the replication technique we use. Our double replications have a weighting scale length of 50 km (roughly the distance between the orbits of Swarm A and C) when transitioning from replications of either track. This is described in more detail by van Irsel et al. (2024, Section 2.3). When performing a weighted replication with plasma flow data, this can result in arbitrary along-arc gradients which affect the first term in Equation 1. In our case, though the along-arc gradient in  $j_{\parallel}$  resulting from this weighting scale length is arbitrary, such gradients have less physical implication on the system as a whole. Following are com-

899      comparisons between double versus single replications of our two double-spacecraft conjunction  
 900      events.

#### 901      *4.3.1 Comparison X: Along-Arc FAC Structure*

902      Comparison X looks at our February 10 conjunction event with Simulation Xa using  
 903      a weighted replication of both Swarm A (east) and C (west), and Simulation Xb which  
 904      uses a replication of Swarm A data only. Their orbits cut roughly through the center of  
 905      the simulation volume at about 47 km apart, which allows us to look at current closure  
 906      on either side of the tracks. Looking at Figure 15, panels b and d, reveals an up-down  
 907      FAC sheet pair that exists in Simulation Xa, but not in Xb. From the western boundary,  
 908      centered around 40 km south-from-center, a roughly -2 to 1  $\mu\text{A}/\text{m}^2$  FAC pair follows  
 909      the arc boundary up until just past the Swarm C FAC data track, from which this  
 910      signature is replicated. Furthermore, the southernmost downward FAC sheet narrows  
 911      and intensifies, when transitioning from the Swarm A to C tracks, from about 25 km wide  
 912      and 1 – 1.5  $\mu\text{A}/\text{m}^2$  in magnitude, to around 10 km and 2  $\mu\text{A}/\text{m}^2$ . Simulation Xb has  
 913      this FAC sheet remain unchanged along the arc.

914      With these differences in replicated FAC maps in mind, Simulation Xa (Figure 15a  
 915      – b), though being the same as Simulation Ia, here shows a different set of current flux  
 916      tubes. They are calculated (in reverse) from ellipses placed at the southernmost upward,  
 917      precipitating current sheet located east of, west of, and in between the two FAC data  
 918      tracks. This helps illustrate the affect on current closure resulting from the difference  
 919      between the two data tracks. Figure 15c – d (Simulation Xb) shows flux tubes that are  
 920      calculated from the same three ellipses.

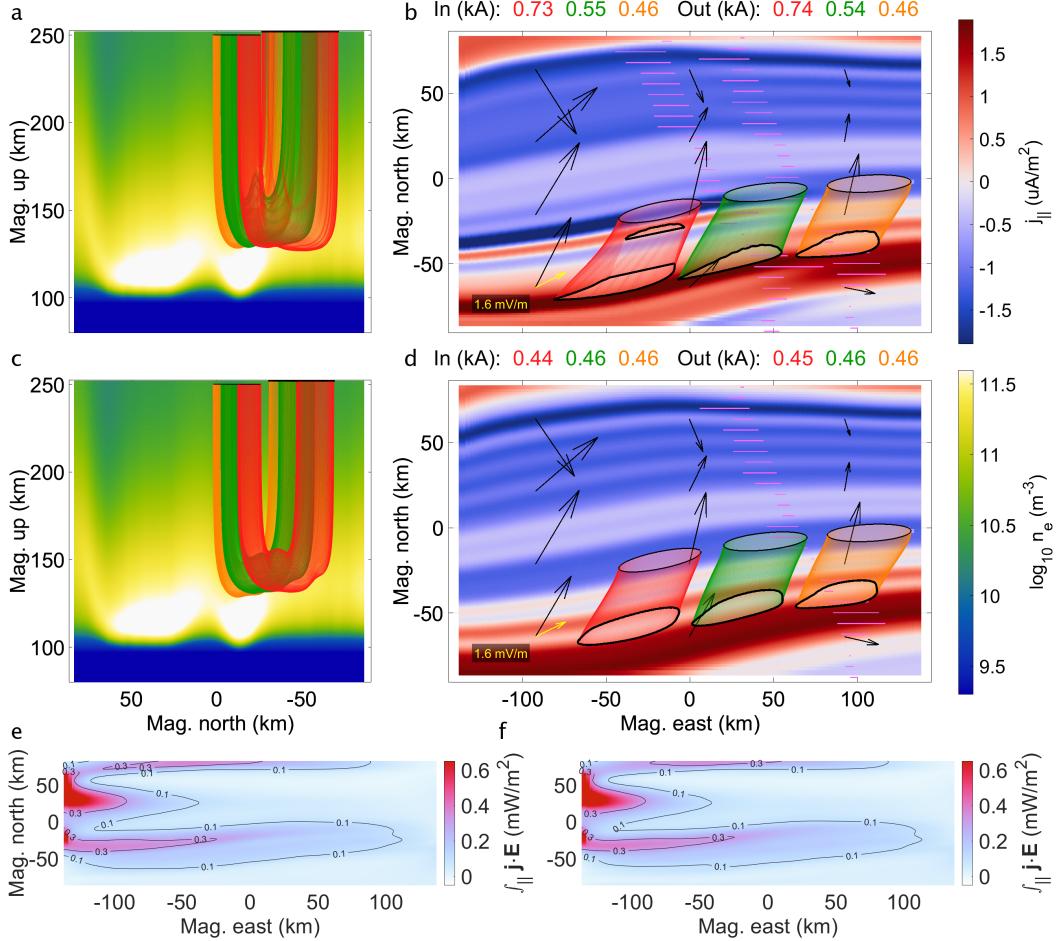
921      The orange flux tube (0.5 kA) lies almost entirely east of the Swarm A track, hence  
 922      it remains mostly unchanged, both in morphology and quantity. The green flux tube,  
 923      however, is encroaching on the aforementioned Swarm C replicated FAC pair and thus  
 924      captures around 0.1 kA more in Simulation Xb. The electric field across the arc has the  
 925      flux tubes directed southwest to northeast, such that the green flux tube has its influx  
 926      end entirely on the western side of Swarm C. Here, the Simulation Xa downward cur-  
 927      rent sheet is stronger, but less than half the width compared to its Simulation Xb coun-  
 928      terpart. The steeper FAC across-arc gradient in Simulation Xa pinches the downward  
 929      green flux tube end into a teardrop shape, while its higher FAC density aids in captur-  
 930      ing that additional 0.1 kA.

931      The red flux tube lies completely on the western side of Swarm C and captures the  
 932      upward part of the FAC sheet pair introduced by Swarm C. At 0.7 kA, this gives it an  
 933      additional 0.3 kA over the red flux tube in Simulation Xb. The adjacent downward cur-  
 934      rent sheet helps close 0.1 kA of this added current, while the remainder is closed with  
 935      a similar teardrop shaped flux tube end.

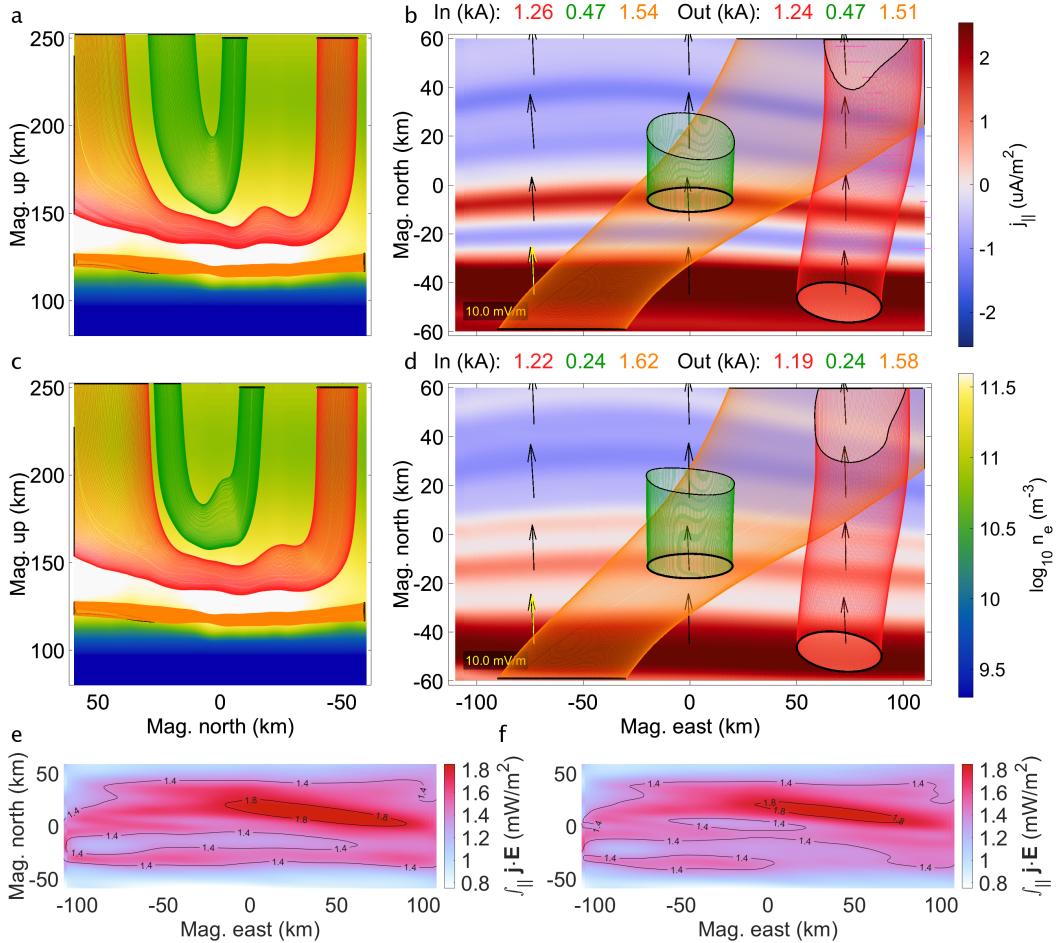
936      Comparison X outlines how a double versus a single FAC data track replication can  
 937      introduce, albeit relatively minor, FAC signatures in the along-arc direction. We have  
 938      to assume such signatures can appear and disappear over distances on the order of 50  
 939      km in every FAC replication. The major FAC structure, however, is conserved, suggest-  
 940      ing the replication methodology holds.

#### 941      *4.3.2 Comparison XI: Along-Arc FAC Structure*

942      Due to limitations of the all-sky imagery of the March 14 conjunction event, the  
 943      simulation region for Comparison XI is almost completely west of both Swarm tracks.  
 944      This prevents us from sourcing current flux tubes on either side of the data tracks, how-  
 945      ever we can still use Comparison XI to provide insight into what confidence can be had  
 946      in the replication technique, and deliberate about the extent to which auroral arc FAC  
 947      varies in the along-arc direction.



**Figure 15.** Comparison X (February 10, 9:51 UT): Top and side views of Simulation Xa with a FAC replication using both Swarm A (east) and C (west) (a, b) versus Simulation Xb with a FAC replication using only Swarm A (c, d) along with height-integrated Joule heating for Simulation Xa (e) and Xb (f). For plot details, see Section 2.8. Data sources: Swarm (2025), SuperDARN (2025), and Simulations (2025).



**Figure 16.** Comparison XI (March 14, 6:49 UT): Top and side views of Simulation XIa with a FAC replication using both Swarm A (east) and C (west) (a, b) versus Simulation XIb with a FAC replication using only Swarm A (c, d) along with height-integrated Joule heating for Simulation XIa (e) and XIb (f). For plot details, see Section 2.8. Data sources: Swarm (2025), SuperDARN (2025), and Simulations (2025).

948 Panels a – b in Figure 16 show results from Simulation XIa, which is driven by a  
949 FAC map replicated from both Swarm A and C data. However, given the locations of  
950 the data tracks, most of this replication uses data from Swarm C, as it is the closest to  
951 the simulation region. With Simulation XIb (panels c – d) using only Swarm A in its  
952 FAC replication, this is essentially a Swarm A versus Swarm C comparison.

953 In contrast to Comparison X, here we see two FAC replications that, though vary-  
954 ing somewhat, are structurally very similar. The southernmost return current sheets for  
955 both Simulations XIa – b are similar in strength, width, and location, as is shown by the  
956 red flux tubes who capture around 1.2 kA in the same place for both simulations. The  
957 return current sheet just above, captured by the green flux tubes, is around half as strong  
958 in Simulation XIb and positioned ~7 km southward, and the orange flux tubes carry a  
959 similar 1.5 – 1.6 kA of Hall current at nearly the same location in both simulations. Over-  
960 all, Comparison XI provides support for the extrapolation of FAC data over a distance  
961 of around 50 km, up to the differences in auroral arc simulations seen here.

962      **4.3.3 Summary: Along-Arc FAC Structure**

963      Two of our six conjunction events benefit from being able to use a second data track  
 964      in their replications and subsequent simulations. Comparisons X and XI show to what  
 965      extent the FAC map can change in just under 50 km, providing important insight into  
 966      the confidence of all of our FAC replications, and consequently the resulting 3-D sim-  
 967      ulations of these auroral arc systems. Overall, contingent on the morphology indicated  
 968      by the imagery and aside from minor FAC signatures, replicating the FAC data using  
 969      arc boundaries defined by auroral imagery is a justifiable method for creating 2-D, con-  
 970      tinuous driver maps for 3-D simulations of auroral arc systems.

971      **5 Discussions & Conclusions**

972      Current closure morphology and Joule heating from resulting closure currents are  
 973      two important aspects of ionospheric physics, particularly surrounding discrete auroral  
 974      arc systems. By carefully incorporating observational data from multi-instrument con-  
 975      junctions into input drivers of auroral arc simulations, we point out three aspects that  
 976      the results are susceptible to: (1) the along-arc structure in FAC and the arc-boundary  
 977      replication technique, (2) the constant background flow, and (3) the specifics of electron  
 978      precipitation. Here, we conclude our findings and discuss possible future studies that can  
 979      advance from this work.

980      Auroral arc systems should be studied in three dimensions to fully understand field-  
 981      aligned current closure and, by extension, Magnetosphere-Ionosphere-Thermosphere cou-  
 982      pling. We show, using several permutations of 3-D, electrostatic, data-driven, auroral  
 983      arc simulations across six conjunction events, that flux tubes of electric current navigate  
 984      around one another in their closure paths; something they cannot do in height-integrated  
 985      (east, north), or cross-arc (north-up) two-dimensional descriptions. These current flux  
 986      tubes tell the story of how FAC, ionospheric electric fields, and Pedersen and Hall con-  
 987      ductivities interplay in a cohesive, self-consistent manner, and they do so with more de-  
 988      tail than 2-D descriptions allow.

989      To produce top-boundary driver maps for our simulations, we demonstrate the use  
 990      of auroral-imagery-guided FAC replication, similar to methods outlined by Clayton et  
 991      al. (2019); van Irsel et al. (2024). We show that this method can produce FAC maps that  
 992      are geophysically consistent with maps of precipitation energetics, and that hold reason-  
 993      ably well for major arc-scale FAC structure. However, more minor FAC structure may  
 994      appear or disappear when moving in the along-arc direction over distances on the order  
 995      of 50 km. Even so, this methodology uses maximal information from imagery derived  
 996      precipitation maps to provide geophysically meaningful extrapolations of FAC surround-  
 997      ing auroral arcs.

998      The 3-D auroral arc simulations covered in this paper have been shown to be very  
 999      sensitive to both the magnitude and the direction of the constant, large-scale, background  
 1000     electric field,  $\bar{\mathbf{E}}$ . Equation 2 shows what the choice of  $\bar{\mathbf{E}}$  implies about the 2-D top-boundary  
 1001     FAC driver map, and thus, how the simulations interpret these maps. We draw the fol-  
 1002     lowing conclusions about how  $\bar{\mathbf{E}}$ , in the absence of neutral winds, affects discrete auro-  
 1003     ral arc systems:

- 1004       • Strong background convection fields can render the use of Hall currents in FAC  
 1005       closure negligible, while weak background convection fields put emphasis on both  
 1006       local polarization fields and FAC closure through the electrojet.
- 1007       • Across-arc electric fields provide shorter closure paths making FAC close through  
 1008       Pedersen current more often.
- 1009       • FAC sheets close with adjacent ones only in the direction of the electric field.

- When part of the electric field is directed along the arc, it lengthens the closure paths and, as current flux tubes cannot intersect, it pushes additional tubes to Hall current altitudes.
- The manner in which the background electric field affects current connectivity, along with the electric field itself, significantly alters Joule heating, and thus the electrostatic load characteristics, of auroral arc systems.

These conclusions support the idea that large-scale convection flow conditions are a dominant driver of the specific morphology of auroral arc systems, with which the arc-scale ionosphere perturbs  $\mathbf{E}$  in accordance with a 3-D conductivity volume.

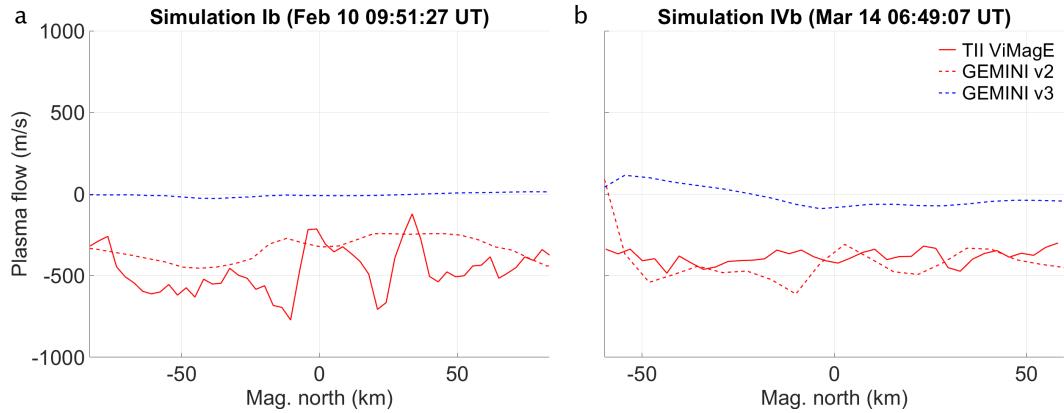
In addition to background convection fields, auroral arc systems are also sensitive to the specifics of precipitating electron energy distributions. We show that the use of unaccelerated Maxwellian spectra can erroneously enhance impact ionization at lower altitudes, resulting in an overestimation of  $E$ -region densities. We compare the use of such spectra against accelerated Maxwellian spectra, which decouple the peak energy from the energy spread. Even for simulations whose source region characteristic energy is estimated to be relatively large,  $T_s \sim 800$  eV, the unaccelerated assumption still greatly overestimates density enhancements at lower altitudes. We show the following:

- The choice of accelerated versus unaccelerated Maxwellian electron precipitation most affects FAC closure in auroral arc systems when the Hall currents play a considerable role in this closure.
- Keeping FAC and total precipitating electron energy drivers constant, the choice of unaccelerated over accelerated precipitation alone can increase the calculated height-integrated Joule heating by 30 – 50 % in some auroral arc systems.
- Unaccelerated Maxwellian auroral precipitation assumptions can greatly enhance electrojet currents compared to accelerated precipitation assumptions.

This work looks at how to determine geophysical, self-consistent solutions to current continuity in auroral arc systems, and what these systems are sensitive to, thus uncovering how important various parameters can be. How then do we know which solution is correct? The existence of TII ion drift data (or other, independent flow data) from the Swarm spacecraft invites comparisons to the calculated GEMINI output flow maps covered in this paper. Figure 17 shows two such comparisons of the magnetic eastward TII flow (assuming no along-track component) across the model space for two of the simulations (Ib and IVb). While we have generated 17 simulations of the six events in Table 1, only the February 10 and March 14 conjunctions include Swarm A TII data; only the former has the crossing directly within the model space. The simulations using PFISR for the background flow for these two cases match better than the corresponding SuperDARN runs, which have smaller background flows.

It is notable among the examples chosen for this study (the six events in Table 1) that there is not a particularly strong correlation between the magnetic and electric field signatures in the raw Swarm data—for most of these events, the  $\nabla \cdot \mathbf{E}$  term in Equation 1 is apparently not the major player for the events in Table 1. Thus this comparison with TII becomes mostly a question of matching the background flow to the TII value, perhaps why the nearer source (PFISR) provides the closest match. For the first example shown, there are some  $\nabla \cdot \mathbf{E}$  signatures in both TII and the GEMINI results, but the GEMINI result is somewhat smoother and slightly offset. Both of these differences may well be artifacts of the image inversion process.

Finally we can consider whether the competition between the  $\nabla \cdot \mathbf{E}$  and  $\nabla \Sigma_{P,H}$  terms in Equation 1 provides a truly unique solution to the problem posed. There is a strong dependence on the chosen  $\bar{\mathbf{E}}$ : choosing the background electric field differently finds different situations. There may be choices, beyond what PFISR and SuperDARN



**Figure 17.** GEMINI versus TII flow comparisons for Simulations Ib (a) and IVb (b). The GEMINI magnetic eastward, “v2”, and northward, “v3”, plasma flows are interpolated through the simulation volume at the Swarm A tracks. TII magnetic eastward ion drift data, “ViMagE”, are converted to geomagnetic coordinates assuming no along-track component. Both simulations use PFISR derived background flow, accelerated Maxwellian precipitation, and double-spacecraft replications. Data sources: Swarm (2025), PFISR (2025), and Simulations (2025).

provide, which more closely track the TII cross-track flow values. We do see that choosing different background flows, e.g. the no-background flow run versus the large-background flow simulations in Comparison III, generates in the GEMINI result a visible  $\nabla \cdot \mathbf{E}$  signature which is masked when the imposed background electric field is strengthened. Future work exploring these comparisons with TII should include (a) events like the one covered by Clayton et al. (2021), with its strong  $\nabla \cdot \mathbf{E}$  signature; and (b) further study of error sources stemming from matching the spacecraft data to inverted imagery, particularly for oblique camera angles which tend to blur and misplace discrete arc structures. We also note the scale of smoothing applied for these runs, as described in Subsections 2.1 and 2.2: this level of smoothing may yet be hiding relevant physics, particularly at sharp arc edge boundaries.

The tools developed herein provide a means for data-driven event case study simulations to be routinely done, assuming sufficient data coverage. Upcoming iterations may consider different, incomplete combinations of input and/or adaptation of our methods into a formal physics-based assimilation scheme. A subject for further studies is the relevant physical gradient limit caused by recombination and collisions in the current closure altitude region: how sharp of gradients can be sustained and be relevant?

In the collective effort to try and understand the nature of aurorae, the instruments that provide our observational data are an ever-existing limitation. It would be optimal to deploy 1000s of spacecraft, radars, and imagers across the northern and southern auroral ovals (Nykyri et al., 2025), but this is impractical. Hence, measurements must be targeted and focused on parameters that are most influential to the physics at hand. This work provides three such aspects to contribute to this focus and aids in making decisions as to what is important and when.

## Appendix A Derivation of Accelerated Bi-Maxwellian Differential Number Flux

In order to implement the impact ionization calculations by Fang et al. (2010), we need the differential (as a function of energy) hemispherical number flux, i.e. electrons/eV/s/cm<sup>2</sup>,

of precipitating energetic auroral electrons at the topside of the ionosphere for every latitude-longitude pair. To derive this flux for an accelerated population we start with a bi-Maxwellian source at the plasmasheet as is done by Fridman and Lemaire (1980):

$$g_s(v_{\parallel,s}, v_{\perp,s}, \varphi) d^3v = n_{e,s} \left( \frac{m_e}{2\pi} \right)^{3/2} \frac{1}{E_{\parallel,s}^{1/2} E_{\perp,s}} \exp \left[ -\frac{m_e v_{\parallel,s}^2}{2E_{\parallel,s}} - \frac{m_e v_{\perp,s}^2}{2E_{\perp,s}} \right] v_{\perp,s} dv_{\parallel} dv_{\perp} d\varphi, \quad (\text{A1})$$

where  $n_{e,s}$  is the source region electron density,  $m_e$  is the mass of an electron,  $E_{\parallel,s}$  and  $E_{\perp,s}$  are the parallel and perpendicular characteristic energies,  $v_{\parallel,s}$  and  $v_{\perp,s}$  are the source region parallel and perpendicular speeds, and  $\varphi$  is the azimuthal coordinate. As electrons precipitate down towards the ionosphere they undergo no collisions—their velocities change in two ways only (Knight, 1973; Fridman & Lemaire, 1980; Kaepller, 2013):

1. The conservation of the first adiabatic invariant, i.e. the mirror force, increases their perpendicular velocity:

$$v_{\perp,s} = \frac{1}{\sqrt{\beta}} v_{\perp,i}, \quad (\text{A2})$$

where  $\beta = B_i/B_s > 1$ , and  $B_i$  and  $B_s$  are the ionospheric and source region magnetic field strengths.

2. The conservation of energy increases the square magnitude speed as they fall through the parallel potential difference,  $U_a$ :

$$v_{\parallel,i}^2 + v_{\perp,i}^2 = v_{\parallel,s}^2 + v_{\perp,s}^2 + \frac{2U_a}{m_e}. \quad (\text{A3})$$

This provides the parallel source region speed as a function of the ionospheric coordinates:

$$v_{\parallel,s} = \pm \sqrt{v_{\parallel,i}^2 + v_{\perp,i}^2 \frac{\beta-1}{\beta} - \frac{2U_a}{m_e}}. \quad (\text{A4})$$

From here, we use Liouville's theorem which tells us that, along a well-defined path through phase space, e.g.  $(\mathbf{x}, \mathbf{v})_s \rightarrow (\mathbf{x}, \mathbf{v})_i$ , the phase space density is held constant such that

$$g_i(\mathbf{x}_i, \mathbf{v}_i) = g_s(\mathbf{x}_s, \mathbf{v}_s). \quad (\text{A5})$$

A good assumption is to say that we may separate spatial and velocity coordinates,  $g(\mathbf{x}, \mathbf{v}) = n(\mathbf{x})f(\mathbf{v})$ , and that locally the densities are constants, i.e.  $n_i(\mathbf{x}) = n_{e,i}$ ,  $n_s(\mathbf{x}) = n_{e,s}$ . This tells us

$$g_i(\mathbf{v}_i) = g_s(\mathbf{v}_s) = g_s(\mathbf{v}_s(\mathbf{v}_i)), \quad (\text{A6})$$

such that

$$g_i(v_{\parallel,i}, v_{\perp,i}) d^2v = n_{e,s} \frac{m_e^{3/2}/\sqrt{2\pi}}{E_{\parallel,s}^{1/2} E_{\perp,s}} \exp \left[ -\frac{m_e \left( v_{\parallel,i}^2 + v_{\perp,i}^2 \frac{\beta-1}{\beta} - \frac{2U_a}{m_e} \right)}{2E_{\parallel,s}} - \frac{m_e v_{\perp,i}^2 / \beta}{2E_{\perp,s}} \right] \frac{v_{\perp,i}}{\sqrt{\beta}} dv_{\parallel} dv_{\perp} \quad (\text{A7})$$

where we've integrated over  $\varphi$ . The ionospheric density is thus

$$n_{e,i} = n_{e,s} \frac{E_{\parallel,s} \sqrt{\beta}}{E_{\parallel,s} + E_{\perp,s}(\beta-1)} \exp \left[ \frac{U_a}{E_{\parallel,s}} \right]. \quad (\text{A8})$$

Note that  $U_a \rightarrow 0$  and  $E_{\parallel,s} \rightarrow E_{\perp,s}$  gives a familiar density relation:  $n_{e,i} = n_{e,s}/\sqrt{\beta}$ . Now that we have the velocity distribution function at the ionosphere, we find the differential number flux using  $J_{\parallel,i}(\mathbf{v}_i) d^3v = v_{\parallel,i} g_i(\mathbf{v}_i) d^3v$  and then we perform the following change of coordinates:

$$v_{\parallel,i} = v \cos \theta = \sqrt{2E/m_e} \cos \theta \text{ and } v_{\perp,i} = v \sin \theta = \sqrt{2E/m_e} \sin \theta, \quad (\text{A9})$$

1115 with  $\theta$  being the pitch angle, and with Jacobian determinant  $1/m_e$ . The energy,  $E$ , has  
1116 the condition

$$E = \frac{m_e}{2} (v_{\parallel,i}^2 + v_{\perp,i}^2) \geq U_a, \quad (\text{A10})$$

1117 as per Equation A3. This gives

$$J_{\parallel,i}(E, \theta) dE d\theta = \frac{n_{e,s}}{\sqrt{m_e}} \frac{1}{E_{\parallel,s}^{1/2} E_{\perp,s}} \frac{\sin 2\theta}{\sqrt{2\pi\beta}} E \exp \left[ -\frac{E - U_a}{E_{\parallel,s}} - \left( \frac{E}{E_{\perp,s}} - \frac{E}{E_{\parallel,s}} \right) \frac{\sin^2 \theta}{\beta} \right] dE d\theta. \quad (\text{A11})$$

1118 With unit-less parameters  $\varepsilon \equiv E/E_{\parallel,s}$ ,  $U_a \equiv u_a/E_{\parallel,s}$ , and  $\delta \equiv E_{\perp,s}/E_{\parallel,s}$ , we get

$$\frac{1}{n_{e,s}} \sqrt{\frac{m_e}{E_{\parallel,s}}} J_{\parallel,i}(E, \theta) dE d\theta = \frac{\sin 2\theta}{\sqrt{2\pi\beta}} \frac{\varepsilon}{\delta} \exp \left[ -(\varepsilon - U_a) - \left( \frac{\varepsilon}{\delta} - \varepsilon \right) \frac{\sin^2 \theta}{\beta} \right] d\varepsilon d\theta. \quad (\text{A12})$$

1119 We now integrate over  $v_{\parallel,i} > 0$ , i.e.  $0 \leq \theta \leq \pi/2$ , and find the hemispherical differen-  
1120 tial number flux.

$$J_{\parallel,i}(\varepsilon) d\varepsilon = n_{e,s} \sqrt{\frac{E_{\parallel,s}}{m_e}} \frac{1}{\delta \sqrt{2\pi\beta}} G \left( \frac{\delta - 1}{\delta\beta} \varepsilon \right) \varepsilon e^{-\varepsilon+u_a} d\varepsilon, \text{ where } G(x) \equiv \frac{e^x - 1}{x}. \quad (\text{A13})$$

1121 For similar parallel and perpendicular source temperatures, we have  $\delta \sim 1$ , and we have  
1122  $\beta \sim 10^3$  for a plasmashell source region (Fridman & Lemaire, 1980), where  $G(x \ll 1) \rightarrow 1 + x/2 + \mathcal{O}(x^2)$  such that

$$J_{\parallel,i}(\varepsilon) d\varepsilon \approx n_{e,s} \sqrt{\frac{E_{\parallel,s}}{m_e}} \frac{1}{\delta \sqrt{2\pi\beta}} \left( 1 + \frac{\delta - 1}{2\delta\beta} \varepsilon \right) \varepsilon e^{-\varepsilon+u_a} d\varepsilon \quad (\text{A14})$$

1124 If we re-cast this in terms of normalized total precipitating energy flux,  $q_p \equiv Q_p/E_{\parallel,s}$ ,  
1125 where

$$q_p = \int_{u_a}^{\infty} \varepsilon J_{\parallel,i}(\varepsilon) d\varepsilon, \quad (\text{A15})$$

1126 we get

$$J_{\parallel,i}(\varepsilon) d\varepsilon = q_p \frac{1 + \chi\varepsilon}{2 + 6\chi + u_a(2 + u_a + (6 + u_a(3 + u_a))\chi)} \varepsilon e^{-\varepsilon+u_a} d\varepsilon, \text{ where } \chi = \frac{\delta - 1}{2\delta\beta}. \quad (\text{A16})$$

1127 We note that in our regime of  $\beta \sim 10^3$  we may ignore the temperature difference at the  
1128 source, so if we take the limit of  $\delta \rightarrow 1$  we get a familiar result

$$J_{\parallel,i}(E) dE = \frac{Q_p}{T_s^2 + (T_s + U_a)^2} \frac{E}{T_s} \exp \left[ -\frac{E - U_a}{T_s} \right] dE, \quad E \geq U_a \quad (\text{A17})$$

1129 where, for clarity, we have defined  $T_s \equiv E_{\parallel,s}$ . These results have been congregated from  
1130 knowledge and derivations obtained in publications by Medicus (1961); Evans (1974);  
1131 Fridman and Lemaire (1980); Strickland et al. (1989); Kaepller (2013).

## 1132 Open Research Section

1133 All 3-D simulation data, imagery inversions, and supporting metadata are avail-  
1134 able at <https://rcweb.dartmouth.edu/lynchhk>. The data for the Poker Flat DASC are  
1135 available at <http://optics.gi.alaska.edu/optics/archive>, for AMISR at <https://data.amisr.com/database>, for SuperDARN at <https://superdarn.ca/data-download>,  
1136 and for the Swarm at <https://swarm-diss.eo.esa.int>. The GEMINI source code and  
1137 documentation is available at <https://github.com/gemini3d> and the replication/vi-  
1138 sualization tools at [https://github.com/317Lab/aurora\\_gemini](https://github.com/317Lab/aurora_gemini).

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